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Portfolio Selection Using Data Envelopment Analysis Cross-Efficiency Evaluation with Undesirable Fuzzy Inputs and Outputs

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Abstract In this paper, we discuss a portfolio selection problem based on fuzzy data envelopment analysis cross-efficiency evaluation wherein both undesirable fuzzy inputs and outputs have been considered. We first propose a data envelopment analysis cross-efficiency model, in which both undesirable inputs and outputs are considered. Furthermore, considering the imprecision of the data, we extend the crisp model to the fuzzy environment and propose a fuzzy data envelopment analysis cross-efficiency model with coexisting undesirable input and output data. We then apply the proposed model to the portfolio selection problem and present a novel mean-semivariance portfolio selection model based on fuzzy data envelopment analysis cross-efficiency scores, in which several realistic constraints are considered, including a budget, cardinality, buy-in thresholds, and no short selling constraints. After that, we employ the genetic algorithm (GA) to solve the proposed model. Finally, a real-life case study is presented to demonstrate the effectiveness of the proposed approaches.

Keywords Data envelopment analysis · Fuzzy undesirable input–output · Cross-efficiency evaluation · Portfolio selection

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1 Introduction

Data envelopment analysis (DEA) is a nonparametric mathematical programming technique proposed by Charnes et al. [9] to measure the relative efficiency of homogeneous decision-making units (DMUs) under the consideration of multiple inputs and outputs. Traditional DEA models can only handle desirable inputs and outputs. Inputs need to be minimized, and outputs need to be maximized [1, 17, 22, 46]. However, there are frequently circumstances in real-life situations where some inputs need to be increased and some outputs need to be decreased to improve the performance of a DMU. For example, for a coal-fired power plant, SO_2 emissions are an undesirable output that should be decreased, and the cost of reducing SO_2 emissions is the undesirable input that should be increased to improve the eco-efficiency. Subsequently, some scholars transformed the values of the undesirable inputs or outputs based on a decreasing function to make them desirable, e.g., Koopmans [28], Lovell et al. [33], Scheel [42], Zanella [52] and Liu et al. [31]. Moreover, many other researchers also focus on the DEA models with undesirable inputs or outputs. For example, Liu and Sharp [32] regarded undesirable inputs as desirable outputs, or undesirable outputs as desirable inputs. Seiford and Zhu [43] used the undesirable outputs as positive desirable outputs by multiplying them by (-1) and using a translation vector. Färe and Grosskopf [16] estimated the DMUs' efficiencies with undesirable factors based on directional distance function. Lozano and Gutierrez [34] handled undesirable outputs in a manner similar to handling the inputs in the DEA model with slacks-based measurements. Sueyoshi and Goto [47] discussed a unified treatment of desirable and undesirable outputs in the nonradial DEA models. Barros et al. [5] applied Russell directional distance function to consider the undesirable output. Toloo and Hančlová [49] selected directional distance function to cope with multivalued undesirable outputs.

It is worthy to note that the above research work is based on accurate measurement of both the input and output data. However, inputs and outputs in real-world problems are often imprecise. Some researchers have proposed various fuzzy methods for dealing with imprecise data in DEA. Since the original study by Sengupta [44, 45], there has been a continuous interest and increased development in fuzzy DEA (FDEA) literature [2, 19, 25–27, 29, 37, 41, 50, 51]. In particular, to solve the FDEA models, many researchers have applied different approaches, such as the tolerance approach, the α-cut approach, the fuzzy ranking approach and the possibility approach. Among these methods, the α-cut approach is widely used and converts the inputs and outputs into the crisp intervals of various level α standards. Hatami-Marbini et al. [20] used the α-cut approach and presented a four-phase FDEA framework based on the theory of the displaced ideal. Chiang and Che [13] proposed a new weight-restricted FDEA methodology by applying the α-level approach and the fuzzy analytical hierarchy procedure. Kao and Lin [24] constructed a pair of two-level programming models for calculating the lower and upper bounds of the α-cuts where the input/output data are fuzzy numbers. Chen et al. [10] incorporated the α-cut technique into the expanding model of fuzzy slack-based measurement to estimate efficiency scores in the Taiwan banking. Puri and Yadav [40] proposed a fuzzy DEA cross-efficiency model with undesirable outputs, which can be solved using the α-cut approach. Mu et al. [38] used FDEA to account for multiple indicators simultaneously with the α-cut approach. Table 1 presents a list of the above FDEA articles.

In the portfolio performance evaluation based on DEA methodology, it was not applied until Murthi et al. [39] proposed a DEA model for measuring efficiency of mutual funds. Later, Joro and Na [23] integrated a nonparametric DEA method with mean, variance and skewness to develop a framework for portfolio performance measure. Branda [8] proposed an efficiency evaluation approach based on traditional DEA models and considered portfolio diversification to identify the investment opportunities. Lim et al. [30] presented a DEA-based mean-variance cross-efficiency model for portfolio selection. Basso and Funari [6] constructed a social responsibility index and proposed some DEA models to evaluate the performance of socially responsible investment funds. More recently, Basso and Funari [7] discussed the role of fund size in the performance evaluation of mutual funds via DEA. Gouveia et al. [18] used the value-based DEA method with multiple criteria decision aiding to measure the Portuguese mutual fund portfolio performance. Zhou et al. [53] presented a segmented DEA approach based on data segment points to evaluate the portfolio performance with cardinality constraints. Chen et al. [11] proposed three variants of DEA models based on various risk measures to evaluate the efficiency of the fuzzy portfolio. Table 2 presents an overview of these literature.

1.1 Research Motivation

Uncertainty is a crucial issue for portfolio selection. In financial markets, many uncertain factors affect inputs and outputs. So, it is difficult to obtain the accurate prediction values of input and output data of the DEA model. Notably, the conventional DEA model is susceptible to perturbation in the input and output data. Therefore, in this paper, we deal with portfolio selection using FDEA cross-efficiency evaluation with coexisting undesirable inputs and outputs. Using FDEA, the present study effectively obtains cross-efficiency scores of all assets based on fuzzy input and output data (coexisting undesirable inputs and outputs). To handle the fuzzy input and output data, we apply the α-cut approach, which considers the fuzzy input and output data corresponding to the different levels of confidence intervals. The cross-efficiency scores of the assets are used to obtain the cross-efficiency mean and semivariances to develop the mean-semivariance model for portfolio selection. The proposed portfolio selection model aims at minimizing portfolio risk (semivariance) constrained to some desired level of portfolio return. Several additional realistic constraints are considered, including the budget, cardinality, buy-in thresholds, and no short selling constraints. Since this kind of mixed-integer nonlinear programming problems cannot be efficiently solved by the conventional optimization approaches, GA is applied to solve the proposed model.

1.2 Novelty of the Proposed Approach

Although there have been a variety of studies on DEA-based portfolio performance evaluation, few researchers have proposed an FDEA model with coexisting undesirable inputs and outputs and then applied it to the portfolio selection problem. Thus, in this paper, we discuss the portfolio selection problem based on FDEA cross-efficiency evaluation, in which both undesirable inputs and outputs are considered. The significant contributions of the proposed research work are as follows:

1. Compared with the literature on DEA with undesirable inputs or outputs [5, 16, 28, 31–34, 42, 43, 47, 49, 52], the present study extends the framework of performance evaluation. Both undesirable inputs and outputs have been considered to address real-world decision

Table 1 FDEA literature

Year	Authors	Approaches	Problems
1992	Sengupta [44]	The tolerance approach	The use of fuzzy set-theoretic measures in DEA
1992	Sengupta [45]	The tolerance approach	Economic efficiency of input–output systems by employing a fuzzy statistical approach
1998	Triantis and Girod [50]	The α-cut approach	The input-reducing technical efficiency performance of a packaging line
2000	Kao and Liu [25]	The α-cut approach	The efficiencies of DMUs with fuzzy observations
2001	Guo and Tanaka [19]	The fuzzy ranking approach	The efficiency evaluation problem with the given fuzzy input and output data
2002	Saati et al. [41]	The α-cut approach	The efficiencies of DMUs with asymmetrical triangular fuzzy number
2003	Kao and Liu [26]	The α-cut approach	The ranking of the 24 university libraries in Taiwan with fuzzy observations
2003	Lertworasirikul et al. [29]	The possibility approach	The performance of activities or organizations using imprecise data represented by fuzzy sets
2010	Hatami-Marbini et al. [20]	The α-cut approach	The efficiencies of fuzzy DMUs in organizations
2010	Chiang and Che [13]	The α-cut approach, the fuzzy analytical hierarchy procedure	The performance of new product development projects
2011	Moheb-Alizadeh et al. [37]	The fuzzy ranking approach	The efficiency of location-allocation models in a fuzzy environment
2011	Azadeh et al. [2]	The fuzzy ranking approach	The efficiencies of single-row facility layout problem in a refrigerator manufacturing company
2011	Wang and Chin [51]	The possibility approach	The selection of a flexible manufacturing system
2012	Kao and Lin [24]	The α-cut approach	The teaching and research efficiencies of chemistry departments in UK universities
2013	Chen et al. [10]	The α-cut approach	The management achievement of Taiwan banking under market risk
2014	Puri and Yadav [40]	The α-cut approach	The performance of public sector banks in India
2016	Khalili-Damghani et al. [27]	FDEA, the dimension-reduction method, the preference ratio method	The performance of emerging markets for international banking
2018	Mu et al. [38]	The α-cut approach	The eco-efficiency of dairy farming

Table 2 DEA on portfolio selection literature

Year	Authors	Approaches	Problems
1997	Murthi et al. [39]	DEA	Efficiency of mutual funds
2006	Joro and Na [23]	DEA	Portfolio performance
2013	Branda [8]	DEA	The efficiency of 25 world financial indices
2014	Lim et al. [30]	DEA, DEA-based mean-variance cross-efficiency model	Portfolio selection
2014	Basso and Funari [6]	DEA	Performance of socially responsible investment funds
2017	Basso and Funari [7]	DEA	Performance evaluation of mutual funds
2017	Gouveia et al. [18]	DEA	The Portuguese mutual fund portfolio performance
2018	Zhou et al. [53]	DEA	The portfolio performance with cardinality constrains
2018	Chen et al. [11]	DEA	The efficiency of the fuzzy portfolio

situations better. A DEA cross-efficiency model is developed to solve performance evaluation problems with coexisting undesirable input and output data, in which a cross-efficiency technique is used to increase the discrimination power of the proposed model.
2. Compared with the literature on FDEA [2, 10, 13, 19, 20, 24–26, 29, 37, 38, 41, 50, 51], the present study extends the literature by considering both undesirable inputs and outputs. An FDEA cross-efficiency model is proposed coexisting both undesirable inputs and outputs, helping to handle the situations wherein some inputs need to be increased, and some outputs need to be decreased to improve the performance of a DMU.
3. Compared with the FDEA model with coexisting undesirable inputs and outputs [27], the present study extends the literature by incorporating cross-efficiency for the evaluation. The consideration of cross-efficiency helps to increase the discrimination power of the proposed model.
4. Compared with the existing work on portfolio selection with DEA [12, 30, 36], the present study extends the framework of assets evaluation by including uncertainty in DEA evaluation, i.e., FDEA is used for assets evaluation instead of DEA.
5. Compared with the existing work on mean-variance portfolio selection with DEA [12, 30, 36], the present study extends the literature by considering mean, and semivariances based on FDEA cross-efficiency scores of assets evaluation wherein both undesirable inputs and outputs are considered to address the real-world investment situations better.

Additionally, a thorough comparison of the proposed work with similar existing works was performed based on many critical attributes, see Table 3 for details.

1.3 Organization of the Paper

The remainder of the paper is organized as follows. In Sect. 2, a DEA cross-efficiency model with coexisting undesirable inputs and outputs is introduced. Section 3 presents the proposed FDEA cross-efficiency model with coexisting undesirable inputs and outputs. Then, the novel mean-semivariance model based on FDEA cross-efficiency is described in Sect. 4. In Sect. 5, the main steps of genetic algorithm are presented. Based on real-market data set, an illustration is provided to validate the proposed approach in Sect. 6. Finally, some concluding remarks are given in Sect. 7.

2 DEA Cross-Efficiency Model with Coexisting Undesirable Inputs and Outputs

In this section, we develop a DEA cross-efficiency model with coexisting undesirable inputs and outputs. In the proposed model, to improve the performance of DMUs, undesirable outputs should be minimized, and undesirable inputs should be maximized.

Assume that there are n DMUs to be measured. Each DMU consumes m different inputs to produce s different outputs. Let the observed desirable and undesirable input vectors of the jth DMU be $X_j^g = (x_{1j}^g, x_{2j}^g, \ldots, x_{m_1 j}^g)$ $(j = 1, 2, \ldots, n)$ and $X_j^b = (x_{m_1+1j}^b, x_{m_1+2j}^b, \ldots, x_{mj}^b)$, respectively. Here, x_{ij}^g $(i = 1, 2, \ldots, m_1)$ represents the amount of the ith desirable input for the jth DMU, and x_{ij}^b $(i = m_1 + 1, 2, \ldots, m)$ represents the amount of the ith undesirable input for the jth DMU. Similarly, let the observed desirable and undesirable output vectors of the jth DMU be $Y_j^g = (y_{1j}^g, y_{2j}^g, \ldots, y_{s_1 j}^g)$ and $Y_j^b = (y_{s_1+1j}^b, y_{s_1+2j}^b, \ldots, y_{sj}^b)$, respectively. Here, q_{ik}^g, q_{ik}^b, p_{rk}^g and p_{rk}^b are the cost of the ith

Table 3 Comparison with similar works

Attributes	Joro and Na [23]	Branda [8]	Lim et al. [30]	Mashayekhi and Omrani [36]	Proposed approach
Objective functions	Single, nonlinear	Single, nonlinear	Single, linear	Multiple, nonlinear	Single, nonlinear
Return	Mean	Mean	Mean	CEBM	CEBM
Risk	Variance	CVaR	Variance	CEBV	CEBSV
Cardinality constraint	×	×	×	√	√
Efficiency of Assets	DEA	DEA	DEA	DEA	FDEA
Environment	Numeric	Numeric	Numeric	Fuzzy	Fuzzy
Defuzzification	×	×	×	√	√
Solution approach	DEA	DEA	DEA	NSGA-II	GA

CEBM Cross-efficiency-based mean, *CEBV* cross-efficiency-based variance, *CEBSV* cross-efficiency-based semivariance, *NSGA-II* nondominated sorting genetic algorithm II, *GA* genetic algorithm desirable and undesirable input and the price of the rth desirable and undesirable output for kth DMU, respectively. In addition, ε_k represents infinitesimal positive value. To account for undesirable inputs and outputs, the DEA model in [30] is extended as follows:

$$\begin{aligned}\max E_k = {} & \sum_{r=1}^{s_1} p_{rk}^g y_{rk}^g - \sum_{r=s_1+1}^{s} p_{rk}^b y_{rk}^b - \sum_{i=1}^{m_1} q_{ik}^g x_{ik}^g \\ & + \sum_{i=m_1+1}^{m} q_{ik}^b x_{ik}^b + \varepsilon_k \text{s.t.} \sum_{r=1}^{s_1} p_{rk}^g y_{rj}^g - \sum_{r=s_1+1}^{s} p_{rk}^b y_{rj}^b \\ & - \sum_{i=1}^{m_1} q_{ik}^g x_{ij}^g + \sum_{i=m_1+1}^{m} q_{ik}^b x_{ij}^b + \varepsilon_k \le 0, \\ & j = 1,2,\ldots,n, p_{rk}^g \ge \frac{1}{(m+s)R_r^{g+}}, r \\ & = 1,2,\ldots,s_1, p_{rk}^b \ge \frac{1}{(m+s)R_r^{b+}}, \\ & r = s_1+1,\ldots,s, q_{ik}^g \ge \frac{1}{(m+s)R_r^{g-}}, \\ & i = 1,2,\ldots,m_1, q_{ik}^b \ge \frac{1}{(m+s)R_r^{b-}}, \quad i = m_1+1,\ldots,m.\end{aligned} \tag{1}$$

The scalars R_i^{g-}, R_i^{b-}, R_r^{g+} and R_r^{b+} can be defined as follows:

$$\begin{aligned}R_i^{g-} & = \max_{j=1,2,\ldots,n}\{x_{ij}^g\} - \min_{j=1,2,\ldots,n}\{x_{ij}^g\}, \ i = 1,2,\ldots,m_1, R_i^{b-} \\ & = \max_{j=1,2,\ldots,n}\{x_{ij}^b\} - \min_{j=1,2,\ldots,n}\{x_{ij}^b\}, \ i = m_1+1,\ldots,m, R_r^{g+} \\ & = \max_{j=1,2,\ldots,n}\{y_{rj}^g\} - \min_{j=1,2,\ldots,n}\{y_{rj}^g\}, \ r = 1,2,\ldots,s_1, R_r^{b+} \\ & = \max_{j=1,2,\ldots,n}\{y_{rj}^b\} - \min_{j=1,2,\ldots,n}\{y_{rj}^b\}, \ r = s_1+1,\ldots,s.\end{aligned} \tag{2}$$

Model (1) can handle coexisting desirable and undesirable inputs and outputs. Here, the desirable outputs and undesirable inputs are expanded and the desirable inputs and undesirable outputs are contracted. Let * represent the optimal solution of model (1). The efficiency scores of other DMUs are obtained by using the weights of kth chosen DMU (q_{ik}^{g*}, q_{ik}^{b*}, p_{rk}^{g*} and p_{rk}^{b*}). The cross-efficiency of DMU j with the weights of DMU k (e_{kj}) can be evaluated as follows:

$$\begin{aligned}e_{kj} = {} & \sum_{r=1}^{s_1} p_{rk}^{g*} y_{rj}^g - \sum_{r=s_1+1}^{s} p_{rk}^{b*} y_{rj}^b - \sum_{i=1}^{m_1} q_{ik}^{g*} x_{ij}^g + \sum_{i=m_1+1}^{m} q_{ik}^{b*} x_{ij}^b \\ & + \varepsilon_k.\end{aligned} \tag{3}$$

We can construct the matrix of cross-efficiencies as $E = (e_{kj})$ $(k, j = 1,2,\ldots,n)$, where e_{kj} is the cross-efficiency of DMU j evaluated by DMU k. The cross-efficiency score of DMU j is defined as the average of the jth column:

$$\overline{e}_j = \frac{1}{n}\sum_{k=1}^{n} e_{kj}. \tag{4}$$

3 Fuzzy DEA Cross-Efficiency Model with Coexisting Undesirable Inputs and Outputs

Conventional DEA models are based on an unrealistic assumption that real situations can be modeled with crisp input and output data. In many situations, inputs and outputs are often imprecise; hence, there is a need to use FDEA models instead of DEA models.

The basic fuzzy form of model (1) with undesirable fuzzy inputs and outputs can be defined in the following FDEA model. It should be noted that the parameters in model (5) are the same as those in model (1), and the symbol ~ represents the fuzziness of the associated parameter.

$$\begin{aligned}\max E_k = {} & \sum_{r=1}^{s_1} p_{rk}^g \tilde{y}_{rk}^g - \sum_{r=s_1+1}^{s} p_{rk}^b \tilde{y}_{rk}^b - \sum_{i=1}^{m_1} q_{ik}^g \tilde{x}_{ik}^g \\ & + \sum_{i=m_1+1}^{m} q_{ik}^b \tilde{x}_{ik}^b + \varepsilon_k \text{s.t.} \\ & \sum_{r=1}^{s_1} p_{rk}^g \tilde{y}_{rj}^g - \sum_{r=s_1+1}^{s} p_{rk}^b \tilde{y}_{rj}^b - \sum_{i=1}^{m_1} q_{ik}^g \tilde{x}_{ij}^g + \sum_{i=m_1+1}^{m} q_{ik}^b \tilde{x}_{ij}^b \\ & + \varepsilon_k \le 0, \\ & j = 1,2,\ldots,n, p_{rk}^g \ge \frac{1}{(m+s)\tilde{R}_r^{g+}}, \\ & r = 1,2,\ldots,s_1, p_{rk}^b \ge \frac{1}{(m+s)\tilde{R}_r^{b+}}, \\ & r = s_1+1,\ldots,s, q_{ik}^g \ge \frac{1}{(m+s)\tilde{R}_r^{g-}}, \\ & i = 1,2,\ldots,m_1, q_{ik}^b \ge \frac{1}{(m+s)\tilde{R}_r^{b-}}, \quad i = m_1+1,\ldots,m.\end{aligned} \tag{5}$$

Note that several different patterns, including triangular, trapezoid, S-curve, exponential, hyperbolic, are used to model the vagueness of the parameters in the existing literature. Among them, the triangular distribution is used most often to represent imprecise data owing to the ease it offers in defining the maximum and minimum limits of deviation of the fuzzy number from its central value, although certain practical applications may prefer other patterns. Moreover, in refs. [38] and [40], the values of each input and output are also regarded as triangular fuzzy numbers. Therefore, in this paper, triangular fuzzy numbers (TFNs) are used to represent the values of each input and output. Let $\tilde{x}_{ij}^g = (x_{ij}^{lg}, x_{ij}^{mg}, x_{ij}^{ug})$ and $\tilde{x}_{ij}^b = (x_{ij}^{lb}, x_{ij}^{mb}, x_{ij}^{ub})$ represent the ith desirable and undesirable inputs corresponding to the jth DMU, respectively. Also let $\tilde{y}_{rj}^g = (y_{rj}^{lg}, y_{rj}^{mg}, y_{rj}^{ug})$ and $\tilde{y}_{rj}^b = (y_{rj}^{lb}, y_{rj}^{mb}, y_{rj}^{ub})$ represent the rth desirable and undesirable outputs of the jth DMU, respectively. We can calculate the lower and upper bounds of the membership functions of these TFNs for each input and output. To that end, here we apply the α-cut approach to transfer the fuzzy numbers to crisp numbers. Using α-cuts, the fuzzy data (uncertainty range) can be represented by different levels of confidence intervals. At a given α-cut level ($0 \le \alpha \le 1$), the upper and lower bounds of the inputs and outputs for an arbitrary α-cut level can be quantified as follows:

$$
\begin{aligned}
&(x_{ij}^{gL})_\alpha = x_{ij}^{lg} + \alpha(x_{ij}^{mg} - x_{ij}^{lg}), \quad i = 1,2,\ldots,m_1,\\
&(x_{ij}^{gU})_\alpha = x_{ij}^{ug} - \alpha(x_{ij}^{ug} - x_{ij}^{mg}), \quad i = 1,2,\ldots,m_1,\\
&(x_{ij}^{bL})_\alpha = x_{ij}^{lb} + \alpha(x_{ij}^{mb} - x_{ij}^{lb}), \quad i = m_1+1,\ldots,m,\\
&(x_{ij}^{bU})_\alpha = x_{ij}^{ub} - \alpha(x_{ij}^{ub} - x_{ij}^{mb}), \quad i = m_1+1,\ldots,m,\\
&(y_{rj}^{gL})_\alpha = y_{rj}^{lg} + \alpha(y_{rj}^{mg} - y_{rj}^{lg}), \quad r = 1,2,\ldots,s_1,\\
&(y_{rj}^{gU})_\alpha = y_{rj}^{ug} - \alpha(y_{rj}^{ug} - y_{rj}^{mg}), \quad r = 1,2,\ldots,s_1,\\
&(y_{rj}^{bL})_\alpha = y_{rj}^{lb} + \alpha(y_{rj}^{mb} - y_{rj}^{lb}), \quad r = s_1+1,\ldots,s,\\
&(y_{rj}^{bU})_\alpha = y_{rj}^{ub} - \alpha(y_{rj}^{ub} - y_{rj}^{mb}), \quad r = s_1+1,\ldots,s.
\end{aligned} \tag{6}
$$

Additionally, the upper and lower bounds of the R_i^{g-}, R_i^{b-}, R_r^{g+} and R_r^{b+} for an arbitrary α-cut level are defined as follows:

$$
\begin{aligned}
&(R_i^{gL-})_\alpha = R_i^{lg-} + \alpha(R_i^{mg-} - R_i^{lg-}), \quad i = 1,2,\ldots,m_1,\\
&(R_i^{gU-})_\alpha = R_i^{ug-} - \alpha(R_i^{ug-} - R_i^{mg-}), \quad i = 1,2,\ldots,m_1,\\
&(R_i^{bL-})_\alpha = R_i^{lb-} + \alpha(R_i^{mb-} - R_i^{lb-}), \quad i = m_1+1,\ldots,m,\\
&(R_i^{bU-})_\alpha = R_i^{ub-} - \alpha(R_i^{ub-} - R_i^{mb-}), \quad i = m_1+1,\ldots,m,\\
&(R_r^{gL+})_\alpha = R_r^{lg+} + \alpha(R_r^{mg+} - R_r^{lg+}), \quad r = 1,2,\ldots,s_1,\\
&(R_r^{gU+})_\alpha = R_r^{ug+} - \alpha(R_r^{ug+} - R_r^{mg+}), \quad r = 1,2,\ldots,s_1,\\
&(R_r^{bL+})_\alpha = R_r^{lb+} + \alpha(R_r^{mb+} - R_r^{lb+}), \quad r = s_1+1,\ldots,s,\\
&(R_r^{bU+})_\alpha = R_r^{ub+} - \alpha(R_r^{ub+} - R_r^{mb+}), \quad r = s_1+1,\ldots,s.
\end{aligned} \tag{7}
$$

Using Eqs. (6) and (7) along with model (5), the following models are proposed to obtain the lower and upper efficiency scores of the assets.

$$
\begin{aligned}
\max\;\; & E_k^L = \sum_{r=1}^{s_1} p_{rk}^g(y_{rk}^{lg} + \alpha(y_{rk}^{mg} - y_{rk}^{lg})) - \sum_{r=s_1+1}^{s} p_{rk}^b(y_{rk}^{ub} - \alpha(y_{rk}^{ub} - y_{rk}^{mb})) - \sum_{i=1}^{m_1} q_{ik}^g(x_{ik}^{ug} - \alpha(x_{ik}^{ug} - x_{ik}^{mg}))\\
&+ \sum_{i=m_1+1}^{m} q_{ik}^b(x_{ik}^{lb} + \alpha(x_{ik}^{mb} - x_{ik}^{lb})) + \varepsilon_k \text{s.t.}\\
& \sum_{r=1}^{s_1} p_{rk}^g(y_{rj}^{lg} + \alpha(y_{rj}^{mg} - y_{rj}^{lg})) - \sum_{r=s_1+1}^{s} p_{rk}^b(y_{rj}^{ub} - \alpha(y_{rj}^{ub} - y_{rj}^{mb})) - \sum_{i=1}^{m_1} q_{ik}^g(x_{ij}^{ug} - \alpha(x_{ij}^{ug} - x_{ij}^{mg}))\\
&+ \sum_{i=m_1+1}^{m} q_{ik}^b(x_{ij}^{lb} + \alpha(x_{ij}^{mb} - x_{ij}^{lb})) + \varepsilon_k \le 0,\\
& j = 1,2,\ldots,n, p_{rk}^g \ge \frac{1}{(m+s)(R_r^{lg+} + \alpha(R_r^{mg+} - R_r^{lg+}))},\\
& r = 1,2,\ldots,s_1, p_{rk}^b \ge \frac{1}{(m+s)(R_r^{ub+} - \alpha(R_r^{ub+} - R_r^{mb+}))},\\
& r = s_1+1,\ldots,s,
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
& q_{ik}^g \ge \frac{1}{(m+s)(R_i^{ug-} - \alpha(R_i^{ug-} - R_i^{mg-}))},\\
& i = 1,2,\ldots,m_1, q_{ik}^b \ge \frac{1}{(m+s)(R_i^{lb-} + \alpha(R_i^{mb-} - R_i^{lb-}))},\\
& i = m_1+1,\ldots,m,
\end{aligned}
$$

and

$$
\begin{aligned}
\max\;\; & E_k^U = \sum_{r=1}^{s_1} p_{rk}^g(y_{rk}^{ug} - \alpha(y_{rk}^{ug} - y_{rk}^{mg}))\\
&- \sum_{r=s_1+1}^{s} p_{rk}^b(y_{rk}^{lb} + \alpha(y_{rk}^{mb} - y_{rk}^{lb})) - \sum_{i=1}^{m_1} q_{ik}^g(x_{ik}^{lg} + \alpha(x_{ik}^{mg} - x_{ik}^{lg}))\\
&+ \sum_{i=m_1+1}^{m} q_{ik}^b(x_{ik}^{ub} - \alpha(x_{ik}^{ub} - x_{ik}^{mb})) + \varepsilon_k \text{s.t.}\\
& \sum_{r=1}^{s_1} p_{rk}^g(y_{rj}^{ug} - \alpha(y_{rj}^{ug} - y_{rj}^{mg})) - \sum_{r=s_1+1}^{s} p_{rk}^b(y_{rj}^{lb} + \alpha(y_{rj}^{mb} - y_{rj}^{lb}))\\
&- \sum_{i=1}^{m_1} q_{ik}^g(x_{ij}^{lg} + \alpha(x_{ij}^{mg} - x_{ij}^{lg}))\\
&+ \sum_{i=m_1+1}^{m} q_{ik}^b(x_{ij}^{ub} - \alpha(x_{ij}^{ub} - x_{ij}^{mb})) + \varepsilon_k \le 0, \quad j = 1,2,\ldots,n,\\
& p_{rk}^g \ge \frac{1}{(m+s)(R_r^{ug+} - \alpha(R_r^{ug+} - R_r^{mg+}))}, \quad r = 1,2,\ldots,s_1,\\
& p_{rk}^b \ge \frac{1}{(m+s)(R_r^{lb+} + \alpha(R_r^{mb+} - R_r^{lb+}))}, \quad r = s_1+1,\ldots,s,\\
& q_{ik}^g \ge \frac{1}{(m+s)(R_i^{lg-} + \alpha(R_i^{mg-} - R_i^{lg-}))}, \quad i = 1,2,\ldots,m_1,\\
& q_{ik}^b \ge \frac{1}{(m+s)(R_i^{ub-} - \alpha(R_i^{ub-} - R_i^{mb-}))}, \quad i = m_1+1,\ldots,m.
\end{aligned} \tag{9}
$$

In this paper, we use cross-efficiency to evaluate the performance of DMUs in fuzzy environment. Let * represent the optimal solution of models (8) and (9). We can calculate the set of weights (q_{ik}^{g*}, q_{ik}^{b*}, p_{rk}^{g*} and p_{rk}^{b*}) by solving models (8) and (9) at a given level of α, which maximizes the efficiency score of kth DMU. Then, the cross-efficiency of DMU j with the weights of DMU k (e_{kj}) at the given α can be expressed as follows:

$$\begin{aligned}(e_{kj})_{\alpha}^{L} &= \sum_{r=1}^{s_1} p_{rk}^{g*}(y_{rj}^{lg} + \alpha(y_{rj}^{mg} - y_{rj}^{lg})) - \sum_{r=s_1+1}^{s} p_{rk}^{b*}(y_{rj}^{ub} \\ &\quad - \alpha(y_{rj}^{ub} - y_{rj}^{mb})) - \sum_{i=1}^{m_1} q_{ik}^{g*}(x_{ij}^{ug} - \alpha(x_{ij}^{ug} - x_{ij}^{mg})) \\ &\quad + \sum_{i=m_1+1}^{m} q_{ik}^{b*}(x_{ij}^{lb} + \alpha(x_{ij}^{mb} - x_{ij}^{lb})) + \varepsilon_k, (e_{kj})_{\alpha}^{U} \\ &= \sum_{r=1}^{s_1} p_{rk}^{g*}(y_{rj}^{ug} - \alpha(y_{rj}^{ug} - y_{rj}^{mg})) - \sum_{r=s_1+1}^{s} p_{rk}^{b*}(y_{rj}^{lb} \\ &\quad + \alpha(y_{rj}^{mb} - y_{rj}^{lb})) - \sum_{i=1}^{m_1} q_{ik}^{g*}(x_{ij}^{lg} + \alpha(x_{ij}^{mg} - x_{ij}^{lg})) \\ &\quad + \sum_{i=m_1+1}^{m} q_{ik}^{b*}(x_{ij}^{ub} - \alpha(x_{ij}^{ub} - x_{ij}^{mb})) + \varepsilon_k.\end{aligned} \tag{10}$$

Once the cross-efficiencies have been obtained, we can construct the matrix called cross-efficiency matrix. The cross-efficiency score of DMU j can be calculated as the average of jth column as follows:

$$(\bar{e}_j)_{\alpha}^{L} = \frac{1}{n}\sum_{k=1}^{n}(e_{kj})_{\alpha}^{L}(\bar{e}_j)_{\alpha}^{U} = \frac{1}{n}\sum_{k=1}^{n}(e_{kj})_{\alpha}^{U} \tag{11}$$

Since the cross-efficiency scores are fuzzy numbers, they cannot be ranked directly. To solve this problem, we can use the ranking index (*RI*), which is a suitable method in [54], to rank the fuzzy efficiency scores:

$$RI(\tilde{e}_j) = \frac{\sum_{k=1}^{\rho} k\hat{\eta}_{jk}}{\sum_{k=1}^{\rho} k}, \tag{12}$$

where $k \in \{0, 1, \ldots, \rho\}$, ρ is the number of α_k, $\hat{\eta}_{jk} = \frac{m_{jk} - L}{\delta_{jk} + U - L + 1}$, $m_{jk} = \frac{(\bar{e}_j)_{\alpha_k}^{U} + (\bar{e}_j)_{\alpha_k}^{L}}{2}$, $\alpha_k = \frac{k}{\rho}$, $\delta_{jk} = (\bar{e}_j)_{\alpha_k}^{U} - (\bar{e}_j)_{\alpha_k}^{L}$, $U = max_{j,k}\{(\bar{e}_j)_{\alpha_k}^{U}\}$, $L = min_{j,k}\{(\bar{e}_j)_{\alpha_k}^{L}\}$.

4 A Mean-Semivariance Model of Portfolio Selection Based on FDEA Cross-Efficiency Evaluation

The traditional use of DEA cross-efficiency evaluation in the portfolio selection problem involves ranking DMUs in decreasing order of cross-efficiency scores and selecting the several top DMUs as the desired portfolio. However, there are two shortcomings for the simple use of DEA cross-efficiency: (i) no consideration of diversification for portfolio selection and (ii) the 'ganging-together' phenomenon [48]. In DEA cross-efficiency evaluation, the DMUs with similar factor levels may have higher cross-efficiency scores simply because they effectively give "high votes" to each other. It leads to selection of a specialized portfolio which consists of relatively similar DMUs and in turn lacks diversification. To eliminate the problems, Lim et al. [30] proposed a DEA MV cross-efficiency model for portfolio selection, which can select a portfolio whose performance is well diversified in terms of its performance on multiple evaluation criteria (for more detail see [30]). Then, Mashayekhi and Omrani [36] proposed an integrated fuzzy multi-objective Markowitz-DEA cross-efficiency model for portfolio selection. Chen et al. [12] used fuzzy mean-semivariance and SR-based DEA cross-efficiency models to develop a comprehensive fuzzy portfolio selection model. Therefore, similar to the above studies, in this paper, we incorporate the FDEA cross-efficiency into Markowitz mean-semivariance model and apply the novel mean-semivariance model to portfolio selection problem.

For a DMU i, Lim et al. [30] defined the return and risk characteristics as its cross-efficiency score and the variance of its cross-efficiencies, respectively. The DEA MV cross-efficiency model is as follows:

$$\begin{aligned}\min \quad & V_{\Omega} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j cov(e_i, e_j) \quad \text{s.t.} \\ & E_{\Omega} = \sum_{i=1}^{n} w_i \bar{e}_i \geq (1-\gamma)E_{\Omega}^{b}, \sum_{i=1}^{n} w_i = 1, w_i \geq 0, \ i \\ & = 1, 2, \ldots, n,\end{aligned} \tag{13}$$

where γ $(0 \leq \gamma \leq 1)$ is the return-risk trade-off parameter, $\bar{e}_i$ is the cross-efficiency score of DMU i, $cov(e_i, e_j)$ is the covariance between DMU i's cross-efficiencies (e_i) and DMU j's cross-efficiencies (e_j), E_{Ω}^{b} is the maximum portfolio return achievable, and w_i is the proportion of asset i $(i = 1, 2, \ldots, n)$.

Note that if the return distributions of assets are not symmetric, the use of variance as a risk measure is not advisable because it leads to predictions of portfolio behavior, which significantly diverge from realistic situations. To handle such situations, many scholars have used semivariance as an alternative risk measure to qualify risk, see for instance Markowitz [35] and Ballestero [4]. Motivated by this information, we have used semivariance to quantify the risk of cross-efficiency. Moreover, to make good investment decision in the complicated financial market, we consider several decision criteria, such as risk, return, budget constraint, cardinality constraint, buy-in thresholds and no short selling. In the following, we formulate a FDEA mean-semivariance cross-efficiency model.

4.1 Objective Function

• **Risk**: We use the semivariance of the *RI* to measure risk. The semivariance of portfolio $w = (w_1, w_2, \ldots, w_n)$ can be obtained as

$$S_\Omega = \sum_{i=1}^{n} w_i [min(0, RI(\tilde{e}_i) - E_\Omega)]^2. \quad (14)$$

4.2 Constraints

- Expected return: The return of the portfolio $w = (w_1, w_2, \ldots, w_n)$ is

$$E_\Omega = \sum_{i=1}^{n} w_i RI(\tilde{e}_i) \geq (1-\gamma) E_\Omega^b, \quad (15)$$

  where E_Ω^b is the maximum portfolio return achievable, which can be obtained by maximizing E_Ω under constraints.
- Budget constraint: Budget constraint represents the full utilization of the available money, i.e.,

$$\sum_{i=1}^{n} w_i = 1. \quad (16)$$

- Cardinality constraint: Cardinality constraint is used to control the number of assets held in the portfolio. The cardinality constraint is described as

$$\sum_{i=1}^{n} z_i = d, \quad (17)$$

  where $z_i \in \{0, 1\}$, if any of asset i is held, $z_i = 1$; otherwise, $z_i = 0$.
- Buy-in thresholds: If any of asset i is held ($z_i = 1$), its proportion w_i must lie no less than ε_i and no more than δ_i, while if no asset i is held ($z_i = 0$), its ratio w_i is zero. Thus, in the presence of cardinality constraint (Eq. 17), buy-in thresholds are represented by

$$\varepsilon_i z_i \leq w_i \leq \delta_i z_i, \; i = 1, 2, \ldots, n. \quad (18)$$

- No short selling: This constraint ensures that short selling is prohibited, and it is expressed as

$$w_i \geq 0, \; i = 1, 2, \ldots, n. \quad (19)$$

4.3 Model Formulation

Based on the above discussions, the FDEA mean-semi-variance cross-efficiency model can be described as follows:

Table 4 Inputs and outputs

Type	Classification	Parameter	Description
Inputs	Desirable (−)	Solvency ratio-I (x_1^g)	Total liability divided by total assets
		Solvency ratio-II (x_2^g)	Total liability divided by shareholders equity
	Undesirable(+)	Receivable turnover (x_3^b)	Revenues for the period divided by receivables
		Inventory turnover (x_4^b)	Revenues for the period divided by inventories
		Asset turnover (x_5^b)	Revenues for the period divided by total assets
		Current ratio (x_6^b)	Total current assets divided by total current liabilities
		Quick ratio (x_7^b)	Quick assets (total current assets minus inventory) divided by total current liabilities
Outputs	Desirable(+)	Return on equity (y_1^g)	Net income divided by shareholders equity
		Return on assets (y_2^g)	Net income divided by the total assets
		Net profit margin (y_3^g)	Net income divided by revenue for the period
		Basic earnings per share (basic EPS) (y_4^g)	Net income minus dividends and preferred stock dividend divided by common shares
		Revenue growth rate (y_5^g)	Current period revenue divided by the previous period revenue minus one
		Net profit growth rate (y_6^g)	Current period net income divided by the previous period net income minus one
		Basic earnings per share growth rate (y_7^g)	Current period basic EPS divided by the previous period basic EPS minus one
	Undesirable(−)	Expense ratio during sales (y_8^b)	Sales expenses plus administrative expenses plus finance expenses divided by operating income
		Income tax/total profit (y_9^b)	Income tax divided by total profit

Table 5 Table caption

Stock codes	Inputs						
	Desirable		Undesirable				
	x_1^g	x_2^g	$x_3^b(\%)$	$x_4^b(\%)$	$x_5^b(\%)$	$x_6^b(\%)$	$x_7^b(\%)$
000100	66.2227	356.8454	778.8013	688.0809	72.5869	110.8627	92.9434
000333	66.5768	224.0133	1553.8232	800.7823	114.9787	142.5879	117.8640
000425	51.6661	107.0620	194.8565	295.0904	62.1482	173.0681	128.2403
000630	61.1346	169.0829	6399.7489	822.8151	178.4447	93.6993	53.4246
000651	68.9094	225.8299	3379.7380	778.0403	74.6400	116.3019	105.0684
000661	29.2609	48.6031	833.9222	58.8693	60.5278	266.3683	173.1366
000725	59.2812	179.0175	591.6974	837.1517	40.6728	200.6191	182.6087
000786	22.4385	29.1170	10832.6525	557.9776	73.3049	190.9471	147.4614
000895	33.0101	52.1426	42773.8981	1330.7524	227.0306	135.2299	95.6965
000983	63.3695	204.6249	819.5472	588.6274	51.2898	69.3887	55.5563
001979	72.1055	350.8046	33762.7276	33.7902	25.8693	171.2904	73.1293
002044	44.6861	85.8397	548.3931	5153.3244	67.4971	102.7857	100.9242
002085	34.9837	58.5033	557.5165	759.6299	105.2580	188.0758	147.8414
002294	10.2829	11.6614	509.6740	195.9485	62.0035	648.4695	565.0916
002311	48.0494	97.6652	4874.6424	1065.1890	277.6880	114.6176	58.2991
002508	33.6731	50.7364	1997.0880	320.6664	97.8592	257.7635	214.9170
002624	47.0197	97.9157	449.7375	326.4449	48.2313	196.4626	166.2527
002625	9.2839	10.2326	181.0701	492.6840	8.4051	1416.5716	1403.3835
300024	28.6330	40.6630	266.8721	83.5528	31.6700	337.4267	211.9594
300124	36.7149	61.2144	374.7144	294.2279	56.1341	224.0999	191.4288
300136	47.8675	92.5124	282.7653	783.3973	80.8521	149.9532	135.8854
300408	20.1615	25.4151	307.5729	307.3118	47.0018	607.3558	527.5952
600004	32.9480	49.4445	848.3120	5326.4273	33.5869	76.4157	74.9450
600010	66.2191	196.8244	2312.9044	264.7358	37.2697	46.5746	24.7858
600018	45.4409	92.3638	1458.0740	161.6352	29.0085	131.9867	67.6935
600023	39.0537	72.5345	965.7560	1768.6584	46.8094	128.1918	113.2601
600100	61.0547	182.5425	364.3548	220.2554	42.8841	102.3942	71.7044
600104	62.3920	200.3358	2626.6009	1705.0280	130.5741	99.7380	86.9387
600111	48.3105	112.5123	864.4753	130.2110	56.3405	253.7424	129.4722
600115	75.1529	321.8958	4279.3857	4073.3137	46.4994	22.7737	20.0535
600153	75.0494	556.9132	6733.0346	276.7722	144.9037	178.6371	75.4870
600297	67.2911	257.5472	5247.5714	894.2047	130.0225	120.5141	95.5650
600332	31.9676	47.9641	1885.9563	403.0791	77.3049	260.0669	215.3179
600436	21.6533	29.5703	919.3652	177.9093	69.5100	366.3381	258.4014
600487	60.8106	165.9077	542.1072	470.1095	108.2105	139.2674	105.0603
600516	26.9418	38.7437	943.4820	166.5191	75.8825	322.9865	281.6503
600535	58.3198	145.0954	276.9098	483.2840	83.2640	132.7098	113.0324
600760	72.3102	267.5217	2173.5020	431.7394	138.5605	111.9845	67.9873
600900	54.7362	121.3009	1585.9826	5590.8596	16.7633	17.0630	16.6325
600998	62.4626	177.6503	637.9247	606.2871	162.9112	141.5972	101.9180
601006	20.9697	26.9367	1129.2204	2556.4709	44.3019	136.3802	127.0943
601021	58.9179	143.4152	10330.7489	11993.8708	54.5136	121.5433	119.8386
601088	33.9426	63.8491	1686.5279	1151.2886	43.6860	114.4419	104.3932
601766	62.1853	191.9251	278.1498	297.6200	59.1501	125.5447	98.4072
601877	53.4135	118.0047	388.3941	622.6548	58.9011	161.4611	136.3574
601888	28.2862	42.1925	3043.0350	736.0977	147.9977	297.0709	241.1435

Table 5 continued

Stock codes	Inputs						
	Desirable		Undesirable				
	x_1^g	x_2^g	$x_3^b(\%)$	$x_4^b(\%)$	$x_5^b(\%)$	$x_6^b(\%)$	$x_7^b(\%)$
601898	57.3681	160.3770	1144.5260	739.0605	33.0651	77.6577	65.8237
601899	57.8539	147.6366	9109.0285	704.7492	105.9172	99.5881	61.0732
601919	67.1817	432.9100	1491.8704	4241.6701	71.5574	90.6884	85.3305
603986	31.7411	46.5209	2059.2363	238.7944	95.6502	258.4061	145.0667

$$\min \quad S_\Omega = \sum_{i=1}^{n} w_i[min(0, RI(\tilde{e}_i) - E_\Omega)]^2, \quad \text{s.t.}$$
$$E_\Omega = \sum_{i=1}^{n} w_i RI(\tilde{e}_i) \geq (1-\gamma)E_\Omega^b, \sum_{i=1}^{n} w_i = 1, \sum_{i=1}^{n} z_i = d, \varepsilon_i z_i \leq w_i \leq \delta_i z_i,\ i = 1,2,\ldots,n, z_i \in \{0,1\},\ i = 1,2,\ldots,n, w_i \geq 0,\ i = 1,2,\ldots,n. \quad (20)$$

5 Genetic Algorithm

Genetic algorithm (GA), which was originally proposed by Holland [21], is a classical practical algorithm based on the mechanism of genetics and natural selection. This paper employs GA to solve the proposed model (20).

5.1 Initialization

At the initialization step, following Bacanin and Tuba [3], GA generates SN random populations using

$$w_{i,j} = \varepsilon_j + \text{rand}(0,1)(\delta_j - \varepsilon_j), \quad (21)$$

where $\text{rand}(0,1)$ is a random number uniformly distributed in [0, 1].

5.2 Constraint Handling

1. Boundary constraint: If the initially generated value for the jth parameter of the ith gene does not fit in the scope $[\varepsilon_j, \delta_j]$, it is being modified:

   $$\text{if} \quad w_{i,j} > \delta_j, \quad \text{then} \quad w_{i,j} = \delta_j, \text{if} \quad w_{i,j} < \varepsilon_j, \quad \text{then} \quad w_{i,j} = \varepsilon_j. \quad (22)$$

2. Cardinality constraint: Decision variables $z_{i,j}$ $(i = 1,2,\ldots,SN, j = 1,2,\ldots,n)$ are generated randomly by applying

   $$z_{i,j} = \begin{cases} 1, & \text{if } \phi < 0.5, \\ 0, & \text{if } \phi \geq 0.5, \end{cases} \quad (23)$$

   where ϕ is random real number between 0 and 1.
3. Budget constraint: For the constraint $\sum_{i=1}^{n} w_i = 1$ we set $\psi = \sum_{i=1}^{n} w_{i,j}$ and put $w_{i,j} = w_{i,j}/\psi$ for all assets that satisfy $j = 1,2,\ldots,n$. The same approach for satisfying this constraint was used in [14].

The implementation procedure of GA is described as Algorithm 1.

Algorithm 1 The genetic algorithm (GA).

```
begin
    t ← 0;
    initialize SN genes with the constraints criterion fulfilled;
    evaluate SN genes;
    find the best chromosomes of SN genes;
    while (t < the maximum iteration number) do
        select two parents from SN genes;
        generate two offspring by crossover and mutation with the constraints criterion fulfilled;
        evaluate the offspring;
        if the offspring is fitter than the worst chromosome of SN genes then
            randomly select a parent except the best one and replace it;
        end if
        find the best chromosomes of SN genes;
        t ← t + 1;
    end while
end
```

Table 6 Output data

Stock codes	Outputs								
	Desirable							Undesirable	
	$y_1^g(\%)$	$y_2^g(\%)$	$y_3^g(\%)$	y_4^g	$y_5^g(\%)$	$y_6^g(\%)$	$y_7^g(\%)$	$y_8^b(\%)$	$y_9^b(\%)$
000100	13.5005	2.2113	3.1769	0.2178	4.7935	65.8309	66.0060	18.4915	25.9938
000333	27.5998	7.5012	7.7317	2.6600	51.3494	17.3325	16.1572	17.5873	14.8415
000425	4.6261	2.0670	3.5314	0.1400	72.4628	374.9802	366.6666	14.1989	10.8658
000630	4.3704	1.5508	0.8991	0.0500	− 4.8963	128.4739	150.0000	2.2334	28.6614
000651	37.6842	10.4706	15.1791	3.7200	36.9186	44.9863	45.3125	15.6202	15.4358
000661	22.2831	12.6181	22.6122	3.8900	41.5822	37.0342	26.2987	52.0860	17.0792
000725	9.6146	3.0691	8.3799	0.2170	36.1486	284.3401	301.8518	12.5654	19.3065
000786	20.6649	14.6169	21.0930	1.3110	36.8837	60.7003	65.3215	10.8599	12.4242
000895	31.3015	19.5353	8.9410	1.3091	− 2.6533	− 1.1182	− 1.9106	7.1075	21.6810
000983	10.7008	3.1762	6.4130	0.4979	46.1187	307.5096	261.5831	18.4334	31.5061
001979	24.0537	4.5124	19.8918	1.5500	18.6901	23.1599	28.0991	4.7512	27.6097
002044	14.1566	5.5637	11.1394	0.2400	102.2495	83.4340	71.4285	33.0955	22.9881
002085	17.6755	9.9493	9.8240	0.4100	7.2899	− 7.6282	− 22.6415	9.7020	16.4165
002294	25.1791	20.9324	34.5921	1.3900	8.3549	3.3698	4.5112	39.5446	16.2326
002311	20.2314	9.3192	3.7671	0.7800	19.7581	40.5555	41.8181	6.7518	17.2465
002508	31.1244	18.4340	20.8224	1.5400	21.0961	21.0786	− 7.7844	29.4547	13.6754
002624	19.2365	8.8026	18.4104	1.1400	28.7551	28.8367	18.7500	37.9135	11.4590
002625	1.9633	0.9620	21.0165	0.0700	− 9.9533	21.0080	− 68.1818	9.2316	28.1309
300024	7.7159	5.2659	18.0682	0.2771	20.7320	5.7437	5.2411	16.5958	12.5309
300124	21.5216	12.0631	22.8448	0.6500	30.5255	11.3443	10.1694	26.4227	9.2232
300136	38.0392	16.6994	25.9294	0.9081	42.3485	70.0602	63.5921	12.7949	16.1813
300408	19.7215	14.6055	34.6933	0.6300	8.3911	2.4531	1.6129	11.5299	14.2633
600004	12.4257	7.1104	23.6816	0.8200	9.6464	15.0367	− 32.2314	8.1360	26.0662
600010	4.2474	1.3985	3.8203	0.0452	73.0160	2309.0349	1638.4615	9.1317	27.4382
600018	19.7321	9.0957	34.3267	0.4978	19.3396	58.8349	66.2658	10.7886	18.9770
600023	7.9705	4.2031	9.2685	0.3200	30.6666	− 34.0763	− 30.4347	4.9768	14.2165
600100	2.4597	0.8327	2.0377	0.0350	− 4.3605	− 88.7339	− 97.5888	19.7277	22.7738
600104	22.5837	6.5119	5.4915	2.9590	14.9739	7.1746	1.9290	10.7888	13.1676
600111	8.1286	3.3819	6.8278	0.1105	99.5628	767.8595	342.0000	8.8222	12.9636
600115	13.6002	2.9982	6.7046	0.4400	3.2071	37.3615	33.3333	9.9851	20.8816
600153	21.3790	2.7528	2.2075	1.1700	50.1478	22.1844	15.8415	2.9955	27.0718
600297	15.2947	3.3306	2.8028	0.5300	18.6743	48.0311	4.0031	5.7657	21.6255
600332	11.7004	7.4828	10.1113	1.2680	4.5845	35.9332	17.9534	26.9872	15.0109
600436	20.4582	13.8170	21.0133	1.3400	60.8500	53.9857	50.5617	18.0739	17.1136
600487	27.6092	7.9203	8.6155	1.6339	34.4535	46.7874	54.0688	11.4238	12.8510
600516	51.1268	28.4687	47.5685	2.1100	248.6204	13132.3664	5175.0000	16.2191	17.7671
600535	16.8760	6.5125	8.7128	1.2700	15.4075	15.0117	16.5137	24.0181	19.2472
600760	18.5827	2.6660	3.6155	0.5100	1585.6044	3510.7987	537.5000	5.5843	12.9787
600900	16.9195	7.4397	44.4187	1.0119	2.4672	6.3848	7.1247	13.4728	16.4317
600998	9.9593	2.8298	1.9919	0.8700	20.1213	62.8718	61.1111	6.2505	23.4742
601006	13.8439	10.2582	23.5428	0.9000	24.6759	85.0695	87.5000	2.6746	22.0230
601021	15.9821	6.1234	11.4996	1.5800	30.1466	32.7255	32.7731	6.7597	23.6020
601088	17.6103	9.5305	21.7289	2.2640	35.8325	82.9970	98.2486	9.4325	23.1512
601766	11.4935	3.4681	6.1662	0.3800	− 8.1444	− 6.4584	− 7.3170	15.3460	15.5056
601877	17.9583	6.7852	12.7946	1.3400	16.1273	14.4443	2.2900	15.6152	11.7402
601888	22.0373	14.0193	10.3759	1.2962	26.3177	44.1881	− 29.9351	15.7988	23.4637

Table 6 continued

Stock codes	Outputs								
	Desirable							Undesirable	
	$y_1^g(\%)$	$y_2^g(\%)$	$y_3^g(\%)$	y_4^g	$y_5^g(\%)$	$y_6^g(\%)$	$y_7^g(\%)$	$y_8^b(\%)$	$y_9^b(\%)$
601898	5.0832	1.7867	5.4806	0.1800	33.7969	51.6547	20.0000	20.8812	27.6701
601899	10.3487	3.6360	3.4347	0.1600	19.9077	92.4738	77.7777	6.0878	28.9059
601919	24.7774	3.6269	5.3399	0.2600	27.1272	153.0772	126.8041	7.6961	15.2962
603986	26.1971	15.4423	19.5862	1.9900	36.3182	127.5604	− 6.1320	17.7094	11.4838

6 Numerical Experiments

6.1 Data Preprocessing

6.1.1 Selection of Input and Output Parameters

In this example, we select 50 firms from the CSI 300 index, which is an index of 300 large-capitalization Shanghai- and Shenzhen-listed Class A shares. We rely on the financial statements for the year 2017 to obtain the required data for inputs and outputs of FDEA. To consider the input and output parameters, we rely on a thorough literature survey [15, 30] along with financial experts' opinions. The performance of each firm is evaluated in terms of two desirable inputs, five undesirable inputs, seven desirable outputs and two undesirable outputs. The selected 16 financial input/output parameters are given in Table 4.

We consider leverages (inputs) that should be minimized to decrease the financial risk and prevent insolvency or even bankruptcy for these firms, consistent with the DEA nomenclature and define them as desirable. In addition, firms require larger asset utilization and liquidity to make efficient services, leading to our DEA-based definition of undesirable inputs. Finally, firms expect a consistent flow of profits to result from their activity (desirable outputs) while improving the performance in the area, i.e., decreasing expense ratio during sales (undesirable outputs).

6.1.2 Fuzzification of Input and Output Data

Tables 5 and 6 present the 16 financial input/output data of 50 firms. From Tables 5 and 6, we can see that the input–output data of each firm are available in crisp form. To eliminate the scale effect, the data given in Tables 5 and 6 are normalized, and the obtained normalized data are given in Tables 7 and 8. To incorporate the uncertainty exists in the real-market, we fuzzify the normalized data as TFNs. The obtained normalized data are represented by a^m, and the corresponding TFN is represented by (a^l, a^m, a^u). As noted in [38], a^l and a^u values are obtained by subtracting and adding 20% of a^m from a^m, respectively. Given space constraints, the corresponding results are omitted here. One company among 50 (000100) has been chosen to represent the numerical findings step by step (till the fuzzification) in the Appendix.

Remark Note that for the crisp negative value, a^l and a^u values are obtained by adding and subtracting 20% of a^m from a^m, respectively, as per the fuzzy algebra rules.

6.2 Performance Assessment of Firms

The computations used to solve models (8) and (9) are made in Matlab. By solving models (8) and (9), we derive a lower and upper bound of the efficiency scores at different α-cut levels ($\alpha = 0, 0.1, 0.2, \ldots, 0.9, 1$). The results are presented in Table 9. Moreover, using Eq. (12), the values of *RI* with respect to various assets are also given in Table 9. Note that the cross-efficiency scores in the case of $\alpha = 1$ are the same as the results based on the crisp input–output data. Given the variation in the satisfaction level α, the lower and upper bounds of efficiency scores are different for almost every DMU. Using the firm with a stock code of 600018 as an example, its efficiency score is -0.5810 under crisp data, while that with fuzzy data fluctuates between -0.5853 and -0.5750.

6.3 Portfolio Selection

In this section, we apply the proposed model on real data for portfolio selection to justify the utility in real-world investment situations. For the purpose, we present two cases: Case I, portfolio selection was performed with equal weights, i.e., the investor wishes to have an equal proportion of investment in the assets of the selected portfolio; Case II, portfolio selection was performed with unequal weights, i.e., the investor wishes to satisfy specific proportion of investment in the assets of the selected portfolio. In both cases, different portfolios were generated by varying the values of $d, \gamma, \varepsilon_i, \delta_i$.

Table 7 Standardized input data

Stock codes	Inputs						
	Desirable		Undesirable				
	x_1^g	x_2^g	x_3^b	x_4^b	x_5^b	x_6^b	x_7^b
000100	0.8812	0.6408	0.0182	0.0574	0.2614	0.0783	0.0662
000333	0.8859	0.4022	0.0363	0.0668	0.4141	0.1007	0.0840
000425	0.6875	0.1922	0.0046	0.0246	0.2238	0.1222	0.0914
000630	0.8135	0.3036	0.1496	0.0686	0.6426	0.0661	0.0381
000651	0.9169	0.4055	0.0790	0.0649	0.2688	0.0821	0.0749
000661	0.3894	0.0873	0.0195	0.0049	0.2180	0.1880	0.1234
000725	0.7888	0.3214	0.0138	0.0698	0.1465	0.1416	0.1301
000786	0.2986	0.0523	0.2533	0.0465	0.2640	0.1348	0.1051
000895	0.4392	0.0936	1.0000	0.1110	0.8176	0.0955	0.0682
000983	0.8432	0.3674	0.0192	0.0491	0.1847	0.0490	0.0396
001979	0.9595	0.6299	0.7893	0.0028	0.0932	0.1209	0.0521
002044	0.5946	0.1541	0.0128	0.4297	0.2431	0.0726	0.0719
002085	0.4655	0.1050	0.0130	0.0633	0.3791	0.1328	0.1053
002294	0.1368	0.0209	0.0119	0.0163	0.2233	0.4578	0.4027
002311	0.6394	0.1754	0.1140	0.0888	1.0000	0.0809	0.0415
002508	0.4481	0.0911	0.0467	0.0267	0.3524	0.1820	0.1531
002624	0.6257	0.1758	0.0105	0.0272	0.1737	0.1387	0.1185
002625	0.1235	0.0184	0.0042	0.0411	0.0303	1.0000	1.0000
300024	0.3810	0.0730	0.0062	0.0070	0.1140	0.2382	0.1510
300124	0.4885	0.1099	0.0088	0.0245	0.2021	0.1582	0.1364
300136	0.6369	0.1661	0.0066	0.0653	0.2912	0.1059	0.0968
300408	0.2683	0.0456	0.0072	0.0256	0.1693	0.4288	0.3759
600004	0.4384	0.0888	0.0198	0.4441	0.1210	0.0539	0.0534
600010	0.8811	0.3534	0.0541	0.0221	0.1342	0.0329	0.0177
600018	0.6046	0.1658	0.0341	0.0135	0.1045	0.0932	0.0482
600023	0.5197	0.1302	0.0226	0.1475	0.1686	0.0905	0.0807
600100	0.8124	0.3278	0.0085	0.0184	0.1544	0.0723	0.0511
600104	0.8302	0.3597	0.0614	0.1422	0.4702	0.0704	0.0619
600111	0.6428	0.2020	0.0202	0.0109	0.2029	0.1791	0.0923
600115	1.0000	0.5780	0.1000	0.3396	0.1675	0.0161	0.0143
600153	0.9986	1.0000	0.1574	0.0231	0.5218	0.1261	0.0538
600297	0.8954	0.4625	0.1227	0.0746	0.4682	0.0851	0.0681
600332	0.4254	0.0861	0.0441	0.0336	0.2784	0.1836	0.1534
600436	0.2881	0.0531	0.0215	0.0148	0.2503	0.2586	0.1841
600487	0.8092	0.2979	0.0127	0.0392	0.3897	0.0983	0.0749
600516	0.3585	0.0696	0.0221	0.0139	0.2733	0.2280	0.2007
600535	0.7760	0.2605	0.0065	0.0403	0.2998	0.0937	0.0805
600760	0.9622	0.4804	0.0508	0.0360	0.4990	0.0791	0.0484
600900	0.7283	0.2178	0.0371	0.4661	0.0604	0.0120	0.0119
600998	0.8311	0.3190	0.0149	0.0505	0.5867	0.1000	0.0726
601006	0.2790	0.0484	0.0264	0.2131	0.1595	0.0963	0.0906
601021	0.7840	0.2575	0.2415	1.0000	0.1963	0.0858	0.0854
601088	0.4516	0.1146	0.0394	0.0960	0.1573	0.0808	0.0744
601766	0.8275	0.3446	0.0065	0.0248	0.2130	0.0886	0.0701
601877	0.7107	0.2119	0.0091	0.0519	0.2121	0.1140	0.0972
601888	0.3764	0.0758	0.0711	0.0614	0.5330	0.2097	0.1718

Table 7 continued

Stock codes	Inputs						
	Desirable		Undesirable				
	x_1^g	x_2^g	x_3^b	x_4^b	x_5^b	x_6^b	x_7^b
601898	0.7634	0.2880	0.0268	0.0616	0.1191	0.0548	0.0469
601899	0.7698	0.2651	0.2130	0.0588	0.3814	0.0703	0.0435
601919	0.8939	0.7773	0.0349	0.3537	0.2577	0.0640	0.0608
603986	0.4224	0.0835	0.0481	0.0199	0.3445	0.1824	0.1034

6.3.1 Portfolio Selection with Equal Weights

We apply the proposed model (20) to the DMUs (assets) with the *RI* data given in Table 9. To present the advantages of the proposed portfolio selection model, we first obtain portfolios using the traditional method for different cardinality constraints, i.e., d =15, 10, and 5, wherein assets are chosen with equal weights in decreasing order of the *RI* data. The obtained portfolio strategies are given in Table 10. Further, we employ GA to solve the proposed model (20) with the trade-off parameter (γ) set to 0.1, 0.2, and 0.3. For all three values of γ, we consider following three cases: (i) $d = 15$, $\varepsilon_i = 0.06667$ and $\delta_i = 0.06667$; (ii) $d = 10$, $\varepsilon_i = 0.1$ and $\delta_i = 0.1$; (iii) $d = 5$, $\varepsilon_i = 0.2$ and $\delta_i = 0.2$, respectively. Moreover, the parameters of GA are set as follows: the number of populations is 50, the crossover probability is 0.9, the mutation probability is 0.1, and the maximum generation number is 100. Finally, the E_{Ω}^b is obtained by maximizing E_{Ω} under constraints (16)-(19). In the following, to compare the obtained portfolios with those obtained through the traditional method, the assets are chosen with equal weights. The obtained portfolio strategies under different cardinality constraints (i.e., d) are presented in Table 11. To present the advantages of the proposed model, we compare obtained portfolios given in Table 11 with the portfolios obtained through the traditional approach given in Table 10. First, we compare the mean and semivariance of the portfolios, and it is clear that the portfolios obtained using the proposed model have remarkable reduction in semivariance and a slight reduction in mean. For example, the portfolio obtained in Table 11 corresponding to $d = 15$ and $\gamma = 0.1$ exhibits a 9.26% decrease in the mean and 41.48% reduction in semivariance in comparison with the portfolio given in Table 10 (obtained using the traditional approach). The same conclusion can be obtained in the case of $d = 10$ and $d = 5$. In addition, the results given in Table 11 clearly highlight the fact that a relatively larger decrease in semivariance can be achieved with a larger value of the return-risk trade-off parameter γ. For example, given $d =$ 10 and comparing the portfolios obtained corresponding to $\gamma = 0.1$ and $\gamma = 0.2$, the latter has significantly smaller semivariance (0.00036 $\rightarrow$ 0.00020, 44.44% reduction) and lower decrease in return (0.44138 $\rightarrow$ 0.40073, 9.21% reduction).

6.3.2 Portfolio Selection with Unequal Weights

To generate portfolios with unequal weights, for $\varepsilon_i = 0$, we set δ_i = 0.3, 0.5 and 0.7, respectively. Note that the parameters of d, γ, and GA, are the same as noted in the above case. Using the GA, we solve the proposed model (20) under different d. The optimal portfolio strategies are given in Tables 12, 13, 14. By comparing the portfolios given in Tables 12, 13, 14 with those given in Table 10, two conclusions can be drawn. First, it can be easily observed that the portfolio risks decrease with smaller reductions in returns. For example, $d = 15$, $\gamma = 0.1$ and $\delta = 0.3$, the portfolio semivariance decreases from 0.00087 to 0.00046, i.e., 47.18% reduction and portfolio return decreases from 0.44608 to 0.41086, i.e., 7.90% reduction. In addition, a larger decrease in semivariance can be

Table 8 Standardized output data

Stock codes	Outputs								
	Desirable							Undesirable	
	y_1^g	y_2^g	y_3^g	y_4^g	y_5^g	y_6^g	y_7^g	y_8^b	y_9^b
000100	0.2641	0.0777	0.0668	0.0560	0.0030	0.0050	0.0128	0.3550	0.8250
000333	0.5398	0.2635	0.1625	0.6838	0.0324	0.0013	0.0031	0.3377	0.4711
000425	0.0905	0.0726	0.0742	0.0360	0.0457	0.0286	0.0709	0.2726	0.3449
000630	0.0855	0.0545	0.0189	0.0129	− 0.0031	0.0098	0.0290	0.0429	0.9097
000651	0.7371	0.3678	0.3191	0.9563	0.0233	0.0034	0.0088	0.2999	0.4899
000661	0.4358	0.4432	0.4754	1.0000	0.0262	0.0028	0.0051	1.0000	0.5421
000725	0.1881	0.1078	0.1762	0.0558	0.0228	0.0217	0.0583	0.2412	0.6128
000786	0.4042	0.5134	0.4434	0.3370	0.0233	0.0046	0.0126	0.2085	0.3943
000895	0.6122	0.6862	0.1880	0.3365	− 0.0017	− 0.0001	− 0.0004	0.1365	0.6882
000983	0.2093	0.1116	0.1348	0.1280	0.0291	0.0234	0.0505	0.3539	1.0000
001979	0.4705	0.1585	0.4182	0.3985	0.0118	0.0018	0.0054	0.0912	0.8763
002044	0.2769	0.1954	0.2342	0.0617	0.0645	0.0064	0.0138	0.6354	0.7296
002085	0.3457	0.3495	0.2065	0.1054	0.0046	− 0.0006	− 0.0044	0.1863	0.5211
002294	0.4925	0.7353	0.7272	0.3573	0.0053	0.0003	0.0009	0.7592	0.5152
002311	0.3957	0.3273	0.0792	0.2005	0.0125	0.0031	0.0081	0.1296	0.5474
002508	0.6088	0.6475	0.4377	0.3959	0.0133	0.0016	− 0.0015	0.5655	0.4341
002624	0.3763	0.3092	0.3870	0.2931	0.0181	0.0022	0.0036	0.7279	0.3637
002625	0.0384	0.0338	0.4418	0.0180	− 0.0063	0.0016	− 0.0132	0.1772	0.8929
300024	0.1509	0.1850	0.3798	0.0712	0.0131	0.0004	0.0010	0.3186	0.3977
300124	0.4209	0.4237	0.4803	0.1671	0.0193	0.0009	0.0020	0.5073	0.2927
300136	0.7440	0.5866	0.5451	0.2334	0.0267	0.0053	0.0123	0.2456	0.5136
300408	0.3857	0.5130	0.7293	0.1620	0.0053	0.0002	0.0003	0.2214	0.4527
600004	0.2430	0.2498	0.4978	0.2108	0.0061	0.0011	− 0.0062	0.1562	0.8273
600010	0.0831	0.0491	0.0803	0.0116	0.0460	0.1758	0.3166	0.1753	0.8709
600018	0.3859	0.3195	0.7216	0.1280	0.0122	0.0045	0.0128	0.2071	0.6023
600023	0.1559	0.1476	0.1948	0.0823	0.0193	− 0.0026	− 0.0059	0.0955	0.4512
600100	0.0481	0.0292	0.0428	0.0090	− 0.0028	− 0.0068	− 0.0189	0.3788	0.7228
600104	0.4417	0.2287	0.1154	0.7607	0.0094	0.0005	0.0004	0.2071	0.4179
600111	0.1590	0.1188	0.1435	0.0284	0.0628	0.0585	0.0661	0.1694	0.4115
600115	0.2660	0.1053	0.1409	0.1131	0.0020	0.0028	0.0064	0.1917	0.6628
600153	0.4182	0.0967	0.0464	0.3008	0.0316	0.0017	0.0031	0.0575	0.8593
600297	0.2992	0.1170	0.0589	0.1362	0.0118	0.0037	0.0008	0.1107	0.6864
600332	0.2289	0.2628	0.2126	0.3260	0.0029	0.0027	0.0035	0.5181	0.4764
600436	0.4001	0.4853	0.4417	0.3445	0.0384	0.0041	0.0098	0.3470	0.5432
600487	0.5400	0.2782	0.1811	0.4200	0.0217	0.0036	0.0104	0.2193	0.4079
600516	1.0000	1.0000	1.0000	0.5424	0.1568	1.0000	1.0000	0.3114	0.5639
600535	0.3301	0.2288	0.1832	0.3265	0.0097	0.0011	0.0032	0.4611	0.6109
600760	0.3635	0.0936	0.0760	0.1311	1.0000	0.2673	0.1039	0.1072	0.4119
600900	0.3309	0.2613	0.9338	0.2601	0.0016	0.0005	0.0014	0.2587	0.5215
600998	0.1948	0.0994	0.0419	0.2237	0.0127	0.0048	0.0118	0.1200	0.7451
601006	0.2708	0.3603	0.4949	0.2314	0.0156	0.0065	0.0169	0.0513	0.6990
601021	0.3126	0.2151	0.2417	0.4062	0.0190	0.0025	0.0063	0.1298	0.7491
601088	0.3444	0.3348	0.4568	0.5820	0.0226	0.0063	0.0190	0.1811	0.7348
601766	0.2248	0.1218	0.1296	0.0977	− 0.0051	− 0.0005	− 0.0014	0.2946	0.4921
601877	0.3513	0.2383	0.2690	0.3445	0.0102	0.0011	0.0004	0.2998	0.3726
601888	0.4310	0.4924	0.2181	0.3332	0.0166	0.0034	− 0.0058	0.3033	0.7447

Table 8 continued

Stock codes	Outputs								
	Desirable							Undesirable	
	y_1^g	y_2^g	y_3^g	y_4^g	y_5^g	y_6^g	y_7^g	y_8^b	y_9^b
601898	0.0994	0.0628	0.1152	0.0463	0.0213	0.0039	0.0039	0.4009	0.8782
601899	0.2024	0.1277	0.0722	0.0411	0.0126	0.0070	0.0150	0.1169	0.9175
601919	0.4846	0.1274	0.1123	0.0668	0.0171	0.0117	0.0245	0.1478	0.4855
603986	0.5124	0.5424	0.4117	0.5116	0.0229	0.0097	− 0.0012	0.3400	0.3645

realized by a larger γ. For example, in the case of $d = 5$ and $\delta = 0.7$, the portfolio in Table 14 exhibits a 98.29% reduction in semivariance, while the return decreases by 20.84% with the value of γ set to 0.3. These conclusions are consistent with the evaluation results obtained for the case of equal weights.

6.4 Comparative Analysis

6.4.1 Comparison with Mashayekhi and Omrani [36]

For substantiating the proposed approach, we conduct a comparative analysis of the proposed approach with [36]. Although most of the inputs and outputs in both the approaches are similar, for the sake of a coherent comparison, the inputs "debt to equity ratio" and "leverage ratio" are dropped from [36], and the outputs "expense ratio during sales" (y_8^b) and "income tax / total profit" (y_9^b) are dropped from the proposed approach. Next, we apply our input–output data on Mashayekhi and Omrani's approach and redo a numerical illustration with the common set of inputs and outputs. The portfolio selection model (Model 9) in [36] is a multi-objective model with four objective functions. For the sake of preserving the coherency of comparison, we drop the two additional objective functions of the return and risk (variance) of the portfolio since, in our proposed approach, the cross-efficiencies and the semivariance of the cross-efficiencies of the DMUs serve as the expected return and risk of the portfolio, respectively. The remaining two objective functions are aggregated using the weighted sum approach with equal weights. The portfolio selection parameters used in [36] are used for both the approaches (viz., cardinality = 10, lower bound = 0.05, and upper bound = 0.2) and the values of α and γ are set as 1 and 0.1, respectively, for the proposed approach. Since crisp values of the inputs and outputs are used in [36], the value of α is set as 1 for the proposed approach to compare both the approaches on the same horizon. Subsequently, portfolio selection is performed for both equal and unequal weights for the proposed approach. The cross-efficiencies and the portfolios obtained from both the approaches are presented in Table 15.

From Table 15, we infer that the portfolios obtained for both equal and unequal weights using the proposed approach are more diverse in comparison with the portfolio obtained from Mashayekhi and Omrani's approach. Moreover, the cross-efficiency mean (risk) is also significantly greater (lesser) for both equal and unequal weights compared to [36]. Besides this, the proposed approach is more robust and dynamic because it employs the concept of desirable and undesirable inputs and outputs, which reflect the true performance of the DMUs. The proposed approach can also handle fuzzy inputs and outputs using the concept of $\alpha-$cuts. The above facts collectively substantiate the proposed approach.

6.4.2 Comparison with Chen et al. [12]

For a stronger validation, we further compare the proposed approach with Chen et al. [12]. Chen et al. use the same set of inputs and outputs as used in [36]. Therefore, on similar

Table 9 The lower, upper bounds at 11 levels and rankings

Stock codes	α											RI	Rank
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0		
000100													
L	− 1.2835	− 1.2835	− 1.2898	− 1.2916	− 1.2835	− 1.2855	− 1.2777	− 1.2777	− 1.2855	− 1.2793	− 1.2854	0.0227	48
U	−− 1.2855	− 1.2793	− 1.2793	− 1.2793	− 1.2793	− 1.2854	− 1.2793	− 1.2793	− 1.2855	− 1.2777	− 1.2854		
000333													
L	− 0.5810	− 0.5810	− 0.5912	− 0.6021	− 0.5810	− 0.5921	− 0.5838	− 0.5838	− 0.5921	− 0.5818	− 0.5919	0.3257	27
U	− 0.5921	− 0.5818	− 0.5818	− 0.5818	− 0.5818	− 0.5919	− 0.5818	− 0.5818	− 0.5921	− 0.5838	− 0.5919		
000425													
L	− 0.6058	− 0.6058	− 0.6140	− 0.6234	− 0.6058	− 0.6154	− 0.6091	− 0.6091	− 0.6154	− 0.6072	− 0.6152	0.3152	29
U	− 0.6154	− 0.6072	− 0.6072	− 0.6072	− 0.6072	− 0.6152	− 0.6072	− 0.6072	− 0.6154	− 0.6091	− 0.6152		
000630													
L	− 0.8121	− 0.8121	− 0.8162	− 0.8078	− 0.8121	− 0.8038	− 0.7912	− 0.7912	− 0.8038	− 0.7997	− 0.8037	0.2326	35
U	−− 0.8038	− 0.7997	− 0.7997	− 0.7997	− 0.7997	− 0.8037	− 0.7997	− 0.7997	− 0.8038	− 0.7912	− 0.8037		
000651													
L	− 0.4728	− 0.4728	− 0.4833	− 0.4934	− 0.4728	− 0.4830	− 0.4737	− 0.4737	− 0.4830	− 0.4725	− 0.4829	0.3733	16
U	− 0.4830	− 0.4725	− 0.4725	− 0.4725	− 0.4725	− 0.4829	− 0.4725	− 0.4725	− 0.4830	− 0.4737	− 0.4829		
000661													
L	− 1.0991	− 1.0991	− 1.0998	− 1.1147	− 1.0991	− 1.1139	− 1.1281	− 1.1281	− 1.1139	− 1.1131	− 1.1139	0.0955	46
U	− 1.1139	− 1.1131	− 1.1131	− 1.1131	− 1.1131	− 1.1139	− 1.1131	− 1.1131	− 1.1139	− 1.1281	− 1.1139		
000725													
L	− 0.8332	− 0.8332	− 0.8401	− 0.8441	− 0.8332	− 0.8373	− 0.8286	− 0.8286	− 0.8373	− 0.8303	− 0.8372	0.2183	38
U	− 0.8373	− 0.8303	− 0.8303	− 0.8303	− 0.8303	− 0.8372	− 0.8303	− 0.8303	− 0.8373	− 0.8286	− 0.8372		
000786													
L	− 0.1937	− 0.1937	− 0.1945	− 0.1948	− 0.1937	− 0.1939	− 0.1943	− 0.1943	− 0.1939	− 0.1931	− 0.1939	0.4975	3
U	− 0.1939	− 0.1931	− 0.1931	− 0.1931	− 0.1931	− 0.1939	− 0.1931	− 0.1931	− 0.1939	− 0.1943	− 0.1939		
000895													
L	− 0.1241	− 0.1241	− 0.1241	− 0.1173	− 0.1241	− 0.1173	− 0.1173	− 0.1173	− 0.1173	− 0.1173	− 0.1173	0.5305	2
U	− 0.1173	− 0.1173	− 0.1173	− 0.1173	− 0.1173	− 0.1173	− 0.1173	− 0.1173	− 0.1173	− 0.1173	− 0.1173		
000983													
L	− 1.3308	− 1.3308	− 1.3344	− 1.3308	− 1.3308	− 1.3273	− 1.3200	− 1.3200	− 1.3273	− 1.3236	− 1.3272	0.0040	50
U	− 1.3273	− 1.3236	− 1.3236	− 1.3236	− 1.3236	− 1.3272	− 1.3236	− 1.3236	− 1.3273	− 1.3200	− 1.3272		
001979													
L	− 0.8359	− 0.8359	− 0.8429	− 0.8389	− 0.8359	− 0.8320	− 0.8210	− 0.8210	− 0.8320	− 0.8250	− 0.8319	0.2205	37
U	− 0.8320	− 0.8250	− 0.8250	− 0.8250	− 0.8250	− 0.8319	− 0.8250	− 0.8250	− 0.8320	− 0.8210	− 0.8319		
002044													

Table 9 continued

Stock codes	α											*RI*	Rank
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0		
L	− 1.0863	− 1.0863	− 1.0886	− 1.0940	− 1.0863	− 1.0918	− 1.0948	− 1.0948	− 1.0918	− 1.0895	− 1.0917	0.1059	45
U	− 1.0918	− 1.0895	− 1.0895	− 1.0895	− 1.0895	− 1.0917	− 1.0895	− 1.0895	− 1.0918	− 1.0948	− 1.0917		
002085													
L	− 0.4368	− 0.4368	− 0.4392	− 0.4386	− 0.4368	− 0.4363	− 0.4321	− 0.4321	− 0.4363	− 0.4339	− 0.4362	0.3923	14
U	− 0.4363	− 0.4339	− 0.4339	− 0.4339	− 0.4339	− 0.4362	− 0.4339	− 0.4339	− 0.4363	− 0.4321	− 0.4362		
002294													
L	− 0.7261	− 0.7261	− 0.7228	− 0.7316	− 0.7261	− 0.7349	− 0.7484	− 0.7484	− 0.7349	− 0.7382	− 0.7349	0.2603	33
U	− 0.7349	− 0.7382	− 0.7382	− 0.7382	− 0.7382	− 0.7349	− 0.7382	− 0.7382	− 0.7349	− 0.7484	− 0.7349		
002311													
L	− 0.2583	− 0.2583	− 0.2634	− 0.2632	− 0.2583	− 0.2582	− 0.2503	− 0.2503	− 0.2582	− 0.2531	− 0.2581	0.4704	5
U	− 0.2582	− 0.2531	− 0.2531	− 0.2531	− 0.2531	− 0.2581	− 0.2531	− 0.2531	− 0.2582	− 0.2503	− 0.2581		
002508													
L	− 0.5711	− 0.5711	− 0.5741	− 0.5839	− 0.5711	− 0.5809	− 0.5852	− 0.5852	− 0.5809	− 0.5779	− 0.5809	0.3289	24
U	− 0.5809	− 0.5779	− 0.5779	− 0.5779	− 0.5779	− 0.5809	− 0.5779	− 0.5779	− 0.5809	− 0.5852	− 0.5809		
002624													
L	− 0.9132	− 0.9132	− 0.9200	− 0.9378	− 0.9132	− 0.9311	− 0.9354	− 0.9354	− 0.9311	− 0.9242	− 0.9310	0.1768	41
U	− 0.9311	− 0.9242	− 0.9242	− 0.9242	− 0.9242	− 0.9310	− 0.9242	− 0.9242	− 0.9311	− 0.9354	− 0.9310		
002625													
L	− 0.6151	−− 0.6151	− 0.6074	− 0.5998	− 0.6151	− 0.6074	− 0.6089	− 0.6089	− 0.6074	− 0.6151	− 0.6075	0.3161	28
U	− 0.6074	− 0.6151	− 0.6151	− 0.6151	− 0.6151	− 0.6075	− 0.6151	− 0.6151	− 0.6074	− 0.6089	− 0.6075		
300024													
L	− 0.5515	− 0.5515	− 0.5537	− 0.5579	− 0.5515	− 0.5557	− 0.5558	− 0.5558	− 0.5557	− 0.5535	− 0.5557	0.3400	21
U	− 0.5557	− 0.5535	− 0.5535	− 0.5535	− 0.5535	− 0.5557	− 0.5535	− 0.5535	− 0.5557	− 0.5558	− 0.5557		
300124													
L	− 0.5495	− 0.5495	− 0.5547	− 0.5675	− 0.5495	− 0.5623	− 0.5645	− 0.5645	− 0.5623	− 0.5571	− 0.5623	0.3375	22
U	− 0.5623	− 0.5571	− 0.5571	− 0.5571	− 0.5571	− 0.5623	− 0.5571	− 0.5571	− 0.5623	− 0.5645	− 0.5623		
300136													
L	− 0.3773	− 0.3773	− 0.3828	− 0.3865	− 0.3773	− 0.3812	− 0.3752	− 0.3752	− 0.3812	− 0.3758	− 0.3811	0.4168	10
U	− 0.3812	− 0.3758	− 0.3758	− 0.3758	− 0.3758	− 0.3811	− 0.3758	− 0.3758	− 0.3812	− 0.3752	− 0.3811		
300408													
L	− 0.2349	− 0.2349	− 0.2345	− 0.2356	− 0.2349	− 0.2359	− 0.2359	− 0.2359	− 0.2359	− 0.2363	− 0.2359	0.4792	4
U	− 0.2359	− 0.2363	− 0.2363	− 0.2363	− 0.2363	− 0.2359	− 0.2363	− 0.2363	− 0.2359	− 0.2359	− 0.2359		
600004													

Table 9 continued

Stock codes	α											*RI*	Rank
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0		
L	− 0.5923	− 0.5923	− 0.5908	− 0.5807	− 0.5923	− 0.5822	− 0.5780	− 0.5780	− 0.5822	− 0.5838	− 0.5823	0.3279	26
U	− 0.5822	− 0.5838	− 0.5838	− 0.5838	− 0.5838	− 0.5823	− 0.5838	− 0.5838	− 0.5822	− 0.5780	− 0.5823		
600010													
L	− 1.0898	− 1.0898	− 1.0955	− 1.0920	− 1.0898	− 1.0863	− 1.0749	− 1.0749	− 1.0863	− 1.0806	− 1.0863	0.1096	44
U	− 1.0863	− 1.0806	− 1.0806	− 1.0806	− 1.0806	− 1.0863	− 1.0806	− 1.0806	− 1.0863	− 1.0749	− 1.0863		
600018													
L	− 0.5815	− 0.5815	− 0.5853	− 0.5848	− 0.5815	− 0.5810	− 0.5750	− 0.5750	− 0.5810	− 0.5772	− 0.5810	0.3294	23
U	− 0.5810	− 0.5772	− 0.5772	− 0.5772	− 0.5772	− 0.5810	− 0.5772	− 0.5772	− 0.5810	− 0.5750	− 0.5810		
600023													
L	− 0.4026	− 0.4026	− 0.4067	− 0.4067	− 0.4026	− 0.4027	− 0.3957	− 0.3957	− 0.4027	− 0.3986	− 0.4026	0.4073	11
U	− 0.4027	− 0.3986	− 0.3986	− 0.3986	− 0.3986	− 0.4026	− 0.3986	− 0.3986	− 0.4027	− 0.3957	− 0.4026		
600100													
L	− 1.1954	− 1.1954	− 1.2015	− 1.2052	− 1.1954	− 1.1991	− 1.1929	− 1.1929	− 1.1991	− 1.1930	− 1.1990	0.0603	47
U	− 1.1991	− 1.1930	− 1.1930	− 1.1930	− 1.1930	− 1.1990	− 1.1930	− 1.1930	− 1.1991	− 1.1929	− 1.1990		
600104													
L	− 0.3533	− 0.3533	− 0.3632	− 0.3715	− 0.3533	−− 0.3619	− 0.3520	− 0.3520	− 0.3619	− 0.3520	− 0.3617	0.4261	9
U	− 0.3619	− 0.3520	− 0.3520	− 0.3520	− 0.3520	− 0.3617	− 0.3520	− 0.3520	− 0.3619	− 0.3520	− 0.3617		
600111													
L	− 0.5241	− 0.5241	− 0.5308	− 0.5354	− 0.5241	− 0.5289	− 0.5213	− 0.5213	− 0.5289	− 0.5222	− 0.5287	0.3527	19
U	− 0.5289	− 0.5222	− 0.5222	− 0.5222	− 0.5222	− 0.5287	− 0.5222	− 0.5222	− 0.5289	− 0.5213	− 0.5287		
600115													
L	− 0.8684	− 0.8684	− 0.8785	− 0.8827	− 0.8684	− 0.8728	− 0.8598	− 0.8598	− 0.8728	− 0.8627	− 0.8726	0.2035	39
U	− 0.8728	− 0.8627	− 0.8627	− 0.8627	− 0.8627	− 0.8726	− 0.8627	− 0.8627	− 0.8728	− 0.8598	− 0.8726		
600153													
L	− 0.9981	− 0.9981	− 1.0060	− 1.0024	− 0.9981	− 0.9947	− 0.9792	− 0.9792	− 0.9947	− 0.9868	− 0.9945	0.1502	43
U	− 0.9947	− 0.9868	− 0.9868	− 0.9868	− 0.9868	− 0.9945	− 0.9868	− 0.9868	− 0.9947	− 0.9792	− 0.9945		
600297													
L	− 0.7249	− 0.7249	− 0.7329	− 0.7335	− 0.7249	− 0.7256	− 0.7129	− 0.7129	− 0.7256	− 0.7176	− 0.7255	0.2673	32
U	− 0.7256	− 0.7176	− 0.7176	− 0.7176	− 0.7176	− 0.7255	− 0.7176	− 0.7176	− 0.7256	− 0.7129	− 0.7255		
600332													
L	− 0.7424	− 0.7424	− 0.7445	− 0.7519	− 0.7424	− 0.7498	− 0.7534	− 0.7534	− 0.7498	− 0.7476	− 0.7497	0.2551	34
U	− 0.7498	− 0.7476	− 0.7476	− 0.7476	− 0.7476	− 0.7497	− 0.7476	− 0.7476	− 0.7498	− 0.7534	− 0.7497		
600436													

Table 9 continued

Stock codes	α											*RI*	Rank
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0		
L	− 0.4915	− 0.4915	− 0.4905	− 0.4903	− 0.4915	− 0.4913	− 0.4936	− 0.4936	− 0.4913	− 0.4923	− 0.4913	0.3675	17
U	− 0.4913	− 0.4923	− 0.4923	− 0.4923	− 0.4923	− 0.4913	− 0.4923	− 0.4923	− 0.4913	− 0.4936	− 0.4913		
600487													
L	− 0.4151	− 0.4151	− 0.4247	− 0.4333	− 0.4151	− 0.4238	− 0.4144	− 0.4144	− 0.4238	− 0.4143	− 0.4237	0.3990	13
U	− 0.4238	− 0.4143	− 0.4143	− 0.4143	− 0.4143	− 0.4237	− 0.4143	− 0.4143	− 0.4238	− 0.4144	− 0.4237		
600516													
L	− 0.0418	− 0.0418	− 0.0418	− 0.0418	− 0.0418	− 0.0418	− 0.0422	− 0.0422	− 0.0418	− 0.0418	− 0.0418	0.5638	1
U	− 0.0418	− 0.0418	− 0.0418	− 0.0418	− 0.0418	− 0.0418	− 0.0418	− 0.0418	− 0.0418	− 0.0422	− 0.0418		
600535													
L	− 0.9255	− 0.9255	− 0.9323	− 0.9403	− 0.9255	− 0.9336	− 0.9298	− 0.9298	− 0.9336	− 0.9269	− 0.9335	0.1760	42
U	− 0.9336	− 0.9269	− 0.9269	− 0.9269	− 0.9269	− 0.9335	− 0.9269	− 0.9269	− 0.9336	− 0.9298	− 0.9335		
600760													
L	− 0.4030	− 0.4030	− 0.4152	− 0.4237	− 0.4030	− 0.4117	− 0.3974	− 0.3974	− 0.4117	− 0.3995	− 0.4115	0.4049	12
U	− 0.4117	− 0.3995	− 0.3995	− 0.3995	− 0.3995	− 0.4115	− 0.3995	− 0.3995	− 0.4117	− 0.3974	− 0.4115		
600900													
L	− 0.4344	− 0.4344	− 0.4413	− 0.4459	− 0.4344	− 0.4391	− 0.4320	− 0.4320	− 0.4391	− 0.4322	− 0.4390	0.3918	15
U	− 0.4391	− 0.4322	− 0.4322	− 0.4322	− 0.4322	− 0.4390	− 0.4322	− 0.4322	− 0.4391	− 0.4320	− 0.4390		
600998													
L	− 0.7148	− 0.7148	− 0.7210	− 0.7192	− 0.7148	− 0.7130	− 0.7011	− 0.7011	− 0.7130	− 0.7068	− 0.7129	0.2725	31
U	− 0.7130	− 0.7068	− 0.7068	− 0.7068	− 0.7068	− 0.7129	− 0.7068	− 0.7068	− 0.7130	− 0.7011	− 0.7129		
601006													
L	− 0.3632	− 0.3632	− 0.3603	− 0.3490	− 0.3632	− 0.3518	− 0.3481	− 0.3481	− 0.3518	− 0.3547	− 0.3519	0.4283	8
U	− 0.3518	− 0.3547	− 0.3547	− 0.3547	− 0.3547	− 0.3519	− 0.3547	− 0.3547	− 0.3518	− 0.3481	− 0.3519		
601021													
L	− 0.3543	− 0.3543	− 0.3597	− 0.3573	− 0.3543	− 0.3520	− 0.3422	− 0.3422	− 0.3520	− 0.3467	− 0.3520	0.4296	7
U	− 0.3520	− 0.3467	− 0.3467	− 0.3467	− 0.3467	− 0.3520	− 0.3467	− 0.3467	− 0.3520	− 0.3422	− 0.3520		
601088													
L	− 0.5485	− 0.5485	− 0.5483	− 0.5415	− 0.5485	− 0.5418	− 0.5379	− 0.5379	− 0.5418	− 0.5421	− 0.5418	0.3458	20
U	− 0.5418	− 0.5421	− 0.5421	− 0.5421	− 0.5421	− 0.5418	− 0.5421	− 0.5421	− 0.5418	− 0.5379	− 0.5418		
601766													
L	− 0.8067	− 0.8067	− 0.8157	− 0.8240	− 0.8067	− 0.8152	− 0.8070	− 0.8070	− 0.8152	− 0.8062	− 0.8150	0.2283	36
U	− 0.8152	− 0.8062	− 0.8062	− 0.8062	− 0.8062	− 0.8150	− 0.8062	− 0.8062	− 0.8152	− 0.8070	− 0.8150		
601877													

Table 9 continued

Stock codes	α											*RI*	Rank
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0		
L	− 0.5081	− 0.5081	− 0.5164	− 0.5262	− 0.5081	− 0.5180	− 0.5120	− 0.5120	− 0.5180	− 0.5098	− 0.5179	0.3576	18
U	− 0.5180	− 0.5098	− 0.5098	− 0.5098	− 0.5098	− 0.5179	− 0.5098	− 0.5098	− 0.5180	− 0.5120	− 0.5179		
601888													
L	− 0.5847	− 0.5847	− 0.5830	− 0.5782	− 0.5847	− 0.5798	− 0.5799	− 0.5799	− 0.5798	− 0.5815	− 0.5798	0.3288	25
U	− 0.5798	− 0.5815	− 0.5815	− 0.5815	− 0.5815	− 0.5798	− 0.5815	− 0.5815	− 0.5798	− 0.5799	− 0.5798		
601898													
L	− 1.2923	− 1.2923	− 1.2959	− 1.2951	− 1.2923	− 1.2916	− 1.2868	− 1.2868	− 1.2916	− 1.2880	− 1.2916	0.0194	49
U	− 1.2916	− 1.2880	− 1.2880	− 1.2880	− 1.2880	− 1.2916	− 1.2880	− 1.2880	− 1.2916	− 1.2868	− 1.2916		
601899													
L	− 0.8974	− 0.8974	− 0.9007	− 0.8930	− 0.8974	− 0.8898	− 0.8798	− 0.8798	− 0.8898	− 0.8865	− 0.8897	0.1948	40
U	− 0.8898	− 0.8865	− 0.8865	− 0.8865	− 0.8865	− 0.8897	− 0.8865	− 0.8865	− 0.8898	− 0.8798	− 0.8897		
601919													
L	− 0.6590	− 0.6590	− 0.6692	− 0.6757	− 0.6590	− 0.6657	− 0.6533	− 0.6533	− 0.6657	− 0.6555	− 0.6655	0.2938	30
U	− 0.6657	− 0.6555	− 0.6555	− 0.6555	− 0.6555	− 0.6655	− 0.6555	− 0.6555	− 0.6657	− 0.6533	− 0.6655		
603986													
L	− 0.2893	− 0.2893	− 0.2927	− 0.2984	− 0.2893	− 0.2952	− 0.2951	− 0.2951	− 0.2952	− 0.2918	− 0.2951	0.4537	6
U	− 0.2952	− 0.2918	− 0.2918	− 0.2918	− 0.2918	− 0.2951	− 0.2918	− 0.2918	− 0.2952	− 0.2951	− 0.2951		

Table 10 The portfolio strategies with different values of d using traditional method

Stock codes	$d = 15$	$d = 10$	$d = 5$	Stock codes	$d = 15$	$d = 10$	$d = 5$
000100	0	0	0	600023	0.0667	0	0
000333	0	0	0	600100	0	0	0
000425	0	0	0	600104	0.0667	0.1	0
000630	0	0	0	600111	0	0	0
000651	0	0	0	600115	0	0	0
000661	0	0	0	600153	0	0	0
000725	0	0	0	600297	0	0	0
000786	0.0667	0.1	0.2	600332	0	0	0
000895	0.0667	0.1	0.2	600436	0	0	0
000983	0	0	0	600487	0.0667	0	0
001979	0	0	0	600516	0.0667	0.1	0.2
002044	0	0	0	600535	0	0	0
002085	0.0667	0	0	600760	0.0667	0	0
002294	0	0	0	600900	0.0667	0	0
002311	0.0667	0.1	0.2	600998	0	0	0
002508	0	0	0	601006	0.0667	0.1	0
002624	0	0	0	601021	0.0667	0.1	0
002625	0	0	0	601088	0	0	0
300024	0	0	0	601766	0	0	0
300124	0	0	0	601877	0	0	0
300136	0.0667	0.1	0	601888	0	0	0
300408	0.0667	0.1	0.2	601898	0	0	0
600004	0	0	0	601899	0	0	0
600010	0	0	0	601919	0	0	0
600018	0	0	0	603986	0.0667	0.1	0
Mean	0.44608 (100%)	0.46959 (100%)	0.50828 (100%)	Semivariance	0.00087 (100%)	0.00082 (100%)	0.00048 (100%)

lines of the previous comparison with [36], we use only the common set of inputs and outputs for a coherent comparison with the proposed approach. The portfolio selection model (Model 10) in [12] is a multi-objective portfolio selection model. Therefore, we drop the additional objective functions of the expected return and semivariance of the portfolio from Model 10. Since both Chen et al. [12] and Mashayekhi and Omrani [36] have used the same DEA model for calculating the cross-efficiencies of the DMUs, similar cross-efficiencies are obtained upon using our input–output data as obtained in the previous comparison with [36] (see Table 15). The same values of the cardinality, lower and upper bounds are used as given in [12], viz., 8, 0.05 and 0.2, respectively. The portfolio obtained from Chen et al.'s approach and the proposed approach for both equal and unequal weights (for $\gamma = 0.1$) are presented in Table 15.

Table 11 The optimal portfolio strategies with different values of d using the proposed model

Stock codes	$d = 15$			$d = 10$			$d = 5$		
	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$
000100	0	0	0	0	0	0	0	0	0
000333	0	0	0.0667	0	0	0	0	0	0
000425	0	0	0.0667	0	0	0	0	0	0
000630	0	0	0	0	0	0	0	0	0
000651	0.0667	0.0667	0.0667	0	0.1	0.1	0	0	0.2
000661	0	0	0	0	0	0	0	0	0
000725	0	0	0	0	0	0	0	0	0
000786	0	0	0	0.1	0	0	0.2	0	0
000895	0	0	0	0	0	0	0.2	0	0
000983	0	0	0	0	0	0	0	0	0
001979	0	0	0	0	0	0	0	0	0
002044	0	0	0	0	0	0	0	0	0
002085	0.0667	0.0667	0	0	0.1	0	0	0	0.2
002294	0	0	0	0	0	0	0	0	0
002311	0.0667	0	0	0.1	0	0	0.2	0.2	0
002508	0	0.0667	0.0667	0	0	0.1	0	0	0
002624	0	0	0	0	0	0	0	0	0
002625	0	0	0.0667	0	0	0	0	0	0
300024	0	0.0667	0.0667	0	0	0.1	0	0	0
300124	0	0.0667	0.0667	0	0	0.1	0	0	0
300136	0.0667	0	0	0.1	0.1	0	0	0	0
300408	0	0	0	0.1	0	0	0.2	0	0
600004	0	0.0667	0.0667	0	0	0	0	0	0
600010	0	0	0	0	0	0	0	0	0
600018	0	0.0667	0.0667	0	0	0.1	0	0	0
600023	0.0667	0	0	0.1	0.1	0	0	0	0
600100	0	0	0	0	0	0	0	0	0
600104	0.0667	0	0	0.1	0.1	0	0	0.2	0
600111	0.0667	0.0667	0.0667	0	0	0.1	0	0	0
600115	0	0	0	0	0	0	0	0	0
600153	0	0	0	0	0	0	0	0	0
600297	0	0	0	0	0	0	0	0	0
600332	0	0	0	0	0	0	0	0	0
600436	0.0667	0.0667	0.0667	0	0.1	0.1	0	0	0
600487	0.0667	0.0667	0	0	0.1	0	0	0	0.2
600516	0	0	0	0	0	0	0	0	0
600535	0	0	0	0	0	0	0	0	0
600760	0.0667	0.0667	0	0.1	0.1	0	0	0	0.2
600900	0.0667	0.0667	0.0667	0	0.1	0	0	0	0.2
600998	0	0	0	0	0	0	0	0	0
601006	0.0667	0	0	0.1	0.1	0	0	0.2	0
601021	0.0667	0	0	0.1	0	0	0	0.2	0
601088	0	0.0667	0.0667	0	0	0.1	0	0	0
601766	0	0	0	0	0	0	0	0	0
601877	0.0667	0.0667	0.0667	0	0	0.1	0	0	0
601888	0	0.0667	0.0667	0	0	0.1	0	0	0
601898	0	0	0	0	0	0	0	0	0

Table 11 continued

Stock codes	$d = 15$			$d = 10$			$d = 5$		
	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$
601899	0	0	0	0	0	0	0	0	0
601919	0	0	0	0	0	0	0	0	0
603986	0.0667	0	0	0.1	0	0	0.2	0.2	0
Mean	0.40475	0.35849	0.34255	0.44138	0.40073	0.34615	0.48626	0.44162	0.39226
	(90.74%)	(80.37%)	(76.79%)	(93.99%)	(85.34%)	(73.71%)	(95.67%)	(86.89%)	(77.17%)
Semivariance	0.00051	0.00030	0.00017	0.00036	0.00020	9.91915e-05	0.00027	0.00011	7.19386e−05
	(58.52%)	(34.49%)	(19.28%)	(44.20%)	(24.41%)	(12.05%)	(56.81%)	(23.48%)	(15.01%)

Table 12 The optimal portfolio strategies with unequal weights in the case of $d = 15$

Stock codes	$\gamma = 0.1$			$\gamma = 0.2$			$\gamma = 0.3$		
	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$
000100	0	0	0	0	0	0	0	0	0
000333	0	0	0	0	0	0.0041	0	0	0
000425	0	0	0	0	0	0.0039	0.0030	0.0036	0
000630	0	0	0	0	0	0	0	0	0
000651	0.0068	0	0	0	0.3139	0.0050	0	0	0.0129
000661	0	0	0	0	0	0	0	0	0
000725	0	0	0	0	0	0	0	0	0
000786	0	0	0.0190	0	0.0023	0	0	0	0
000895	0	0	0.0017	0.0269	0	0	0	0	0
000983	0	0	0	0	0	0	0	0	0
001979	0	0	0	0	0	0	0	0	0
002044	0	0	0	0	0	0	0	0	0
002085	0.0088	0	0	0.0112	0	0	0	0.0762	0.0110
002294	0	0	0	0	0	0	0	0	0
002311	0.0041	0.0029	0	0	0	0	0	0	0
002508	0	0	0	0	0	0.0037	0.0038	0.0045	0
002624	0	0	0	0	0	0	0	0	0
002625	0	0.0019	0	0	0	0	0	0	0
300024	0	0	0	0	0.0064	0	0.2051	0.0143	0.0060
300124	0	0	0.0094	0	0.0186	0	0	0	0.1345
300136	0.0022	0.1180	0.5002	0	0	0	0	0	0.0027
300408	0	0.0042	0.0046	0	0	0.0031	0	0	0
600004	0	0	0	0.0022	0	0.0060	0.2587	0.0295	0
600010	0	0	0	0	0	0	0	0	0
600018	0	0	0	0.1162	0	0	0.1130	0.0079	0
600023	0.1553	0	0	0	0.1078	0	0	0.0030	0.0048
600100	0	0	0	0	0	0	0	0	0
600104	0.0158	0.2056	0	0.0083	0	0	0.0156	0	0.0061
600111	0.0041	0.0042	0.0021	0.1901	0.0075	0	0	0.0148	0.5166
600115	0	0	0	0	0	0	0	0	0
600153	0	0	0	0	0	0	0	0	0

Table 12 continued

Stock codes	$\gamma = 0.1$			$\gamma = 0.2$			$\gamma = 0.3$		
	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$
600297	0	0	0	0	0	0	0	0	0
600332	0	0	0	0	0	0	0	0	0
600436	0.2092	0.0329	0	0	0.0330	0.0052	0.0331	0.0212	0.0159
600487	0.0096	0.0050	0	0.0346	0.0050	0.5298	0	0.0417	0.0121
600516	0	0	0.0017	0	0	0	0.0042	0	0
600535	0	0	0	0	0	0	0	0	0
600760	0.0082	0.0275	0.0294	0	0.0713	0.0131	0.0201	0.0437	0.0072
600900	0.0868	0.0258	0.2795	0.1073	0.1267	0.3120	0.0562	0.0116	0
600998	0	0	0	0	0	0	0.0032	0	0
601006	0.0916	0.0083	0.1185	0.0068	0.0197	0.0555	0.0050	0	0.0564
601021	0.2851	0.4251	0.0017	0.0051	0.0248	0.0408	0	0	0
601088	0.0015	0	0.0049	0.1682	0.0077	0.0031	0	0.4676	0.0034
601766	0	0	0	0.0030	0	0	0	0	0
601877	0	0.0019	0.0164	0.2801	0.2361	0	0.0031	0.1473	0.1452
601888	0	0	0.0035	0.0320	0	0.0105	0.2542	0.1133	0.0650
601898	0	0	0	0	0	0	0	0	0
601899	0	0	0	0	0	0	0	0	0
601919	0	0.0100	0	0	0	0.0044	0	0	0
603986	0.1110	0.1267	0.0075	0.0080	0.0193	0	0.0218	0	0
Mean	0.41086	0.42485	0.41088	0.36234	0.38086	0.39699	0.34288	0.35431	0.35674
	(92.10%)	(95.24%)	(92.11%)	(81.23%)	(85.38%)	(88.99%)	(76.86%)	(79.43%)	(79.97%)
Semivariance	0.00046	0.00038	0.00025	0.00029	0.00021	0.00021	0.00015	0.00014	0.00011
	(52.82%)	(43.94%)	(28.75%)	(33.14%)	(24.35%)	(24.23%)	(17.10%)	(16.53%)	(12.72%)

From the results in Table 15, we infer that the return (risk) for the portfolios obtained using the proposed approach for both equal and unequal weights are significantly greater (lesser) than obtained using Chen et al.'s approach. Note that Chen et al. [12] have used the variance of the cross-efficiency of the risk measure. For similar reasons mentioned in the previous comparison, the proposed approach is superior to [12].

7 Conclusion

This paper discussed an FDEA model for assets evaluation, wherein it is possible to consider both undesirable inputs and outputs simultaneously. Furthermore, we developed a novel FDEA cross-efficiency evaluation-based mean-semivariance model for portfolio selection. To illustrate the proposed approach, a case study based on 50 firms was

Table 13 The optimal portfolio strategies with unequal weights in the case of $d = 10$

Stock codes	$\gamma = 0.1$			$\gamma = 0.2$			$\gamma = 0.3$		
	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$
000100	0	0	0	0	0	0	0	0	0
000333	0	0	0	0	0	0	0	0	0
000425	0	0	0	0	0	0	0	0	0
000630	0	0	0	0	0	0	0	0	0
000651	0.0250	0	0	0	0	0	0.2973	0.0667	0.6478
000661	0	0	0	0	0	0	0	0	0
000725	0	0	0	0	0	0	0	0	0
000786	0	0	0	0	0	0	0	0	0
000895	0	0	0	0	0	0	0	0	0
000983	0	0	0	0	0	0	0	0	0
001979	0	0	0	0	0	0	0	0	0
002044	0	0	0	0	0	0	0	0	0
002085	0	0	0	0	0	0	0.0520	0.0176	0.0835
002294	0	0	0	0	0	0	0	0	0
002311	0	0	0	0	0	0	0	0	0
002508	0	0	0	0	0	0	0	0	0
002624	0	0	0	0	0	0	0	0	0
002625	0	0	0	0	0	0	0	0	0
300024	0	0	0	0	0	0	0	0.0567	0.0053
300124	0	0	0	0	0	0.0075	0.0048	0.0023	0
300136	0.0030	0	0	0.2974	0	0	0	0	0
300408	0.1192	0	0	0	0	0.0137	0	0	0
600004	0	0	0	0	0	0	0	0	0
600010	0	0	0	0	0	0	0	0	0
600018	0	0	0.0050	0	0	0	0	0	0
600023	0.1027	0	0.0537	0.0154	0.2149	0	0	0	0
600100	0	0	0	0	0	0	0	0	0
600104	0.0017	0.0312	0.0018	0.0097	0	0	0	0	0.0053
600111	0	0	0	0.0222	0	0	0.0493	0.4752	0.0118
600115	0	0	0	0	0	0	0	0	0
600153	0	0	0	0	0	0	0	0	0
600297	0	0	0	0	0	0	0	0	0
600332	0	0	0	0	0	0	0	0	0
600436	0	0	0	0	0.0037	0	0.1398	0.0906	0.1100
600487	0.0034	0.0138	0	0.1274	0.0031	0.2626	0.0132	0.0113	0
600516	0	0.0872	0	0	0	0	0	0	0
600535	0	0	0	0	0	0	0	0	0
600760	0.0947	0.0016	0.0075	0.2181	0.4448	0.5668	0.0079	0	0.0077
600900	0	0.0016	0.0413	0.0125	0.0039	0.1019	0.2911	0.0279	0.0404
600998	0	0.0016	0	0	0	0	0	0	0
601006	0.1497	0.3319	0.2834	0.1814	0.2325	0.0072	0	0	0
601021	0.2206	0.4473	0.5369	0	0.0590	0.0029	0	0	0
601088	0	0.0022	0.0041	0	0.0054	0	0.0301	0.0816	0.0118
601766	0	0	0	0	0	0	0	0	0
601877	0	0	0	0.0104	0	0.0102	0.1144	0.1701	0.0765
601888	0	0	0.0040	0	0	0.0089	0	0	0
601898	0	0	0	0	0	0	0	0	0

Table 13 continued

Stock codes	$\gamma = 0.1$			$\gamma = 0.2$			$\gamma = 0.3$		
	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$
601899	0	0	0	0	0.0015	0	0	0	0
601919	0	0	0	0	0	0	0	0	0
603986	0.2799	0.0814	0.0624	0.1055	0.0313	0.0183	0	0	0
Mean	0.43588	0.44176	0.42654	0.41551	0.41300	0.40246	0.37564	0.35724	0.37357
	(92.82%)	(94.07%)	(90.83%)	(88.48%)	(87.95%)	(85.71%)	(79.99%)	(76.08%)	(79.55%)
Semivariance	0.00030	0.00023	0.00019	0.00019	0.00014	0.00012	0.00011	3.82359e-05	4.37873e-05
	(35.91%)	(28.22%)	(22.51%)	(23.05%)	(17.13%)	(13.99%)	(13.10%)	(4.64%)	(5.32%)

Table 14 The optimal portfolio strategies with unequal weights in the case of $d = 5$

Stock codes	$\gamma = 0.1$			$\gamma = 0.2$			$\gamma = 0.3$		
	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$
000100	0	0	0	0	0	0	0	0	0
000333	0	0	0	0	0	0	0	0	0
000425	0	0	0	0	0.0033	0	0	0	0
000630	0	0	0	0	0	0	0	0	0
000651	0	0	0	0	0	0	0	0	0
000661	0	0	0	0	0	0	0	0	0
000725	0	0	0	0	0	0	0	0	0
000786	0.1862	0	0	0	0	0	0	0	0
000895	0	0	0	0	0	0	0	0	0
000983	0	0	0	0	0	0	0	0	0
001979	0	0	0	0	0	0	0	0	0
002044	0	0	0	0	0	0	0	0	0
002085	0	0	0	0	0	0	0	0.3365	0.0168
002294	0	0	0	0	0	0	0	0	0
002311	0.2851	0	0.5213	0	0	0	0	0	0
002508	0	0	0	0	0	0	0	0	0
002624	0	0	0	0	0	0	0	0	0
002625	0	0	0	0	0	0	0	0	0
300024	0	0	0	0	0	0	0	0	0
300124	0	0	0	0	0	0	0	0	0
300136	0	0.0041	0	0.2235	0	0	0	0	0
300408	0.1865	0.4058	0	0	0	0	0	0	0
600004	0	0	0	0	0	0	0	0	0
600010	0	0	0	0	0	0	0	0	0
600018	0	0	0	0	0	0	0	0	0
600023	0	0.0183	0	0	0	0	0.2481	0.2661	0.0029
600100	0	0	0	0	0	0	0	0	0
600104	0	0	0.0108	0	0.0112	0.5549	0	0	0
600111	0	0	0	0	0	0	0	0	0
600115	0	0	0	0	0	0	0	0	0
600153	0	0	0	0	0	0	0	0	0

Table 14 continued

Stock codes	$\gamma = 0.1$			$\gamma = 0.2$			$\gamma = 0.3$		
	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 0.7$
600297	0	0	0	0	0	0	0	0	0
600332	0	0	0	0	0	0	0	0	0
600436	0	0	0	0	0	0.0028	0	0	0
600487	0	0	0	0	0	0.0157	0.2481	0.0335	0.5728
600516	0	0	0	0	0	0	0	0	0.0066
600535	0	0	0	0	0	0	0	0	0
600760	0	0	0	0.1378	0	0	0.1585	0	0.4009
600900	0	0	0	0	0	0.0145	0.2481	0.2661	0
600998	0	0	0	0	0	0	0	0	0
601006	0	0.1060	0.0437	0.2235	0.3285	0.4121	0	0	0
601021	0.1060	0	0	0.2235	0.3285	0	0	0.0979	0
601088	0	0	0	0	0	0	0	0	0
601766	0	0	0	0	0	0	0	0	0
601877	0	0	0.0065	0	0	0	0	0	0
601888	0	0	0	0	0	0	0	0	0
601898	0	0	0	0	0	0	0	0	0
601899	0	0	0	0	0	0	0	0	0
601919	0	0	0	0	0	0	0	0	0
603986	0.2362	0.4658	0.4177	0.1916	0.3285	0	0.0970	0	0
Mean	0.46882 (92.24%)	0.46036 (90.57%)	0.46038 (90.58%)	0.42766 (84.14%)	0.43668 (85.91%)	0.42592 (83.80%)	0.40552 (79.78%)	0.40003 (78.70%)	0.40237 (79.16%)
Semivariance	0.00022 (45.27%)	0.00019 (39.38%)	0.00014 (30.20%)	9.77829e-05 (20.40%)	8.92137e-05 (18.61%)	3.77478e-05 (7.87%)	5.72717e-05 (11.95%)	3.81883e-05 (7.97%)	8.19042e-06 (1.71%)

considered. The numerical results showed that the obtained portfolios using the proposed model have a remarkable reduction in semivariance and a slight reduction in mean than those obtained through the traditional method. Also, a more significant decrease in semivariance can be realized by a larger γ. The obtained portfolio selection strategies empirically support the effectiveness of the proposed approach for stock portfolio selection. The proposed model is capable of generating portfolios per the preferences of the investor corresponding to d, γ, ε_i, and δ_i. If the investor is not satisfied with the obtained portfolios, more portfolios can be generated by varying different model parameters such as d, γ, ε_i, and δ_i. The obtained portfolios are not only efficient but also in line with the preferences of the

Table 15 Comparison with Mashayekhi and Omrani [36] and Chen et al. [12]

Stock codes	Cross-efficiency		Portfolio					
	Chen et al. [12] and Mashayekhi and Omrani [36]	Proposed approach	Mashayekhi and Omrani [36]	Proposed approach for $\gamma = 0.1$ Equal weights	Proposed approach for $\gamma = 0.1$ Unequal weights	Chen et al. [12]	Proposed approach for $\gamma = 0.1$ Equal weights	Proposed approach for $\gamma = 0.1$ Unequal weights
000100	0.02638	0.07811	0	0.0	0	0	0	0
000333	0.38708	0.24478	0	0.0	0	0	0	0
000425	0.78224	0.3067	0	0.0	0	0	0	0
000630	0.17423	0.17875	0	0.0	0	0	0	0
000651	0.68045	0.26905	0	0.0	0	0	0	0
000661	0.75546	0.71968	0	0.1	0.1	0	0.125	0.2
000725	0.62870	0.20961	0	0.0	0	0	0	0
000786	0.38935	0.69392	0	0.1	0.05	0.05	0.125	0.05
000895	0.11851	0.61898	0	0.1	0.05	0	0	0
000983	0.58461	0.17269	0	0.0	0	0	0	0
001979	0.16944	0.1224	0	0.0	0	0	0	0
002044	0.12708	0.44357	0	0.0	0	0	0	0
002085	−0.03424	0.49354	0.2	0.0	0	0	0	0
002294	0.30023	0.81686	0	0.1	0.2	0.2	0.125	0.2
002311	0.10350	0.38432	0	0.0	0	0	0	0
002508	0.25925	0.57318	0	0.0	0	0	0	0
002624	0.19804	0.39612	0	0.0	0	0	0	0
002625	−0.28007	0.75764	0	0.1	0.2	0	0.125	0.2
300024	0.02290	0.55108	0.2	0.0	0	0	0	0
300124	0.12851	0.49327	0.1319	0.0	0	0	0	0
300136	0.31659	0.39877	0	0.0	0	0.05	0	0
300408	0.12347	0.6736	0.2	0.1	0.05	0.2	0.125	0.05
600004	0.02782	0.57173	0	0.0	0	0	0	0
600010	3.76719	0.13933	0	0.0	0	0	0	0
600018	0.23811	0.3844	0.2	0.0	0	0.05	0	0
600023	−0.09093	0.45189	0	0.0	0	0	0	0
600100	−0.37168	0.15469	0	0.0	0	0	0	0
600104	0.39091	0.30197	0	0.0	0	0	0	0
600111	0.74338	0.34936	0	0.0	0	0	0	0
600115	−0.02676	0.0381	0	0.0	0	0	0	0
600153	0.02183	0.00087	0	0.0	0	0	0	0
600297	−0.07181	0.12239	0	0.0	0	0	0	0
600332	0.20544	0.56033	0	0.0	0	0	0	0
600436	0.36687	0.69788	0	0.1	0.05	0.06	0.125	0.05
600487	0.32018	0.26406	0	0.0	0	0	0	0
600516	12.89852	0.80563	0	0.1	0.2	0	0.125	0.2
600535	0.15932	0.26756	0	0.0	0	0	0	0
600760	1.44056	0.56811	0.0681	0.0	0	0.2	0	0
600900	0.15216	0.34593	0	0.0	0	0	0	0
600998	0.13142	0.20073	0	0.0	0	0	0	0
601006	0.36917	0.68831	0	0.1	0.05	0.194	0.125	0.05
601021	0.17798	0.39142	0	0.0	0	0	0	0
601088	0.61954	0.59887	0	0.0	0	0	0	0
601766	−0.08138	0.16207	0	0.0	0	0	0	0

Table 15 continued

Stock codes	Cross-efficiency		Portfolio					
	Chen et al. [12] and Mashayekhi and Omrani [36]	Proposed approach	Mashayekhi and Omrani [36]	Proposed approach for $\gamma = 0.1$		Chen et al. [12]	Proposed approach for $\gamma = 0.1$	
				Equal weights	Unequal weights		Equal weights	Unequal weights
601877	0.16176	0.33009	0	0.0	0	0	0	0
601888	0.11414	0.62144	0	0.1	0.05	0	0	0
601898	−0.04086	0.22424	0	0.0	0	0	0	0
601899	0.06737	0.23209	0	0.0	0	0	0	0
601919	0.17243	0.09329	0	0.0	0	0	0	0
603986	0.33768	0.61452	0	0.0	0	0	0	0
Return			0.18513	0.70939	0.7477	0.51222	0.73169	0.75765
Risk			0.15477*	0.00180**	0.00242**	0.88933*	0.00099**	0.00295**
Sharpe ratio						0.57596	-	-

*Variance of the cross-efficiencies; **Semivariance of the cross-efficiencies

investor. For future research, we would like to extend the research by considering nonfinancial indicators in the FDEA model to incorporate more realistic situations of performance evaluation. Besides, the inputs and outputs can also be considered as trapezoidal fuzzy variables. The proposed mean-semivariance model can also be extended to a multiperiod one.

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Compliance with ethical standards

Conflicts of interest The authors declare no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Appendix

One company among 50 (000100) has been chosen to represent the numerical findings step by step (till the fuzzification).

Step 1 The normalization of data. The normalized data are defined as the initial data divided by max data, where max data are the max one among the 50 firms. For example, 75.1529 is the max Solvency ratio-I (x_1^g) of the 50 firms.

Step 2 The fuzzification of data. The obtained normalized data are represented by a^m, and the corresponding TFN is represented by (a^l, a^m, a^u). As noted in [38], a^l and a^u values are obtained by subtracting and adding 20% of a^m from a^m, respectively.

Data preprocessing of the input data (000100)

Data	Inputs						
	Desirable		Undesirable				
	x_1^g	x_2^g	x_3^b	x_4^b	x_5^b	x_6^b	x_7^b
Initial data	66.2227	356.8454	778.8013	688.0809	72.5869	110.8627	92.9434
Max data	75.1529	556.9132	42773.8981	11993.8708	277.6880	1416.5716	1403.3835
Normalized data	0.8812	0.6408	0.0182	0.0574	0.2614	0.0783	0.0662
a^m	0.8812	0.6408	0.0182	0.0574	0.2614	0.0783	0.0662
a^l	0.7049	0.5126	0.0146	0.0459	0.2091	0.0626	0.0530
a^u	1.0574	0.7689	0.0218	0.0688	0.3137	0.0939	0.0795

References

1. Avkiran, N.K.: Investigating technical and scale efficiencies of Australian universities through data envelopment analysis. Socio-Econ. Plan. Sci. **35**(1), 57–80 (2001)
2. Azadeh, A., Moghaddam, M., Asadzadeh, S.M., Negahban, A.: An integrated fuzzy simulation-fuzzy data envelopment analysis algorithm for job-shop layout optimization: the case of injection process with ambiguous data. Eur. J. Oper. Res. **214**(3), 768–779 (2011)
3. Bacanin, N., Tuba, M.: Firefly algorithm for cardinality constrained mean-variance portfolio optimization problem with entropy diversity constraint. Sci. World J. **2014**, 1–16 (2014)
4. Ballestero, E.: Mean-semivariance efficient frontier: a downside risk model for portfolio selection. Appl. Math. Finan. **12**(1), 1–15 (2005)
5. Barros, C.P., Managi, S., Matousek, R.: The technical efficiency of the Japanese banks: non-radial directional performance measurement with undesirable output. Omega **40**(1), 1–8 (2012)
6. Basso, A., Funari, S.: Constant and variable returns to scale DEA models for socially responsible investment funds. Eur. J. Oper. Res. **235**(3), 775–783 (2014)
7. Basso, A., Funari, S.: The role of fund size in the performance of mutual funds assessed with DEA models. Eur. J. Finan. **23**(6), 457–473 (2017)
8. Branda, M.: Diversification-consistent data envelopment analysis with general deviation measures. Eur. J. Oper. Res. **226**(3), 626–635 (2013)
9. Charnes, A., Cooper, W.W., Rhodes, E.: Measuring the efficiency of decision makingunits. Eur. J. Oper. Res. **2**(6), 429–444 (1978)
10. Chen, Y.C., Chiu, Y.H., Huang, C.W., Tu, C.H.: The analysis of bank business performance and market risk-Applying Fuzzy DEA. Econ. Model. **32**, 225–232 (2013)
11. Chen, W., Gai, Y., Gupta, P.: Efficiency evaluation of fuzzy portfolio in different risk measures via DEA. Ann. Oper. Res. **269**(1–2), 103–127 (2018)
12. Chen, W., Li, S.S., Zhang, J., Mehlawat, M.K.: A comprehensive model for fuzzy multi-objective portfolio selection based on DEA cross-efficiency model. Soft Comput. (2018). https://doi.org/10.1007/s00500-018-3595-x
13. Chiang, T.A., Che, Z.H.: A fuzzy robust evaluation model for selecting and ranking NPD projects using Bayesian belief network and weight-restricted DEA. Expert Syst. Appl. **37**(11), 7408–7418 (2010)
14. Cura, T.: Particle swarm optimization approach to portfolio optimization. Nonlinear Anal. Real World Appl. **10**, 2396–2406 (2009)
15. Edirisinghe, N.C.P., Zhang, X.: Portfolio selection under DEA-based relative financial strength indicators: case of US industries. J. Oper. Res. Soc. **59**(6), 842–856 (2008)
16. Färe, R., Grosskopf, S.: Modeling undesirable factors in efficiency evaluation: comment. Eur. J. Oper. Res. **157**(1), 242–245 (2004)
17. Fried, H.O., Lovell, C.K., Schmidt, S.S., Yaisawarng, S.: Accounting for environmental effects and statistical noise in data envelopment analysis. J. Prod. Anal. **17**(1–2), 157–174 (2002)
18. Gouveia, M. D. C., Neves, E. D., Dias, L. C., & Antunes, C. H.: Performance evaluation of Portuguese mutual fund portfolios using the value-based DEA method. J. Oper. Res. Soc. **69**(10), 1628–1639 (2018)
19. Guo, P., Tanaka, H.: Fuzzy DEA: a perceptual evaluation method. Fuzzy Sets Syst. **119**(1), 149–160 (2001)
20. Hatami-Marbini, A., Saati, S., Tavana, M.: An ideal-seeking fuzzy data envelopment analysis framework. Appl. Soft Comput. **10**(4), 1062–1070 (2010)
21. Holland, J.H.: Adaptation in natural and artificial systems, vol. 1. The University of Michigan Press, Ann Arbor, MI (1975)
22. Hu, J.L., Kao, C.H.: Efficient energy-saving targets for APEC economies. Energy Policy **35**(1), 373–382 (2007)
23. Joro, T., Na, P.: Portfolio performance evaluation in a mean-variance-skewness framework. Eur. J. Oper. Res. **175**(1), 446–461 (2006)
24. Kao, C., Lin, P.H.: Efficiency of parallel production systems with fuzzy data. Fuzzy Sets Syst. **198**, 83–98 (2012)
25. Kao, C., Liu, S.T.: Fuzzy efficiency measures in data envelopment analysis. Fuzzy Sets Syst. **113**(3), 427–437 (2000)
26. Kao, C., Liu, S.T.: A mathematical programming approach to fuzzy efficiency ranking. Int. J. Prod. Econ. **86**(2), 145–154 (2003)
27. Khalili-Damghani, K., Tavana, M., Santos-Arteaga, F.J.: A comprehensive fuzzy DEA model for emerging market assessment and selection decisions. Appl. Soft Comput. **38**, 676–702 (2016)
28. Koopmans, T.C.: Activity analysis of production and allocation. In: Koopmans, T.C. (ed.) Activity Analysis of Production and Allocation, pp. 33–97. Wiley, New York (1951)
29. Lertworasirikul, S., Fang, S.C., Joines, J.A., Nuttle, H.L.: Fuzzy data envelopment analysis (DEA): a possibility approach. Fuzzy Sets Syst. **139**(2), 379–394 (2003)
30. Lim, S., Oh, K.W., Zhu, J.: Use of DEA cross-efficiency evaluation in portfolio selection: an application to Korean stock market. Eur. J. Oper. Res. **236**(1), 361–368 (2014)
31. Liu, X., Chu, J., Yin, P., Sun, J.: DEA cross-efficiency evaluation considering undesirable output and ranking priority: a case study of eco-efficiency analysis of coal-fired power plants. J. Clean. Prod. **142**, 877–885 (2017)
32. Liu, W., Sharp, J.: DEA models via goal programming. In: Westerman, G. (ed.) Data Envelopment Analysis in the Service Sector, pp. 79–101. DeutscherUniversitats-verlag, Wiesbaden (1999)
33. Lovell, C.K., Pastor, J.T., Turner, J.A.: Measuring macroeconomic performance in the OECD: a comparison of European and non-European countries. Eur. J. Oper. Res. **87**(3), 507–518 (1995)
34. Lozano, S., Gutierrez, E.: Slacks-based measure of efficiency of airports with airplanes delays as undesirable outputs. Comput. Oper. Res. **38**(1), 131–139 (2011)
35. Markowitz, H.: Portfolio Selection: Efficient Diversification of Investments. Wiley, New Yor (1959)
36. Mashayekhi, Z., Omrani, H.: An integrated multi-objective Markowitz-DEA cross-efficiency model with fuzzy returns for portfolio selection problem. Appl. Soft Comput. **38**, 1–9 (2016)
37. Moheb-Alizadeh, H., Rasouli, S.M., Tavakkoli-Moghaddam, R.: The use of multi-criteria data envelopment analysis (MCDEA) for location-allocation problems in a fuzzy environment. Expert Syst. Appl. **38**(5), 5687–5695 (2011)
38. Mu, W., Kanellopoulos, A., van Middelaar, C.E., Stilmant, D., Bloemhof, J.M.: Assessing the impact of uncertainty on benchmarking the eco-efficiency of dairy farming using fuzzy data envelopment analysis. J. Clean. Prod. **189**, 709–717 (2018)
39. Murthi, B.P.S., Choi, Y.K., Desai, P.: Efficiency of mutual funds and portfolio performance measurement: a non-parametric approach. Eur. J. Oper. Res. **98**(2), 408–418 (1997)
40. Puri, J., Yadav, S.P.: A fuzzy DEA model with undesirable fuzzy outputs and its application to the banking sector in India. Expert Syst. Appl. **41**(14), 6419–6432 (2014)
41. Saati, S.M., Memariani, A., Jahanshahloo, G.R.: Efficiency analysis and ranking of DMUs with fuzzy data. Fuzzy Optim. Decis. Making **1**(3), 255–267 (2002)
42. Scheel, H.: Undesirable outputs in efficiency valuations. Eur. J. Oper. Res. **132**(2), 400–410 (2001)

43. Seiford, L.M., Zhu, J.: Modeling undesirable factors in efficiency evaluation. Eur. J. Oper. Res. **142**(1), 16–20 (2002)
44. Sengupta, J.K.: A fuzzy systems approach in data envelopment analysis. Comput. Math. Appl. **24**(8–9), 259–266 (1992)
45. Sengupta, J.K.: Measuring efficiency by a fuzzy statistical approach. Fuzzy Sets Syst. **46**(1), 73–80 (1992)
46. Sherman, H.D.: Hospital efficiency measurement and evaluation: empirical test of a new technique. Med. Care **22**, 922–938 (1984)
47. Sueyoshi, T., Goto, M.: Data envelopment analysis for environmental assessment: comparison between public and private ownership in petroleum industry. Eur. J. Oper. Res. **216**(3), 668–678 (2012)
48. Tofallis, C.: Improving discernment in DEA using profiling. Omega **24**(3), 361–364 (1996)
49. Toloo, M., Hančlová, J., : Multi-valued measures in DEA in the presence of undesirable outputs. Omega (2019). https://doi.org/10.1016/j.omega.2019.01.010
50. Triantis, K., Girod, O.: A mathematical programming approach for measuring technical efficiency in a fuzzy environment. J. Prod. Anal. **10**(1), 85–102 (1998)
51. Wang, Y.M., Chin, K.S.: Fuzzy data envelopment analysis: a fuzzy expected value approach. Expert Syst. Appl. **38**(9), 11678–11685 (2011)
52. Zanella, A. (2014). Assessment of performance in the presence of undesirable outputs: the promotion of livability and sustainable development of cities and countries using Data Envelopment Analysis. In: Ph.D. thesis
53. Zhou, Z., Jin, Q., Xiao, H., Wu, Q., Liu, W.: Estimation of cardinality constrained portfolio efficiency via segmented DEA. Omega **76**, 28–37 (2018)
54. Zhou, Z., Zhao, L., Lui, S., Ma, C.: A generalized fuzzy DEA/AR performance assessment model. Math. Comput. Modell. **55**(11–12), 2117–2128 (2012)

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