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Adaptive Fuzzy Fault-Tolerant Control of High-Order Nonlinear Systems: A Fully Actuated System Approach

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Abstract An adaptive tracking control problem of high-order nonlinear strict-feedback system (SFS) with non-affine nonlinear faults is considered in this paper. Based on high-order fully actuated (HOFA) systems theory, dynamic surface control technique and universal approximation of fuzzy logic systems, a novel adaptive fuzzy fault-tolerant tracking controllers are directly constructed, it does not need to convert the high-order system into first-order one. By using Lyapunov function theory, the proposed controller design approach can guarantee that the closed-loop system is stable; at the same time, the tracking error can converge to a compact neighborhood with respect to zero. The simulation example has been verified the feasibility and effectiveness of the control approach in this paper.

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1 Introduction

Compared with linear systems, almost all actual industrial systems are nonlinear systems, such as hypersonic aircraft [1], ship autopilot system [2], aircraft flight control system [3], and robotic manipulator system [4]. However, because the analysis of nonlinear systems is far more complicated than linear systems, and there is a lack of effective mathematical tools that can be processed uniformly, there are few results on nonlinear systems compared with linear systems. It was not until the emergence of neural networks (NNs) and fuzzy logic systems (FLSs) that this deadlock was broken. NNs and FLSs have been proven to be a universal approximator that can approximate a continuous unknown nonlinear function with arbitrary precision. Since then, adaptive control methods of unknown nonlinear systems have been extensively developed in [4–17]. For the switched nonlinear systems, an adaptive fuzzy finite-time tracking control approach was investigated in [5]. Based on event triggered, a fuzzy adaptive tracking fixed-time control problem was studied for non-strict-feedback nonlinear systems in [16].

Based on the state-space method, also called the first-order method, all the above control methods are very effective for dealing with the control problems of nonlinear systems, but it requires a system to be a first-order differential system. According to the laws of physics, such as Newton's laws of motion, Euler's equations, Lagrange's equations, Kirchhoff's law, etc., many models of real industrial systems are higher-order differential equations.

For example, the rigid robotic systems were modeled as second-order differential dynamics equations in [18], and single-link flexible manipulators were described as fourth-order differential equations in [19]. The state-space method can solve the control problems of high-order nonlinear systems, but it is necessary to transform the high-order system into a first-order one by lowering the order and maximizing the number of equations, which greatly increases the complexity of the controller design, and the system after processing by maximizing the number of equations, the physical meaning of some states may be lost. How to directly design a simpler controller for high-order systems is more challenging. Professor Guangren Duan first proposed the high-order fully actuated (HOFA) system method, namely the high-order method, which provides a new dawn for the controller design of high-order systems, such as [20–29]. The adaptive tracking controllers and stabilizing controllers were first designed for three types of high-order system models with parametric uncertainties in [23]. In [24], the high-order backstepping control and robust control approaches were discussed for an uncertain high-order strict-feedback system (SFS), an uncertain second-order SFS, and a single HOFA model with nonlinear uncertainties. Although these results require fault-free operating conditions, but they provide a new idea for directly designing a fault-tolerant controller of high-order unknown nonlinear systems in this paper.

For actual industrial systems, faults are inevitable and unpredictable. Faults may lead to poor control performance and even system instability. Therefore, considering the controller design of the unknown nonlinear system with faults is theoretical and practical significance. Some adaptive control methods of nonlinear systems with faults were investigated in [30–33]. An adaptive decentralized fault-tolerant control (FTC) approach was proposed for uncertain interconnected nonlinear systems in [31]. In [30–33], all the control methods only consider linear faults, which are invalid for nonlinear faults. In fact, most faults in practical systems are nonlinear functions of controller u and state x [34–38]. To the best of the authors' knowledge, so far there are few FTC results that considered nonlinear faults in nonlinear system control. For instance, the FTC algorithm proposed in [39] has solved the control problem of nonlinear systems with affine nonlinear faults, which are functions of the state x. But what about non-affine nonlinear faults, the control methods above are obviously ineffective. For the above literature, all the FTC approaches were studied for first-order nonlinear systems. Therefore, how to directly design an adaptive controller of the high-order nonlinear SFS with non-affine nonlinear faults is still an open problem, which is of great theoretical and practical value.

Based on the above motivation, we study the adaptive tracking FTC method for the high-order nonlinear SFS with non-affine nonlinear faults. The main contributions are summarized as follows:

(1) Based on the state-space method, for the control methods of second- or high-order systems in the existing literature, it is all need to converting the system to first-order one. But for actual industrial systems, some states have lost the physical meanings in the process of model transformation. In this paper, the adaptive controller can be designed directly for high-order unknown nonlinear systems, it does not need to convert the high-order system into first-order one. Thus, the proposed high-order backstepping method needs fewer steps than the usual state-space backstepping method; at the same time, the computation complexity has been greatly reduced.

(2) For the actual industrial system, faults are inevitable due to some unpredictable reasons. For the existing results on faults, most control methods only consider linear faults [30–33], and only a few control methods consider affine nonlinear faults [39]. However, in most cases, the faults exhibit non-affine properties. Therefore, it is of practical significance to investigate high-order nonlinear systems with non-affine nonlinear faults. And according to the authors' knowledge, it is the first time to solve the adaptive fuzzy FTC of high-order nonlinear SFS with non-affine nonlinear faults.

The outline of this paper is state as follows: Sect. 2 provides the problem description and preliminaries. High-order backstepping controller is constructed in Sect. 3. Section 4 shows the stability analysis. Finally, the simulation results and conclusion are provided in Sect. 5 and 6, respectively.

2 Problem Description and Preliminaries

2.1 Problem Formulation

The high-order nonlinear SFS is considered as follows:

$$\begin{cases} x_1^{(q_1)} = f_1(x_1^{(0\sim q_1-1)}) + g_1(x_1^{(0\sim q_1-1)})x_2, \\ x_2^{(q_2)} = f_2(x_i^{(0\sim q_i-1)}|_{i=1\sim 2}) + g_2(x_i^{(0\sim q_i-1)}|_{i=1\sim 2})x_3, \\ \qquad \vdots \\ x_{n-1}^{(q_{n-1})} = f_{n-1}(x_i^{(0\sim q_i-1)}|_{i=1\sim n-1}) + g_{n-1}(x_i^{(0\sim q_i-1)}|_{i=1\sim n-1})x_n, \\ x_n^{(q_n)} = f_n(x_i^{(0\sim q_i-1)}|_{i=1\sim n}) + g_n(x_i^{(0\sim q_i-1)}|_{i=1\sim n})u + l(t-T_0)v\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right), \\ \qquad y = x_1, \end{cases}, \tag{1}$$

where $x_i \in R$, $i = 1, 2, \ldots, n$ are the state variables, $f_j(x_i^{(0\sim q_i-1)}\big|_{i=1\sim j}) \in R$ and $g_j(x_i^{(0\sim q_i-1)}\big|_{i=1\sim j}) \in R$, $j =$

$1, 2..., n$ denote unknown nonlinear functions and known nonlinear functions, respectively. $y \in R$ and $u \in R$ are the output and input of the considered system. It is assumed that $g_j(x_i^{(0\sim q_i-1)}|_{i=1\sim j}) \neq 0$. $v\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right) \in R$ represents an unknown external disturbance caused by a fault. $l(t-T_0) \in R$ denotes the time profile of the fault that occurs at some unknown time:

$$l(t-T_0) = \begin{cases} 0, & t < T_0, \\ 1-e^{-\delta(t-T_0)}, & t \geq T_0, \end{cases} \tag{2}$$

where $\delta > 0$ is the evolution rate of the unknown fault. The reference signal y_r is a smooth function, y_r and its derivatives $\dot{y}_r, \ldots, y_r^{(q_1)}$ are all bounded.

The objective is to construct adaptive fuzzy controller for high-order nonlinear SFS with non-affine nonlinear faults (1), such that the output of the system can track the ideal signal y_r, and the closed-loop system is stable.

Assumption 1 ([40]:) For system (1), the inequality

$$\begin{aligned} &\left|f_n(x_i^{(0\sim q_i-1)}|_{i=1\sim n}) + l(t-T_0)v\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right)\right| \\ &\quad \leq \eta\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right) \end{aligned} \tag{3}$$

holds, where $\eta\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right)$ is an unknown non-negative function.

Remark 1 It should be pointed out that almost all the results on adaptive control problems of nonlinear systems are based on the state-space method, i.e., the first-order method. According to the laws of physics, many models of real industrial systems are high-order dynamic differential equations. The state-space method can also solve the control problem of high-order nonlinear systems, but it needs to transform the system into a first-order one first, so this method is relatively cumbersome. In the past 2 years, professor Guangren Duan first proposed the HOFA system method [20–29]. This method can directly design the controller of the high-order nonlinear system, but it requires the nonlinear function to be known [20, 22–28]. Due to the natural environment or technical means, many practical systems cannot be accurately modeled. Therefore, it is of great theoretical and practical significance to study the direct design controller of high-order systems with unknown nonlinear functions.

Remark 2 For most of the existing results on adaptive FTC approaches of nonlinear systems, basically only linear faults are considered [30–33], i.e., lock-in-place model and loss of effectiveness model. Compared to linear faults, there are few results considering nonlinear faults. In fact, most faults in practical systems are nonlinear functions of controller u and state x [39]. The FTC algorithm proposed in [39] has solved the control problem of nonlinear systems with affine nonlinear faults, which are functions of the state x. But what about non-affine faults, the control methods above are obviously ineffective. However, in most cases, the faults exhibit non-affine properties. Therefore, the faults considered in this paper are more general than the above-mentioned results.

2.2 Preliminaries

For convenience, we define the following symbols that can be used in the following paper. I_m represents the identity matrix, and

$$x^{(0\sim q)} = \begin{bmatrix} x \\ \dot{x} \\ \vdots \\ x^{(q)} \end{bmatrix},$$

$$x_k^{(q_0\sim q_k)}\Big|_{k=i\sim j} = \begin{bmatrix} x_i^{(q_0\sim q_k)} \\ x_{i+1}^{(q_0\sim q_k)} \\ \vdots \\ x_j^{(q_0\sim q_k)} \end{bmatrix}, \quad j \geq i,$$

$$A^{0\sim q-1} = [A_0 \quad A_1 \quad \cdots \quad A_{q-1}],$$

$$\Phi\left(A^{0\sim q-1}\right) = \begin{bmatrix} 0 & I & & \\ & & \ddots & \\ & & & I \\ -A_0 & -A_1 & \cdots & A_{q-1} \end{bmatrix}.$$

FLSs are used to approximate the unknown nonlinear functions of the system (1). The inference rules of knowledge base are in the following [40]:

R^l: If x_1 is F_l^l and x_2 is F_2^l and ... and x_n is F_n^l,
then y is G^l, $l = 1, 2, \ldots, N$,

where $x = [x_1, \ldots, x_n]^T$ and y are the input and output of an FLS. N is the rules number. Fuzzy sets F_i^l and G^l are associated with the fuzzy membership functions $\mu_{F_i^l}(x)$ and $\mu_{G^l}(y)$, respectively.

By using product inference, center average defuzzification along with singleton fuzzifier, the FLS is designed as follows:

$$y(x) = \frac{\sum_{l=1}^{N} \bar{y}_l \prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N}[\prod_{i=1}^{n} \mu_{F_i^l}(x_i)]}, \tag{4}$$

where $\bar{y}_l = \max_{y\in R} \mu_{G^l}(y)$.

The fuzzy basis functions are designed as follows:

$$\varphi_l = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N}\left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)} \tag{5}$$

then, (4) can be rewritten as $y(x) = \theta^{\mathrm{T}}\varphi(x)$, where $\theta^{\mathrm{T}} = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_N] = [\theta_1, \theta_2, \ldots, \theta_N]$ and $\varphi(x) = [\varphi_1(x), \ldots, \varphi_N(x)]^{\mathrm{T}}$.

Lemma 1 *Let $f(x)$ be a continuous smooth function defined on a compact set U, for any positive approximation error ε, there exists a FLS $\theta^{\mathrm{T}}\varphi(x)$ such that*

$$\sup_{x\in U}\left|f(x) - \theta^{\mathrm{T}}\varphi(x)\right| \le \varepsilon, \tag{6}$$

where ε satisfies $|\varepsilon| \le \varepsilon^*$, ε^* is a positive constant.

Proposition 1 *([22, 23]:) For an arbitrarily chosen $F \in R^{q_i\times q_i}$, all the matrix $A^{0\sim q_i-1}$ and the nonsingular matrix $V \in R^{q_i\times q_i}$ satisfying*

$$\Phi\left(A^{0\sim q_i-1}\right) = VFV^{-1} \tag{7}$$

are given by

$$A^{0\sim q_i-1} = -ZF^{q_i}V^{-1}(Z,F), \tag{8}$$

$$V(Z,F) = \begin{bmatrix} Z \\ ZF \\ \vdots \\ ZF^{q_i-1} \end{bmatrix}, \tag{9}$$

where $Z \in R^{1\times q_i}$ is an arbitrary parameter matrix satisfying

$$\det V(Z,F) \ne 0. \tag{10}$$

Then, to solve the matrix $P\left(A_i^{0\sim q_i}\right)$ satisfying the following Lyapunov matrix equation (13), some notations related to a square matrix $\Phi \in R^{q_i\times q_i}$ are introduced as follows:

$$\det(sI + \Phi) \triangleq \sum_{i=0}^{q_i} c_i^{\Phi} s^i, \tag{11}$$

$$adj\left(sI + \Phi^{\mathrm{T}}\right) \triangleq \sum_{i=0}^{q_i-1} C_i^{\Phi} s^i. \tag{12}$$

Proposition 2 *([23]:) If $\Phi \in R^{q_i\times q_i}$ is Hurwitz, then the following Lyapunov equation*

$$\Phi^{\mathrm{T}}P + P\Phi = -I \tag{13}$$

has a unique solution given by

$$P = \sum_{i=0}^{q_i-1} C_i^{\Phi} P_0^{-1} \Phi^i \tag{14}$$

with

$$P_0 = \sum_{i=0}^{q_i} c_i^{\Phi} \Phi^i. \tag{15}$$

3 High-Order Backstepping Controller Design

Bases on HOFA theory, the backstepping controller design approach can be directly given for the high-order nonlinear SFS (1) without converting the system into a first-order one.

Suppose $A_i^{0\sim q_i-1} \in R^{1\times q_i}$, $i = 1, 2, \ldots, n$ are a set of matrices which make $\Phi(A_i^{0\sim q_i-1}) \in R^{q_i\times q_i}$, $i = 1, 2, \ldots, n$ stable, and

$$P_i\left(A_i^{0\sim q_i-1}\right) = \left[\, P_{iF}\left(A_i^{0\sim q_i-1}\right) \quad P_{iM}\left(A_i^{0\sim q_i-1}\right) \quad P_{iL}\left(A_i^{0\sim q_i-1}\right) \,\right] \in R^{q_i\times q_i}$$

is the unique positive definite solution to the Lyapunov equation:

$$\begin{aligned} &\Phi^{\mathrm{T}}\left(A_i^{0\sim q_i-1}\right)P_i\left(A_i^{0\sim q_i-1}\right) \\ &\quad + P_i\left(A_i^{0\sim q_i-1}\right)\Phi\left(A_i^{0\sim q_i-1}\right) = -I_{q_i}, \end{aligned} \tag{16}$$

where $P_{iF}\left(A_i^{0\sim q_i-1}\right) \in R^{q_i\times 1}$ and $P_{iL}\left(A_i^{0\sim q_i-1}\right) \in R^{q_i\times 1}$.

For the backstepping control method in this paper, the following first-order filter is introduced

$$l_i\dot{\bar{\alpha}}_i + \bar{\alpha}_i = \alpha_i, \quad i = 2, \ldots, n, \tag{17}$$

where $\bar{\alpha}_i$ is the output, the backstepping virtual controller α_i is the input, and l_i is a positive design parameter.

Let

$$\varpi_i = \bar{\alpha}_i - \alpha_i \tag{18}$$

denotes the filter error. Then, it is easy to get

$$l_i G_i(\cdot) = l_i\dot{\varpi}_i + \varpi_i, \tag{19}$$

where $G_i(\cdot)$ represents the continuous function.

Step 1: Let

$$\xi_1^{(0\sim q_1-1)} = x_1^{(0\sim q_1-1)} - y_{\mathrm{r}}^{(0\sim q_1-1)} \tag{20}$$

and

$$\xi_2^{(0\sim q_2-1)} = x_2^{(0\sim q_2-1)} - \bar{\alpha}_2^{(0\sim q_2-1)}. \tag{21}$$

(21) can be decomposed into

$$\xi_2 = x_2 - \bar{\alpha}_2, \tag{22}$$

then the q_1th derivative of ξ_1 is given by

$$\begin{aligned} \xi_1^{(q_1)} &= f_1(x_1^{(0\sim q_1-1)}) + g_1(x_1^{(0\sim q_1-1)})(\xi_2 + \varpi_2 + \alpha_2) \\ &\quad - y_{\mathrm{r}}^{(q_1)}. \end{aligned} \tag{23}$$

Design the first virtual control α_2 as follows:

$$\begin{aligned}\alpha_2 = &-g_1^{-1}\left(x_1^{(0\sim q_1-1)}\right)\left(A_1^{0\sim q_1-1}\xi_1^{(0\sim q_1-1)}\right.\\ &\left.+\hat{\theta}_1^{\mathrm{T}}\varphi_1\left(x_1^{(0\sim q_1-1)}\right)-y_{\mathrm{r}}^{(q_1)}\right).\end{aligned} \tag{24}$$

Substituting (24) into (23) gives

$$\begin{aligned}\xi_1^{(q_1)} = &-A_1^{0\sim q_1-1}\xi_1^{(0\sim q_1-1)}+\tilde{\theta}_1^{\mathrm{T}}\varphi_1\left(x_1^{(0\sim q_1-1)}\right)+\varepsilon_1\\ &+g_1(x_1^{(0\sim q_1-1)})(\xi_2+\varpi_2).\end{aligned} \tag{25}$$

(25) can be further written as follows:

$$\dot{\xi}_1^{(0\sim q_1-1)} = \Phi_1\left(A_1^{0\sim q_1-1}\right)\xi_1^{(0\sim q_1-1)}+\begin{bmatrix}0\\ b_1\end{bmatrix}, \tag{26}$$

where

$$b_1 = \tilde{\theta}_1^{\mathrm{T}}\varphi_1\left(x_1^{(0\sim q_1-1)}\right)+\varepsilon_1+g_1\left(x_1^{(0\sim q_1-1)}\right)(\xi_2+\varpi_2). \tag{27}$$

Select Lyapunov function candidate as follows:

$$V_1 = \left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_1\left(A_1^{0\sim q_1-1}\right)\xi_1^{(0\sim q_1-1)}+\tilde{\theta}_1^{\mathrm{T}}\tilde{\theta}_1. \tag{28}$$

Differentiating V_1 with respect to time produces

$$\begin{aligned}\dot{V}_1 &= \left(\dot{\xi}_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_1\left(A_1^{0\sim q_1-1}\right)\xi_1^{(0\sim q_1-1)}\\ &\quad+\left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_1\left(A_1^{0\sim q_1-1}\right)\dot{\xi}_1^{(0\sim q_1-1)}-2\tilde{\theta}_1^{\mathrm{T}}\dot{\hat{\theta}}_1\\ &=\left(\Phi_1\left(A_1^{0\sim q_1-1}\right)\xi_1^{(0\sim q_1-1)}+\begin{bmatrix}0\\ b_1\end{bmatrix}\right)^{\mathrm{T}}P_1\left(A_1^{0\sim q_1-1}\right)\xi_1^{(0\sim q_1-1)}\\ &\quad+\left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_1\left(A_1^{0\sim q_1-1}\right)\left(\Phi_1\left(A_1^{0\sim q_1-1}\right)\xi_1^{(0\sim q_1-1)}+\begin{bmatrix}0\\ b_1\end{bmatrix}\right)-2\tilde{\theta}_1^{\mathrm{T}}\dot{\hat{\theta}}_1\\ &=\left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}\left(\Phi_1^{\mathrm{T}}\left(A_1^{0\sim q_1-1}\right)P_1\left(A_1^{0\sim q_1-1}\right)\right.\\ &\quad\left.+P_1\left(A_1^{0\sim q_1-1}\right)\Phi_1\left(A_1^{0\sim q_1-1}\right)\right)\xi_1^{(0\sim q_1-1)}\\ &\quad+2\tilde{\theta}_1^{\mathrm{T}}\left(\left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_{1L}\left(A_1^{0\sim q_1-1}\right)\varphi_1\left(x_1^{(0\sim q_1-1)}\right)-\dot{\hat{\theta}}_1\right)\\ &\quad+2\left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_{1L}\left(A_1^{0\sim q_1-1}\right)g_1(x_1^{(0\sim q_1-1)})(\xi_2+\varpi_2)\\ &\quad+2\left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_{1L}\left(A_1^{0\sim q_1-1}\right)\varepsilon_1.\end{aligned} \tag{29}$$

By designing the adaptive law

$$\dot{\hat{\theta}}_1 = \left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_{1L}\left(A_1^{0\sim q_1-1}\right)\varphi_1\left(x_1^{(0\sim q_1-1)}\right)-\gamma_1\hat{\theta}_1. \tag{30}$$

$\dot{V}_1$ can be transformed into

$$\begin{aligned}\dot{V}_1 = &-\left\|\xi_1^{(0\sim q_1-1)}\right\|^2+2\gamma_1\tilde{\theta}_1^{\mathrm{T}}\hat{\theta}_1+2\left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_{1L}\left(A_1^{0\sim q_1-1}\right)\varepsilon_1\\ &+2\left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_{1L}\left(A_1^{0\sim q_1-1}\right)g_1\left(x_1^{(0\sim q_1-1)}\right)(\xi_2+\varpi_2).\end{aligned} \tag{31}$$

By using the following inequalities:

$$\begin{aligned}&2\left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_{1L}\left(A_1^{0\sim q_1-1}\right)g_1\left(x_1^{(0\sim q_1-1)}\right)(\xi_2+\varpi_2)\\ &\quad\le\frac{1}{4}\left\|\xi_1^{(0\sim q_1-1)}\right\|^2+8\left\|P_{1L}\left(A_1^{0\sim q_1-1}\right)\right\|^2g_1^2\left(x_1^{(0\sim q_1-1)}\right)\varpi_2^2\\ &\qquad+8\left\|P_{1L}\left(A_1^{0\sim q_1-1}\right)\right\|^2g_1^2\left(x_1^{(0\sim q_1-1)}\right)\left\|\xi_2^{(0\sim q_2-1)}\right\|^2\end{aligned} \tag{32}$$

$$\begin{aligned}&2\left(\xi_1^{(0\sim q_1-1)}\right)^{\mathrm{T}}P_{1L}\left(A_1^{0\sim q_1-1}\right)\varepsilon_1\\ &\quad\le\frac{1}{4}\left\|\xi_1^{(0\sim q_1-1)}\right\|^2+4\left\|P_{1L}\left(A_1^{0\sim q_1-1}\right)\right\|^2\varepsilon_1^{*2}.\end{aligned} \tag{33}$$

(31) becomes

$$\begin{aligned}\dot{V}_1\le&-\frac{1}{2}\left\|\xi_1^{(0\sim q_1-1)}\right\|^2+2\gamma_1\tilde{\theta}_1^{\mathrm{T}}\hat{\theta}_1+8\left\|P_{1L}\left(A_1^{0\sim q_1-1}\right)\right\|^2g_1^2\left(x_1^{(0\sim q_1-1)}\right)\varpi_2^2\\ &+8\left\|P_{1L}\left(A_1^{0\sim q_1-1}\right)\right\|^2g_1^2\left(x_1^{(0\sim q_1-1)}\right)\left\|\xi_2^{(0\sim q_2-1)}\right\|^2+c_1,\end{aligned} \tag{34}$$

where

$$c_1 = 4\left\|P_{1L}\left(A_1^{0\sim q_1-1}\right)\right\|^2\varepsilon_1^{*2}.$$

Step i: Let

$$\xi_i^{(0\sim q_i-1)} = x_i^{(0\sim q_i-1)}-\bar{\alpha}_i^{(0\sim q_i-1)} \tag{35}$$

$\xi_i^{(q_i-1)}$ can be written as follows:

$$\xi_i^{(q_i-1)} = x_i^{(q_i-1)}-\bar{\alpha}_i^{(q_i-1)}. \tag{36}$$

Differentiating (36) with respect to time, and using (1), yield

$$\begin{aligned}\xi_i^{(q_i)} =& f_i(x_j^{(0\sim q_j-1)}|_{j=1\sim i})+g_i\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right)x_{i+1}\\ &-\bar{\alpha}_i^{(q_i)}.\end{aligned} \tag{37}$$

Let

$$\xi_{i+1}^{(0\sim q_{i+1}-1)} = x_{i+1}^{(0\sim q_{i+1}-1)}-\bar{\alpha}_{i+1}^{(0\sim q_{i+1}-1)} \tag{38}$$

which can be equivalently decomposed into

$$\xi_{i+1} = x_{i+1}-\bar{\alpha}_{i+1}. \tag{39}$$

Substituting (39) into (37) gives

$$\begin{aligned}\xi_i^{(q_i)} =& f_i\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right)-\bar{\alpha}_i^{(q_i)}\\ &+g_i\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right)(\xi_{i+1}+\varpi_{i+1}+\alpha_{i+1}).\end{aligned} \tag{40}$$

By designing the virtual controller α_{i+1} as follows:

$$\alpha_{i+1} = -\left(g_i(x_j^{(0\sim q_j-1)}|_{j=1\sim i})\right)^{-1}\left(A_i^{0\sim q_i-1}\xi_i^{(0\sim q_i-1)} + \hat{\theta}_i^{\mathrm{T}}\varphi\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right) - \bar{\alpha}_i^{(q_i)}\right). \tag{41}$$

$\xi_i^{(q_i)}$ can be further transformed into the following equation:

$$\xi_i^{(q_i)} = -A_i^{0\sim q_i-1}\xi_i^{(0\sim q_i-1)} + \tilde{\theta}_i^{\mathrm{T}}\varphi(x_j^{(0\sim q_j-1)}|_{j=1\sim i}) + \varepsilon_i + g_i\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right)(\xi_{i+1} + \varpi_{i+1}). \tag{42}$$

Eq (42) can be further rewritten in the state-space form as follows:

$$\dot{\xi}_i^{(0\sim q_i-1)} = \Phi_i\left(A_i^{0\sim q_i-1}\right)\xi_i^{(0\sim q_i-1)} + \begin{bmatrix} 0 \\ b_i \end{bmatrix}, \tag{43}$$

where

$$b_i = \tilde{\theta}_i^{\mathrm{T}}\varphi\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right) + \varepsilon_i + g_i\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right)(\xi_{i+1} + \varpi_{i+1}). \tag{44}$$

Select Lyapunov function candidate as follows:

$$V_i = \left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim q_i-1}\right)\xi_i^{(0\sim q_i-1)} + \varpi_i^2 + \tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i. \tag{45}$$

Taking the derivative of V_i yields

$$\begin{aligned}
\dot{V}_i &= \left(\dot{\xi}_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim q_i-1}\right)\xi_i^{(0\sim q_i-1)} + \left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} \\
&\quad P_i\left(A_i^{0\sim q_i-1}\right)\dot{\xi}_i^{(0\sim q_i-1)} + 2\varpi_i\dot{\varpi}_i - 2\tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i \\
&= \left(\Phi_i\left(A_i^{0\sim q_i-1}\right)\xi_i^{(0\sim q_i-1)} + \begin{bmatrix} 0 \\ b_i \end{bmatrix}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim q_i-1}\right)\xi_i^{(0\sim q_i-1)} \\
&\quad + \left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim q_i-1}\right)\left(\Phi_i\left(A_i^{0\sim q_i-1}\right)\xi_i^{(0\sim q_i-1)} + \begin{bmatrix} 0 \\ b_i \end{bmatrix}\right) \\
&\quad + 2\varpi_i\left(G_i(\cdot) - \frac{\varpi_i}{l_i}\right) - 2\tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i \\
&= \left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}}\left(\Phi_i^{\mathrm{T}}\left(A_i^{0\sim q_i-1}\right)P_i\left(A_i^{0\sim q_i-1}\right) + P_i\left(A_i^{0\sim q_i-1}\right)\Phi_i\left(A_i^{0\sim q_i-1}\right)\right)\xi_i^{(0\sim q_i-1)} \\
&\quad + 2\tilde{\theta}_i^{\mathrm{T}}\left(\left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim q_i-1}\right)\varphi\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right) - \dot{\hat{\theta}}_i\right) \\
&\quad + 2\left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim q_i-1}\right)g_i\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right)(\xi_{i+1} + \varpi_{i+1}) \\
&\quad + 2\left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim q_i-1}\right)\varepsilon_i + 2\varpi_i\left(G_i(\cdot) - \frac{\varpi_i}{l_i}\right).
\end{aligned} \tag{46}$$

The adaptive law is designed as follows:

$$\dot{\hat{\theta}}_i = \left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim q_i-1}\right)\varphi\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right) - \gamma_i\hat{\theta}_i. \tag{47}$$

Together with (47), (46) is given as follows:

$$\begin{aligned}
\dot{V}_i &= -\left\|\xi_i^{(0\sim q_i-1)}\right\|^2 + 2\gamma_i\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i + 2\varpi_i\left(G_i(\cdot) - \frac{\varpi_i}{l_i}\right) \\
&\quad + 2\left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim q_i-1}\right)g_i\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right) \\
&\quad (\xi_{i+1} + \varpi_{i+1}) + 2\left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim q_i-1}\right)\varepsilon_i.
\end{aligned} \tag{48}$$

By using Yang's inequality, the following inequalities hold:

$$\begin{aligned}
&2\left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim q_i-1}\right)g_i\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right)(\xi_{i+1} + \varpi_{i+1}) \\
&\quad \leq \frac{1}{4}\left\|\xi_i^{(0\sim q_i-1)}\right\|^2 + 8\left\|P_{iL}\left(A_i^{0\sim q_i-1}\right)\right\|^2 g_i^2\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right) \\
&\quad \left\|\xi_{i+1}^{(0\sim q_{i+1}-1)}\right\|^2 + 8\left\|P_{iL}\left(A_i^{0\sim q_i-1}\right)\right\|^2 g_i^2\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right)\varpi_{i+1}^2,
\end{aligned} \tag{49}$$

$$\begin{aligned}
&2\left(\xi_i^{(0\sim q_i-1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim q_i-1}\right)\varepsilon_i \\
&\quad \leq \frac{1}{4}\left\|\xi_i^{(0\sim q_i-1)}\right\|^2 + 4\left\|P_{iL}\left(A_i^{0\sim q_i-1}\right)\right\|^2 \varepsilon_i^{*2},
\end{aligned} \tag{50}$$

$$2\varpi_i G_i(\cdot) \leq \beta + \frac{G_i^2(\cdot)\varpi_i^2}{\beta}. \tag{51}$$

Substituting (49)–(51) into (48) yields

$$\begin{aligned}
\dot{V}_i &\leq -\frac{1}{2}\left\|\xi_i^{(0\sim q_i-1)}\right\|^2 + 2\gamma_i\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i - \left(\frac{2}{l_i} - \frac{G_i^2(\cdot)}{\beta}\right)\varpi_i^2 \\
&\quad + 8\left\|P_{iL}\left(A_i^{0\sim q_i-1}\right)\right\|^2 g_i^2\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right)\left\|\xi_{i+1}^{(0\sim q_{i+1}-1)}\right\|^2 \\
&\quad + 8\left\|P_{iL}\left(A_i^{0\sim q_i-1}\right)\right\|^2 g_i^2\left(x_j^{(0\sim q_j-1)}|_{j=1\sim i}\right)\varpi_{i+1}^2 + c_i,
\end{aligned} \tag{52}$$

where

$$c_i = 4\left\|P_{iL}\left(A_i^{0\sim q_i-1}\right)\right\|^2 \varepsilon_i^{*2} + \beta. \tag{53}$$

Step n: Similarly, let

$$\xi_n^{(0\sim q_n-1)} = x_n^{(0\sim q_n-1)} - \bar{\alpha}_n^{(0\sim q_n-1)} \tag{54}$$

which gives

$$\xi_n^{(q_n-1)} = x_n^{(q_n-1)} - \bar{\alpha}_n^{(q_n-1)}. \tag{55}$$

From the system (1), taking the derivative of (55) yields

$$\xi_n^{(q_n)} = f_n\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}\right) + g_n\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}\right)u + l(t-T_0)v\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right) - \bar{\alpha}_n^{(q_n)}. \tag{56}$$

By designing the actual controller

$$u = -\left(g_n\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}\right)\right)^{-1}\left(A_n^{0\sim q_n-1}\xi_n^{(0\sim q_n-1)} + \hat{\theta}_n^{\mathrm{T}}\varphi\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u_{\mathrm{f}}\right) - \bar{\alpha}_n^{(q_n)}\right), \tag{57}$$

then the Eq. (56) can be converted to

$$\xi_n^{(q_n)} = -A_n^{0\sim q_n-1}\xi_n^{(0\sim q_n-1)} + f_n\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}\right) - \hat{\theta}_n^{\mathrm{T}}\varphi\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u_{\mathrm{f}}\right) + l(t-T_0)v\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right). \tag{58}$$

Then, it can be converted into a state-space form:

$$\dot{\xi}_n^{(0\sim q_n-1)} = \Phi_n\left(A_n^{0\sim q_n-1}\right)\xi_n^{(0\sim q_n-1)} + \begin{bmatrix} 0 \\ b_n \end{bmatrix}, \tag{59}$$

where

$$b_n = f_n\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}\right) - \hat{\theta}_n^{\mathrm{T}}\varphi\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u_{\mathrm{f}}\right) + l(t-T_0)v\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right). \tag{60}$$

Design the following Lyapunov function

$$V_n = \left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_n\left(A_n^{0\sim q_n-1}\right)\xi_n^{(0\sim q_n-1)} + \varpi_n^2 + \tilde{\theta}_n^{\mathrm{T}}\tilde{\theta}_n. \tag{61}$$

Differentiating V_n with respect to time produces

$$\begin{aligned}\dot{V}_n &= \left(\dot{\xi}_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_n\left(A_n^{0\sim q_n-1}\right)\xi_n^{(0\sim q_n-1)} + \left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_n\left(A_n^{0\sim q_n-1}\right)\dot{\xi}_n^{(0\sim q_n-1)} + 2\varpi_n\dot{\varpi}_n - 2\tilde{\theta}_n^{\mathrm{T}}\dot{\hat{\theta}}_n \\ &= \left(\Phi_n\left(A_n^{0\sim q_n-1}\right)\xi_n^{(0\sim q_n-1)} + \begin{bmatrix} 0 \\ b_n \end{bmatrix}\right)^{\mathrm{T}} P_n\left(A_n^{0\sim q_n-1}\right)\xi_n^{(0\sim q_n-1)} \\ &\quad + \left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_n\left(A_n^{0\sim q_n-1}\right)\left(\Phi_n\left(A_n^{0\sim q_n-1}\right)\xi_n^{(0\sim q_n-1)} + \begin{bmatrix} 0 \\ b_n \end{bmatrix}\right) \\ &\quad + 2\varpi_n\left(G_n(\cdot) - \frac{\varpi_n}{l_n}\right) - 2\tilde{\theta}_n^{\mathrm{T}}\dot{\hat{\theta}}_n \\ &= \left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}}\left(\Phi_n^{\mathrm{T}}\left(A_n^{0\sim q_n-1}\right)P_n\left(A_n^{0\sim q_n-1}\right) + P_n\left(A_n^{0\sim q_n-1}\right)\Phi_n\left(A_n^{0\sim q_n-1}\right)\right)\xi_n^{(0\sim q_n-1)} \\ &\quad + 2\left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_{nL}\left(A_n^{0\sim q_n-1}\right)\left(f_n\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}\right) - \hat{\theta}_n^{\mathrm{T}}\varphi\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u_{\mathrm{f}}\right) + l(t-T_0)v\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right)\right) \\ &\quad + 2\varpi_n\left(G_n(\cdot) - \frac{\varpi_n}{l_n}\right) - 2\tilde{\theta}_n^{\mathrm{T}}\dot{\hat{\theta}}_n.\end{aligned} \tag{62}$$

By using Assumption 1 and Young's inequality, one has

$$\begin{aligned}&2\left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_{nL}\left(A_n^{0\sim q_n-1}\right)\left(f_n\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}\right) + l(t-T_0)v\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right)\right) \\ &\le 2\left|\left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_{nL}\left(A_n^{0\sim q_n-1}\right)\right|\left|\eta\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right)\right| \\ &\le \frac{1}{a}\left(\left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_{nL}\left(A_n^{0\sim q_n-1}\right)\right)^2\eta^2\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right) + a \\ &= 2\left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_{nL}\left(A_n^{0\sim q_n-1}\right)\bar{\eta}\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right) + a,\end{aligned} \tag{63}$$

where

$$\bar{\eta}\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right) = \frac{1}{2a}\left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_{nL}\left(A_n^{0\sim q_n-1}\right)\eta^2\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right). \tag{64}$$

Define the approximation error as follows:

$$\varepsilon_n = \bar{\eta}\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u\right) - \theta_n^{\mathrm{T}}\varphi\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u_{\mathrm{f}}\right), \tag{65}$$

where $|\varepsilon_n| \le \varepsilon_n^*$, ε_n^* being a positive constant, u_{f} is the output of filtered signal

$$u_{\mathrm{f}} = H_{\mathrm{L}}(s)u \approx u \tag{66}$$

and $H_{\mathrm{L}}(s)$ is the Butterworth low-pass filter. Then, (62) is expressed as follows:

$$\begin{aligned}\dot{V}_n &\le -\left\|\xi_n^{(0\sim q_n-1)}\right\|^2 + 2\varpi_n\left(G_n(\cdot) - \frac{\varpi_n}{l_n}\right) \\ &\quad + 2\tilde{\theta}_n^{\mathrm{T}}\left(\left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_{nL}\left(A_n^{0\sim q_n-1}\right)\varphi\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u_{\mathrm{f}}\right) - \dot{\hat{\theta}}_n\right) \\ &\quad + 2\left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_{nL}\left(A_n^{0\sim q_n-1}\right)\varepsilon_n + a.\end{aligned} \tag{67}$$

It is true that

$$2\left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_{nL}\left(A_n^{0\sim q_n-1}\right)\varepsilon_n \le \frac{1}{2}\left\|\xi_n^{(0\sim q_n-1)}\right\|^2 + 2\left\|P_{nL}\left(A_n^{0\sim q_n-1}\right)\right\|^2\varepsilon_n^{*2}, \tag{68}$$

$$2\varpi_n G_n(\cdot) \le \beta + \frac{G_n^2(\cdot)\varpi_n^2}{\beta}. \tag{69}$$

By designing the adaptive law

$$\dot{\hat{\theta}}_n = \left(\xi_n^{(0\sim q_n-1)}\right)^{\mathrm{T}} P_{nL}\left(A_n^{0\sim q_n-1}\right)\varphi\left(x_i^{(0\sim q_i-1)}|_{i=1\sim n}, u_{\mathrm{f}}\right) - \gamma_n\hat{\theta}_n \tag{70}$$

and according to (68) and (69), $\dot{V}_n$ can be obtained as follows:

$$\dot{V}_n \leq -\frac{1}{2}\left\|\xi_n^{(0\sim q_n-1)}\right\|^2 + 2\gamma_n \tilde{\theta}_n^{\mathrm{T}} \hat{\theta}_n - \left(\frac{2}{l_n} - \frac{G_n^2(\cdot)}{\beta}\right)\varpi_n^2 + c_n, \tag{71}$$

where

$$c_n = 2\left\|P_{nL}\left(A_n^{0\sim q_n-1}\right)\right\|^2 \varepsilon_n^{*2} + a + \beta.$$

4 Stability Analysis

So far, according to fully actuated system approach, the adaptive tracking FTC has been completed for high-order nonlinear SFS with non-affine nonlinear faults. Then, a theorem can be summarized as follows.

Theorem 1 *Consider the fuzzy adaptive tracking control of high-order nonlinear SFS with non-affine nonlinear faults* (1), *composed of the virtual controllers* (24) *and* (41), *the actual controllers* (57) *and adaptive laws* (30), (47), *and* (70), *if there exist the positive design parameters* γ_i *and* l_i *satisfy* $\frac{1}{2} - \eta_i > 0$ *and* $\frac{2}{l_i} - \frac{G_i^2}{\beta} - \delta_i > 0$, *then all the signals in the closed-loop system are bounded, and the satisfactory tracking control performance is achieved.*

Proof Select the whole Lyapunov function V as follows:

$$V = \sum_{i=1}^{n} V_i. \tag{72}$$

According to the inequalities (34), (52) and (71), the derivative of V can be given as follows:

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{n}\left(\frac{1}{2} - \eta_i\right)\left\|\xi_i^{(0\sim q_i-1)}\right\|^2 + 2\sum_{i=1}^{n}\gamma_i \tilde{\theta}_i^{\mathrm{T}} \hat{\theta}_i \\ & -\sum_{i=2}^{n}\left(\frac{2}{l_i} - \frac{G_i^2(\cdot)}{\beta} - \delta_i\right)\varpi_i^2 + c_0,\end{aligned} \tag{73}$$

where $c_0 = \sum_{i=1}^{n} c_i$, $\eta_1 = 0$,

$$\eta_i = 8\left\|P_{i-1,L}\left(A_{i-1}^{0\sim q_{i-1}-1}\right)\right\|^2 g_{i-1}^2\left(x_j^{(0\sim q_j-1)}\big|_{j=1\sim i-1}\right), \quad i = 2,\ldots,n,$$

$$\delta_i = 8\left\|P_{i-1,L}\left(A_{i-1}^{0\sim q_{i-1}-1}\right)\right\|^2 g_{i-1}^2\left(x_j^{(0\sim q_j-1)}\big|_{j=1\sim i-1}\right), \quad i = 2,\ldots,n.$$

and $G_i(\cdot)$ satisfy the inequalities $|G_i(\cdot)| \leq \bar{G}_i$ with $\bar{G}_i$ being some positive constants. □

Based on Young's inequality, one has

$$\tilde{\theta}_i^{\mathrm{T}} \hat{\theta}_i = \tilde{\theta}_i^{\mathrm{T}}\left(\theta_i - \tilde{\theta}_i\right) = \tilde{\theta}_i^{\mathrm{T}} \theta_i - \tilde{\theta}_i^{\mathrm{T}} \tilde{\theta}_i \leq -\frac{1}{2}\tilde{\theta}_i^{\mathrm{T}} \tilde{\theta}_i + \frac{1}{2}\theta_i^{\mathrm{T}} \theta_i. \tag{74}$$

Substituting (74) into (73) gives

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{n}\left(\frac{1}{2} - \eta_i\right)\left\|\xi_i^{(0\sim q_i-1)}\right\|^2 - \sum_{i=1}^{n}\gamma_i \tilde{\theta}_i^{\mathrm{T}} \tilde{\theta}_i \\ & -\sum_{i=2}^{n}\left(\frac{2}{l_i} - \frac{\bar{G}_i^2}{\beta} - \delta_i\right)\varpi_i^2 + \sum_{i=1}^{n}\gamma_i \theta_i^{\mathrm{T}} \theta_i + c_0 \\ \leq & -bV + c,\end{aligned} \tag{75}$$

where $c = \sum_{i=1}^{n}\gamma_i \theta_i^{\mathrm{T}} \theta_i + c_0$ and $b = \min\left\{\frac{1}{2} - \eta_i, \gamma_i, \frac{2}{l_i} - \frac{\bar{G}_i^2}{\beta} - \delta_i\right\}$.

5 Simulation Example

In order to illustrate the effectiveness of the proposed control approach, a numerical example is considered in the following

$$\begin{cases} \ddot{x}_1 = f_1(x_1^{(0\sim 2)}) + g_1(x_1^{(0\sim 2)})x_2, \\ \ddot{x}_2 = f_2(x_i^{(0\sim q_i-1)}|_{i=1\sim 2}) + g_2(x_i^{(0\sim q_i-1)}|_{i=1\sim 2})u + l(t - T_0)v\left(x_i^{(0\sim q_i-1)}|_{i=1\sim 2}, u\right), \\ y = x_1, \end{cases} \tag{76}$$

where $f_1(x_1^{(0\sim 2)}) = \sin(\dot{x}_1)\mathrm{e}^{-x_1^4}$, $f_2(x_i^{(0\sim q_i-1)}|_{i=1\sim 2}) = \dot{x}_2 \mathrm{e}^{0.5x_1\dot{x}_1} + \dot{x}_1\sin(x_1 x_2)$, $g_1(x_1^{(0\sim 2)}) = 2 + \sin(x_1\dot{x}_1)$, $g_2(x_i^{(0\sim q_i-1)}|_{i=1\sim 2}) = 3 + 0.5\cos(x_1\dot{x}_1)\sin(x_2\dot{x}_1)$. Select the fault function as $v\left(x_i^{(0\sim q_i-1)}|_{i=1\sim 2}, u\right) = 15(x_1\dot{x}_1 x_2\dot{x}_2 + \sin(u)) + 15$, and the time profile of fault as follows:

$$l(t - T_0) = \begin{cases} 0, & t < T_0, \\ 1 - \mathrm{e}^{-\delta(t-T_0)}, & t \geq T_0, \end{cases}$$

where $\delta = 8$ and $T_0 = 10$ s. The Butterworth low-pass filter is chosen as $H_{\mathrm{L}}(s) = \frac{1}{s^2 + 1.414s + 1}$, and the reference signal is chosen as $y_{\mathrm{r}} = \sin(t)$.

Choose

$$F_1 = \begin{bmatrix} -6 & 1 \\ 0 & -6 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -5 & -1 \\ 1 & -5 \end{bmatrix},$$
$$Z_1 = [1 \quad 0], \quad Z_2 = [1 \quad 1]$$

by Proposition 1, we have

$$V_1 = \begin{bmatrix} Z_1 \\ Z_1 F_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -6 & 1 \end{bmatrix},$$

$$V_2 = \begin{bmatrix} Z_2 \\ Z_2 F_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -4 & -6 \end{bmatrix}$$

and

$$A_1^{0\sim 2} = -Z_1 F_1^2 V_1^{-1} = [36 \quad 12],$$

$$A_2^{0\sim 2} = -Z_2 F_2^2 V_2^{-1} = [26 \quad 10].$$

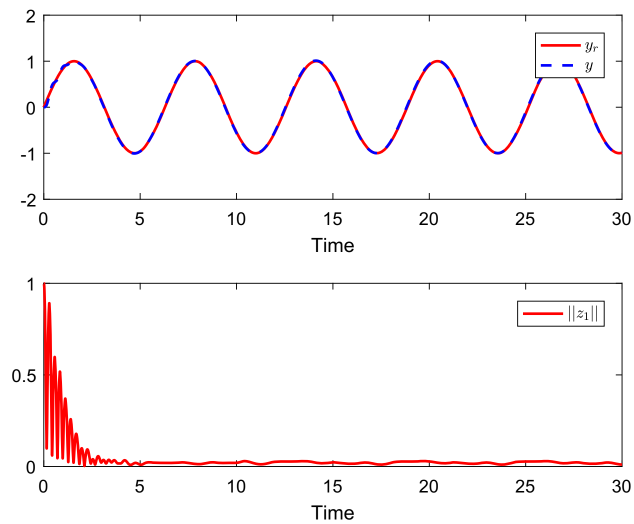

Fig. 1 Tracking performance trajectories

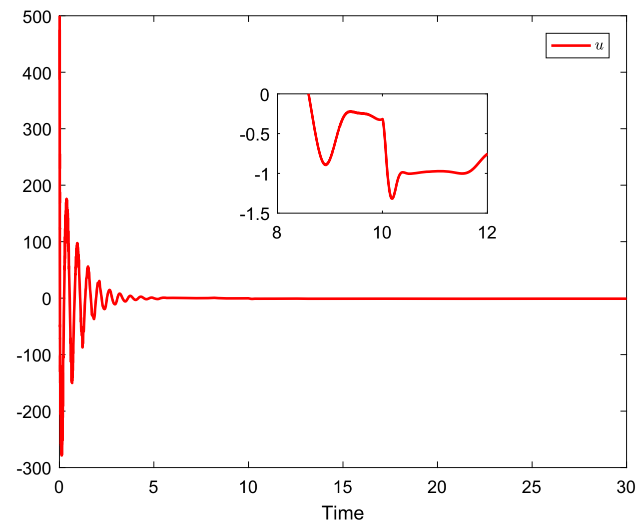

Fig. 3 Trajectory of control signal u

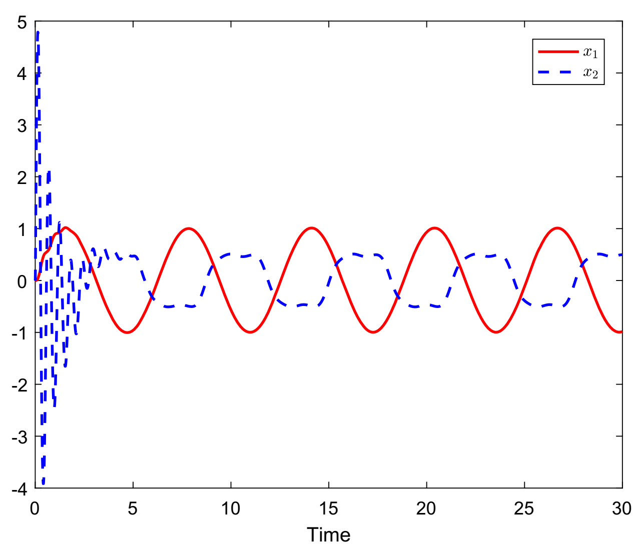

Fig. 2 States trajectories

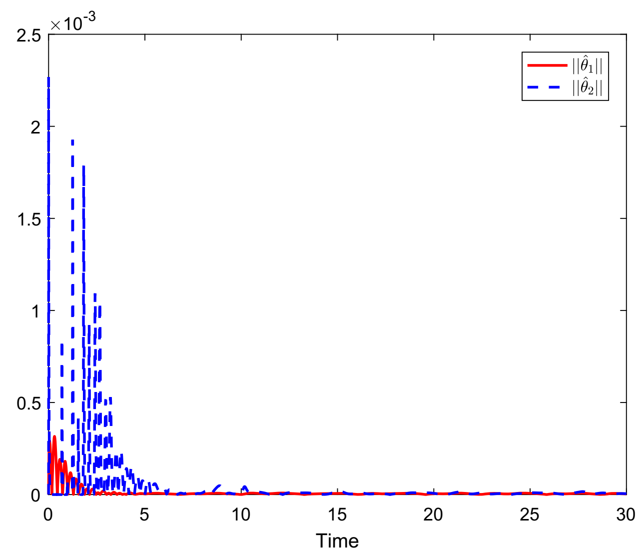

Fig. 4 Trajectories of Norm of adaptive laws estimation $||\hat{\theta}_1||$ and $||\hat{\theta}_2||$

Select the initial values as $x_1(0) = x_2(0) = 0$ and $\theta_1^{\mathrm{T}}(0) = \theta_2^{\mathrm{T}}(0) = [0, 0, 0, 0, 0, 0, 0]$, and choose the design parameters as $\gamma_1 = \gamma_2 = 60$ and $l_2 = 0.01$.

By using the proposed fuzzy adaptive tracking control approach of high-order nonlinear SFS with non-affine nonlinear faults, the simulation results are given in Figs. 1. 2, 3, 4, and 5. The tracking trajectories are displayed in Fig. 1. From Fig. 1, it is clearly seen that the proposed control method in this paper has satisfactory tracking control performance. The states trajectories are shown in Fig. 2. Figure 3 shows the response of the adaptive fuzzy tracking controller. The norm of adaptive laws estimation are shown in Fig. 4. The input and output of first-order filter are shown in Fig. 5. Figures 1, 2, 3, 4, and 5 show that the stability of the high-order nonlinear SFS is guaranteed by using the proposed fuzzy adaptive tracking control method. Besides, the tracking control performance is achieved.

6 Conclusion

An novel adaptive fuzzy FTC method has been investigated for high-order nonlinear SFS with non-affine nonlinear fault. The fuzzy logic systems can be used as approximators of unknown nonlinear functions in the system. There was no need to convert a high-order system to

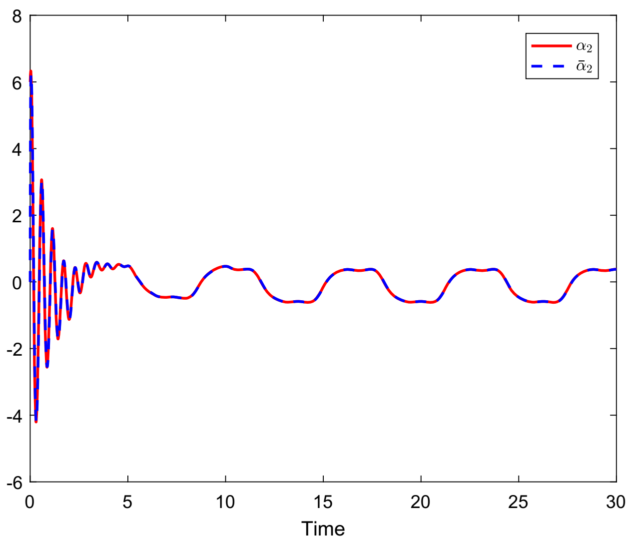

Fig. 5 Trajectories of the filter's input α_2 and output $\bar{\alpha}_2$

first-order one, the controllers have been designed directly for the high-order system, and the control performance can be achieved. In the further, our research scope will be extended to the cooperative control of high-order nonlinear multi-agent systems by using the fully actuated system approach.

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Data Availability The data used to support the findings of this study are available from the corresponding author upon request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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