



A New Reliability Approach for the Fuzzy and Random Structure Based on the Uniformly Distributed Membership Level

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Received: 2 September 2021 / Revised: 7 January 2022 / Accepted: 21 February 2022 / Published online: 23 June 2022
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Abstract For the uncertain structural system with fuzzy uncertainties as well as random ones, a new approach is proposed to assess the reliability of the mixed uncertain structure, which the membership levels of the different fuzzy input variables are treated as random variables with independent and uniform distributions. The possible combination of the membership levels of the different fuzzy variables was considered, which contains more available information. By analyzing the relationship between the proposed approach and traditional approaches, the rationality and advancement of the proposed approach are verified. In order to improve the computation efficiency of the proposed approach, a highly efficient kriging-based solution is established in this paper. Furthermore, two practical engineering examples are used to demonstrate the feasibility of the proposed approach and the efficiency of the proposed solution.

Keywords Fuzzy uncertainty · Membership function · Random uncertainty · Reliability analysis · Failure probability

List of Symbols

M	Output response
$g(\cdot)$	Performance function
X	Input variables
X_F	Fuzzy variables
X_R	Random variables
$\mu(\cdot)$	Membership function of the fuzzy variable
$f(\cdot)$	Probability density function
n_f	Number of the fuzzy variables
n_r	Number of the random variables
n	Number of the input variables
λ	Cut level
$X_F(\lambda)$	Fuzzy variable at cut level λ
$X_F^L(\lambda)$	Lower bound of the fuzzy variable $X_F(\lambda)$
$X_F^U(\lambda)$	Upper bound of the fuzzy variable $X_F(\lambda)$
P_f	Failure probability
$P\{\cdot\}$	Probability operator
Y	Interval variables
η	Non-probabilistic reliability index
$M^{(2)}$	Second level performance response
nwP_f	Failure probability of the proposed approach
nwP_f^U	Upper bound of the failure probability nwP_f
nwP_f^L	Lower bound of the failure probability nwP_f

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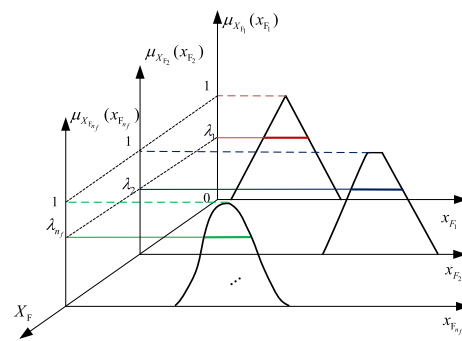
1 Introduction

Random uncertainty extensively exists in engineering applications, which comes from the inherent variation and the unapparent causal relationship. However, due to a lack of sufficient data and insufficient knowledge, there is a lot of non-random uncertainty in structural systems [1–3].

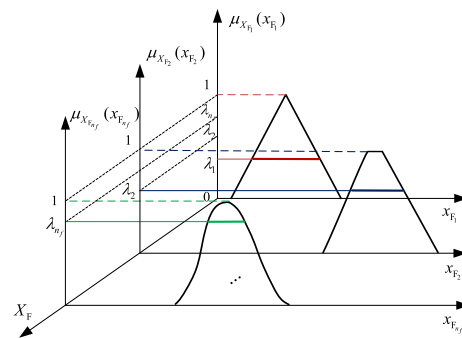
Numerous non-random methods have been developed to deal with the non-random uncertainty, such as interval analysis [4, 5], Dempster–Shafer evidence theory, DSET [6–8], and fuzzy theory [9–11]. Among these methods, fuzzy theory was proved to be an effective method to deal with the fuzzy uncertainty [12].

Based on probability and mathematical statistics theory, the structural reliability analysis approach under the random uncertainty has been widely researched and applied in engineering and academia, such as the first-order reliability method, FORM [13], Monte Carlo simulation, MCS [14], variance reduction techniques [15], etc. However, these random reliability methods are not capable of analyzing the fuzzy uncertain structural system. For this purpose, based on fuzzy theory [9, 16], extensive efforts have been done by researchers to establish fuzzy reliability methods [11, 17–20]. Among them, the membership levels method [21–23] is common, which represents fuzzy variables by membership functions based on their possibility of occurrence or level of confidence. In light of this, a series of reliability methods have been developed for dealing with fuzzy variables, such as the vertex method [24], optimization techniques [25, 26]. Recently, Li et al. [27] proposed a new approach for the fuzzy reliability problem only containing fuzzy uncertainty based on probability perspective, which measures the reliability of a structural system with the failure probability. And Deng and Deng [11] proposed a new approach to measure the information volume of fuzzy membership function including the first-order and the higher-order information volume, which can be used to assess the safety of the fuzzy structural system.

The above research methods consider only one case of fuzzy uncertainties or random ones, but do not accommodate a combination of the two types of uncertainties. However, fuzzy and random uncertainties often present in engineering applications simultaneously. To address this issue, Baudrit et al. [28, 29] a hybrid approach for the system with fuzzy and random uncertainties, which can use to extract the final random fuzzy set and average lower and upper cumulative distributions of the output with respect to a threshold. However, the generalized reliability and distribution of output are more concerned by designers [30]. Möller et al. [31] formulated a fuzzy FORM (FFORM) to analyze the mixed uncertain structural system. The membership function of the reliability index can be estimated using this method, but the failure probability cannot be obtained from the safety index values. Adduri and Penmetsa [32, 33] presented a new technique based on the response surface models and transformations of possibility functions, which does not apply to the performance



(a) The cases of the membership level of the different fuzzy inputs taking the same value ($\lambda_1 = \lambda_2 = \dots = \lambda_n$)



(b) The cases of the membership level of different fuzzy inputs taking the different values ($\lambda_1 \neq \lambda_2 \neq \dots \neq \lambda_n$)

Fig. 1 The diagram for the combination of the membership levels of the different fuzzy variables

function including cross items of the fuzzy and the random uncertainties. In order to improve the computational efficiency of the membership function of the fuzzy reliability, Li et al. [34] proposed the saddlepoint approximation based on the line sampling method, Ebenuwa and Tee [35] presented a numerical strategy using a fuzzy-based optimized subset simulation (SS) approach, and You et al. [36] employed a random fuzzy support vector machine based on the particle swarm optimization (PSO) method to obtain the membership function of reliability.

These methods above obtaining the analysis result are the membership function of reliability, while engineering designers are more likely to understand probability than membership function. In addition, the above methods regard membership levels of different fuzzy uncertain inputs as the same value as shown in Fig. 1a, without considering the case that membership levels of different fuzzy variables have different values as shown in Fig. 1b. In the Fig. 1, $\mu_{x_{Fi}}(x_{Fi})$ ($i = 1, 2, \dots, n_f$) denote the membership functions of the fuzzy input variables

$\mathbf{X}_F = (X_{F1}, X_{F2}, \dots, X_{Fn_f})$, and λ_i ($i = 1, 2, \dots, n_f$) denotes the membership lever (cut level) of X_{Fi} .

In this paper, a new mixed uncertain reliability approach is proposed, which given the assessment result is an interval value of failure probability or reliability. And the possible combinations of the different fuzzy variables with different membership levels can be considered. The membership level taking value $[0, 1]$ of the fuzzy variable is treated as the random variable obeying standard uniform distribution. According to the fuzzy cut set theory, the fuzzy variable can be represented as the interval variable at each membership level. Then, the fuzzy and random mixed uncertain structural system is converted into the interval variables (at the given membership level) and the random mixed structural system. The non-probabilistic reliability model [37] is employed to deal with the interval and random mixed uncertain system. Considering to the membership level obeying standard uniform distribution, the second level performance function [37] (relative to the original performance function) containing original random variables and standard uniformly variables is successive established. By taking the membership levels of the different fuzzy input variables as the same uniform distribution, the relationship between the traditional approaches and the proposed approach is rigorously verified in theory. By the several engineering examples, the rationality and feasibility of the proposed approach are testified.

The remainder of this paper is organized as follows: In Sect. 2 the description of uncertainty propagation from fuzzy and random variables to the structural response is introduced. A new reliability approach based on uniformly distributed membership level is proposed for the mixed uncertain system and the relationship with the traditional approaches is discussed in Sect. 3. In Sect. 4 a highly efficient kriging-based solution is introduced for the proposed approach. In Sect. 5 two practical examples are employed to test the rationality and superiority of the proposed approach, and the applicability of the kriging-based solution. Finally, the conclusion comes at the end of this paper.

2 The Propagation of the Fuzzy and Random Uncertainties

Since the fuzzy and random variables exist in a structural system simultaneously, the output response of a structure would be fuzzy and random.

Denote the performance function of a structural system determined by the failure criterion as follows:

$$M = g(\mathbf{X}) = g(\mathbf{X}_F, \mathbf{X}_R) = g(X_{F1}, X_{F2}, \dots, X_{Fn_f}, X_{R1}, X_{R2}, \dots, X_{Rn_r}), \tag{1}$$

where M is the output response and $g(\mathbf{X})$ is the performance function of a structural system. $\mathbf{X} = (X_1, X_2, \dots, X_n)$ are input variables, in which $\mathbf{X}_F = (X_{F1}, X_{F2}, \dots, X_{Fn_f})$ are fuzzy input variables represented by membership functions $\mu_{X_{Fi}}(x_{Fi})$ ($i = 1, 2, \dots, n_f$), and $\mathbf{X}_R = (X_{R1}, X_{R2}, \dots, X_{Rn_r})$ are random input variables with the corresponding probability density functions (PDFs) $f_{X_{Rj}}(x_{Rj})$ ($j = 1, 2, \dots, n_r$), and $n_f + n_r = n$.

According to the content of cut sets, at each cut level (membership level) λ , the variation of the fuzzy variable is defined by a lower bound and an upper bound, i.e., $X_F(\lambda) \in [X_F^L(\lambda), X_F^U(\lambda)]$ at cut level λ . For the reliability model in the presence of both the fuzzy variables and the random ones, the output response will also be an interval at each cut level. The effect of the fuzzy uncertainties on the performance function at the cut level λ is expressed by Eq. (2). For the case of two random variables, the effect of the fuzzy variables on the performance function is shown in Fig. 2.

$$g(\mathbf{X}_F(\lambda), \mathbf{X}_R) \in [g_{\min}(\mathbf{X}_F(\lambda), \mathbf{X}_R), g_{\max}(\mathbf{X}_F(\lambda), \mathbf{X}_R)]. \tag{2}$$

Furthermore, according to random reliability analysis and probabilistic knowledge, the bounds of the failure probability of the mixed structural system can be obtained as:

$$P_f^L(\lambda) = P\{g_{\max}(\mathbf{X}_F(\lambda), \mathbf{X}_R) \leq 0\}, \tag{3}$$

$$P_f^U(\lambda) = P\{g_{\min}(\mathbf{X}_F(\lambda), \mathbf{X}_R) \leq 0\}, \tag{4}$$

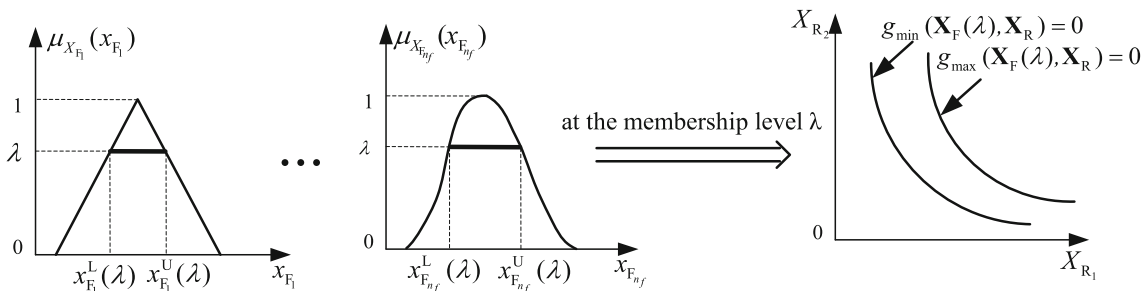


Fig. 2 The effect of the fuzzy uncertainties on the performance function

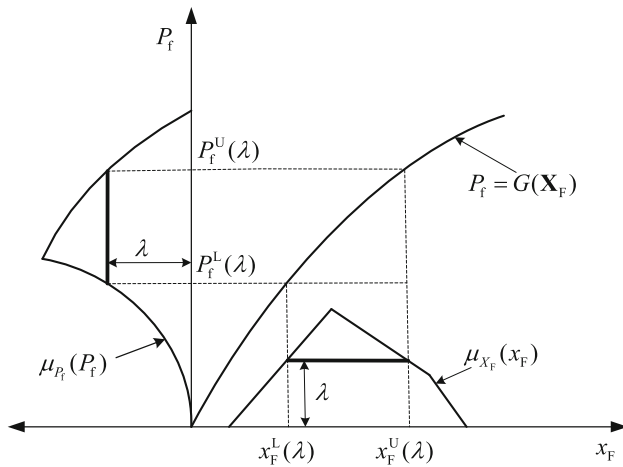


Fig. 3 The propagation of the fuzzy uncertainties for the failure probability

where $P\{\cdot\}$ means the probability of the given inequality.

The membership function of the failure probability for the traditional approach can be obtained by calculating the lower and upper bounds of the failure probability at each membership level. Figure 3 shows the fuzzy uncertainties propagation from input variables to failure probability of the mixed uncertain system, in which, the possibility distribution of the fuzzy failure probability is described by the fuzzy membership function $\mu_{P_f}(P_f)$. $P_f = G(\mathbf{X}_F)$ means $P_f = P\{g(\mathbf{X}_F, \mathbf{X}_R) \leq 0\} = \int_{g(\mathbf{X}_F, \mathbf{X}_R) \leq 0} f_{\mathbf{X}_R}(\mathbf{x}_R) d\mathbf{x}_R = G(\mathbf{X}_F)$. $P_f^L(\lambda)$ and $P_f^U(\lambda)$ mean the lower bound and upper of the fuzzy failure probabilities with respect to membership level λ .

3 A New Reliability Approach for the Mixed Uncertain Structural System

The traditional approaches for a mixed uncertain structural system take the membership levels of the different fuzzy variables as the same value. As discussed above, for each cut level λ , the fuzzy variable X_{F_i} degenerates into an interval variable $[X_{F_i}^L(\lambda), X_{F_i}^U(\lambda)]$, so it changes into a problem that contains the interval variables and random ones at membership level λ . The membership levels are taken multiple values in region $[0, 1]$. By calculating the bounds of the failure probabilities at corresponding membership levels, the membership function of the failure probability can be obtained.

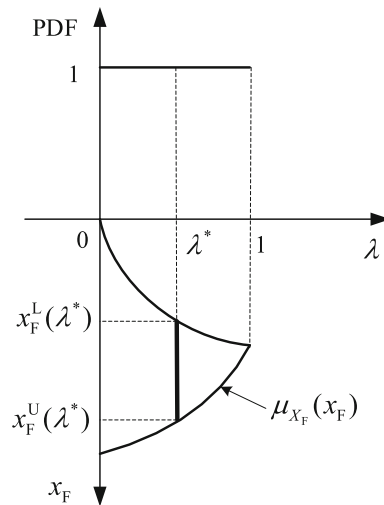


Fig. 4 The membership interval of the fuzzy variable at the realization λ^*

The membership levels of the fuzzy variables are treated as the random variables obeying standard uniform distribution, i.e., $\lambda_i \sim U(0, 1)$. The relationship between the membership interval of the fuzzy variable and the random variable λ is shown in Fig. 4.

When λ and \mathbf{X}_R are fixed at their realization values λ^* and \mathbf{X}_R^* , respectively, the performance function can be rewritten as:

$$M = g(\mathbf{X}_F(\lambda^*), \mathbf{X}_R^*), \tag{5}$$

where $\mathbf{X}_F(\lambda^*)$ are interval variables at the membership levels λ^* , i.e., $\mathbf{X}_F(\lambda^*) \in [X_{F_i}^L(\lambda^*), X_{F_i}^U(\lambda^*)]$.

There are only interval variables in the performance function Eq. (5). The non-probabilistic reliability model [38, 39] based on interval analysis is a good tool to deal with interval uncertain problem. Next, the detail of the non-probabilistic reliability model will be briefly reviewed as follows.

3.1 A Review of the Non-probabilistic Reliability Model

Denote $\mathbf{Y} = (Y_1, Y_2, \dots, Y_m)$ as the set of the interval variables for a structural system whose performance function is expressed as:

$$M_I = g(\mathbf{Y}) = g(Y_1, Y_2, \dots, Y_m), \tag{6}$$

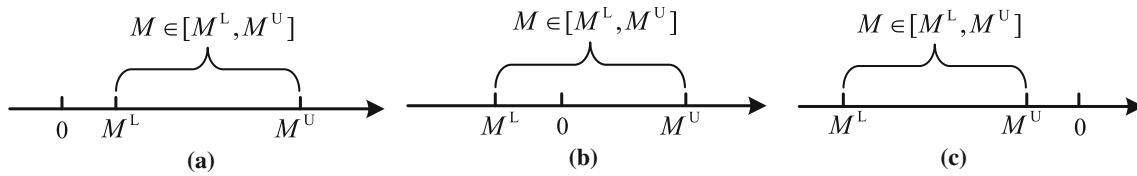


Fig. 5 The significance of the non-probabilistic reliability index

where $Y_k \in [Y_k^L, Y_k^U]$ ($k = 1, 2, \dots, m$), Y_k^L and Y_k^U are the lower and upper bounds of the interval variable Y_k , respectively. Thus, the center and radius of the interval variable can be formulated as $Y_k^C = (Y_k^L + Y_k^U)/2$ and $Y_k^R = (Y_k^U - Y_k^L)/2$, respectively.

The response M_I is also an interval variable with the lower and upper bounds $[M_I^L, M_I^U]$ on the condition that $g(\cdot)$ is a continuum function of the input interval variables \mathbf{Y} . The center and radius of the interval response M are expressed by $M_I^C = (M_I^L + M_I^U)/2$ and $M_I^R = (M_I^U - M_I^L)/2$, respectively.

The following formulation is proposed to measure the safety of the structural system with interval variables [38, 39], which is called non-probabilistic reliability index:

$$\eta = M_I^C / M_I^R = (M_I^L + M_I^U) / (M_I^U - M_I^L). \tag{7}$$

According to reliability theory, the hypersurface $g(\mathbf{Y}) = 0$ is defined as the limit state surface, which divides the input variable space into the failure domain $\Omega_f = \{\mathbf{Y} : g(\mathbf{Y}) < 0\}$ and the safe domain $\Omega_s = \{\mathbf{Y} : g(\mathbf{Y}) > 0\}$. According to Eq. (7), the following conclusions can be drawn.

(1) If the inequality $\eta > 1$ holds, namely, $M_I^L > 0$, then $g(\mathbf{Y}) > 0$ for any Y_k . This means that the structural system must be safe, and the bigger η is, the more reliable the structural system is, as shown in Fig. 5a.

(2) If the inequality $\eta < -1$ holds, namely $M_I^U < 0$, then $g(\mathbf{Y}) < 0$ for any Y_k , which indicates an unavoidable failure, as shown in Fig. 5c.

(3) If the inequality $-1 \leq \eta \leq 1$, then $g(\mathbf{Y}) > 0$ or $g(\mathbf{Y}) < 0$, which means whether the structural system is safe or not cannot be distinguished, and this case is shown in Fig. 5b.

If the performance function Eq. (6) is linear, it can be rewritten as:

$$M_I = g(\mathbf{Y}) = a_0 + \sum_{k=1}^m a_k Y_k \quad (k = 1, 2, \dots, m). \tag{8}$$

The analytic expression of the non-probabilistic reliability index η can be derived as:

$$\eta = \frac{a_0 + \sum_{k=1}^m a_k Y_k^C}{\sum_{k=1}^m |a_k| Y_k^R}. \tag{9}$$

3.2 The Mixed Uncertain Reliability Approach Based on the Uniformly Distributed Membership Level

According to the discussion in Sect. 3.1, the non-probabilistic reliability index of Eq. (5) can be obtained as:

$$\eta(\lambda^*, \mathbf{X}_R^*) = M^C(\lambda^*, \mathbf{X}_R^*) / M^R(\lambda^*, \mathbf{X}_R^*). \tag{10}$$

Considering realizations of λ^* and \mathbf{X}_R^* vary in the range of the random variables λ and \mathbf{X}_R , respectively, a new performance function can be formulated, which is called the second level performance function to distinguish from the original performance function in this paper, as:

$$M^{(2)} = \eta(\lambda, \mathbf{X}_R). \tag{11}$$

$M^{(2)}$ is the function of the random variables \mathbf{X}_R and random variables λ following uniform distribution in $[0, 1]$. As mentioned in Sect. 3.1, if inequality $\eta(\lambda, \mathbf{X}_R) > 1$ holds, the structural system must be safe; if inequality $\eta(\lambda, \mathbf{X}_R) < -1$ holds, the structural system must be failure. Following the two conditions, two limit state equations can be obtained as:

$$M^{(2)} - 1 = 0, \quad \text{i.e. } \eta(\lambda, \mathbf{X}_R) - 1 = 0, \tag{12}$$

$$M^{(2)} + 1 = 0, \quad \text{i.e. } \eta(\lambda, \mathbf{X}_R) + 1 = 0. \tag{13}$$

The upper bound of the failure probability of the structural system can be obtained by the limit state equation $\eta(\lambda, \mathbf{X}_R) - 1 = 0$, while the lower bound of failure probability can be obtained by the limit state equation $\eta(\lambda, \mathbf{X}_R) + 1 = 0$. According to random reliability theory, the bounds can be expressed, respectively, as:

$$\begin{aligned} \text{nw}P_f^U &= P\{\eta(\lambda, \mathbf{X}_R) - 1 \leq 0\} \\ &= \int_{\eta(\lambda, \mathbf{X}_R) - 1 \leq 0} f_\lambda(\lambda) f_{\mathbf{X}_R}(\mathbf{x}_R) d\lambda d\mathbf{x}_R, \end{aligned} \tag{14}$$

$$\begin{aligned} \text{nw}P_f^L &= P\{\eta(\lambda, \mathbf{X}_R) + 1 \leq 0\} \\ &= \int_{\eta(\lambda, \mathbf{X}_R) + 1 \leq 0} f_\lambda(\lambda) f_{\mathbf{X}_R}(\mathbf{x}_R) d\lambda d\mathbf{x}_R, \end{aligned} \tag{15}$$

where $f_\lambda(\lambda)$ and $f_{\mathbf{X}_R}(\mathbf{x}_R)$ are the joint PDF of the random variables λ and \mathbf{X}_R , respectively, and $d\lambda = d\lambda_1 d\lambda_2 \dots d\lambda_{n_f}$, $d\mathbf{x}_R = dx_{R1} dx_{R2} \dots dx_{Rn_r}$.

For an uncertain structural system with fuzzy and random variables, the precise failure probability cannot be

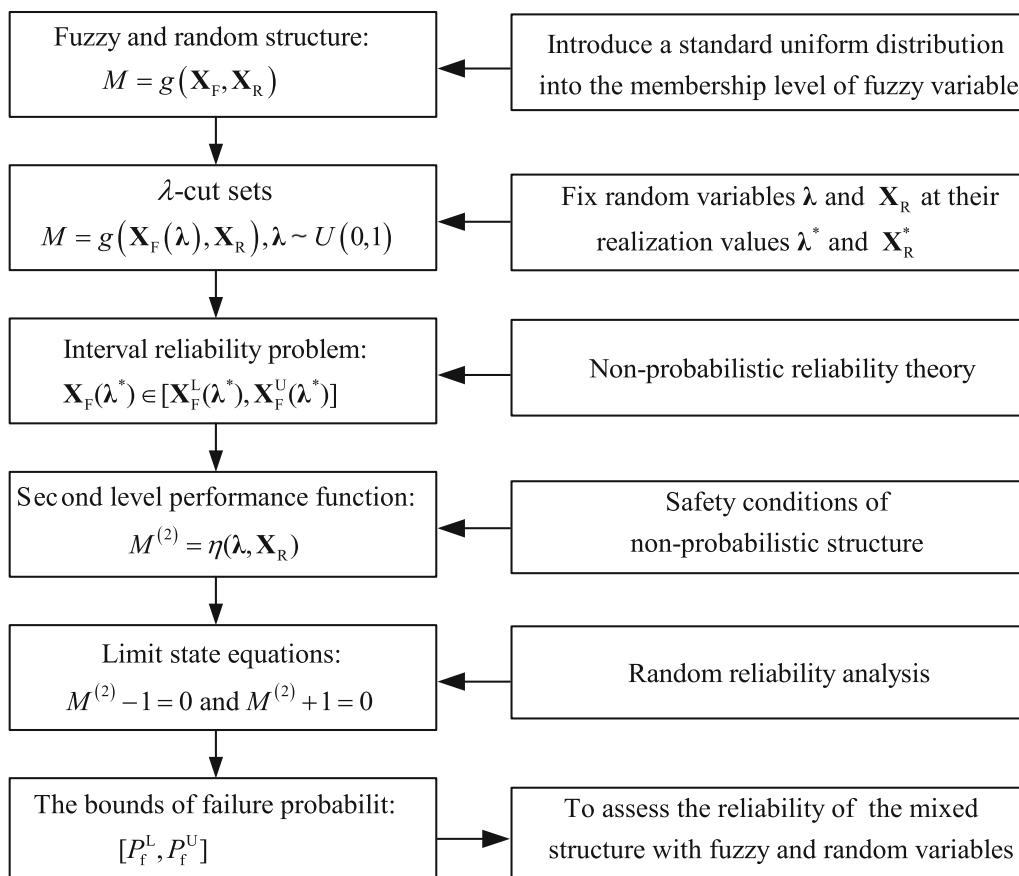


Fig. 6 The flowchart of the proposed new approach

obtained like the random reliability problem. This proposed approach uses the interval of failure probability $[nwP_f^L, nwP_f^U]$ to assess the reliability of the mixed system instead of the traditional membership function of failure probability. This proposed approach can implement the combination of the membership levels of the different fuzzy variables.

If the performance function of the mixed uncertain structural system Eq. (1) is linear, it can be expressed as:

$$M = g(\mathbf{X}_F, \mathbf{X}_R) = \sum_{i=1}^{n_f} a_i X_{Fi} + \sum_{j=1}^{n_r} b_j X_{Rj}. \quad (16)$$

After introducing the standard uniform distributions into the membership levels of the fuzzy input variables, Eq. (16) can be rewritten as:

$$M = g(\boldsymbol{\lambda}, \mathbf{X}_R) = \sum_{i=1}^{n_f} a_i X_{Fi}(\lambda_i) + \sum_{j=1}^{n_r} b_j X_{Rj}. \quad (17)$$

The second level performance function, i.e., the non-probabilistic index function, of Eq. (17) can be derived as:

$$M^{(2)} = \eta(\boldsymbol{\lambda}, \mathbf{X}_R) = \frac{\sum_{i=1}^{n_f} a_i X_{Fi}^C(\lambda_i) + \sum_{j=1}^{n_r} b_j X_{Rj}}{\sum_{i=1}^{n_f} |a_i| X_{Fi}^R(\lambda_i)}. \quad (18)$$

Assuming the membership function of the fuzzy variable X_{Fi} be the normal type, i.e.,

$$\mu_{X_{Fi}}(x_{Fi}) = \exp\left\{-\frac{(x_{Fi} - m_{X_{Fi}})^2}{2\sigma_{X_{Fi}}^2}\right\}, \quad (19)$$

where $m_{X_{Fi}}$ and $\sigma_{X_{Fi}}$ are the mean value and deviation of the fuzzy variable X_{Fi} , respectively.

The center and radius of the interval variable $X_{Fi}(\lambda_i)$ at the membership level λ_i can be expressed as:

$$\begin{cases} X_{Fi}^C(\lambda_i) = m_{X_{Fi}}, \\ X_{Fi}^R(\lambda_i) = \sqrt{-2 \ln \lambda_i} \sigma_{X_{Fi}}. \end{cases} \quad (20)$$

Submitting Eq. (20) into Eq. (18), the second level performance function can be expressed as:

$$M^{(2)} = \eta(\boldsymbol{\lambda}, \mathbf{X}_R) = \frac{\sum_{i=1}^{n_f} a_i m_{X_{Fi}} + \sum_{j=1}^{n_r} b_j X_{Rj}}{\sum_{i=1}^{n_f} |a_i| \sqrt{-2 \ln \lambda_i} \sigma_{X_{Fi}}}. \quad (21)$$

For Eq. (21), random reliability approaches can be used to implement the reliability analysis.

The flowchart of the proposed approach for the mixed uncertain structural system is given in Fig. 6.

3.3 The Relationship Between the Proposed Approach and the Traditional Ones

In order to discuss the relationship between the proposed approach and traditional approaches, a special situation of the proposed approach where the membership levels of the fuzzy variables are taken as the same value, i.e., $\lambda_1 = \lambda_2 = \dots = \lambda_{n_f} = \lambda \sim U(0, 1)$, is considered. And the process of taking λ by the traditional approaches can be seen as a random process which takes the value following the uniform distribution in $[0, 1]$.

Firstly, the upper bound of the failure probability is discussed. Because $\lambda_1 = \lambda_2 = \dots = \lambda_{n_f} = \lambda$, the upper bound of the failure probability Eq. (14) is rewritten as:

$$nwP_f^U = \int_{\eta(\lambda, \mathbf{X}_R) - 1 \leq 0} f_\lambda(\lambda) f_{\mathbf{X}_R}(\mathbf{x}_R) d\lambda d\mathbf{x}_R. \tag{22}$$

In Eq. (22), changing the integral order, it can be expressed as:

$$nwP_f^U = \int_0^1 \left[\int_{\eta(\lambda, \mathbf{X}_R) - 1 \leq 0} f_{X_{R1}}(x_{R1}) f_{X_{R2}}(x_{R2}) \dots f_{X_{Rn_r}}(x_{Rn_r}) dx_{R1} dx_{R2} \dots dx_{Rn_r} \right] f(\lambda) d\lambda. \tag{23}$$

The integral term inner bracket of the right side of Eq. (23) can be expressed as:

$$\int_{\eta(\lambda, \mathbf{X}_R) - 1 \leq 0} f_{X_{R1}}(x_{R1}) f_{X_{R2}}(x_{R2}) \dots f_{X_{Rn_r}}(x_{Rn_r}) dx_{R1} dx_{R2} \dots dx_{Rn_r} = P\{\eta(\lambda, \mathbf{X}_R) - 1 \leq 0\}. \tag{24}$$

According to the defined of the non-probabilistic reliability index, $\eta(\lambda, \mathbf{X}_R) - 1$ can be expanded as:

$$\begin{aligned} \eta(\lambda, \mathbf{X}_R) - 1 &= \frac{M^L(\lambda, \mathbf{X}_R) + M^U(\lambda, \mathbf{X}_R)}{M^U(\lambda, \mathbf{X}_R) - M^L(\lambda, \mathbf{X}_R)} - 1 \\ &= \frac{2M^L(\lambda, \mathbf{X}_R)}{M^U(\lambda, \mathbf{X}_R) - M^L(\lambda, \mathbf{X}_R)}. \end{aligned} \tag{25}$$

It can be seen that $M^U(\lambda, \mathbf{X}_R) - M^L(\lambda, \mathbf{X}_R)$ in Eq. (25) is always large than zero. If inequality $\eta(\lambda, \mathbf{X}_R) - 1 \leq 0$ satisfies, inequality $M^L(\lambda, \mathbf{X}_R) \leq 0$ must hold, i.e., $g_{\min}(\mathbf{X}_R, \mathbf{X}_F(\lambda)) \leq 0$ holds. So the following equation can be derived as:

$$\begin{aligned} P\{\eta(\lambda, \mathbf{X}_R) - 1 \leq 0\} &= P\{M^L(\lambda, \mathbf{X}_R) \leq 0\} \\ &= P\{g_{\min}(\mathbf{X}_R, \mathbf{X}_F(\lambda)) \leq 0\}. \end{aligned} \tag{26}$$

Then, Eq. (24) can be rewritten as:

$$\begin{aligned} &\int \dots \int_{\eta(\lambda, \mathbf{X}_R) - 1 \leq 0} f_{X_{R1}}(x_{R1}) f_{X_{R2}}(x_{R2}) \dots f_{X_{Rn_r}}(x_{Rn_r}) dx_{R1} dx_{R2} \dots dx_{Rn_r} \\ &= P\{g_{\min}(\mathbf{X}_R, \mathbf{X}_F(\lambda)) \leq 0\} = P_f^U(\lambda), \end{aligned} \tag{27}$$

where $P_f^U(\lambda)$ is the upper bound of the failure probability at the membership level λ . And it also denotes the left branch of the membership function of the failure probability for the traditional approaches.

Substituting Eq. (27) into Eq. (23), the following equation can be obtained as:

$$nwP_f^U = \int_0^1 P_f^U(\lambda) f(\lambda) d\lambda = E[P_f^U(\lambda)]. \tag{28}$$

The same conclusion for the lower bound of the failure probability can be derived as:

$$nwP_f^L = \int_0^1 P_f^L(\lambda) f(\lambda) d\lambda = E[P_f^L(\lambda)], \tag{29}$$

where $P_f^L(\lambda)$ is the right branch of the membership function of the failure probability.

In the traditional approaches, the membership level is discretized into uniform values in $[0, 1]$, and the membership function of the failure probability can be regarded as the failure probability interval at the different membership levels. Through the above proof, the following conclusion can be obtained as: the lower and upper bounds of the failure probability obtained under the membership levels of the different fuzzy variables taking as the same random value are numerically equal to the mathematical expectation of the left and right branches of the membership function of the failure probability calculated by the traditional approaches, respectively.

The proposed approach provides the interval value of failure probability, which can guide the engineer to realize the failure of the mixed structural system from the probabilistic perspective. Furthermore, when the membership levels of the different fuzzy variables are taken different values, the results more accurately reflect the essence of the mixed uncertain structural system and provide more valuable information for the reliability analysis.

4 The Solution for the Proposed Approach

Since there are only random variables λ and \mathbf{X}_R in the second level performance function, the solution for the proposed approach is a random reliability problem. However, it is worth pointing that if the original performance function is non-monotonic with respect to the fuzzy variables, the optimization method should be employed to calculate the corresponding non-probabilistic reliability

index. The approximation methods of the random reliability, such as the FORM and the second-order reliability method (SORM) are widely adopted in the random reliability analysis. But these methods are not appropriate for the case of multiple most probable points (MPPs), besides, they depend on MPP convergence and the estimated results are affected by the precision of the performance function approximation. MCS is a basic reference approach and is widely used in reliability analysis. However, for the implicit structural systems where the finite element method (FEM) is needed to obtain output, the applications of MCS are restricted by the large computational cost. The surrogate model approaches which can use small sample points to well fit the original performance function are a good choice for the reliability analysis, among which the kriging method has been deeply researched and widely applied to the structural reliability analysis [40–43].

In order to efficiently solve the proposed approach, the kriging method [42] as an available alternative is employed to construct the surrogate model of the second level performance function. Using the constructed surrogate model to evaluate the bounds of the failure probability no longer calls the original performance function. The expected feasibility function, EFF [40] is used to ensure the surrogate model accuracy in this paper.

The procedure of the kriging-based solution to solve the proposed approach is simply introduced as follows:

- (1) Use the Latin hypercube sampling (LHS) to generate $N = (n + 1)(n + 2)/2$ samples $(\lambda_t, \mathbf{x}_{Rt}) = (\lambda_{1,t}, \dots, \lambda_{n_f,t}, x_{R1,t}, \dots, x_{Rn_r,t})$ ($t = 1, 2, \dots, N$) of the random variables λ and random input variables \mathbf{X}_R according to their joint PDF, where $n = n_f + n_r$.
- (2) Calculate the non-probabilistic reliability indices η_t ($t = 1, 2, \dots, N$) corresponding to these sample points. If performance function M is non-monotonic with respect to the fuzzy variables, the optimization method is employed to compute η_t in this step.
- (3) Construct an initial surrogate model of the second level performance function, which is denoted as $\hat{M}^{(2)}$.
- (4) Find the point U with maximum EFF using the global optimizer. And judge whether the maximum value is less than the given threshold (taking 0.001 in this paper). If so, turn to step 6 directly.
- (5) Compute non-probabilistic reliability index at this point U . And add this new sample to experiment points and loop back to step 3.
- (6) Use the surrogate model $\hat{M}^{(2)}$ to evaluate the bounds of the failure probability by MCS.

The procedure of the kriging-based solution for the proposed approach is shown in Fig. 7.

The kriging-based solution developed by this paper is one of the efficient approaches. Certainly, other efficient random reliability approaches all can be used to solve the interval of failure probability.

5 Examples

Two engineering examples in this subsection are used to demonstrate the rationality advantages of the proposed approach.

Example 1 A roof truss shown in Fig. 8 is investigated, the top boom and the compression bars are reinforced by concrete, and the bottom boom and the tension bars are made of steel. Assume the uniformly distributed load q is applied on the roof truss, and the uniformly distributed load can be transformed into the nodal load $P = ql/4$. The perpendicular deflection Δ_C of the node C can be obtained by the mechanical analysis, and it is the function of the input variables, i.e., $\Delta_C = \frac{ql^2}{2} \left(\frac{3.81}{A_C E_C} + \frac{1.13}{A_S E_S} \right)$, where A_C , A_S , E_C , E_S and l respectively represent the sectional area, elastic modulus, length of the concrete, and that of the steel bars. Considering the safety and the applicability, Δ_C of the node C not exceeding 3 cm is taken as the constraint condition, the performance function can be constructed by $g = 0.03 - \Delta_C$. The variables A_C , A_S and q are assumed as the independently normal random variables with the distribution parameters given in Table 1. l (m), E_C (MPa) and E_S (MPa) are assumed as the fuzzy input variables with the membership functions as follows:

$$\mu_l(l) = \exp\left\{-\frac{(l-12)^2}{2 \times 0.12^2}\right\};$$

$$\mu_{E_C}(E_C) = \begin{cases} (E_C - 1.88 \times 10^{10})/1.2 \times 10^9 & 1.88 \times 10^{10} \leq E_C \leq 2 \times 10^{10}, \\ (E_C - 2.12 \times 10^{10})/(-1.2 \times 10^9) & 2 \times 10^{10} \leq E_C \leq 2.12 \times 10^{10}, \end{cases}$$

$$\mu_{E_S}(E_S) = \begin{cases} (E_S - 0.94 \times 10^{11})/6 \times 10^9 & 0.94 \times 10^{11} \leq E_S \leq 1 \times 10^{11}, \\ (E_S - 1.06 \times 10^{11})/(-6 \times 10^9) & 1 \times 10^{11} \leq E_S \leq 1.06 \times 10^{11}. \end{cases}$$

The results of the fuzzy reliability analysis computed by the kriging-based solution and MCS method are listed in Table 2.

Table 2 shows that the kriging-based solution is highly efficient and accurate to solve the proposed approach. It can be seen from Table 2 that the interval width of the failure probability under the membership levels of the different fuzzy variables taking different random variables is less than the one where the membership levels of the different fuzzy variables take the same random variable, which means the former uncertainty is less than the latter one because of the width of an interval variable represents an uncertain extent. This illustrates more available

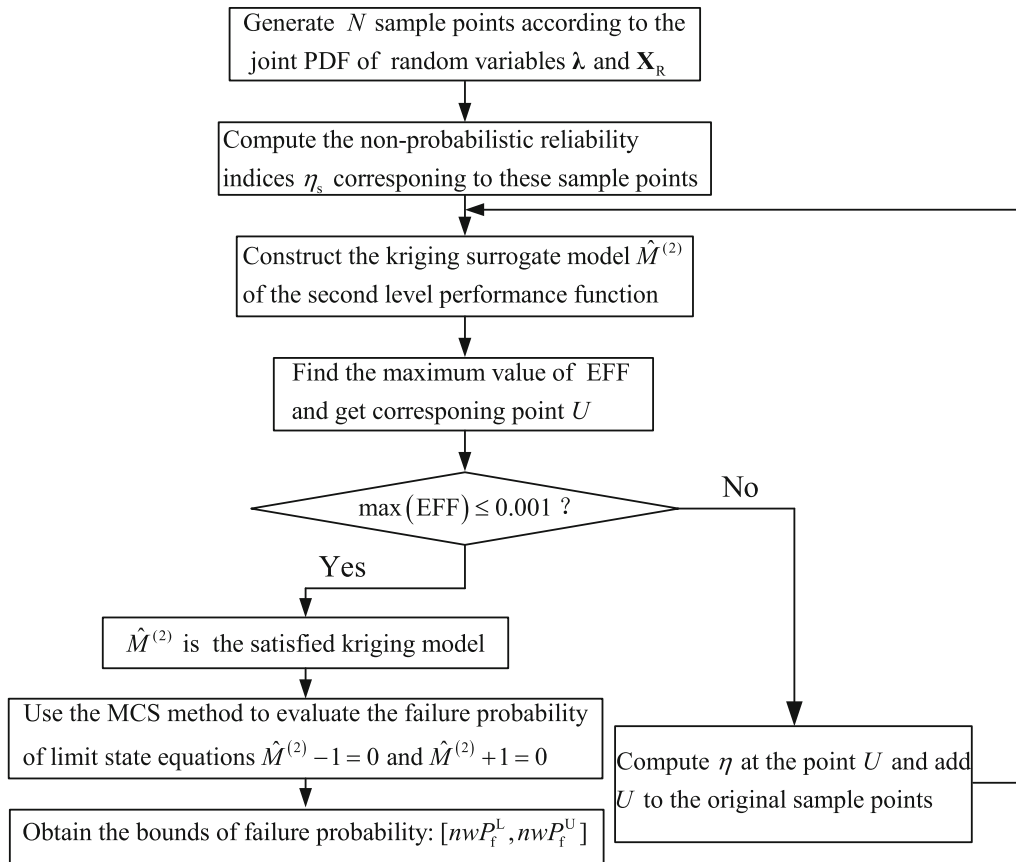


Fig. 7 The flowchart of the kriging-based solution

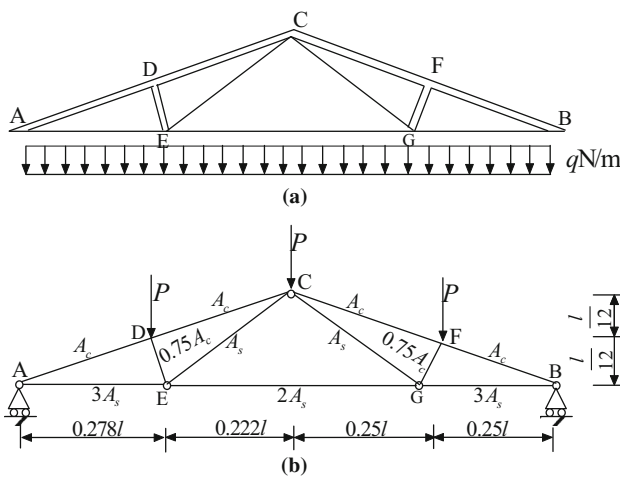


Fig. 8 The schematic diagram of the roof truss

information is adopted for the case of the membership levels treated as independently standard uniform distributions. The proposed approach considers the possible combination of the membership levels of the different fuzzy variables.

Furthermore, in order to illustrate the relationship between the proposed approach and the traditional

approaches more clearly, the bounds of the failure probability obtained by the proposed approach and the membership function of the failure probability by traditional approaches are shown in Fig. 9, where $nwP_f(\lambda_1 = \lambda_2 = \dots = \lambda_{n_f})$ denotes the failure probability with the membership levels of the fuzzy variables treated as the same random variable i.e., $\lambda_1 = \lambda_2 = \dots = \lambda_{n_f}$; $nwP_f(\lambda)$ denotes the failure probability with the membership levels of the different fuzzy variables being treated as the independently standard uniform distributions. The proposed approach assesses the failure of the mixed structural system from the probabilistic perspective, which is more familiar to the engineer than the membership function of failure probability.

To show the effect of the fuzzy variables l, E_C and E_S on the output response of the structural system, the upper and lower bounds of cumulative distribution function (CDF) of g are computed by the kriging-based solution and plotted in Fig. 10. The gap between the two curves reflects the effect of the fuzzy variables on g . Since the gap between the two curves is great, the effect of the fuzzy variables l, E_C and E_S is significant. The significance of this graph is to remind the engineer to pay more attention to the fuzzy variables,

Table 1 The distribution parameters of the input random variables for Example 1

Random variable	Mean value	Standard deviation	Coefficient of variation
q (N/m)	20,000	2000	0.10
A_S (m ²)	9.82×10^{-4}	5.892×10^{-5}	0.06
A_C (m ²)	0.04	0.0048	0.12

Table 2 The results of the reliability analysis for Example 1

	$\lambda_1 = \lambda_2 = \dots = \lambda_{n_f}$ *		$\lambda = (\lambda_1, \lambda_2, \dots, \lambda_{n_f})^{**}$	
	MCS	Kriging-based solution	MCS	Kriging-based solution
nwP_f^L	1.816×10^{-4}	1.7863×10^{-4}	1.4574×10^{-4}	1.4350×10^{-4}
nwP_f^U	5.9834×10^{-3}	5.7985×10^{-3}	4.9725×10^{-3}	5.1296×10^{-3}
Sample points	5×10^6	694	5×10^6	755

*The membership levels of the different fuzzy variables taken the same value

**The membership levels of the different fuzzy variables taken different values

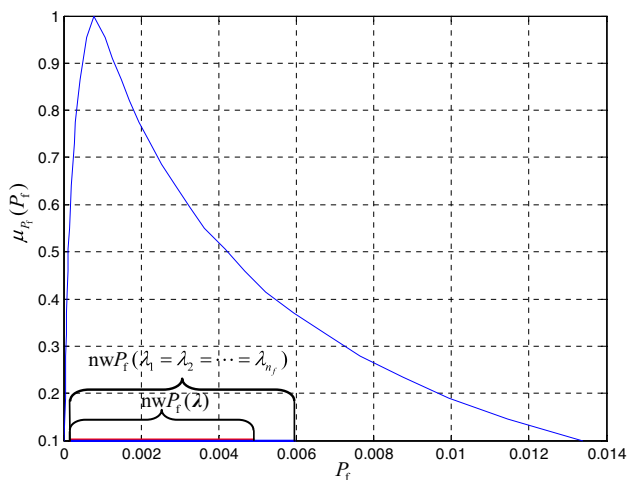


Fig. 9 The failure probability results for the roof truss

and collect more information to decrease the uncertainty of the fuzzy variables for reducing the gap of the CDFs.

Example 2 A ten-bar truss as shown in Fig. 11 is considered to estimate its failure probability. The length and sectional areas of horizontal and vertical bars are denoted as L and A_i ($i = 1, \dots, 6$), the length and sectional areas of the diagonal bar are $\sqrt{2}L$ and A_i ($i = 7, \dots, 10$) respectively, the elastic modulus of all bars is E ; P_1 and P_2 are the external loads. Assume that the input variables L , A_i ($i = 1, \dots, 10$) and E are the normal input random variables with the distribution parameters listed in Table 3. P_1 (kN), P_2 (kN) and P_3 (kN) are assumed as the fuzzy variables with the triangular membership functions as follows:

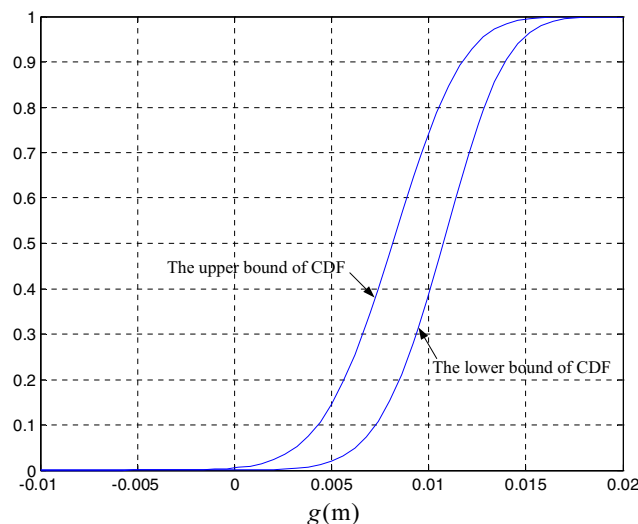


Fig. 10 The CDF bound of the output response for the roof truss

$$\mu_{P_1}(P_1) = \begin{cases} (P_1 - 76 \times 10^3)/4 \times 10^3 & 76 \times 10^3 \leq P_1 \leq 80 \times 10^3, \\ (P_1 - 84 \times 10^3)/(-4 \times 10^3) & 80 \times 10^3 \leq P_1 \leq 84 \times 10^3, \end{cases}$$

$$\mu_{P_2}(P_2) = \begin{cases} (P_2 - 9.5 \times 10^3)/0.5 \times 10^3 & 9.5 \times 10^3 \leq P_2 \leq 10 \times 10^3, \\ (P_2 - 10.5 \times 10^3)/(-0.5 \times 10^3) & 10 \times 10^3 \leq P_2 \leq 10.5 \times 10^3, \end{cases}$$

$$\mu_{P_3}(P_3) = \begin{cases} (P_3 - 9.5 \times 10^3)/0.5 \times 10^3 & 9.5 \times 10^3 \leq P_3 \leq 10 \times 10^3, \\ (P_3 - 10.5 \times 10^3)/(-0.5 \times 10^3) & 10 \times 10^3 \leq P_3 \leq 10.5 \times 10^3. \end{cases}$$

According to the mechanics analysis, taking the perpendicular displacement of the node 2 not exceeding 4 mm

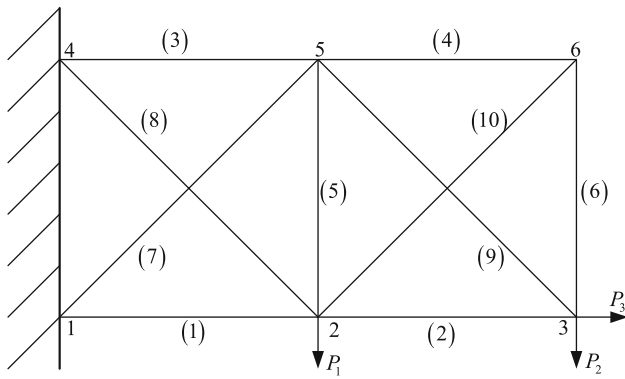


Fig. 11 The diagram of the ten-bar truss

Table 3 The distribution parameters of the input random variables for Example 2

Random variable	Mean value	Standard deviation	Coefficients of variation
L (m ²)	1	0.1	0.1
A_i (m ²)	0.001	5×10^{-5}	0.05
E (GPa)	100	10	0.1

as the constraint condition, the following performance function can be constructed, $Y = g = 0.004 - |\Delta_2|$, where Δ_2 is an implicit function of input variables, i.e., $\Delta_2 = \Delta(L, A_i, E, P_1, P_2, P_3)$ ($i = 1, 2, \dots, 10$), and is determined by the FEM. The finite element model of the ten-bar truss is shown in Fig. 12.

The comparisons of the failure probability of the ten-bar truss obtained by the kriging-based solution and MCS method are listed in Table 4.

It can be seen from Table 4 that the kriging-based solution is applicable to the proposed approach for the implicit performance function. The membership function of the failure probability of this example obtained by the traditional approaches and the interval of failure probability obtained by the proposed approach are also shown in Fig. 13, same as Example 1, $nwP_f(\lambda_1 = \lambda_2 = \dots = \lambda_{n_f})$ and $nwP_f(\lambda)$ represent the cases where the membership levels of the different fuzzy variables take the same and different values, respectively.

It can be seen from Table 4 and Fig. 14 that the results of the membership levels of the fuzzy variables taking the same value are very near to that obtained by taking different values of the membership levels of the different fuzzy variables. According to the results, it can be reasonably speculated that the effect of one of the fuzzy variables on the failure probability is dominant, and the other two fuzzy variables are secondary. The CDF bounds of output response are plotted in Fig. 13. To discuss the effect of fuzzy variables on the output response, P_1 is fixed

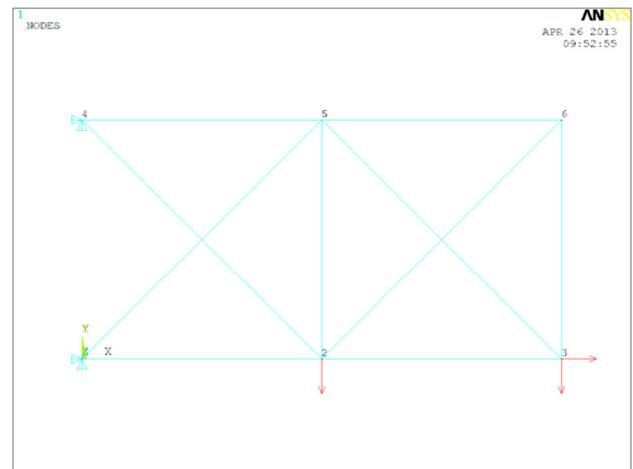


Fig. 12 The finite element model of the ten-bar truss

Table 4 The results of the reliability analysis for Example 2

	$\lambda_1 = \lambda_2 = \dots = \lambda_{n_f}$ *		$\lambda = (\lambda_1, \lambda_2, \dots, \lambda_{n_f})$ **	
	MCS	Kriging-based solution	MCS	Kriging-based solution
nwP_f^L	0.002566	0.002551	0.002451	0.002369
nwP_f^U	0.007403	0.007384	0.007194	0.007223
Sample points	10^6	983	10^6	1055

*The membership levels of the different fuzzy variables taken the same value

**The membership levels of the different fuzzy variables taken different values

at nominal value 80×10^3 (i.e., the membership level of P_1 is equal to 1) to eliminate its fuzzy uncertainty. Then the CDF bounds of the output response of the structural system are calculated by the kriging-based solution and plotted in Fig. 15. Comparing Figs. 14 and 15, it can be seen that the gap between the two CDF curves in Fig. 15 is obviously less than the one in Fig. 14, which means that the uncertain reduction of the output response is very obvious after eliminating the uncertainty of the fuzzy variable P_1 . In other words, the effect of the fuzzy variable P_1 on the output response is very large, which verifies our previous speculation.

The designer can identify the effect of the fuzzy variables on the output response of the structural system by the gap between the CDF bounds and the CDF values, and pay more attention to the most critical input variables.

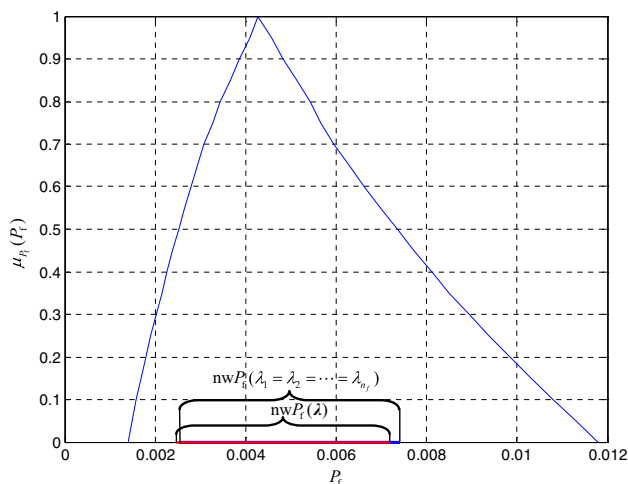


Fig. 13 The failure probability results for the ten-bar truss

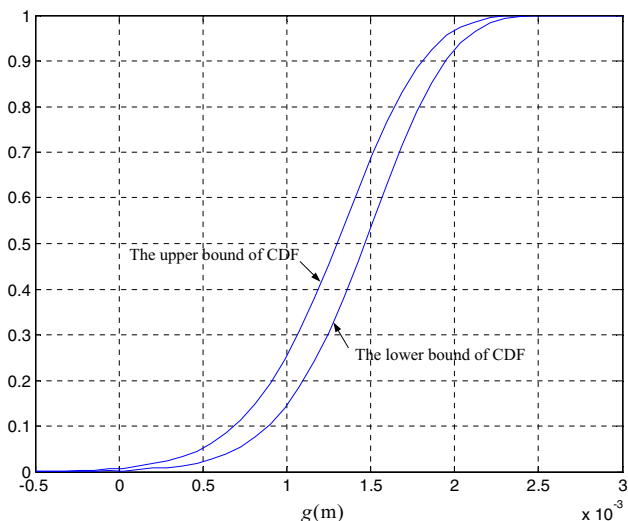


Fig. 14 The CDF bound of the output response for the ten-bar truss

6 Conclusions

In the current study, a new reliability approach is proposed for the mixed uncertain structural system. By introducing the standard uniform distribution into the membership level of the fuzzy variable, the reliability of the mixed uncertain structures is measured by the probability interval instead of the traditional reliability membership function. Compared with the reliability membership function, the obtained probability value from the proposed approach is easier to be understood by engineers. Another advantage of the proposed approach is that the combination of different membership levels of the fuzzy variables with different values is considered, and the fuzzy input information is used more fully, which is proved by engineering examples in this paper. When a fuzzy variable is fixed at a given

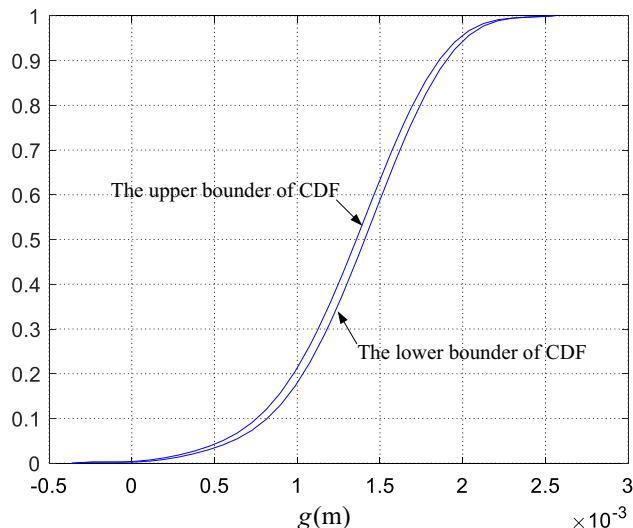


Fig. 15 The CDF bound of the output response when P_1 fixed at 80×10^3

value, the CDF bounds of output response can be obtained by the proposed approach. Then the influence of the fuzzy uncertainty on output response can be qualitatively identified. Additionally, the established kriging-based solution process is given in this paper, which can calculate the proposed approach efficiently.

It is important to identify the influence of input uncertainties on output response in reliability assessment and engineering design. As was stated previously, the proposed approach can only qualitatively identify the influence of the fuzzy uncertainties on the output response. Therefore, in future work, based on the proposed approach, the quantitative approach that can measure the influence of the fuzzy input variables and random input variables on output response should be developed.

Acknowledgements Authors gratefully acknowledge the supports of Basic Scientific Research Projects of Dalian University of Technology [Grant No. DUT21RC(3)071].

References

1. Helton, J.C., Johnson, J.D., Oberkampf, W.L., Sallaberry, C.J.: Representation of analysis results involving aleatory and epistemic uncertainty. *Int. J. Gen. Syst.* **39**(6), 605–646 (2010)
2. Keshtegar, B., Bagheri, M., Meng, D., Kolahchi, R., Trung, N.T.: Fuzzy reliability analysis of nanocomposite ZnO beams using hybrid analytical-intelligent method. *Eng. Comput.* **37**(4), 2575–2590 (2020)
3. Adduri, P.R., Penmetsa, R.C.: System reliability analysis for mixed uncertain variables. *Struct. Saf.* **31**(5), 375–382 (2009)
4. Jaulin, L., Kieffer, M., Didrit, O., Walter, E.: *Applied Interval Analysis*. Springer, New York (2001)
5. Tooranloo, H.S., Ayatollah, A.S.: Pathology the internet ranking service quality using failure mode and effect analysis in interval-

- valued intuitionistic fuzzy environment. *Int. J. Fuzzy Syst.* **19**(1), 1–15 (2016)
6. Chen, L.Y., Deng, Y., Cheong, K.H.: Probability transformation of mass function: a weighted network method based on the ordered visibility graph. *Eng. Appl. Artif. Intell.* **105**, 104438 (2021)
 7. Wang, H.Y., Wang, J.S., Wang, G.: Clustering validity function fusion method of FCM clustering algorithm based on Dempster-Shafer evidence theory. *Int. J. Fuzzy Syst.* **24**, 650–675 (2021)
 8. Deng, Y.: Uncertainty measure in evidence theory. *Sci. China Inf. Sci.* **63**(11), 1–19 (2020)
 9. Zadeh, L.A.: Fuzzy sets. *Inf. Control* **8**(3), 338–353 (1965)
 10. Zadeh, L.A.: Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets Syst.* **1**(1), 3–28 (1978)
 11. Deng, J.X., Deng, Y.: Information volume of fuzzy membership function. *Int. J. Comput. Commun.* (2021). <https://doi.org/10.15837/ijccc.2021.1.4106>
 12. Shi, Y., Lu, Z., Zhao, L.: Global sensitivity analysis of the failure probability upper bound to random and fuzzy inputs. *Int. J. Fuzzy Syst.* **21**(2), 454–467 (2019)
 13. Zhao, Y.G., Ono, T.: A general procedure for first/second-order reliability method (FORM/SORM). *Struct. Saf.* **21**(2), 95–112 (1999)
 14. Wang, Z., Broccardo, M., Song, J.: Hamiltonian Monte Carlo methods for subset simulation in reliability analysis. *Struct. Saf.* **76**, 51–67 (2019)
 15. Pulido, J.E., Jacobs, T.L., Lima, E.: Structural reliability using Monte Carlo simulation with variance reduction techniques on elastic–plastic structures. *Comput. Struct.* **43**(3), 419–430 (1992)
 16. Cai, K.Y., Wen, C.Y., Zhang, M.L.: Fuzzy variables as a basis for a theory of fuzzy reliability in the possibility context. *Fuzzy Sets Syst.* **42**(2), 145–172 (1991)
 17. Guh, Y.Y., Yang, M.S., Po, R.W., Lee, E.S.: Establishing performance evaluation structures by fuzzy relation-based cluster analysis. *Comput. Math. Appl.* **56**(2), 572–582 (2008)
 18. Zhang, F., Wang, X.H., Hou, X.T., et al.: Variance-based global sensitivity analysis of a hybrid thermoelectric generator fuzzy system. *Appl. Energ.* **80**(12), 118208 (2022)
 19. Möller, B., Graf, W., Beer, M.: Fuzzy structural analysis using a-level optimization. *Comput. Mech.* **26**(6), 547–565 (2000)
 20. Cremona, C., Gao, Y.: The possibilistic reliability theory: theoretical aspects and applications. *Struct. Saf.* **19**(2), 173–201 (1997)
 21. Wang, C., Qiu, Z.P., Xu, M.H., Li, Y.L.: Novel numerical methods for reliability analysis and optimization in engineering fuzzy heat conduction problem. *Struct. Multidiscip. Optim.* **56**(6), 1247–1257 (2017)
 22. Gauger, U., Turrin, S., Hanss, M., Gaul, L.: A new uncertainty analysis for the transformation method. *Fuzzy Sets Syst.* **159**(11), 1273–1291 (2008)
 23. Giannini, O., Hanss, M.: An interdependency index for the outputs of uncertain systems. *Fuzzy Sets Syst.* **159**(11), 1292–1308 (2008)
 24. Dong, W.M., Wong, F.S.: Fuzzy weighted averages and implementation of the extension principle. *Fuzzy Sets Syst.* **21**(2), 183–199 (1987)
 25. Smith, S.A., Krishnamurthy, T., Mason, B.H.: Optimized vertex method and hybrid reliability. In: *The 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*. AIAA-2002-1465 (2002)
 26. Penmetsa, R.C., Grandhi, R.V.: Uncertainty propagation using possibility theory and function approximations. *Mech. Based Des. Struct.* **31**(2), 257–279 (2003)
 27. Li, G.J., Lu, Z.Z., Xu, J.: A fuzzy reliability approach for structures based on the probability perspective. *Struct. Saf.* **54**, 10–18 (2015)
 28. Baudrit, C., Dubois, D., Guyonnet, D.: Joint propagation and exploitation of probabilistic and possibilistic information in risk assessment. *IEEE Trans. Fuzzy Syst.* **14**(5), 593–608 (2006)
 29. Guyonnet, D., Bourguine, B., Dubois, D., Fargier, H., Côme, B., Chilès, J.P.: A hybrid approach for addressing uncertainty in risk assessments. *Environ. Eng.* **29**(1), 68–78 (2003)
 30. Song, S.F., Lu, Z.Z., Li, W., Cui, L.J.: The uncertainty importance measures of the structural system in view of mixed uncertain variables. *Fuzzy Sets Syst.* **243**(16), 25–35 (2014)
 31. Möller, B., Graf, W., Beer, M.: Safety assessment of structures in view of fuzzy randomness. *Comput. Struct.* **81**(15), 1567–1582 (2003)
 32. Adduri, P.R., Penmetsa, R.C.: Membership function development for reliability analysis with mixed uncertain variables. In: *The 46th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, AIAA 2005-20722 (2005)
 33. Adduri, P.R., Penmetsa, R.C.: System reliability analysis in the presence of random and non-random variables. In: *The 11th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*. AIAA 2006-6929 (2006)
 34. Li, L.Y., Lu, Z.Z., Song, S.F.: Saddlepoint approximation based line sampling method for uncertainty propagation in fuzzy and random reliability analysis. *Sci. China Technol. Sci.* **53**(8), 2252–2260 (2010)
 35. Ebenutwa, A.U., Tee, K.F.: Fuzzy-based optimised subset simulation for reliability analysis of engineering structures. *Struct. Infrastruct. Eng.* **25**(3), 413–425 (2019)
 36. You, L.F., Zhang, J.G., Zhou, S., Wu, J.: A novel mixed uncertainty support vector machine method for structural reliability analysis. *Acta Mech.* **232**(12), 1497–1513 (2021)
 37. Guo, S.X., Lu, Z.Z.: Hybrid probabilistic and non-probabilistic model of structural reliability. *Chin. J. Mech. Strength* **24**(4), 524–526 (2002)
 38. Li, G.J., Lu, Z.Z., Tian, L.F., Xu, J.: The importance measure on the non-probabilistic reliability index of uncertain structures. *Proc. Inst. Mech. Eng. O* **227**(6), 651–661 (2013)
 39. Guo, S.X., Lu, Z.Z., Feng, Y.S.: A non-probabilistic model of structural reliability based on interval analysis. *Chin. J. Comput. Mech.* **18**(1), 56–60 (2001)
 40. Romero, V.J., Swiler, L.P., Giunta, A.A.: Construction of response surface based on progressive-lattice-sampling experimental designs with application to uncertainty propagation. *Struct. Saf.* **26**(2), 201–219 (2004)
 41. Zhai, Z., Li, H., Wang, X.: An adaptive sampling method for Kriging surrogate model with multiple outputs. *Eng. Comput.* (2020). <https://doi.org/10.1007/s00366-020-01145-1>
 42. Bellary, S.A.I., Samad, A., Couckuyt, I., Dhaene, T.: A comparative study of Kriging variants for the optimization of a turbomachinery system. *Eng. Comput.* **32**(1), 49–59 (2015)
 43. Li, G.J., Lu, Z.Z., Li, L.Y., Ren, B.: Aleatory and epistemic uncertainties analysis based on non-probabilistic reliability and its kriging solution. *Appl. Math. Model.* **40**(9–10), 5703–5716 (2016)



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