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Finite-Time Tracking Control for a Class of MIMO Nonstrict-Feedback Nonlinear Systems Via Adaptive Fuzzy Method

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Abstract This paper addresses the finite-time adaptive fuzzy control for multi-input and multi-output (MIMO) nonstrict-feedback nonlinear systems with fuzzy dead zones. Combining the semi-global practical finite-time stability criterion with the condition of variable partition, a feasible finite-time adaptive fuzzy control scheme is proposed for the developed MIMO system. The designed control algorithm guarantees that the tracking error converges to a small neighborhood of the origin in finite time. Compared with existing research results, the main advantage of this paper lies in that the finite-time control issue is considered for MIMO nonstrict-feedback systems with input nonlinearities. Two simulation examples are provided to illustrate the effectiveness of the suggested approach.

Keywords Finite-time · Adaptive fuzzy control · Multi-input and multi-output · Nonstrict-feedback nonlinear systems · Fuzzy dead zones

1 Introduction

Due to the fact that most of the practical systems are multivariable, tightly coupled and nonlinear, many backstepping-based adaptive control methods have been widely developed for uncertain MIMO nonlinear systems in the past decades. By utilizing fuzzy logic systems or neural networks (NNs) to approximate the unknown nonlinear functions, many feasible works have been carried out such as [1–8]. A serious of adaptive fuzzy or NN control schemes for deterministic MIMO nonlinear systems with unknown nonlinearities were considered in [1–5]. And, the controlled MIMO nonlinear systems developed in [6–8] were extended to MIMO stochastic systems. It should be pointed out that although some significant results have been achieved in the aforementioned studies, these researches have three limitations. First, the fuzzy dead zone problems are not taken into consideration. Second, the proposed control methods depend on the system strict-feedback structures, if the structures of the controlled nonlinear systems are in nonstrict-feedback forms, the designed controller cannot work. Third, the aforementioned control strategies are considered with the general infinite time control problems, and they cannot guarantee the desired finite-time performance.

Dead zone nonlinearity is common in many practical systems, and the existence of such nonlinearity may cause severe deterioration of the system performance or even the instability of system. In [9], a smooth inverse function of the dead zone was introduced to achieve the adaptive output control design. Without constructing the dead zone inverse, an adaptive compensation algorithm was employed in [10] for uncertain dynamical systems preceded by a non-symmetric dead zone input. In [11], observer-based adaptive fuzzy-neural control was studied for a class of single input and single output (SISO) systems with completely unknown functions and unknown dead zone inputs. Then, combining with backstepping techniques, dead zone nonlinearities were considered in references [12–14] for SISO strict-feedback system, switched nonlinear system and MIMO strict-feedback nonlinear system, respectively. However, the results obtained in [9–14] are feasible under the presupposition that the dead

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zone output of the actuator is certain and precise in character, that is, the actuator output is a deterministic value for a given actuator input signal. To remove the restrictive condition posed on dead zone model in [9–14], the authors of [15] employed a center-of-gravity method to overcome the difficulty caused by the uncertainty in the dead zone input of the nonlinear system. However, the systems under consideration in the above existing works were assumed to be in strict-feedback forms or in pure-feedback forms, that is, the controlled nonlinear systems were in low-triangular structures.

In nonstrict-feedback nonlinear systems, all the state variables are contained in the system functions, therefore, the controller design for this form of nonlinear system is different from pure-feedback or strict-feedback systems, and it yields much more difficulty in stability analysis and controller design, especially for MIMO nonlinear systems. To deal with the difficulty caused by nonstrict-feedback structure, a variable separation method was applied in [16] for nonstrict-feedback stochastic nonlinear systems with input saturation and prescribed performance. The problems of adaptive neural control were considered in [17] for nonstrict-feedback systems with dynamic uncertainties and in [18] for nonstrict-feedback stochastic systems, respectively. Then, in [19–21], approximation-based adaptive tracking control was developed for a class of nonstrict-feedback systems with input nonlinearities such as dead zone, time delays and input saturation. It is noted that the controlled systems in [16–21] are all SISO rather than MIMO. The problem of adaptive fuzzy control for a class of MIMO nonstrict-feedback nonlinear systems was investigated in [22]. Based on the common Lyapunov function method, adaptive fuzzy tracking controllers were designed in [23] for MIMO uncertain switched nonstrict-feedback nonlinear systems with arbitrary switchings. Although, the existing results in [16–23] are for nonstrict-feedback nonlinear systems, the control schemes proposed in these works are considered with the infinite time stability and tracking control problems.

In recent years, the issue of finite-time control has received increasing attention. Based on Lyapunov theory, the finite-time stability for nonlinear systems was first considered in [24, 25]. Since then, many useful and valuable results have been achieved in [26–30] for the study of the finite-time stability of nonlinear systems based on the Lyapunov function method. However, the earlier results in [24–30] for finite-time controller design were obtained under certain restrictive conditions, if these completely unknown nonlinear functions that do not meet some growth conditions are taken into account, the control algorithms designed in [24–30] will not work. Recently, a novel contributing criterion of finite-time semi-global practical stability was established in [31] for nonlinear pure-feedback systems, and many follow-up studies [32–34] were discussed based on the stability criterion in [31]. However, the existing useful finite-time control schemes are only considered for SISO strict-feedback systems [32, 33] or SISO nonstrict-feedback systems [34], but not for more complicated systems such as MIMO nonstrict-feedback systems with fuzzy input nonlinearities.

However, on one hand, the control algorithms in [16, 18] were the stabilization problems for SISO nonstrict-feedback systems rather than the finite-time tracking problem. On the other hand, the existing result in [3] for MIMO system requires that the structure of the system is strict-feedback, that is, each subsystem function in the controlled system cannot contain the whole state variables. In addition, meeting practical constraints may degrade the system performance. To the best of our knowledge, there are still few results available for the finite-time tracking control of MIMO nonstrict-feedback system with fuzzy dead zones, which is the motivation of our work.

Based on the above discussion, this paper concentrates on the finite-time adaptive fuzzy control problem for MIMO nonstrict-feedback nonlinear systems with fuzzy dead zones. Compared with the existing works, the main contributions of our proposed control scheme are summarized as follows.

(i) Compared with existing results in [22, 23] for infinite time control of MIMO nonstrict-feedback uncertain nonlinear systems, this paper investigates the finite-time control problem of MIMO nonstrict-feedback systems subject to fuzzy dead zones for the first time.

(ii) The existing adaptive control methods designed in [16, 18] for SISO nonstrict-feedback systems only guarantee the stabilization of the system, and the adaptive fuzzy or neural control strategies in [19–21] for SISO nonstrict-feedback systems are concerned with the tracking problem in infinite time. In our paper, the desired tracking performance is obtained in finite time. On one hand, stabilization problem, i.e., $y_{dj} = 0$, is a special case of tracking problem, on the other hand, the finite-time stability analysis is different from the infinite time stability analysis. Therefore, conventional adaptive fuzzy control scheme cannot be directly used for the finite-time tracking control in our paper.

(iii) Compared with previous results in [31–34], in which the finite-time control issue is considered for kinds of SISO nonlinear systems, this paper extends the results to more general MIMO nonstrict-feedback systems with fuzzy dead zones.

Under the proposed control approach, system performance is guaranteed in finite time.

2 Problem Formulation and Preliminaries

2.1 Problem Statement

Consider the following MIMO nonlinear nonstrict-feedback systems with M subsystems. The jth $(j = 1, 2, \ldots, M)$ subsystem is described as

$$\begin{cases} \dot{x}_{j,p} = x_{j,p+1} + f_{j,p}(X), \quad 1 \le p \le n_j - 1, \\ \dot{x}_{j,n_j} = \psi_j(u_j) + f_{j,n_j}(X), \\ y_j = x_{j,1}, \end{cases} \tag{1}$$

where $X = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_M^{\mathrm{T}}]^{\mathrm{T}}$ denotes the state vector with $x_j = [x_{j,1}, x_{j,2}, \ldots, x_{j,n_j}]^{\mathrm{T}} \in R^{n_j}$ and $\bar{x}_{j,p} = [x_{j,1}, x_{j,2}, \ldots, x_{j,p}]^{\mathrm{T}} \in R^p$, $u_j \in R$ and $y_j \in R$ denote the jth nonlinear subsystem control input and output, respectively. $f_{j,p}(\cdot)(j = 1, 2, \ldots, M, p = 1, 2, \ldots, n_j)$ are unknown nonlinear smooth functions with $f_{j,p}(0) = 0$. $\psi_j(u_j)$ denotes the fuzzy dead zone input and is expressed as the following form

$$\psi_j(u_j) = \begin{cases} \tilde{k_j}\left(u_j - b_{j,r}\right), \quad u_j \ge b_{j,r}, \\ 0, \quad b_{j,l} < u_j < b_{j,r}, \\ \tilde{k_j}\left(u_j - b_{j,l}\right), \quad u_j \le b_{j,l}, \end{cases} \tag{2}$$

where $b_{j,r} > 0$ and $b_{j,l} < 0$ are the breakpoints of the input nonlinearity. The slope $\tilde{k_j}$ is given as

$$\tilde{k_j} = \frac{k_{j,1}}{\varsigma_{j,1}} + \frac{k_{j,2}}{\varsigma_{j,2}} + \cdots + \frac{k_{j,\gamma}}{\varsigma_{j,\gamma}},$$

where $k_{j,q}(j = 1, 2, \ldots, M, q = 1, 2, \ldots, \gamma)$ are possible values, which are taken by $\tilde{k_j}(j = 1, 2, \ldots, M)$, $\varsigma_{j,q}$ denotes the fuzzy grade of $k_{j,q}$ with $k_{j,q} \ne 0$. Generally, it is assumed that $k_{j,q} > 0$ and $k_{j,1} < k_{j,2} < \cdots < k_{j,\gamma}$. "$(k_{j,q}/\varsigma_{j,q})$" represents the relationship between the mapping of $k_{j,q}$ and $\varsigma_{j,q}$, and "+" denotes a collection in the universe of discourse $U_j = \{k_{j,1}, k_{j,2}, \ldots, k_{j,\gamma}\}$.

As the value of the dead zone output $\psi_j(u_j)$ is vague, we cannot design controller directly. To cope with the term $\psi_j(u_j)$, we apply the following center-of-gravity method.

$$\mathrm{Defu}\left(\psi_j(u_j)\right) = \begin{cases} \bar{k_j}\left(u_j - b_{j,r}\right), \quad u_j \ge b_{j,r}, \\ 0, \quad b_{j,l} < u_j < b_{j,r}, \\ \bar{k_j}\left(u_j - b_{j,l}\right), \quad u_j \le b_{j,l}, \end{cases} \tag{3}$$

where $\mathrm{Defu}(\cdot)$ is a center-of-gravity defuzzification operation for $\psi_j(u_j)$, and the defuzzified value $\bar{k_j}$ satisfies

$$\bar{k_j} = \frac{\sum_{q=1}^{\gamma} \varsigma_{j,q} k_{j,q}}{\sum_{q=1}^{\gamma} \varsigma_{j,q}}.$$

Then, the defuzzified dead zone model can be further expressed as

$$\mathrm{Def}u\left(\psi_j(u_j)\right) = \bar{k_j} u_j + \bar{d_j}(t), \tag{4}$$

where

$$\bar{d_j}(t) = \begin{cases} -\bar{k_j} b_{j,r}, \quad u_j \ge b_{j,r}, \\ -\bar{k_j} u_j, \quad b_{j,l} < u_j < b_{j,r}, \\ -\bar{k_j} b_{j,l}, \quad u_j \le b_{j,l}. \end{cases}$$

Obviously, $\bar{d_j}(t)$ is bounded and satisfies

$$|\bar{d_j}(t)| \le \max\{k_{j,\gamma}|b_{j,r}|, k_{j,\gamma}|b_{j,l}|\}.$$

The objective of this paper is to construct an adaptive fuzzy controller for MIMO system (1) to guarantee that the system output y_j locates a small area of the desired reference signal y_{dj} in finite time. For convenience of the subsequent controller design process, the following assumptions are required.

Assumption 1 The desired trajectory $y_{dj}(j = 1, 2\ldots, M)$ and their time derivatives up to the n_jth order $y_{dj}^{(n_j)}$ are continuous and bounded. Furthermore, there exists a positive constant d_0 such that $|y_{dj}(t)| \le d_0$.

Assumption 2 For nonlinear function $f_{j,p}(X)$, there exists a strict increasing smooth function $\chi_{j,p}(\cdot) : R^+ \to R^+$ with $\chi_{j,p}(0) = 0$ satisfying

$$|f_{j,p}(X)| \le \chi_{j,p}(\|X\|).$$

Remark 1 In Assumption 2, the monotony property of $\chi_{j,p}(\cdot)$ implies $\chi_{j,p}(\sum_{k=1}^{n} b_k) \le \sum_{k=1}^{n} \chi_{j,p}(n b_k)$ with $b_k \ge 0$. Based on the property of $\chi_{j,p}(s)$, there exists a smooth function $z_{j,p}(s)$ such that $\chi_{j,p}(s) = s z_{j,p}(s)$, which yields

$$\chi_{j,p}\left(\sum_{k=1}^{n} b_k\right) \le \sum_{k=1}^{n} n b_k z_{j,p}(n b_k). \tag{5}$$

2.2 Preliminaries for Finite Time Stability Analysis

To facilitate the finite-time control design, some useful definition and Lemmas are introduced.

Definition 1 [31] Let $\omega = 0$ be the equilibrium of the nonlinear system $\dot{\omega} = f(\omega, u)$. If for all initial condition $\omega(t_0) = \omega_0$, there exists a positive constant τ and a settling time $T(\tau, \omega_0) < +\infty$ such that $\|\omega(t)\| < \tau$ for all $t \geq t_0 + T$, then the nonlinear system is semi-global practical finite-time stable (SGPFS).

Lemma 1 [35] *For $b_j \in R$, $0 < \upsilon \leq 1$, the following relation holds*:

$$\left(\sum_{j=1}^{m} |b_j|\right)^{\upsilon} \leq \sum_{j=1}^{m} |b_j|^{\upsilon} \leq m^{1-\upsilon}\left(\sum_{j=1}^{m} |b_j|\right)^{\upsilon}. \tag{6}$$

Lemma 2 [32] *Consider the differential equation $\dot{\varpi}(t) = -a\varpi(t) + bv(t)$. If $a,\ b$ are positive constants and $v(t) > 0$, then $\varpi(t_0) \geq 0$ means $\varpi(t) \geq 0$ for $\forall t \geq t_0$.*

Lemma 3 [36] *For $\forall \sigma > 0$ and $\forall \iota \in R$, the following inequality holds*:

$$0 \leq |\iota| - \iota \tanh\left(\frac{\iota}{\sigma}\right) \leq \kappa_1 \sigma, \quad \kappa_1 = 0.2785. \tag{7}$$

Lemma 4 [37] *For any real variables α, β and any constants $\lambda > 0, v > 0, o > 0$, the following inequality holds*

$$|\alpha|^{\lambda}|\beta|^{v} \leq \frac{\lambda}{\lambda + v} o|\alpha|^{\lambda+v} + \frac{v}{\lambda + v} o^{-\frac{\lambda}{v}}|\beta|^{\lambda+v}.$$

Lemma 5 [31] *Suppose that there exist scalars $a_1 > 0$, $0 < \upsilon < 1$ and $b_1 > 0$, for the system $\dot{\omega} = f(\omega, u)$, the time derivation of the smooth positive definite function $V(\omega)$ satisfies the following equation*

$$\dot{V}(\omega) \leq -a_1 V^{\upsilon}(\omega) + b_1, \quad t \geq 0, \tag{8}$$

and let $T_r = \frac{1}{(1-\upsilon)\kappa a_1}[V^{1-\upsilon}(\omega(0)) - (\frac{b_1}{(1-\kappa)a_1})^{\frac{1-\upsilon}{\upsilon}}]$ with $V(\omega(0))$ being the initial condition of $V(\omega)$, and $0 < \kappa \leq 1$, then the nonlinear system $\dot{\omega} = f(\omega, u)$ is SGPFS for $\forall t \geq T_r$.

Remark 2 It should be pointed out that although some similar works have been done for finite-time adaptive control in [31–34] or for nonlinear systems with fuzzy dead zones in [15, 38], where the presented control algorithms are considered for SISO nonlinear systems and they cannot guarantee the system tracking performance for MIMO systems. The system (1) under study is in MIMO nonstrict-feedback form, which is more complicated and general than SISO systems. So far, there is no finite-time adaptive fuzzy control results to be reported for MIMO system (1).

2.3 Fuzzy Logic Systems

To approximate the continuous function *f*(*x*) defined on a compact set Ω, a fuzzy logic system needs to be designed. Define the following If-Then fuzzy rules:

R^l: If x_1 is F_1^l and ... and x_n is F_n^l, then y is G^l, $l = 1, 2, \ldots, K$, where $x = [x_1, \ldots, x_n]^T$ and y are the input and output of the fuzzy system, respectively. Fuzzy sets F_j^l and G^l are associate with the fuzzy membership functions $\mu_{F_j^l}(x_j)$ and $\mu_{G^l}(y)$, respectively. K is the number of rules. The output of the fuzzy system can be expressed as [39]

$$y(x) = \frac{\sum_{l=1}^{K} \pi_l \prod_{j=1}^{n} \mu_{F_j^l}(x_j)}{\sum_{l=1}^{K}\left[\prod_{j=1}^{n} \mu_{F_j^l}(x_j)\right]},$$

where $\pi_l = \max_{y \in R} \mu_{G^l}(y)$. Simultaneously, we define the fuzzy basis function as

$$\eta_l(x) = \frac{\prod_{j=1}^{n} \mu_{F_j^l}(x_j)}{\sum_{l=1}^{K}\left[\prod_{j=1}^{n} \mu_{F_j^l}(x_j)\right]},$$

then the fuzzy logic system can be expressed as

$$y(x) = \pi^{\mathrm{T}} \eta(x), \tag{9}$$

where

$$\pi = [\pi_1, \pi_2, \ldots, \pi_K]^{\mathrm{T}}, \eta(x) = [\eta_1(x), \eta_2(x), \ldots, \eta_K(x)]^{\mathrm{T}}.$$

Lemma 6 [39] *Let f(x) be a continuous function defined on a compact set Ω. Then, for any positive constant τ, there exists a fuzzy logic system (9) such that*

$$\sup_{x \in \Omega} |f(x) - \pi^{\mathrm{T}} \eta(x)| \leq \tau. \tag{10}$$

3 Adaptive Fuzzy Controller Design

In this section, a backstepping-based control design scheme will be presented. Define $Z_{j,p} = [\bar{x}_{j,p}^{\mathrm{T}}, \bar{\hat{\phi}}_{j,p}^{\mathrm{T}}, \bar{y}_{dj}^{(p)\mathrm{T}}]^{\mathrm{T}}$ with $\bar{\hat{\phi}}_{j,p} = [\hat{\phi}_{j,1}, \ldots, \hat{\phi}_{j,p}]^{\mathrm{T}}$ and $\bar{y}_{dj}^{(p)} = [y_{dj}, \dot{y}_{dj}, \ldots, y_{dj}^{(p)}]^{\mathrm{T}}$. $\hat{\phi}_{j,p}$ is the estimation of an unknown constant $\phi_{j,p}$, which is defined as $\phi_{j,p} = \|\pi_{j,p}\|^2$, and $\tilde{\phi}_{j,p} = \phi_{j,p} - \hat{\phi}_{j,p} (j = 1, \ldots, M, p = 1, \ldots, n_j)$. In the following, the virtual control signals $\zeta_{j,p}(j = 1, \ldots, M, p =$

$1,\ldots,n_j-1)$ and the controller $u_j(j=1,\ldots,M)$ are respectively designed as

$$\zeta_{j,p}=-\rho_{j,p}\vartheta_{j,p}^{2\upsilon-1}-\frac{\hat{\phi}_{j,p}}{2\mu_{j,p}^2}\vartheta_{j,p}\eta_{j,p}^{\mathrm{T}}(Z_{j,p})\eta_{j,p}(Z_{j,p}), \tag{11}$$

$$u_j=-\frac{1}{\bar{k}_j}\left[\rho_{j,n_j}\vartheta_{j,n_j}^{2\upsilon-1}+\frac{\hat{\phi}_{j,n_j}}{\mu_{j,n_j}^2}\vartheta_{j,n_j}\eta_{j,n_j}^{\mathrm{T}}(Z_{j,n_j})\eta_{j,n_j}(Z_{j,n_j})\right], \tag{12}$$

with $\rho_{j,p}$ and $\mu_{j,p}$ being positive constants. $\upsilon=[(2n-1)/(2n+1)]$ with $n\geq 2$ being the natural number, $(1/2)<\upsilon<1$. $\vartheta_{j,p}$ is defined in the following coordinate transformation

$$\vartheta_{j,p}=x_{j,p}-\zeta_{j,p-1} \tag{13}$$

with $\zeta_{j,0}=y_{dj}$. The parameter adaptive laws are designed as

$$\dot{\hat{\phi}}_{j,p}=\frac{\delta_{j,p}}{2\mu_{j,p}^2}\vartheta_{j,p}^2\eta_{j,p}^{\mathrm{T}}(Z_{j,p})\eta_{j,p}(Z_{j,p})-\varepsilon_{j,p}\hat{\phi}_{j,p},\ \hat{\phi}_{j,p}(0)\geq 0, \tag{14}$$

where $\delta_{j,p}$ and $\varepsilon_{j,p}$ are positive design constants.

Lemma 7 *From $\vartheta_{j,p}=x_{j,p}-\zeta_{j,p-1}(1\leq j\leq M, 1\leq p\leq n_j)$, the following inequality holds*

$$\|X\|\leq\sum_{j=1}^{M}\sum_{p=1}^{n_j}\Gamma_{j,p}\left(\vartheta_{j,p},\hat{\phi}_{j,p}\right)|\vartheta_{j,p}|+\bar{d}_0, \tag{15}$$

where $\bar{d}_0=Md_0$, $\Gamma_{j,p}(\vartheta_{j,p},\hat{\phi}_{j,p})=1+\rho_{j,p}\vartheta_{j,p}^{2\upsilon-2}+(\hat{\phi}_{j,p}/(2\mu_{j,p}^2))$ with $1\leq j\leq M$, $1\leq p\leq n_j-1$, and $\hat{\phi}_{j,n_j}=1$.

Theorem 1 *Consider the MIMO nonstrict-feedback nonlinear systems (1), under Assumptions 1–2, the controller u_j (12), together with the intermediate control signal $\zeta_{j,p}$ (11) and parameter adaptive law $\dot{\hat{\phi}}_{j,p}$ (14).*

(i) For the defuzzified value $\bar{k}_j$, all the signals of the closed-loop system are bounded, and the tracking errors converge to a small neighborhood of the origin in finite time.

(ii) For the fuzzy value $\tilde{k}_j$, $\tilde{k}_j\in[\bar{k}_j-\Xi_j,\bar{k}_j+\Xi_j]$, Ξ_j is a bounded disturbance, if $0<\Xi_j\leq(k_{j,1}/2)$, then all the signals of the closed-loop system are bounded, and the tracking errors converge to a small neighborhood of the origin in finite time.

3.1 Proof Case 1: Consider the Defuzzification Value $\bar{k}_j$

Step 1 Design the following Lyapunov function candidate

$$V_{j,1}=\frac{1}{2}\vartheta_{j,1}^2+\frac{1}{2\delta_{j,1}}\tilde{\phi}_{j,1}^2. \tag{16}$$

Based on $\vartheta_{j,1}=x_{j,1}-y_{dj}$, the time derivative of $V_{j,1}$ is calculated as

$$\begin{aligned}\dot{V}_{j,1}&=\vartheta_{j,1}(x_{j,2}+f_{j,1}(X)-\dot{y}_{dj})-\frac{\tilde{\phi}_{j,1}}{\delta_{j,1}}\dot{\hat{\phi}}_{j,1}\\&=\vartheta_{j,1}(\vartheta_{j,2}+\zeta_{j,1}+f_{j,1}(X)-\dot{y}_{dj})-\frac{\tilde{\phi}_{j,1}}{\delta_{j,1}}\dot{\hat{\phi}}_{j,1}.\end{aligned} \tag{17}$$

By applying Assumption 2, (5) and Lemma 7, we obtain

$$\begin{aligned}&\vartheta_{j,1}f_{j,1}(X)\\&\leq|\vartheta_{j,1}||f_{j,1}(X)|\leq|\vartheta_{j,1}|\chi_{j,1}(\|X\|)\\&\leq|\vartheta_{j,1}|\chi_{j,1}\left(\sum_{l=1}^{M}\sum_{s=1}^{n_l}\Gamma_{l,s}(\hat{\phi}_{l,s})|\vartheta_{l,s}|+\bar{d}_0\right)\\&\leq|\vartheta_{j,1}|\sum_{l=1}^{M}\sum_{s=1}^{n_l}\chi_{j,1}\left(\varrho_0|\vartheta_{l,s}|\Gamma_{l,s}(\hat{\phi}_{l,s})\right)+|\vartheta_{j,1}|\chi_{j,1}(\varrho_0\bar{d}_0)\\&\leq\sum_{l=1}^{M}\sum_{s=1}^{n_l}|\vartheta_{j,1}|\varrho_0|\vartheta_{l,s}|\Gamma_{l,s}(\hat{\phi}_{l,s})\\&\quad\times z_{j,1}\left(\varrho_0|\vartheta_{l,s}|\Gamma_{l,s}(\hat{\phi}_{l,s})\right)+|\vartheta_{j,1}|\chi_{j,1}(\varrho_0\bar{d}_0)\\&\leq\frac{1}{2}\bar{K}\vartheta_{j,1}^2+\sum_{l=1}^{M}\sum_{s=1}^{n_l}\frac{1}{2}\vartheta_{l,s}^2\bar{\chi}_{j,1}^2(\vartheta_{l,s},\hat{\phi}_{l,s})+|\vartheta_{j,1}|\chi_{j,1}(\varrho_0\bar{d}_0),\end{aligned} \tag{18}$$

where $\bar{\chi}_{j,1}^2(\vartheta_{l,s},\hat{\phi}_{l,s})=\varrho_0^2\Gamma_{l,s}^2(\hat{\phi}_{l,s})z_{j,1}^2\left(\varrho_0|\vartheta_{l,s}|\Gamma_{l,s}(\hat{\phi}_{l,s})\right)$, $\bar{K}=\sum_{l=1}^{M}n_l$, and $\varrho_0=1+\sum_{l=1}^{M}n_l$.

Let $\Theta_{j,1}=\chi_{j,1}(\varrho_0\bar{d}_0)$, and applying Lemma 3, we have

$$|\vartheta_{j,1}|\Theta_{j,1}-\vartheta_{j,1}\Theta_{j,1}\tanh\left(\frac{\vartheta_{j,1}\Theta_{j,1}}{\sigma_{j,1}}\right)\leq\kappa_1\sigma_{j,1}, \tag{19}$$

where $\sigma_{j,1}$ is a positive constant. Substituting (18) and (19) into (17) results in

$$\begin{aligned}\dot{V}_{j,1}\leq{}&\vartheta_{j,1}(\vartheta_{j,2}+\zeta_{j,1}-\dot{y}_{dj})+\frac{1}{2}\bar{K}\vartheta_{j,1}^2+\kappa_1\sigma_{j,1}\\&+\sum_{l=1}^{M}\sum_{s=1}^{n_l}\frac{1}{2}\vartheta_{l,s}^2\bar{\chi}_{j,1}^2(\vartheta_{l,s},\hat{\phi}_{l,s})-\frac{\tilde{\phi}_{j,1}}{\delta_{j,1}}\dot{\hat{\phi}}_{j,1}\\&+\vartheta_{j,1}\Theta_{j,1}\tanh\left(\frac{\vartheta_{j,1}\Theta_{j,1}}{\sigma_{j,1}}\right).\end{aligned} \tag{20}$$

Step 2 From (13), the time derivative of $\vartheta_{j,2}$ is given as

$$\begin{aligned}\dot{\vartheta}_{j,2} &= \dot{x}_{j,2} - \dot{\zeta}_{j,1} \\ &= x_{j,3} + f_{j,2}(X) - \frac{\partial \zeta_{j,1}}{\partial x_{j,1}}(x_{j,2} + f_{j,1}(X)) \\ &\quad - \sum_{m=0}^{1} \frac{\partial \zeta_{j,1}}{\partial y_{dj}^{(m)}} y_{dj}^{(m+1)} - \frac{\partial \zeta_{j,1}}{\partial \hat{\phi}_{j,1}} \dot{\hat{\phi}}_{j,1}.\end{aligned} \tag{21}$$

Choose the following Lyapunov function candidate

$$V_{j,2} = \frac{1}{2}\vartheta_{j,2}^2 + \frac{1}{2\delta_{j,2}}\tilde{\phi}_{j,2}^2. \tag{22}$$

The time derivative of $V_{j,2}$ is calculated as

$$\begin{aligned}\dot{V}_{j,2} &= \vartheta_{j,2}\Bigg[x_{j,3} + f_{j,2}(X) - \frac{\partial \zeta_{j,1}}{\partial x_{j,1}}(x_{j,2} + f_{j,1}(X)) \\ &\quad - \sum_{m=0}^{1} \frac{\partial \zeta_{j,1}}{\partial y_{dj}^{(m)}} y_{dj}^{(m+1)} - \frac{\partial \zeta_{j,1}}{\partial \hat{\phi}_{j,1}} \dot{\hat{\phi}}_{j,1}\Bigg] - \frac{\tilde{\phi}_{j,2}}{\delta_{j,2}}\dot{\hat{\phi}}_{j,2}.\end{aligned} \tag{23}$$

Similarly to the analysis in (18), we have

$$\begin{aligned}-\vartheta_{j,2}\frac{\partial \zeta_{j,1}}{\partial x_{j,1}} f_{j,1}(X) &\le |\vartheta_{j,2}|\left|\frac{\partial \zeta_{j,1}}{\partial x_{j,1}}\right|\chi_{j,1}(\|X\|) \\ &\le \frac{1}{2}\bar{K}\vartheta_{j,2}^2\left(\frac{\partial \zeta_{j,1}}{\partial x_{j,1}}\right)^2 \\ &\quad + \sum_{l=1}^{M}\sum_{s=1}^{n_l}\frac{1}{2}\vartheta_{l,s}^2\bar{\chi}_{j,1}^2(\vartheta_{l,s},\hat{\phi}_{l,s}) \\ &\quad + |\vartheta_{j,2}|\left|\frac{\partial \zeta_{j,1}}{\partial x_{j,1}}\right|\chi_{j,1}(\varrho_0\bar{d}_0),\end{aligned} \tag{24}$$

$$\begin{aligned}\vartheta_{j,2}f_{j,2}(X) &\le \frac{1}{2}\bar{K}\vartheta_{j,2}^2 + \sum_{l=1}^{M}\sum_{s=1}^{n_l}\frac{1}{2}\vartheta_{l,s}^2\bar{\chi}_{j,2}^2(\vartheta_{l,s},\hat{\phi}_{l,s}) \\ &\quad + |\vartheta_{j,2}|\chi_{j,2}(\varrho_0\bar{d}_0),\end{aligned} \tag{25}$$

where $\bar{\chi}_{j,m}^2(\vartheta_{l,s},\hat{\phi}_{l,s}) = \varrho_0^2\Gamma_{l,s}^2(\hat{\phi}_{l,s})z_{j,m}^2\Big(\varrho_0|\vartheta_{l,s}|\Gamma_{l,s}(\hat{\phi}_{l,s})\Big)$, $m = 1, 2$.

Let $\Theta_{j,2} = |\frac{\partial \zeta_{j,1}}{\partial x_{j,1}}|\chi_{j,1}(\varrho_0\bar{d}_0) + \chi_{j,2}(\varrho_0\bar{d}_0)$. From Lemma 3, one has

$$|\vartheta_{j,2}|\Theta_{j,2} - \vartheta_{j,2}\Theta_{j,2}\tanh\left(\frac{\vartheta_{j,2}\Theta_{j,2}}{\sigma_{j,2}}\right) \le \kappa_1\sigma_{j,2}, \tag{26}$$

where $\sigma_{j,2}$ is a positive constant. Substituting (24)–(26) into (23), we have

$$\begin{aligned}\dot{V}_{j,2} &\le \vartheta_{j,2}\Bigg[\vartheta_{j,3} + \zeta_{j,2} - \frac{\partial \zeta_{j,1}}{\partial x_{j,1}}x_{j,2} - \sum_{m=0}^{1}\frac{\partial \zeta_{j,1}}{\partial y_{dj}^{(m)}}y_{dj}^{(m+1)} \\ &\quad - \frac{\partial \zeta_{j,1}}{\partial \hat{\phi}_{j,1}}\dot{\hat{\phi}}_{j,1}\Bigg] + \frac{1}{2}\bar{K}\vartheta_{j,2}^2\left(\frac{\partial \zeta_{j,1}}{\partial x_{j,1}}\right)^2 + \frac{1}{2}\bar{K}\vartheta_{j,2}^2 + \kappa_1\sigma_{j,2} \\ &\quad + \sum_{m=1}^{2}\sum_{l=1}^{M}\sum_{s=1}^{n_l}\frac{1}{2}\vartheta_{l,s}^2\bar{\chi}_{j,m}^2(\vartheta_{l,s},\hat{\phi}_{l,s}) - \frac{\tilde{\phi}_{j,2}}{\delta_{j,2}}\dot{\hat{\phi}}_{j,2} \\ &\quad + \vartheta_{j,2}\Theta_{j,2}\tanh\left(\frac{\vartheta_{j,2}\Theta_{j,2}}{\sigma_{j,2}}\right).\end{aligned} \tag{27}$$

Step p ($3 \le p \le n_j - 1$): Similarly, the time derivative of $\vartheta_{j,p}$ is shown as

$$\begin{aligned}\dot{\vartheta}_{j,p} &= \dot{x}_{j,p} - \dot{\zeta}_{j,p-1} \\ &= x_{j,p+1} + f_{j,p}(X) - \sum_{m=1}^{p-1}\frac{\partial \zeta_{j,p-1}}{\partial x_{j,m}}(x_{j,m+1} + f_{j,m}(X)) \\ &\quad - \sum_{m=0}^{p-1}\frac{\partial \zeta_{j,p-1}}{\partial y_{dj}^{(m)}}y_{dj}^{(m+1)} - \sum_{m=1}^{p-1}\frac{\partial \zeta_{j,p-1}}{\partial \hat{\phi}_{j,m}}\dot{\hat{\phi}}_{j,m}.\end{aligned} \tag{28}$$

Consider the following Lyapunov function candidate

$$V_{j,p} = \frac{1}{2}\vartheta_{j,p}^2 + \frac{1}{2\delta_{j,p}}\tilde{\phi}_{j,p}^2. \tag{29}$$

Then, differentiating $V_{j,p}$ yields

$$\begin{aligned}\dot{V}_{j,p} &= \vartheta_{j,p}\Bigg[x_{j,p+1} + f_{j,p}(X) - \sum_{m=1}^{p-1}\frac{\partial \zeta_{j,p-1}}{\partial x_{j,m}}(x_{j,m+1} + f_{j,m}(X)) \\ &\quad - \sum_{m=0}^{p-1}\frac{\partial \zeta_{j,p-1}}{\partial y_{dj}^{(m)}}y_{dj}^{(m+1)} - \sum_{m=1}^{p-1}\frac{\partial \zeta_{j,p-1}}{\partial \hat{\phi}_{j,m}}\dot{\hat{\phi}}_{j,m}\Bigg] - \frac{\tilde{\phi}_{j,p}}{\delta_{j,p}}\dot{\hat{\phi}}_{j,p}.\end{aligned} \tag{30}$$

Similarly to the analysis in (24) and (25), we have

$$\begin{aligned}&-\vartheta_{j,p}\sum_{m=1}^{p-1}\frac{\partial \zeta_{j,p-1}}{\partial x_{j,m}}f_{j,m}(X) \\ &\le \frac{1}{2}\bar{K}\vartheta_{j,p}^2\sum_{m=1}^{p-1}\left(\frac{\partial \zeta_{j,p-1}}{\partial x_{j,m}}\right)^2 + \sum_{m=1}^{p-1}\sum_{l=1}^{M}\sum_{s=1}^{n_l}\frac{1}{2}\vartheta_{l,s}^2\bar{\chi}_{j,m}^2(\vartheta_{l,s},\hat{\phi}_{l,s}) \\ &\quad + |\vartheta_{j,p}|\sum_{m=1}^{p-1}\left|\frac{\partial \zeta_{j,p-1}}{\partial x_{j,m}}\right|\chi_{j,m}(\varrho_0\bar{d}_0),\end{aligned} \tag{31}$$

$$\begin{aligned}\vartheta_{j,p}f_{j,p}(X) &\le \frac{1}{2}\bar{K}\vartheta_{j,p}^2 + \sum_{l=1}^{M}\sum_{s=1}^{n_l}\frac{1}{2}\vartheta_{l,s}^2\bar{\chi}_{j,p}^2(\vartheta_{l,s},\hat{\phi}_{l,s}) \\ &\quad + |\vartheta_{j,p}|\chi_{j,p}(\varrho_0\bar{d}_0),\end{aligned} \tag{32}$$

where $\bar{\chi}_{j,m}^2(\vartheta_{l,s},\hat{\phi}_{l,s}) = \varrho_0^2\Gamma_{l,s}^2(\hat{\phi}_{l,s})z_{j,m}^2\Big(\varrho_0|\vartheta_{l,s}|\Gamma_{l,s}(\hat{\phi}_{l,s})\Big)$, $m = 1, \ldots, p$.

Let $\Theta_{j,p} = \sum_{m=1}^{p-1} |\frac{\partial \zeta_{j,p-1}}{\partial x_{j,m}}| \chi_{j,m}(\varrho_0 \bar{d}_0) + \chi_{j,m}(\varrho_0 \bar{d}_0)$. According to Lemma 3, one has

$$|\vartheta_{j,p}|\Theta_{j,p} - \vartheta_{j,p}\Theta_{j,p} \tanh\left(\frac{\vartheta_{j,p}\Theta_{j,p}}{\sigma_{j,p}}\right) \le \kappa_1 \sigma_{j,p}, \tag{33}$$

where $\sigma_{j,p} > 0$ is a constant. Substituting (31)–(33) into (30), one has

$$\begin{aligned}\dot{V}_{j,p} \le{}& \vartheta_{j,p}\Bigg[\vartheta_{j,p+1} + \zeta_{j,p} - \sum_{m=1}^{p-1} \frac{\partial \zeta_{j,p-1}}{\partial x_{j,m}} x_{j,m+1} \\ &- \sum_{m=0}^{p-1} \frac{\partial \zeta_{j,p-1}}{\partial y_{dj}^{(m)}} y_{dj}^{(m+1)} - \sum_{m=1}^{p-1} \frac{\partial \zeta_{j,p-1}}{\partial \hat{\phi}_{j,m}} \dot{\hat{\phi}}_{j,m}\Bigg] - \frac{\tilde{\phi}_{j,p}}{\delta_{j,p}} \dot{\hat{\phi}}_{j,p} \\ &+ \frac{1}{2}\bar{K}\vartheta_{j,p}^2 \sum_{m=1}^{p-1} \left(\frac{\partial \zeta_{j,p-1}}{\partial x_{j,m}}\right)^2 + \frac{1}{2}\bar{K}\vartheta_{j,p}^2 + \kappa_1 \sigma_{j,p} \\ &+ \sum_{m=1}^{p} \sum_{l=1}^{M} \sum_{s=1}^{n_l} \frac{1}{2} \vartheta_{l,s}^2 \bar{\chi}_{j,m}^2(\vartheta_{l,s}, \hat{\phi}_{l,s}) \\ &+ \vartheta_{j,p}\Theta_{j,p} \tanh\left(\frac{\vartheta_{j,p}\Theta_{j,p}}{\sigma_{j,p}}\right).\end{aligned} \tag{34}$$

Step n_j: Choose the following Lyapunov function

$$V_{j,n_j} = \frac{1}{2}\vartheta_{j,n_j}^2 + \frac{1}{2\delta_{j,n_j}} \tilde{\phi}_{j,n_j}^2. \tag{35}$$

As $\vartheta_{j,n_j} = x_{j,n_j} - \zeta_{j,n_j-1}$, then the time derivative of V_{j,n_j} is given as

$$\begin{aligned}\dot{V}_{j,n_j} ={}& \vartheta_{j,n_j}\Bigg[\bar{k}_j u_j + f_{j,n_j}(X) - \sum_{m=1}^{n_j-1} \frac{\partial \zeta_{j,n_j-1}}{\partial x_{j,m}} (x_{j,m+1} + f_{j,m}(X)) \\ &- \sum_{m=0}^{n_j-1} \frac{\partial \zeta_{j,n_j-1}}{\partial y_{dj}^{(m)}} y_{dj}^{(m+1)} - \sum_{m=1}^{n_j-1} \frac{\partial \zeta_{j,n_j-1}}{\partial \hat{\phi}_{j,m}} \dot{\hat{\phi}}_{j,m}\Bigg] - \frac{\tilde{\phi}_{j,n_j}}{\delta_{j,n_j}} \dot{\hat{\phi}}_{j,n_j}.\end{aligned} \tag{36}$$

A similar procedure in (31)–(34) is employed for step n_j, we have

$$\begin{aligned}\dot{V}_{j,n_j} \le{}& \vartheta_{j,n_j}\Bigg[\bar{k}_j u_j - \sum_{m=1}^{n_j-1} \frac{\partial \zeta_{j,n_j-1}}{\partial x_{j,m}} x_{j,m+1} - \sum_{m=0}^{n_j-1} \frac{\partial \zeta_{j,n_j-1}}{\partial y_{dj}^{(m)}} y_{dj}^{(m+1)} \\ &- \sum_{m=1}^{n_j-1} \frac{\partial \zeta_{j,n_j-1}}{\partial \hat{\phi}_{j,m}} \dot{\hat{\phi}}_{j,m}\Bigg] + \frac{1}{2}\bar{K}\vartheta_{j,n_j}^2 \sum_{m=1}^{n_j-1} \left(\frac{\partial \zeta_{j,n_j-1}}{\partial x_{j,m}}\right)^2 \\ &+ \frac{1}{2}\bar{K}\vartheta_{j,n_j}^2 + \kappa_1 \sigma_{j,n_j} + \sum_{m=1}^{n_j} \sum_{l=1}^{M} \sum_{s=1}^{n_l} \frac{1}{2} \vartheta_{l,s}^2 \bar{\chi}_{j,m}^2(\vartheta_{l,s}, \hat{\phi}_{l,s}) \\ &+ \vartheta_{j,n_j}\Theta_{j,n_j} \tanh\left(\frac{\vartheta_{j,n_j}\Theta_{j,n_j}}{\sigma_{j,n_j}}\right) - \frac{\tilde{\phi}_{j,n_j}}{\delta_{j,n_j}} \dot{\hat{\phi}}_{j,n_j}.\end{aligned} \tag{37}$$

Subsequently, design the following Lyapunov function for the whole systems

$$V = \sum_{j=1}^{M} \sum_{p=1}^{n_j} V_{j,p} = \sum_{j=1}^{M} \left(\sum_{p=1}^{n_j} \frac{1}{2}\vartheta_{j,p}^2 + \sum_{p=1}^{n_j} \frac{1}{2\delta_{j,p}} \tilde{\phi}_{j,p}^2\right). \tag{38}$$

From $\dot{V}_{j,1}$ (20), $\dot{V}_{j,2}$ (27), $\dot{V}_{j,p}(3 \le p \le n_j - 1)$ (34) and $\dot{V}_{j,n_j}$ (37), we obtain

$$\begin{aligned}\dot{V} \le{}& -\sum_{j=1}^{M} \sum_{p=1}^{n_j} \frac{\tilde{\phi}_{j,p}}{\delta_{j,p}} \dot{\hat{\phi}}_{j,p} + \sum_{j=1}^{M} \vartheta_{j,1}\Bigg[\vartheta_{j,2} + \zeta_{j,1} - \dot{y}_{dj} \\ &+ \frac{1}{2}\bar{K}\vartheta_{j,1} + \Theta_{j,1} \tanh\left(\frac{\vartheta_{j,1}\Theta_{j,1}}{\sigma_{j,1}}\right)\Bigg] \\ &+ \sum_{j=1}^{M} \sum_{p=2}^{n_j-1} \vartheta_{j,p}\Bigg[\vartheta_{j,p+1} + \zeta_{j,p} - \sum_{m=1}^{p-1} \frac{\partial \zeta_{j,p-1}}{\partial x_{j,m}} x_{j,m+1} \\ &- \sum_{m=0}^{p-1} \frac{\partial \zeta_{j,p-1}}{\partial y_{dj}^{(m)}} y_{dj}^{(m+1)} - \sum_{m=1}^{p-1} \frac{\partial \zeta_{j,p-1}}{\partial \hat{\phi}_{j,m}} \dot{\hat{\phi}}_{j,m} \\ &+ \frac{1}{2}\bar{K}\vartheta_{j,p} + \frac{1}{2}\bar{K}\vartheta_{j,p} \sum_{m=1}^{p-1} \left(\frac{\partial \zeta_{j,p-1}}{\partial x_{j,m}}\right)^2 \\ &+ \Theta_{j,p} \tanh\left(\frac{\vartheta_{j,p}\Theta_{j,p}}{\sigma_{j,p}}\right)\Bigg] \\ &+ \sum_{j=1}^{M} \vartheta_{j,n_j}\Bigg[\bar{k}_j u_j - \sum_{m=1}^{n_j-1} \frac{\partial \zeta_{j,n_j-1}}{\partial x_{j,m}} x_{j,m+1} \\ &- \sum_{m=0}^{n_j-1} \frac{\partial \zeta_{j,n_j-1}}{\partial y_{dj}^{(m)}} y_{dj}^{(m+1)} - \sum_{m=1}^{n_j-1} \frac{\partial \zeta_{j,n_j-1}}{\partial \hat{\phi}_{j,m}} \dot{\hat{\phi}}_{j,m} \\ &+ \frac{1}{2}\bar{K}\vartheta_{j,n_j} \sum_{m=1}^{n_j-1} \left(\frac{\partial \zeta_{j,n_j-1}}{\partial x_{j,m}}\right)^2 + \frac{1}{2}\bar{K}\vartheta_{j,n_j} \\ &+ \Theta_{j,n_j} \tanh\left(\frac{\vartheta_{j,n_j}\Theta_{j,n_j}}{\sigma_{j,n_j}}\right)\Bigg] + \sum_{j=1}^{M} \sum_{p=1}^{n_j} \kappa_1 \sigma_{j,p} \\ &+ \sum_{j=1}^{M} \sum_{p=1}^{n_j} \sum_{m=1}^{p} \sum_{l=1}^{M} \sum_{s=1}^{n_l} \frac{1}{2} \vartheta_{l,s}^2 \bar{\chi}_{j,m}^2(\vartheta_{l,s}, \hat{\phi}_{l,s}).\end{aligned} \tag{39}$$

By rearranging sequence, one has

$$\sum_{j=1}^{M}\sum_{p=1}^{n_j}\sum_{m=1}^{p}\sum_{l=1}^{M}\sum_{s=1}^{n_l}\frac{1}{2}\vartheta_{l,s}^2\bar{\chi}_{j,m}^2(\vartheta_{l,s},\hat{\phi}_{l,s}) = \sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{1}{2}\vartheta_{j,p}^2\sum_{l=1}^{M}\sum_{s=1}^{n_l}\sum_{m=1}^{s}\bar{\chi}_{l,m}^2(\vartheta_{j,p},\hat{\phi}_{j,p}). \tag{40}$$

On the other hand, for $j=1,\ldots,M, p=1,\ldots,n_j-1$, the following inequality holds

$$\vartheta_{j,p}\vartheta_{j,p+1}\le\frac{\vartheta_{j,p}^2}{2}+\frac{\vartheta_{j,p+1}^2}{2}. \tag{41}$$

Then substituting (40) and (41) into (39), we have

$$\begin{aligned}\dot{V}\le&\sum_{j=1}^{M}\vartheta_{j,1}\Bigg[\zeta_{j,1}+\frac{1}{2}\bar{K}\vartheta_{j,1}+\vartheta_{j,1}\\&+\frac{1}{2}\vartheta_{j,1}\sum_{l=1}^{M}\sum_{s=1}^{n_l}\sum_{m=1}^{s}\bar{\chi}_{l,m}^2(\vartheta_{j,1},\hat{\phi}_{j,1})\\&-\dot{y}_{dj}+\Theta_{j,1}\tanh\left(\frac{\vartheta_{j,1}\Theta_{j,1}}{\sigma_{j,1}}\right)\Bigg]-\sum_{j=1}^{M}\frac{\vartheta_{j,1}^2}{2}\\&+\sum_{j=1}^{M}\sum_{p=2}^{n_j-1}\vartheta_{j,p}\Bigg[\zeta_{j,p}+\frac{3}{2}\vartheta_{j,p}-\sum_{m=1}^{p-1}\frac{\partial\zeta_{j,p-1}}{\partial x_{j,m}}x_{j,m+1}\\&-\sum_{m=0}^{p-1}\frac{\partial\zeta_{j,p-1}}{\partial y_{dj}^{(m)}}y_{dj}^{(m+1)}-\sum_{m=1}^{p-1}\frac{\partial\zeta_{j,p-1}}{\partial\hat{\phi}_{j,m}}\dot{\hat{\phi}}_{j,m}\\&+\frac{1}{2}\bar{K}\vartheta_{j,p}+\frac{1}{2}\bar{K}\vartheta_{j,p}\sum_{m=1}^{p-1}\left(\frac{\partial\zeta_{j,p-1}}{\partial x_{j,m}}\right)^2\\&+\frac{1}{2}\vartheta_{j,p}^2\sum_{l=1}^{M}\sum_{s=1}^{n_l}\sum_{m=1}^{s}\bar{\chi}_{l,m}^2(\vartheta_{j,p},\hat{\phi}_{j,p})\\&+\Theta_{j,p}\tanh\left(\frac{\vartheta_{j,p}\Theta_{j,p}}{\sigma_{j,p}}\right)\Bigg]-\sum_{j=1}^{M}\sum_{p=2}^{n_j-1}\frac{\vartheta_{j,p}^2}{2}\\&+\sum_{j=1}^{M}\vartheta_{j,n_j}\Bigg[\bar{k}_ju_j-\sum_{m=1}^{n_j-1}\frac{\partial\zeta_{j,n_j-1}}{\partial x_{j,m}}x_{j,m+1}+\vartheta_{j,n_j}\\&-\sum_{m=0}^{n_j-1}\frac{\partial\zeta_{j,n_j-1}}{\partial y_{dj}^{(m)}}y_{dj}^{(m+1)}-\sum_{m=1}^{n_j-1}\frac{\partial\zeta_{j,n_j-1}}{\partial\hat{\phi}_{j,m}}\dot{\hat{\phi}}_{j,m}\\&+\frac{1}{2}\bar{K}\vartheta_{j,n_j}\sum_{m=1}^{n_j-1}\left(\frac{\partial\zeta_{j,n_j-1}}{\partial x_{j,m}}\right)^2+\frac{1}{2}\bar{K}\vartheta_{j,n_j}\\&+\frac{1}{2}\vartheta_{j,n_j}^2\sum_{l=1}^{M}\sum_{s=1}^{n_l}\sum_{m=1}^{s}\bar{\chi}_{l,m}^2(\vartheta_{j,n_j},\hat{\phi}_{j,n_j})\\&+\Theta_{j,n_j}\tanh\left(\frac{\vartheta_{j,n_j}\Theta_{j,n_j}}{\sigma_{j,n_j}}\right)\Bigg]-\sum_{j=1}^{M}\frac{\vartheta_{j,n_j}^2}{2}\\&+\sum_{j=1}^{M}\sum_{p=1}^{n_j}\kappa_1\sigma_{j,p}-\sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}}{\delta_{j,p}}\dot{\hat{\phi}}_{j,p}.\end{aligned} \tag{42}$$

Subsequently, $\dot{V}$ is rewritten as

$$\begin{aligned}\dot{V}\le&\sum_{j=1}^{M}\vartheta_{j,1}(\zeta_{j,1}+\bar{f}_{j,1}(Z_{j,1}))\\&+\sum_{j=1}^{M}\sum_{p=2}^{n_j-1}\vartheta_{j,p}(\zeta_{j,p}+\bar{f}_{j,p}(Z_{j,p}))\\&+\sum_{j=1}^{M}\vartheta_{j,n_j}(\bar{k}_ju_j+\bar{f}_{j,n_j}(Z_{j,n_j}))-\sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\vartheta_{j,p}^2}{2}\\&+\sum_{j=1}^{M}\sum_{p=1}^{n_j}\kappa_1\sigma_{j,p}-\sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}}{\delta_{j,p}}\dot{\hat{\phi}}_{j,p},\end{aligned} \tag{43}$$

where

$$\begin{aligned}\bar{f}_{j,1}(Z_{j,1})=&\frac{1}{2}\bar{K}\vartheta_{j,1}+\vartheta_{j,1}-\dot{y}_{dj}+\Theta_{j,1}\tanh\left(\frac{\vartheta_{j,1}\Theta_{j,1}}{\sigma_{j,1}}\right)\\&+\frac{1}{2}\vartheta_{j,1}\sum_{l=1}^{M}\sum_{s=1}^{n_l}\sum_{m=1}^{s}\bar{\chi}_{l,m}^2(\vartheta_{j,1},\hat{\phi}_{j,1}),\\\bar{f}_{j,p}(Z_{j,p})=&\frac{3}{2}\vartheta_{j,p}-\sum_{m=1}^{p-1}\frac{\partial\zeta_{j,p-1}}{\partial x_{j,m}}x_{j,m+1}\\&-\sum_{m=0}^{p-1}\frac{\partial\zeta_{j,p-1}}{\partial y_{dj}^{(m)}}y_{dj}^{(m+1)}-\sum_{m=1}^{p-1}\frac{\partial\zeta_{j,p-1}}{\partial\hat{\phi}_{j,m}}\dot{\hat{\phi}}_{j,m}\\&+\frac{1}{2}\bar{K}\vartheta_{j,p}+\frac{1}{2}\bar{K}\vartheta_{j,p}\sum_{m=1}^{p-1}\left(\frac{\partial\zeta_{j,p-1}}{\partial x_{j,m}}\right)^2\\&+\frac{1}{2}\vartheta_{j,p}^2\sum_{l=1}^{M}\sum_{s=1}^{n_l}\sum_{m=1}^{s}\bar{\chi}_{l,m}^2(\vartheta_{j,p},\hat{\phi}_{j,p})\\&+\Theta_{j,p}\tanh\left(\frac{\vartheta_{j,p}\Theta_{j,p}}{\sigma_{j,p}}\right),\\\bar{f}_{j,n_j}(Z_{j,n_j})=&\vartheta_{j,n_j}-\sum_{m=1}^{n_j-1}\frac{\partial\zeta_{j,n_j-1}}{\partial x_{j,m}}x_{j,m+1}\\&-\sum_{m=0}^{n_j-1}\frac{\partial\zeta_{j,n_j-1}}{\partial y_{dj}^{(m)}}y_{dj}^{(m+1)}-\sum_{m=1}^{n_j-1}\frac{\partial\zeta_{j,n_j-1}}{\partial\hat{\phi}_{j,m}}\dot{\hat{\phi}}_{j,m}\\&+\frac{1}{2}\bar{K}\vartheta_{j,n_j}\sum_{m=1}^{n_j-1}\left(\frac{\partial\zeta_{j,n_j-1}}{\partial x_{j,m}}\right)^2+\frac{1}{2}\bar{K}\vartheta_{j,n_j}\\&+\frac{1}{2}\vartheta_{j,n_j}^2\sum_{l=1}^{M}\sum_{s=1}^{n_l}\sum_{m=1}^{s}\bar{\chi}_{l,m}^2(\vartheta_{j,n_j},\hat{\phi}_{j,n_j})\\&+\Theta_{j,n_j}\tanh\left(\frac{\vartheta_{j,n_j}\Theta_{j,n_j}}{\sigma_{j,n_j}}\right).\end{aligned}$$

From Lemma 6, for any given $\tau_{j,p}>0$, there is a fuzzy logic system $\pi_{j,p}^T\eta_{j,p}(Z_{j,p})$ such that

$$\bar{f}_{j,p}(Z_{j,p})=\pi_{j,p}^T\eta_{j,p}(Z_{j,p})+\Phi_{j,p}(Z_{j,p}),|\Phi_{j,p}(Z_{j,p})|\le\tau_{j,p} \tag{44}$$

with $\Phi_{j,p}$ being the approximation error. According to Young's inequality, we have

$$\vartheta_{j,p}\bar{f}_{j,p}(Z_{j,p}) \leq \frac{\phi_{j,p}}{2\mu_{j,p}^2}\vartheta_{j,p}^2\eta_{j,p}^T\eta_{j,p} + \frac{\mu_{j,p}^2}{2} + \frac{\vartheta_{j,p}^2}{2} + \frac{\tau_{j,p}^2}{2}, \tag{45}$$

where $\phi_{j,p} = \|\pi_{j,p}\|^2$, $j = 1, \ldots, M, p = 1, \ldots, n_j$. Substituting (45) into (43), one has

$$\begin{aligned}\dot{V} \leq & \sum_{j=1}^{M}\vartheta_{j,1}\left(\zeta_{j,1} + \frac{\hat{\phi}_{j,1}}{2\mu_{j,1}^2}\vartheta_{j,1}^2\eta_{j,1}^{\mathrm{T}}\eta_{j,1}\right)\\ & + \sum_{j=1}^{M}\sum_{p=2}^{n_j-1}\vartheta_{j,p}(\zeta_{j,p} + \frac{\hat{\phi}_{j,p}}{2\mu_{j,p}^2}\vartheta_{j,p}^2\eta_{j,p}^{\mathrm{T}}\eta_{j,p})\\ & + \sum_{j=1}^{M}\vartheta_{j,n_j}(\bar{k}_j u_j + \frac{\hat{\phi}_{j,n_j}}{2\mu_{j,n_j}^2}\vartheta_{j,n_j}^2\eta_{j,n_j}^{\mathrm{T}}\eta_{j,n_j})\\ & - \sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}}{\delta_{j,p}}\left(\dot{\hat{\phi}}_{j,p} - \frac{\delta_{j,p}}{2\mu_{j,p}^2}\vartheta_{j,p}^2\eta_{j,p}^{\mathrm{T}}\eta_{j,p}\right)\\ & + \sum_{j=1}^{M}\sum_{p=1}^{n_j}\left(\kappa_1\sigma_{j,p} + \frac{\mu_{j,p}^2}{2} + \frac{\tau_{j,p}^2}{2}\right).\end{aligned} \tag{46}$$

From (14) and Lemma 2, we know that $\hat{\phi}_{j,p}(0) \geq 0$ implies that $\hat{\phi}_{j,p}(t) \geq 0$ for all $t \geq 0$. Therefore, the following result holds

$$-\frac{\hat{\phi}_{j,n_j}}{\mu_{j,n_j}^2}\vartheta_{j,n_j}^2\eta_{j,n_j}^{\mathrm{T}}\eta_{j,n_j} \leq -\frac{\hat{\phi}_{j,n_j}}{2\mu_{j,n_j}^2}\vartheta_{j,n_j}^2\eta_{j,n_j}^{\mathrm{T}}\eta_{j,n_j}. \tag{47}$$

It is noted that

$$\frac{\tilde{\phi}_{j,p}}{\delta_{j,p}}\hat{\phi}_{j,p} \leq -\frac{\tilde{\phi}_{j,p}^2}{2\delta_{j,p}} + \frac{\phi_{j,p}^2}{2\delta_{j,p}}. \tag{48}$$

Substituting (11), (12), (14), (47) and (48) into (46), we can obtain

$$\begin{aligned}\dot{V} \leq & -\sum_{j=1}^{M}\sum_{p=1}^{n_j}\left(\rho_{j,p}\vartheta_{j,p}^{2\upsilon} + \frac{\varepsilon_{j,p}}{2\delta_{j,p}}\tilde{\phi}_{j,p}^2\right)\\ & + \sum_{j=1}^{M}\sum_{p=1}^{n_j}\left(\frac{\varepsilon_{j,p}}{2\delta_{j,p}}\phi_{j,p}^2 + \kappa_1\sigma_{j,p} + \frac{\mu_{j,p}^2}{2} + \frac{\tau_{j,p}^2}{2}\right)\\ \leq & -\rho_0\sum_{j=1}^{M}\sum_{p=1}^{n_j}\vartheta_{j,p}^{2\upsilon} - \rho_0\sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}^2}{2\delta_{j,p}}\\ & + \sum_{j=1}^{M}\sum_{p=1}^{n_j}\left(\frac{\varepsilon_{j,p}}{2\delta_{j,p}}\phi_{j,p}^2 + \kappa_1\sigma_{j,p} + \frac{\mu_{j,p}^2}{2} + \frac{\tau_{j,p}^2}{2}\right),\end{aligned} \tag{49}$$

where $\rho_0 = \min\{\rho_{j,p}, \varepsilon_{j,p}, 1 \leq j \leq M, 1 \leq p \leq n_j\}$.

Based on Lemma 4, let $\alpha = 1$, $\beta = \left(\sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}^2}{2\delta_{j,p}}\right)^{\upsilon}$, $\lambda = 1 - \upsilon$, $v = \upsilon$ and $o = \upsilon^{\frac{\upsilon}{1-\upsilon}}$, we get

$$\left(\sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}^2}{2\delta_{j,p}}\right)^{\upsilon} \leq (1-\upsilon)\upsilon^{\frac{\upsilon}{1-\upsilon}} + \sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}^2}{2\delta_{j,p}}. \tag{50}$$

Substituting (50) into (49), one has

$$\begin{aligned}\dot{V} \leq & -\rho_0\sum_{j=1}^{M}\sum_{p=1}^{n_j}\vartheta_{j,p}^{2\upsilon} - \rho_0\left(\sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}^2}{2\delta_{j,p}}\right)^{\upsilon} + \xi_0\\ \leq & -2^{\upsilon}\rho_0\sum_{j=1}^{M}\sum_{p=1}^{n_j}\left(\frac{\vartheta_{j,p}^2}{2}\right)^{\upsilon} - \rho_0\left(\sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}^2}{2\delta_{j,p}}\right)^{\upsilon} + \xi_0,\end{aligned} \tag{51}$$

where $\xi_0 = (1-\upsilon)\upsilon^{\frac{\upsilon}{1-\upsilon}} + \sum_{j=1}^{M}\sum_{p=1}^{n_j}\left(\frac{\varepsilon_{j,p}}{2\delta_{j,p}}\phi_{j,p}^2 + \kappa_1\sigma_{j,p} + \frac{\mu_{j,p}^2}{2} + \frac{\tau_{j,p}^2}{2}\right)$. Then applying Lemma 1, we have

$$\begin{aligned}\dot{V} \leq & -2^{\upsilon}\rho_0\left(\sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\vartheta_{j,p}^2}{2}\right)^{\upsilon} - \rho_0\left(\sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}^2}{2\delta_{j,p}}\right)^{\upsilon} + \xi_0\\ \leq & -\rho_0 V^{\upsilon} + \xi_0.\end{aligned} \tag{52}$$

Let $T_r = \frac{1}{(1-\upsilon)\kappa\rho_0}[V^{1-\upsilon}(\zeta_j(0), \tilde{\phi}_j(0)) - (\frac{\xi_0}{(1-\kappa)\rho_0})^{\frac{1-\upsilon}{\upsilon}}]$ with $\zeta_j(0) = [\zeta_{j,1}(0), \ldots, \zeta_{j,n_j}(0)]^T$, $\tilde{\phi}_j(0) = [\tilde{\phi}_{j,1}(0), \ldots \tilde{\phi}_{j,n_j}(0)]^T$ and $0 < \kappa \leq 1$. According to Lemma 5, for $t \geq T_r$, $V^{\upsilon}(\zeta_j, \tilde{\phi}_j) \leq \frac{\xi_0}{(1-\kappa)\rho_0}$, which implies that all the signals in the closed-loop system are SGPFS. In addition, from the definition of V, for $\forall t \geq T_r$, we have

$$|y_j - y_{dj}| \leq \sqrt{2}\left(\frac{\xi_0}{(1-\kappa)\rho_0}\right)^{\frac{1}{2\upsilon}}, \tag{53}$$

which means that the tracking error remains in a small neighborhood of the origin after the finite time T_r.

3.2 Case 2: Consider the Fuzzy Value $\tilde{k_j} \in [\bar{k_j} - \Xi_j, \bar{k_j} + \Xi_j]$

The discussion in step $p(1 \leq p \leq n_j - 1)$ is the same as that in (16)–(34). For V_{j,n_j} defined in (35), $\dot{V}_{j,n_j}$ is calculated as

$$\dot{V}_{j,n_j} \leq \vartheta_{j,n_j}\Bigg[\tilde{k}_j u_j - \sum_{m=1}^{n_j-1}\frac{\partial\zeta_{j,n_j-1}}{\partial x_{j,m}}x_{j,m+1} - \sum_{m=0}^{n_j-1}\frac{\partial\zeta_{j,n_j-1}}{\partial y_{dj}^{(m)}}y_{dj}^{(m+1)} - \sum_{m=1}^{n_j-1}\frac{\partial\zeta_{j,n_j-1}}{\partial\hat{\phi}_{j,m}}\dot{\hat{\phi}}_{j,m}\Bigg] + \frac{1}{2}\bar{K}\vartheta_{j,n_j}^2\sum_{m=1}^{n_j-1}\left(\frac{\partial\zeta_{j,n_j-1}}{\partial x_{j,m}}\right)^2 + \frac{1}{2}\bar{K}\vartheta_{j,n_j}^2 + \kappa_1\sigma_{j,n_j} + \sum_{m=1}^{n_j}\sum_{l=1}^{M}\sum_{s=1}^{n_l}\frac{1}{2}\vartheta_{l,s}^2\bar{\chi}_{j,m}^2(\vartheta_{l,s},\hat{\phi}_{l,s}) + \vartheta_{j,n_j}\Theta_{j,n_j}\tanh\left(\frac{\vartheta_{j,n_j}\Theta_{j,n_j}}{\sigma_{j,n_j}}\right) - \frac{\tilde{\phi}_{j,n_j}}{\delta_{j,n_j}}\dot{\hat{\phi}}_{j,n_j}. \tag{54}$$

Then, similarly to the above process in (38)–(45), we have

$$\dot{V} \leq \sum_{j=1}^{M}\vartheta_{j,1}\left(\zeta_{j,1} + \frac{\hat{\phi}_{j,1}}{2\mu_{j,1}^2}\vartheta_{j,1}^2\eta_{j,1}^{\mathrm{T}}\eta_{j,1}\right) + \sum_{j=1}^{M}\sum_{p=2}^{n_j-1}\vartheta_{j,p}\left(\zeta_{j,p} + \frac{\hat{\phi}_{j,p}}{2\mu_{j,p}^2}\vartheta_{j,p}^2\eta_{j,p}^{\mathrm{T}}\eta_{j,p}\right) + \sum_{j=1}^{M}\vartheta_{j,n_j}\left(\tilde{k}_j u_j + \frac{\hat{\phi}_{j,n_j}}{2\mu_{j,n_j}^2}\vartheta_{j,n_j}^2\eta_{j,n_j}^{\mathrm{T}}\eta_{j,n_j}\right) - \sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}}{\delta_{j,p}}\left(\dot{\hat{\phi}}_{j,p} - \frac{\delta_{j,p}}{2\mu_{j,p}^2}\vartheta_{j,p}^2\eta_{j,p}^{\mathrm{T}}\eta_{j,p}\right) + \sum_{j=1}^{M}\sum_{p=1}^{n_j}\left(\kappa_1\sigma_{j,p} + \frac{\mu_{j,p}^2}{2} + \frac{\tau_{j,p}^2}{2}\right). \tag{55}$$

As $\bar{k_j} \in [k_{j,1}, k_{j,\gamma}]$ and $0 < \Xi_j \leq \frac{k_{j,1}}{2}$, the following inequality holds

$$\frac{\tilde{k_j}}{\bar{k_j}} \geq \frac{\bar{k_j} - \Xi_j}{\bar{k_j}} \geq 1 - \frac{k_{j,1}}{2\bar{k_j}} \geq \frac{1}{2}. \tag{56}$$

Combining (47) with (56), we obtain

$$-\frac{\tilde{k_j}}{\bar{k_j}}\frac{\hat{\phi}_{j,n_j}}{\mu_{j,n_j}^2}\vartheta_{j,n_j}^2\eta_{j,n_j}^{\mathrm{T}}\eta_{j,n_j} \leq -\frac{\hat{\phi}_{j,n_j}}{2\mu_{j,n_j}^2}\vartheta_{j,n_j}^2\eta_{j,n_j}^{\mathrm{T}}\eta_{j,n_j}. \tag{57}$$

Substituting (11), (12), (14), (48), (56) and (57) into (55), the time derivative of V_{j,n_j} is rewritten as

$$\dot{V} \leq -\sum_{j=1}^{M}\sum_{p=1}^{n_j-1}\rho_{j,p}\vartheta_{j,p}^{2\upsilon} - \sum_{j=1}^{M}\frac{\rho_{j,n_j}}{2}\vartheta_{j,n_j}^{2\upsilon} - \sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\varepsilon_{j,p}}{2\delta_{j,p}}\tilde{\phi}_{j,p}^2 + \sum_{j=1}^{M}\sum_{p=1}^{n_j}\left(\frac{\varepsilon_{j,p}}{2\delta_{j,p}}\phi_{j,p}^2 + \kappa_1\sigma_{j,p} + \frac{\mu_{j,p}^2}{2} + \frac{\tau_{j,p}^2}{2}\right)$$
$$\leq -\rho_0\sum_{j=1}^{M}\sum_{p=1}^{n_j}\vartheta_{j,p}^{2\upsilon} - \rho_0\sum_{j=1}^{M}\sum_{p=1}^{n_j}\frac{\tilde{\phi}_{j,p}^2}{2\delta_{j,p}} + \sum_{j=1}^{M}\sum_{p=1}^{n_j}\left(\frac{\varepsilon_{j,p}}{2\delta_{j,p}}\phi_{j,p}^2 + \kappa_1\sigma_{j,p} + \frac{\mu_{j,p}^2}{2} + \frac{\tau_{j,p}^2}{2}\right), \tag{58}$$

where $\rho_0 = \min\{\rho_{j,p}(1 \leq j \leq M, 1 \leq p \leq n_j - 1), \frac{\rho_{j,n_j}}{2}, \varepsilon_{j,p}(1 \leq j \leq M, 1 \leq p \leq n_j)\}$.

Then, by repeating the same way applied in the process in (50)–(52), we can get

$$\dot{V} \leq -\rho_0 V^{\upsilon} + \xi_0 \tag{59}$$

with ξ_0 being defined in (51). Consequently, similarly to the previous analysis in Case 1, for a fuzzy value $\tilde{k_j} \in [\bar{k_j} - \Xi_j, \bar{k_j} + \Xi_j]$, all the signals in the closed-loop system are SGPFS, and the tracking error converges to a small neighborhood of the origin and remains there after the finite time T_r. The proof is completed. □

Remark 3 From (53) and the definitions of ξ_0 and ρ_0, we know that the tracking error $|y_j - y_{dj}|$ is governed by the main design parameters $\rho_{j,p}$, $\delta_{j,p}$ and $\mu_{j,p}$. Obviously, smaller tracking error can be achieved by selecting larger parameters $\rho_{j,p}$ and $\delta_{j,p}$ and smaller parameter $\mu_{j,p}$.

Remark 4 Note that several results on MIMO nonlinear systems have been obtained in [3, 14, 22]. The main differences between our result and the ones in [3, 14, 22] are concluded as follows.

- (i) The tracking errors in [3, 14, 22] are guaranteed to be small enough as the time variable goes to infinity. Unlike these works, in this paper, the proposed control method makes the tracking errors as small as possible in finite time.
- (ii) From the controlled MIMO system models, the presented control methods in [3, 14] are developed in the sense of MIMO strict-feedback systems, and this paper is concerned with the finite-time tracking control problem for MIMO nonlinear systems in nonstrict-feedback structures.
- (iii) Compared with [22], where adaptive fuzzy control methods are investigated for MIMO nonstrict-feedback systems, the effects of fuzzy dead zones are considered in our paper. Therefore, the developed MIMO nonlinear system in this paper is more general.

4 Simulation Examples

In this section, the effectiveness of the proposed finite-time control approach is testified by the following two examples.

Example 1 Consider the following two continuous stirred tank reactor process system [22], as shown in Fig. 1. The model can be described by the following differential equations:

$$\begin{aligned}
\dot{x}_{1,1} &= d_{1,1}x_{1,2}, \quad y_1 = x_{1,1},\\
\dot{x}_{1,2} &= d_{1,2}u_1,\\
\dot{x}_{2,1} &= d_{2,1}x_{2,2} + \varphi_{2,1}(x_{1,1}, x_{2,1}) + \varDelta_{2,1},\\
\dot{x}_{2,2} &= d_{2,2}u_2 + \varphi_{2,2}(x_{2,1}, x_{2,2}), \quad y_2 = x_{2,1},\\
\dot{x}_{3,1} &= d_{3,1}x_{3,2} + \varphi_{3,1}(x_{2,1}, x_{3,1}) + \varDelta_{3,1},\\
\dot{x}_{3,2} &= d_{3,2}u_3 + \varphi_{3,2}(x_{3,1}, x_{3,2}), \quad y_3 = x_{3,1},
\end{aligned} \tag{60}$$

where $d_{1,1} = 1$, $d_{1,2} = 1$, $d_{2,1} = UA/\rho c_\rho V$, $d_{2,2} = F_{j2}/V_j$, $d_{3,1} = UA/\rho c_\rho V$, $d_{3,2} = F_{j1}/V_j$, $\varGamma = (F + F_R)/V$, $\varPsi = F_0/V$, $\varphi_{2,1} = ((F + F_R)/V)T_1^d -$ $[((F + F_R)/V)$ $(x_{2,1} + T_2^d)] -$ $[(\alpha\lambda/\rho c_\rho)(x_{1,1} + C_{A2}^d)e$ $-(E/(R(x_{2,1} +$ $T_2^d)))] -$ $(UA/\rho c_\rho V)(x_{2,1} + T_2^d - T_{j2}^d)$, $\varphi_{2,2} = (F_{j2}/V_j)$ $(T_{j20}^d -$ $x_{2,2} - T_{j2}^d) + (UA/\rho_j c_j V_j)(x_{2,1} + \varGamma_2^d - x_{2,2} - T_{j2}^d)$, $\varphi_{3,1} = F_0 T_0^d/V - ((F + F_R)/V)$ $(x_{3,1} + T_1^d) + (F_R/V)$ $(x_{2,1} + T_2^d) - \alpha\lambda C_{A1} e$ $-E/(R(x_{3,1} + T_1^d))/$ $\rho c_\rho -$ $UA(x_{3,1} + T_1^d - x_{3,2} -$ $T_{j1}^d)/\rho_j c_j V_j$, $\varphi_{3,2} = (F_{j1}$ $/V_j)(T_{j10}^d -$ $x_{3,2} - T_{j,1}^d) + (UA/\rho_j c_j V_j)$ $(x_{3,1} + T_1^d - x_{3,2} - T_{j1}^d)$, $C_{A1} =$ $(V/(F + F_R))(x_{1,2} +$ $((F + F_R)/V)(x_{1,1} + C_{A2}^d) +$ $\alpha(x_{1,1} +$ $C_{A2}^d)e^{-E/(R(x_{2,1}+T_2^d))})$. The terms $\varDelta_{2,1}$ and $\varDelta_{3,1}$ denote the unknown connections between the subsystems, and $\varDelta_{2,1} = \varGamma x_{3,1}$ and $\varDelta_{3,1} = \varPsi\varpi$. In addition, $V_{j1} = V_{j2} = V$, $V_1 = V_2 = V$, $F_0 = F_2 = F$, $\varpi = e^{-0.15t}\sin(t)$ and the values of the parameters are selected same as [22].

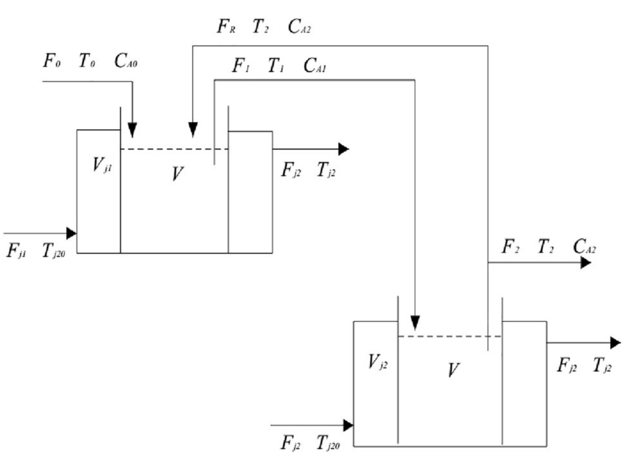

Fig. 1 Two continuous stirred tank reactor

In this paper, we consider the practical two continuous stirred tank reactor process system with fuzzy dead zones, as the parameters of the dead zones are usually uncertain. By defining the following coordinate changes: $\bar{x}_{1,1} = x_{1,1}$, $\bar{x}_{1,2} = d_{1,1}x_{1,2}$, $\bar{x}_{2,1} = x_{2,1}$, $\bar{x}_{2,2} = d_{2,1}x_{2,2}$, $\bar{x}_{3,1} = x_{3,1}$, and $\bar{x}_{3,2} = d_{3,1}x_{3,2}$, the state space equations of (60) can be rewritten as

$$\begin{aligned}
\dot{\bar{x}}_{1,1} &= \bar{x}_{1,2}, y_1 = \bar{x}_{1,1},\\
\dot{\bar{x}}_{1,2} &= \bar{u}_1,\\
\dot{\bar{x}}_{2,1} &= \bar{x}_{2,2} + \bar{\varphi}_{2,1}(\bar{x}_{1,1}, \bar{x}_{2,1}) + \bar{\varDelta}_{2,1},\\
\dot{\bar{x}}_{2,2} &= \bar{u}_2 + \bar{\varphi}_{2,2}(\bar{x}_{2,1}, \bar{x}_{2,2}), y_2 = \bar{x}_{2,1},\\
\dot{\bar{x}}_{3,1} &= \bar{x}_{3,2} + \bar{\varphi}_{3,1}(\bar{x}_{2,1}, \bar{x}_{3,1}) + \bar{\varDelta}_{3,1},\\
\dot{\bar{x}}_{3,2} &= \bar{u}_3 + \bar{\varphi}_{3,2}(\bar{x}_{3,1}, \bar{x}_{3,2}), y_3 = \bar{x}_{3,1},
\end{aligned} \tag{61}$$

where

$$\begin{aligned}
&\bar{u}_1 = d_{1,1}d_{1,2}\psi_1(u_1), \bar{u}_2 = d_{2,1}d_{2,2}\psi_2(u_2),\\
&\bar{u}_3 = d_{3,1}d_{3,2}\psi_3(u_3), \bar{\varphi}_{2,1}(\bar{x}_{1,1}, \bar{x}_{2,1}) = \varphi_{2,1}(x_{1,1}, x_{2,1}),\\
&\bar{\varphi}_{2,2}(\bar{x}_{2,1}, \bar{x}_{2,2}) = d_{2,1}\varphi_{2,2}(x_{2,1}, d_{2,1}x_{2,2}),\\
&\bar{\varphi}_{3,1}(\bar{x}_{2,1}, \bar{x}_{3,1}) = \varphi_{3,1}(x_{2,1}, x_{3,1}),\\
&\bar{\varphi}_{3,2}(\bar{x}_{3,1}, \bar{x}_{3,2}) = d_{3,1}\varphi_{3,2}(x_{3,1}, d_{3,1}x_{3,2}),\\
&\bar{\varDelta}_{2,1} = \varDelta_{2,1} + \bar{\varphi}_{2,1}\bar{x}_{1,2}\bar{x}_{2,2}\bar{x}_{3,2},\\
&\bar{\varDelta}_{3,1} = \varDelta_{3,1} + \bar{\varphi}_{2,1}\bar{x}_{3,1}\bar{x}_{1,2}\bar{x}_{2,2}\sqrt[3]{\bar{x}_{3,2}}.
\end{aligned}$$

$\psi_1(u_1)$, $\psi_2(u_2)$ and $\psi_3(u_3)$ are the outputs of the dead zones. The parameters of the dead zones are designed as $b_{1,r} = b_{3,r} = 0.8$, $b_{1,l} = b_{3,l} = -0.8$, $b_{2,r} = 1$ and $b_{2,l} = -1$. We assume that the universe of discourse of $\tilde{k_j}$ is $U_j = \{1, 1.5, 2\}$, i.e., $k_{1,1} = k_{2,1} = k_{3,1} = 1$, $k_{1,2} = k_{2,2} = k_{3,2} = 1.5$, $k_{1,3} = k_{2,3} = k_{3,3} = 2$. The fuzzy grade of $k_{j,q}$ is designed as $\mu_j(k_{j,q}) = \frac{1}{\sqrt{2\pi}\sigma_j}\exp(\frac{-(k_{j,q}-1.5)^2}{2\sigma_j^2})$ with $\sigma_j = \frac{1}{6}$. The reference signals are designed as $y_{d1} = \sin(1.5t)$, $y_{d2} = \sin(t) + \sin(1.5t)$, and $y_{d3} = \sin(t)$.

Remark 5 It should be pointed out that $\bar{\varDelta}_{2,1}$ and $\bar{\varDelta}_{3,1}$ in [3] are chosen as $\bar{\varDelta}_{2,1} = \varDelta_{2,1}$ and $\bar{\varDelta}_{3,1} = \varDelta_{3,1}$, because the applied control scheme in [3] requires that the controlled system is in strict-feedback structure. Unlike [3], this paper is concerned with the tracking control issue for MIMO nonlinear systems with nonstrict-feedback structures. Therefore, the existing control approaches cannot be applied to this system as they are considered just for nonlinear strict-feedback systems, which means that our proposed scheme can be used to more general classes of MIMO systems.

In the simulation, nine fuzzy sets are defined over interval $[-8, 8]$ for $\bar{x}_{1,1}$, $\bar{x}_{1,2}$, $\bar{x}_{2,1}$, $\bar{x}_{2,2}$, $\bar{x}_{3,1}$ and $\bar{x}_{3,2}$, and by choosing partitioning points as -8, -6, -4, -2, 0, 2, 4, 6, 8. The fuzzy membership function are designed as follows:

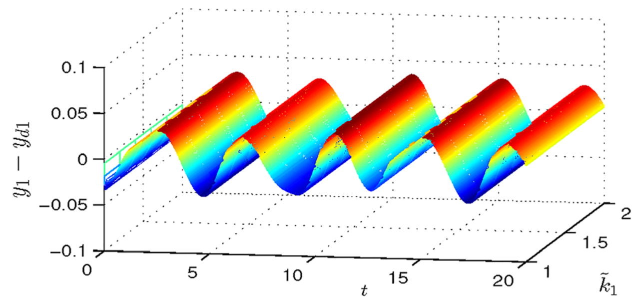

Fig. 2 Tracking error $y_1 - y_{d1}$ for Example 1

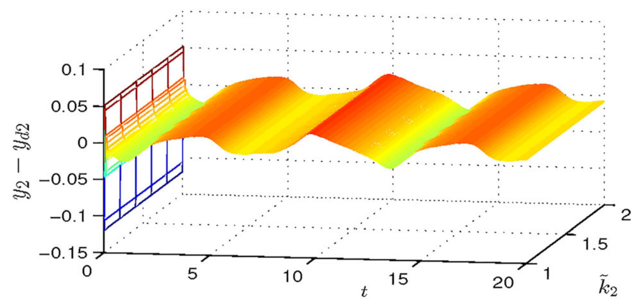

Fig. 3 Tracking error $y_2 - y_{d2}$ for Example 1

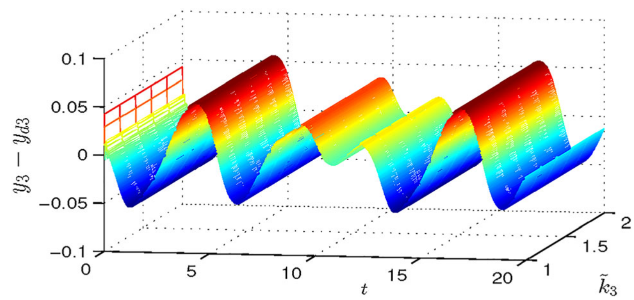

Fig. 4 Tracking error $y_3 - y_{d3}$ for Example 1

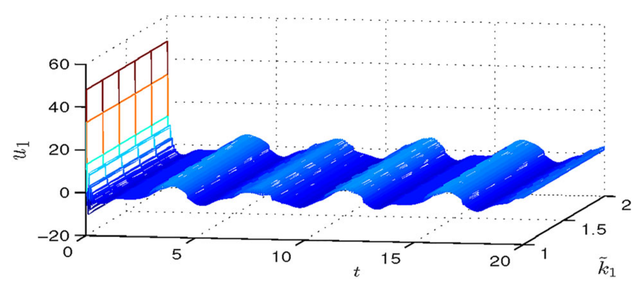

Fig. 5 Control signal u_1 for Example 1

$$\mu_{F^l_{j,p}}(\bar{x}_{j,p}) = \exp(-0.5(\bar{x}_{j,p} - w_l)^2), l = 1, 2, \ldots, 9$$

with $w_1 = -8$, $w_2 = -6$, $w_3 = -4$, $w_4 = -2$, $w_5 = 0$, $w_6 = 2$, $w_7 = 4$, $w_8 = 6$, $w_9 = 8$.

The design parameters in the virtual control signals, actuator controller and parameters adaptive laws are

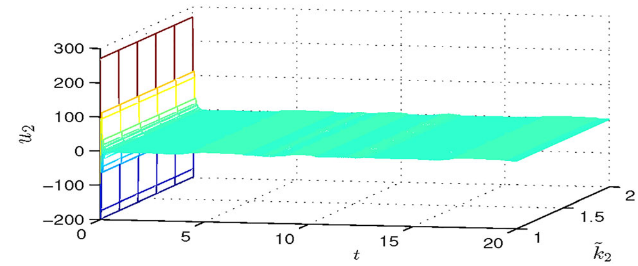

Fig. 6 Control signal u_2 for Example 1

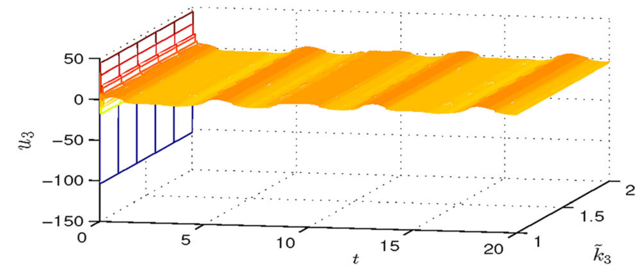

Fig. 7 Control signal u_3 for Example 1

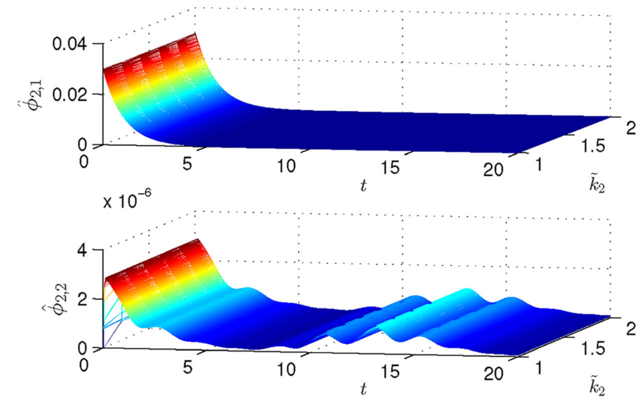

Fig. 8 Adaptive parameters $\hat{\phi}_{2,1}$ and $\hat{\phi}_{2,2}$ for Example 1

designed as $\rho_{1,1} = 58$, $\rho_{1,2} = 60$, $\rho_{2,1} = \rho_{3,1} = 52$, $\rho_{2,2} = \rho_{3,2} = 54$, $\mu_{2,1} = \mu_{2,2} = \mu_{3,1} = \mu_{3,2} = 2$, $\delta_{2,1} = \delta_{2,2} = \delta_{3,1} = \delta_{3,2} = 1$, $\varepsilon_{2,1} = \varepsilon_{2,2} = \varepsilon_{3,1} = \varepsilon_{3,2} = 1$, and $\upsilon = \frac{9999}{10001}$. The initial conditions are chosen as $\bar{x}_{1,1}(0) = 0.01$, $\bar{x}_{1,2}(0) = 0.05$, $\bar{x}_{2,1}(0) = -0.1$, $\bar{x}_{2,2}(0) = 0.2$, $\bar{x}_{3,1}(0) = 0.05$, $\bar{x}_{3,2}(0) = 0.1$, $\hat{\phi}_{2,1}(0) = \hat{\phi}_{3,1}(0) = 0.03$, $\hat{\phi}_{2,2}(0) = \hat{\phi}_{3,2}(0) = 0$. The simulation results are depicted in Figs. 2, 3, 4, 5, 6, 7, 8 and 9. Figures 2, 3 and 4 address the trajectories of tacking errors for $\tilde{k}_1$, $\tilde{k}_2$ and $\tilde{k}_3$, respectively. Figures 5, 6 and 7 illustrate the trajectories of control signals u_1, u_2 and u_3, respectively. Figures 8 and 9 show the trajectories of adaptive parameters $\hat{\phi}_{2,1}$, $\hat{\phi}_{2,2}$, $\hat{\phi}_{3,1}$ and $\hat{\phi}_{3,2}$. From these

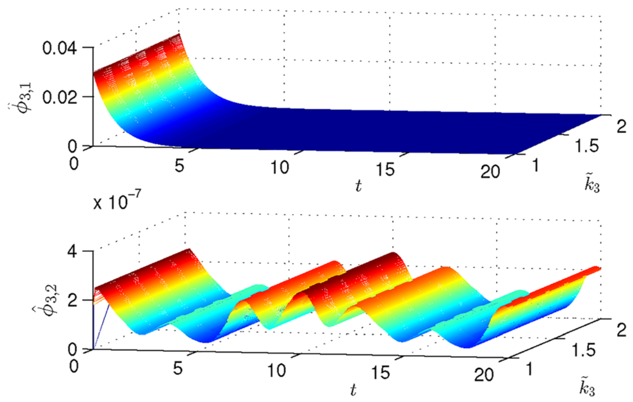

Fig. 9 Adaptive parameters $\hat{\phi}_{3,1}$ and $\hat{\phi}_{3,2}$ for Example 1

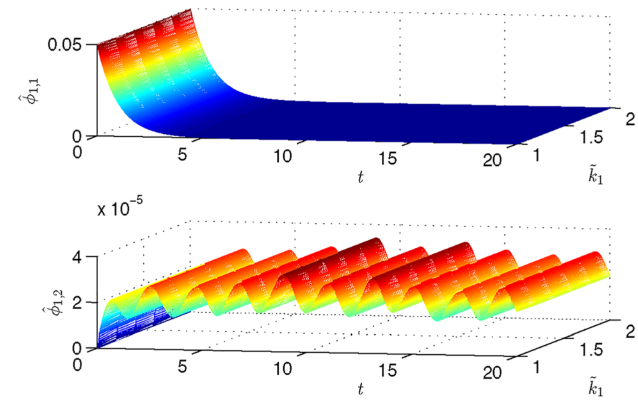

Fig. 12 Adaptive parameters $\hat{\delta}_{1,j}(j = 1, 2)$ for Example 2

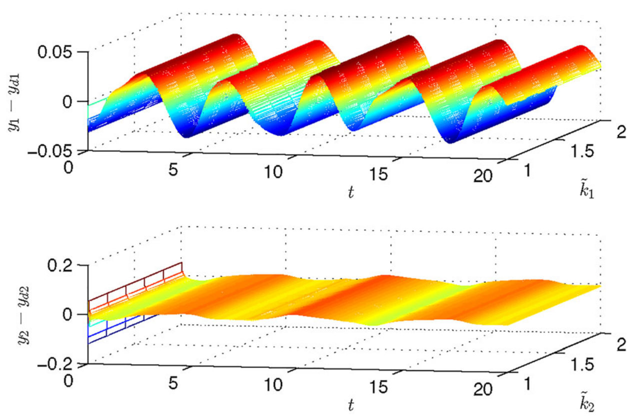

Fig. 10 Tracking errors $y_j - y_{dj}(j = 1, 2)$ for Example 2

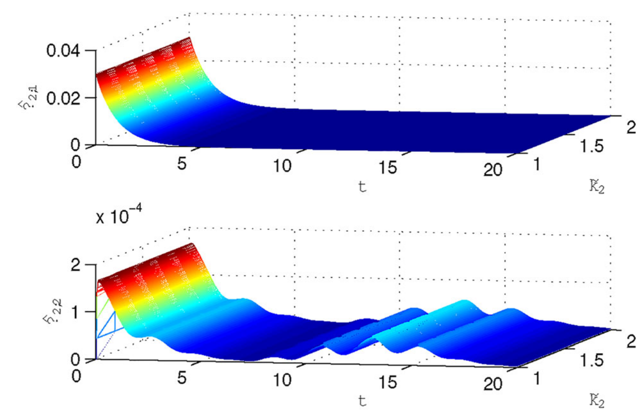

Fig. 13 Adaptive parameters $\hat{\delta}_{2,j}(j = 1, 2)$ for Example 2

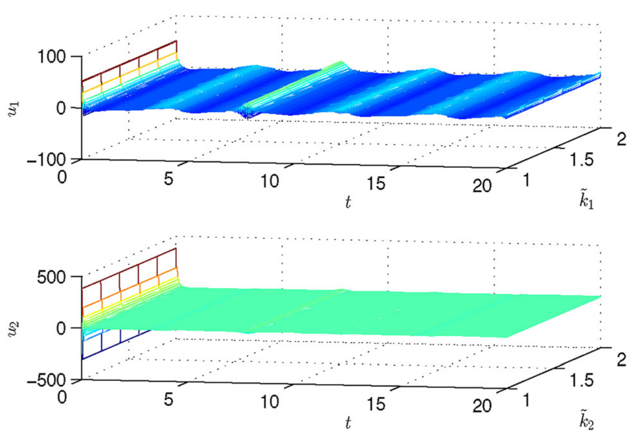

Fig. 11 Control signals $u_j(j = 1, 2)$ for Example 2

simulation results, it is observed that the proposed finite-time control scheme cannot only guarantee the stability of the controlled MIMO nonlinear system in finite time but also obtain desired tracking control performances.

Example 2 Consider the following MIMO nonstrict-feedback system with fuzzy dead zones described by:

$$\begin{cases} \dot{x}_{1,1} = x_{1,2} + \dfrac{x_{1,1}x_{2,1}}{2 + x_{1,2}^2 x_{2,2} + x_{2,1}^2}, \\ \dot{x}_{1,2} = \psi_1(u_1) + x_{1,1}x_{1,2}x_{2,1} + x_{1,2}x_{2,2}^2, \\ y_1 = x_{1,1}, \\ \dot{x}_{2,1} = x_{2,2} + 2x_{2,1}^2 + x_{1,1}x_{1,2}\sin(x_{2,2}), \\ \dot{x}_{2,2} = \psi_2(u_2) + x_{1,1}x_{2,2} + x_{2,1}^2\sin(x_{1,2}), \\ y_2 = x_{2,1}, \end{cases} \tag{62}$$

where $\psi_1(u_1)$ and $\psi_2(u_2)$ are the outputs of the dead zones. The parameters of the dead zones are selected as $b_{1,r} = 0.8$, $b_{1,l} = -0.8$, $b_{2,r} = 0.6$, $b_{2,l} = -0.6$. We assume that the universe of discourse of $\tilde{k_j}$ is $U_j = \{1, 1.5, 2\}$, i.e., $k_{1,1} = 1$, $k_{1,2} = 1.5$, $k_{1,3} = 2$, $k_{2,1} = 1$, $k_{2,2} = 1.5$, $k_{2,3} = 2$. The fuzzy grade of $k_{j,q}$ is represented as $\mu_j(k_{j,q}) = \frac{1}{\sqrt{2\pi}\sigma_j}\exp(\frac{-(k_{j,q}-1.5)^2}{2\sigma_j^2})$ with $\sigma_j = \frac{1}{6}$. The reference signals are given as $y_{d1} = \sin(1.5t)$, $y_{d2} = \sin(t) + \sin(1.5t)$.

In this simulation, seven fuzzy sets are defined over interval $[-3,3]$ by choosing the partitioning points as -3, -2, -1, 0, 1, 2, 3 for all state variables $x_{j,p}(j=1,2,p=1,2)$. The fuzzy membership functions are designed as follows

$$\mu_{F^l_{j,p}}(x_{j,p}) = \exp(-0.5(x_{j,p}-w_l)^2), l=1,2,\ldots,7,$$

where $w_1=-3$, $w_2=-2$, $w_3=-1$, $w_4=0$, $w_5=1$, $w_6=2$, $w_7=3$. Choose the design virtual control signals $\zeta_{j,1}(j=1,2)$ (11), the actual controllers $u_j(j=1,2)$ (12), and the parameter adaptive laws $\hat{\phi}_{j,p}(j=1,2,p=1,2)$ (14).

The design parameters are chosen as $\rho_{1,1}=63$, $\rho_{1,2}=65$, $\rho_{2,1}=62$, $\rho_{2,2}=64$, $\mu_{1,1}=1.2$, $\mu_{1,2}=1.2$, $\mu_{2,1}=1.5$, $\mu_{2,2}=1.5$, $\delta_{1,1}=0.5$, $\delta_{1,2}=0.6$, $\delta_{2,1}=0.7$, $\delta_{2,2}=0.6$, $\varepsilon_{1,1}=1$, $\varepsilon_{1,2}=1$, $\varepsilon_{2,1}=1$, $\varepsilon_{2,2}=1$, $\upsilon=\frac{9999}{10001}$. The simulation is carried out with the initial conditions $x_{1,1}(0)=0.01$, $x_{1,2}(0)=0.05$, $x_{2,1}(0)=-0.1$, $x_{2,2}(0)=0.2$, $\hat{\phi}_{1,1}(0)=0.05$, $\hat{\phi}_{1,2}(0)=0$, $\hat{\phi}_{2,1}(0)=0.03$, $\hat{\phi}_{2,2}(0)=0$. The simulation results are shown in Figs. 10 and 11. The trajectories of tracking errors and control signals are depicted in Figs. 10 and 11, respectively. The trajectories of adaptive parameters $\hat{\phi}_{1,j}$ and $\hat{\phi}_{2,j}$ for $j=1,2$ are given in Figs. 12 and 13, respectively.

5 Conclusion

The finite-time adaptive fuzzy control problem has been investigated for a class of MIMO nonstrict-feedback nonlinear systems with fuzzy dead zones in this paper. By applying the variable partition approach and fuzzy logic systems, an adaptive fuzzy control scheme is formed for the considered MIMO nonlinear system. Under the proposed control strategy, the tracking error is located in a small neighborhood of the origin in finite time. Finally, the validity of the proposed control method is verified by the simulation results. In our follow-up study, we will investigate the finite-time control for MIMO stochastic nonstrict-feedback nonlinear systems.

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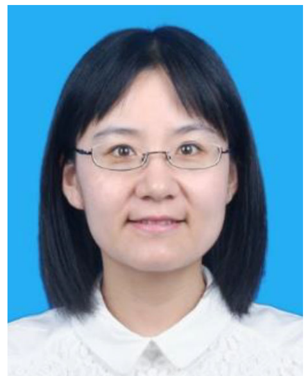

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