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ELECTRE TRI-C with Hesitant Fuzzy Sets and Interval Type 2 Trapezoidal Fuzzy Numbers Using Stochastic Parameters: Application to a Brazilian Electrical Power Company Problem

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Abstract ELECTRE TRI-C is a method for sorting problems with imprecise evaluations and stable criteria weights, typically for a single decision-maker. While some extensions have addressed uncertain criteria weights and outranking functions using hesitant fuzzy sets (HFS) and interval type 2 trapezoidal fuzzy numbers (IT2TrfN), there is a gap in handling situations where multiple decision-makers provide uncertain information. This paper presents an extension of the ELECTRE TRI-C method incorporating a stochastic framework to model HFS and IT2TrfN, thereby accommodating subjective judgments from multiple decision-makers. The extended method was validated by sorting 49 projects based on their criticality in a Brazilian electrical power company, involving three decision-makers. The application shows strong correlations in project rankings among decision-makers, but with some exceptions. However, significant variations in acceptability ratings for sorting among decision-makers lead to notable error dispersion, highlighting differences between ranking and sorting outcomes. The key contributions of our approach are as follows: (1) Integration of subjective judgments from multiple decision-makers using IT2TrFN and Monte Carlo Simulation for constructing outranking functions; (2) Aggregation of preferences from multiple decision-makers using HFS; (3) Stochastic processing of both quantitative and qualitative criteria; (4) Integration of linear equations to represent weight constraints; and (5) Introduction of a novel visualization method for comprehensive analysis of stochastic results, enhancing robustness analysis. The proposal's advantages over alternative methods are also highlighted.

Keywords Group decision · ELECTRE TRI-C · Hesitant fuzzy sets · Interval type 2 trapezoidal fuzzy number · Robustness analysis

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1 Introduction

The multi-criteria decision-making/analysis (MCDM/A) field faces challenges in sorting decision problems, particularly when handling uncertainty from multiple decision-makers' inputs [1]. Despite available methods like ELECTRE TRI, Flowsort, TODIM-FSE, and others, effectively managing uncertainty remains a persistent challenge [2–8]. Decision-makers, information sources, and evaluation processes often convey varying levels of uncertainty due to factors, such as subjective judgments, conflicting preferences, and incomplete data.

ELECTRE TRI, an MCDM/A framework renowned for their efficacy in sorting tasks with imprecise evaluations by employing predefined membership functions known as outranking functions to represent categories, stands out as a prominent family of methods [2, 9]. Within this framework, ELECTRE TRI-C offers a single decision-maker (DM) sorting approach, utilizing deterministic outranking functions to model central reference alternatives representing categories [9]. This method ranks among the top five cited methods in the literature on multi-criteria sorting [1].

Extensions of ELECTRE TRI methods incorporate fuzzy sets to handle uncertain evaluations and weights, including fuzzy sets, interval-valued fuzzy sets, interval-valued intuitionistic fuzzy sets, and intuitionistic fuzzy numbers [10–14]. Notably, these extensions have addressed hesitant fuzzy sets (HFS) and interval type 2 trapezoidal fuzzy numbers (IT2TrFN) [15–18]. HFS is particularly suitable for modeling discrete uncertainty, often arising from judgments related to a limited number of decision-makers [19]. Interval Type 2 Triangular Fuzzy Numbers (IT2TrFN) are well suited for modeling uncertain or ill-defined membership functions, especially when built from information provided by a single decision-maker [20, 21]. However, existing extensions of these methods primarily focus on either HFS or IT2TrFN [16, 17], leaving a significant gap in simultaneously addressing both alongside accommodating quantitative and qualitative criteria. Meanwhile, stochastic methods such as SMAA-TRI tackle uncertainty, geared toward quantitative criteria, but it is not well suited for situations involving group decision-making and qualitative criteria [22, 23]. Hence, there arises a necessity for hybrid MCDM/A approaches that can effectively handle uncertainty in both data and preferences, accommodating qualitative and quantitative criteria, as well as multiple decision-makers [24–26].

In this study, we introduce a novel extension of the ELECTRE TRI-C method aimed at handling uncertainties and subjective judgments from multiple decision-makers (DMs). We define a fictitious "Supra Decision-Maker" (SDM) to aggregate information from the DMs. Essentially, IT2TrFN are employed to model uncertainty in outranking functions, which are utilized for constructing central reference alternatives' membership functions. HFS are employed to aggregate the outranking functions of individual DMs into the outranking function of the SDM. The parameters of the outranking function and criteria weights are randomized, encapsulating the entire method in a constructive process that iterates for a predefined number of rounds. We introduce a novel visualization method that facilitates comprehensive analysis of stochastic results, thereby enhancing robustness analysis. To the best of our knowledge, there is currently no extension of ELECTRE TRI-C that simultaneously incorporates IT2TrFN, HFS, and stochastic wrapping features. Our approach represents a novel integration of these techniques, offering a comprehensive solution for multi-criteria decision-making under uncertainty.

We apply this method to address a sorting problem within a Brazilian electrical power company. Energy-related decision-making involves considering technical, economic, social/political, and environmental criteria, leading to the application of MCDM/A methods in various contexts [24, 25, 27–30]. Hybrid fuzzy MCDA approaches are common due to the inherent uncertainties in data and preferences [24, 25], often necessitating sensitivity analysis for adjusting criteria weights [31]. Despite their lesser prevalence, adaptations incorporating hesitant fuzzy sets and Interval Type 2 Fuzzy Numbers (IT2TrFNs) for uncertainty modeling have been proposed [32–37]. Sorting problems also play a role in hybrid approaches for classification and selection [38], while potential opportunities for using outranking-based methods have also been identified [29].

The paper is structured as follows: Sect. 2 discusses related literature, Sect. 3 presents our proposed approach, Sect. 4 illustrates its application in the Brazilian company, Sect. 5 evaluates its advantages and limitations, and Sect. 6 concludes with future research suggestions.

2 Related Work

Practitioners and researchers have devoted significant attention to the ELECTRE TRI family of sorting methods, categorizing alternatives using predefined upper and lower reference alternatives (ELECTRE TRI-B, ELECTRE TRI-nB) [39], or central reference alternatives (ELECTRE TRI-C, ELECTRE TRI-nC) [40]. These methods involve comparing alternatives to predefined boundaries or central references using an outranking function, which is essentially a fuzzy set defined by precise preference thresholds, and precise criteria weights. Hence, ELECTRE does not inherently incorporate uncertainty associated with these parameters and preferences [2].

The use of hesitant fuzzy sets (HFS) in decision-making offers a versatile approach for handling uncertainty and ambiguity. Unlike traditional fuzzy sets, HFS allows decision-makers to express multiple membership degrees for each element, capturing varying levels of uncertainty or conflicting opinions [19]. In extensions of ELECTRE, hesitant fuzzy sets (HFS) facilitate the aggregation of decision-makers' preferences, offering a comprehensive representation of the decision-making process [41]. Interval type 2 fuzzy sets (IT2FS), instead, expand upon traditional fuzzy sets by introducing uncertainty regarding the membership functions themselves [42]. In IT2FS, the discourse universe characteristic parameters of membership functions can be represented as intervals rather than precise

Table 1 Fuzzy set/number extensions of ELECTRE methods

Reference	Key points
[43]	Extension of ELECTRE TRI-B for individual results and intuitionistic fuzzy credibility indices for group consensus
[44]	Usage of trapezoidal fuzzy numbers (TrFNs) in ELECTRE TRI-C to handle outranking functions
[17]	TrFN adaptation of outranking functions in ELECTRE TRI-C, followed by HFS processing for several DMs
[16, 41, 45–47]	Extension of ELECTRE methods to consider HFS
[48, 49]	Modification of ELECTRE-I method based on hesitant fuzzy sets provided by DMs
[50]	m-polar HFS adaptation of ELECTRE-I for selecting best option with hesitancy in evaluations
[51, 52]	Use of HFS for modeling uncertain judgments
	Suggestion of using Interval Type 2 Fuzzy Sets (IT2FS) for better modeling of outranking functions
[18, 20, 21, 53, 54]	Implementation of Interval Type 2 Trapezoidal Fuzzy Numbers (IT2TrFN) in extensions of ELECTRE methods

values, allowing decision-makers to account for uncertainty in both membership degrees and the shape/location of the membership functions. By complementing hesitant fuzzy sets (HFS) and interval type 2 fuzzy sets (IT2FS), uncertainty in criteria weights, alternative evaluations, and threshold parameters can be effectively modeled, thereby enhancing decision-making by capturing and managing inherent vagueness and ambiguity.

In Table 1, we summarize and structure the reviewed literature regarding HFS and IT2FS extensions of ELECTRE methods, namely the ELECTRE TRI-C method. Reference [54] extends the ELECTRE III ranking method by modeling the criteria weights, the assessment of alternatives with respect to criteria, and the thresholds using Gaussian Interval Type 2 fuzzy sets (GIT2FSs). Reference [43] proposes an approach where each decision-maker (DM) utilizes an extension of ELECTRE TRI-B, with individual decisions aggregated using group intuitionistic fuzzy credibility indices to compute the final group consensus. Reference [44] proposes the use of trapezoidal fuzzy numbers (TrFNs) in ELECTRE TRI-C to manage uncertain parameters and determine the membership degree of alternatives to categories, reducing the proliferation of fuzzy numbers required for evaluations. Reference [16] advocates for using HFS to model DMs' judgments regarding alternatives and criteria weights. Reference [17] proposes a TrFN adaptation of outranking functions in ELECTRE TRI-C, followed by HFS processing, particularly in contexts involving several DMs. These extensions are deterministic, implying sensitivity and/or robustness analysis as a post-processing activity.

ELECTRE methods have also integrated HFS [45–47]. References [48, 49] present a modified ELECTRE-I method for ranking and selecting the best alternative in renewable energy and manufacturing problems, relying on multiple decision-makers providing hesitant fuzzy sets to evaluate alternatives. Reference [50] develops an m-polar HFS adaptation of ELECTRE-I for selecting the best option when evaluations exhibit hesitancy. HFS can also model uncertain judgments, while interval type 2 fuzzy sets (IT2FS) effectively capture subjective threshold parameters [51, 52]. Extensions implementing interval type 2 trapezoidal fuzzy numbers (IT2TrFN) demonstrate their utility in modeling uncertainty in outranking functions [18, 20, 21, 53].

Existing extensions of the ELECTRE TRI-C method typically focus on either HFS or iIT2TrFN, lacking integration of both alongside quantitative and qualitative criteria. A gap exists for a methodology that combines HFS and IT2TrFN within ELECTRE TRI-C, while also incorporating stochastic parameter modeling to enhance robustness analysis. In addition, although stochastic approaches like SMAA-TRI address uncertainty [22, 23], they primarily handle quantitative criteria. Monte Carlo simulation offers an opportunity to address this gap by optimizing stochastic parameters [55] or analyzing robustness of solutions [56], while HFS and IT2TrFN into ELECTRE TRI-C improve decision analysis in group contexts.

Hence, there is a notable gap in the literature regarding an extension of ELECTRE TRI-C that integrates IT2TrFN, HFS, and stochastic wrapping features simultaneously.

3 Extending ELECTRE TRI-C

3.1 Overall Approach

This novel approach extends ELECTRE TRI-C by introducing a stochastic framework to model uncertain HFS and IT2TrfN in contexts with multiple decision-makers. Illustrated in Fig. 1, it also involves a fictitious "Supra Decision-Maker" (SDM) for whom information from the DMs is aggregated.

Firstly, the process begins by defining the sorting problem, including identifying the group of DMs, alternatives, criteria, categories, and initial values of imprecision parameters (p_j, q_j), and criteria weights (w_j) for ELECTRE TRI-C. Secondly, membership functions (μ_j) representing the central reference actions within categories are constructed for each DM and criterion, utilizing imprecision parameters. Thirdly,

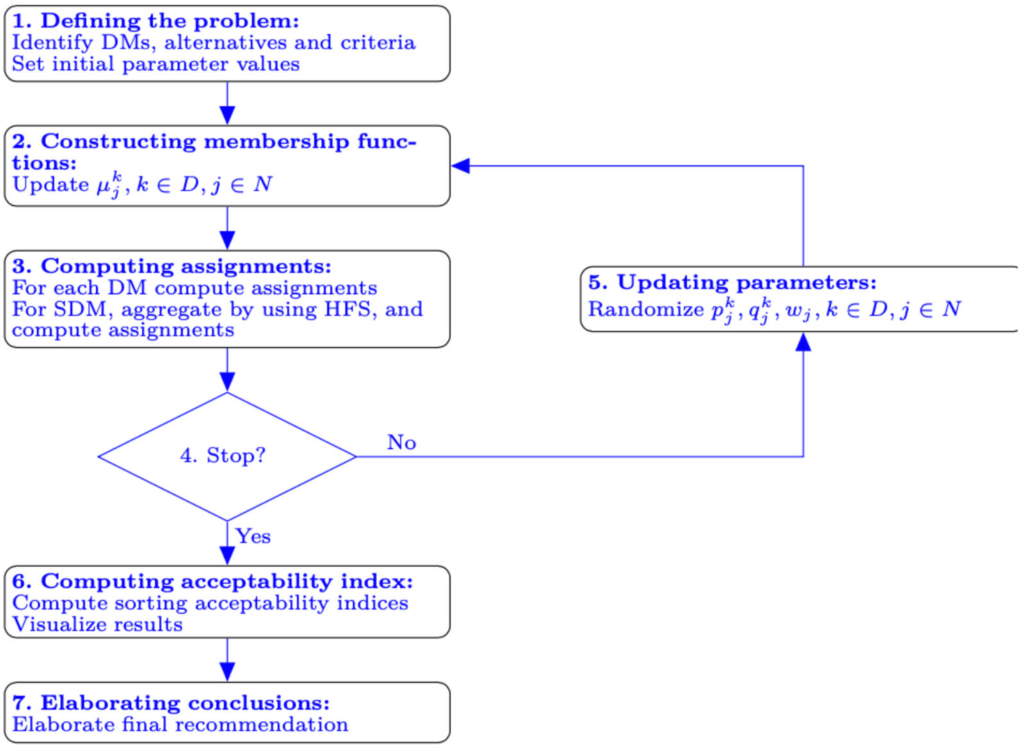

Fig. 1 Group decision sorting approach

the ELECTRE TRI-C method is applied for each DM and the SDM. Fourth, the process continues until a predefined number of iterations is reached. Fifth, if additional rounds are needed, imprecision parameters and weights are randomized, and ITsTrFNs are updated accordingly. Sixth, if the stochastic rounds are completed, sorting acceptability indices are computed, and results are visualized. Finally, conclusions are drawn based on the results from both DMs and the SDM, leading to a final recommendation.

In the following sections, ELECTRE TRI-C and the main techniques and process for the proposed extension are briefly described.

3.2 ELECTRE TRI-C

ELECTRE TRI-C is a multi-criteria sorting method that compares actions to central reference actions representing predefined categories [9]. Each category is associated with a single central reference action. We define sets of cardinal numbers $M = \{1, \ldots, m\}$, $N = \{1, \ldots, n\}$, $D = \{1, \ldots, K\}$, and $Z = \{1, 2, \ldots, H\}$. Let $A = \{a_i \mid i \in M\}$ represent the set of actions to be categorized, $B = \{b_h \mid h \in Z\}$ the set of central reference actions, with each b_h representing a category C_h, and $F = \{g_j \mid j \in N\}$ the family of criteria used to evaluate each action. In ELECTRE methods, the outranking relation, indicating "b is at least as good as a," can be assessed for each criterion using the partial direct concordance index $c_j(b, a)$:

$$c_j(b,a) = \begin{cases} 0 & \text{if } g_j(a) - g_j(b) > p_j, \\ 1 & \text{if } g_j(a) - g_j(b) \leq q_j, \\ \dfrac{g_j(b) - g_j(a) + p_j}{p_j - q_j} & \text{otherwise} , \end{cases} . \tag{1}$$

where p_j, q_j are two imprecision parameters called the direct preference threshold and the direct indifference threshold, respectively [2]. Whenever inverse thresholds p'_j, q'_j can be defined [57], a *partial inverse concordance index*, which considers the inverse thresholds, may be defined as

$$cinv_j(a,b) = \begin{cases} 0 & \text{if } g_j(b) - g_j(a) > p'_j, \\ 1 & \text{if } g_j(b) - g_j(a) \leq q'_j, \\ \dfrac{g_j(a) - g_j(b) + p'_j}{p'_j - q'_j} & \text{otherwise} . \end{cases} . \tag{2}$$

The partial direct and inverse concordance indices allow to compute the global concordance indices:

$$\sigma_D(b_h, a) = \sum_{j=1}^{m} w_j c_j(b_h, a), , \tag{3}$$

$$\sigma_I(a,b_h) = \sum_{j=1}^{m} w_j cinv_j(a,b_h). . \quad (4)$$

Thus, two assignment rules can be defined [44]:

1. *Descending Rule*. Let $\lambda \in [0.5, 1]$ be a minimum credibility level. Decrease h from $H+1$ until the first t such that $\sigma_I(a,b_t) \geq \lambda$:
   (a) If $t = H+1$, assign a to C_H.
   (b) If $t = 0$, assign a to C_1.
   (c) For $0 < t < H+1$, if $\min\{\sigma_I(a,b_t), \sigma_D(b_t,a)\} > \min\{\sigma_I(a,b_{t+1}), \sigma_D(b_{t+1},a)\}$ then assign a to C_t; otherwise, assign a to C_{t+1}.
2. *Ascending Rule*. Let $\lambda \in [0.5, 1]$ be a minimum credibility level. Increase h from 0 until the first t such that $\sigma_D(b_t,a) \geq \lambda$:
   (a) If $t = 1$, assign a to C_1.
   (b) If $t = H+1$, assign a to C_H.
   (c) For $0 < t < H+1$, if $\min\{\sigma_I(a,b_t), \sigma_D(b_t,a)\} > \min\{\sigma_I(a,b_{t-1}), \sigma_D(b_{t-1},a)\}$, then assign a to C_t; otherwise, assign a to C_{t-1}.

3.3 Hesitant Fuzzy Sets

HFS is particularly suitable for modeling discrete uncertainty, often arising from judgments related to a limited number of decision-makers [19]. HFS facilitate the aggregation of decision-makers' preferences, offering a comprehensive representation of the decision-making process [41]. Let X be a reference set. A hesitant fuzzy set (HFS) H on X is defined in terms of a function $h_H(x)$, expressed as $H = \{\langle x, h_H(x)\rangle \mid x \in X\}$, that returns a subset [0, 1] when it is applied to X.

$h_H(x) = \{\gamma \mid \gamma \in [0,1]\}$ is a set of some different values in [0, 1] where γ represents the possible membership degree of the element $x \in X$ to H. $h_H(x)$ is called a hesitant fuzzy element (HFE), a basic unit of HFS.

As an example, let us consider $X = \{x_1, x_2\}$ to be a set such that $h_H(x_1) = \{0.3, 0.2\}$ and $h_H(x_2) = \{0.1, 0.3, 0.4\}$, then an HFS can be written as follows: $H = \{\langle x_1, \{0.3, 0.2\}\rangle, \langle x_2, \{0.1, 0.3, 0.4\}\rangle\}$.

For an HFE h, a function $s(h) = \frac{1}{l_h}\sum_{\gamma \in h}\gamma$ is called the score function of h, where l_h is the number of elements in h. In addition, for h_1 and h_2, if $s(h_1) > s(h_2) \Rightarrow h_1 \succ h_2$ (superior) and $s(h_1) = s(h_2) \Rightarrow h_1 \sim h_2$ (indifferent).

3.4 Interval Type 2 Trapezoidal Fuzzy Numbers

Interval type 2 fuzzy sets (IT2FS) expand upon traditional fuzzy sets by introducing uncertainty regarding the membership functions themselves [42]. In IT2FS, membership functions are represented as intervals rather than precise values, allowing decision-makers to account for uncertainty in both membership degrees and the shape/location of the membership functions.

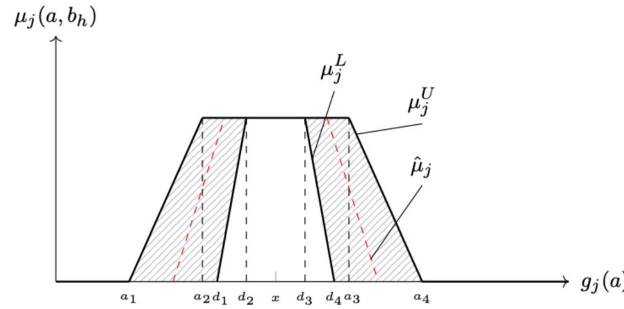

Fig. 2 Example of upper and lower fuzzy sets in a IT2TrFN

Let X denotes a finite non-empty set and $Int([0, 1])$ represents the set of all closed subintervals of [0, 1]. A mapping $I : X \to Int([0,1])$ is referred to as an Interval Type 2 Fuzzy Set (IT2FS) on X [20]. If $I(x)$ is a traditional Trapezoidal Fuzzy Number (TrFN) defined on the closed and bounded interval [0, 1] and then I is termed as a traditional Interval Type 2 Trapezoidal Fuzzy Number (IT2TrFn) on X [58].

In a traditional IT2TrFN, uncertainty is characterized by two non-negative TrFNs, illustrated in Fig. 2, where μ_j^U and μ_j^L represent the upper and lower TrFNs, respectively [52]. The second-order uncertainty corresponds to the hatched area between these TrFNs, defined as the footprint of uncertainty (FOU) [53].

Discrete fuzzy numbers (DFNs) [59] can be utilized for criteria assessed using qualitative scales. In this context, a discrete membership function is defined as a set $M = (x_1, \mu_1), (x_2, \mu_2), \ldots, (x_r, \mu_r)$, where x_i and μ_i (for $i = 1, 2, \ldots, r$) represent the grade on the qualitative scale and the membership degree, respectively, assigned by a decision-maker to an alternative with such an evaluation. Consequently, the footprint of uncertainty (FOU) can be depicted by intervals of the form $[\underline{\mu}, \bar{\mu}]$, as illustrated in Fig. 3. In this figure, the set of blue bullets represents the upper membership function, while the red bullets depict the lower membership function.

3.5 Stochastic Modeling

SMAA-TRI is a multi-criteria sorting method [60] which has been applied in different problems [61–64] and where some elements can be used in our approach. This method allows to analyze the stability of solutions built by the sorting process whenever the ELECTRE parameters are modeled by stochastic variables. Therefore, let us consider the criteria weights $\mathbf{w} = (w_j)_{1\times m}$ are modeled by stochastic variables. The same process may be applied to model p_j, q_j, p'_j, q'_j. This can be done by defining intervals where these variables can

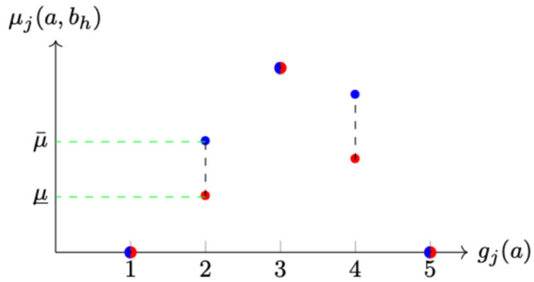

Fig. 3 Example of FOU in discrete criteria

be generated. For instance, in Fig. 2, the interval $[a_1, b_1]$ may be used to generate the p_j values in cases of quantitative criteria. Equally, intervals $[\underline{\mu}, \bar{\mu}]$ in Fig. 3 may be used to generate stochastic variables that help to define discrete fuzzy numbers, in cases of qualitative criteria.

A *sorting acceptability index* π_i^h describes the share of possible parameter values that an alternative $i (i = 1, \ldots, n)$ has to be assigned to a category C_h. This index is in the range [0, 1]. A value 0 indicates that there is no evidence that i does belong to the category C_h and 1 means that the action surely belongs to that category. This index is computed by Monte Carlo Simulation (MCS), used to generate the primary membership functions in the FOU areas, modeling the intra DMs subjectivity. Note that this process also allows to perform robustness analysis, avoiding the necessity to run post-process sensitivity analysis, as proposed by other approaches [52].

3.6 Group and Individual Credibility Indices

Let us denote a set of DMs $D = \{DM_1, DM_2, \ldots, DM_K\}$ and a fictitious "Supra Decision-Maker" (SDM) who is intended to "aggregate" the individual information coming from the other DMs. Let $b \in B$ and $a \in A$ such that for each DM_k (k=1, ..., K) and g_j, the following fuzzy number is defined:

$$\mu_j^k(a, b) = \min\{c_j^k(b, a), cinv_j^k(a, b)\}. \tag{5}$$

that can be interpreted as a fuzzy indifference relation, constructed from two outranking relations [65]. Note that $c_j^k(b, a), cinv_j^k(a, b)$ may be generated by construction of membership functions inside the FOU areas, both in quantitative and qualitative criteria (see, for instance, the red dashed lines in Fig. 2). Thus, let us define the following HFEs:

$$hdir_j(b_h, a) = \{c_j^k(b_h, a) \mid k = 1, \ldots, K\},, \tag{6}$$

$$hinv_j(a, b_h) = \{cinv_j^k(a, b_h) \mid k = 1, \ldots, K\}.. \tag{7}$$

For each DM k, the individual credibility indices are computed as follows:

$$\sigma_D^k(b_h, a) = \sum_{j=1}^{m} w_j^k c_j^k(b_h, a),, \tag{8}$$

$$\sigma_I^k(a, b_h) = \sum_{j=1}^{m} w_j^k cinv_j^k(a, b_h).. \tag{9}$$

Instead, at the SDM level, the credibility indices are computed as follows:

$$\sigma_D(b_h, a) = \sum_{j=1}^{m} w_j sdir_j(b_h, a),, \tag{10}$$

$$\sigma_I(a, b_h) = \sum_{j=1}^{m} w_j sinv_j(a, b_h),, \tag{11}$$

where $w_j \geq 0, \sum_{j=1}^{n} w_j = 1$ is a set of weights that the set of DMs accept to correctly represent the importance of the criteria. Functions $sdir_j(b_h, a)$ and $sinv_j(a, b_h)$ are HFS computed by aggregating $hdir_j(b_h, a)$ and $hinv_j(a, b_h)$, respectively. For instance, the *max* score function may be considered (Reference [66] provides other kinds of aggregation):

$$\begin{aligned} sdir_j(b_h, a) &= \max_K\{c_j^k(b_h, a)\}, \\ sinv_j(a, b_h) &= \max_K\{cinv_j^k(b_h, a)\}. \end{aligned} \tag{12}$$

In which follows, this approach is applied to a real problem.

4 Application

4.1 Defining the Problem

In a study presented in [67], a Brazilian electrical power company undertook the sorting of 49 projects based on their criticality, defined as the level of specialized resources, time, and strategic involvement that the company could allocate to each project. The decision-making process involved several stakeholders: the PMO manager (DM1) held the responsibility of making the final decisions regarding project allocation, while two experienced project managers (DM2 and DM3) assisted in the analysis.

The criteria family comprises

- g_1: Project Complexity, which increases with the project's budget and the number of departments involved.
- g_2: Resources, measuring the number of task hours required for project completion.
- g_3: Expected rate of development, assessing the urgency in project development and implementation.

Table 2 Criteria, scales, and weights

Criterion	Weight			Verbal scale	Scale
	DM1	DM2	DM3		
g_1	0.20	0.15	0.30	High	4
				Medium	3
				Low	2
				Very low	1
g_2	0.20	0.15	0.25		Man-hours
g_3	0.20	0.20	0.15	Urgent	5
				Critical	4
				Competitive	3
				Regular	2
				Low	1
g_4	0.30	0.15	0.15	Very high	4
				High	3
				Medium	2
				Low	1
g_5	0.10	0.35	0.15	High	4
				Medium	3
				Low	2
				Negligible	1

- g_4: Contribution to the achievement of organizational strategy.
- g_5: Technological level involved in project development.

Table 2 presents the criteria, scales, and weights determined by the three DMs involved in the process [67]. It is worth noting that the SDM weights could be derived from the information in this table by utilizing a hesitant average score function [47].

The evaluations of each project are presented in Table 3. Three categories are defined according to [67]: C_1 for non-critical projects, C_2 for critical projects, and C_3 for very critical projects. Additionally, a senior manager assisted in defining the three reference alternatives, as depicted in the table.

4.2 Building Outranking Functions

To establish the outranking functions, the $g_j(b_h)$ values are extracted from the central reference actions, for instance, from information in Table 3. For each DM k and b_h, we need the parameters contributing to the construction of the direct and inverse outranking functions helping to build the membership function for a category h in a criterion j.

In a previous study [67], the DMs were not available when our approach began development. Thus, parameters are defined based on the type of criterion, distinguishing two cases:

Table 3 Evaluation of projects and central reference actions

Project	g_1	g_2	g_3	g_4	g_5
1	2	640	2	3	2
2	3	480	2	2	1
3	3	640	1	3	2
4	1	720	2	2	2
5	1	160	3	3	2
6	2	160	2	3	2
7	3	620	2	1	2
8	2	640	1	2	1
9	1	320	2	3	3
10	2	320	2	3	2
11	2	160	3	3	1
12	1	160	1	1	2
13	2	640	3	3	2
14	2	240	3	3	2
15	1	160	2	2	1
16	1	160	2	3	1
17	3	320	3	2	1
18	3	640	2	3	1
19	3	960	2	3	2
20	1	160	2	2	2
21	2	640	2	2	2
22	1	160	2	2	3
23	3	1280	1	2	2
24	1	320	3	2	2
25	2	160	1	2	2
26	1	160	2	2	1
27	2	1280	1	3	2
28	3	640	2	2	2
29	2	800	1	2	2
30	2	160	3	2	2
31	2	160	1	2	1
32	3	320	1	2	2
33	2	240	2	3	1
34	2	320	2	2	1
35	1	120	2	3	1
36	2	160	1	3	1
37	2	3200	1	2	1
38	2	3200	1	2	1
39	2	960	1	2	1
40	2	3840	1	2	1
41	2	2880	1	2	1
42	1	1200	3	2	1
43	1	1200	3	2	1
44	3	7680	4	3	3
45	1	2400	3	2	1
46	3	5760	4	3	1

Table 3 continued

Project	g_1	g_2	g_3	g_4	g_5
47	3	2880	4	3	1
48	2	7680	3	3	1
49	1	7680	3	3	1
b_1	1	300	2	1	1
b_2	2	2000	3	2	2
b_3	3	5000	4	3	3

g_2 : We define m_2 as the mean average and σ_2^m as the standard deviation of the set of differences between evaluations on this criterion (see [17] for a detailed explanation). Then, a crisp value $p_2 = \sigma_2$ is initially set. The other thresholds are set as $p_2 = p_2', p_2 = 2q_2, p_2' = 2q_2'$. For SDM, we define $p_2^{SDM} = \max_{k\in\{1,2,3\}} p_2^k$. Thus, the expression in Eq. (12) can be applied at runtime.

Other : Membership functions are defined in Table 4. For instance, if an alternative is evaluated with a score of 2 on criterion g_1, DM1 would judge that the credibility of this alternative belonging to C_1 equals 0.3. These Discrete Fuzzy Numbers (DFNs) may be elicited through an interactive process with the DM or by another method (see, for instance, [68]).

4.3 Updating Parameters

For each parameter, a stochastic variable uniformly distributed within a closed interval is considered. Random values for each parameter are then generated, and Monte Carlo Simulation (MCS) is applied. Two cases are distinguished:

g_2 : A random preference threshold is defined by the stochastic variable $\xi_{p_2} \sim U(\sigma_2 - \psi^k m_2, \sigma_2 + \psi^k m_2)$, where $\psi^k \in 0.8, 0.9, 1.0$ is a scaling factor that depends on DM $k \in \{1, 2, 3\}$. The other thresholds are directly set by the equations $p_2 = p_2', p_2 = 2q_2, p_2' = 2q_2'$. For SDM, we define $p_2^{SDM} = \max_{k\in\{1,2,3\}} p_2^k$. Thus, the expression in Eq. (12) can be applied at runtime for each simulation round.

Other : To simulate FOU areas, intervals of 0.1 point above and below a given membership degree in Table 4 are defined. For example, for criterion g_1 and DM1, when the score 2 is considered, a stochastic variable $\xi_\mu \sim U(0.2, 0.4)$ is defined. For each criterion, only the extreme scale grades and the ones having a membership degree equal to 1 are fixed, without uncertainty. All other grades follow the uncertainty rule mentioned here. The hesitant SDM membership degrees are constructed at runtime.

Weights are generated as stochastic variables uniformly distributed, $U(0, 1)$, constrained by linear equations reflecting their preferences, as summarized in Table 5.

Table 4 Membership degrees of categories for qualitative criteria

g	Scale	C_1			C_2			C_3		
		DM1	DM2	DM3	DM1	DM2	DM3	DM1	DM2	DM3
g_1	1	1	1	1	0.1	0.1	0.2	0	0	0
	2	0.3	0.2	0.1	1	1	1	0.1	0.1	0.2
	3	0	0	0	0.3	0.4	0.1	1	1	1
	4	0	0	0	0	0	0	1	1	1
g_3	1	1	1	1	0	0	0	0	0	0
	2	1	1	1	0.3	0.5	0.4	0.1	0.1	0.1
	3	0.3	0.3	0.2	1	1	1	0.7	0.3	0.4
	4	0	0	0	0.3	0.3	0.3	1	1	1
	5	0	0	0	0	0	0	1	1	1
g_4	1	1	1	1	0.3	0.1	0.1	0	0	0
	2	0.2	0.2	0.2	1	1	1	0.3	0.1	0.2
	3	0	0	0	0.2	0.2	0.3	1	1	1
	4	0	0	0	0	0	0	1	1	1
g_5	1	1	1	1	0.3	0.3	0.1	0	0	0
	2	0.6	0.5	0.4	1	1	1	0.3	0.1	0.2
	3	0.2	0.1	0	0.3	0.4	0.1	1	1	1
	4	0	0	0	0	0	0	1	1	1

Table 5 Weight restrictions

DM1		DM2	
$w_4 \geq w_j, j \neq 4$ $w_5 \leq w_j, j \neq 5$ $w_j \in [0,1], j = 1,2,3,4,5$	(13)	$w_3 \geq w_j, j = 1,2,4$ $w_5 \geq w_j, j \neq 5$ $w_j \in [0,1], j = 1,2,3,4,5$	(14)
DM3		**Supra DM**	
$w_1 \geq w_j, j \neq 4$ $w_2 \geq w_j, j = 3,4,5$ $w_j \in [0,1], j = 1,2,3,4,5$	(15)	$w_1 \geq w_j, j \neq 1$ $w_3 \leq w_j, j = 2,4,5$ $w_j \in [0,1], j = 1,2,3,4,5$	(16)

4.4 Applying ELECTRE TRI-C and Results

We conduct simulations using a Python implementation of the method outlined in Sect. 4.3, generating 3000 parameter instances for each decision-maker (DM) and the Supra Decision-Maker (SDM). Table 6 displays the results, with darker shades indicating higher acceptability. For each project, the acceptability levels for every DM and SDM, on each category are shown. Strongly highlighted cells have acceptability values equal to or greater than 0.75.

It seems that DM1 predominantly assigns most projects between C_2 and C_3, with high acceptability values (indicated by strong red cells). Conversely, DM2, DM3, and the SDM assign most projects to C_1 and C_2. Notably, the C_3 category is sparsely populated in DM2.

As the criteria weights are independently computed for each decision-maker, the projects assigned by the SDM do not necessarily reflect the "mean assignment" among the DMs. Therefore, it was anticipated that SDM assignments would not be closely aligned with those of the individual DMs.

4.5 Drawing Up Conclusions

To summarize the results and further elaborate conclusions, we calculate two measures: the median acceptability value and the dispersion error across the three decision-makers and the SDM for each alternative in the three categories. Figure 4 provides a more comprehensive view of the classification than Table 6, displaying error bars for each project's acceptability indices. For example, consider the error bar of project P_{49} in category C_1, where the median acceptability level among the decision-makers is 0.64, with upper and lower bounds of 0.74 and 0.0, respectively. In category C_2, the median value is 0.33, while in C_3 it is 0.0. This figure allows for a clearer observation of the dispersion of decision-makers' preferences for each project and category.

A smaller error bar indicates that, based on the decision-makers' preferences, a project is consistently assigned by all four decision-makers. This is evident in projects P_{32}, P_{29}, and P_{28}, which have strong arguments for being assigned to C_2. For projects where ambiguity remains high, a closer analysis is necessary. Therefore, the higher the median value and the narrower the error bar, the clearer the project's assignment to a category.

A very high median value allows to identify where most of projects are assigned. This is the case, for example, of projects $P_{44}, P_{41}, P_{40}, P_{39}, P_{38}, P_{37}, P_{34}$. Note that, despite of the long error bar, projects P_{46}, P_{47} are assigned to C_3, mainly due to the very low median values in the other categories. This a thumb rule to achieve conclusions. For instance, $P_{48}, P_{45}, P_{43}, P_{42}$ assignments can be discerned using the rule. However, ambiguity can be also found on some cases: $P_{24}, P_{22}, P_{19}, P_{18}, P_9, P_5, P_4, P_3$, where two categories could be chosen as assignment. Thus, these measures (error bar and median) provide a tool for drawing up conclusions, but also to identify ambiguous cases where an in-depth analysis could be necessary.

In Table 7, the Kendall Tau correlation among the acceptability levels across the 49 projects for each decision-maker (DM) and category is presented.

It is worth noting that, except for the correlations between DM1 and DM2 and between DM2 and DM3 in category C_2, the correlations are statistically significant. A high or significant value of Kendall Tau indicates a strong ordinal relationship, but the relationship between the variables could be more complex than a simple linear one. Even if the project orderings are similar for two DMs, the acceptability values for one DM could differ significantly from those of another. This discrepancy is why the dispersion error across the three decision-makers for each alternative in the three categories may be very high in some cases, as observed in Fig. 4. The average dispersion error reaches 0.54, 0.60, and 0.24 for C_1, C_2, and C_3, respectively. In other words, the ranking results can be very different from the outcomes generated in the sorting process.

Findings from this application reveal (1) Strong correlation among acceptability levels for different projects across decision-makers and categories, except in specific cases; (2) Varied acceptability ratings among decision-makers despite similar project rankings, leading to high

Table 6 Sorting acceptability indices of projects

Project	DM1			DM2			DM3			SDM		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
P_1	0.01	0.58	0.41	0.01	0.99	0.00	0.01	0.99	0.00	0.00	1.00	0.00
P_2	0.03	0.97	0.00	0.98	0.02	0.00	0.12	0.70	0.19	0.02	0.82	0.16
P_3	0.02	0.50	0.48	0.13	0.87	0.00	0.03	0.58	0.39	0.00	0.38	0.62
P_4	0.32	0.68	0.00	0.26	0.74	0.00	1.00	0.00	0.00	0.96	0.04	0.00
P_5	0.04	0.61	0.35	0.01	0.99	0.00	0.96	0.04	0.00	0.89	0.11	0.00
P_6	0.02	0.60	0.38	0.03	0.97	0.00	0.02	0.98	0.00	0.01	0.99	0.00
P_7	0.98	0.02	0.00	0.20	0.80	0.00	0.12	0.68	0.20	0.03	0.81	0.16
P_8	0.09	0.91	0.00	1.00	0.00	0.00	0.15	0.85	0.00	0.08	0.92	0.00
P_9	0.26	0.57	0.17	0.08	0.59	0.33	0.99	0.01	0.00	0.83	0.17	0.00
P_{10}	0.01	0.60	0.39	0.02	0.98	0.00	0.02	0.98	0.00	0.00	0.99	0.00
P_{11}	0.00	0.43	0.57	0.65	0.35	0.00	0.03	0.97	0.00	0.06	0.94	0.00
P_{12}	1.00	0.00	0.00	0.77	0.23	0.00	1.00	0.00	0.00	1.00	0.00	0.00
P_{13}	0.00	0.38	0.62	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00
P_{14}	0.00	0.40	0.60	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00
P_{15}	0.45	0.55	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00
P_{16}	0.35	0.51	0.14	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00
P_{17}	0.00	1.00	0.00	0.63	0.37	0.00	0.03	0.76	0.22	0.01	0.81	0.17
P_{18}	0.01	0.44	0.55	0.98	0.02	0.00	0.11	0.55	0.34	0.01	0.53	0.46
P_{19}	0.00	0.33	0.67	0.00	1.00	0.00	0.00	0.48	0.52	0.00	0.34	0.66
P_{20}	0.40	0.60	0.00	0.33	0.67	0.00	1.00	0.00	0.00	0.98	0.02	0.00
P_{21}	0.02	0.98	0.00	0.02	0.98	0.00	0.01	0.99	0.00	0.00	1.00	0.00
P_{22}	0.38	0.62	0.00	0.09	0.66	0.25	1.00	0.00	0.00	0.90	0.10	0.00
P_{23}	0.00	1.00	0.00	0.07	0.93	0.00	0.00	0.69	0.31	0.00	0.78	0.22
P_{24}	0.08	0.92	0.00	0.01	0.99	0.00	0.96	0.04	0.00	0.93	0.07	0.00
P_{25}	0.08	0.92	0.00	0.21	0.79	0.00	0.06	0.94	0.00	0.01	0.99	0.00
P_{26}	0.45	0.55	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00
P_{27}	0.00	0.61	0.39	0.07	0.93	0.00	0.00	1.00	0.00	0.00	1.00	0.00
P_{28}	0.01	0.98	0.00	0.01	0.99	0.00	0.01	0.73	0.26	0.00	0.78	0.22
P_{29}	0.03	0.97	0.00	0.14	0.86	0.00	0.02	0.98	0.00	0.00	1.00	0.00
P_{30}	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.01	0.99	0.00
P_{31}	0.12	0.88	0.00	1.00	0.00	0.00	0.24	0.76	0.00	0.13	0.87	0.00
P_{32}	0.05	0.95	0.00	0.17	0.83	0.00	0.04	0.76	0.20	0.00	0.79	0.21
P_{33}	0.03	0.62	0.35	0.99	0.01	0.00	0.15	0.85	0.00	0.08	0.92	0.00
P_{34}	0.05	0.95	0.00	0.99	0.01	0.00	0.15	0.85	0.00	0.09	0.91	0.00
P_{35}	0.36	0.51	0.13	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00
P_{36}	0.07	0.66	0.28	1.00	0.00	0.00	0.22	0.78	0.00	0.11	0.89	0.00
P_{37}	0.00	1.00	0.00	0.99	0.01	0.00	0.00	1.00	0.00	0.00	1.00	0.00
P_{38}	0.00	1.00	0.00	0.99	0.01	0.00	0.00	1.00	0.00	0.00	1.00	0.00
P_{39}	0.04	0.96	0.00	1.00	0.00	0.00	0.07	0.93	0.00	0.03	0.97	0.00
P_{40}	0.00	1.00	0.00	0.98	0.02	0.00	0.00	1.00	0.00	0.00	1.00	0.00
P_{41}	0.00	1.00	0.00	0.99	0.01	0.00	0.00	1.00	0.00	0.00	1.00	0.00
P_{42}	0.02	0.98	0.00	0.91	0.09	0.00	0.98	0.02	0.00	1.00	0.00	0.00
P_{43}	0.02	0.98	0.00	0.91	0.09	0.00	0.98	0.02	0.00	1.00	0.00	0.00
P_{44}	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00
P_{45}	0.01	0.99	0.00	0.79	0.21	0.00	0.72	0.28	0.00	0.88	0.12	0.00
P_{46}	0.00	0.00	1.00	0.34	0.62	0.04	0.00	0.04	0.96	0.00	0.10	0.90
P_{47}	0.00	0.00	1.00	0.35	0.62	0.03	0.00	0.07	0.93	0.00	0.23	0.77
P_{48}	0.00	0.00	1.00	0.43	0.57	0.00	0.00	0.90	0.10	0.00	0.83	0.17
P_{49}	0.00	0.37	0.63	0.71	0.29	0.00	0.57	0.43	0.00	0.74	0.26	0.00

error dispersion across them; (3) Discrepancies attributed to differences in membership functions, their positions across criteria axes, and constraints on criteria weights; (4) Imposing weight constraints may lead to information anomalies, a concern for future research; and (5) The crucial role of membership function extraction in fuzzy representation.

5 Discussion

Membership functions are essential in fuzzy representation, serving as the basis of fuzzy set theory. However, their extraction is context dependent, leading to evaluation complexities. Various strategies have been proposed in the literature for the extraction and analysis of fuzzy sets [69].

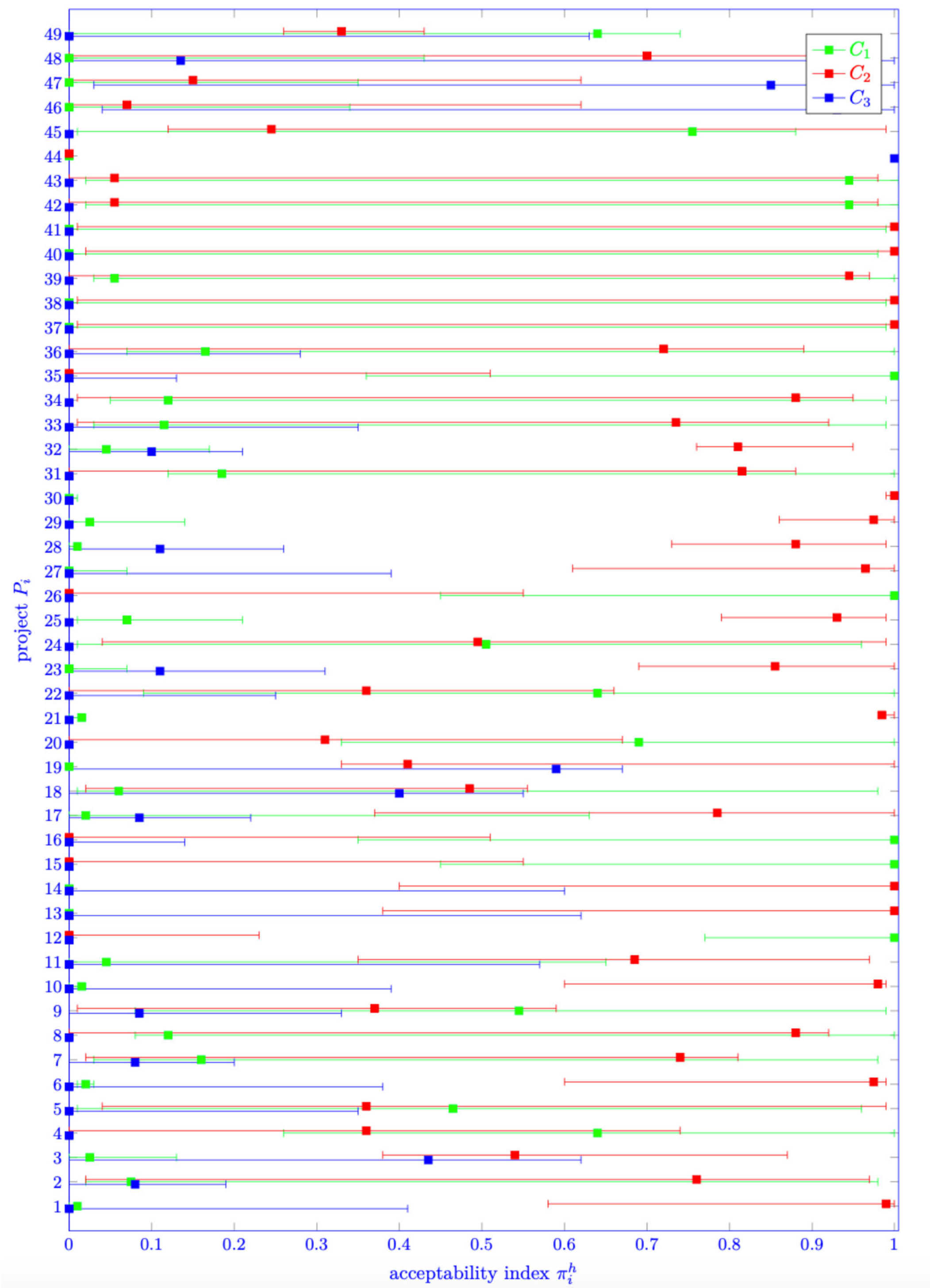

Fig. 4 Stochastic classification of projects

Some of these include selecting different shaped functions (such as Gaussian, triangular, trapezoidal, linear, and sigmoidal) [70, 71], extracting membership functions from expert information [69], optimizing membership function parameters for specific problems [72], or extracting fuzzy membership functions where data are available [73]. Therefore, we anticipate a need for future research to refine the definition of membership functions in the construction of IT2 fuzzy numbers for decision-makers, distinct from trapezoidal or triangular fuzzy sets.

In our application, the method automatically generates the 45 outranking functions required for application and

Table 7 Kendall Tau correlation of acceptability levels

	C_1			C_2			C_3		
Category	DM1	DM2	DM3	DM1	DM2	DM3	DM1	DM2	DM3
DM1		0.33**	0.84**		−0.33	0.26**		0.29*	0.29*
DM2			0.33**			0.20			0.32*
DM3									

*: $p < 0.05$; **: $p < 0.01$

computation of credibility indices in each round, a significant advantage over other fuzzy set extensions of ELECTRE methods that require elicitation of 245 HFSs [16, 17]. However, a limitation may arise if decision-makers are unavailable to provide information for building the outranking functions, highlighting a potential area for further research, particularly in energy-related companies involved in intensive project evaluation, development, and monitoring activities. Future research may explore methods for automatically extracting HFS and IT2TrFN when project lifecycle data are available [73].

The discrepancy between the ranking correlation and sorting outcomes could be explained by the fact that the outranking functions defined in the criteria for the three DMs may be similar, but they are positioned differently along the criteria axis, and the criteria weights are not significantly different among the DMs. This could be intensified by restrictions on the criteria weights, which could steer the admissible set of weights toward areas where "information anomalies" may appear [74]. An information anomaly is the phenomenon whereby additional information on the criteria weights, through the imposition of a constraint, might lead to a worse approximation of true preferences, also excluding weight variability. Future research should be conducted to analyze the specific effects of weight restrictions on the sorting process, considering the possibility of information anomalies.

Recently, authors have highlighted the complementarity of fuzzy sets, outranking relations, and stochastic analysis as a means to consider both uncertainty and sensitivity/robustness analysis [75–77]. Generally, these approaches are grounded in the stochastic modeling of standard or interval type 2 fuzzy sets (IT2FS). Several authors have applied Monte Carlo Simulation in contexts where IT2FS is used for modeling uncertainty [56]. However, to our knowledge, the only existing ELECTRE method capable of accommodating stochastic threshold modeling, even with deterministic scores, is SMAA-TRI [22], an extension of ELECTRE TRI-B. We have drawn inspiration from the rationale of SMAA-TRI to develop our approach. Our methodology, compared to SMAA-TRI, introduces an ELECTRE TRI-C extension tailored for group decision-making, accommodating both quantitative and qualitative criteria, and complex weight relations through linear equations. To the best of our knowledge, no ELECTRE TRI extension incorporating HFS and IT2TrFN has been published to date, enabling the modeling of intra- and inter-DM uncertainty in our approach. In our opinion, an approach worth considering to future enhancement of our methodology, as proposed by [55], involves utilizing the Simulated Annealing metaheuristic. This method aims to approximate the optimal distance between a given alternative, represented by an IT2FS, and a reference point (central reference action).

The application of our approach to a problem within a Brazilian electrical power company showcases its adaptability to group decision-making with minimal effort compared to alternatives. In previous work, [78] used PROMSORT, a deterministic method based on PROMETHEE, for project sorting [79]. PROMSORT assigns alternatives to categories using predefined limits and pairwise comparisons. Unlike our approach, PROMSORT lacks robustness analysis and uses deterministic parameters, requiring additional post-processing. However, in this method, some alternatives assigned to the categories may serve as reference actions. This suggests that our approach could be utilized to extend ELECTRE TRI-nc, a method in which n alternatives may represent a category [80]. In such a scenario, future research should consider extracting and/or constructing multiple IT2TrFNs for a given category from information provided by the DMs.

Graphical visualization techniques are highly beneficial for comprehending and identifying solutions in choice, ranking, or sorting problems [81]. Recently, scholars have emphasized the importance of evaluating the effectiveness of graphical visualizations when integrated into MCDM/A methods. Reference [82] investigate how using ad hoc visual tools enhance managerial judgment and decision-making. Reference [83] utilize eye-tracking experiments to offer analysts insights into the application of graphical visualization within the FITradeoff method. Reference [84] assesses the reliability of interactive visualization tools for healthcare data analytics and medical diagnosis, addressing ambiguities from multiple expert opinions. Reference [85] combine the VIKOR method with dependency nested rings, qualifier radar charts, and scatter graphs to offer decision-makers insights in contexts with a high number of alternatives. Reference [86] propose that future research could explore novel visualization paradigms for enhance transparency and flexibility of visualization tools. Therefore, future research is necessary to assess the effectiveness of the proposed graphical visualization in facilitating the decision-making process.

6 Conclusion

This paper introduces an innovative extension of the ELECTRE TRI-C method tailored to accommodate uncertain information and subjective judgments from multiple decision-makers. At the core of this approach lies a stochastic wrapper that incorporates a constructive procedure for generating outranking functions, catering to both quantitative and qualitative criteria. Therefore, robustness analysis is seamlessly integrated within the process, distinguishing it from other methodologies where such analyses are conducted post MCDM/A processing.

The key contributions of our approach are as follows: (1) Enabling the construction of both continuous and discrete outranking functions by integrating the subjective judgments of multiple decision-makers using Interval Type 2 fuzzy numbers (IT2TrFN) and Monte Carlo Simulation. (2) Aggregating this data using HFS to simulate a supra-decision-maker, thereby enhancing decision-making accuracy. (3) Offering flexibility for stochastic processing of both quantitative and qualitative criteria, facilitating robust decision-making in uncertain environments. (4) Integrating stochastic weights and linear equations to effectively represent constraints between them. (5) Introducing a novel visualization method that provides a holistic view of the stochastic results obtained from the group of decision-makers, aiding in insightful analysis.

Applied within a Brazilian electrical power company tasked with categorizing projects into three distinct groups, our approach proves useful to company managers by showcasing how projects are classified individually by each decision-maker and collectively by the entire group, as represented by a supra-decision-maker. This allows managers to identify projects assigned to each category and recognize candidate projects where ambiguity persists, furnishing decision-makers with actionable insights.

Findings in this application are as follows: (1) Significant correlation is observed among project rankings across various DMs, except for specific cases. (2) Despite similar project rankings, sorting acceptability ratings may significantly differ among DMs, resulting in high error dispersion across them for each alternative in different categories. (3) Discrepancies in sorting results could be attributed to differences in outranking functions position across each criterion axis. (4) Imposing weight constraints could lead to information anomalies, where additional information might result in very similar or overlapping weight spaces, which is a research concern for our future work. (5) IT2TrFN extraction is crucial in fuzzy representation, thus the availability of DMs information for constructing outranking functions could lead to the early detection of similarities, but this needs future research.

Future research could delve into methods for extracting/ building HFS and IT2TrFN from expert opinions and available data. Additionally, further investigation should be conducted to analyze the specific effects of weight restrictions on the sorting process, taking into account the potential presence of information anomalies. An approach worth considering for enhancing our methodology in future involves using methods to approximate the optimal distance between a given alternative, represented by an IT2FS, and a reference point (central reference action). We think that our approach could be utilized to extend methods as ELECTRE TRI-nc where *n* alternatives may represent a category. Finally, further research is necessary to assess the effectiveness of the proposed graphical visualization in facilitating the decision-making process.

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Data availability Non applicable.

Declarations

Competing interests The authors declare that no potential conflict of interest exists regarding the proposal presented in this manuscript.

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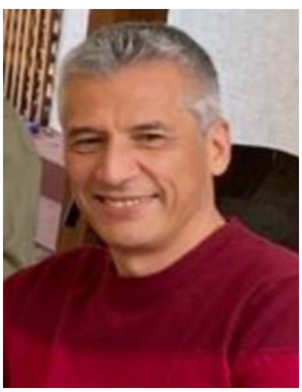

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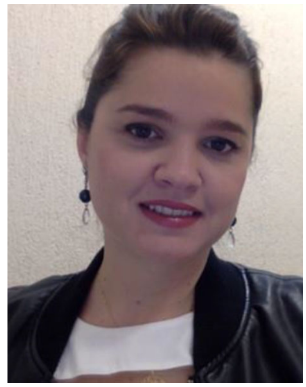

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