Int. J. Fuzzy Syst. (2021) 23(3):794–803
https://doi.org/10.1007/s40815-020-01018-1

Adaptive Fixed-Time Fuzzy Control of Uncertain Nonlinear Quantized Systems

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Received: 30 June 2020 / Revised: 16 October 2020 / Accepted: 17 November 2020 / Published online: 5 February 2021

Abstract This article settles the issue of adaptive fixed-time control of uncertain nonlinear quantized systems. Different from the traditional study about fixed-time control for uncertain nonlinear systems, quantitative control issue is considered in this paper, and the nonlinear term can be unknown. The new adaptive control tactic of fixed-time tracking control is proposed via fuzzy logic systems approaching unknown nonlinearity, which overcomes the existing limitation of the upper boundary of system settling time relies on the initial condition. The closed-loop system stability is guaranteed in a fixed time. At the end of this paper, the availability of the strategy is proved by a numerical simulation.

Keywords Fixed-time stability · Backstepping technique · Adaptive fuzzy control · Nonlinear quantized systems

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1 Introduction

With the development of hybrid systems, digital control and network control systems (see [1–6]), the importance of quantization methods has increased in recent years. These systems need to transmit information between components via wireless media. Because of the wireless communication network has physical constraints, it is necessary to introduce quantization technology to reduce communication rate. Quantitative control of linear and nonlinear plants has received much attention in the past several years, which result in many fruitful results [4–10]. Among the above results [4–6], are for linear quantized systems [7–11], are for nonlinear quantized systems. Theoretically, the above works merely ensure the expected performance as the time variable approaches infinity. In addition, the above works require the control system is fully known.

Due to the influence of uncertain factors, the actual systems have nonlinearity. The nonlinear system functions, including the bounding functions, may be completely unknown. As we all know, the adaptive fuzzy or neural control approaches are very useful tool to solve unknown nonlinear problems, for instance, see [12–15]. It is worth mentioning that the above works are realized on the basis of infinite time stability, and the results of the asymptotic stability has been unable to meet people's needs. Different from asymptotic stability, the good features of finite time control are fast response speed, high tracking accuracy and strong anti-interference ability. Obviously, finite time stability make the systems have better transient performance. In recent years, the researches about finite time control has been well developed. In [16], the terminal sliding mode control method was raised. On that basis [17], established a terminal sliding manifold to surmount the singularity around the equilibrium point. Sliding mode control is a useful tool to stabilize complex nonlinear systems. In [18], the event-triggered fuzzy sliding mode control issue about a networked control system based on a semi-Markov process was studied. The sliding mode control problem of nonlinear stochastic Markov jump systems with uncertain time-varying delays was studied in [19]. The Lyapunov-type finite-time stability theory was established firstly in [20, 21] to solve the chattering phenomenon resulted from the sliding mode controller. On that basis [22, 23], studied the finite-time stability of nonlinear systems. In order to guarantee the capability of the system under low communication rates, the quantization scheme was proposed on basis of finite time stability. In [24], the quantitative control problem of finite-time synthesis of nonlinear semi-Markov switching systems based on T–S fuzzy means was studied. In [25], the issue of finite-time quantitative cost-guaranteed fuzzy control of nonlinear systems was discussed. The issue of finite-time adaptive fuzzy control for nonlinear quantized systems with unknown time delays was discussed in [26]. Even though some works about finite time on basis of nonlinear quantized systems has made some progress, the determination of time relies on the initial state. Therefore, when the initial conditions are unknown, the designer cannot accurately estimate the system stability time, which restricts the application of finite time control. When initial conditions are unknown or uncertain, the finite time control methods described above have been unable to guarantee that the systems achieve the desired performance at a predetermined time.

To solve this problem, a sufficient condition for stability in fixed time was raised for the first time in [27]. From [27], the stability time of the system is connected with a constant that has nothing to do with the initial state, and the constant is only determined by the design parameters. The fixed-time stability is especially important for either hybrid or switching systems [28–30] with dwell time. According to the theorem of stability in fixed time in [27], the fixed time controller was constructed for nonlinear systems in [31, 32] respectively. In [33], the issue of fixed-time stability of strict-feedback uncertain nonlinear systems was investigated [34] proposed a fixed-time adaptive control strategy for nonlinear systems by neural network. In [35], a fixed-time terminal sliding mode control means was raised for momentum wheel system. A global fixed-time consistency protocol of second-order multi-agent systems was discussed in [36]. In [35, 36], the sinusoid continuous functions were introduced to eliminate the singularity. In order to suppress chaotic oscillation in power system, a fast fixed time non-singular terminal sliding mode control means was proposed in [37]. However, it should be pointed out that the results in [31–37] need to meet the hypothesis that the nonlinear function is known. Moreover, since the signals between the factory and the controller are realized remotely through the communication channel with limited bandwidth, the quantization of the control signals cannot be ignored. In [38], the fixed-time attitude tracking control issue of rigid spacecraft with external disturbances and input quantization was investigated. It should be pointed out that the nonlinear function of the system in [38] is known. When the nonlinear terms of uncertain nonlinear systems are completely unknown, the fixed time control schemes mentioned above have been unable to apply. It is a challenge to construct a quantitative feedback controller to avoid chattering and ensure the system is stable at fixed time. Inspired by the above, the adaptive fixed-time control issue of quantized uncertain nonlinear systems is investigated in this paper. The main merits are generalized below.

(1) Compared with the existing fixed-time works [31–37] for nonlinear systems, the quantitative control issue is considered. Compared with [38], the nonlinear function of the system is completely unknown in this paper. By applying the fuzzy logic system to approximate unknown nonlinearities, a novel fixed-time fuzzy control strategy is raised. The proposed control strategy ensures the system performance in fixed time.
(2) In the finite-time control schemes [20–26], the convergence time is dependent on the initial conditions. In practice, the initial condition of the system is not easy to obtain, which makes the schemes in [20–26] difficult to implement. Instead, by constructing a novel fuzzy fixed-time controller, this limitation can be overcome in this paper. The upper bound of convergence time is merely affected by design parameters. The proposed control strategy is more interesting in application.

This article is organized below. The preliminary knowledge and problem description are introduced in the second part. The third part gives the major research results of this article. Simulation studies are conducted in the fourth part. Finally, the fifth part summarizes the work.

2 Preliminaries and Problem Statement

According to some lemmas and definitions of fixed-time control, important results of this article are obtained in this part.

2.1 Preliminaries

Definition 1 [39] Think about a nonlinear system:

$$\dot{x} = f(x, t), x(0) = x_0 \tag{1}$$

where $x \in R^n$ represents state variable, $f(x, t)$ is unknown smooth nonlinear function and satisfy $x(0) = 0$, $f(0,0) = 0$. Suppose that system (1) is stable in Lyapunov sense. If there exist convergence time T, for $\forall t \geq T$, $x(t) = 0$, then system (1) is finite-time stability.

Definition 2 [39] If system (1) is stable in finite time, and convergence time T has a definite upper bound T_m , and the upper bound don't affect by the initial state, then the system (1) is fixed-time stability.

Lemma 1 [34] *If the design constants* $\varsigma, \epsilon > 0$, $p > 1, q \in (0,1), \eta \in (0, \infty)$, *one has*:

$$\dot{V} \leq -\varsigma V^p(x) - \epsilon V^q(x) + \eta, \forall x \in R^n \tag{2}$$

the system (1) is globally fixed-time stability, and settling time T_m reckons as:

$$T \leq T_m = \frac{1}{\varsigma\phi(p-1)} + \frac{1}{\epsilon\phi(1-q)} \tag{3}$$

the residual set of system (1) solution as follows:

$$x \in \left\{ V(x) \leq min\left\{ \left(\frac{\eta}{(1-\phi)\varsigma}\right)^{\frac{1}{p}}, \left(\frac{\eta}{(1-\phi)\epsilon}\right)^{\frac{1}{q}} \right\} \right\}. \tag{4}$$

Lemma 2 [40] *For any given* $(x, y) \in R^2$, $\alpha > 1, \beta > 1, \varepsilon > 0$, *and* $(\alpha - 1)(\beta - 1) = 1$, *the following relation hold*:

$$xy \leq \frac{\varepsilon^\alpha}{\alpha}|x|^\alpha + \frac{1}{\beta\varepsilon^\beta}|y|^\beta.$$

Lemma 3 [41] *For any given positive constants* b_1, b_2, b_3, *and real variables* x *and* y, *we have*:

$$|x|^{b_1}|y|^{b_2} \leq \frac{b_1}{b_1+b_2}b_3|x|^{b_1+b_2} + \frac{b_2}{b_1+b_2}b_3^{-\frac{b_1}{b_2}}|y|^{b_1+b_2}. \tag{5}$$

Lemma 4 [42] *For* $\iota k \in R$, *and* $k = 1, \cdots, n, 0 < \varphi \leq 1$, *we have*:

$$\left(\sum_{k=1}^{n}|\iota k|\right)^\varphi \leq \sum_{k=1}^{n}|\iota k|^\varphi \leq n^{1-\varphi}\left(\sum_{k=1}^{n}|\iota k|\right)^\varphi.$$

2.2 Problem Statement

This article thinks about an uncertain nonlinear quantized system below:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i), 1 \leq i \leq n-1, \\ \dot{x}_n = Q(u(t)) + f_n(\bar{x}_n), \\ y = x_1. \end{cases} \tag{6}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$ stands for system states, $(i = 1, 2, \ldots, n)$, and $y \in R$ expresses output of system. $f_i(.) : R^i \to R(i = 1, 2, \ldots, n)$ is unknown smooth functions. $u(t) \in R$ and $Q(u(t))$ denote a controller and a quantizer, respectively.

In this article, according to [43], hysteretic quantizer is presented to avert chattering. It follows from [44] that quantizer $Q(u(t))$ is described as follows:

$$Q(u(t)) = \begin{cases} u_i sgn(u), \frac{u_i}{1+\delta} < |u| \leq u_i, \dot{u} < 0, or \\ u_i < |u| \leq \frac{u_i}{1-\delta}, \dot{u} > 0 \\ u_i(1+\delta)sgn(u), u_i < |u| \leq \frac{u_i}{1-\delta}, \dot{u} < 0, or \\ \frac{u_i}{1-\delta} < |u| \leq \frac{u_i(1+\delta)}{1-\delta}, \dot{u} > 0 \\ 0, 0 \leq |u| < \frac{u_{min}}{1+\delta}, \dot{u} < 0, or \\ \frac{u_{min}}{1+\delta} \leq |u| \leq u_{min}, \dot{u} > 0 \\ Q(u(t^-)), \dot{u} = 0 \end{cases} \tag{7}$$

where $u_i = \rho^{1-i}u_{min}$, $i = 1, 2, \ldots, n$, $u_{min} > 0$ and $0 < \rho < 1$, $\delta = \frac{1-\rho}{1+\rho}$. Thus, $Q(u(t)) \in U = \{0, \pm u_i, \pm u_i(1+\delta)$, $i = 1, 2, \ldots\}$. u_{min} governs the range of dead-zone. The map of $Q(u(t))$ displays in Fig 1.

Remark 1 Different from the traditional research about fixed time control of nonlinear systems, the quantization of signals is considered firstly. An fixed-time adaptive control strategy is provided for uncertain nonlinear quantized systems.

Lemma 5 [45] *Decomposing the quantized value* $Q(u(t))$ *as follows*:

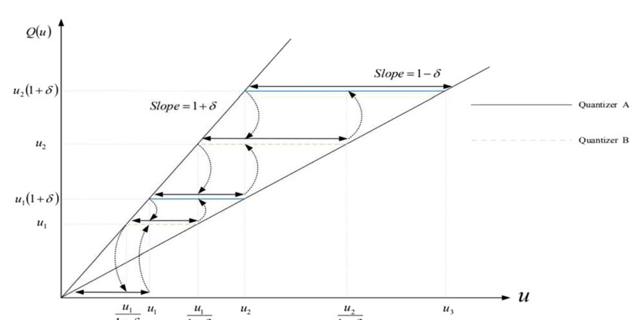

Fig. 1 Quantizer $Q(u(t))$

$$Q(u(t)) = \varpi(u)u(t) + d(t) \tag{8}$$

where

$$1 - \delta \leq \varpi(u) \leq 1 + \delta, |d(t)| \leq u_{\min}. \tag{9}$$

2.3 Fuzzy Logic Systems

Since the system (6) contains unknown functions, we need to introduce fuzzy logic system to approximate them. Using the singleton point fuzzifier, the center-average defuzzifier and the product inference, we can obtain fuzzy rules:

R^l: IF x_1 is $F_1^l, \ldots, x_n$ is F_n^l,

Then: y_G is G^l, $l = 1, 2, \ldots, N$ where $x = [x_1, x_2, ..., x_n]^T \in R^n$ denotes input variable, and $y_G \in R$ stands for fuzzy system output. The membership functions of fuzzy set F_i^l are signified by $\mu_{F_i^l}(x_i)$. N expresses total amount of the rules.

We can select Gaussian functions with the exponential as the membership functions:

$$\mu_{F_i^l}(x) = exp\left[-\frac{(x_i - a_i^l)}{b_i^l}\right]$$

where a_i^l and b_i^l stands for the center and the width of a fuzzy membership function, respectively.

Applying the singleton function, the center-average defuzzification and the product inference [46], $y_G(x)$ is described as:

$$y_G(x) = \frac{\sum_{l=1}^{N} \Phi_l \prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} [\prod_{i=1}^{n} \mu_{F_i^l}(x_i)]} \tag{10}$$

where

$$\Phi_l = \max_{y_G \in R} \mu_{G^l}(y), \Phi = (\Phi_1, \Phi_2, \ldots, \Phi_N)^T.$$

Let

$$\omega_l(x) = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} [\prod_{i=1}^{n} \mu_{F_i^l}(x_i)]}$$

and $\omega(x) = (\omega_1(x), \omega_2(x), \ldots, \omega_N(x))^T$. Then, fuzzy logic system is expressed as:

$$y_G(x) = \Phi^T \omega(x). \tag{11}$$

Lemma 6 [47] *If $f(x) \in \Omega$ is a continuous function, for any given $\varepsilon > 0$, fuzzy logic system (11) makes the relational expression true*:

$$\sup_{x \in \Omega} |f(x) - \Phi^T \omega(x)| \leq \varepsilon. \tag{12}$$

The purpose of this manuscript is constructed an fuzzy adaptive controller to ensure all closed-loop signals remain stable for a fixed time, and system output signal y follows the reference signal y_d.

3 Adaptive Controller Design

A fixed-time adaptive controller for the system (6) is constructed via backstepping approach in this part. Then, the error variable is defined as follows:

$$\begin{aligned} z_1 &= x_1 - y_d, \\ z_i &= x_i - \alpha_{i-1}, i = 2, \cdots, n. \end{aligned} \tag{13}$$

where α_{i-1} stands for virtual controller. For convenience of symbol operation, the ith time derivative of y_d is expressed by $y_d^{(i)}$, and let $\bar{y}_d^{(i)} = [y_d, y_d^{(1)}, \ldots, y_d^{(i)}]^T$, $i = 1, 2, \ldots, n$.

Remark 2 Based on backstepping technique, approximating unknown nonlinear function $\bar{f}_i$ by fuzzy logic system $\Phi_i(X_i)$. To achieve control objectives, we define a constant $\theta_i = \|\Phi_i\|^2, i = 1, 2, \cdots, n$, $\hat{\theta}_1$ stands for estimate of θ_1, and the parameter estimation error is $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$.

Step 1. Consider the nonlinear uncertain system (6), according to $z_1 = x_1 - y_d$, we have:

$$\dot{z}_1 = x_2 + f_1(x_1) - \dot{y}_d. \tag{14}$$

Think about the following Lyapunov function:

$$V_1 = \frac{z_1^2}{2} + \frac{\tilde{\theta}_1^2}{2\lambda_1} \tag{15}$$

where $\lambda_1 > 0$ is a design constant.

By (13), (14), one has:

$$\dot{V}_1 = z_1(z_2 + \alpha_1 + f_1(x_1) - \dot{y}_d) - \frac{\tilde{\theta}_1 \dot{\hat{\theta}}_1}{\lambda_1}. \tag{16}$$

Using Young's inequality, one has:

$$z_1 z_2 \leq \frac{z_1^2}{2} + \frac{z_2^2}{2}. \tag{17}$$

Substituting (17) into (16) yields:

$$\dot{V}_1 \leq z_1(f_1 + \frac{z_1}{2} + \alpha_1 - \dot{y}_d) + \frac{z_2^2}{2} - \frac{\tilde{\theta}_1 \dot{\hat{\theta}}_1}{\lambda_1}. \tag{18}$$

Let $\bar{f}_1 = f_1 + z_1 - \dot{y}_d$, then (18) can be rewritten as:

$$\dot{V}_1 \leq z_1\alpha_1 + z_1\bar{f}_1 + \frac{z_2^2}{2} - \frac{z_1^2}{2} - \frac{\tilde{\theta}_1 \dot{\hat{\theta}}_1}{\lambda_1}. \tag{19}$$

Since $\bar{f}_1$ is unknown, it cannot be obtained. Applying Lemma 6, for $\forall \varepsilon_1 > 0$, there is a fuzzy logic system

$\Phi_1^T \omega_1(X_1)$ as follows:

$$\bar{f}_1 = \Phi_1^T \omega_1(X_1) + \delta_1(X_1), |\delta_1(X_1)| \le \varepsilon_1 \tag{20}$$

where $X_1 = (x_1, y_d, \dot{y}_d)$.

By applying Young's inequality, one has:

$$z_1 \bar{f}_1 \le \frac{1}{2a_1^2} z_1^2 \theta_1 \omega_1^T \omega_1 + \frac{1}{2} a_1^2 + \frac{1}{2} z_1^2 + \frac{1}{2} \varepsilon_1^2. \tag{21}$$

Then, we choose the virtual controller as:

$$\begin{aligned} \alpha_1 = & - c_1 z_1^{2\sigma-1} - k_1 z_1^{2\beta-1} \\ & - \frac{1}{2a_1^2} \hat{\theta}_1 z_1 \omega_1^T \omega_1 \end{aligned} \tag{22}$$

where $c_1 > 0$, $k_1 > 0$ and $a_1 > 0$ are design constants.

Next, we set up the adaptive law as:

$$\dot{\hat{\theta}}_1 = \frac{\lambda_1}{2a_1^2} z_1^2 \omega_1^T \omega_1 - \gamma_1 \hat{\theta}_1 - \kappa_1 \hat{\theta}_1, \hat{\theta}_1(0) \ge 0 \tag{23}$$

where γ_1 and κ_1 are positive design constants.

Substituting (21)–(23) into (19), we have:

$$\begin{aligned} \dot{V}_1 \le & - c_1 z_1^{2\sigma} - k_1 z_1^{2\beta} + \frac{z_2^2}{2} + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} + \frac{\gamma_1}{\lambda_1} \tilde{\theta}_1 \hat{\theta}_1 \\ & + \frac{\kappa_1}{\lambda_1} \tilde{\theta}_1 \hat{\theta}_1. \end{aligned} \tag{24}$$

It is noted that:

$$\frac{\gamma_1}{\lambda_1} \tilde{\theta}_1 \hat{\theta}_1 \le - \frac{\gamma_1}{2\lambda_1} \tilde{\theta}_1^2 + \frac{\gamma_1}{2\lambda_1} \theta_1^2, \tag{25}$$

$$\frac{\kappa_1}{\lambda_1} \tilde{\theta}_1 \hat{\theta}_1 \le - \frac{\kappa_1}{2\lambda_1} \tilde{\theta}_1^2 + \frac{\kappa_1}{2\lambda_1} \theta_1^2. \tag{26}$$

Equation (24) can be described as:

$$\dot{V}_1 \le - c_1 z_1^{2\sigma} - k_1 z_1^{2\beta} - \frac{\gamma_1}{2\lambda_1} \tilde{\theta}_1^2 - \frac{\kappa_1}{2\lambda_1} \tilde{\theta}_1^2 + \eta_1 + \frac{z_2^2}{2} \tag{27}$$

where $\eta_1 = \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} + \frac{\gamma_1}{2\lambda_1} \theta_1^2 + \frac{\kappa_1}{\lambda_1} \theta_1^2$.

Step $i (2 \le i \le n-1)$. Based on (13) and Step 1, we have:

$$\dot{z}_i = f_i(\bar{x}_i) + x_{i+1} - \dot{\alpha}_{i-1} \tag{28}$$

where

$$\begin{aligned} \dot{\alpha}_{i-1} = & \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left[f_j(\bar{x}_j) + x_{j+1} \right] + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \\ & + \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)}. \end{aligned} \tag{29}$$

Now, think about the Lyapunov function:

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2\lambda_i} \tilde{\theta}_i^2 \tag{30}$$

where λ_i is a positive design constant.

Then its derivative can be obtained:

$$\dot{V}_i = \dot{V}_{i-1} + z_i (z_{i+1} + \alpha_i + f_i(\bar{x}_i) - \dot{\alpha}_{i-1}) - \frac{1}{\lambda_i} \tilde{\theta}_i \dot{\hat{\theta}}_i. \tag{31}$$

According to Lemma 2, we have:

$$z_i z_{i+1} \le \frac{1}{2} z_i^2 + \frac{1}{2} z_{i+1}^2. \tag{32}$$

Substituting (32) into (31), one has:

$$\begin{aligned} \dot{V}_i \le & - \sum_{j=1}^{i-1} (c_j z_j^{2\sigma} + k_j z_j^{2\beta} + \frac{\gamma_j}{2\lambda_j} \tilde{\theta}_j^2 + \frac{\kappa_j}{2\lambda_j} \tilde{\theta}_j^2) + \sum_{j=1}^{i-1} \\ & \eta_j + z_i \bar{f}_i + z_i \alpha_i + \frac{1}{2} z_{i+1}^2 - \frac{1}{2} z_i^2 - \frac{1}{\lambda_i} \tilde{\theta}_i \dot{\hat{\theta}}_i \end{aligned} \tag{33}$$

where

$$\bar{f}_i = f_i(\bar{x}_i) - \dot{\alpha}_{i-1} + \frac{3}{2} z_i^2.$$

Similarly, $\Phi_i^T \omega_i(X_i)$ is used for approximating $\bar{f}_i$, and $X_i = [\bar{x}_i^T, \bar{\hat{\theta}}_{i-1}^T, \bar{y}_d^{(i)T}]^T \in \Omega_{Z_i} \subset R^{3i}$ with $\hat{\theta}_{i-1} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{i-1}]^T$. Applying Lemma 6, we have:

$$\bar{f}_i = \Phi_i^T \omega_i(X_i) + \delta_i(X_i), |\delta_i(X_i)| \le \varepsilon_i \tag{34}$$

where ε_i is any given positive constant.

Using Young's inequality, we have:

$$z_i \bar{f}_i \le \frac{1}{2a_i^2} z_i^2 \theta_i \omega_i^T \omega_i + \frac{1}{2} a_i^2 + \frac{1}{2} z_i^2 + \frac{1}{2} \varepsilon_i^2. \tag{35}$$

Then, the virtual controller is constructed as:

$$\alpha_i = -c_i z_i^{2\sigma-1} - k_i z_i^{2\beta-1} - \frac{1}{2a_i^2} \hat{\theta}_i z_i \omega_i^T \omega_i \tag{36}$$

where $c_i > 0$, $k_i > 0$ and $a_i > 0$ are design constants.

Next, we set up adaptation law as:

$$\dot{\hat{\theta}}_i = \frac{\lambda_i}{2a_i^2} z_i^2 \omega_i^T \omega_i - \gamma_i \hat{\theta}_i - \kappa_i \hat{\theta}_i, \hat{\theta}_i(0) \ge 0 \tag{37}$$

where γ_i and κ_i are positive design constants.

Much like (25), (26), we have:

$$\frac{\gamma_i}{\lambda_i} \tilde{\theta}_i \hat{\theta}_i \le - \frac{\gamma_i}{2\lambda_i} \tilde{\theta}_i^2 + \frac{\gamma_i}{2\lambda_i} \theta_i^2, \tag{38}$$

$$\frac{\kappa_i}{\lambda_i} \tilde{\theta}_i \hat{\theta}_i \le - \frac{\kappa_i}{2\lambda_i} \tilde{\theta}_i^2 + \frac{\kappa_i}{2\lambda_i} \theta_i^2. \tag{39}$$

Substituting (35)–(39) into (33) yields:

$$\dot{V}_i \leq -\sum_{j=1}^{i}(c_j z_j^{2\sigma} + k_j z_j^{2\beta} + \frac{\gamma_j}{2\lambda_j}\tilde{\theta}_j^2 + \frac{\kappa_j}{2\lambda_j}\tilde{\theta}_j^2) + \sum_{j=1}^{i}\eta_j + \frac{1}{2}z_{i+1}^2 \tag{40}$$

where $\eta_j = \frac{\gamma_j}{2\lambda_j}\theta_j^2 + \frac{\kappa_j}{2\lambda_j}\theta_j^2 + \frac{1}{2}a_j^2 + \frac{1}{2}\varepsilon_j^2, j = 1, 2, \cdots, i$.

Step n. According to the analysis in step i, one has:

$$\dot{z}_n = f_n(\bar{x}_n) + Q(u(t)) - \dot{\alpha}_{n-1} \tag{41}$$

where

$$\dot{\alpha}_{n-1} = \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left(f_j(\bar{x}_j) + x_{j+1}\right) + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j + \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)}.$$

Now, think about the Lyapunov function:

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2\lambda_n}\tilde{\theta}_n^2 \tag{42}$$

where λ_n is a positive design constant.

From its derivative and Lemma 5, one has:

$$\dot{V}_n = \dot{V}_{n-1} + z_n(f_n(\bar{x}_n) + \varpi(u)u(t) + d(t) - \dot{\alpha}_{n-1}) - \frac{\tilde{\theta}_n}{\lambda_n}\dot{\hat{\theta}}_n \tag{43}$$

Based on Young's inequality, we have:

$$z_n d(t) \leq \frac{1}{2}z_n^2 + \frac{1}{2}u_{\min}^2. \tag{44}$$

According to (40) with $(i = n-1)$, (44), one has:

$$\dot{V}_n \leq -\sum_{j=1}^{n-1}(c_j z_j^{2\sigma} + k_j z_j^{2\beta} + \frac{\gamma_j}{2\lambda_j}\tilde{\theta}_j^2 + \frac{\kappa_j}{2\lambda_j}\tilde{\theta}_j^2) + \sum_{j=1}^{n-1}\eta_j + z_n\bar{f}_n - \frac{1}{2}z_n^2 + z_n\varpi(u)u(t) + \frac{1}{2}u_{\min}^2 - \frac{1}{\lambda_n}\tilde{\theta}_n\dot{\hat{\theta}}_n \tag{45}$$

where

$$\bar{f}_n = f_n(\bar{x}_n) - \dot{\alpha}_{n-1} + \frac{3}{2}z_n. \tag{46}$$

Similarly, for $\forall\varepsilon_n > 0$, fuzzy logic system $\Phi_n^T\omega_n(X_n)$ is adopted to approximate $\bar{f}_n$.

From Lemma 6, we have:

$$\bar{f}_n = \Phi_n^T\omega_n(X_n) + \delta_n(X_n), |\delta_n(X_n)| \leq \varepsilon_n. \tag{47}$$

Applying Young's inequality, we have:

$$z_n\bar{f}_n \leq \frac{1}{2a_n^2}z_n^2\theta_n\omega_n^T\omega_n + \frac{1}{2}a_n^2 + \frac{1}{2}z_n^2 + \frac{1}{2}\varepsilon_n^2. \tag{48}$$

Now, the actual controller is constructed as:

$$u = -c_n {z_n}^{2\sigma-1} - k_n {z_n}^{2\beta-1} - \frac{z_n\hat{\theta}_n\omega_n^T\omega_n}{2a_n^2} \tag{49}$$

where $k_n > 0$,$c_n > 0$ and $a_n > 0$ are design constants.

Next, we set up adaptation law as:

$$\dot{\hat{\theta}}_n = \frac{\lambda_n}{2a_n^2}z_n^2\omega_n^T\omega_n - \gamma_n\hat{\theta}_n - \kappa_n\hat{\theta}_n, \hat{\theta}_n(t_0) \geq 0 \tag{50}$$

where γ_n is a positive design constant.

Combining (48), (9), we can obtain:

$$z_n\varpi(u)u(t) \leq -c_n {z_n}^{2\sigma} - k_n {z_n}^{2\beta} - \frac{1}{2a_n^2}z_n^2\hat{\theta}_n\omega_n^T\omega_n. \tag{51}$$

Substituting (48)–(51) into (45) gives:

$$\dot{V}_n \leq -\sum_{j=1}^{n-1}(c_j z_j^{2\sigma} + k_j z_j^{2\beta} + \frac{\gamma_j}{2\lambda_j}\tilde{\theta}_j^2 + \frac{\kappa_j}{2\lambda_j}\tilde{\theta}_j^2) + \sum_{j=1}^{n-1}\eta_j - c_n z_n^{2\sigma} - k_n z_n^{2\beta} + \frac{1}{2}u_{\min}^2 + \frac{1}{2}a_n^2 + \frac{1}{2}\varepsilon_n^2 + \frac{\gamma_n}{\lambda_n}\tilde{\theta}_n\hat{\theta}_n + \frac{\kappa_n}{\lambda_n}\tilde{\theta}_n\hat{\theta}_n. \tag{52}$$

Furthermore, as is the same case of (38) and (39), we have:

$$\frac{\gamma_n}{\lambda_n}\tilde{\theta}_n\hat{\theta}_n \leq -\frac{\gamma_n}{2\lambda_n}\tilde{\theta}_n^2 + \frac{\gamma_n}{2\lambda_n}\theta_n^2, \tag{53}$$

$$\frac{\kappa_n}{\lambda_n}\tilde{\theta}_n\hat{\theta}_n \leq -\frac{\kappa_n}{2\lambda_n}\tilde{\theta}_n^2 + \frac{\kappa_n}{2\lambda_n}\theta_n^2 \tag{54}$$

Substituting (53), (54) into (52), one has:

$$\dot{V}_n \leq -\sum_{j=1}^{n}(c_j z_j^{2\sigma} + \frac{\gamma_j}{2\lambda_j}\tilde{\theta}_j^2) - \sum_{j=1}^{n}(k_j z_j^{2\beta} + \frac{\kappa_j}{2\lambda_j}\tilde{\theta}_j^2) + \tau \tag{55}$$

where $\tau = \sum_{j=1}^{n}\eta_j + \frac{1}{2}u_{\min}^2$ and $\eta_j = \frac{\gamma_j}{2\lambda_j}\theta_j^2 + \frac{\kappa_j}{2\lambda_j}\theta_j^2 + \frac{1}{2}a_j^2 + \frac{1}{2}\varepsilon_j^2, j = 1, 2, \ldots, n$.

So far, the following theorem summarizes the above work.

Theorem 1 *Think about the uncertain nonlinear system (6), preceded by hysteresis quantizer (7). Based on controller (49), intermediate virtual controller (36) and adaptive law (37), system (6) is globally fixed-time stability. The tracking error converges to a small neighborhood of the origin. Within a fixed time, the closed-loop system* *stability is guaranteed and convergence time has a certain upper bound.*

Proof Applying Lemma 3, one has:

$$\Big(\sum_{j=1}^{n}\frac{\tilde{\theta}_j^2}{2\lambda_j}|Big)^{\sigma} \le \sum_{j=1}^{n}\frac{\tilde{\theta}_j^2}{2\lambda_j} + (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}, \tag{56}$$

$$\Big(\sum_{j=1}^{n}\frac{\tilde{\theta}_j^2}{2\lambda_j}\Big)^{\beta} \le \sum_{j=1}^{n}\frac{\tilde{\theta}_j^2}{2\lambda_j} + (1-\beta)\beta^{\frac{\beta}{1-\beta}}. \tag{57}$$

Combining (55), (56), (57), we have:

$$\begin{aligned}\dot{V}_n \le & -\sum_{j=1}^{n} c_j z_j^{2\sigma} - \sum_{j=1}^{n}\gamma_j\Big(\frac{\tilde{\theta}_j^2}{2\lambda_j}\Big)^{\sigma} - \sum_{j=1}^{n} k_j z_j^{2\beta} \\ & - \sum_{j=1}^{n}\kappa_j\Big(\frac{\tilde{\theta}_j^2}{2\lambda_j}\Big)^{\beta} + \varrho \\ \le & -c\sum_{j=1}^{n}\Big(\frac{1}{2}z_j^2\Big)^{\sigma} - c\sum_{j=1}^{n}\Big(\frac{\tilde{\theta}_j^2}{2\lambda_j}\Big)^{\sigma} - k\sum_{j=1}^{n}\Big(\frac{1}{2}z_j^2\Big)^{\beta} \\ & - k\sum_{j=1}^{n}\Big(\frac{\tilde{\theta}_j^2}{2\lambda_j}\Big)^{\beta} + \varrho\end{aligned} \tag{58}$$

where $c = min\{2^{\sigma}c_j, \gamma_j, j = 1, \ldots, n\}$, $k = min\{2^{\beta}k_j, \kappa_j, j = 1, \cdots, n\}$, $\varrho = \sum_{j=1}^{n}\eta_j + \frac{1}{2}u_{\min}^2 + (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}\gamma_j + (1-\beta)\beta^{\frac{\beta}{1-\beta}}\kappa_j$ and $\eta_j = \frac{\gamma_j}{2\lambda_j}\theta_j^2 + \frac{\kappa_j}{2\lambda_j}\theta_j^2 + \frac{1}{2}a_j^2 + \frac{1}{2}\varepsilon_j^2, j = 1, 2, \ldots, n$.

By using Lemma 4, one has:

$$\begin{aligned}\dot{V}_n \le & -c\left(\sum_{j=1}^{n}\left(\frac{z_j^2}{2} + \frac{\tilde{\theta}_j^2}{2\lambda_j}\right)\right)^{\sigma} - k\left(\sum_{j=1}^{n}\left(\frac{z_j^2}{2} + \frac{\tilde{\theta}_j^2}{2\lambda_j}\right)\right)^{\beta} + \varrho \\ \le & -cV^{\sigma}(x) - kV^{\beta}(x) + \varrho.\end{aligned} \tag{59}$$

Applying Lemma 1, we can obtain system (6) is practical fixed-time stable and converges to a compact set

$$x \in \left\{V(x) \le min\left\{\Big(\frac{\varrho}{(1-\phi)c}\Big)^{\frac{1}{\sigma}}, \Big(\frac{\varrho}{(1-\phi)k}\Big)^{\frac{1}{\beta}}\right\}\right\} \tag{60}$$

Then, the setting time can be obtained:

$$T \le T_m = \frac{1}{c\phi(\sigma-1)} + \frac{1}{k\phi(1-\beta)}. \tag{61}$$

In addition, applying the definition of V_n, we have:

$$|y - y_d| \le 2\Big(\frac{\varrho}{(1-\phi)c}\Big)^{\frac{1}{2\sigma}}. \tag{62}$$

By choosing appropriate constant parameters, the tracking error can reduce to an ideal range in a fixed time. □

Remark 3 From formula (61), the convergence time boundary T is not affected by the initial state. The raised adaptive fixed-time control tactic overcomes the limitation of the finite-time control scheme relies on initial state and guarantees tracking performance within a fixed-time.

4 Simulation Results

Example 1 The following second-order uncertain nonlinear quantized system is thought about:

$$\begin{aligned}\dot{x}_1 &= x_2 + f_1(\bar{x}_1) \\ \dot{x}_2 &= Q(u) + f_2(\bar{x}_2) \\ & y\end{aligned} \tag{63}$$

where y stands for system output, x_1 and x_2 denote the systems states, $Q(u)$ represents hysteretic quantizer as in (7).

Then, the fuzzy membership functions are defined below:

$$\mu_{F_i^1}(x) = \exp\left(\frac{-(x+2)^2}{2}\right),$$
$$\mu_{F_i^2}(x) = \exp\left(\frac{-(x+1.5)^2}{2}\right),$$
$$\mu_{F_i^3}(x) = \exp\left(\frac{-(x+1)^2}{2}\right),$$
$$\mu_{F_i^4}(x) = \exp\left(\frac{-(x+0.5)^2}{2}\right),$$
$$\mu_{F_i^5}(x) = \exp\left(\frac{-x^2}{2}\right),$$
$$\mu_{F_i^6}(x) = \exp\left(\frac{-(x-0.5)^2}{2}\right),$$
$$\mu_{F_i^7}(x) = \exp\left(\frac{-(x-1)^2}{2}\right),$$
$$\mu_{F_i^8}(x) = \exp\left(\frac{-(x-1.5)^2}{2}\right),$$
$$\mu_{F_i^9}(x) = \exp\left(\frac{-(x-2)^2}{2}\right).$$

According to Theorem 1, the adaptive law, the virtual controller and the real controller are established as:

$$\alpha_1 = -c_1 z_1^{2\sigma-1} - k_1 z_1^{2\beta-1} - \frac{z_1}{2a_1^2}\hat{\theta}_1\omega_1^T(X_1)\omega_1(X_1) \tag{64}$$

$$u = -c_2 z_2^{2\sigma-1} - k_2 z_2^{2\beta-1} - \frac{z_2\hat{\theta}_2\omega_2^T(X_2)\omega_2(X_2)}{2a_2^2} \tag{65}$$

$$\dot{\hat{\theta}}_i = \frac{\lambda_i}{2a_i^2}z_i^2\omega_i^T(X_i)\omega_i(X_i) - \gamma_i\hat{\theta}_i - \kappa_i\hat{\theta}_i, i = 1, 2 \tag{66}$$

where $\sigma = 118/100$, $\beta = 90/101$, $z_1 = x_1 - y_d, z_2 = x_2 - \alpha_1, X_1 = [x_1, y_d, \dot{y}_d]^T$ and $X_2 = [\bar{x}_2^T, \hat{\theta}_1, \bar{y}_d^{(2)T}]^T$ (Fig. 2).

In simulation, $f_i(\bar{x}_i)$ is chosen as $f_1(\bar{x}_1) = (1 - \sin^2(x_1))x_1$, $f_2(\bar{x}_2) = cos(x_1)^2 x_1^2 + sin(x_1)^2 x_2^2 - x_1 x_2^2$.

To test the correctness of Theorem 1, we need to let $y_d = sin(0.5t) + 0.5sin(t)$. The related parameters can be set to $\delta = 0.5$, $\mu_{min} = 0.2$, $c_1 = c_2 = 8$, $c_3 = c_4 = 6$, $a_1 = 0.8$, $a_2 = 1.2$, $\lambda_1 = 20$, $\lambda_2 = 25$, $\gamma_1 = 1$, $\gamma_2 = 2$, $\kappa_1 = 1$, $\kappa_2 = 2$. we choose initial conditions as $[x_1(0), x_2(0)]^T = [0.05, -0.5]^T$, and $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0.1, 0.2]^T$. Simulation results are displayed in Figs. 3, 4,5, 6, and 7. From the simulation results, it is shown that all the signals are stabilized in fixed time.

Remark 4 To further demonstrate the suitability and availability of the proposed control tactics, we have compared the results with the control results in [42]. According to Fig. 7, we can know that the proposed fixed time control scheme has faster convergence speed and better tracking performance. In addition, different from [18, 19], the control strategy can guarantee the stability of the system at fixed time. Moreover, the upper bound of the stability time is only influenced by the design parameters.

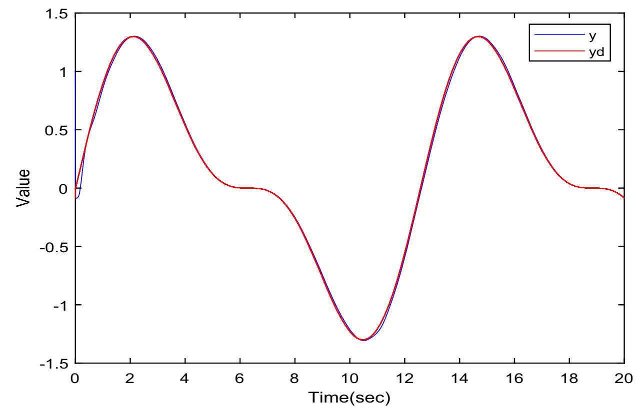

Fig. 3 The tracking performance of y and y_d

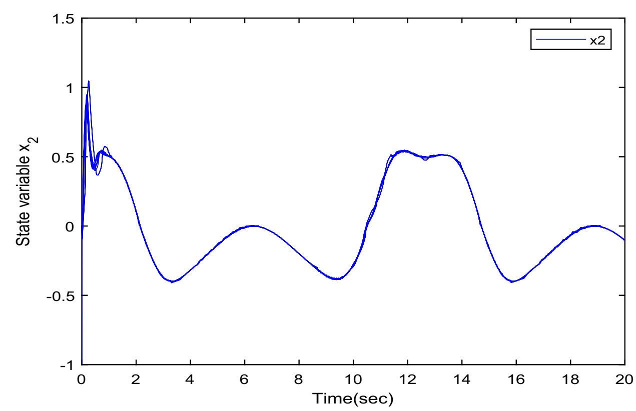

Fig. 4 State variable x_2

5 Conclusion

This article provides an adaptive fixed-time control scheme for uncertain nonlinear quantized systems. The fuzzy logic system is used to approximate the unknown nonlinear function. To build the relationship of $u(t)$ and $Q(u(t))$, a nonlinear decomposition of quantized input is introduced. The fixed time controller is constructed to get over limitation that system convergence time relies on initial state, and ensures tracking error converges to a small neighborhood of the origin within a fixed time. Simultaneously, the closed-loop system signals are bounded, and convergence time has a definite upper bound. Finally, the main result is proved by simulation results. Moreover, the time delay problem is not considered in the proposed control strategy. How to design a quantitative feedback controller to ensure the fixed time stability of nonlinear systems with time delay is our future research direction.

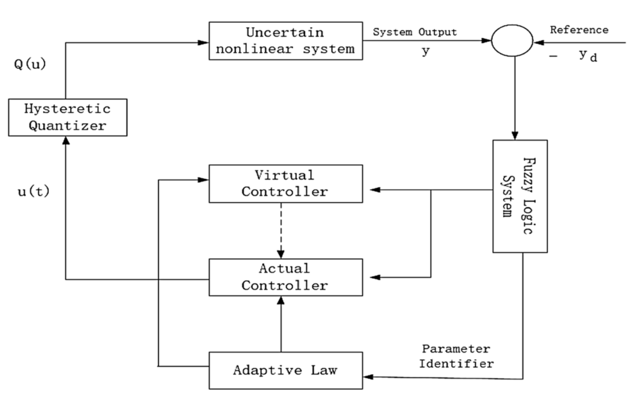

Fig. 2 Control design process

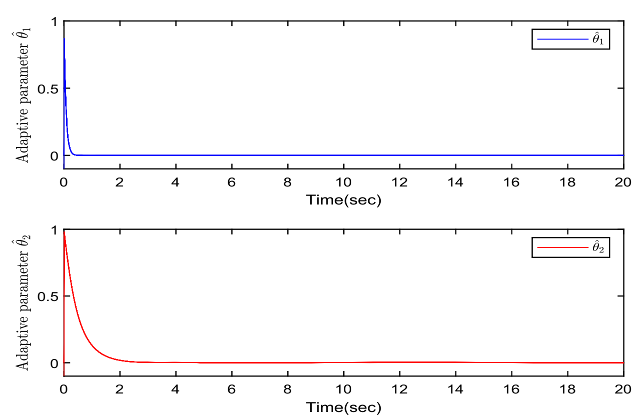

Fig. 5 The adaptive parameter $\hat{\theta}_1$ and $\hat{\theta}_2$

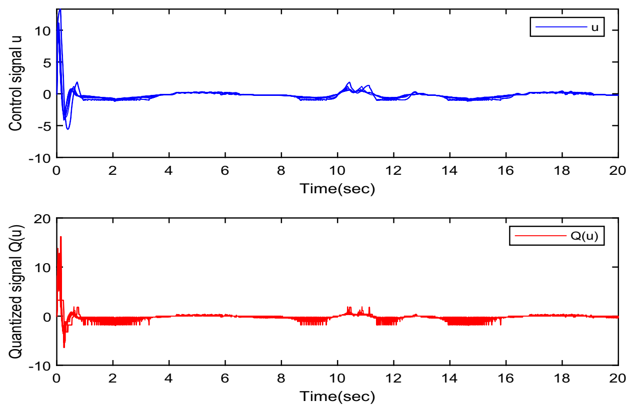

Fig. 6 u and $Q(u)$

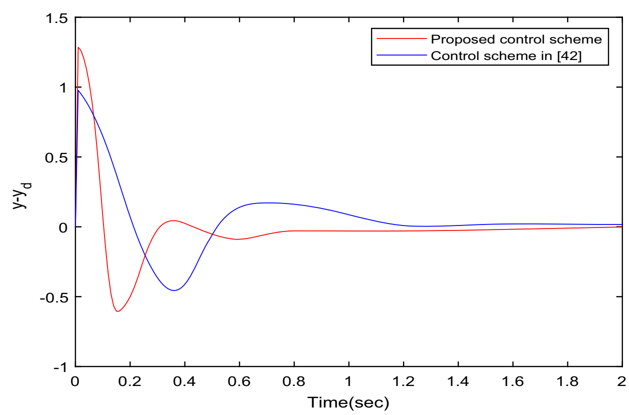

Fig. 7 The tracking error $y - y_d$

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