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Finite-Time Adaptive Fuzzy Control for Nonlinear Systems with Unknown Backlash-Like Hysteresis

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Abstract This study considers the tracking control problem of the nonstrict-feedback nonlinear system with unknown backlash-like hysteresis, and a finite-time adaptive fuzzy control scheme is developed to address this problem. More precisely, the fuzzy systems are employed to approximate the unknown nonlinearities, and the design difficulties caused by the nonlower triangular structure are also overcome by using the property of fuzzy systems. Besides, the effect of unknown hysteresis input is compensated by approximating an intermediate variable. With the aid of finite-time stability theory, the proposed control algorithm could guarantee that the tracking error converges to a smaller region. Finally, a simulation example is provided to further verify the above theoretical results.

Keywords Nonstrict-feedback nonlinear systems · Adaptive fuzzy control · Finite-time control · Unknown backlash-like hysteresis

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1 Introduction

As we know, nonlinear systems are widespread in practice, especially with the advancement of science and technology, there are more and more nonlinear phenomena in actual control systems. Therefore, the control problem of nonlinear systems has naturally become a hot topic. In the past few decades, a series of nonlinear system control methods have been developed, such as backstepping control, sliding mode control, robust control, and adaptive control, which greatly enriched the nonlinear system control theory. Besides, many scholars have successfully dealt with a lot of complex control problems by combining multiple control technologies [1–6] or fusing some intelligent control methods (fuzzy control [7–9] and neural network control [10, 11]). For example, a robust adaptive control strategy for the nonlinear system with actuator failures is given in [5]. By means of the approximation abilities of the fuzzy logic system (FLS) and neural network (NN), two adaptive control strategies for switched nonlinear systems are proposed in [8] and [10], respectively. It should be noted that a common feature of literature [7–11] is that the controlled systems are all in the form of the lower triangle structure, so the final controllers can be calculated according to the idea of backstepping recursion. However, this method is not suitable for the more general nonstrict-feedback nonlinear system, in which the unknown i-th subsystem function contains all state variables. A variable separation method based on the structural characteristics and monotonically increasing nature of the boundary function is adopted to solve this problem in [12]. Moreover, the authors of [13] and [14] successfully overcome this difficulty by using the properties of FLS and NN.

On the other hand, due to the limitations of physical components, actual control systems often have many nonlinear characteristics such as saturation [15, 16], dead zone [17, 18], and hysteresis [19–21] that can adversely affect system performance. In order to make the control method of nonlinear systems more effective in practical engineering, it is necessary to fully understand these nonlinear characteristics. Different from the time delay in the usual sense, the hysteresis phenomenon is often a dynamic process and has the characteristics of nonsmooth, multi-mapping, and memory. In recent years, the sensing and driving technology with smart materials as the core has been widely used in many fields due to its strong adaptability to environmental changes and excellent performance, such as aerospace, micro-robots, and bioengineering. However, the hysteresis characteristics existing in many smart materials can not only cause system oscillations, reduce the control accuracy of the system, and even cause the system to fail to operate normally. Therefore, the research on the control problems of nonlinear systems with hysteresis input is beneficial to realize the high requirements on the system performance. In view of the rich practical application background of nonlinear systems with hysteresis, the related research has always been favored by scholars [22–27]. Huang et al. [24] and Yu et al. [26] consider the output feedback control problems for switched and stochastic nonlinear systems with unknown hysteresis input, respectively. For the nonlinear multi-agent system with unknown actuator hysteresis, an adaptive event-triggered control scheme is given in [27].

However, in practical applications, the system cannot maintain a long or even infinite operating cycle, especially for some systems that require relatively high time performance. Therefore, it is necessary to study how to obtain the ideal tracking performance in a finite time. Finite-time control is usually regarded as a time-optimal control strategy and has a good robust performance. Since the finite-time Lyapunov stability theory is proposed, finite-time control has been developed rapidly and widely used, and a large number of excellent results have been reported [28–32]. Based on the above discussions, the tracking control problem of nonstrict-feedback nonlinear systems with unknown backlash-like hysteresis is taken into account in this study. With the help of FLS and finite-time Lyapunov stability theory, a finite-time adaptive fuzzy control algorithm is developed to resolve this problem, the characteristics of which compared with the existing results are listed as follows:

(1) Different from [22], since the system considered in this study is a nonstrict-feedback form, it is more general and has greater model versatility for the actual system. Moreover, the effect caused by the unknown hysteresis input is compensated by approximating an intermediate variable, and this method can avoid the singularity problem, which is different from [23].

(2) Based on finite-time stability theory, the proposed control scheme in this study could guarantee that the tracking error converges to near the origin in a finite time. Meanwhile, a higher tracking accuracy can be realized.

2 System Description and Preliminaries

Consider the following nonstrict-feedback nonlinear system with unknown hysteresis input

$$\begin{cases} \dot{\zeta}_i = \zeta_{i+1} + f_i(\zeta) + d_i(t), \\ 1 \le i \le n-1 \\ \dot{\zeta}_n = u(v) + f_n(\zeta) + d_n(t), \\ y = \zeta_1, \end{cases} \tag{1}$$

where ζ_i is the system state with $\zeta = [\zeta_1, \ldots, \zeta_n]^T$, $y \in R$ is the system output, $f_i(\zeta)$ is the unknown smooth nonlinear function, $d_i(t)$ is the unknown time-varying disturbance, $u \in R$ is the output of the unknown backlash-like hysteresis described by

$$\frac{du}{dt} = k\left|\frac{dv}{dt}\right|(\varsigma v - u) + c\frac{dv}{dt}, \tag{2}$$

where v is the input of the backlash-like hysteresis; k, ς and c are unknown constants, $\varsigma > 0$ is the slope of the lines and satisfies $\varsigma > c$. Figure 1 shows the backlash-like hysteresis curves generated by the model (2), where $k = 1$, $\varsigma = 3.1635$, $b = 0.345$, the input signals are $v(t) = \kappa \sin(2.3t)$ with $\kappa = 4.5$ and $\kappa = 6.5$, and the initial values are $v(0) = 0$ and $u(0) = 0$.

As stated in [19], (2) can be expressed as

$$\begin{aligned} u(v) &= \varsigma v(t) + d(v), \\ d(v) &= [u_0 - \varsigma v_0]e^{-k(v-v_0)sgn(\dot{v})} \\ &\quad + e^{-kvsgn(\dot{v})}\int_{v_0}^{v}[c-\varsigma]e^{k\eta sgn(\dot{v})}d\eta, \end{aligned}$$

where $v_0 = v(0)$, $u_0 = u(0)$, $d(v)$ satisfies $|d(v)| \le D$ with the unknown constant D. Therefore, the system (1) is rewritten as

$$\begin{cases} \dot{\zeta}_i = \zeta_{i+1} + f_i(\zeta) + d_i(t), \\ 1 \le i \le n-1 \\ \dot{\zeta}_n = \varsigma v(t) + d(v) + f_n(\zeta) + d_n(t), \\ y = \zeta_1, \end{cases} \tag{3}$$

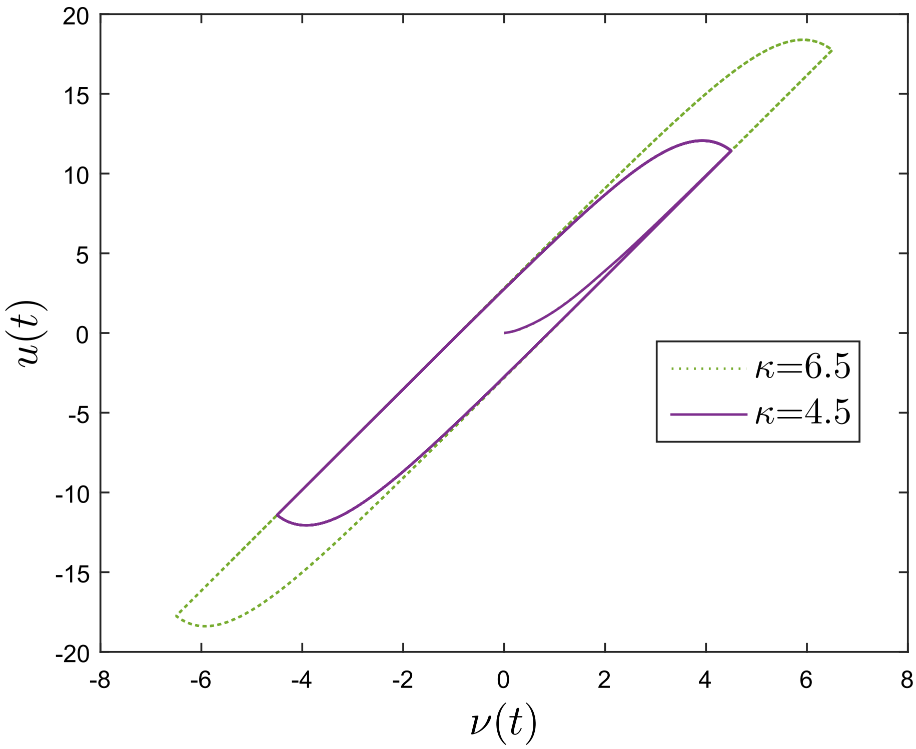

Fig. 1 Backlash-like hysteresis curves

The control objective of this study is to construct an adaptive fuzzy controller for the system (1) such that the system output y can track the reference signal y_r in a finite time, and all signals in the closed-loop system are bounded. To this end, we need to introduce the following assumptions and knowledge.

Assumption 1 The reference signal y_r and its time derivative up to the nth order $y_r^{(n)}$ are continuous and bounded.

Assumption 2 There exists the unknown constant $\bar{d}_i$ such that $|d_i(t)| \leq \bar{d}_i$, $i = 1, \ldots, n$.

Definition 1 [33] Consider the nonlinear system $\dot{\zeta} = f(\zeta(t))$, the equilibrium $\zeta = 0$ is practical finite-time stable, if for any $\zeta(0) \in \zeta_0$, there exist a constant $\varepsilon > 0$ and the settling time $T(\varepsilon, \zeta_0) < \infty$ such that

$$\|\zeta(t)\| < \varepsilon, \forall t > T.$$

Lemma 1 [34] *For positive constants* α, β, $0 < p < 1$ *and* $0 < \Gamma < \infty$, *if there is a positive-definite function* $V(\zeta)$ *satisfying*

$$\dot{V}(\zeta) \leq -\alpha V^p(\zeta) - \beta V(\zeta) + \Gamma,$$

then the trajectory of $\dot{\zeta} = f(\zeta(t))$ *is practical finite-time stable, and* $V(\zeta)$ *satisfies*

$$V^p(\zeta) \leq \frac{\Gamma}{(1-\omega)\alpha}, \forall t \geq T$$

with $0 < \omega < 1$ *and* $T = \frac{1}{\beta(1-p)} \ln \frac{\beta V^{1-p}(\zeta_0) + \omega\alpha}{\omega\alpha}$.

Lemma 2 [35] *For real variables* μ, ϖ *and constant* $\pi_i > 0 (i = 1, 2, 3)$, *one has*

$$|\mu|^{\pi_1} |\varpi|^{\pi_2} \leq \frac{\pi_1}{\pi_1 + \pi_2} \pi_3 |\mu|^{\pi_1 + \pi_2} + \frac{\pi_2}{\pi_1 + \pi_2} \pi_3^{-\frac{\pi_1}{\pi_2}} |\varpi|^{\pi_1 + \pi_2}.$$

Lemma 3 [36]: *For any* $\zeta_i \in R$, $i = 1, \ldots, n$, $0 < p \leq 1$, *one has*

$$\left(\sum_{i=1}^{n} |\zeta_i| \right)^p \leq \sum_{i=1}^{n} |\zeta_i|^p.$$

In the subsequent control design, FLS would be utilized to estimate the unknown nonlinear function.

IF-THEN Rules: $\mathcal{R}^i$: *If* ζ_1 *is* $\mathcal{F}_1^i$ *and ...and* ζ_n *is* $\mathcal{F}_n^i$, *then* y *is* $\mathcal{G}^i$, $i = 1, \ldots, N$.

The FLS could be formulated as

$$y(\zeta) = \frac{\sum_{i=1}^{N} h_i \prod_{j=1}^{n} \mu_{\mathcal{F}_j^i}(\zeta_j)}{\sum_{i=1}^{N} \left[\prod_{j=1}^{n} \mu_{\mathcal{F}_j^i}(\zeta_j)\right]}.$$

Denote $\phi_i(\zeta) = \frac{\prod_{j=1}^{n} \mu_{\mathcal{F}_j^i}(\zeta_j)}{\sum_{i=1}^{N}\left[\prod_{j=1}^{n} \mu_{\mathcal{F}_j^i}(\zeta_j)\right]}$, $\Phi(\zeta) = [\phi_1(\zeta), \ldots, \phi_N(\zeta)]^T$, $\mathcal{H} = [h_1, \ldots, h_N]^T$, *then one has*

$$y(\zeta) = \mathcal{H}^T \Phi(\zeta).$$

Lemma 4 [28]: *For any continuous function* $h(\zeta)$ *defined on the compact set U, there is a FLS* $\mathcal{H}^T\Phi(\zeta)$ *satisfying*

$$\sup_{\zeta \in U} |h(\zeta) - \mathcal{H}^T\Phi(\zeta)| \le \epsilon$$

with $\epsilon > 0$ *being an arbitrary constant.*

3 Adaptive Fuzzy Control Design

The control design begins with the following coordinate transformations

$$s_1 = \zeta_1 - y_r, \tag{4}$$

$$s_i = \zeta_i - \alpha_{i-1}, i = 2, \ldots, n, \tag{5}$$

where α_{i-1} is the virtual control law. Next, we denote $\theta = \max\{\|\mathcal{H}_i\|^2, i = 1, \ldots, n\}$ and $\rho = \frac{1}{\varsigma}$, $\hat{\theta}$ and $\hat{\rho}$ are estimates of θ and ρ, respectively. In accordance with the backstepping technique, the virtual and actual controllers are designed as

$$\alpha_1 = -c_1 s_1 - \frac{\hat{\theta} s_1}{2a_1^2 \Phi_1^T(Z_1)\Phi_1(Z_1)}, \tag{6}$$

$$\alpha_i = -c_i s_i - s_{i-1} - \frac{\hat{\theta} s_i}{2a_i^2 \Phi_i^T(Z_i)\Phi_i(Z_i)}, i = 2, \ldots, n, \tag{7}$$

$$v = \hat{\rho}\alpha_n \tag{8}$$

with the constant $c_i, a_i > 0$ $(i = 1, \ldots, n)$. Meanwhile, the corresponding adaptive laws are

$$\dot{\hat{\rho}} = -\gamma s_n \alpha_n - \sigma\hat{\rho}, \tag{9}$$

$$\dot{\hat{\theta}} = \lambda\tau_n - r\hat{\theta}, \tag{10}$$

where $\sigma, r > 0$ are known constants, and τ_n will be defined later.

Step 1: Based on (3), (4), and (5), the derivative of s_1 is calculated as

$$\begin{aligned} \dot{s}_1 &= \dot{\zeta}_1 - \dot{y}_r \\ &= s_2 + \alpha_1 + f_1(\zeta) + d_1(t) - \dot{y}_r. \end{aligned} \tag{11}$$

Define

$$V_1 = \frac{1}{2}s_1^2 + \frac{1}{2\lambda}\tilde{\theta}^2 \tag{12}$$

with $\tilde{\theta} = \theta - \hat{\theta}$. Then, it is deduced that

$$\begin{aligned} \dot{V}_1 &= s_1\left(s_2 + \alpha_1 + f_1(\zeta) + d_1(t) - \dot{y}_r\right) - \frac{1}{\lambda}\tilde{\theta}\dot{\hat{\theta}} \\ &= s_1\left(-k_1 s_1^{2p-1} - s_1 + s_2 + \alpha_1 + \bar{f}_1(X_1) + d_1(t)\right) - \frac{1}{\lambda}\tilde{\theta}\dot{\hat{\theta}}, \end{aligned} \tag{13}$$

where $p = \frac{\breve{m}}{\breve{n}} < 1$ with positive odd integers $\breve{m}$, $\breve{n}$, $\bar{f}_1(X_1) = f_1(\zeta) - \dot{y}_r + k_1 s_1^{2p-1} + s_1$ and $X_1 = [\zeta, y_r, \dot{y}_r]^T$. According to Lemma 4, the FLS $\mathcal{H}_1^T\Phi_1(X_1)$ could be employed to approximate the unknown term $\bar{f}_1(X_1)$, i.e.,

$$\begin{aligned} &\bar{f}_1(X_1) = \mathcal{H}_1^T\Phi_1(X_1) + \Delta_1(X_1), \\ &|\Delta_1(X_1)| \le \epsilon_1 \end{aligned}$$

with an approximation error $\Delta_1(X_1)$ and $\epsilon_1 > 0$. Based on Young's inequality and the property of FLS $0 < \Phi_1^T(\cdot)\Phi_1(\cdot) \le 1$, one has

$$\begin{aligned} s_1\bar{f}_1(X_1) &= s_1\mathcal{H}_1^T\Phi_1 + s_1\Delta_1(X_1) \\ &\le \frac{\theta s_1^2 \Phi_1^T(X_1)\Phi_1(X_1)}{2a_1^2} + \frac{1}{2}a_1^2 + \frac{1}{2}s_1^2 + \frac{1}{2}\epsilon_1^2 \\ &\le \frac{\theta s_1^2}{2a_1^2\Phi_1^T(Z_1)\Phi_1(Z_1)} + \frac{1}{2}a_1^2 + \frac{1}{2}s_1^2 + \frac{1}{2}\epsilon_1^2, \end{aligned} \tag{14}$$

$$s_1 d_1(t) \le \frac{1}{2}s_1^2 + \frac{1}{2}\bar{d}_1^2, \tag{15}$$

where $Z_1 = [\zeta_1, y_r, \dot{y}_r]^T$. Substituting (14) and (15) into (13) produces

$$\begin{aligned} \dot{V}_1 \le\ & s_1\left(-k_1 s_1^{2p-1} + \alpha_1 + \frac{\hat{\theta} s_1}{2a_1^2\Phi_1^T(Z_1)\Phi_1(Z_1)}\right) \\ &+ s_1 s_2 + \frac{1}{2}(\bar{d}_1^2 + a_1^2 + \epsilon_1^2) \\ &+ \tilde{\theta}\left(\frac{s_1^2}{2a_1^2\Phi_1^T(Z_1)\Phi_1(Z_1)} - \frac{1}{\lambda}\dot{\hat{\theta}}\right). \end{aligned} \tag{16}$$

Furthermore, by substituting the first virtual controller (6) into (16), we have

$$\dot{V}_1 \le -k_1 s_1^{2p} - c_1 s_1^2 + s_1 s_2 + \tilde{\theta}\left(\tau_1 - \frac{1}{\lambda}\dot{\hat{\theta}}\right) + \iota_1 \tag{17}$$

with $\tau_1 = \frac{s_1^2}{2a_1^2\Phi_1^T(Z_1)\Phi_1(Z_1)}$ and $\iota_1 = \frac{1}{2}(\bar{d}_1^2 + a_1^2 + \epsilon_1^2)$.

Step $i (i = 2, \ldots, n-1)$: It follows from (3) and (5) that

$$\begin{aligned} \dot{s}_i &= \dot{\zeta}_i - \dot{\alpha}_{i-1} \\ &= s_{i+1} + \alpha_i + f_i(\zeta) + d_i(t) - \dot{\alpha}_{i-1}. \end{aligned} \tag{18}$$

When the following Lyapunov function candidate is considered

$$V_i = V_{i-1} + \frac{1}{2}s_i^2, \tag{19}$$

then one has

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + s_i\Big(s_{i+1} + \alpha_i + f_i(\zeta) + d_i(t) - \dot{\alpha}_{i-1}\Big)\\ &\le -\sum_{j=1}^{i-1} k_j s_j^{2p} - \sum_{j=1}^{i-1} c_j s_j^2 + s_i s_{i+1} + \tilde{\theta}(\tau_{i-1} - \frac{1}{\lambda}\dot{\hat{\theta}}) + \iota_{i-1}\\ &\quad + s_i\Big(-k_i s_i^{2p-1} - s_i + s_{i-1} + \alpha_i + \bar{f}_i(X_i) + d_i(t)\Big),\end{aligned} \tag{20}$$

where $\bar{f}_i(X_i) = f_i(\zeta) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\zeta_j}(\zeta_{j+1} + f_j(\zeta) + d_j(t)) - \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} + k_i s_i^{2p-1} + s_i$ and $X_i = [\zeta, y_r, \ldots, y_r^{(i)}, \hat{\theta}]^T$. Based on Lemma 4, there exists a FLS $\mathcal{H}_i^T\Phi_i(X_i)$ that can estimate the unknown term $\bar{f}_i(X_i)$. For any $\epsilon_i > 0$,

$$\bar{f}_i(X_i) = \mathcal{H}_i^T\Phi_i(X_i) + \Delta_i(X_i),$$

where $|\Delta_i(X_i)| \le \epsilon_i$ is an approximation error. As the same case of (14) and (15), the following inequalities hold

$$\begin{aligned} s_i\bar{f}_i(X_i) &= s_i\mathcal{H}_i^T\Phi_i(X_i) + s_i\Delta_i(X_i)\\ &\le \frac{\theta s_i^2\Phi_i^T(X_i)\Phi_i(X_i)}{2a_i^2} + \frac{1}{2}a_i^2 + \frac{1}{2}s_i^2 + \frac{1}{2}\epsilon_i^2\\ &\le \frac{\theta s_i^2}{2a_i^2\Phi_i^T(Z_i)\Phi_i(Z_i)} + \frac{1}{2}a_i^2 + \frac{1}{2}s_i^2 + \frac{1}{2}\epsilon_i^2,\end{aligned} \tag{21}$$

$$s_i d_i(t) \le \frac{1}{2}s_i^2 + \frac{1}{2}\bar{d}_i^2, \tag{22}$$

where $Z_i = [\zeta_1, \ldots, \zeta_i, y_r, \ldots, y_r^{(i)}, \hat{\theta}]^T$. Substituting them into (20), we have

$$\begin{aligned}\dot{V}_i \le& -\sum_{j=1}^{i-1} k_j s_j^{2p} - \sum_{j=1}^{i-1} c_j s_j^2 + s_i s_{i+1}\\ &+ s_i\left(-k_i s_i^{2p-1} + s_{i-1} + \alpha_i + \frac{\hat{\theta}s_i}{2a_i^2\Phi_i^T(Z_i)\Phi_i(Z_i)}\right)\\ &+ \tilde{\theta}\left(\tau_{i-1} + \frac{s_i^2}{2a_i^2\Phi_i^T(Z_i)\Phi_i(Z_i)} - \frac{1}{\lambda}\dot{\hat{\theta}}\right)\\ &+ \iota_{i-1} + \frac{1}{2}(\bar{d}_i^2 + a_i^2 + \epsilon_i^2).\end{aligned} \tag{23}$$

Then, substituting (7) into (23) yields

$$\dot{V}_i \le -\sum_{j=1}^{i} k_j s_j^{2p} - \sum_{j=1}^{i} c_j s_j^2 + s_i s_{i+1} + \tilde{\theta}\left(\tau_i - \frac{1}{\lambda}\dot{\hat{\theta}}\right) + \iota_i \tag{24}$$

with $\tau_i = \tau_{i-1} + \frac{s_i^2}{2a_i^2\Phi_i^T(Z_i)\Phi_i(Z_i)}$ and $\iota_i = \iota_{i-1} + \frac{1}{2}(\bar{d}_i^2 + a_i^2 + \epsilon_i^2)$.

Step n **:** From (3) and (5), we have

$$\begin{aligned}\dot{z}_n &= \dot{\zeta}_n - \dot{\alpha}_{n-1}\\ &= \varsigma v + d(v) + f_n(\zeta) + d_n(t) - \dot{\alpha}_{n-1}.\end{aligned} \tag{25}$$

Choose

$$V_n = V_{n-1} + \frac{1}{2}s_n^2 + \frac{\varsigma}{2\gamma}\tilde{\rho}^2 \tag{26}$$

with $\tilde{\rho} = \rho - \hat{\rho}$. Then, it could be obtained that

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + s_n\Big(\varsigma v + d(v) + f_n(\zeta) + d_n(t) - \dot{\alpha}_{n-1}\Big) - \frac{\varsigma}{\gamma}\tilde{\rho}\dot{\hat{\rho}}\\ &\le -\sum_{j=1}^{n-1} k_j s_j^{2p} - \sum_{j=1}^{n-1} c_j s_j^2 + \tilde{\theta}\Big(\tau_{n-1} - \frac{1}{\lambda}\dot{\hat{\theta}}\Big) + \iota_{n-1}\\ &\quad + s_n\Big(-k_n s_n^{2p-1} - s_n + s_{n-1} + \varsigma v + d(v) + \bar{f}_n(X_n) + d_n(t)\Big) - \frac{\varsigma}{\gamma}\tilde{\rho}\dot{\hat{\rho}}\end{aligned} \tag{27}$$

with $\bar{f}_n(X_n) = f_n(\zeta) - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\zeta_j}(\zeta_{j+1} + f_j(\zeta) + d_j(t)) - \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} + k_n s_n^{2p-1} + s_n$ and $X_n = [\zeta, y_r, \ldots, y_r^{(n)}, \hat{\theta}]^T$. According to Lemma 4, for the arbitrary constant $\epsilon_n > 0$, one has

$$\bar{f}_n(X_n) = \mathcal{H}_n^T\Phi_n(X_n) + \Delta_n(X_n),$$

where the approximation error $\Delta_n(X_n)$ satisfies $|\Delta_n(X_n)| \le \epsilon_n$. Similar to (14) and (15), the following inequalities hold

$$\begin{aligned} s_n\bar{f}_n(X_n) &= s_n\mathcal{H}_n^T\Phi_n(X_n) + s_n\Delta_n(X_n)\\ &\le \frac{\theta s_n^2\Phi_n^T(X_n)\Phi_n(X_n)}{2a_n^2} + \frac{1}{2}a_n^2 + \frac{1}{2}s_n^2 + \frac{1}{2}\epsilon_n^2\\ &\le \frac{\theta s_n^2}{2a_n^2\Phi_n^T(Z_n)\Phi_n(Z_n)} + \frac{1}{2}a_n^2 + \frac{1}{2}s_n^2 + \frac{1}{2}\epsilon_n^2,\end{aligned} \tag{28}$$

$$s_n(d(v) + d_n(t)) \le \frac{1}{2}s_n^2 + \frac{1}{2}(D + \bar{d}_n)^2, \tag{29}$$

where $Z_n = X_n$. Substituting them into (27) leads to

$$\begin{aligned}\dot{V}_n \le& -\sum_{j=1}^{n-1} k_j s_j^{2p} - \sum_{j=1}^{n-1} c_j s_j^2 + s_n\\ &\left(-k_n s_n^{2p-1} + s_{n-1} + \varsigma v + \frac{\hat{\theta}s_n}{2a_n^2\Phi_n^T(Z_n)\Phi_n(Z_n)}\right)\\ &+ \tilde{\theta}\left(\tau_n - \frac{1}{\lambda}\dot{\hat{\theta}}\right) - \frac{\varsigma}{\gamma}\tilde{\rho}\dot{\hat{\rho}} + \iota_n,\end{aligned} \tag{30}$$

where $\tau_n = \tau_{n-1} + \frac{s_n^2}{2a_n^2\Phi_n^T(Z_n)\Phi_n(Z_n)}$ and

$\imath_n = \imath_{n-1} + \frac{1}{2}\left((D+\bar{d}_n)^2 + a_n^2 + \epsilon_n^2\right)$. By combining (8)–(10), it could be deduced that

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n-1} k_j s_j^{2p} - \sum_{j=1}^{n-1} c_j s_j^2 + s_n \varsigma \hat{\rho} \alpha_n + s_n \\ & \left(-k_n s_n^{2p-1} + s_{n-1} + \frac{\hat{\theta} s_n}{2a_n^2 \Phi_n^T(Z_n)\Phi_n(Z_n)}\right) \\ & + \frac{r}{\lambda}\tilde{\theta}\hat{\theta} + s_n \varsigma \tilde{\rho} \alpha_n + \frac{\sigma\varsigma}{\gamma}\tilde{\rho}\hat{\rho} + \imath_n \\ = & -\sum_{j=1}^{n-1} k_j s_j^{2p} - \sum_{j=1}^{n-1} c_j s_j^2 + s_n \alpha_n + s_n \\ & \left(-k_n s_n^{2p-1} + s_{n-1} + \frac{\hat{\theta} s_n}{2a_n^2 \Phi_n^T(Z_n)\Phi_n(Z_n)}\right) \\ & + \frac{r}{\lambda}\tilde{\theta}\hat{\theta} + \frac{\sigma\varsigma}{\gamma}\tilde{\rho}\hat{\rho} + \imath_n \\ = & -\sum_{j=1}^{n} k_j s_j^{2p} - \sum_{j=1}^{n} c_j s_j^2 + \frac{r}{\lambda}\tilde{\theta}\hat{\theta} + \frac{\sigma\varsigma}{\gamma}\tilde{\rho}\hat{\rho} + \imath_n. \end{aligned} \tag{31}$$

4 Main Results

Theorem 1 *For the nonlinear system (1) with unknown hysteresis input (2), under Assumption 1 and 2, applying the proposed control scheme (6)–(10), the following conclusions hold:*

(1) *the tracking error could converge to near the origin in a finite time;*
(2) *all signals in the closed-loop system are bounded.*

Proof Recalling the definitions of θ and ρ, one has

$$\frac{r}{\lambda}\tilde{\theta}\hat{\theta} \leq -\frac{r}{2\lambda}\tilde{\theta}^2 + \frac{r}{2\lambda}\theta^2,$$
$$\frac{\sigma\varsigma}{\gamma}\tilde{\rho}\hat{\rho} \leq -\frac{\sigma\varsigma}{2\gamma}\tilde{\rho}^2 + \frac{\sigma\varsigma}{2\gamma}\rho^2.$$

By substituting them into (31), we could get

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n} k_j s_j^{2p} - \sum_{j=1}^{n} c_j s_j^2 - \frac{r}{2\lambda}\tilde{\theta}^2 + \frac{r}{2\lambda}\theta^2 - \frac{\sigma\varsigma}{2\gamma}\tilde{\rho}^2 + \frac{\sigma\varsigma}{2\gamma}\rho^2 \\ & - r(\frac{\tilde{\theta}^2}{2\lambda})^p + r(\frac{\tilde{\theta}^2}{2\lambda})^p - \sigma(\frac{\varsigma}{2\gamma}\tilde{\rho}^2)^p + \sigma(\frac{\varsigma}{2\gamma}\tilde{\rho}^2)^p + \imath_n. \end{aligned} \tag{32}$$

Based on Lemma 2, one has

$$r(\frac{\tilde{\theta}^2}{2\lambda})^p \leq p\frac{r}{2\lambda}\tilde{\theta}^2 + r(1-p), \tag{33}$$

$$\sigma(\frac{\varsigma}{2\gamma}\tilde{\rho}^2)^p \leq p\frac{\sigma\varsigma}{2\gamma}\tilde{\rho}^2 + \sigma(1-p). \tag{34}$$

Substituting (33) and (34) into (32) and combining Lemma 3, we could further derived that

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n} k_j s_j^{2p} - r(\frac{\tilde{\theta}^2}{2\lambda})^p - \sigma(\frac{\varsigma}{2\gamma}\tilde{\rho}^2)^p \\ & -\sum_{j=1}^{n} c_j s_j^2 - (1-p)\frac{r}{2\lambda}\tilde{\theta}^2 - (1-p)\frac{\sigma\varsigma}{2\gamma}\tilde{\rho}^2 \\ & + \imath_n + \frac{r}{2\lambda}\theta^2 + \frac{\sigma\varsigma}{2\gamma}\rho^2 + (r+\sigma)(1-p) \\ \leq & -\alpha V_n^p - \beta V_n + \Gamma \end{aligned} \tag{35}$$

with $\alpha = \min\{2^p k_j, r, \sigma\}$, $\beta = \min\{2c_j, (1-p)r, (1-p)\sigma\}$ and $\Gamma = \imath_n + \frac{r}{2\lambda}\theta^2 + \frac{\sigma\varsigma}{2\gamma}\rho^2 + (r+\sigma)(1-p)$.

It follows from (35) and Lemma 1 that

$$V_n^p(t) \leq \frac{\Gamma}{(1-\omega)\alpha}, \forall t \geq T,$$

and $T = \frac{1}{\beta(1-p)}\ln\frac{\beta V_n^{1-p}(\zeta_0)+\omega\alpha}{\omega\alpha}$.

Then combining with the definition V_n, it is not difficult to deduce that

$$|s_1| \leq \sqrt{2\left(\frac{\Gamma}{(1-\omega)\alpha}\right)^{\frac{1}{p}}}, \forall t \geq T,$$

that is, the tracking error could converge to near the origin in the limited time by choosing the suitable parameters.

On the other hand, from (35), we have $\dot{V}_n(t) \leq -\beta V_n(t) + \Gamma$ and $V_n(t) \leq \left(V_n(0) - \frac{\Gamma}{\beta}\right)e^{-\beta t} + \frac{\Gamma}{\beta}$, which means that s_i, $\tilde{\theta}$, $\hat{\theta}$, $\tilde{\rho}$ and $\hat{\rho}$ are bounded. It follows from (6) to (8) that α_i $(i = 1, \ldots, n)$ and v are bounded because of the boundedness of the included variables. Moreover, combining (4) and (5), we could obtained that ζ_i $(i = 1, \ldots, n)$ is bounded. Consequently, all signals in the closed-loop system remain bounded. This completes the proof.

Remark 1 The challenges of this study mainly include two aspects: on the one hand, the controlled system in this study is a nonstrict-feedback form, which makes the standard backstepping technology not directly applicable to the control design of such systems. To this end, we have solved this problem with the help of the property of the fuzzy logic system $0 < \Phi_i^T(\cdot)\Phi_i(\cdot) \leq 1$ (Details can be seen in formulas (14), (21), and (28)). On the other hand, how to deal with the problems caused by the unknown hysteresis input also needs to be considered in control design. Inspired by Reference [23], we introduced a new intermediate variable $\rho = \frac{1}{\varsigma}$. Furthermore, the effect caused by the unknown hysteresis input can be compensated by approximating the new variable ρ. It should be pointed out that, compared with the method in Reference [23], the proposed method does not discuss whether the parameter $\hat{\rho}$ will be equal to 0, because $\hat{\rho}$ will not appear in the denominator.

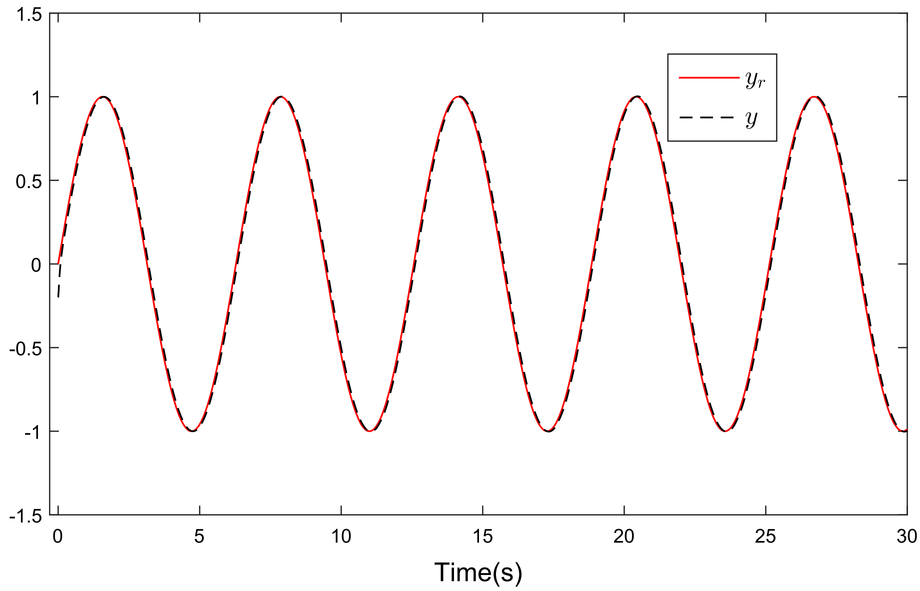

Fig. 2 The system output y and the reference signal y_r

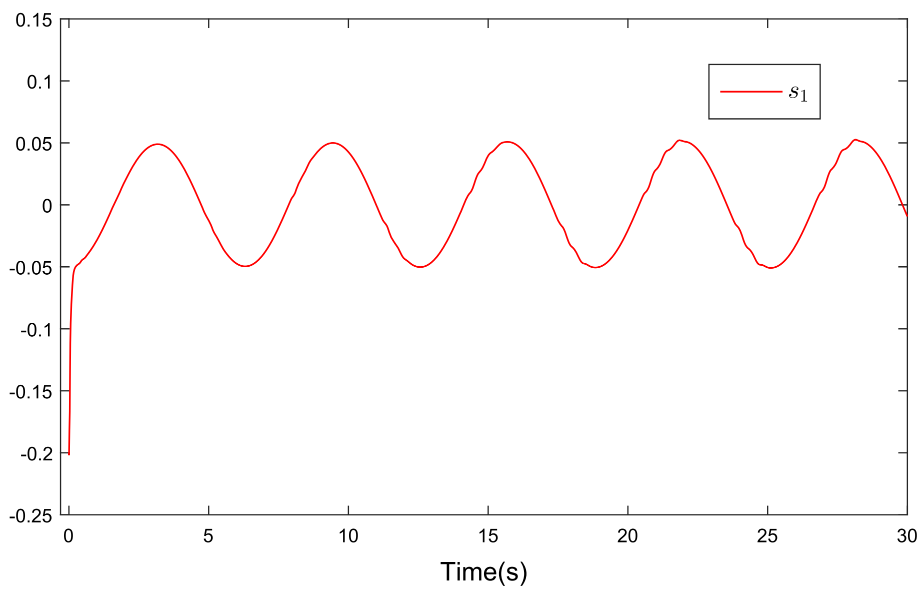

Fig. 3 The tracking error s_1

5 Simulation Example

This section aims to further intuitively verify the proposed control algorithm through simulation results.

Consider the following nonlinear system with unknown actuator hysteresis

$$\begin{cases} \dot{\zeta}_1 = \zeta_2 + f_1(\zeta) + d_1(t), \\ \dot{\zeta}_2 = u(v) + f_2(\zeta) + d_2(t), \\ y = \zeta_1, \end{cases} \tag{36}$$

where $f_1(\zeta) = 0.05\zeta_2 \cos(\zeta_1)$, $f_2(\zeta) = 0.1\zeta_1 \sin(\zeta_2)$, $d_1(t) = 0.1 \cos t$, $d_2(t) = 0.5 \sin t$; u is the output of the backlash-like hysteresis described by (2) with $k = 1$, $\varsigma = 3.1635$ and $c = 0.345$. The reference trajectory is $y_r = \sin t$.

In this simulation, the following membership functions are selected

$$\mu_{\mathcal{F}_j^i} = e^{-\frac{1}{2}(\zeta_j + \chi_i)^2}$$

with $\chi_i = 9, 7, 5, 3, 1, 0, -1, -3, -5, -7, -9$, $j = 1, 2$ and $i = 1, \ldots, 11$. To fulfill the control objective, the controllers are designed in accordance with the aforementioned design procedures

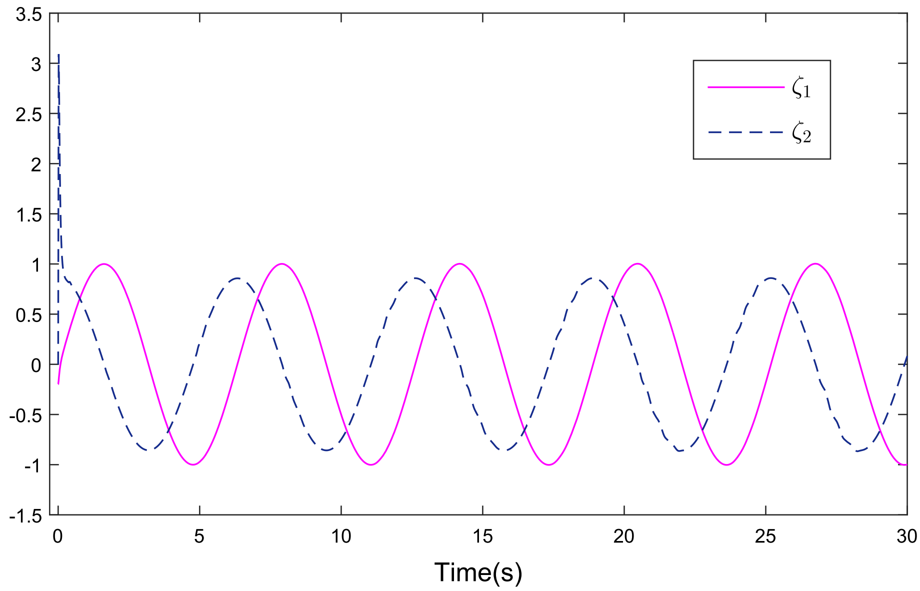

Fig. 4 States ζ_1 and ζ_2

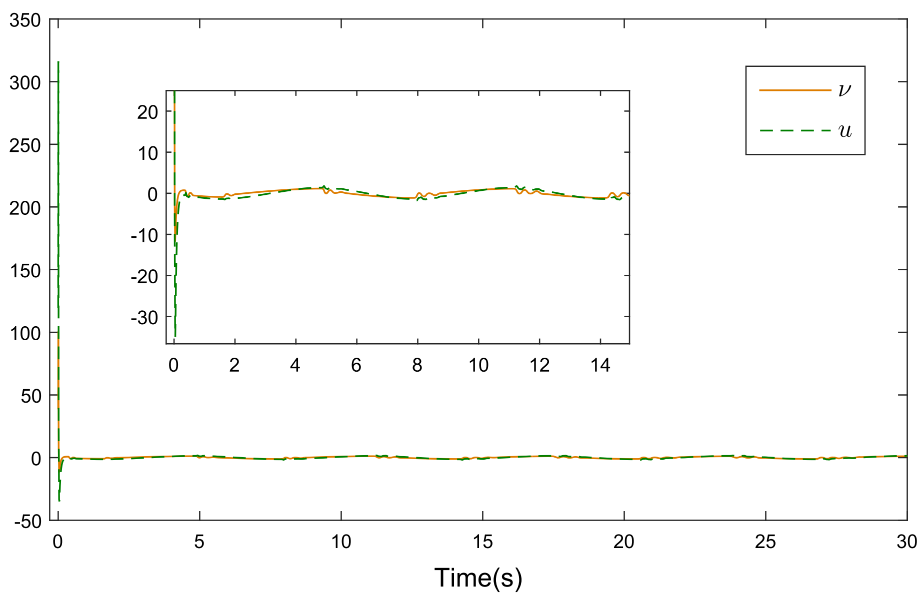

Fig. 5 Control signals u and v

$$\alpha_1 = -c_1 s_1 - \frac{\hat{\theta} s_1}{2a_1^2 \Phi_1^T(Z_1)\Phi_1(Z_1)},$$

$$\alpha_2 = -c_2 s_2 - s_1 - \frac{\hat{\theta} s_2}{2a_2^2 \Phi_2^T(Z_2)\Phi_2(Z_2)},$$

$$v = \hat{\rho}\alpha_2,$$

and adaptive laws can be calculated as

$$\dot{\hat{\rho}} = -\gamma s_2 \alpha_2 - \sigma\hat{\rho},$$

$$\dot{\hat{\theta}} = \lambda\tau_2 - r\hat{\theta}$$

with $\tau_2 = \frac{s_1^2}{2a_1^2\Phi_1^T(Z_1)\Phi_1(Z_1)} + \frac{s_2^2}{2a_2^2\Phi_2^T(Z_2)\Phi_2(Z_2)}$.

The initial conditions are $\zeta_1(0) = -0.2$, $\zeta_2(0) = 0$, $\hat{\theta}(0) = \hat{\rho}(0) = 1$. The parameters to be designed are $c_1 = 16$, $c_2 = 10$, $a_1 = a_2 = 1$, $\lambda = 1$, $r = 0.1$, $\gamma = 6$, $\sigma = 0.1$. The simulation results are exhibited in Figures 2, 3, 4, 5, and 6. The system output y and the reference signal y_r are plotted in Figure 2. Figure 3 presents the curve of the tracking error s_1. As shown in Figure 2, the maximum tracking error is less than 0.05 after 2 second. Thus, it can be seen from these two figures the system output y could track the reference trajectory y_r. The trajectories of system

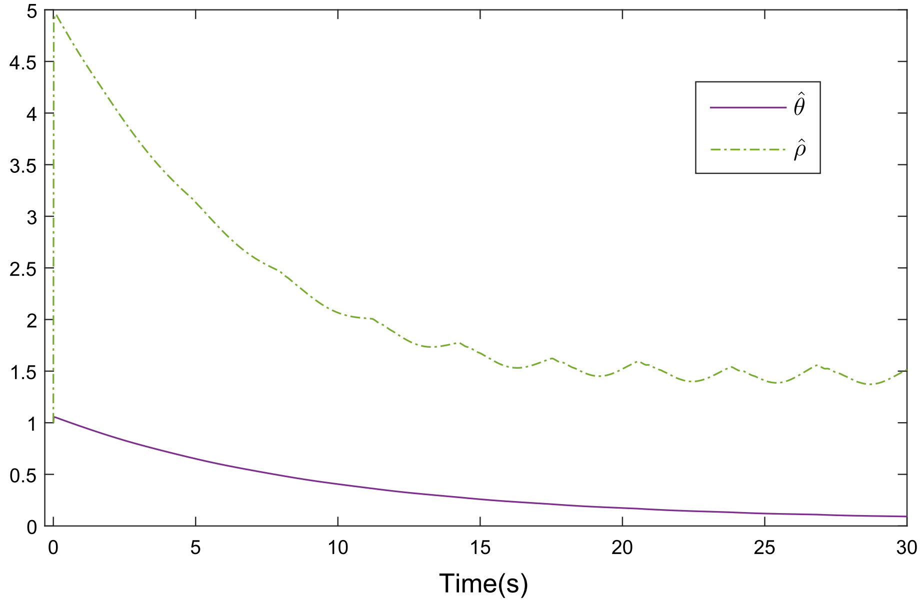

Fig. 6 Adaptive parameters $\hat{\theta}$ and $\hat{\rho}$

states ζ_1 and ζ_2 are displayed in Figure 4. The control signals are exhibited in Figure 5. Then combined with Figure 2, we could get that the proposed control scheme can still achieve tracking control performance in the presence of unknown hysteresis input. Figure 6 gives the trajectories of adaptive parameters $\hat{\theta}$ and $\hat{\rho}$. As these figures show, all signals in the closed-loop system are bounded. Consequently, the proposed scheme could achieve the control objective.

6 Conclusions

In this study, a finite-time tracking control strategy is proposed for the nonstrict-feedback nonlinear system with unknown actuator hysteresis. Based on the property of FLS, the design difficulties caused by the nonstrict-feedback structure are successfully resolved. In the presence of unknown nonlinear input, better tracking performance could be obtained in a finite time by combining FLS and adaptive backstepping technology. In the future, how to address the fixed-time tracking control problem for the nonlinear system with unknown nonlinear input is very meaningful and challenging work.

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