



Stabilization of Fractional-Order T–S Fuzzy Systems with Time Delays via an H_∞ Performance Model

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Abstract Takagi–Sugeno (T–S) fuzzy model is an effective technology for describing complex nonlinear industrial processes and dynamic systems with unmeasurable parameters. However, to stabilize this type of system, feedback linearization techniques and complex adaptive schemes usually need to be adopted. This paper focuses on the stabilization of uncertain fractional-order (FO) systems with unmeasurable states, external disturbances, and time delays, where certain “IF-THEN” rules based on an FO T–S model are proposed to describe FO systems. A new FO H_∞ performance model is established to provide stabilization sufficient conditions. A fuzzy observer is implemented to reconstruct system states, and a feedback controller is established to stabilize the integrated system. Benefiting from a continuous frequency distribution model, the stabilization can be discussed through integer-order stabilization theories, and several stabilization conditions are obtained. Moreover, the proposed control approach avoids the usage of feedback linearization techniques, which effectively reduces the computational complexity. Apart from theoretical analysis, through a numerical experiment, the feasibility of derived results is verified.

Keywords Fractional-order system · Takagi–Sugeno fuzzy model · Frequency distributed model · Time delay · Fuzzy observer

1 Introduction

Recently, fractional calculus has been broadly investigated because of its unique heritability and memorability in system modeling and controlling, and has been used in many fields, such as bioengineering, physics, economics, and electronics [1–4]. Compared with integer-order systems, it can be found that nonlinear systems described by fractional calculus are more consistent with real dynamic systems. Consequently, stabilization of fractional-order nonlinear systems (FONSs) has been a research hotspot [5–8], and researchers have provided different control methods, including Takagi–Sugeno (T–S) fuzzy control [9–11], sliding mode control [12, 13], neural network control [14, 15], adaptive fuzzy control [16, 17], sampling control [18, 19], and backstepping control [20, 21]. It is well known that with the help of “IF-THEN” rules, some FONSs can be described through a fractional-order (FO) T–S fuzzy model. The main characteristic of this model is linearizing the local of complex nonlinear systems, and the whole system can be obtained through “mixed” linear subsystems. Compared with ordinary fuzzy systems, T–S systems usually has fewer fuzzy rules, and some interesting consequences have been reported recently, e.g., Refs. [10, 22, 23]. An FO T–S fuzzy model was obtained to precisely approximate unknown dynamics systems in [22]. A switched fuzzy control for FONSs was introduced by T–S fuzzy models in [23]. Adaptive T–S fuzzy schemes for FONSs were discussed in [10]. However, one of the basic assumptions in above works is that system states are measurable.

In fact, because of sensor faults and the objective existence of measurement errors in practical systems, usually, only part of states are available. In addition, there exist some states that are hard to be measured directly, such as

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the temperature of the combustion furnace and wind speed. A common method to deal with this problem is designing an observer, whose main advantages include saving the cost of sensors, improving the accuracy of output values, and providing a good window to observe system states. For FONSs with unmeasurable states, some interesting works have been reported, e.g., Refs. [10, 24, 25]. T–S fuzzy tracking control of FONSs based on an observer was introduced in [24]. An adaptive T–S fuzzy scheme for uncertain FONSs was considered in [10]. A predefined finite time impulsive observer for chaotic FONSs was designed in [25]. Unfortunately, the prevalent time delays are not considered in the above literature.

Note that time delays can result in system instability and performance deterioration in actual application. So, it is a challenging but meaningful work to study the stabilization of FO delayed systems, and some interesting results have been reported [26–31]. A direct Lyapunov–Krasovskii function was proposed for FO time-delay systems in [26, 27]. A new Lyapunov–Krasovskii functional was established for FO time-varying delay systems in [28]. Stability analysis of FO time-delay systems was obtained by Lyapunov–Krasovskii functions and linear matrix inequalities in [29]. Stable sufficient conditions for an FO time-delay interconnected system were provided by calculating linear matrix inequality (LMI) in [30]. The positivity and stability of FO systems with time delays were verified in [31]. However, in the above literature, it is often necessary to solve some complex linear programming problems to design feedback controllers, which may have no solution or cannot be optimally solved [32, 33]. How to simplify the computation is worth further investigation.

Inspired by the above analysis, this article presents a new T–S fuzzy control method with FO H_∞ performance, which effectively solves the stabilization problem of FONSs with time delays, unmeasurable states, and bounded external disturbances. This method not only implements a fuzzy observer to reconstruct system states but also proposes a new FO H_∞ performance model to guarantee the observer’s performance. In addition, it is proven that systems are stable when the FO H_∞ performance is decreased to the prescribed attenuation level by utilizing a frequency distribution model. The main contributions of this article are given as follows: (1) A new FO H_∞ performance model is proposed, which can avoid solving a traditional nonlinear Hamilton–Jacobi partial differential equation. It is also shown that even the FO H_∞ tracking performance is decreased to a prescribed attenuation level, the system is stable. In addition, to obtain the stabilization condition, only a simple LMI needs to be solved, which

effectively reduces the computational complexity compared with related works, e.g., Refs. [30, 34]; (2) By introducing an FO H_∞ performance model, a fuzzy observer is designed to reconstruct unmeasurable states, which can improve the accuracy of the output and provide a good window to observe system states; (3) Based on a frequency distribution model, an indirect Lyapunov stabilization condition is derived.

The frame of the article is arranged below. In Sect. 2, a T–S fuzzy model of FONSs is introduced, and some basic lemmas and concepts are provided. In Sect. 3, the main results are introduced, in which the performance analysis of a fuzzy observer is proposed in Sect. 3.1, and the stabilization of FONSs with time delays, unmeasurable states, and bounded external disturbances is analyzed in Sect. 3.2. Section 4 verifies the efficacy and feasibility of the control scheme by a simulation example. Finally, Sect. 5 concludes the article.

Throughout the article, some notations must be presented in advance. \mathbb{R}^n denotes n -dimension vector space. $\mathbb{R}^{n \times m}$ represents the space of matrices with n row and m column. \mathbb{R}^+ is the set of nonnegative real numbers. $\Gamma(\cdot)$ represents the Gamma function. The exponent “ T ” denotes the rank transformation of a vector or matrix. For a matrix $A = [a_{ij}] \in \mathbb{R}^{n \times m}$ means that A is a nonnegative matrix, denoted by $A \succeq 0 (A \succ 0)$, i.e., $a_{ij} \geq 0 (a_{ij} > 0)$, and $[A]_{ij} = a_{ij}$. Particularly, for $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ is a Metzler matrix if $a_{ij} \geq 0$ holds for all $i \neq j$.

2 Problem Description and Preliminaries

2.1 Fractional-Order Calculus

In this section, to facilitate subsequent theoretical analysis, some fractional definitions and related lemmas need introduced first for system modeling, controller design, and stabilization analysis.

Definition 1 [29] The fractional integral of a continuous $\theta(t)$ is given by

$$\mathcal{I}_t^\gamma \theta(t) = \frac{1}{\Gamma(\gamma)} \int_0^t (t - \hbar)^{\gamma-1} \theta(\hbar) d\hbar,$$

where $\gamma \in (0, 1)$.

Definition 2 [29] The γ -th Caputo fractional derivative is

$$D_t^\gamma \varphi(t) = \frac{1}{\Gamma(1 - \gamma)} \int_0^t \frac{\varphi'(t)}{(t - \sigma)^\gamma} d\sigma, \tag{1}$$

where $\gamma \in (0, 1)$.

Definition 3 [29] The Mittag–Leffler function is

$$E_{a,b}(k) = \sum_{i=0}^{+\infty} \frac{k^i}{\Gamma(ia + b)}, \quad (2)$$

with $k \in \mathbb{C}$, and $a, b \geq 0$.

Lemma 1 [35] *Supposing $V(t)$ is a continuous function, one has*

$$\frac{1}{2} D^\gamma (V^T(t)V(t)) \leq V^T(t)D^\gamma V(t). \quad (3)$$

Lemma 2 [35] *By considering the frequency distributed model, an FONS $D^\gamma \delta(\vartheta) = f(\delta, \vartheta)$ with $\gamma \in (0, 1)$ can be represented as*

$$\begin{cases} \frac{\partial \ell(\beta, \vartheta)}{\partial \vartheta} = -\beta \ell(\beta, \vartheta) + f(\delta, \vartheta), \\ \delta(\vartheta) = \int_0^\infty \kappa_\gamma(\beta) \ell(\beta, \vartheta) d\beta, \end{cases} \quad (4)$$

with $\kappa_\gamma(\beta)$ being a weighting function satisfying $\kappa_\gamma(\beta) = \frac{\sin(\gamma\pi)}{\beta^\gamma \pi}$.

2.2 Problem Description

In this section, a T–S fuzzy model is introduced, which is described by fuzzy “IF-THEN” rules. A type of T–S time-delay FONSs are considered, whose fuzzy rule is expressed as

Plant rule l :

IF $\mu_1(t)$ is F_{l1} and \dots and $\mu_G(t)$ is F_{lG} , THEN

$$\begin{cases} D^\gamma x(t) = A_l x(t) + A_{ld} x(t - d_l(t)) + B_l u(t) + H\omega(t), \\ y(t) = C_l x(t) + C_{ld} x(t - d_l(t)) + J\omega(t), \quad l = 1, 2, \dots, L, \\ x(\sigma) = \psi(\sigma), \quad \sigma \in [-\tau, 0], \end{cases} \quad (5)$$

with $\gamma \in (0, 1)$, $t \in \mathbb{R}^+$, $x = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$ being the unmeasurable state vector, $y = [y_1, y_2, \dots, y_q]^T \in \mathbb{R}^q$ being the measurable output, $\omega = [\omega_1, \omega_2, \dots, \omega_n]^T \in \mathbb{R}^n$ being a bounded external disturbance, $u = [u_1, u_2, \dots, u_m]^T \in \mathbb{R}^m$ being the control input, μ_g being a premise variable, F_{lg} being a fuzzy membership function, $g = 1, \dots, G$, G being the number of membership functions, $l = 1, \dots, L$, L being the number of rules, $A_l \in \mathbb{R}^{n \times n}$, $A_{ld} \in \mathbb{R}^{n \times n}$, $B_l \in \mathbb{R}^{n \times m}$, $C_l \in \mathbb{R}^{q \times n}$, $C_{ld} \in \mathbb{R}^{q \times n}$, $H \in \mathbb{R}^{n \times n}$, and $J \in \mathbb{R}^{q \times n}$ being constant matrices, $d_l(t)$ being a bounded time delay which

satisfies the following Assumption 1. In the following, $l = 1, 2, \dots, L$ if the range of l is not specified.

Assumption 1 Time delays $d_l(t)$ satisfies $0 < d_l(t) < \tau$ for a known positive constant τ , and the initial condition $\psi(\sigma) \in [-\tau, 0]$ is a known continuous function.

Using the T–S fuzzy method to “mix” multiple FO linear systems, one has

$$\begin{aligned} D^\gamma x(t) &= \frac{\sum_{l=1}^L v_l(\mu(t)) [A_l x(t) + A_{ld} x(t - d_l(t)) + B_l u(t) + H\omega(t)]}{\sum_{l=1}^L v_l(\mu(t))} \\ &= \sum_{l=1}^L \tilde{h}_l(\mu(t)) [A_l x(t) + A_{ld} x(t - d_l(t)) + B_l u(t) + H\omega(t)], \end{aligned} \quad (6)$$

and

$$\begin{aligned} y(t) &= \frac{\sum_{l=1}^L v_l(\mu(t)) [C_l x(t) + C_{ld} x(t - d_l(t)) + J\omega(t)]}{\sum_{l=1}^L v_l(\mu(t))} \\ &= \sum_{l=1}^L \tilde{h}_l(\mu(t)) [C_l x(t) + C_{ld} x(t - d_l(t)) + J\omega(t)], \end{aligned} \quad (7)$$

where $\tilde{h}_l(\mu) = \frac{v_l(\mu)}{\sum_{l=1}^L v_l(\mu)}$, $v_l(\mu) = \prod_{g=1}^G F_{lg}(\mu_g)$ is the membership degree, in which μ_g belongs to F_{lg} . Moreover, it holds $v_l(\mu) \geq 0$, $\sum_{l=1}^L v_l(\mu) > 0$ for all t , and as a result $\tilde{h}_l(\mu) \geq 0$, in which $\sum_{l=1}^L \tilde{h}_l(\mu) = 1$.

The objective of this article is to provide a feedback controller that satisfies an FO H_∞ performance and achieves the stabilization of the system (6). To accomplish this purpose, a new FO H_∞ performance model is proposed, and several stabilization sufficient conditions are provided.

The following FO H_∞ performance related to $x(t)$ is given as follows:

$${}_0\mathcal{I}_{t_f}^\gamma [x^T(t)Qx(t)] \leq x^T(0)Px(0) + \rho_0^2 {}_0\mathcal{I}_{t_f}^\gamma [\omega^T(t)\omega(t)], \quad (8)$$

with $Q \in \mathbb{R}^{n \times n}$, $P \in \mathbb{R}^{n \times n}$ being two positive matrixes, ρ being the prescribed attenuation level, and t_f being the terminal time of control.

Note that system states are unmeasurable, they can be reconstructed through the following observer.

Observer rule l :

IF $\mu_1(t)$ is F_{l1} and \dots and $\mu_G(t)$ is F_{lG} , THEN

$$\begin{cases} D^\gamma \hat{x}(t) = A_l \hat{x}(t) + A_{ld} \hat{x}(t - d_l(t)) + B_l u(t) + L_l (y(t) - \hat{y}(t)), \\ \hat{y}(t) = C_l \hat{x}(t) + C_{ld} \hat{x}(t - d_l(t)), \end{cases} \tag{9}$$

with $\hat{x}(t)$ being the observation of $x(t)$, L_l being an observer gain for the l th rule.

Then, consider the whole fuzzy observer, one has

$$D^\gamma \hat{x}(t) = \sum_{l=1}^L \tilde{h}_l(\mu(t)) \left[A_l \hat{x}(t) + A_{ld} \hat{x}(t - d_l(t)) + B_l u(t) + L_l (y(t) - \hat{y}(t)) \right]. \tag{10}$$

In addition, a feedback controller is provided through the parallel distributed compensation (PDC) method.

Control rule j :

IF $\mu_j(t)$ is F_{j1} and ... and $\mu_G(t)$ is F_{jG} , THEN

$$u(t) = K_j \hat{x}(t), \quad j = 1, 2, \dots, L \tag{11}$$

with K_j being a control gain.

The whole fuzzy feedback controller is

$$u(t) = \sum_{j=1}^L \tilde{h}_j(\mu) K_j \hat{x}(t). \tag{12}$$

Remark 1 A traditional integer-order H_∞ performance model was proposed in [24]. Compared with this model, the considered system (8) generalizes the integer-order research result to an FO case and considers the effect of time delays, which is a new mathematical tool with wider application and is expected to exhibit better performance.

If $d_l(t) = 0$, the same conclusion can be obtained by the proposed model. If $d_l(t) > 0$, values of time delays do not influence final results. The performance of the fuzzy observer (10) is only dependent on its internal matrices. In this case, it is difficult to directly find control gains K_j and observer gains L_l , which will be designed later.

3 Main Results

3.1 Performance Analysis of the Fuzzy Observer

The stabilization analysis of observation errors is introduced to ensure the fuzzy observer performance superiority, as shown in Fig. 1. In addition, for simplicity and convenience, the time variable t will be omitted in some subsequent functions.

The observation error is defined as

$$e(t) = x(t) - \hat{x}(t). \tag{13}$$

By differentiating (13), according to (6) and (10), one has

$$\begin{aligned} D^\gamma e(t) &= D^\gamma x(t) - D^\gamma \hat{x}(t) \\ &= \sum_{l=1}^L \tilde{h}_l \left[A_l x(t) - A_l \hat{x}(t) + A_{ld} x(t - d_l) - A_{ld} \hat{x}(t - d_l) + H \omega(t) - L_l C_l (x(t) - \hat{x}(t)) - L_l C_{ld} (x(t - d_l) - \hat{x}(t - d_l)) - L_l J \omega(t) \right] \\ &= \sum_{l=1}^L \tilde{h}_l \left[(A_l - L_l C_l) e(t) + (A_{ld} - L_l C_{ld}) e(t - d_l) + (H - L_l J) \omega(t) \right]. \end{aligned} \tag{14}$$

Defining $A_{l1} = A_l - L_l C_l \in \mathbb{R}^{n \times n}$, $A_{l2} = A_{ld} - L_l C_{ld} \in \mathbb{R}^{n \times n}$, and $E_l = H - L_l J \in \mathbb{R}^{n \times n}$, one obtains

$$D^\gamma e(t) = \sum_{l=1}^L \tilde{h}_l \left[A_{l1} e(t) + A_{l2} e(t - d_l) + E_l \omega(t) \right]. \tag{15}$$

In this paper, the Lyapunov method is utilized to stabilize the system (6). The following theorem indicates the main results.

Theorem 1 For the FONS (15) with time delays, if there exist two positive matrices $P, S > 0$ such that

$$Q + A_{l1}^T P + P A_{l1} + \frac{1}{\rho^2} P E_l E_l^T P + P A_{l2} S^{-1} A_{l2}^T P + K S < 0, \tag{16}$$

then the FO H_∞ performance of observer errors $e(t)$ is decreased to a specified ρ^2 , i.e.,

$${}_0 \mathcal{I}_{t_f}^\gamma \left[e^T(t) Q e(t) \right] \leq e^T(0) P e(0) + \rho^2 {}_0 \mathcal{I}_{t_f}^\gamma \left[\omega^T(t) \omega(t) \right]. \tag{17}$$

Proof: By integrating (13), one can obtain

$${}_0 \mathcal{I}_{t_f}^\gamma \left[e^T Q e \right] = {}_0 \mathcal{I}_{t_f}^\gamma \left[(x - \hat{x})^T Q (x - \hat{x}) \right], \tag{18}$$

where $t_f \geq 0$.

If the observation error is 0 and $t_f = 0$, one has

$${}_0 \mathcal{I}_{t_f}^\gamma \left[e^T Q e \right] = e^T(0) P e(0) + \rho^2 {}_0 \mathcal{I}_{t_f}^\gamma \left[\omega^T \omega \right]. \tag{19}$$

Otherwise,

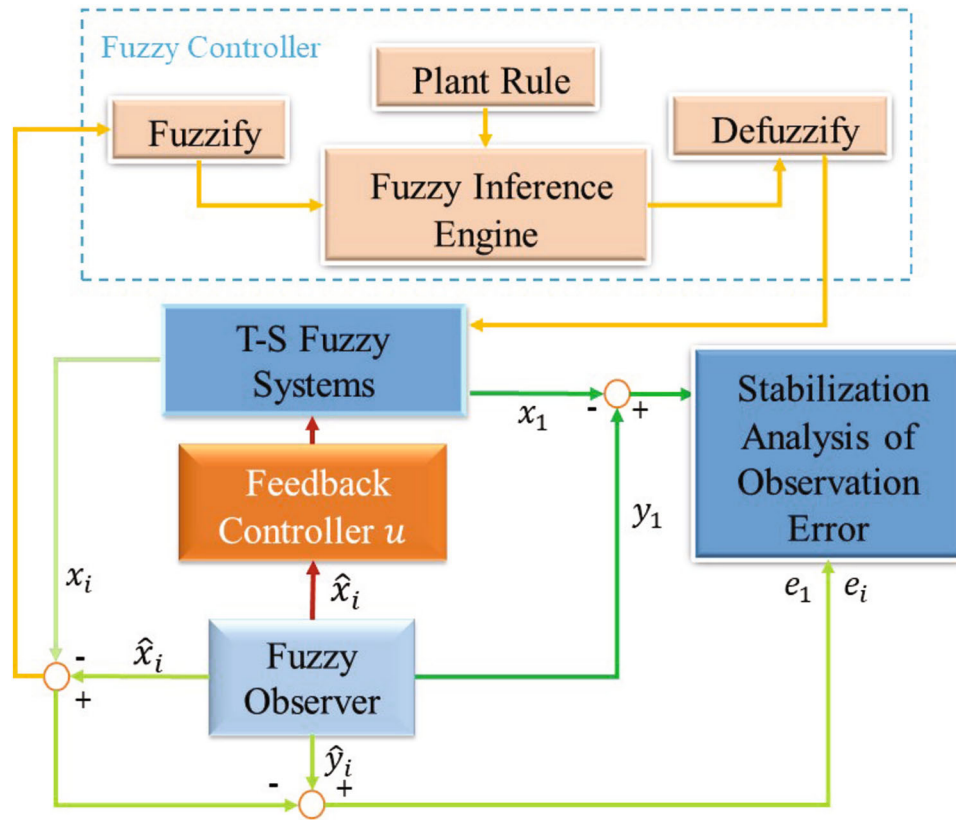


Fig. 1 The structure of the proposed T-S fuzzy system

$$\begin{aligned}
 & {}_0\mathcal{I}_{t_f}^\gamma [e^T Q e] \\
 &= e^T(0) P e(0) - e^T(t_f) P e(t_f) + {}_0\mathcal{I}_{t_f}^\gamma [e^T Q e + {}_0 D_{t_f}^\gamma (e^T P e)] \\
 &\leq e^T(0) P e(0) + {}_0\mathcal{I}_{t_f}^\gamma [e^T Q e + {}_0 D_{t_f}^\gamma (e^T P e)] \\
 &= e^T(0) P e(0) + {}_0\mathcal{I}_{t_f}^\gamma \left\{ e^T Q e + \sum_{l=1}^L \tilde{h}_l [(A_{11} e)^T P e \right. \\
 &\quad + (A_{12} e(t-d_l))^T P e + (E_l \omega)^T P e + e^T P A_{11} e \\
 &\quad + e^T P A_{12} e(t-d_l) + e^T P E_l \omega - \rho^2 \omega^T \omega \\
 &\quad \left. - \frac{1}{\rho^2} e^T P E_l E_l^T P e + \rho^2 \omega^T \omega + \frac{1}{\rho^2} e^T P E_l E_l^T P e \right\} \\
 &= e^T(0) P e(0) + {}_0\mathcal{I}_{t_f}^\gamma \left\{ e^T Q e + \sum_{l=1}^L \tilde{h}_l [e^T A_{11}^T P e + e^T P A_{11} e \right. \\
 &\quad + e^T (t-d_l) A_{12}^T P e + e^T P A_{12} e(t-d_l) + \frac{1}{\rho^2} e^T P E_l E_l^T P e \\
 &\quad \left. - \left(\frac{1}{\rho} E_l^T P e - \rho \omega \right)^T \left(\frac{1}{\rho} E_l^T P e - \rho \omega \right) + \rho^2 \omega^T \omega \right\} \\
 &\leq e^T(0) P e(0) + {}_0\mathcal{I}_{t_f}^\gamma \left\{ e^T Q e + \sum_{l=1}^L \tilde{h}_l [e^T A_{11}^T P e + e^T P A_{11} e \right. \\
 &\quad \left. + \rho^2 \omega^T \omega + \frac{1}{\rho^2} e^T P E_l E_l^T P e + 2e^T P A_{12} e(t-d_l) \right\} \\
 &\leq e^T(0) P e(0) + {}_0\mathcal{I}_{t_f}^\gamma \sum_{l=1}^L \tilde{h}_l \left[e^T (A_{11}^T P + P A_{11} + \frac{1}{\rho^2} P E_l E_l^T P \right. \\
 &\quad \left. + P A_{12} S^{-1} A_{12}^T P + Q) e + e^T (t-d_l) S e(t-d_l) + \rho^2 \omega^T \omega \right].
 \end{aligned}$$

When $e(t) = 0$, the observation objective is completed. Otherwise, there exists a constant $K > 1$, such that

$$e^T(t-d_l) S e(t-d_l) < K e^T(t) S e(t). \tag{20}$$

From (20), one can obtain

$$\begin{aligned}
 {}_0\mathcal{I}_{t_f}^\gamma [e^T Q e] &< e^T(0) P e(0) + {}_0\mathcal{I}_{t_f}^\gamma \sum_{l=1}^L \tilde{h}_l \left[e^T (Q^T \right. \\
 &\quad \left. + A_{11}^T P + P A_{11} + \frac{1}{\rho^2} P E_l E_l^T P \right. \\
 &\quad \left. + P A_{12} S^{-1} A_{12}^T P + K S) e + \rho^2 \omega^T \omega \right].
 \end{aligned}$$

If (16) holds, one has

$${}_0\mathcal{I}_{t_f}^\gamma [e^T(t) Q e(t)] < e^T(0) P e(0) + \rho^2 {}_0\mathcal{I}_{t_f}^\gamma [\omega^T(t) \omega(t)]. \tag{21}$$

To sum up, the FO H_∞ performance (17) holds, which is guaranteed to the specified ρ^2 . That completes the proof. ■

In addition, the design thought of the proposed scheme is summarized in Fig. 2.

Remark 2 According to [10], from a view of energy point, the influence of all bounded external disturbances on the state vector should be decreased to a specified standard ρ^2 . To achieve this goal, the general practice is utilizing an

adaptive law to adjust fuzzy systems and a control scheme to weaken the influence of external interference. Different from the above method, the FO H_∞ performance with a fuzzy observer is considered for all bounded inputs, which can avoid complex updating rules of the adaptive law and effectively reduce the influence of bounded external disturbances.

Remark 3 Note that the fuzzy observer (10) is designed to reconstruct unmeasurable states, which can improve the accuracy of output values. The performance of the fuzzy observer is excellent enough through the stabilization analysis of $e(t)$. Furthermore, to receive excellent control performance, control problems can be converted into the minimized sum of squares problems by utilizing the FO H_∞ performance model. However, it should be pointed out that this model can only simplify the calculation, but it inevitably needs to solve a simple linear matrix inequality problem (LMIP).

3.2 Stabilization Analysis

In the subsection, the stabilization analysis for FONSs with unmeasurable states, bounded external disturbances, and time delays is given. Obviously, stabilization of the system (5) can be achieved by equivalently discussing the system (6).

Through the suitable fuzzy observer (10), the system (6) is reconstructed as

$$D^\gamma \hat{x}(t) = \sum_{l=1}^L \tilde{h}_l \left[A_l \hat{x}(t) + A_{ld} \hat{x}(t - d_l) + B_l u(t) + L_l J \omega(t) \right]. \tag{22}$$

Based on the PDC in [36, 37], by using the controller (12), it follows from (22) that

$$\begin{aligned} D^\gamma \hat{x}(t) &= \sum_{l=1}^L \sum_{j=1}^L \tilde{h}_l \tilde{h}_j \left[A_l \hat{x}(t) + A_{ld} \hat{x}(t - d_l) + B_l K_j \hat{x}(t) + L_l J \omega(t) \right], \\ &= \sum_{l=1}^L \sum_{j=1}^L \tilde{h}_l \tilde{h}_j \left[(A_l + B_l K_j) \hat{x}(t) + A_{ld} \hat{x}(t - d_l) + L_l J \omega(t) \right]. \end{aligned} \tag{23}$$

Defining $B_{lj} = A_l + B_l K_j \in \mathbb{R}^{n \times n}$, $F_l = L_l J \in \mathbb{R}^{n \times n}$, the system (23) is expressed as

$$D^\gamma \hat{x}(t) = \sum_{l=1}^L \sum_{j=1}^L \tilde{h}_l \tilde{h}_j \left[B_{lj} \hat{x}(t) + A_{ld} \hat{x}(t - d_l) + F_l \omega(t) \right]. \tag{24}$$

Based on the above stabilization condition of FONSs, Theorem 2 is proved in a similar way.

Theorem 2 For the FONS (24) with time delays, if there exist two positive matrices $P_0, S_0 > 0$ such that

$$\begin{aligned} Q + B_{ij}^T P_0 + P_0 B_{ij} + \frac{1}{\rho^2} P_0 F_l F_l^T P_0 \\ + P_0 A_{ld} S_0^{-1} A_{ld}^T P_0 + K S_0 < 0, \end{aligned} \tag{25}$$

then the FO H_∞ performance of states \hat{x} is decreased to a specified ρ^2 , i.e.,

$${}_0 \mathcal{I}_{t_f}^\gamma \left[\hat{x}^T(t) Q \hat{x}(t) \right] \leq \hat{x}^T(0) P_0 \hat{x}(0) + \rho^2 {}_0 \mathcal{I}_{t_f}^\gamma \left[\omega^T(t) \omega(t) \right], \tag{26}$$

with $0 < \gamma < 1$.

Proof: Using the similar method as Theorem 1, if the state $\hat{x}(t) = 0$ or $t_f = 0$, one has

$${}_0 \mathcal{I}_{t_f}^\gamma \left[\hat{x}^T(t) Q \hat{x}(t) \right] = \hat{x}^T(0) P_0 \hat{x}(0) + \rho^2 {}_0 \mathcal{I}_{t_f}^\gamma \left[\omega^T(t) \omega(t) \right].$$

Otherwise,

$$\begin{aligned} {}_0 \mathcal{I}_{t_f}^\gamma \left[\hat{x}^T Q \hat{x} \right] &= \hat{x}^T(0) P_0 \hat{x}(0) - \hat{x}^T(t_f) P_0 \hat{x}(t_f) \\ &\quad + {}_0 \mathcal{I}_{t_f}^\gamma \left[\hat{x}^T Q \hat{x} + {}_0 D_{t_f}^\gamma (\hat{x}^T P_0 \hat{x}) \right] \\ &\leq \hat{x}^T(0) P_0 \hat{x}(0) + {}_0 \mathcal{I}_{t_f}^\gamma \left[\hat{x}^T Q \hat{x} + {}_0 D_{t_f}^\gamma (\hat{x}^T P_0 \hat{x}) \right] \\ &= \hat{x}^T(0) P_0 \hat{x}(0) + {}_0 \mathcal{I}_{t_f}^\gamma \left[\hat{x}^T Q \hat{x} + D_{t_f}^\gamma \hat{x}^T P_0 \hat{x} + \hat{x}^T P_0 D_{t_f}^\gamma \hat{x} \right] \\ &\leq \hat{x}^T(0) P_0 \hat{x}(0) + {}_0 \mathcal{I}_{t_f}^\gamma \left\{ \hat{x}^T Q \hat{x} + \sum_{l=1}^L \sum_{j=1}^L \tilde{h}_l \tilde{h}_j \left[\hat{x}^T B_{lj}^T P_0 \hat{x} \right. \right. \\ &\quad \left. \left. + \hat{x}^T P_0 B_{lj} \hat{x} + \rho^2 \omega^T \omega + \frac{1}{\rho^2} \hat{x}^T P_0 F_l F_l^T P_0 \hat{x} \right. \right. \\ &\quad \left. \left. + 2 \hat{x}^T P_0 A_{ld} \hat{x}(t - d_l) \right\} \\ &< \hat{x}^T(0) P_0 \hat{x}(0) + {}_0 \mathcal{I}_{t_f}^\gamma \left\{ \sum_{l=1}^L \sum_{j=1}^L \tilde{h}_l \tilde{h}_j \left[\hat{x}^T (Q + B_{lj}^T P_0 + P_0 B_{lj} \right. \right. \right. \\ &\quad \left. \left. + \frac{1}{\rho^2} P_0 F_l F_l^T P_0 + P_0 A_{ld} S_0^{-1} A_{ld}^T P_0 + K S_0) \hat{x} + \rho^2 \omega^T \omega \right\}. \end{aligned}$$

From (25), one can obtain

$${}_0 \mathcal{I}_{t_f}^\gamma \left[\hat{x}^T Q \hat{x} \right] < \hat{x}^T(0) P_0 \hat{x}(0) + \rho^2 {}_0 \mathcal{I}_{t_f}^\gamma \left[\omega^T \omega \right].$$

Considering all cases, the FO H_∞ performance (26) holds, which is decreased to the specified ρ^2 . This proof is finished. ■

According to the preceding derivation, the system (24) is stable. In addition, noting that the system (24) contains external disturbances, a simple closed-loop system

$$D^\gamma \hat{x}(t) = \sum_{l=1}^L \sum_{j=1}^L \tilde{h}_l \tilde{h}_j \left[B_{lj} \hat{x}(t) + A_{ld} \hat{x}(t - d_l) \right] \tag{27}$$

is considered.

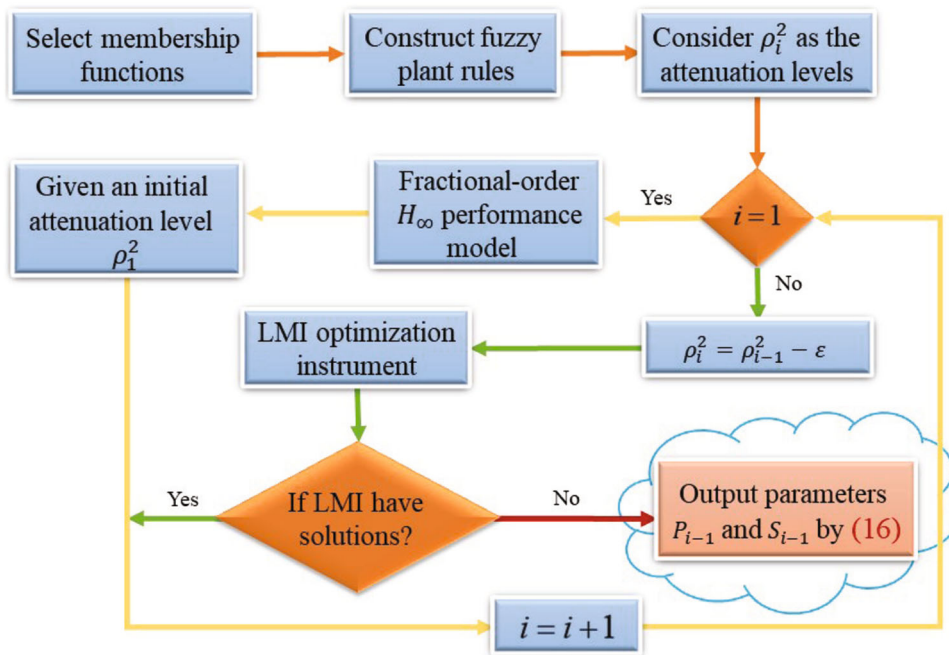


Fig. 2 Design procedures

According to the preceding derivation, the following conclusion can be reached.

Theorem 3 *If the system (24) satisfies the FO H_∞ performance (26), then the closed-loop system (27) is quadratically stable.*

Proof: From (4), the following frequency distribution model is used to reconstruct the system (27), one can obtain

$$\begin{cases} \frac{\partial z_{\omega_1}}{\partial t} = -\omega_1 z_{\omega_1} + f_d, \\ \hat{x}(t) = \int_0^\infty z_{\omega_1} d\omega_1, \end{cases} \quad (28)$$

where

$$f_d = D^\gamma \hat{x}(t) = \sum_{l=1}^L \sum_{j=1}^L \tilde{h}_l \tilde{h}_j^T [B_{lj} \hat{x}(t) + A_{ld} \hat{x}(t - d_l)].$$

Defining the Lyapunov function as

$$V_1(t) = \int_0^\infty z_{\omega_1}^T P_1 z_{\omega_1} d\omega_1, \quad (29)$$

with P_1 being a weighting matrix.

By differentiating (29), one has

$$\begin{aligned} \dot{V}_1(t) &= \int_0^\infty (-\omega_1 z_{\omega_1}^T + f_d^T) P_1 z_{\omega_1} d\omega_1 \\ &\quad + \int_0^\infty z_{\omega_1}^T P_1 (-\omega_1 z_{\omega_1} + f_d) d\omega_1 \\ &= -\int_0^\infty \omega_1 z_{\omega_1}^T P_1 z_{\omega_1} d\omega_1 + f_d^T P_1 \int_0^\infty z_{\omega_1} d\omega_1 \\ &\quad - \int_0^\infty z_{\omega_1}^T P_1 \omega_1 z_{\omega_1} d\omega_1 + \int_0^\infty z_{\omega_1}^T d\omega_1 P_1 f_d \\ &= -2 \int_0^\infty \omega_1 z_{\omega_1}^T P_1 z_{\omega_1} d\omega_1 + f_d^T P_1 \hat{x}(t) + \hat{x}^T(t) P_1 f_d. \end{aligned} \quad (30)$$

Substituting $f_d = D^\gamma \hat{x}(t)$ into (30) yields

$$\begin{aligned} \dot{V}_1(t) &< D^\gamma \hat{x}^T(t) P_1 \hat{x}(t) + \hat{x}^T(t) P_1 D^\gamma \hat{x}(t) \\ &= \sum_{l=1}^L \sum_{j=1}^L \tilde{h}_l \tilde{h}_j^T \hat{x}^T(t) [B_{lj}^T P_1 + P_1 B_{lj} \\ &\quad + P_1 A_{ld} S^{-1} A_{ld}^T P_1 + KS] \hat{x}(t). \end{aligned} \quad (31)$$

From Theorem 2, one has

$$\dot{V}_1(t) < 0. \quad (32)$$

Then the closed-loop system (27) is quadratically stable. This proof is finished. ■

Obviously, if the matrix inequality (25) holds and $\dot{V}_1(t) < 0$, the system (6) is stable.

Remark 4 The Lyapunov function is a serviceable theoretical instrument for solving stabilization problems. However, for some complex systems, it is challenging to discover suitable Lyapunov functions, which makes the effectiveness of Lyapunov theories worse. Note that the system (27) is a complex polynomial, which will increase the computational complexity when designing Lyapunov functions directly. Therefore, FONS (27) can be transformed into a simple integer-order system (28) by the frequency distribution model, which is helpful for theoretical analysis.

Remark 5 Theorems 1 and 2 about the FO H_∞ performance can stabilize any FONSs with unmeasurable states, bounded external disturbances, and time delays. Compared with some related literature, such as [38–40], this FO H_∞ performance design can avoid the calculation of nonlinear partial differential equations, and apply to more complex FO systems. Moreover, the stabilization of the closed-loop system (27) without external perturbations can be achieved through a suitable Lyapunov function in Theorem 3.

4 Simulation Results

Next, a numerical simulation example is proposed to verify the effectiveness of the control strategy.

Considering the following FO time-delay T-S fuzzy system,

$$\begin{cases} D^\nu x(t) = \sum_{l=1}^4 \tilde{h}_l(\mu) [A_l x(t) + A_{ld} x(t - d_l) + B_l u(t) + H\omega(t)], \\ y(t) = \sum_{l=1}^4 \tilde{h}_l(\mu) [C_l x(t) + C_{ld} x(t - d_l) + J\omega(t)], \quad l = 1, 2, \dots, 4, \end{cases} \tag{33}$$

where fuzzy rules are given by

- Rule 1: IF $x_1(t)$ is $F_1^1(x_1(t))$;
 - Rule 2: IF $x_1(t)$ is $F_2^1(x_1(t))$;
 - Rule 3: IF $x_1(t)$ is $F_3^1(x_1(t))$;
 - Rule 4: IF $x_1(t)$ is $F_4^1(x_1(t))$;
- THEN,

$$\begin{cases} D^\nu x(t) = A_l x(t) + A_{ld} x(t - d_l) + B_l u(t) + H\omega(t), \\ y(t) = C_l x(t) + C_{ld} x(t - d_l) + J\omega(t), \quad l = 1, 2, \dots, 4, \end{cases} \tag{34}$$

with $n = 3, m = 3, q = 3, x = [x_1, x_2, x_3]^T$ being the state vector, $y = [y_1, y_2, y_3]^T$ being the output of the system, $u = [u_1, u_2, u_3]^T$ being the control input, F_l^1 being a fuzzy membership function. $A_l \in \mathbb{R}^{3 \times 3}, A_{ld} \in \mathbb{R}^{3 \times 3}, B_l \in \mathbb{R}^{3 \times 3}, C_l \in \mathbb{R}^{3 \times 3}, C_{ld} \in \mathbb{R}^{3 \times 3}, H \in \mathbb{R}^{3 \times 3}$, and $J \in \mathbb{R}^{3 \times 3}$, external disturbances $\omega_1(t) = 0.1 \sin 2t, \omega_2(t) = 0.1 \cos 2t, \omega_3(t) =$

$0.1 \cos 2t$, and the bounded time delay $d_l(t) = 2\cos^2 t$. In the following, $l = 1, \dots, 4$ if the range of l is not specified. To convenience the design, trigonometric membership functions are supplied in fuzzy rules, as shown in Fig. 3a.

Note that the state $x(t)$ is unmeasurable, it can be reconstructed through the fuzzy observer (10). Observer gains are found as $L_1 = 0.8, L_2 = 15, L_3 = 18, L_4 = 20$. The trajectory of $x_i(t)$ and $\hat{x}_i(t)$ is shown in Fig. 3b–d.

Next, based on Theorems 2 and 3, the stabilization of the system (34) can be achieved. First, matrices A_l and A_{ld} are chosen as

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.82 & 0.44 & -1.06 \\ -1.16 & -0.41 & 0.4 \\ -0.11 & -0.34 & -0.44 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -0.14 & -0.35 & 0.45 \\ 0.16 & -0.43 & -1.04 \\ -0.44 & -0.52 & 0.14 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 0.35 & 0.80 & -2.16 \\ -1.43 & -0.48 & -1.46 \\ 0.03 & 0.05 & -0.08 \end{bmatrix}, \\ A_4 &= \begin{bmatrix} -1.50 & -1.43 & 0.39 \\ -0.11 & -1.02 & -0.43 \\ 0.55 & 0.31 & 0.42 \end{bmatrix}. \\ A_{1d} &= \begin{bmatrix} 0.47 & 0.41 & 0.84 \\ 0.69 & 0.80 & 0.54 \\ 0.03 & 0.68 & 0.02 \end{bmatrix}, \\ A_{2d} &= \begin{bmatrix} 0.73 & 0.64 & 0.54 \\ 0.82 & 0.63 & 0.06 \\ 0.56 & 0.28 & 0.59 \end{bmatrix}, \\ A_{3d} &= \begin{bmatrix} 0.02 & 0.08 & 0.20 \\ 0.16 & 0.71 & 0.84 \\ 0.03 & 0.58 & 0.02 \end{bmatrix}, A_{4d} = \begin{bmatrix} 0.32 & 0.68 & 0.33 \\ 0.27 & 0.07 & 0.53 \\ 0.65 & 0.37 & 0.59 \end{bmatrix}. \end{aligned}$$

Matrices B_l are given as

$$\begin{aligned} B_1 &= \begin{bmatrix} 0.64 & 0.54 & 0.38 \\ 0.16 & 0.15 & 0.84 \\ 0.58 & 0.10 & 0.23 \end{bmatrix}, B_2 = \begin{bmatrix} 0.47 & 0.14 & 0.78 \\ 0.01 & 0.29 & 0.84 \\ 0.64 & 0.58 & 0.43 \end{bmatrix}, \\ B_3 &= \begin{bmatrix} 0.02 & 0.73 & 0.13 \\ 0.03 & 0.14 & 0.71 \\ 0.14 & 0.70 & 0.24 \end{bmatrix}, B_4 = \begin{bmatrix} 0.18 & 0.56 & 0.47 \\ 0.24 & 0.24 & 0.43 \\ 0.50 & 0.72 & 0.80 \end{bmatrix}. \end{aligned}$$

The values of all elements of C_l and C_{ld} are in the range of $[0, 1]$, and the matrix formed by randomly selected values is as

$$\begin{aligned}
 C_1 &= \begin{bmatrix} 0.18 & 0.28 & 0.05 \\ 0.65 & 0.46 & 0.43 \\ 0.65 & 0.07 & 0.67 \end{bmatrix}, C_2 = \begin{bmatrix} 0.83 & 0.36 & 0.06 \\ 0.02 & 0.01 & 0.69 \\ 0.46 & 0.23 & 0.21 \end{bmatrix}, \\
 C_3 &= \begin{bmatrix} 0.42 & 0.16 & 0.64 \\ 0.16 & 0.55 & 0.35 \\ 0.50 & 0.58 & 0.08 \end{bmatrix}, C_4 = \begin{bmatrix} 0.12 & 0.72 & 0.07 \\ 0.81 & 0.43 & 0.34 \\ 0.05 & 0.89 & 0.10 \end{bmatrix}. \\
 C_{1d} &= \begin{bmatrix} 0.8619 & 0.7173 & 0.2998 \\ 0.0046 & 0.7687 & 0.1599 \\ 0.6749 & 0.0844 & 0.7001 \end{bmatrix}, \\
 C_{2d} &= \begin{bmatrix} 0.3314 & 0.1638 & 0.7693 \\ 0.8106 & 0.1455 & 0.4797 \\ 0.1818 & 0.1361 & 0.4499 \end{bmatrix}, \\
 C_{3d} &= \begin{bmatrix} 0.0450 & 0.2510 & 0.0760 \\ 0.7530 & 0.4132 & 0.1399 \\ 0.5221 & 0.3018 & 0.0233 \end{bmatrix}, \\
 C_{4d} &= \begin{bmatrix} 0.1839 & 0.0497 & 0.3909 \\ 0.1400 & 0.8027 & 0.3893 \\ 0.3173 & 0.8448 & 0.2377 \end{bmatrix}.
 \end{aligned}$$

In this simulation example, the control gain K_j is shown in Table 1. Therefore, the system (34) is stable through the condition (26) in Theorem 2, which can be observed in Fig. 4a and b. Obviously, trajectories of $x(t)$ and $y(t)$ converge to zero asymptotically, which shows that the control method is effective. The time responses of $x_1(t), x_2(t), x_3(t)$ are plotted in Fig. 4a, and the time responses of $y_1(t), y_2(t), y_3(t)$ are plotted in Fig. 4b. Figure 5a shows the time response of the controller (12), which shows that the control performance is superior.

Furthermore, we compare the performance of the proposed feedback controller (12) with the controller in [28]. For the fairness of comparison, the controller in [28] also selects the same control gain k_j in Table 1. Therefore, we compare the performance of different controllers in Fig. 5, which explicitly shows that the designed feedback controller (12) has better control performance with less energy loss. Simultaneously, it is obvious that trajectories of $x(t)$ and $y(t)$ converge to zero faster by using the proposed feedback controller (12), which is shown in Fig. 4.

Note that the system (34) contains external disturbances, a closed-loop system is expressed as

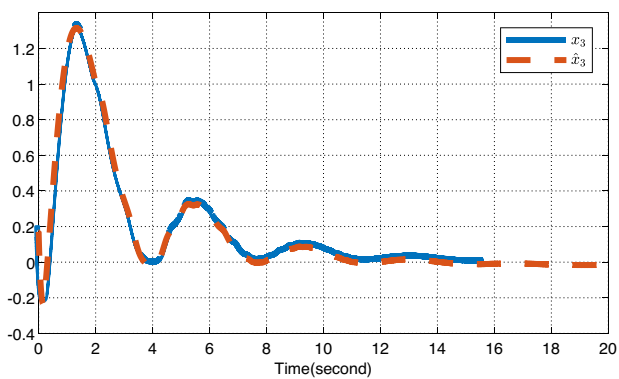
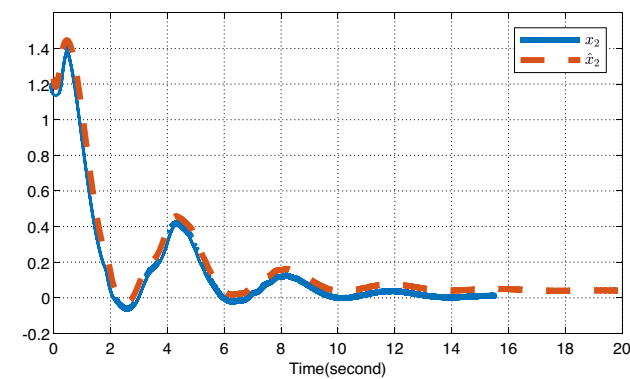
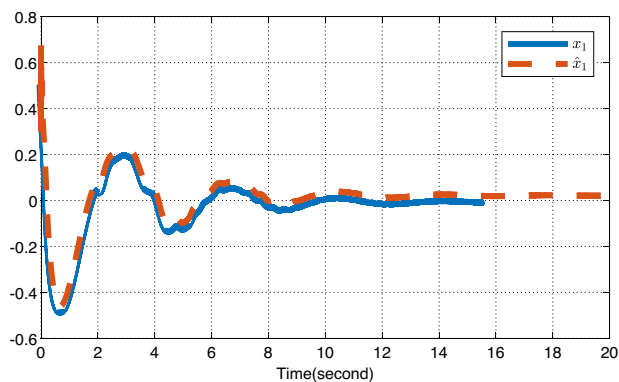
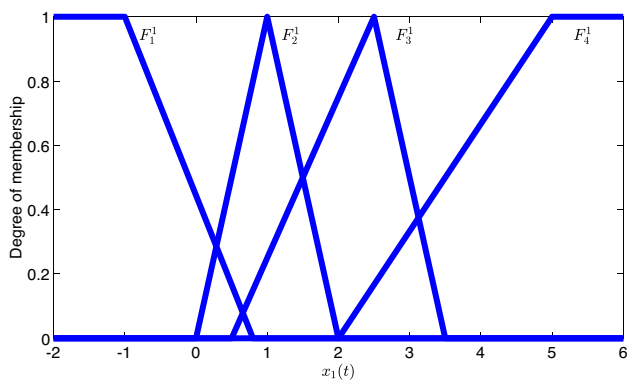
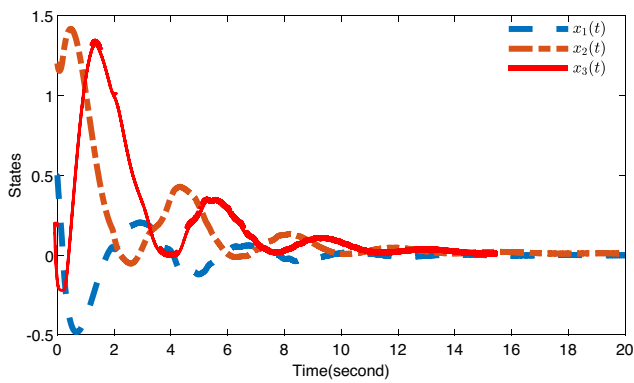


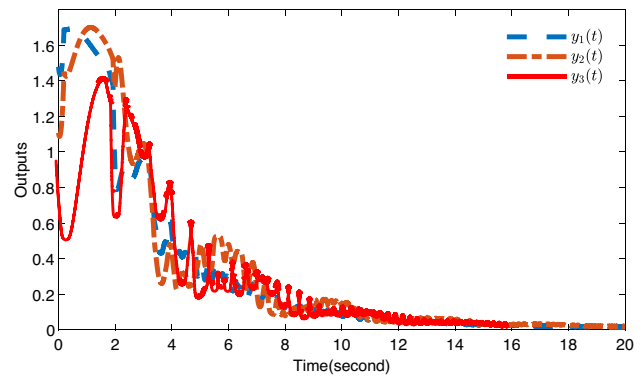
Fig. 3 Simulation results: **a** Fuzzy membership functions for $x_1(t)$; **b** The trajectory of $x_1(t)$ and $\hat{x}_1(t)$; **c** The trajectory of $x_2(t)$ and $\hat{x}_2(t)$; **d** The trajectory of $x_3(t)$ and $\hat{x}_3(t)$

Table 1 Simulation results for LMIP in [37]

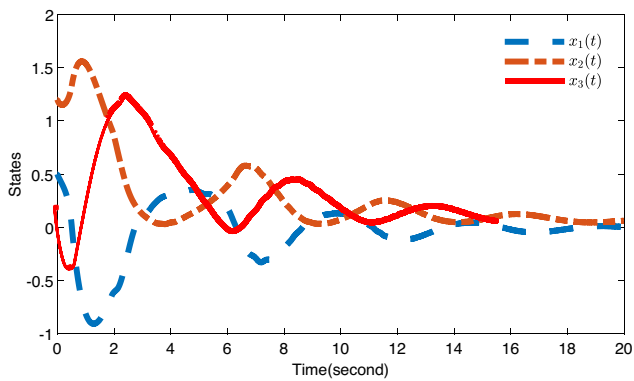
Variable	Value	Variable	Value
K_1	$\begin{bmatrix} 1.7135 & 1.8560 & 2.6011 \\ 1.8560 & 20.1034 & 2.8173 \\ 2.6011 & 2.8173 & 3.9483 \end{bmatrix}$	K_2	$\begin{bmatrix} 0.0013 & 0.0615 & 0.0493 \\ 0.0615 & 0.2569 & 0.2238 \\ 0.0493 & 0.2238 & 0.1735 \end{bmatrix}$
K_3	$\begin{bmatrix} 0.0003 & 0.0023 & 0.0062 \\ 0.0023 & 0.0001 & 0.0049 \\ 0.0062 & 0.0049 & 0.0167 \end{bmatrix}$	K_4	$\begin{bmatrix} 1.7135 & 1.8560 & 2.6011 \\ 1.8560 & 20.1034 & 2.8173 \\ 2.6011 & 2.8173 & 3.9483 \end{bmatrix}$
P_1	$\begin{bmatrix} 1.6311 & 1.7668 & 2.4760 \\ 1.7668 & 19.1366 & 2.6819 \\ 2.4760 & 2.6819 & 3.7584 \end{bmatrix}$	P_2	$\begin{bmatrix} 0.0095 & 0.0617 & 0.0495 \\ 0.0617 & 0.2738 & 0.2321 \\ 0.0495 & 0.2321 & 0.1721 \end{bmatrix}$
P_3	$\begin{bmatrix} 0.0006 & 0.0024 & 0.0067 \\ 0.0024 & 0.0034 & 0.0061 \\ 0.0067 & 0.0061 & 0.0124 \end{bmatrix}$	P_4	$\begin{bmatrix} 1.9926 & 3.9886 & 4.0611 \\ 3.9886 & 9.7662 & 1.3525 \\ 4.0611 & 1.3525 & 2.4604 \end{bmatrix}$
S_1	$\begin{bmatrix} 9.0037 & 6.6236 & 4.1293 \\ 6.6236 & 1.0511 & 1.0581 \\ 4.1293 & 1.0581 & 9.3841 \end{bmatrix}$	S_2	$\begin{bmatrix} 1.3953 & 1.4387 & 1.0452 \\ 1.4387 & 1.2999 & 6.0448 \\ 1.0452 & 6.0448 & 1.2730 \end{bmatrix}$
S_3	$\begin{bmatrix} 3.6540 & 2.1339 & 4.0927 \\ 2.1339 & 3.6624 & 1.2655 \\ 4.0927 & 1.2655 & 3.5528 \end{bmatrix}$	S_4	$\begin{bmatrix} 4.5774 & 3.2474 & 1.0618 \\ 3.2474 & 4.2465 & 4.6875 \\ 1.0618 & 4.6875 & 5.3815 \end{bmatrix}$



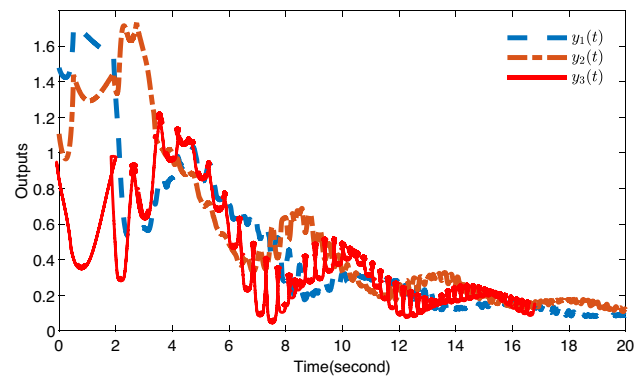
(a)



(b)



(c)



(d)

Fig. 4 Simulation results: **a** State variables $x_1(t)$, $x_2(t)$, and $x_3(t)$; **b** Output $y_1(t)$, $y_2(t)$, and $y_3(t)$; **c** State variables $x_1(t)$, $x_2(t)$, and $x_3(t)$ in [28]; **d** Output $y_1(t)$, $y_2(t)$, and $y_3(t)$ in [28]

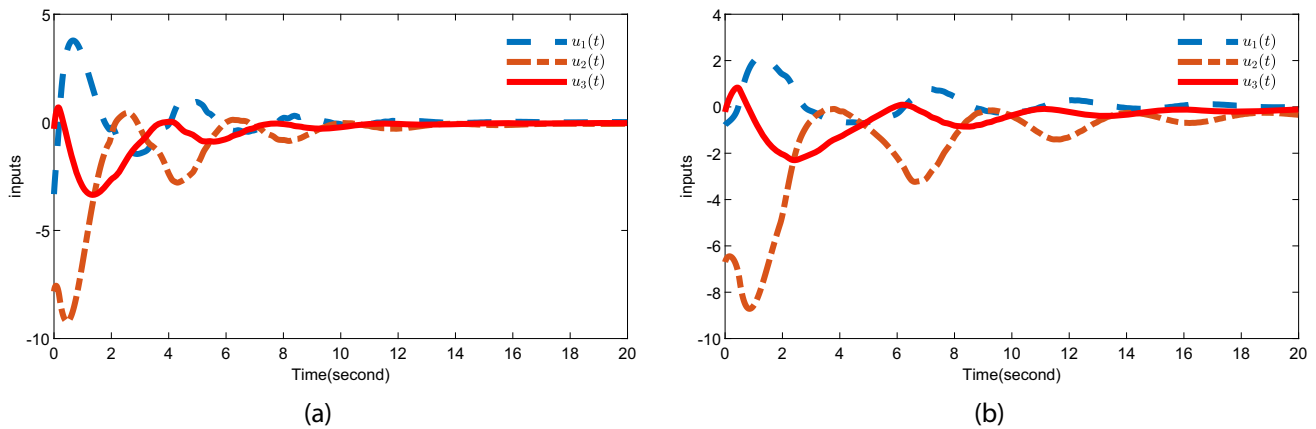


Fig. 5 Simulation results: **a** State feedback controller $u(t)$ by (12); **b** State feedback controller $u(t)$ in [28]

$$D^\gamma x(t) = A_l x(t) + A_{ld} x(t - d_l) + B_l u(t). \quad (35)$$

Based on Theorem 3, to verify that the system (35) is quadratic stable, a sufficient condition must be satisfied, which is similar to the condition (25). The sufficient condition for the stabilization analysis is that there exist two positive matrices $P \in \mathbb{R}^{3 \times 3}$ and $S \in \mathbb{R}^{3 \times 3}$ such that

$$A_l^T P + P A_l + P A_{ld} S^{-1} A_{ld}^T P + K S < 0. \quad (36)$$

The inequality in (36) is transformed to the following LMIP by the Schur complements,

$$\begin{bmatrix} A_l^T P + P A_l + K S & P A_{ld} \\ A_{ld}^T P & -S \end{bmatrix} < 0. \quad (37)$$

To solve (37), the LMI optimization instrument is used for LMIP in [41], and the result is shown in Table 1. Based on the previous experimental data, Theorem 3 can be verified. Simulation results ensure the internal stabilization of the model and its boundedness.

5 Conclusions

In this article, the stabilization analysis of FONs with time delays, unmeasurable states, and bounded external perturbations has been discussed by utilizing the T–S fuzzy method. Several sufficient stabilization conditions are proposed by introducing an FO H_∞ performance model and a frequency distribution model. It is shown that the effect of uncertain parameters and time delays is solved through a new FO H_∞ performance model. It is also proven that the frequency distribution model can use the integer-order stabilization theory to stabilize FO systems, which effectively reduces the computational complexity. Moreover, an observer is implemented to reconstruct unmeasurable states, and a fuzzy feedback controller is

established to satisfy the FO H_∞ performance and stabilize the system. The advantage of the design scheme is that the method avoids the use of feedback linearization techniques and complex adaptive schemes, which is a simple and systematic stabilization analysis algorithm. To sum up, it is an open question to consider the stabilization of FO real systems with uncertainties. Furthermore, how to establish sufficient conditions for the γ -passivity of FONs is also an important content of further research.

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Data availability Data sharing is not applicable to this article since no associated the date.

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