Int. J. Fuzzy Syst. (2023) 25(6):2250–2265
https://doi.org/10.1007/s40815-023-01497-y

Concept Representation and Trust Relationship Modeling in Fuzzy Social Networks

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Received: 16 February 2021 / Revised: 30 November 2021 / Accepted: 1 March 2023 / Published online: 11 April 2023

Abstract Social networks (SNs) are changing all aspects of people's way of life, especially their decision making and behavioral styles. Trust, as an essential and important relationship in social network analysis, has gained increasingly more focus. Furthermore, it is important to design an accurate representation and computational model for a trust-enhanced social network. To develop the practical applications of social network analysis, we compare and discuss the properties of trust in SNs and propose the main challenges to measure trust. A fuzzy context-based social network description model is proposed based on these challenges. Multigranularity linguistic variables are used in this model to describe trust relationships among agents. Trust relationship is mapped to a tuple that is named the trust score and contains two parameters: the degree and the strength of trust. We design a trust propagation operator, using t-norm and t-conorm, to estimate the trust propagation score. Then, a trust relationship model for group decision making in the new social network environment is proposed. Finally, an illustrative example of group decision making with incomplete preference information in SNs is given. We show how to use trust relationship to estimate unknown evaluations and complete group decisions in this example. The proposal can realize qualitative descriptions and quantitative measures of trust in social networks. The main differences or innovations of our trust-enhanced social network model are that we distinguish trust relationships according to context and quantify uncertainty in the trust network with the paradigm of computing with words.

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Keywords Social networks (SNs) · Context-based social network · Multigranularity linguistic set · Property of trust · Trust propagation operator

1 Introduction

People live in large social contexts, such as schools, workplaces, neighborhoods, and online communities. In addition, they form smaller groups in which they experience a higher level of communication than the rest of the social context [1]. Uncovering the community structure of a social network and modeling it are important tasks in social network analysis (SNA). SNA studies the relationships between social entities such as the members of a group, corporations or nations [2]. The phenomenon or data reflected by their relationship model are the focus of network analysis. Agents' interaction in the social environment can be expressed as a pattern or rule based on relationship. The regular pattern based on this relationship reflects the social structure, and the quantitative analysis of this structure is the starting point of SNA. The focus of SNA is relationships and the relationship model, which is conceptually different from traditional statistical analysis and data processing methods.

Trust in general is a multifaceted concept. It is subjective, dynamic and context specific [3]. Trust is defined as an entity behaving in an expected manner, despite the lack of an ability to monitor or control the environment in which it operates [4]. Trust measures have been studied in many disciplines from different perspectives. Businesses use the trust relationship in the social network environment to effectively recommend customers and increase the purchase rate of customers. An important practical application of SNA is trust-enhanced recommender systems (or trust-aware recommender systems).

Trust is a representative relationship in SNs. When the strength of a relationship is related to the concept of "trust," the social network is referred to as a trust network [5]. For example, WeChat, as a trust network, shows that users accept advice that comes from individuals they trust. A recommendation mechanism induced by both objective and subjective trust will be a more rational approach to conduct measurements. Trust modeling is a meaningful topic for users who have not been exposed to social networks to determine whether a strange user is trustworthy [6]. Levin, Cross [7] discussed implications of trust relationship for theory and practice. A social network is defined by a directed graph. An adjacent matrix can only describe whether trust relationship between each pair of decision makers exists or not in the graph. Dong et al. [8] defined a weighted adjacent matrix to describe trust strength. However, the vagueness of trust strength cannot be reflected completely. Victor et al. [9], Wu et al. [10], Gong et al. [11], Cai et al. [12] and Wu et al. [13] studied trust models that extract some effective social factors from the information in a social network. All these trust models try to interpret trust as a gradual phenomenon. The use of bilattices results for (trust, distrust)-couples is defined as trust score or trust function in [9, 10, 13]. Although the degree of trust and distrust of an agent can reflect his/her uncertainty in some degree, the hypothesis of the coexistence of trust and distrust remains to be discussed.

SNA is becoming an important technology in human behavioral modeling. We can exploit plenty of valuable information from SNA. The provision of a bridge between a social network's conceptual properties and quantitative model is the premise of future research on SNA. Although existing SNA has been developed, the foundation of the preliminary work is not very solid, which will affect the further application of quantitative models. Existing computational models have developed trust propagation methods for unlinked individuals/organizations via trusted third parties (TTPs) that have direct trust in each other [5]. Wu et al. [10] constructed a uninorm operator that propagates trust and distrust simultaneously. Victor et al. [9] introduced several bilattice-based trust models and their propagation operators. Wu et al. [2] proposed a new dual trust propagator which successfully describes the phenomenon that the distrust value increases during the propagation process. Gong et al. [11] proposed two weighted trust aggregation operators to accomplish a multitrust transitive aggregation mode. However, the difference between direct trust and indirect trust is not taken into account in trust propagation. In another aspect, the objective fact that information attenuation also exists in the process of trust transmission is ignored, which leads to inconsistency with the facts. These trust propagation methods are not effective when dealing with complex trust information, such as interval-valued trust information and linguistic trust information.

How to compute and predict trust between agents more accurately and effectively is still an open problem [14]. In this paper, we discuss how to use fuzzy graph-based approaches to quantify human trust behavior in SNs and give patterns or rules based on trust to reflect the social structure. The remainder of this paper is organized as follows. In Sect. 2, we present a brief review of multigranularity linguistic variables and trust models. In Sect. 3, we propose a fuzzy context-based social network description model. A weighted direct graph can help agents to describe their trust relationships in a visual way which is the basis of trust relationship modeling. The characteristics of SNs are fully described by setting the properties of nodes and edges in the graph. In Sect. 4, a trust relationship model is carried out to compute and predict direct or indirect trust which is a key parameter to support social network group decision making (SN-GDM). In Sect. 5, we provide a framework to trust-based decision model and our proposal is applied to solve an SN-GDM scenario in an incomplete information context. An illustrative example is given. We also compare our proposal with existing methods. In Sect. 5, the advantages and limitations are discussed.

2 Preliminaries

In order to make the paper self-contained, we review some basic concepts and operations of multigranularity linguistic variables and trust models.

2.1 Multigranularity Linguistic Variables

Some activities in the real world cannot be assessed in a quantitative form but rather in a qualitative way. In such a case, a better approach may be the use of linguistic assessments instead of numerical ones. Linguistic variables can be represented as (s_i, α), where s_i is a linguistic term and α is a numeric value representing the symbolic translation [15]. This form can be translated to a value $\beta \in [0, g]$ which is used to represent the value of linguistic 2-tuples.

The translation function Δ^{-1} and retranslation function Δ are as follows [15]:

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5), \tag{1}$$

$$\Delta(\beta) = (s_i, \alpha), \text{with} \begin{cases} s_i & i = \text{round}(\beta) \\ \alpha & = \beta - i \end{cases}, \tag{2}$$

$$\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g], \tag{3}$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta. \tag{4}$$

Different experts may have different levels of knowledge about a problem; therefore, multigranularity linguistic information can be used to express their opinions. $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set characterized by its cardinality or granularity, where $\#(S^g) = g + 1$. They use several linguistic term sets with different granularities of uncertainty [16]. If a high precision is needed, then it is possible to select a high granularity value. On the contrary, a low granularity value can be used [17].

Definition 1 [18, 19] Let $S^g = \left\{s_0^g, \cdots, s_g^g\right\}$ be a linguistic term set where $s_0^g < s_1^g < \cdots < s_g^g$. The linguistic term is represented as $s_i^g \in S^g$, where superscript $\sup(s_i^g) = g$ measures the uncertainty; subscript $sub(s_i^g) = i$ is the value of the term to measure order in the set.

Definition 2 [18, 19] Let $S^g = \left\{s_0^g, s_1^g, \cdots, s_g^g\right\}$ be a linguistic term set in hierarchical structure and R be a real number set. We define the 2-scale numerical function $2 - SNF : S^g \rightarrow R$, which is constituted by two parts: order function and vagueness function (O, V).

(O, V) should satisfy these conditions:

(1) In order to normalize the values of labels in different levels, we require $O \in [0, 1]$ and $V \in [0, 1]$;
(2) If the linguistic term A is vaguer than the term B, then $V(A) > V(B)$.

If we suppose the term set to be a symmetrical one with uniform distribution, then we can get the functions (5) and (6):

$$O(s_i^g) = i/g, \tag{5}$$

$$V(s_i^g) = 2/g, \tag{6}$$

Fusion mechanisms need to integrate assessments expressed in multigranularity linguistic variables, accommodating groups of experts with different expertise or uncertainty levels. These two parameters (O,V) to represent a multigranularity linguistic variable can remain necessary information.

Even though the linguistic approaches are appropriate to describe vague concepts associated with natural language, due to the expert's granules of knowledge, the employment of a single linguistic term might not be enough to express the expert's assessment. To avoid the situation that a selected linguistic term from a predefined set might not match the expert's opinion, the use of complex linguistic expressions instead of single linguistic terms is proposed [20]. These related methods [21–23] dealt with the problem of expert's granules of knowledge in another way. Our paper will apply the method [18, 19] to exhibit vagueness and imprecision of trust relationship.

2.2 Trust Model

A binary (crisp) relation is a mapping $R : Y \times Y \rightarrow \{0, 1\}$, i.e., if an agent has a connection with another agent, then there is a link between them. Trust networks based on social relationships are important information sources for choices or decisions based on opinions from people one knows well or with whom one shares common interests. Han et al. [24] indicated that trust and distrust are two distinct but coexisting concepts.

Definition 3 [9]. Trust value (t, d) is an element of $[0, 1]^2$, where t is called the degree of trust, and d is the degree of distrust.

A trust score space $\mathcal{BL} = ([0, 1]^2, \leq_t, \leq_k, \neg)$ consists of the set $[0, 1]^2$ of trust scores (t_i, d_i), a trust ordering $\leq_t$, a knowledge ordering $\leq_k$, and a negation $\neg$ defined by

$(t_1, d_1) \leq_t (t_2, d_2)$ iff $t_1 \leq t_1$ and $d_1 \geq d_2$;
$(t_1, d_1) \leq_k (t_2, d_2)$ iff $t_1 \leq t_1$ and $d_1 \leq d_2$;
$\neg(t_1, d_1) = (d_1, t_1)$

Wu et al. [13] provided two functions (7) and (8) to define the trust score and knowledge deficit

$$TS(t, d) = t - d \tag{7}$$

$$KD(t, d) = |1 - t - d| \tag{8}$$

Although social and economic networks generally use binary relations, binary networks do not allow us to extract complex knowledge of the relationship intensity between agents. Implicit trust plays a significant role in the overall dynamics of social networks. A fuzzy relation is defined as a mapping $R : Y \times Y \rightarrow [0, 1]$, where $\mu_R(y_i, y_j)$ denotes the degree of membership of the relationship between the pair of actors (y_i, y_j). Zadeh et al. [25] used m-ary fuzzy relations to describe an adjacency matrix. Such relations represent social relationships among m individuals when a group of m individuals is considered:

$$\mu(y_1, y_2, \ldots, y_m) = \begin{cases} 1 \, if \; y_1, y_2, \ldots, y_m \; are \; related \; to \; each \; other \\ (0, 1) \; if \; y_1, y_2, \ldots, y_m \; are \; related \; to \; each \; other \; to \; some \; extent \\ 0 \; if \; y_1, y_2, \ldots, y_m \; are \; not \; related \; to \; each, \; other \end{cases} \tag{9}$$

Genç et al. [26] proposed linguistic summary forms by considering both attributes of social and economic agents and the relations between them. The processes in polyadic quantifiers have been extended to semifuzzy cases.

If trust is used to support decision making, it is important to have an accurate estimate of trust when trust is not directly available. Victor et al. [9] defined the concept of a propagation operator and gave an example of function (10). Trust propagation is often exploited to enable a source user to estimate trust in an unknown target user based on a trust chain of users that links them together.

$$P((t_{0i}, d_{0i}), (t_{im}, d_{im})) = (\mathcal{T}(t_{0i}, t_{im}), \mathcal{T}(t_{0i}, d_{im})) \tag{10}$$

with $\mathcal{T}$ being a t-norm.

In Fig. 1, there is no direct orthopair of trust/distrust values between experts E_1 and E_3. Through the path via expert E_2, we can calculate the trust/distrust values between experts E_1 and E_3.

Kuter,Golbeck [27] described a trust inference algorithm that uses a probabilistic sampling technique to estimate our confidence in the trust information from some designated sources. The confidence of n for n' as the conditional probability $P(n|n')$ is defined as follows: Given that n conveys some information to n', the probability that n believes in the correctness of that information is $P(n|n')$.

3 Representation of a Social Network

SNA enables us to examine the structural and locational properties including prestige, centrality, trust relationship, etc. Firstly, we explain why current models are not fully suitable for the measurement of trust in a social network. Then, we construct a fuzzy context-based social network where multigranularity linguistic variables are used to describe the trust relationships among agents.

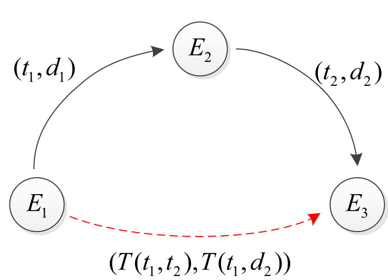

Fig. 1 An example of trust propagation

3.1 New Challenges to Measure Trust

Although people's understanding of trust relationships has been studied for a long time, the knowledge is not unified. The pattern or rule based on trust is developed based on people's understanding of trust. We face some challenges when we attempt to quantify it. We discuss them in detail.

1. Context Specific (or Context Dependence)

Trust is context specific in its scope. Sherchan et al. [3] gave an example. John, as a professional doctor, receives Mike's trust. Mike will ask John about his health. However, he does not ask John about vehicle maintenance and repair because he does not trust John as an expert in vehicle maintenance and repair. Because of this property, the trust between a pair of agents should be multidimensional. We focus on the relationship composed of multidimensional factors and how such relationship affects the behavior of network members. A single-dimensional network is not enough to describe trust relationships. Our trust model should represent the multidimension of trust between a pair of agents.

2. Asymmetry

The trust relationship between A and B is not equal. It is common for one side to trust the other side slightly more or slightly less. Hence, trust is directed and asymmetric. Yager [14] discussed the relationship in two situations, which are symmetry and asymmetry, and primarily constructed an undirected graph to represent an SN. Our model assumes that the trust network should be a directed graph.

3. Transitivity/Nontransitivity

Social networks consist of direct and indirect trust relationships (recommended trust relationships) between nodes [11]. Transitivity captures the property "friend of a friend is my friend." Therefore, most computational models of trust prediction [9, 11, 13, 14] include the property of transitivity. However, Sherchan et al. [3] stated that trust is not transitive. If Mary trusts Jack and Jack trusts Jim, we cannot conclude that Mary trusts Jim. Whether the trust model is based on transitivity or nontransitivity is a key problem. However, there is no consensus yet.3

4. Propagation

Although there is no consensus on the issue of transitivity/nontransitivity, people widely admit that through an indirect chain of TTPs, trust can be propagated to an unknown person. A trust propagation chain may involve more than three agents. There are often more than two trust propagation chains from A to B. Designing a reasonable operator to calculate the degree of trust from A to B and giving an explanation of the real environment will be difficult.

5. Subjective

Trust is not a crisp and complete relation. Gong et al. [11] pointed out that trust relationships are characterized by subjective uncertainty and are difficult to quantify accurately. This property makes it difficult to quantify the trust network. Fuzzy set theory has the ability to model and analyze imprecise relations and connections between individuals or groups [25]. Sherchan et al. [3] illustrated the importance of combining computing with words to model SNs. A linguistic term set may not be sufficient to represent such subjectivity because a trust network is a multiagent network. The information that the agents provide is more related to their own opinions and feelings than to specific and measurable facts and objects. Modeling social relationships in GDM by integrating and exploiting relationship information, e.g., trust between agents, is facing major challenges inherent to GDM problems [20]. Multigranular fuzzy linguistic modeling methods make information transformed and presented in an organized way [17]. Therefore, we combine multigranular fuzzy linguistic modeling methods to complete the representation and measure of trust.

In the following sections, we construct a representation model that is a fuzzy direct graph, where linguistic variables such as strong or very strong are applied to quantify the degree of trust of the arc in the graph. Context specificity can be reflected by the multidimensions of trust of an arc.

3.2 Fuzzy Context-Based Social Network Description Method

Given a weighted direct graph $G = \{V, E\}$, let $V = \{v_1, v_2, \cdots, v_n\}$ be a set of nodes and each of the nodes has an associated vector of attribute (feature) values. Node $v_i \in V$ represents an agent in an SN. c_{ik} is the value of attribute C_k according to agent v_i. The vector of attributes $C_i = \{c_{i1}, c_{i2}, \cdots, c_{im}\}$ shows the diversity of a person's characters. Therefore, an SN is also diverse. Attribute matrix $C = [c_{ik}]_{n\times m}$ is important to help understand the multidimensional property of social networks. SNA is based on a source of multidimensional information, which has the property of being context specific. This property is reflected in the structure of SNs. In fact, as the attribute space varies, the trust relationship also varies. We name G a fuzzy context-based social network.

E is a set of arcs. $R(v_i, v_j)$ can be seen as defining the weight on arcs (v_i, v_j). A fuzzy relationship on $V \times V$ is in the form of a fuzzy multigranularity linguistic subset, where $R(v_i, v_j)$ indicates the degree of trust from v_i to v_j. Different agents may have different confidence or preferences to describe their degrees of trust. Therefore, it is reasonable to map the relationship on $V \times V$ to a fuzzy multigranularity linguistic subset.

Social networks consist of direct and indirect relationships between nodes. A direct relationship from v_i to v_j means there is an arc $(v_i, v_j) \in E$ in G. For each node v_i, let $NG_j = \{v_j : <v_i, v_j>\}$ represent the set of nodes neighboring v_j, which has an arc (v_i, v_j).

An indirect relationship from v_i to v_j means there is no arc $(v_i, v_j) \in E$, but we can find a chain from v_i to v_j. The definition of a relationship chain is as follows.

Definition 4 For two agents v_i and v_j, if there is a path $v_i \rightarrow v_{\sigma(1)} \rightarrow v_{\sigma(2)} \cdots \rightarrow v_{\sigma(q)} \rightarrow v_j$, where $v_{\sigma(k)} \in V(k = 1, 2, \cdots, q)$, then the path from individual v_i to v_j is reachable. It is denoted as a relationship chain $v_i \Rightarrow v_j$

If two nonadjacent nodes do not have direct interaction experience, then there is no arc among them. Some research assumes that trust is transitive and supposes there is recommended trust between two nonadjacent nodes if there is a trust propagation chain between them. Recommended trust means that the trust relationship between the two nonadjacent nodes can be obtained through a chain connecting them. Regardless of whether trust is transitive or nontransitive, the property of propagation is accepted anyway. Therefore, the definition of a relationship chain is in fact an interaction chain that shows the interactions among the members in the chain. Whether it is a trust propagation chain depends on another property of SNs, which is context specific.

We believe the trust between a pair of agents should be multidimensional. In addition, we use the attribute matrix $C = [c_{ik}]_{n\times m}$ to help us understand the multidimensional property of the trust network. In path $v_i \rightarrow v_{\sigma(1)} \rightarrow v_{\sigma(2)} \cdots \rightarrow v_{\sigma(q)} \rightarrow v_j$, if the trust from v_i to $v_{\sigma(1)}$ and the trust from $v_{\sigma(1)}$ to $v_{\sigma(2)}$ are in different dimensions, the trust relationship cannot be transitive. Some existing research assumes that these trust relationships are in the same dimension. This assumption makes modeling easier but also deviates from the understanding of the essence of social networks. Take the example in Sect. 3.1 again. Mike and John have interactions in the dimension of health consulting, so there is an arc from Mike to John in this dimension. However, Mike and John have no interactions in the dimension of mechanical consulting, so there is no arc from Mike to John in that dimension. One day John introduced his colleague Alice to Mike. Mike cannot transmit trust to Alice in the dimension of mechanical consulting. Therefore, the fuzzy relationship $R(v_i, v_j)$ on $V \times V$ is extended to $R^{C_k}(v_i, v_j)$, where v_i's trust to v_j is based on the same attribute C_k. In the above example, the same occupation makes the trust transitive from Mike to Alice. The attribute $C_k = occupation$ and $c_{Alice,k} = c_{John,k} = doctor$. In other words, Mike trusts Alice in the dimension of health consulting.

3.3 Trust Score

For a pair of agents v_i and v_j, $R(v_i, v_j)$ is quantified by the trust score λ_{ij}^c. λ_{ij}^c is defined as a trust score from v_i to v_j according to context c. Because humans do not merely reason in terms of "trusting" and "not trusting", but rather trusting someone "very much" or "more or less" [9]. We apply linguistic variables in trust score. However, an SN is a multiagent network, and different preferences of agents make multigranularity linguistic variables better. We adopt the computational model of [19, 28]. $\wedge(\lambda_{ij}^c) \to (O, V)$, in which O is the degree of trust, and V is the uncertainty of trust (the confidence of the agent). The trust score will be denoted by

$$\wedge(\lambda_{ij}^c) = \{(O(\lambda_{ij}^c), V(\lambda_{ij}^c))|O(\lambda_{ij}^c), V(\lambda_{ij}^c) \in [0,1]\} \equiv [0,1]^2 \tag{11}$$

Given two trust scores, λ_1 and λ_2, the comparison rules are as follows:

1. If $O(\lambda_1) < O(\lambda_2)$, then we obtain $\lambda_1 \prec \lambda_2$;
2. If $O(\lambda_1) = O(\lambda_2)$ and $V(\lambda_1) > V(\lambda_2)$, then we obtain $\lambda_1 \prec \lambda_2$;
3. If $O(\lambda_1) = O(\lambda_2) \text{and} V(\lambda_1) = V(\lambda_2)$ then we obtain $\lambda_1 \sim \lambda_2$.

Comparison rules are based on the assumption that the value of the degree of trust is the primary factor determining the rank of the trust score. When the value of the degree of trust is the same, we prefer the trust score with a smaller value of V because a smaller value of V means more confidence in the value of the degree of trust.

Here, we set two special values of λ_{ij}^c, which are named absolute trust and absolute distrust. Absolute trust is represented as $\Lambda(\lambda_{ij}^c) = (1, 0)$, which means that the degree of trust is the largest value and the confidence is complete. Absolute distrust is represented as $\Lambda(\lambda_{ij}^c) = (0, 0)$, which means that these two agents have no interactions, and the value reflecting uncertainty is the largest.

We define $(\lambda_{ij}^c)_\alpha$ as the α—cut set trust score, which means $V\left(\lambda_{ij}^c\right) \le \alpha$. $(\lambda_{ij}^c)_\alpha$ can help us eliminate trust relationships that do not have much credibility. By setting the value of α, we can modify an SN's strength. The lower the value of α is, the stronger the SN's tie. We can cut the arcs whose $V\left(\lambda_{ij}^c\right) > \alpha$ to ensure that the remaining arcs are reliable to a certain extent.

4 Trust Relationship Modeling

In this section, we design a trust propagation operator based on the trust score defined in Sect. 3.3 firstly. Then, we propose a weight identification method that is useful for decisions of social network environment.

4.1 Trust Propagation Operator

Definition 4 defines the concept of a relationship chain. However, the relationship chain cannot improve the transitivity of trust. Because we consider trust to be context specific. Only when the interactions among the three agents are in the same attribute dimension, trust can be transmitted. A relationship chain is transformed to a trust propagation chain. The definition is as follows.

Definition 5 For two individuals v_i and v_j, according to a specific context c, if there is a path $v_i \xrightarrow{c} v_{\sigma(1)} \xrightarrow{c} v_{\sigma(2)} \cdots \xrightarrow{c} v_{\sigma(q)} \xrightarrow{c} v_j$, where $v_{\sigma(k)} \in V(k = 1, 2, \cdots, q)$ and $\xrightarrow{c}$ represents an arc according to context c, then the path from individual v_i to v_j is reachable. It is denoted as a trust propagation chain according to context c: $v_i \overset{c}{\Rightarrow} v_j$.

Trust propagation operator $P\left(\lambda_{ij}^c, \lambda_{jk}^c\right)$ is used to obtain trust score λ_{ik}^c, where there is a path $v_i \to v_j \to v_k$. First, we introduce the properties of a trust propagation operator $P\left(\lambda_{ij}^c, \lambda_{jk}^c\right)$.

1. Completely transitive: If agent v_j fully trusts agent v_k, then the trust relationship from v_i to v_j is completely transmitted to v_k. In real life, if a friend whom you fully trust tells you to trust someone and you have no other information about this person, you will likely choose to trust him. In other words, P is used to denote an operator for trust score propagation in this situation: $P\left(\lambda_{ij}^c, \lambda_{jk}^c\right) = \lambda_{jk}^c$.
2. Completely nontransitive (or trust block): If agent v_i fully distrusts agent v_j, then the trust relationship from v_j to v_k is blocked. In real life, if a friend whom you fully trust tells you to distrust someone and you have no other information about this person, you likely will choose to distrust him. P is used to denote an operator for trust score propagation in this situation: $\Lambda(P\left(\lambda_{ij}^c, \lambda_{jk}^c\right)) = (0, V\left(\lambda_{jk}^c\right))$
3. Associativity: $P\left(\lambda_{ij}^c, P\left(\lambda_{jk}^c, \lambda_{kl}^c\right)\right) \sim P\left(P\left(\lambda_{ij}^c, \lambda_{jk}^c\right), \lambda_{kl}^c\right)$. In path $v_i \to v_j \to v_k \to v_l$, the subsequence of propagation will not affect the final trust value from v_i to v_l.

4. Monotonicity: If the members in a chain trust each other more than those in another chain, the result of propagation will be larger. $P\left(\lambda_{ij}^c, \lambda_{jk}^c\right) \succ P\left(\lambda_{i'j'}^c, \lambda_{j'k'}^c\right)$ if $\lambda_{ij}^c \succ \lambda_{i'j'}^c$ and $\lambda_{jk}^c \succ \lambda_{j'k'}^c$.

Then, we design a propagation operator P that satisfies the above properties.

Definition 6 The trust propagation operator $P\left(\lambda_{ij}^c, \lambda_{jk}^c\right)$ associates two trust scores $\Lambda(\lambda_{ij}^c) = \left(O(\lambda_{ij}^c), V(\lambda_{ij}^c)\right)$, $\Lambda(\lambda_{jk}^c) = \left(O(\lambda_{jk}^c), V(\lambda_{jk}^c)\right)$ with the following trust score output:

$$\Lambda(P\left(\lambda_{ij}^c, \lambda_{jk}^c\right)) = \left(\mathcal{T}_p\left(O(\lambda_{ij}^c), O(\lambda_{jk}^c)\right), \mathcal{S}(V\left(\lambda_{ij}^c\right), V\left(\lambda_{jk}^c\right))\right) \tag{12}$$

where $\mathcal{T}_p$ is the product t-norm function $\mathcal{T}_p\left(O(\lambda_{ij}^c), O(\lambda_{jk}^c)\right) = O(\lambda_{ij}^c) \times O(\lambda_{jk}^c)$, and $\mathcal{S}$ is the t-conorm function $\mathcal{S}(V\left(\lambda_{ij}^c\right), V\left(\lambda_{jk}^c\right)) = Max(V\left(\lambda_{ij}^c\right), V\left(\lambda_{jk}^c\right))$. P has two neutral elements $(1, 0)$ and $(0, 1)$

Functions $\mathcal{T}_P : [0,1]^2 \rightarrow [0,1]$ and $\mathcal{S} : [0,1]^2 \rightarrow [0,1]$ are arbitrary associative, commutative functions having a neutral element $e = 1 (e = 0)$, which is increasing in each of its arguments [29]. Now, we prove that P can satisfy the properties of being completely transitive, trust block, associative, and monotonic.

1. Completely transitive:

Proof Agent v_i fully trusts agent v_j means absolute trust and $\lambda_{ij}^c = (1, 0)$

$$\begin{aligned}\Lambda(P\left(\lambda_{ij}^c, \lambda_{jk}^c\right)) &= \left(\mathcal{T}_p\left(1, O(\lambda_{jk}^c)\right), \mathcal{S}(0, V\left(\lambda_{jk}^c\right))\right) \\ &= \left(O(\lambda_{jk}^c), V\left(\lambda_{jk}^c\right)\right) = \lambda_{jk}^c\end{aligned}$$

2. Completely nontransitive trust block

Proof Agent v_i fully distrusts agent v_j means absolute distrust and $\lambda_{ij}^c = (0, 0)$

$$\begin{aligned}\Lambda(P\left(\lambda_{ij}^c, \lambda_{jk}^c\right)) &= \left(\mathcal{T}_p\left(0, O(\lambda_{jk}^c)\right), \mathcal{S}(0, V\left(\lambda_{jk}^c\right))\right) \\ &= (0, V\left(\lambda_{jk}^c\right))\end{aligned}$$

3. Associativity:

Proof
$$\begin{aligned}\Lambda\left(P\left(\lambda_{ij}^c, P\left(\lambda_{jk}^c, \lambda_{kl}^c\right)\right)\right) &= \left(O\left(\lambda_{ij}^c\right) \times O\left(\lambda_{jk}^c\right) \times O(\lambda_{kl}^c), \mathrm{Max}\left(V\left(\lambda_{ij}^c\right), \mathrm{Max}\left(V\left(\lambda_{jk}^c\right), V(\lambda_{kl}^c)\right)\right)\right) \\ &= \Lambda\left(P\left(P\left(\lambda_{ij}^c, \lambda_{jk}^c\right), \lambda_{kl}^c\right)\right)\end{aligned}$$

4. Monotonicity:

Proof
$$\lambda_{ij}^c > \lambda_{i'j'}^c \Rightarrow \left\{\begin{matrix} O\left(\lambda_{ij}^c\right) > O\left(\lambda_{i'j'}^c\right) \\ O\left(\lambda_{ij}^c\right) = O\left(\lambda_{i'j'}^c\right) \text{ and } V\left(\lambda_{ij}^c\right) < V\left(\lambda_{i'j'}^c\right) \end{matrix}\right\}$$
$$\lambda_{jk}^c > \lambda_{j'k'}^c \Rightarrow \left\{\begin{matrix} O\left(\lambda_{jk}^c\right) > O\left(\lambda_{j'k'}^c\right) \\ O\left(\lambda_{jk}^c\right) = O\left(\lambda_{j'k'}^c\right) \text{ and } V\left(\lambda_{jk}^c\right) < V\left(\lambda_{j'k'}^c\right) \end{matrix}\right\}$$
$$\Rightarrow \left.\begin{matrix} O\left(\lambda_{ij}^c\right) > O\left(\lambda_{i'j'}^c\right) \\ O\left(\lambda_{ij}^c\right) \times O\left(\lambda_{jk}^c\right) = O\left(\lambda_{i'j'}^c\right) \times O\left(\lambda_{j'k'}^c\right) \text{ and } V\left(\lambda_{ij}^c\right) \times V\left(\lambda_{jk}^c\right) < V\left(\lambda_{i'j'}^c\right) \times V\left(\lambda_{j'k'}^c\right) \end{matrix}\right\}$$
$$\Rightarrow P\left(\lambda_{ij}^c, \lambda_{jk}^c\right) > P\left(\lambda_{i'j'}^c, \lambda_{j'k'}^c\right)$$

Now, let us discuss the problem that there is more than one path from v_i to v_j. Let p be the number of trust propagation chains from agent v_i to v_j. We denote $\rho_l(v_i \rightarrow v_j)$ as the lth chain in G, from agent v_i toward v_j. Figure 2 shows a parallel network of p trust propagation chains between two agents, where each chain consists of at least one node. These chains are parallel, and we can calculate $\lambda_{ij}^c(\rho_l)$ according to path $\rho_l(v_i \rightarrow v_j)$.

However, when there is more than one path from v_i to v_j, we can obtain different trust scores via the trust propagation chain. How should the trust score be calculated? Mui et al. [30] gave two possible methods: additive and multiplicative. The form of an additive estimate for λ_{ij}^c is

$$\lambda_{ij}^c = \sum_{l=1}^{p} \lambda_{ij}^c(\rho_l)/p \tag{13}$$

Function (13) combines the parallel information about λ_{ij}^c and may be one option. However, we propose a new method to estimate the trust score considering the propagation chain's "reliability." In the parallel network of p chains from v_i to v_j, some chains are tied with strong connections, and some chains are tied with weak connections. A stronger tie means this chain is more reliable.

An important concept in trust propagation chain analysis is the strength of the path $v_i \xrightarrow{c} v_{\sigma(1)} \xrightarrow{c} v_{\sigma(2)} \cdots \xrightarrow{c} v_{\sigma(q)} \xrightarrow{c} v_j$. The function to calculate the strength of the path is proposed by [31]. We modified the strength function in [31] to suit our trust score function. The strength $ST(\rho_l)$ of path ρ_l is defined as

$$ST(\rho_l) = O(\lambda_{i\sigma(1)}^c) \times O(\lambda_{\sigma(1)\sigma(2)}^c) \times \cdots \times O\left(\lambda_{\sigma(q)j}^c\right) \tag{14}$$

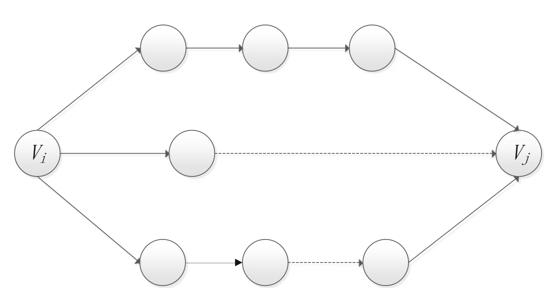

Fig. 2 Illustration of parallel paths from agent v_i to v_j

We can say that the strength of path ρ_l is determined by the lowest degree of trust of the pair in the chain. Two nodes for which there is a path ρ with $ST(\rho) > 0$ between them are called connected. In other words,v_i and v_j are connected according to the lowest degree of trust.

We select the path ρ^* whose strength is $\max_{l=1top} ST(\rho_l)$ as the strongest path that provides the strongest connection between two nodes. The propagation trust value of path ρ^* is most reliable as the trust score from v_i to v_j.

However, when there is a direct arc from v_i to v_j, we prioritize the direct arc. After all, the more people that participate in this trust propagation chain, the greater the uncertainty of this chain, and the value of the degree of trust is not as reliable. The transmission of uncertainty in a group is more complex and needs further research. Therefore, we prefer a direct arc.

We conclude the following situations.

1. If v_i has no knowledge about v_j, there is no path in G and $\lambda_{ij}^c = (0, 1)$.
2. If v_i has no direct arc with v_j but it has at least a propagation path to v_j, we set $\lambda_{ij}^c = \lambda_{ij}^c(\rho^*)$.
3. If v_i connects directly with v_j and there are other indirect paths to v_j, we prefer the trust score of the direct arc, not the propagation trust score from an indirect path.

4.2 Weight Identification in Social Network Group Decision Making

As a weighted directed graph, a node's centrality is an important parameter. The idea of what kind of power an individual or an organization has in its social network, or what kind of central position it occupies, is one of the earliest contents discussed by network analysis. The centrality of an individual measures the degree to which the individual is in the center of the network, reflecting the importance of the node in the network. In a trust-enhanced social network, it represents how much trust he/she received from other agents in the SN. More important he/she is, much trust he/she receives. The centrality of a node is closely related to its importance in the network [14]. The measure of the centrality of node v_j is the aggregated trust of the nodes connected to it by arcs. The definition of the weight of node v_j, which is also the centrality.

$$w(v_j) = \sum_{v_i \in NG_j} O\left(\lambda_{ij}^c\right) \tag{15}$$

The previous t-norm and uninorm-based trust propagation operators treat trust values with different knowledge deficits equally [5]. We extend functions (14) to (15) by combining the uncertainty of the trust to increase the reliability of the degree of trust. In other words, one agent receiving a high degree of trust from another agent may not increase the weight because this knowledge is not reliable. We use a level-set representation and obtain an $\alpha-$ cut weight measure considering the uncertainty of agents. Here, $\alpha-$ cut weight can be expressed as

$$w_\alpha(v_j) = \sum_{v_i \in NG_j} O(\left(\lambda_{ij}^c\right)_\alpha) \tag{16}$$

$w_\alpha(v_j)$ is the aggregation trust of nodes connected to v_j with an uncertainty of at least α.

Currently, information is produced and processed by many people, such as trust-enhanced recommender systems, and trust is used to support decision making. SN-GDM is very popular and is particularly relevant to decision contexts involving historical interconnections between individuals within a group. A key issue in SN-GDM problems is how to aggregate individual preferences into a collective one to derive a final solution. Let an SN contain a set of agents $V = \{v_1, v_2, \cdots, v_n\}$ and the opinion set $A = \{a_1, a_2, \cdots, a_n\}$. Based on their historical interactions, we construct an SN. When aggregating individual opinions, the weight of a node is closely related to its centrality.

Then, the collective opinion A^c is $F(a_j, w_\alpha(v_j))$, where $F(\cdot)$ is an aggregation operator. Here, we utilize a weighted average operator, and the form of collective opinion is as follows.

$$A^c = \sum_{j=1}^{n} w_\alpha(v_j) \cdot a_j \tag{17}$$

4.3 Discussion

This paper constructs a novel trust relationship model and applies the model to SN-GDM based on properties of trusted relationship which is closer to human cognition. People's cognition of trust relationship is the basis of decision making model. Therefore, the assumptions of the properties of trust relationship are very important. Therefore, we give the comparisons of the proposal and the existing references from two aspects. One is from the understanding of trust relationship, and the other is from the measuring and computing of trust relationship in an SN. The detailed comparisons are provided in Tables 1 and 2.

The explanations of the differences of our trust-enhanced SN are as follows.

1. Multidimensional Feature of a Social Network

Trust can increase or decrease with time. This is the property of being dynamic which is ignored by existing references. However, our model seeks to describe the change not with time but with context. Few studies have been carried out in this direction. This improvement is very important. The multidimensional feature of social networks can explain the contradiction of transitive and nontransitive networks.

We assume that agent A can trust agent B to some degree in one context and can distrust agent B to some degree in another context in an SN. However, these two kinds of relationships cannot exist at the same time. This feature strongly illustrates the multidimensional feature of social networks. Because of the multidimensional features of social networks, people simply see that one person shows trust and distrust in another at the same time. However, they do not realize that trust and distrust in fact exist in different dimensions.

2. Uncertainty in a Trust Network

Victor et al. [9] defined a knowledge deficit $KD(t, d)$ to evaluate the degree of uncertainty of the trust function. $KD(t, d)$ is similar to our function O. Victor et al. [9] thought $KD(\lambda) = 0$ (i.e., $t + d = 1$) means perfect knowledge; otherwise, there is uncertainty in the knowledge of trust. However, in our model, we think that trust and distrust cannot exist at the same time. Therefore, we do not agree with the definition of the trust function, which uses a tuple $\lambda = (t, d)$.

The ability of fuzzy sets to represent the degree of relation between individuals changes the depth of the analysis and provides new more realistic results [25]. In such a case, a binary fuzzy relation can be perceived as a generalization of a binary relation R_b. Its membership function is:

$$\mu(R_b) = \mu_b : A \times A \rightarrow [0, 1] \tag{18}$$

Compared with crisp values used to describe absolute trust or distrust, fuzzy sets provide great improvements.

$$R_b : A \times A \rightarrow \{0, 1\} \tag{19}$$

Humans usually employ words in most of their computing and reasoning processes without the necessity of any precise number [34]. We apply the words and propositions drawn from natural language to emulate human trust relationships and describe uncertainty in trust networks.

$$\mu(R_b) = \mu_b : A \times A \rightarrow S^g. \tag{20}$$

5 An Application of Trust Relationship Model

In this section, we provide a framework of a trust-based decision model to SN-GDM scenario in an incomplete information context. An illustrative example is given to illustrate the proposed method. We discuss and compare several trust-based decision methodologies with the proposal.

5.1 A Framework to SN-GDM with Incomplete Preferences

In a real group decision process, there is often the problem of missing preference values. We develop a trust-enhanced social network for SN-GDM with incomplete preferences. A key issue that needs to be addressed in this type of decision making environment is to estimate unknown preference values [13]. One agent can use other agents' knowledge to estimate the unknown preference values in his/her personal decision matrix. We can use the trust relationship model to estimate agents' unknown preferences in this problem. We need to complete two tasks: (1) to estimate the unknown preference values and (2) to aggregate agents' preferences.

The application of trust relationship model for incomplete SN-GDM consists of the following five steps: (1) computing trust degrees; (2) collecting preference; (3) estimating unknown preference values. (4) aggregating preferences; and (5) ranking alternatives. A framework is shown in Fig. 3.

5.2 An Illustrative Example

In this subsection, we give an illustrative example of decision making with incomplete preference information. Let $I = \{I_1, I_2, I_3\}$ be a set of items needed to be recommended. There are a set of agents $V = \{v_1, v_2, \cdots, v_6\}$ and a set of criteria to be considered $C = \{c_1, c_2, c_3\}$. c_1 is outward appearance. c_2 is intrinsic performance. c_3 is product upgrade in the future. The evaluation information is $a_{ijk} \in [0, 10]$, which is the evaluation of I_i according to criteria c_k byv_j. The higher the value is, the higher the evaluation. These agents construct an SN. We obtain the trust relationship matrix $R = \left[r_{ij}\right]_{6\times 6}$, where $r_{ij} = R(v_i, v_j)$. We select one label from a constructed ordered linguistic term set $S^g = \left\{s_0^g, \cdots, s_g^g\right\}$ to describe trust relationship.

Three agents supply evaluation matrix in Table 3.

We assume that the trust relationship $R(v_i, v_j)$ on $V \times V$ is context specific. In other words, their trusts vary according to criteria. For example, someone is the expert in designing products and he may be trustable in the criterion of intrinsic performance, while another one is the expert in fashion and he may be trustable in the criterion of outward appearance. $R^{C_k}(v_i, v_j)$ is the trust from v_i to v_j according to criterion C_k. $R^{C_k}(v_i, v_j)$ is in the form of multigranularity linguistic variables. We can obtain three trust relationship matrices and construct SNs accordingly. We take the trust relationship based on attribute C_1 as an example (see Table 4). We present the process of using the trust relationship to estimate the unknown preference values in Table 3.

We construct an SN (see Fig. 4.) according to the information in Table 4.

Step 1. Computing the trust score.
We adopt the computational model of multigranularity linguistic variables to obtain $\Lambda(\lambda_{ij}^{c}) \rightarrow (O, V)$ (Table 5).

Table 1 Comparisons of the proposal and the existing trust relationship modeling

Trust relationship models	Properties of trust relationship				Representation of trust score	Propagation operator	Comparison rules
	Asymmetry	Transitivity	Propagation	Subjective			
Bilattice-based trust model [32]	✔	✔	✔	✔	Numerical values between the interval [0,1]	T-norm to propagate trust and t-conorm to propagate distrust	Not embody the idea of propagation
Gradual trust and distrust model [9]	✔	✔	✔	✔	Trust score space	Use standard negator and product t-norm at same time	Propagate both trust and distrust at the same time
A Consensus model based on the construction and propagation of trust/distrust relationships [33]	✔	✔	✔	✔	Weighted average of degree of consistency and degree of inconsistency	Spread trust and distrust at the same time	Calculate the trust function in the hesitant fuzzy cases
A visual interaction consensus model [2]	✔	✔	✔	✔	Numerical values between the interval [0,1]	Einstein sum operator to propagate trust and the Einstein product operator to propagate distrust	Distrust increases in the process of propagating
The proposal	✔	✔	✔	✔	Multigranularity linguistic variables	Order and vagueness of trust score are propagated at the same time	Order and vagueness of trust score are considered at the same time

Table 2 Comparisons of the proposal and the existing SN-GDMs

SN-GDMs	Context specific	Properties of propagation in trust chain				Propagation in multitrust chain	Weight identification
		Completely transitive	Trust block	Associative	Monotonic		
Victor et al. [32]	✔	✗	✗	✔	✗	Only single path is considered	The knowledge awarding averaging trust score aggregation operator associated with knowledge reward
Wu et al. [10]	✔	✔	✔	✔	✔	The shortest path should be selected	Using basic unit-interval monotone membership function
Wu et al. [2]	✔	✔	✔	✔	✔	The shortest path should be selected	Determining the importance score of the experts by using the trust/distrust relationships matrix
Pei et al. [33]	✔	✔	✔	✔	✔	The shortest path should be selected	Using OWA-based procedure and trust function values
The proposal	✔	✔	✔	✔	✔	The strongest path should be selected	The centrality which represents how much trust an individual received from other agents in the SN

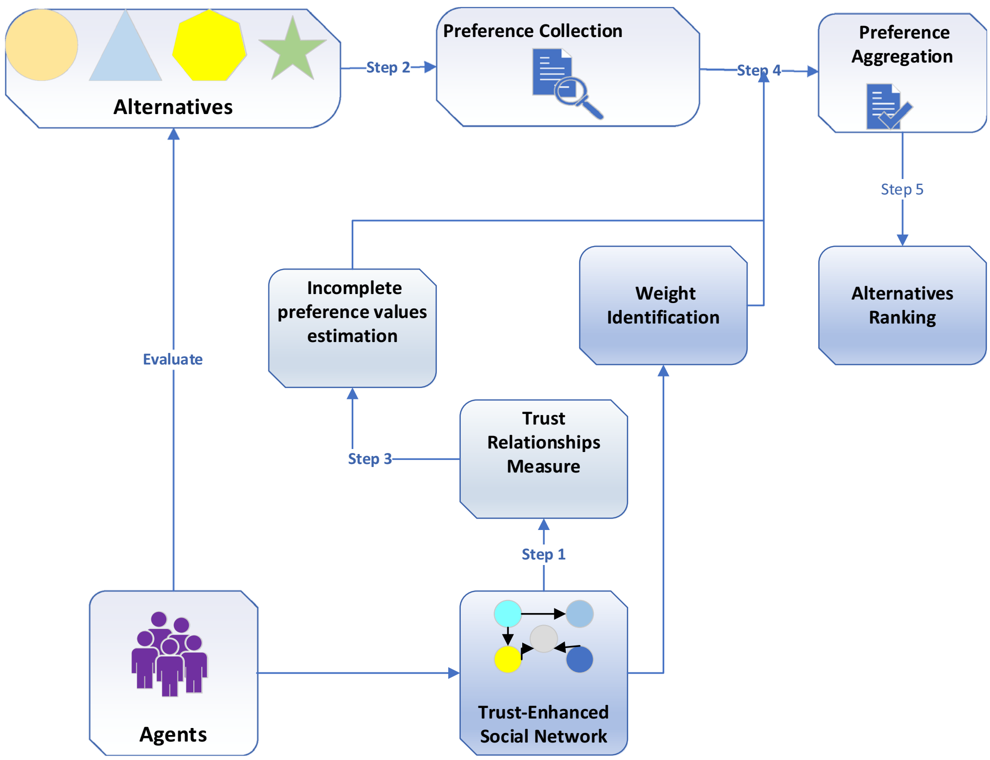

Fig. 3 A framework to SN-GDM problem with incomplete preferences

Table 3 Evaluation matrices for items

Item/criterion	c_1	c_2	c_3
v_1			
I_1	–	7	9
I_2	7	–	9
I_3	–	6	–
v_2			
I_1	6	8	7
I_2	8	6	5
I_3	7	9	8
v_3			
I_1	7	6	5
I_2	6	8	9
I_3	5	4	5
v_4			
I_1	7	6	5
I_2	5	7	6
I_3	8	8	7
v_5			
I_1	7	8	8
I_2	6	7	6
I_3	8	8	5
v_6			
I_1	8	7	8
I_2	6	6	6
I_3	4	8	7

Step 2 Estimating unknown preference values.
The trust score of a particular agent can be used to predict the trust-enhanced evaluation matrices of the other agents in a trust propagation chain when the evaluation matrix is incomplete. In Table 3, a_{111} and a_{311} are unknown. Therefore, we need to use related agents' trust to predict these two values.v_1's evaluations are unknown.v_1 has arcs or trust propagation chains from v_1 to v_3,v_4, v_5, and v_6 that show that there are trust relationships from v_1 to them. We predict p_{111} and p_{311} by aggregating their evaluations. We assume that the agent with a high trust degree from v_1 will be assigned a high weight. Therefore, we set weight of v_j as

$$w_j = O((\Lambda\left(\lambda_{1j}^{c}\right))/\sum_{j=3}^{6} O((\Lambda\left(\lambda_{1j}^{c}\right)) \tag{21}$$

First, we calculate the trust scores through the trust propagation chains from v_1. There are two trust scores ($\lambda_{14}^{C_1}$ and $\lambda_{15}^{C_1}$) that are calculated by trust propagation chains.
There is one trust propagation chain $\rho_1 : v_1 \rightarrow v_3 \rightarrow v_4$.
Therefore, $\lambda_{14}^{C_1} = (\frac{1}{2}, \frac{1}{2})$.
There are two trust propagation chains $\rho_1 : v_1 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5$ and $\rho_2 : v_1 \rightarrow v_6 \rightarrow v_5$.

Table 4 Trust relationship matrices R^{C_1}

	v_1	v_2	v_3	v_4	v_5	v_6
v_1	–	–	s_4^4	–	–	s_2^4
v_2	s_3^4	–	–	–	–	–
v_3	–	–	–	s_3^6	–	–
v_4	–	–	–	–	s_5^6	–
v_5	–	–	–	–	-	–
v_6	–	–	–	–	s_7^8	–

Table 5 Trust score matrices

	v_1	v_2	v_3	v_4	v_5	v_6
v_1			$(1,\frac{1}{2})$	–	–	$(\frac{1}{2},\frac{1}{2})$
v_2	$(\frac{3}{4},\frac{1}{2})$	–	–	–	–	–
v_3	–	–	–	$(\frac{1}{2},\frac{1}{3})$	–	–
v_4	–	–	–	–	$(\frac{5}{6},\frac{1}{3})$	–
v_5	–	–	–	–	-	–
v_6	–	–	–	–	$(\frac{7}{8},\frac{1}{4})$	–

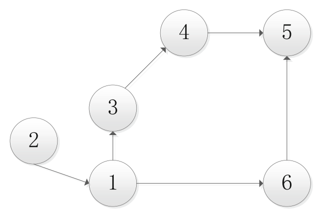

Fig. 4 An SN based on attribute C_1

Table 6 Trust score from v_1 to v_3,v_4, v_5, and v_6

	v_3	v_4	v_5	v_6
v_1	$(1,\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(\frac{7}{16},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$

$$ST(\rho_1) = 0.4167$$

$$ST(\rho_2) = 0.4375$$

ρ_2 is stronger, and we select ρ_2 to calculate $\lambda_{15}^{C_1} = (\frac{7}{16},\frac{1}{2})$.

Then, we obtain the trust scores from v_1 to v_3,v_4, v_5, and v_6 (see Table 6).

We predict the evaluation of v_1.

$$a_{111} = \sum_{j=3}^{6} O\left(\lambda_{1j}^{c}\right) \times a_{1j1} / \sum_{j=3}^{6} O\left(\lambda_{1j}^{c}\right) = 7.2$$

$$a_{311} = \sum_{j=3}^{6} O\left(\lambda_{1j}^{c}\right) \times a_{2j1} / \sum_{j=3}^{6} O\left(\lambda_{1j}^{c}\right) = 5.95$$

Step 3 Determining the weights of agents.
According to expression (16), the $\alpha-$ cut centrality values of agents are given in Table 7.

Step 4 Aggregation Process.

We compute the collective overall evaluation values $a_i^{c_1}, (i = 1, 2, 3)$ of the three items:$a_1^{c_1} = 7.1$

$a_2^{c_1} = 6.1$

$a_3^{c_1} = 6.6$

We repeat the above steps to complete the evaluation matrix.

We construct the SNs (see Figs. 5 and 6) according to trust relationship matrices R^{C_2} and R^{C_3}.

According to expression (16), the $\alpha-$ cut centrality values of agents in context of c_2 and c_3 are given in Table 8.

Table 7 The centralities of agents ($\alpha = 1/2$)

	w_1	w_2	w_3	w_4	w_5	w_6
Original value	3/4	0	1	1/2	5/6 + 7/8	1/2
Normalized value	0.17	0.00	0.22	0.11	0.38	0.11

We obtain $a_{212} = 7.1$ by analyzing the SN in context c_2 and $a_{313} = 6.3$ by analyzing the SN in context c_3.

We obtain the global evaluation of the three items in three SNs decision environments (see Table 9).

Using the average operator, we obtain

$$a_1 = \frac{7.2 + 7.0 + 6.9}{3} = 7.0$$

$$a_2 = 6.7$$

$$a_3 = 6.5$$

The final ranking of items is:

$I_1 \succ I_2 \succ I_3$

5.3 Comparisons

In the following, we give a comparative example using the method in [8] to solve this problem.

Firstly, we obtain the trust scores from v_1 to v_3,v_4, v_5, and v_6 (see Table 10).

Then, the complete evaluation of v_1 is obtained according to the weighted average (see Table 11).

Calculate the weight of each agent (see Table 12). Different from this paper, this method has equal weight on different criteria.

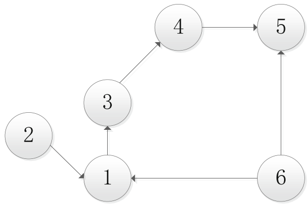

Fig. 5 An SN based on attribute C_2

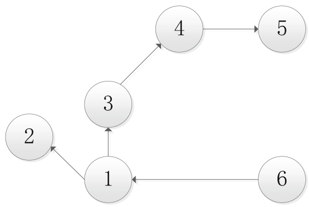

Fig. 6 An SN based on attribute C_3

Finally, the global evaluation of the three items in three SN decision environments is obtained (see Table 13).

Using the average operator, we obtain

$a_1 = 6.92$

$a_2 = 6.58$

$a_3 = 6.42$

The final ranking of items is:

$I_1 \succ I_2 \succ I_3$

Remark: Through comparison, we find that although the ranking of the items obtained by these two methods is the same. It is worth noting that the values of the incomplete evaluation matrix and the collective evaluation matrix are inconsistent. This shows that in different decision making environments, the ranking of final items will change accordingly.

In this example, we illustrate one key point that social networks are context specific. The same group of people constructs three different SNs according to attributes. We construct three SNs according to three criteria. We only trust the judgment of the areas where experts are good at. Therefore, we predict unknown evaluations based on different weights of agents, while other methods, like the method in [8], can also estimate the missing information, but do not distinguish multidimensional degree of trust between a pair of agents. Our computational model can be applied in more areas.

Table 8 The centralities of agents in the context of c_2 and c_3 ($\alpha = 1/3$)

	w_1	w_2	w_3	w_4	w_5	w_6
c_2	0.28	0	0.21	0.14	0.37	0
c_3	0.18	0.21	0.16	0.21	0.24	0

Table 9 Global evaluation matrices for items

Item/criterion	c_1	c_2	c_3
I_1	7.1	7.0	6.9
I_2	6.1	7.2	6.8
I_3	6.6	6.6	6.3

Table 10 Trust score from v_1 to v_3,v_4, v_5, and $\boldsymbol{v}_6$

	v_3	v_4	v_5	v_6
v_1	0.41	0.21	0.17	0.21

Table 11 Evaluation matrix for items

v_1	c_1	c_2	c_3
I_1	7.21	7.0	6.9
I_2	6.1	7.2	6.8
I_3	5.93	6.6	5.84

Table 12 The weight of each agent

	w_1	w_2	w_3	w_4	w_5	w_6
c_1	0.1	0	0.23	0.18	0.38	0.11
c_2	0.1	0	0.23	0.18	0.38	0.11
c_3	0.1	0	0.23	0.18	0.38	0.11

Table 13 Global evaluation matrices for items

Item/criterion	c_1	c_2	c_3
I_1	7.13	6.97	6.66
I_2	5.83	7.14	6.77
I_3	6.66	6.94	5.66

6 Conclusion

This paper proposes a fuzzy context-based social network description method after analyzing the properties of trust. The weighted direct graph allows us to design a trust propagation model to reflect the new challenges of measuring trust. In this section, we point out the contributions and limitations of our proposal. The contributions of our paper are in two aspects:

1. Our proposal reveals multidimensional feature of a social network which can explain the property of transitivity or nontransitivity.
2. Our proposal allows to address complex real-world trust relationships where humans exhibit vagueness and imprecision.

The significant opportunities also exist for future research:

1. Consistency of incomplete preferences
   Cabrerizo et al. [35] pointed out that the missing values of incomplete fuzzy preference relations should be consistent with the complete fuzzy preference relations. So Cabrerizo et al. [35] proposed a process to adjust the established value to maximize the consistency level. However, our proposal lacks this step. As a future work, we would add a consistency test to perfect the process of estimating incomplete information.
2. Dynamic change of trust relationship
   Many facets of social networks—including their agents, interconnections, discussed topics, interests and trends—are dynamic [25]. When trust increases or decreases with new experiences, our fuzzy graph needs to adjust to suit this situation. We consider the trust prediction in the context space more and in time series less. Further research should develop in this aspect. Perhaps a dynamic network is better at describing trust relationships.
3. Uncertainty in a trust-enhanced social network
   The mathematical decision models should be closer to human common sense in the representation of uncertainty and in the process of human reasoning in decision making [36]. We must rethink an SN as a fuzzy system and extend fuzzy models to represent uncertainty not only in trust relationship, but also in centrality of an SN. This expansion will have greater managerial and academic impact.

Acknowledgements This work was funded by National Natural Science Foundation of China (NSFC) (71871121), Future Network Scientific Research Fund Project (FNSRFP-2021-YB-19), HRSA, US Department of Health & Human Services (No. H49MC0068), the Startup Foundation for Introducing Talent of NUIST (Grant No. 1441182001002), the General Project of Philosophy and Social Science Research in Colleges and Universities in Jiangsu Province (Grant No. 2020SJA0174), and the Natural Science Foundation of Jiangsu Higher Education Institution of China) (Grant No. 21KJB410001).

Declarations

Conflict of interest This section is to certify that we have no potential conflict of interest. This article does not contain any studies with human participants or animals performed by any of the authors.

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