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Non-weighted Asynchronous H_∞ Filtering for Continuous-Time Switched Fuzzy Systems

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Abstract This paper focuses on the non-weighted asynchronous H_∞ filtering problem for a class of continuous-time switched nonlinear systems. The nonlinearities of subsystems are described by Takagi–Sugeno (T-S) fuzzy models. Using the information of switching instants, the filters are designed to be time-scheduled and separately in the asynchronous and synchronous time intervals. Based on a new time-scheduled fuzzy multiple Lyapunov function (TSFMLF), sufficient conditions are achieved to guarantee the switched filtering error system is globally asymptotically stable with a non-weighted H_∞ performance. Finally, an example is presented to demonstrated the effectiveness of the theoretical results.

Keywords Non-weighted asynchronous H_∞ filtering · Continuous-time switched nonlinear systems · Takagi–Sugeno (T-S) fuzzy models · Time-scheduled fuzzy multiple Lyapunov function (TSFMLF)

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1 Introduction

Researching on switched systems has been widely expanded in recent decades on account of their special characteristics. Such systems consist of several discrete- or continuous-time subsystems and switching laws governing them. Switched systems have great theoretical and practical values and exist extensively in engineering applications, such as dc/dc convertors [1], mobile robots [2], aircraft systems [3] and so on.

On the other hand, nonlinearity exists widely in real systems, so the research on switched nonlinear systems has also attracted the attention of scholars in recent years. In [4], Takagi and Sugeno introduced a T-S fuzzy model, which can approximate the smooth nonlinear systems by blending the local linear models. Now, it is well known as an efficient approach to handle the nonlinearity. Recently, some efforts have extended the T-S fuzzy model to the investigation of switched nonlinear systems and obtained many meaningful results [5–15].

Meanwhile, to obtain reliable state estimates of dynamic systems, the H_∞ filtering problem has also become a hot research issue. Zheng et al., Zhang et al., Zheng and Zhang, Shi et al., Xiang et al. [12–16] have designed the synchronous H_∞ filters for switched systems. However, due to model detection, sensor response delay and other reasons, the filters and subsystems may not be matched immediately in practice systems. Therefore, it is meaningful to research the case of asynchronous filtering [7, 8, 17–21]. It is worth noting that the H_∞ performance indices obtained in the most of above efforts are weighted ones. Referring to [21–26], the non-weighted ones are more anticipated in mathematical analysis and practical use. To the best of our knowledge, there are few efforts on non-weighted asynchronous H_∞ filtering for continuous-time switched T-S fuzzy systems.

In addition, the Lyapunov function is the main tool for analyzing switched systems. Hong et al., Zhang et al., Mahmoud and Shi, Wang et al. [7, 18, 27, 28] have researched the asynchronous H_∞ filtering problems for switched systems based on the multiple Lyapunov function (MLF) approach, which is time-independent. Generally speaking, time-scheduled Lyapunov functions are more flexible and relaxed than the time-independent ones. Shi et al., Xiang et al., Li et al. [8, 21, 29] have used the interpolation to improve the MLF and proposed some novel time-scheduled multiple Lyapunov functions, which can further reduce the conservatism. Aiming at the switched T-S fuzzy systems, Zhang et al., Mao and Zhang, Zheng and Zhang, Zheng et al. [13, 30–32] have introduced the fuzzy multiple Lyapunov functions (FMLF), which are more applicable to the fuzzy characteristic of such systems. However, the proposed FMLFs are still time-independent and have rooms to improve.

The main contribution of our paper can be sorted as follows: (1) The non-weighted asynchronous H_∞ filtering problem for continuous-time switched T-S fuzzy systems is researched. (2) The synchronous and asynchronous filters are time-scheduled and designed separately, which can reduce the conservatism. (3) A new TSFMLF is proposed, which is more general than the FMLF. The remainder of this article is organized as follows: the system models and some preliminaries are introduced in Sect. 2. Section 3 derives out sufficient conditions for non-weighted asynchronous H_∞ filter design. In Sect. 4, a single-link robot arm system is provided as the simulation example. In the end, Sect. 5 concludes the paper.

Notations: $\mathbb{N}(\mathbb{N}^+)$ stands for the set of non-negative (positive) integers. $P \geq 0(>0)$ means that P is a semi-positive definite (positive definite) matrix. $\|\cdot\|$ and $\mathbb{R}^n$ refer to the Euclidean vector norm and n-dimensional Euclidean space, respectively. "$*$" is the ellipsis for the terms that are introduced by symmetry. The superscript "T" represents matrix transposition. $L_2[0,\infty)$ is the space of square integrable infinite sequence. A function $\alpha: [0,\infty) \to [0,\infty)$ is said to be class $\mathcal{K}$ if it is continuous, strictly increasing and $\alpha(0)=0$. Also, a function $\beta: [0,\infty) \to [0,\infty)$ is of class $\mathcal{KL}$ if $\beta(\cdot,t)$ is of class $\mathcal{K}$ for each fixed $t \geq 0$ and $\beta(s,t)$ decreases to 0 as $t \to \infty$ for each fixed $s \geq 0$.

2 System Descriptions and Preliminaries

Consider a class of continuous-time switched nonlinear systems described as follows:

$$\begin{aligned} \dot{x}(t) &= g_{\sigma(t)}(x(t), w(t)), \\ y(t) &= u_{\sigma(t)}(x(t), w(t)), \\ z(t) &= s_{\sigma(t)}(x(t), w(t)). \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ and $y(t) \in \mathbb{R}^{n_y}$ denote the state vector and output vector, respectively. $z(t) \in \mathbb{R}^{n_z}$ is the objective signal to be estimated and $w(t) \in \mathbb{R}^{n_w}$ the disturbance input which belongs to $L_2[0,\infty)$. $\sigma(t)$ is piecewise continuous switching signal which values belong to the finite set $\mathcal{S} = \{1, 2, \ldots, N\}$, where $N \in \mathbb{N}^+$ denotes the number of subsystems. $g_{\sigma(t)}(\cdot), u_{\sigma(t)}(\cdot)$ and $s_{\sigma(t)}(\cdot)$ are nonlinear functions. Besides, the $\sigma(t_k)$ subsystem is said to be activated when $t \in [t_k, t_{k+1})$.

In this paper, using the T-S fuzzy model approach, each subsystem is described by the following IF-THEN fuzzy rules:

Rule p for subsystem i: IF $\theta_{i1}(t)$ is M_{ip1} and $\cdots$ and $\theta_{im}(t)$ is M_{ipm}, then

$$\begin{cases} \dot{x}(t) = A_{ip}x(t) + B_{ip}w(t), \\ y(t) = C_{ip}x(t) + D_{ip}w(t), \\ z(t) = H_{ip}x(t) + L_{ip}w(t), \end{cases} \tag{2}$$

where $i \in \mathcal{S}$ and r_i is the number of fuzzy rules, $\theta_{i1}(t), \ldots, \theta_{im}(t)$ are the premise variable, $M_{ip1}, \ldots, M_{ipm}$ are the fuzzy sets, A_{ip}, B_{ip}, andC_{ip}, D_{ip}, H_{ip}, L_{ip} are real matrices of the pth local model of the ith subsystem.

After "fuzzy blending", one can infer the final model of ith subsystem as

$$\begin{cases} \dot{x}(t) = \sum_{p=1}^{r_i} h_{ip}(t)[A_{ip}x(t) + B_{ip}w(t)], \\ y(t) = \sum_{p=1}^{r_i} h_{ip}(t)[C_{ip}x(t) + D_{ip}w(t)], \\ z(t) = \sum_{p=1}^{r_i} h_{ip}(t)[H_{ip}x(t) + L_{ip}w(t)], \end{cases} \tag{3}$$

where $h_{ip}(t) = \frac{\prod_{n=1}^{m} M_{ipn}(t)}{\sum_{p=1}^{r_i}\prod_{n=1}^{m} M_{ipn}(t)}$ is the normalized membership function and satisfies $h_{ip}(t) \geq 0, \sum_{p=1}^{r_i} h_{ip}(t) = 1$. Besides, let $\mathcal{H}_{ij}(t) = \sum_{p=1}^{r_i}\sum_{q=1}^{r_j} h_{ip}(t)h_{jq}(t)$.

Given a positive scalar τ, we call τ is the minimum dwell time if the switching signals satisfy that $t_{k+1} - t_k \geq \tau, \forall k \in \mathbb{N}^+$. We denote $\mathcal{T}_k$ as the mismatched time between filter and subsystem after each switching occurring. The maximum mismatched time is defined as $\mathcal{T}_{\max}$ and satisfies $0 \leq \mathcal{T}_k \leq \mathcal{T}_{\max} \leq \tau, \forall k \in \mathbb{N}^+$. Without loss of generality, supposed that $\sigma(t_k) = i, \sigma(t_{k-1}) = j$. Then, given a positive scalar ϕ, the full-order fuzzy filter is constructed as follows:

$$\begin{cases} \dot{x}_F(t) = \sum_{p=1}^{r_j} h_{jp}(t)[\bar{A}_{Fjp}(t)x(t) + \bar{B}_{Fjp}(t)y(t)] \\ z_F(t) = \sum_{p=1}^{r_j} h_{jp}(t)[\bar{C}_{Fjp}(t)x(t) + \bar{D}_{Fjp}(t)y(t)], \end{cases} \quad t \in [t_k, t_k + \mathcal{T}_k)$$
$$\begin{cases} \dot{x}_F(t) = \sum_{p=1}^{r_i} h_{ip}(t)[\hat{A}_{Fip}(t)x(t) + \hat{B}_{Fip}(t)y(t)] \\ z_F(t) = \sum_{p=1}^{r_i} h_{ip}(t)[\hat{C}_{Fip}(t)x(t) + \hat{D}_{Fip}(t)y(t)], \end{cases} \quad t \in [t_k + \mathcal{T}_k, t_k + \mathcal{T}_k + \phi) \tag{4}$$
$$\begin{cases} \dot{x}_F(t) = \sum_{p=1}^{r_i} h_{ip}(t)[\hat{A}_{Fip}(\phi)x(t) + \hat{B}_{Fip}(\phi)y(t)] \\ z_F(t) = \sum_{p=1}^{r_i} h_{ip}(t)[\hat{C}_{Fip}(\phi)x(t) + \hat{D}_{Fip}(\phi)y(t)], \end{cases} \quad t \in [t_k + \mathcal{T}_k + \phi, t_{k+1})$$

where $\bar{A}_{Fip}(t)$, $\bar{B}_{Fip}(t)$, $\bar{C}_{Fip}(t)$, $\bar{D}_{Fip}(t)$, $\hat{A}_{Fip}(t)$, $\hat{B}_{Fip}(t)$, $\hat{C}_{Fip}(t)$, $\hat{D}_{Fip}(t)$ are time-scheduled filter gains to be determined. Then, let $\tilde{x}(t) = [x^T(t), x_F^T(t)]^T, e(t) = z(t) - z_F(t)$, the following filtering error system can be obtained:

$$\begin{cases} \dot{\tilde{x}}(t) = \bar{\mathcal{A}}_{ij}(t)x(t) + \bar{\mathcal{B}}_{ij}(t)w(t) \\ e(t) = \bar{\mathcal{C}}_{ij}(t)x(t) + \bar{\mathcal{D}}_{ij}(t)w(t), \end{cases} \quad t \in [t_k, t_k + \mathcal{T}_k)$$
$$\begin{cases} \dot{\tilde{x}}(t) = \hat{\mathcal{A}}_{ii}(t)x(t) + \hat{\mathcal{B}}_{ii}(t)w(t) \\ e(t) = \hat{\mathcal{C}}_{ii}(t)x(t) + \hat{\mathcal{D}}_{ii}(t)w(t), \end{cases} \quad t \in [t_k + \mathcal{T}_k, t_k + \mathcal{T}_k + \phi) \tag{5}$$
$$\begin{cases} \dot{\tilde{x}}(t) = \hat{\mathcal{A}}_{ii}(\phi)x(t) + \hat{\mathcal{B}}_{ii}(\phi)w(t) \\ e(t) = \hat{\mathcal{C}}_{ii}(\phi)x(t) + \hat{\mathcal{D}}_{ii}(\phi)w(t), \end{cases} \quad t \in [t_k + \mathcal{T}_k + \phi, t_{k+1})$$

where

$$\bar{\mathcal{A}}_{ij}(t) = \mathcal{H}_{ij}(t)\begin{bmatrix} A_{ip} & 0 \\ \bar{B}_{Fjq}(t)C_{ip} & \bar{A}_{Fjq}(t) \end{bmatrix}$$
$$\bar{\mathcal{B}}_{ij}(t) = \mathcal{H}_{ij}(t)\begin{bmatrix} B_{ip} \\ \bar{B}_{Fjq}(t)D_{ip} \end{bmatrix}$$
$$\bar{\mathcal{C}}_{ij}(t) = \mathcal{H}_{ij}(t)\begin{bmatrix} H_{ip} - \bar{D}_{Fjq}(t)C_{ip}, & -\bar{C}_{Fjq}(t) \end{bmatrix}$$
$$\bar{\mathcal{D}}_{ij}(t) = \mathcal{H}_{ij}(t)[L_{ip} - \bar{D}_{Fjq}(t)D_{ip}]$$
$$\hat{\mathcal{A}}_{ii}(t) = \mathcal{H}_{ii}(t)\begin{bmatrix} A_{ip} & 0 \\ \hat{B}_{Fiq}(t)C_{ip} & \hat{A}_{Fiq}(t) \end{bmatrix}$$
$$\hat{\mathcal{B}}_{ii}(t) = \mathcal{H}_{ii}(t)\begin{bmatrix} B_{ip} \\ \hat{B}_{Fiq}(t)D_{ip} \end{bmatrix}$$
$$\hat{\mathcal{C}}_{ii}(t) = \mathcal{H}_{ii}(t)\begin{bmatrix} H_{ip} - \hat{D}_{Fiq}(t)C_{ip}, & -\hat{C}_{Fiq}(t) \end{bmatrix}$$
$$\hat{\mathcal{D}}_{ii}(t) = \mathcal{H}_{ii}(t)[L_{ip} - \hat{D}_{Fiq}(t)D_{ip}]$$
$$\hat{\mathcal{A}}_{ii}(\phi) = \mathcal{H}_{ii}(t)\begin{bmatrix} A_{ip} & 0 \\ \hat{B}_{Fiq}(\phi)C_{ip} & \hat{A}_{Fiq}(\phi) \end{bmatrix}$$
$$\hat{\mathcal{B}}_{ii}(\phi) = \mathcal{H}_{ii}(t)\begin{bmatrix} B_{ip} \\ \hat{B}_{Fiq}(\phi)D_{ip} \end{bmatrix}$$
$$\hat{\mathcal{C}}_{ii}(\phi) = \mathcal{H}_{ii}(t)\begin{bmatrix} H_{ip} - \hat{D}_{Fiq}(\phi)C_{ip}, & -\hat{C}_{Fiq}(\phi) \end{bmatrix}$$
$$\hat{\mathcal{D}}_{ii}(\phi) = \mathcal{H}_{ii}(t)[L_{ip} - \hat{D}_{Fiq}(\phi)D_{ip}].$$

Definition 1 [33] A switched system is globally asymptotically stable if under any initial condition $x(t_0)$, there exists a class $\mathcal{KL}$ function $\beta(\cdot)$ such that the inequality $\|x(t)\| \leq \beta(\|x(t_0)\|), \forall t \geq 0$ is satisfied.

Definition 2 [24] For $\bar{\gamma} > 0$, the switched system (1) is said to have a non-weighted L_2 gain no great than $\bar{\gamma}$, if under zero initial condition, the system is globally asymptotically stable and inequality $\int_0^\infty y^T(t)y(t)dt \leq \bar{\gamma}^2 \int_0^\infty w^T(t)w(t)dt$ holds for all $w(t) \in Ł_2[0, \infty)$.

Assumption 1 [23, 26, 34] The switching instants $t_0, \ldots, t_k, \ldots$ can be detected instantaneously online.

Remark 1 In the above assumption, although the time when switching occurs can be instantaneously detected, the activated subsystem at this moment is still unknown and needs some time to identify. Therefore, the subsystem and filter may be mismatched in the identified time interval. This identified time is also be called mode-identifying time in [34]. Besides, the case that the switching instants cannot be detected is also discussed in Corollary 1.

Lemma 1 [35] *Given real matrices U, W and symmetrical matrix G of appropriate dimensions, for any F satisfying* $F^TF \leq I$, *the inequality* $G + UFW + W^TF^T U^T < 0$ *will hold if and only if there exists a scalar* $\varepsilon > 0$ *such that* $G + \varepsilon UU^T + \varepsilon^{-1}W^TW < 0$.

3 Non-weighted L_2 Gain Analysis

The non-weighted L_2 gain analysis for the filtering error system (5) is derived in this section. First, we denote $\bar{T}(s,t)$ and $\hat{T}(s,t)$ as the total mismatched time and matched time for the time interval [s, t], respectively.

Lemma 2 [26] *Given a time interval* [s, t], *the total mismatched time* $\bar{T}(s,t)$ *satisfies*

$$\bar{T}(s,t) \le \left(1 + \frac{t - s - \mathcal{T}_{\max}}{\tau}\right)\mathcal{T}_{\max}. \tag{6}$$

Proof The time interval is shown in Fig. 1. Let N denotes the number of switching in the time interval [s, t], and $t'_1, \ldots, t'_{N+1}$ are the switching instants, $\mathcal{T}'_1, \ldots, \mathcal{T}'_{N+1}$ are mismatched times. From Fig. 1 it can be seen that the total mismatched time $\bar{T}(s,t) = \sum_{i=1}^{N} \mathcal{T}'_i + \bar{T}(t'_N, t) = \sum_{i=1}^{N} \mathcal{T}'_i + \min\{t - t'_N, \mathcal{T}'_{N+1}\} \le N\mathcal{T}_{\max} + \min\{t - t'_N, \mathcal{T}'_{N+1}\}$. Then, when $t - s < N\tau + \mathcal{T}_{\max}$, we choose $\bar{T}(s,t) \le N\mathcal{T}_{\max} + t - t'_N = N\mathcal{T}_{\max} + t - s - (t'_1 - s + t'_2 - t'_1 + \cdots + t'_N - t'_{N-1}) \le N\mathcal{T}_{\max} + t - s - N\tau \le (1 + \frac{t-s-\mathcal{T}_{\max}}{\tau})\mathcal{T}_{\max}$. In the other case when $t - s \ge N\tau + \mathcal{T}_{\max}$, we choose $\bar{T}(s,t) \le N\mathcal{T}_{\max} + \mathcal{T}'_{N+1} \le (N+1)\mathcal{T}_{\max} \le (1 + \frac{t-s-\mathcal{T}_{\max}}{\tau})\mathcal{T}_{\max}$. The proof is completed. □

Lemma 3 *For the switched filtering error system* (5), let $\alpha > 0, \beta > 0, \gamma > 0, \phi > \frac{\alpha \mathcal{T}_{\max}}{\beta}$ *as given scalars, if there exist non-negative functions* $V_{\sigma(t)}(\tilde{x}(t)) : \mathbb{R}^N \to \mathbb{R}$, *such that* $\forall(\sigma(t_k) = i, \sigma(t_{k-1}) = j) \in \mathcal{S} \times \mathcal{S}, i \ne j$,

$$V_i(\tilde{x}(t_k)) \le V_j(\tilde{x}(t_k^-)), \tag{7}$$

$$V_i(\tilde{x}(t_k + \mathcal{T}_k)) \le V_i(\tilde{x}((t_k + \mathcal{T}_k)^-)), \tag{8}$$

$$\dot{V}_i(\tilde{x}(t)) \le \begin{cases} \alpha V_i(\tilde{x}(t)) - \Gamma(t), & t \in \bar{T}(t_k, t_{k+1}) \\ -\beta V_i(\tilde{x}(t)) - \Gamma(t), & t \in \hat{T}(t_k, t_{k+1}) \end{cases} \tag{9}$$

where $\Gamma(t) = e^T(t)e(t) - \gamma^2 w^T(t)w(t)$. *Then, for any switching signal satisfying* $\tau \ge \phi + \mathcal{T}_{\max}$, *the system* (5) *is globally asymptotically stable with a non-weighted* L_2 *gain no greater than*

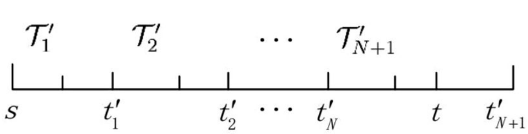

Fig. 1 Time interval

$$\bar{\gamma} = \sqrt{\frac{\beta\tau e^{(\alpha+\beta)\mathcal{T}_{\max}(1 - \frac{\mathcal{T}_{\max}}{\tau})}}{\beta\tau - (\alpha+\beta)\mathcal{T}_{\max}}}\gamma. \tag{10}$$

Proof First, we will prove the stability of the system (5) with $w(t) \equiv 0$. Due to $e^T(t)e(t) \ge 0$, from (9) one can get

$$\dot{V}_i(\tilde{x}(t)) \le \begin{cases} \alpha V_i(\tilde{x}(t)), & t \in \bar{T}(t_k, t_{k+1}) \\ -\beta V_i(\tilde{x}(t)), & t \in \hat{T}(t_k, t_{k+1}) \end{cases}. \tag{11}$$

Combining (9) with (7), (8), for any $k \in \mathbb{N}^+$, we can obtain

$$\begin{aligned} & V_{\sigma(t_{k+1})}(\tilde{x}(t_{k+1})) \\ & \le V_{\sigma(t_k)}(\tilde{x}(t_{k+1})) \\ & \le e^{\alpha\mathcal{T}_k - \beta(t_{k+1} - t_k - \mathcal{T}_k)} V_{\sigma(t_k)}(\tilde{x}(t_k)) \\ & \le e^{(\alpha+\beta)\mathcal{T}_k - \beta\tau} V_{\sigma(t_k)}(\tilde{x}(t_k)) \\ & \le e^{(\alpha+\beta)\mathcal{T}_{\max} - \beta(\phi + \mathcal{T}_{\max})} V_{\sigma(t_k)}(\tilde{x}(t_k)) \\ & < V_{\sigma(t_k)}(\tilde{x}(t_k)). \end{aligned} \tag{12}$$

Therefore, there must exists a constant $0 < \zeta < 1$ such that $V_{\sigma(t_{k+1})}(\tilde{x}(t_{k+1})) < \zeta V_{\sigma(t_k)}(\tilde{x}(t_k)) < \cdots < \zeta^{k+1} V_{\sigma(t_0)}(\tilde{x}(t_0))$. As $t \to +\infty$, $V_{\sigma(t)}(\tilde{x}(t))$ will converge to zero. Then, we can conclude that filtering error system (5) is globally asymptotically stability.

Next, we will derive the non-weighted L_2 gain with disturbance $w(t) \ne 0$. Combining (7) (8) (9), we can get $V(t) \le e^{\alpha\bar{T}(t_0,t) - \beta\hat{T}(t_0,t)} V(t_0) - \int_{t_0}^{t} e^{\alpha\bar{T}(s,t) - \beta\hat{T}(s,t)} \Gamma(s)ds$. Since that $V(t_0) = 0$ and $V(t) \ge 0$, let $t_0 = 0$, one can achieve

$$\begin{aligned} & \int_0^t e^{\alpha\bar{T}(s,t) - \beta\hat{T}(s,t)} e^T(s)e(s)ds \\ & \le \int_0^t e^{\alpha\bar{T}(s,t) - \beta\hat{T}(s,t)} \gamma^2 w^T(s)w(s)ds. \end{aligned} \tag{13}$$

Integrating the left-hand side for t from 0 to ∞, we have

$$\begin{aligned} & \int_0^\infty \int_0^t e^{\alpha\bar{T}(s,t) - \beta\hat{T}(s,t)} e^T(s)e(s)dsdt \\ & = \int_0^\infty \int_0^t e^{(\alpha+\beta)\bar{T}(s,t) - \beta(t-s)} e^T(s)e(s)dsdt \\ & \ge \int_0^\infty \int_0^t e^{-\beta(t-s)} e^T(t)e(t)dsdt \\ & \ge \int_0^\infty e^T(s)e(s)(\int_s^\infty e^{-\beta(t-s)}dt)ds \\ & \ge \frac{1}{\beta}\int_0^\infty e^T(s)e(s)ds. \end{aligned} \tag{14}$$

Similarly, with Lemma 2, integrate of the right-hand side is

$$\begin{aligned}
&\int_0^\infty \int_0^t e^{\alpha\bar{T}(s,t)-\beta\hat{T}(s,t)}\gamma^2 w^T(s)w(s)dsdt \\
&= \gamma^2 \int_0^\infty \int_0^t e^{(\alpha+\beta)\bar{T}(s,t)-\beta(t-s)} w^T(s)w(s)dsdt \\
&\le \gamma^2 \int_0^\infty \int_0^t e^{(\alpha+\beta)(1+\frac{t-s-\mathcal{T}_{\max}}{\tau})\mathcal{T}_{\max}-\beta(t-s)} \\
&w^T(s)w(s)dsdt \\
&\le \gamma^2 e^{(\alpha+\beta)\mathcal{T}_{\max}(1-\frac{\mathcal{T}_{\max}}{\tau})} \\
&\int_0^\infty \int_0^t e^{\frac{(\alpha+\beta)\mathcal{T}_{\max}-\beta\tau}{\tau}(t-s)} w^T(s)w(s)dsdt \\
&\le \frac{\gamma^2 \tau e^{(\alpha+\beta)\mathcal{T}_{\max}(1-\frac{\mathcal{T}_{\max}}{\tau})}}{\beta\tau-(\alpha+\beta)\mathcal{T}_{\max}} \int_0^\infty w^T(s)w(s)ds.
\end{aligned} \tag{15}$$

Combining (13), (14) and (15), one can get $\int_0^\infty e^T(s)e(s)ds \le \frac{\beta\tau e^{(\alpha+\beta)\mathcal{T}_{\max}(1-\frac{\mathcal{T}_{\max}}{\tau})}}{\beta\tau-(\alpha+\beta)\mathcal{T}_{\max}}\gamma^2 \int_0^\infty w^T(s)w(s)ds$. Then, we can achieve the non-weighted L_2 gain $\bar{\gamma}$ as (10). The proof is completed.

Remark 2 It is noting that the filter is designed under minimum dwell time constraint in this paper. Meanwhile, the value of Lyapunov function is non-increasing in switching and matching instant. Under this circumstance, the non-weighted L_2 gain performance can be achieved with the inequality in Lemma 2.

4 Asynchronous H_∞ Filter

This section is aimed to design a filter such that the asynchronous switched filtering error system (5) is globally asymptotically stable with a non-weighted L_2 gain performance.

In this paper, we construct a TSFMLF in following forms:

$$V_i(\tilde{x}(t)) = \tilde{x}^T(t)\mathcal{P}_i(t)\tilde{x}(t),\ \forall\sigma(t_k) = i \in \mathcal{S}, \tag{16}$$

where

$$\mathcal{P}_i(t) = \begin{cases} \bar{P}_i(t) = \sum_{p=1}^{r_i} h_{ip}(t)\bar{P}_{ip}(t), \\ \qquad t \in [t_k, t_k+\mathcal{T}_k), \\ \hat{P}_i(t) = \sum_{p=1}^{r_i} h_{ip}(t)\hat{P}_{ip}(t), \\ \qquad t \in [t_k+\mathcal{T}_k, t_k+\mathcal{T}_k+\phi), \\ \hat{P}_i(\phi) = \sum_{p=1}^{r_i} h_{ip}(t)\hat{P}_{ip}(\phi), \\ \qquad t \in [t_k+\mathcal{T}_k+\phi, t_{k+1}). \end{cases} \tag{17}$$

Considering the asynchronous characteristics, we define three time intervals for each switching: $D_1 = [t_k, t_k+\mathcal{T}_{\max})$, $D_2 = [t_k+\mathcal{T}_k, t_k+\mathcal{T}_k+\phi)$, $D_3 = [t_k+\mathcal{T}_k+\phi, t_{k+1})$. Based on the linear interpolation approach, D_1 and D_2 are divided into $\bar{L}$ and $\hat{L}$ subintervals, respectively. Then, the TSFMLF matrices can be constructed as

$$\bar{P}_{ip}(t) = [1-\bar{\eta}(t)]\bar{P}_{ip,\bar{g}} + \bar{\eta}(t)\bar{P}_{ip,\bar{g}+1}, \tag{18}$$

$$\hat{P}_{ip}(t) = [1-\hat{\eta}(t)]\hat{P}_{ip,\hat{g}} + \hat{\eta}(t)\hat{P}_{ip,\hat{g}+1}, \tag{19}$$

$$\hat{P}_{ip}(\phi) = \hat{P}_{ip,\hat{L}}. \tag{20}$$

where $\bar{g} = 0,1,\ldots,\bar{L}-1, \bar{\eta}(t) = (t-t_k-\bar{\psi}_{\bar{g}})/\bar{h}, \bar{\psi}_{\bar{g}} = \bar{g}\bar{h}, \bar{h} = \mathcal{T}_{\max}/\bar{L}, \hat{g} = 0,1,\ldots,\hat{L}-1, \hat{\eta}(t) = (t-t_k-\mathcal{T}_k-\hat{\psi}_{\hat{g}})/\hat{h}, \hat{\psi}_{\hat{g}} = \hat{g}\hat{h}, \hat{h} = \phi/\hat{L}$.

Remark 3 If we choose $\bar{P}_{ip,0} = \ldots = \bar{P}_{ip,\bar{L}} = \bar{P}_{ip}$, $\hat{P}_{ip,\hat{0}} = \ldots = \hat{P}_{ip,\hat{L}} = \hat{P}_{ip}$, i.e., $\bar{P}_i(t) = \sum_{p=1}^{r_i} h_{ip}(t)\bar{P}_{ip}$, $\hat{P}_i(t) = \hat{P}_{ip}(\phi) = \sum_{p=1}^{r_i} h_{ip}(t)\hat{P}_{ip}$, then the TSFMLF will be reduced to the FMLF in [13, 31, 36], which is time-independent. Therefore, the proposed TSFMLF is more general.

Based on the TSFMLF approach, the following sufficient condition can be achieved.

Theorem 1 *For the switched fuzzy system* (3), *given constants* $\alpha > 0, \beta > 0, \gamma > 0, \mu_{ip} \ge 0, \phi > \frac{\alpha\mathcal{T}_{\max}}{\beta}$ *and positive integers* $\bar{L}, \hat{L}$, *if there exist matrices* $\bar{X}_i$, $\bar{Y}_i$, $\bar{Z}_i$, $\hat{X}_i$, $\hat{Y}_i$, $\hat{Z}_i$, $\bar{P}^1_{ip,\bar{l}}$, $\bar{P}^2_{ip,\bar{l}}$, $\bar{P}^3_{ip,\bar{l}}$, $\hat{P}^1_{ip,\hat{l}}$, $\hat{P}^2_{ip,\hat{l}}$, $\hat{P}^3_{ip,\hat{l}}$, $\bar{A}_{fip,\bar{l}}$, $\bar{B}_{fip,\bar{l}}$, $\bar{C}_{fip,\bar{l}}$, $\bar{D}_{fip,\bar{l}}$, $\hat{A}_{fip,\hat{l}}$, $\hat{B}_{fip,\hat{l}}$, $\hat{C}_{fip,\hat{l}}$, $\hat{D}_{fip,\hat{l}}$, $\bar{l} \in \{0,\ldots,\bar{L}\}$, $\hat{l} \in \{0,\ldots,\hat{L}\}$, *such that* $\forall d \in \{0,1\}$, $v \in \{1,\ldots,r_i-1\}$, $\bar{g} \in \{0,\ldots,\bar{L}-1\}$, $\hat{g} \in \{0,\ldots,\hat{L}-1\}$, $(i,j) \in \mathcal{S}\times\mathcal{S}, i \ne j$

$$|\dot{h}_{ip}(t)| \le \mu_{ip}, \tag{21}$$

$$\bar{P}_{ip,\bar{l}} = \begin{bmatrix} \bar{P}^1_{ip,\bar{l}} & \bar{P}^2_{ip,\bar{l}} \\ * & \bar{P}^3_{ip,\bar{l}} \end{bmatrix} > 0, \tag{22}$$

$$\hat{P}_{ip,\hat{l}} = \begin{bmatrix} \hat{P}^1_{ip,\hat{l}} & \hat{P}^2_{ip,\hat{l}} \\ * & \hat{P}^3_{ip,\hat{l}} \end{bmatrix} > 0, \tag{23}$$

$$\bar{\Omega}_{ijpq,\bar{g}+d}(-\alpha) < 0, \tag{24}$$

$$\hat{\Omega}_{iipq,\hat{g}+d}(\beta) + \hat{\Omega}_{iiqp,\hat{g}+d}(\beta) < 0,\ p \le q, \tag{25}$$

$$\hat{\Omega}_{iipq,\hat{L}}(\beta) + \hat{\Omega}_{iiqp,\hat{L}}(\beta) < 0,\ p \le q, \tag{26}$$

$$\bar{P}_{ip,0} \le \hat{P}_{jq,\hat{L}}, \tag{27}$$

$$\hat{P}_{ip,0} \le \bar{P}_{ip,\bar{l}}, \tag{28}$$

$$\bar{P}_{ir_i,\bar{l}} \le \bar{P}_{iv,\bar{l}}, \tag{29}$$

$$\hat{P}_{ir_i,\hat{l}} \le \hat{P}_{iv,\hat{l}}, \tag{30}$$

where

$$\tilde{\Omega}_{ijpq,m}(\alpha) = \begin{bmatrix} \tilde{\Omega}^{11}_{ij,m} & \tilde{\Omega}^{12}_{ij,m} & \tilde{\Omega}^{13}_{ij,m} & \tilde{\Omega}^{14}_{ij,m} & \tilde{\Omega}^{15}_{ij,m} & \tilde{\Omega}^{16}_{ij,m} \\ * & \tilde{\Omega}^{22}_{ij,m} & \tilde{\Omega}^{23}_{ij,m} & \tilde{\Omega}^{24}_{ij,m} & \tilde{\Omega}^{25}_{ij,m} & \tilde{\Omega}^{26}_{ij,m} \\ * & * & \tilde{\Omega}^{33}_{ij,m} & \tilde{\Omega}^{34}_{ij,m} & \tilde{\Omega}^{15}_{ij,m} & 0 \\ * & * & * & \tilde{\Omega}^{44}_{ij,m} & \tilde{\Omega}^{25}_{ij,m} & 0 \\ * & * & * & * & -\gamma^2 I & \tilde{\Omega}^{56}_{ij,m} \\ * & * & * & * & * & -I \end{bmatrix},$$

$p \in \{1,..,r_i\},\ q \in \{1,\ldots,r_j\}$,

$$\bar{\Psi}^n_m = \sum_{v=1}^{r_i-1} \mu_{iv}(\bar{P}^n_{iv,m} - \bar{P}^n_{ir_i,m}) + \frac{\bar{L}(\bar{P}^n_{ip,\bar{g}+1} - \bar{P}^n_{ip,\bar{g}})}{\mathcal{T}_{\max}},$$

$$\hat{\Psi}^n_m = \begin{cases} \sum_{v=1}^{r_i-1} \mu_{iv}(\hat{P}^n_{iv,m} - \hat{P}^n_{ir_i,m}) \\ + \dfrac{\hat{L}(\hat{P}^n_{ip,\hat{g}+1} - \hat{P}^n_{ip,\hat{g}})}{\phi}, m = \hat{g} + d, \\ \sum_{v=1}^{r_i-1} \mu_{iv}(\hat{P}^n_{iv,m} - \hat{P}^n_{ir_i,m}), m = \hat{L}, \end{cases}$$

$$\tilde{\Omega}^{11}_{ij,m} = A^T_{ip}\tilde{X}^T_j + C^T_{ip}\tilde{B}^T_{fjq,m} + \tilde{X}_j A_{ip} + \tilde{B}_{fjq,m}C_{ip} + \bar{\Psi}^1_m + \alpha\bar{P}^1_{ip,m},$$

$$\tilde{\Omega}^{12}_{ij,m} = A^T_{ip}\tilde{Z}^T_j + C^T_{ip}\tilde{B}^T_{fjq,m} + \tilde{A}_{fjq,m} + \tilde{\Psi}^2_m + \alpha\tilde{P}^2_{ip,m},$$

$$\tilde{\Omega}^{13}_{ij,m} = \tilde{P}^1_{ip,m} - \tilde{X}_j + A^T_{ip}\tilde{X}^T_j + C^T_{ip}\tilde{B}^T_{fjq,m},$$

$$\tilde{\Omega}^{14}_{ij,m} = \tilde{P}^2_{ip,m} - \tilde{Y}_j + A^T_{ip}\tilde{Z}^T_j + C^T_{ip}\tilde{B}^T_{fjq,m},$$

$$\tilde{\Omega}^{15}_{ij,m} = \tilde{X}_j B_{ip} + \tilde{B}_{fjq,m}D_{ip},$$

$$\tilde{\Omega}^{16}_{ij,m} = H^T_{ip} - C^T_{ip}\tilde{D}^T_{fjq,m},$$

$$\tilde{\Omega}^{22}_{ij,m} = \tilde{A}^T_{fjq,m} + \tilde{A}_{fjq,m} + \tilde{\Psi}^3_m + \alpha\tilde{P}^3_{ip,m},$$

$$\tilde{\Omega}^{23}_{ij,m} = (\tilde{P}^2_{ip,m})^T - \tilde{Z}_j + \tilde{A}^T_{fjq,m},$$

$$\tilde{\Omega}^{24}_{ij,m} = \tilde{P}^3_{ip,m} - \tilde{Y}_j + \tilde{A}^T_{fjq,m},$$

$$\tilde{\Omega}^{25}_{ij,m} = \tilde{Z}_j B_{ip} + \tilde{B}_{fjq,m}D_{ip},\ \tilde{\Omega}^{26}_{ij,m} = -\tilde{C}^T_{fjq,m},$$

$$\tilde{\Omega}^{33}_{ij,m} = -\tilde{X}_j - \tilde{X}^T_j,\ \tilde{\Omega}^{34}_{ij,m} = -\tilde{Y}_j - \tilde{Z}^T_j,$$

$$\tilde{\Omega}^{44}_{ij,m} = -\tilde{Y}_j - \tilde{Y}^T_j,\ \tilde{\Omega}^{56}_{ij,m} = L^T_{ip} - D^T_{ip}\tilde{D}^T_{fjq,m}.$$

Then, for any switching signal satisfying $\tau \ge \phi + \mathcal{T}_{\max}$, the filter error system (5) is globally asymptotically stable with a non-weighted L_2 gain no greater than (10). Besides, the filter gains can be achieved as

$$\begin{cases} \begin{bmatrix} \bar{A}_{Fip}(t) & \bar{B}_{Fip}(t) \\ \bar{C}_{Fip}(t) & \bar{D}_{Fip}(t) \end{bmatrix} = [1-\bar{\eta}(t)] \begin{bmatrix} \bar{Y}^{-1}_i \bar{A}_{fip,\bar{g}} & \bar{Y}^{-1}_i \bar{B}_{fip,\bar{g}} \\ \bar{C}_{fip,\bar{g}} & \bar{D}_{fip,\bar{g}} \end{bmatrix} + \bar{\eta}(t) \begin{bmatrix} \bar{Y}^{-1}_i \bar{A}_{fip,\bar{g}+1} & \bar{Y}^{-1}_i \bar{B}_{fip,\bar{g}+1} \\ \bar{C}_{fip,\bar{g}+1} & \bar{D}_{fip,\bar{g}+1} \end{bmatrix}, \\ \begin{bmatrix} \hat{A}_{Fip}(t) & \hat{B}_{Fip}(t) \\ \hat{C}_{Fip}(t) & \hat{D}_{Fip}(t) \end{bmatrix} = [1-\hat{\eta}(t)] \begin{bmatrix} \hat{Y}^{-1}_i \hat{A}_{fip,\hat{g}} & \hat{Y}^{-1}_i \hat{B}_{fip,\hat{g}} \\ \hat{C}_{fip,\hat{g}} & \hat{D}_{fip,\hat{g}} \end{bmatrix} + \hat{\eta}(t) \begin{bmatrix} \hat{Y}^{-1}_i \hat{A}_{fip,\hat{g}+1} & \hat{Y}^{-1}_i \hat{B}_{fip,\hat{g}+1} \\ \hat{C}_{fip,\hat{g}+1} & \hat{D}_{fip,\hat{g}+1} \end{bmatrix}, \\ \begin{bmatrix} \hat{A}_{Fip}(\phi) & \hat{B}_{Fip}(\phi) \\ \hat{C}_{Fip}(\phi) & \hat{D}_{Fip}(\phi) \end{bmatrix} = \begin{bmatrix} \hat{Y}^{-1}_i \hat{A}_{fip,\hat{L}} & \hat{Y}^{-1}_i \hat{B}_{fip,\hat{L}} \\ \hat{C}_{fip,\hat{L}} & \hat{D}_{fip,\hat{L}} \end{bmatrix}. \end{cases} \tag{31}$$

Proof First, we define $\Delta(A,B,P(t),\alpha) = A^T B^T + BA + \dot{P}(t) + \alpha P(t)$. In the mismatched interval when $t \in \bar{T}(t_k, t_{k+1})$, we have

$$\begin{aligned} &\dot{V}_i(\tilde{x}(t)) - \alpha V_i(\tilde{x}(t)) + \Gamma(t) \\ &= [\tilde{x}^T(t), w^T(t)]\left(\begin{bmatrix} \Delta(\bar{\mathcal{A}}_{ij}(t), \bar{P}_i(t), \bar{P}_i(t), -\alpha & \bar{P}_i(t)\bar{\mathcal{B}}_{ij}(t) \\ * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \bar{\mathcal{C}}^T_{ij}(t) \\ \bar{\mathcal{D}}^T_{ij}(t) \end{bmatrix} [\bar{\mathcal{C}}_{ij}(t), \bar{\mathcal{D}}_{ij}(t)] \right) \begin{bmatrix} \tilde{x}(t) \\ w(t) \end{bmatrix} \\ &= \xi(t)^T \left[\bar{\Theta}_1(t) + \bar{\Theta}^T_2(t)\bar{\Theta}_2(t) \right] \xi(t), \end{aligned} \tag{32}$$

where

$$\bar{\Theta}_1(t) = \begin{bmatrix} \Delta(\bar{\mathcal{A}}_{ij}(t), \bar{P}_i(t), \bar{P}_i(t), -\alpha) & \bar{P}_i(t)\bar{\mathcal{B}}_{ij}(t) \\ * & -\gamma^2 I \end{bmatrix},$$

$$\bar{\Theta}_2(t) = [\bar{\mathcal{C}}_{ij}(t), \bar{\mathcal{D}}_{ij}(t)],\ \xi(t) = [\tilde{x}^T(t), w^T(t)]^T.$$

Then, we will introduce the following inequality:

$$\begin{bmatrix} \Delta(\bar{\mathcal{A}}_{ij}(t), \bar{P}_i(t), \bar{P}_i(t), -\alpha) & \bar{P}_i(t)\bar{\mathcal{B}}_{ij}(t) & \bar{\mathcal{C}}^T_{ij}(t) \\ * & -\gamma^2 I & \bar{\mathcal{D}}^T_{ij}(t) \\ * & * & -I \end{bmatrix} < 0. \tag{33}$$

Applying the Schur complement, (33) implies $\bar{\Theta}_1(t) + \bar{\Theta}^T_2(t)\bar{\Theta}_2(t) < 0$, then the inequality $\dot{V}_i(\tilde{x}(t)) - \alpha V_i(\tilde{x}(t)) + \Gamma(t) < 0$ can be obtained.

Next, to design the filter, we define a new matrix:

$$\bar{\Omega}(t) = \begin{bmatrix} \Delta(\bar{\mathcal{A}}_{ij}(t), \bar{E}_j, \bar{P}_i(t), -\alpha) & \bar{P}_i(t) - \bar{E}_j + \bar{\mathcal{A}}_{ij}^T(t)\bar{E}_j^T & \bar{E}_j\bar{\mathcal{B}}_{ij}(t) & \bar{\mathcal{C}}_{ij}^T(t) \\ * & -\bar{E}_j - \bar{E}_j^T & \bar{E}_j\bar{\mathcal{B}}_{ij}(t) & 0 \\ * & * & -\gamma^2 I & \bar{\mathcal{D}}_{ij}^T(t) \\ * & * & * & -I \end{bmatrix} < 0, \quad (34)$$

where $\bar{E}_j = \begin{bmatrix} X_j & Y_j \\ Z_j & Y_j \end{bmatrix}$. Pre- and post-multiplying (34) by $\begin{bmatrix} I & \bar{\mathcal{A}}_{ij}^T(t) & 0 & 0 \\ 0 & \bar{\mathcal{B}}_{ij}^T(t) & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$ and its transpose respectively will get (33). Therefore, we can sum up that if (34) holds, the condition $\dot{V}_i(\tilde{x}(t)) \le \alpha V_i(\tilde{x}(t)) - \Gamma(t)$ of Lemma 3 will hold. Similarly, when $t \in \hat{T}(t_k, t_{k+1})$, we can define matrices $\hat{\Omega}(t), \hat{\Omega}(\phi)$ which have the same structure as $\bar{\Omega}(t)$. If $\hat{\Omega}(t) < 0, \hat{\Omega}(\phi) < 0$ hold, it can be derived out that $\dot{V}_i(\tilde{x}(t)) \le -\beta V_i(\tilde{x}(t)) - \Gamma(t)$ hold.

Then, we will prove the non-weighted H_∞ performance with the filter (31). First, according to $\sum_{p=1}^{r_i} \dot{h}_{ip}(t) = \sum_{v=1}^{r_i-1} \dot{h}_{ip}(t) + \dot{h}_{ir_i}(t) = 0$, with (21) and (29), one can get

$$\begin{aligned} &\sum_{p=1}^{r_i} \dot{h}_{ip}(t)\bar{P}_{ip}(t) \\ &= \sum_{v=1}^{r_i-1} \dot{h}_{iv}(t)\bar{P}_{iv}(t) + \dot{h}_{ir_i}(t)\bar{P}_{ir_i}(t) \\ &= \sum_{v=1}^{r_i-1} \dot{h}_{iv}(t)[\bar{P}_{iv}(t) - \bar{P}_{ir_i}(t)] \\ &= \sum_{v=1}^{r_i-1} \dot{h}_{iv}(t)\{[1-\bar{\eta}(t)](\bar{P}_{iv,\bar{g}} - \bar{P}_{ir_i,\bar{g}}) \\ &+ \bar{\eta}(t)(\bar{P}_{iv,\bar{g}+1} - \bar{P}_{ir_i,\bar{g}+1})\} \\ &\le \sum_{v=1}^{r_i-1} \mu_{iv}\{[1-\bar{\eta}(t)](\bar{P}_{iv,\bar{g}} - \bar{P}_{ir_i,\bar{g}}) \\ &+ \bar{\eta}(t)(\bar{P}_{iv,\bar{g}+1} - \bar{P}_{ir_i,\bar{g}+1})\}. \end{aligned} \quad (35)$$

With (18), we have

$$\dot{\bar{P}}_{ip}(t) = \dot{\bar{\eta}}(t)(\bar{P}_{ip,\bar{g}+1} - \bar{P}_{ip,\bar{g}}) = \frac{\bar{L}(\bar{P}_{ip,\bar{g}+1} - \bar{P}_{ip,\bar{g}})}{\mathcal{T}_{\max}}. \quad (36)$$

Substituting (5), (31), (35) and (36) to $\bar{\Omega}(t)$ and combining with (22) and (24), one can obtain

$$\begin{aligned} &\bar{\Omega}(t) \\ &\le \mathcal{H}_{ij}(t)\{[1-\bar{\eta}(t)]\bar{\Omega}_{ijpq,\bar{g}}(-\alpha) + \bar{\eta}(t)\bar{\Omega}_{ijpq,\bar{g}+1}(-\alpha)\} \\ &< 0, \end{aligned} \quad (37)$$

thus the condition (34) hold. Afterward, from the above proof, we can conclude that $\dot{V}_i(\tilde{x}(t)) \le \alpha V_i(\tilde{x}(t)) - \Gamma(t)$ hold. Similarly, with (21), (23), (25), (26), (30) and (31), one can derive out

$$\begin{aligned} &\hat{\Omega}(t) \\ &\le \mathcal{H}_{ii}(t)\{[1-\hat{\eta}(t)]\hat{\Omega}_{iipq,\hat{g}}(\beta) + \hat{\eta}(t)\hat{\Omega}_{iipq,\hat{g}+1}(\beta)]\} \\ &\le [1-\hat{\eta}(t)][\sum_{p=q=1}^{r_i} h_{ip}^2(t)(\frac{\hat{\Omega}_{iipp,\hat{g}}(\beta) + \hat{\Omega}_{iiqq,\hat{g}}(\beta)}{2}) \\ &\quad + \sum_{p=1}^{r_i}\sum_{q>p}^{r_i} h_{ip}(t)h_{iq}(t)(\hat{\Omega}_{iipq,\hat{g}}(\beta) + \hat{\Omega}_{iiqp,\hat{g}}(\beta))] \\ &\quad + \hat{\eta}(t)[\sum_{p=q=1}^{r_i} h_{ip}^2(t)(\frac{\hat{\Omega}_{iipp,\hat{g}+1}(\beta) + \hat{\Omega}_{iiqq,\hat{g}+1}(\beta)}{2}) \\ &\quad + \sum_{p=1}^{r_i}\sum_{q>p}^{r_i} h_{ip}(t)h_{iq}(t)(\hat{\Omega}_{iipq,\hat{g}+1}(\beta) + \hat{\Omega}_{iiqp,\hat{g}+1}(\beta))] \\ &< 0, \end{aligned} \quad (38)$$

$$\begin{aligned} &\hat{\Omega}(\phi) \\ &\le \mathcal{H}_{ii}(t)\hat{\Omega}_{iipq,\hat{L}}(\beta) \\ &\le \sum_{p=q=1}^{r_i} h_{ip}^2(t)(\frac{\hat{\Omega}_{iipp,\hat{L}}(\beta) + \hat{\Omega}_{iiqq,\hat{L}}(\beta)}{2}) \\ &+ \sum_{p=1}^{r_i}\sum_{q>p}^{r_i} h_{ip}(t)h_{iq}(t)(\hat{\Omega}_{iipq,\hat{L}}(\beta) + \hat{\Omega}_{iiqp,\hat{L}}(\beta)) \\ &< 0, \end{aligned} \quad (39)$$

which can conclude that $\dot{V}_i(\tilde{x}(t)) \le -\beta V_i(\tilde{x}(t)) - \Gamma(t)$ hold.

Finally, with (27) and (28), one can get

$$\begin{aligned} &V_i(\tilde{x}(t_k)) - V_j(\tilde{x}(t_k^-)) \\ &= \tilde{x}^T(t_k)[\bar{P}_i(t_k) - \hat{P}_j(\phi)]\tilde{x}(t_k) \\ &= \mathcal{H}_{ij}(t)\tilde{x}^T(t_k)[\bar{P}_{ip,0} - \hat{P}_{jq,\hat{L}}]\tilde{x}(t_k) \\ &\le 0, \end{aligned} \quad (40)$$

$$\begin{aligned} &V_i(\tilde{x}(t_k + \mathcal{T}_k)) - V_i(\tilde{x}((t_k + \mathcal{T}_k)^-)) \\ &= \tilde{x}^T(t_k + \mathcal{T}_k)[\hat{P}_i(t_k + \mathcal{T}_k) - \bar{P}_i(t_k + \mathcal{T}_k)]\tilde{x}(t_k + \mathcal{T}_k) \\ &= \sum_{p=1}^{r_i} h_{ip}(t)\tilde{x}^T(t_k + \mathcal{T}_k)[\hat{P}_{ip,0} - \bar{P}_{ip,\bar{l}}]\tilde{x}(t_k + \mathcal{T}_k) \\ &\le 0. \end{aligned} \quad (41)$$

Therefore, the conditions (7)–(9) of Lemma 3 are all satisfied. According to Lemma 3, the filter error system (5) is globally asymptotically stable with a non-weighted L_2 gain no greater than $\bar{\gamma}$ as (10) for any switching signal satisfying $\tau \geq \phi + \mathcal{T}_{\max}$. The proof is completed. □

Considering the case when switching instants cannot be detected instantaneously, the filter will not be updated in the mismatched interval after switching occurring. This means that the synchronous and asynchronous filters are not designed separately and will become the traditional forms. On this occasion, the asynchronous filter gains in (4) will become $\bar{A}_{Fjp}(t) = \hat{A}_{Fjp}(\phi)$, $\bar{B}_{Fjp}(t) = \hat{B}_{Fjp}(\phi)$, $\bar{C}_{Fjp}(t) = \hat{C}_{Fjp}(\phi)$, $\bar{D}_{Fjp}(t) = \hat{D}_{Fjp}(\phi)$. Accordingly, the asynchronous Lyapunov function matrix will become $\bar{P}_i(t) = \hat{P}_j(\phi)$. Then, we can get the following corollary:

Corollary 1 *For the switched fuzzy system (3), given constants $\alpha > 0, \beta > 0, \gamma > 0, \mu_{ip} \geq 0, \phi > \frac{\alpha \mathcal{T}_{\max}}{\beta}$ and a positive integer $\hat{L}$, if there exist matrices $\hat{X}_i$, $\hat{Y}_i$, $\hat{Z}_i$, $\hat{P}^1_{ip,\hat{l}}$, $\hat{P}^2_{ip,\hat{l}}$, $\hat{P}^3_{ip,\hat{l}}$, $\hat{A}_{fip,\hat{l}}$, $\hat{B}_{fip,\hat{l}}$, $\hat{C}_{fip,\hat{l}}$, $\hat{D}_{fip,\hat{l}}$, $\hat{l} \in \{0,\ldots,\hat{L}\}$, such that $\forall d \in \{0,1\}$, $v \in \{1,\ldots,r_i-1\}$, $\hat{g} \in \{0,\ldots,\hat{L}-1\}$, $(i,j) \in \mathcal{S}\times\mathcal{S}, i \neq j$*

$$|\dot{h}_{ip}(t)| \leq \mu_{ip}, \tag{42}$$

$$\hat{P}_{ip,\hat{l}} = \begin{bmatrix} \hat{P}^1_{ip,\hat{l}} & \hat{P}^2_{ip,\hat{l}} \\ * & \hat{P}^3_{ip,\hat{l}} \end{bmatrix} > 0, \tag{43}$$

$$\hat{\Omega}_{ijpq,\hat{L}}(-\alpha) < 0, \tag{44}$$

$$\hat{\Omega}_{iipq,\hat{g}+d}(\beta) + \hat{\Omega}_{iiqp,\hat{g}+d}(\beta) < 0, \; p \leq q, \tag{45}$$

$$\hat{\Omega}_{iipq,\hat{L}}(\beta) + \hat{\Omega}_{iiqp,\hat{L}}(\beta) < 0, \; p \leq q, \tag{46}$$

$$\hat{P}_{ip,0} \leq \hat{P}_{jq,\hat{L}}, \tag{47}$$

$$\hat{P}_{ir_i,\hat{l}} \leq \hat{P}_{iv,\hat{l}}. \tag{48}$$

Then, for any switching signal satisfying $\tau \geq \phi + \mathcal{T}_{\max}$, the filter error system (5) is globally asymptotically stable with a non-weighted L_2 gain no greater than (10). Besides, the filter gains can be achieved as

$$\begin{cases} \begin{bmatrix} \hat{A}_{Fip}(t) & \hat{B}_{Fip}(t) \\ \hat{C}_{Fip}(t) & \hat{D}_{Fip}(t) \end{bmatrix} = [1-\hat{\eta}(t)] \begin{bmatrix} \hat{Y}_i^{-1}\hat{A}_{fip,\hat{g}} & \hat{Y}_i^{-1}\hat{B}_{fip,\hat{g}} \\ \hat{C}_{fip,\hat{g}} & \hat{D}_{fip,\hat{g}} \end{bmatrix} + \hat{\eta}(t) \begin{bmatrix} \hat{Y}_i^{-1}\hat{A}_{fip,\hat{g}+1} & \hat{Y}_i^{-1}\hat{B}_{fip,\hat{g}+1} \\ \hat{C}_{fip,\hat{g}+1} & \hat{D}_{fip,\hat{g}+1} \end{bmatrix}, \\ \begin{bmatrix} \hat{A}_{Fip}(\phi) & \hat{B}_{Fip}(\phi) \\ \hat{C}_{Fip}(\phi) & \hat{D}_{Fip}(\phi) \end{bmatrix} = \begin{bmatrix} \hat{Y}_i^{-1}\hat{A}_{fip,\hat{L}} & \hat{Y}_i^{-1}\hat{B}_{fip,\hat{L}} \\ \hat{C}_{fip,\hat{L}} & \hat{D}_{fip,\hat{L}} \end{bmatrix}. \end{cases} \tag{49}$$

Proof Similar proof to Theorem 1 and omitted here. □

Supposed that the synchronous and asynchronous filters are still designed separately. As discussed in Remark 3, we can change the TSFMLF to FMLF in [13, 31, 36], and the corresponding filter gains will become time-independent. Then, based on the FMLF approach, the following corollary can be established.

Corollary 2 *For the switched fuzzy system (3), given constants $\alpha > 0, \beta > 0, \gamma > 0, \mu_{ip} \geq 0, \phi > \frac{\alpha \mathcal{T}_{\max}}{\beta}$ and positive integers $\bar{L}, \hat{L}$, if there exist matrices $\bar{X}_i$, $\bar{Y}_i$, $\bar{Z}_i$, $\hat{X}_i$, $\hat{Y}_i$, $\hat{Z}_i$, $\bar{P}^1_{ip}$, $\bar{P}^2_{ip}$, $\bar{P}^3_{ip}$, $\hat{P}^1_{ip}$, $\hat{P}^2_{ip}$, $\hat{P}^3_{ip}$, $\bar{A}_{fip}$, $\bar{B}_{fip}$, $\bar{C}_{fip}$, $\bar{D}_{fip}$, $\hat{A}_{fip}$, $\hat{B}_{fip}$, $\hat{C}_{fip}$, $\hat{D}_{fip}$, such that $\forall v \in \{1,\ldots,r_i-1\}$, $(i,j) \in \mathcal{S}\times\mathcal{S}, i \neq j$*

$$|\dot{h}_{ip}(t)| \leq \mu_{ip}, \tag{50}$$

$$\bar{P}_{ip} = \begin{bmatrix} \bar{P}^1_{ip} & \bar{P}^2_{ip} \\ * & \bar{P}^3_{ip} \end{bmatrix} > 0, \tag{51}$$

$$\hat{P}_{ip} = \begin{bmatrix} \hat{P}^1_{ip} & \hat{P}^2_{ip} \\ * & \hat{P}^3_{ip} \end{bmatrix} > 0, \tag{52}$$

$$\bar{\Omega}_{ijpq}(-\alpha) < 0, \tag{53}$$

$$\hat{\Omega}_{iipq}(\beta) + \hat{\Omega}_{iiqp}(\beta) < 0, \; p \leq q, \tag{54}$$

$$\bar{P}_{ip} \leq \hat{P}_{jq}, \tag{55}$$

$$\hat{P}_{ip} \leq \bar{P}_{ip}, \tag{56}$$

$$\bar{P}_{ir_i} \leq \bar{P}_{iv}, \tag{57}$$

$$\hat{P}_{ir_i} \leq \hat{P}_{iv}, \tag{58}$$

where $\tilde{\Psi}^n = \sum_{v=1}^{r_i-1} \mu_{iv}(\tilde{P}^n_{iv} - \tilde{P}^n_{ir_i})$, other variables are similar to Theorem 1. Then, for any switching signal satisfying $\tau \geq \phi + \mathcal{T}_{\max}$, the filter error system (5) is globally asymptotically stable with a non-weighted L_2 gain no greater than (10). Besides, the filter gains can be achieved as

$$\begin{cases} \begin{bmatrix} \bar{A}_{Fip} & \bar{B}_{Fip} \\ \bar{C}_{Fip} & \bar{D}_{Fip} \end{bmatrix} = \begin{bmatrix} \bar{Y}_i^{-1}\bar{A}_{fip} & \bar{Y}_i^{-1}\bar{B}_{fip} \\ \bar{C}_{fip} & \bar{D}_{fip} \end{bmatrix}, \\ \begin{bmatrix} \hat{A}_{Fip} & \hat{B}_{Fip} \\ \hat{C}_{Fip} & \hat{D}_{Fip} \end{bmatrix} = \begin{bmatrix} \hat{Y}_i^{-1}\hat{A}_{fip} & \hat{Y}_i^{-1}\hat{B}_{fip} \\ \hat{C}_{fip} & \hat{D}_{fip} \end{bmatrix}. \end{cases} \tag{59}$$

Proof Similar proof to Theorem 1 and omitted here.

If $\mathcal{T}_{\max} = 0$, this means the filters and subsystems are always synchronously matched. Therefore, from Theorem 1, we can also achieve the synchronous H_∞ filter condition based on the TSFMLF approach.

Corollary 3 *For the switched fuzzy system* (3), *given constants* $\beta > 0, \gamma > 0, \mu_{ip} \geq 0$ *and positive integer* $\hat{L}$, *if there exist matrices* $\hat{X}_i$, $\hat{Y}_i$, $\hat{Z}_i$, $\hat{P}^1_{ip,\hat{l}}$, $\hat{P}^2_{ip,\hat{l}}$, $\hat{P}^3_{ip,\hat{l}}$, $\hat{A}_{fip,\hat{l}}$, $\hat{B}_{fip,\hat{l}}$, $\hat{C}_{fip,\hat{l}}$, $\hat{D}_{fip,\hat{l}}$, $\hat{l} \in \{0,\ldots,\hat{L}\}$, *such that* $\forall d \in \{0,1\}$, $v \in \{1,\ldots,r_i-1\}$, $\hat{g} \in \{0,\ldots,\hat{L}-1\}$, $(i,j) \in \mathcal{S}\times\mathcal{S}, i \neq j$

$$|\dot{h}_{ip}(t)| \leq \mu_{ip}, \tag{60}$$

$$\hat{P}_{ip,\hat{l}} = \begin{bmatrix} \hat{P}^1_{ip,\hat{l}} & \hat{P}^2_{ip,\hat{l}} \\ * & \hat{P}^3_{ip,\hat{l}} \end{bmatrix} > 0, \tag{61}$$

$$\hat{\Omega}_{iipq,\hat{g}+d}(\beta) + \hat{\Omega}_{iiqp,\hat{g}+d}(\beta) < 0, \ p \leq q, \tag{62}$$

$$\hat{\Omega}_{iipq,\hat{L}}(\beta) + \hat{\Omega}_{iiqp,\hat{L}}(\beta) < 0, \ p \leq q, \tag{63}$$

$$\hat{P}_{ip,0} \leq \hat{P}_{jq,\hat{L}}, \tag{64}$$

$$\hat{P}_{ir_i,\hat{l}} \leq \hat{P}_{iv,\hat{l}}, \tag{65}$$

where the variables are same as Theorem 1. *Then, for any switching signal satisfying* $\tau \geq \phi$, *the filter error system* (5) *is globally asymptotically stable with a non-weighted* L_2 *gain no greater than* γ. *Besides, the synchronous filter gains can be achieved as*

$$\begin{cases} \begin{bmatrix} \hat{A}_{Fip}(t) & \hat{B}_{Fip}(t) \\ \hat{C}_{Fip}(t) & \hat{D}_{Fip}(t) \end{bmatrix} = [1-\hat{\eta}(t)] \begin{bmatrix} \hat{Y}_i^{-1}\hat{A}_{fip,\hat{g}} & \hat{Y}_i^{-1}\hat{B}_{fip,\hat{g}} \\ \hat{C}_{fip,\hat{g}} & \hat{D}_{fip,\hat{g}} \end{bmatrix} + \hat{\eta}(t) \begin{bmatrix} \hat{Y}_i^{-1}\hat{A}_{fip,\hat{g}+1} & \hat{Y}_i^{-1}\hat{B}_{fip,\hat{g}+1} \\ \hat{C}_{fip,\hat{g}+1} & \hat{D}_{fip,\hat{g}+1} \end{bmatrix}, \\ \begin{bmatrix} \hat{A}_{Fip}(\phi) & \hat{B}_{Fip}(\phi) \\ \hat{C}_{Fip}(\phi) & \hat{D}_{Fip}(\phi) \end{bmatrix} = \begin{bmatrix} \hat{Y}_i^{-1}\hat{A}_{fip,\hat{L}} & \hat{Y}_i^{-1}\hat{B}_{fip,\hat{L}} \\ \hat{C}_{fip,\hat{L}} & \hat{D}_{fip,\hat{L}} \end{bmatrix}. \end{cases} \tag{66}$$

Proof Similar proof to the Theorem 1 and omitted here.□

In practice application, the designed filters may be affected by uncertainty. Furthermore, we consider the following switched T-S fuzzy system contains the parameter uncertainty:

$$\begin{cases} \dot{x}(t) = \sum_{p=1}^{r_i} h_{ip}(t)[(A_{ip} + \Delta A_{ip})x(t) \\ \qquad + (B_{ip} + \Delta B_{ip})w(t)], \\ y(t) = \sum_{p=1}^{r_i} h_{ip}(t)[C_{ip}x(t) + D_{ip}w(t)], \\ z(t) = \sum_{p=1}^{r_i} h_{ip}(t)[H_{ip}x(t) + L_{ip}w(t)]. \end{cases} \tag{67}$$

The uncertainty term is $[\Delta A_{ip}, \Delta B_{ip}] = U_{ip}F_i(t)[W_{1ip}, W_{2ip}]$, where U_{ip}, W_{1ip}, W_{2ip} are known real constant matrices, $F_i(t)$ is an unknown time-varying matrix function and satisfies $F_i^T(t)F_i(t) \leq I$. Then, with the same filter structure as (4), the sufficient condition can be obtained.

Corollary 4 *For the switched fuzzy system* (67), *given constants* $\alpha > 0, \beta > 0, \gamma > 0, \varepsilon > 0, \mu_{ip} \geq 0, \phi > \frac{\alpha\mathcal{T}_{\max}}{\beta}$ *and positive integers* $\bar{L}, \hat{L}$, *if there exist matrices* $\bar{X}_i$, $\bar{Y}_i$, $\bar{Z}_i$, $\hat{X}_i$, $\hat{Y}_i$, $\hat{Z}_i$, $\bar{P}^1_{ip,\bar{l}}$, $\bar{P}^2_{ip,\bar{l}}$, $\bar{P}^3_{ip,\bar{l}}$, $\hat{P}^1_{ip,\hat{l}}$, $\hat{P}^2_{ip,\hat{l}}$, $\hat{P}^3_{ip,\hat{l}}$, $\bar{A}_{fip,\bar{l}}$, $\bar{B}_{fip,\bar{l}}$, $\bar{C}_{fip,\bar{l}}$, $\bar{D}_{fip,\bar{l}}$, $\hat{A}_{fip,\hat{l}}$, $\hat{B}_{fip,\hat{l}}$, $\hat{C}_{fip,\hat{l}}$, $\hat{D}_{fip,\hat{l}}$, $\bar{l} \in \{0,\ldots,\bar{L}\}$, $\hat{l} \in \{0,\ldots,\hat{L}\}$, *such that* $\forall d \in \{0,1\}$, $v \in \{1,\ldots,r_i-1\}$, $\bar{g} \in \{0,\ldots,\bar{L}-1\}$, $\hat{g} \in \{0,\ldots,\hat{L}-1\}$, $(i,j) \in \mathcal{S}\times\mathcal{S}, i \neq j$ *inequalities* (21), (22), (23), (27), (28), (29), (30) *and*

$$\bar{\Upsilon}_{ijpq,\bar{g}+d}(-\alpha) < 0, \tag{68}$$

$$\hat{\Upsilon}_{iipq,\hat{g}+d}(\beta) + \hat{\Upsilon}_{iiqp,\hat{g}+d}(\beta) < 0, \ p \leq q, \tag{69}$$

$$\hat{\Upsilon}_{iipq,\hat{L}}(\beta) + \hat{\Upsilon}_{iiqp,\hat{L}}(\beta) < 0, \ p \leq q, \tag{70}$$

hold, where

$$\tilde{\Upsilon}_{ijpq,m}(\alpha) = \begin{bmatrix} \tilde{\Omega}_{ijpq,m}(\alpha) & \tilde{W}_{ijpq} & \tilde{U}_{ijpq} \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon^{-1}I \end{bmatrix}, \tag{71}$$

$$\tilde{W}_{ijpq} = [W_{1ip}, 0, 0, 0, W_{1ip}, 0]^T, \tag{72}$$

$$\tilde{U}_{ijpq} = [U_{ip}^T\tilde{X}_j^T, U_{ip}^T\tilde{Z}_j^T, U_{ip}^T\tilde{X}_j^T, U_{ip}^T\tilde{Z}_j^T, 0, 0]^T \tag{73}$$

and other variables are same as Theorem 1. *Then, for any switching signal satisfying* $\tau \geq \phi + \mathcal{T}_{\max}$, *the filter error system is globally asymptotically stable with a non-weighted* L_2 *gain no greater than* (10). *Besides, the filter gains can be achieved as* (31).

Proof Applying the Schur complement, with (71), inequality $\tilde{\Upsilon}_{ijpq,m}(\alpha) < 0$ implies

$$\tilde{\Omega}_{ijpq,m}(\alpha) + \varepsilon^{-1}\tilde{W}_{ijpq}\tilde{W}_{ijpq}^T + \varepsilon\tilde{U}_{ijpq}\tilde{U}_{ijpq}^T < 0. \quad (74)$$

Then, according to Lemma 1, one can conclude that if (74) satisfied, then the following inequality will hold:

$$\tilde{\Omega}_{ijpq,m}(\alpha) + \tilde{U}_{ijpq}F_i(t)\tilde{W}_{ijpq}^T + \tilde{W}_{ijpq}F_i^T(t)\tilde{U}_{ijpq}^T < 0. \quad (75)$$

Meanwhile, replacing A_{ip}, B_{ip} with $A_{ip} + \Delta A_{ip}, B_{ip} + \Delta B_{ip}$ in $\tilde{\Omega}_{ijpq,m}(\alpha)$ of Theorem 1, one will get the left-hand side of inequality (75). Therefore, from Theorem 1, this corollary can be proved. □

5 Numerical Example

A practical single-link robot arm system is given in this section to demonstrate the effectiveness of our results. From [5, 8, 15], the system model can be described as follows:

$$\ddot{x}(t) = -\frac{m_i g l}{J_i}sin(x(t)) - \frac{D_i}{J_i}\dot{x}(t) + \frac{1}{J_i}u(t), \quad (76)$$

where $i \in \{1, 2, 3\}$, m_i, D_i and J_i are the mass, damping and inertia of the arm, respectively. $x(t)$ denotes the angle of the arm and we set $x_1(t) = x(t), x_2(t) = \dot{x}(t)$. $u(t)$ is the control input. Referring to [15], the system parameters can be achieved as

$$A_{11} = \begin{bmatrix} 0 & 1 \\ -4.9 & -3 \end{bmatrix}, A_{12} = \begin{bmatrix} 0 & 1 \\ -4.2 & -3 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 0 & 1 \\ -4.3 & -3.4 \end{bmatrix}, A_{22} = \begin{bmatrix} 0 & 1 \\ -2.5 & -3.4 \end{bmatrix},$$
$$A_{31} = \begin{bmatrix} 0 & 1 \\ -5.5 & -3.2 \end{bmatrix}, A_{32} = \begin{bmatrix} 0 & 1 \\ -2.9 & -3.2 \end{bmatrix},$$
$$B_{ip} = [0 \quad 0.1]^T, C_{ip} = [1 \quad 0], D_{ip} = 0.2, H_{ip} = [0 \quad 1],$$
$$L_{ip} = -0.2, \; i \in \{1, 2, 3\}, p \in \{1, 2\},$$

and the normalized membership functions are $h_{i1} = (\sin^2(x_1(t)) + \sin^2(x_2(t)))/2$, $h_{i2} = 1 - h_{i1}$. Using the method of [37], we can set $\mu_{ip} = 4$.

First, given $\alpha = 0.01$, $\beta = 0.02$, $\mathcal{T}_{\max} = 0.1$, $\phi = 0.1$, $\bar{L} = \hat{L} = 1$, $\tau = 0.2$. According to Theorem 1, we will find the minimum non-weighted L_2 gain parameters $\gamma = 0.0531$, $\bar{\gamma} = 0.1063$. Due to the space constraints, the filter gains are omitted here. Assuming the disturbance input $w(t) = 2\sin(\pi t)\exp(-t)$, under the switching signal shown in Fig. 2, the output signal and its estimation are shown in Fig. 3. Figure 4 shows the trajectory of the filtering error. The simulation results verify the effectiveness of our proposed filters.

Then, consider the synchronous H_∞ filtering problem and change $\mathcal{T}_{\max} = 0$. According to Corollary 3, the non-

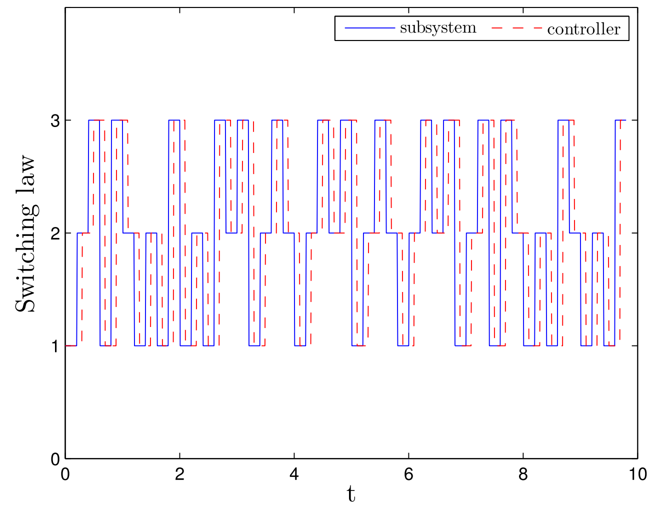

Fig. 2 Switching signal $\sigma(t)$

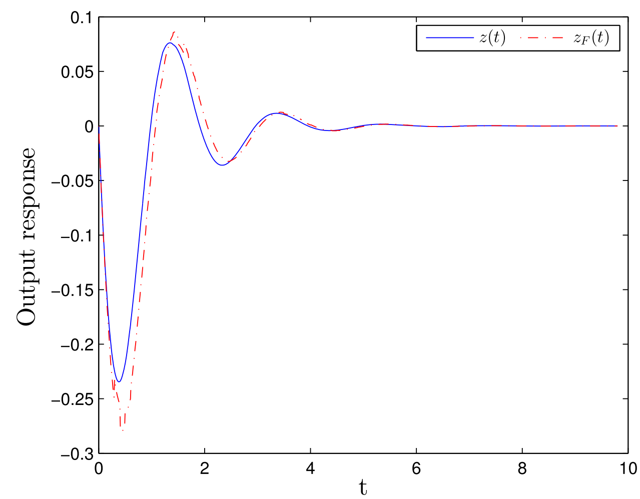

Fig. 3 Output signal $z(t)$ and estimation $z_F(t)$ without uncertainty

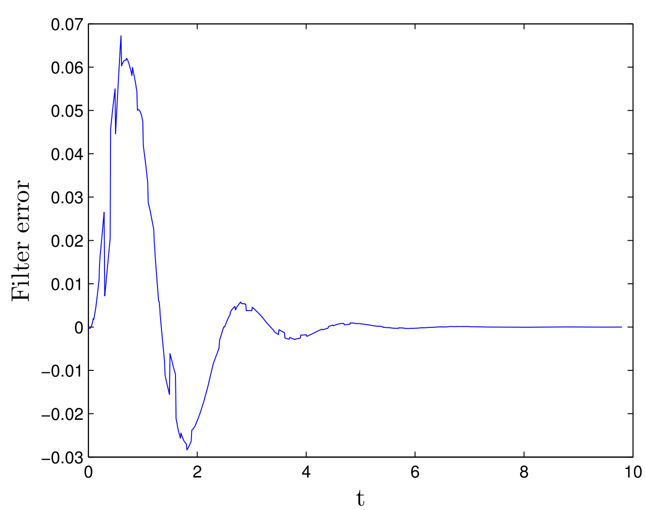

Fig. 4 Filtering error $e(t)$ without uncertainty

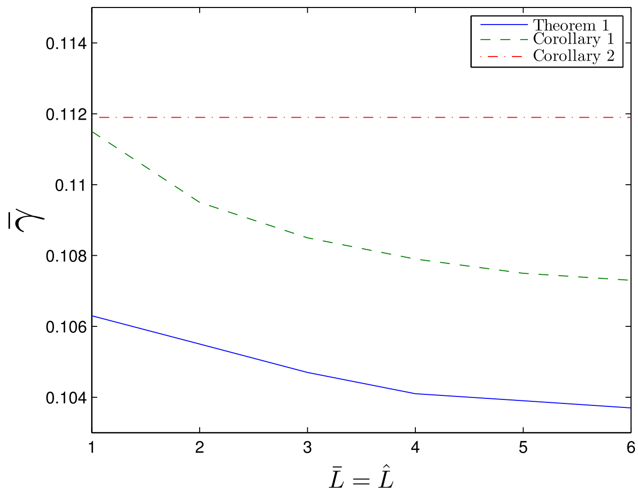

Fig. 5 Optimized non-weighted L_2 gain

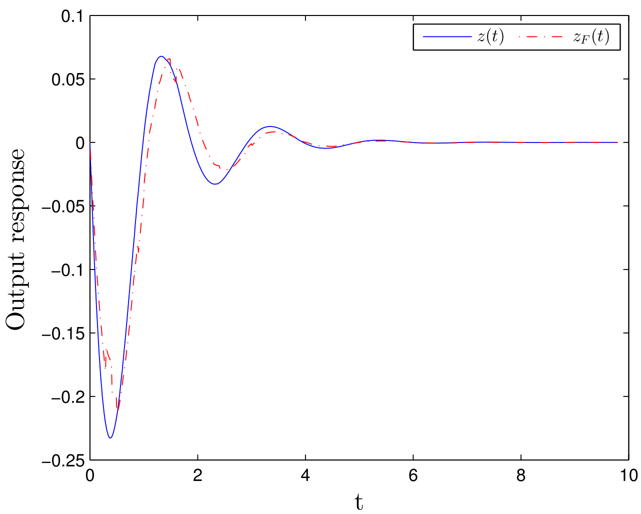

Fig. 6 Output signal $z(t)$ and estimation $z_F(t)$ with uncertainty

weighted L_2 gain can be found as $\bar{\gamma} = 0.0525$, which is less than the asynchronous one. Therefore, the synchronous switching has better non-weighted H_∞ performance.

In addition, if we change $\bar{L}, \hat{L}$, the obtained optimized non-weighted L_2 gains $\bar{\gamma}$ are displayed in Fig. 5. It is obvious that Theorem 1 can get smaller L_2 gains than Corollaries 1 and 2, and thus is less conservative. Actually, Corollaries 1 and 2 can be seen as special cases of Theorem 1. It also reminds us, from the view of practical application, obtaining the information of switching instants online can reduce the conservatism.

Finally, supposed that the switched system has uncertainty and the corresponding parameters are

$$U_{ip} = \begin{bmatrix} 0.1 & 0.5 \\ 0.2 & 0.3 \end{bmatrix}, W_{1ip} = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.2 \end{bmatrix}, W_{2ip} = [0.2 \quad 0]^T,$$

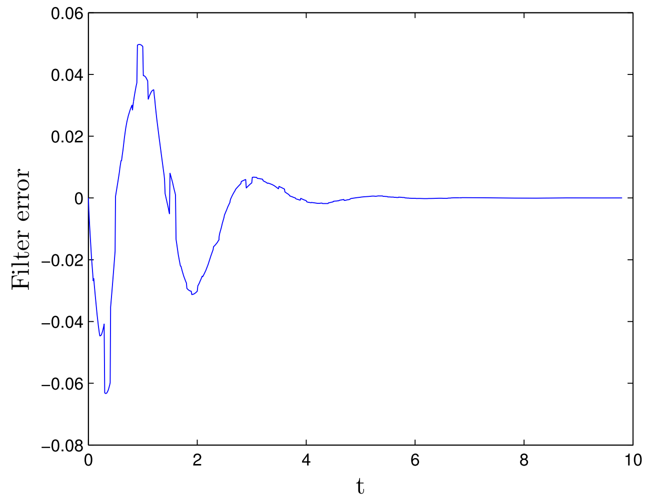

Fig. 7 Filtering error $e(t)$ with uncertainty

where $i \in \{1, 2, 3\}, p \in \{1, 2\}$. Let $\alpha = 0.01$, $\beta = 0.02$, $\varepsilon = 0.5$, $\mathcal{T}_{\max} = 0.1$, $\phi = 0.1$, $\bar{L} = \hat{L} = 1$, $\tau = 0.2$. According to Corollary 3, the feasible solution can be found with $\gamma = 0.3568$, $\bar{\gamma} = 0.7141$. Under the same disturbance and switching signal, the simulation results are displayed in Figs. 6 and 7.

6 Conclusion

This article investigates the non-weighted asynchronous H_∞ filtering problem for continuous-time switched T-S fuzzy systems. First, under the assumption that the switching instants can be detected instantaneously online, we have designed the filters separately in mismatched and matched intervals. Based on this idea, a more general TSFMLF approach is proposed to achieve the sufficient conditions for the filtering error system with non-weighted H_∞ performance. In addition, the conditions under the case when the switching instants cannot be detected instantaneously have also been derived. Comparing these two cases can be known that obtaining the information of switching instants can further reduce the conservatism. Furthermore, we have extended the TSFMLF to research the synchronous switching behavior and parameter uncertainty. Finally, an example is presented to illustrate the effectiveness of our schemes.

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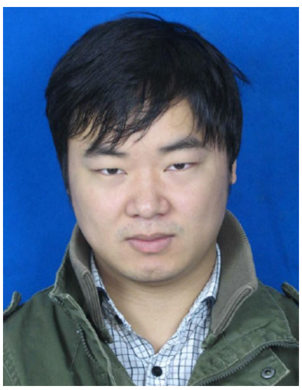
References

1. Ma, Y., Kawakami, H., Tse, C.: Bifurcation analysis of switched dynamical systems with periodically moving borders. IEEE Trans. Circuits Syst. I Regul. Pap. **51**(6), 1184–1193 (2004). https://doi.org/10.1109/TCSI.2004.829240
2. Lee, T.C., Jiang, Z.P.: Uniform asymptotic stability of nonlinear switched systems with an application to mobile robots. IEEE Trans. Autom. Control **53**(5), 1235–1252 (2008). https://doi.org/10.1109/TAC.2008.923688
3. Lian, J., Li, C., Xia, B.: Sampled-data control of switched linear systems with application to an f-18 aircraft. IEEE Trans. Ind. Electron. **64**(2), 1332–1340 (2017). https://doi.org/10.1109/TIE.2016.2618872
4. Takagi, T., Sugeno, M.: Fuzzy identification of systems and its applications to modeling and control. In: Readings in Fuzzy Sets for Intelligent Systems, vol. 1, Elsevier, pp. 387–403, (1993), https://doi.org/10.1016/B978-1-4832-1450-4.50045-6
5. Wang, G., Xie, R., Zhang, H., Yu, G., Dang, C.: Robust exponential H_∞ filtering for discrete-time switched fuzzy systems with time-varying delay. Circuits Syst. Signal Process. **35**(1), 117–138 (2016). https://doi.org/10.1007/s00034-015-0062-0
6. Zhang, M., Shi, P., Liu, Z., Ma, L., Su, H.: H_∞ filtering for discrete-time switched fuzzy systems with randomly occurring time-varying delay and packet dropouts. Signal Process. **143**, 320–327 (2018). https://doi.org/10.1016/j.sigpro.2017.09.009
7. Hong, Y., Zhang, H., Zheng, Q.: Asynchronous H_∞ filtering for switched T-S fuzzy systems and its application to the continuous stirred tank reactor. Int. J. Fuzzy Syst. **20**(5), 1470–1482 (2018). https://doi.org/10.1007/s40815-018-0454-y
8. Shi, S., Fei, Z., Shi, P., Ahn, C.K.: Asynchronous filtering for discrete-time switched T-S fuzzy systems. IEEE Trans. Fuzzy Syst. PP(c):1–11, https://doi.org/10.1109/TFUZZ.2019.2917667(2019)
9. Du, S., Qiao, J., Zhao, X., Wang, R.: Stability and L_1-gain analysis for switched positive T-S fuzzy systems under asynchronous switching. J. Frankl. Inst. **355**(13), 5912–5927 (2018). https://doi.org/10.1016/j.jfranklin.2018.06.005
10. Du, S., Qiao, J.: Stability analysis and L_1-gain controller synthesis of switched positive T-S fuzzy systems with time-varying delays. Neurocomputing **275**, 2616–2623 (2018). https://doi.org/10.1016/j.neucom.2017.11.026
11. Zheng, Q., Xu, S., Zhang, Z.: Nonfragile quantized H_∞ filtering for discrete-time switched T-S fuzzy systems with local nonlinear models. IEEE Trans. Fuzzy Syst. (2020). https://doi.org/10.1109/TFUZZ.2020.2979675
12. Zheng, Q., Guo, X., Zhang, H.: Mixed H_∞ and passive filtering for a class of nonlinear switched systems with unstable subsystems. Int. J. Fuzzy Syst. **20**(3), 769–781 (2018). https://doi.org/10.1007/s40815-017-0340-z
13. Zhang, L., Dong, X., Qiu, J., Alsaedi, A., Hayat, T.: H_∞ filtering for a class of discrete-time switched fuzzy systems. Nonlinear Anal. Hybrid Syst. **14**, 74–85 (2014). https://doi.org/10.1016/j.nahs.2014.05.009
14. Zheng, Q., Zhang, H.: H_∞ filtering for a class of nonlinear switched systems with stable and unstable subsystems. Signal Process. **141**, 240–248 (2017). https://doi.org/10.1016/j.sigpro.2017.06.021
15. Shi, S., Fei, Z., Wang, T., Xu, Y.: Filtering for switched T-S fuzzy systems with persistent dwell time. IEEE Trans. Cybern. **49**(5), 1923–1931 (2019). https://doi.org/10.1109/TCYB.2018.2816982
16. Xiang, W., Xiao, J., Iqbal, M.N.: H_∞ filtering for short-time switched discrete-time linear systems. Circuits Syst. Signal Process. **31**(6), 1927–1949 (2012). https://doi.org/10.1007/s00034-012-9416-z
17. Shi, S., Ma, Y., Ren, S.: Asynchronous filtering for 2-D switched systems with missing measurements. Multidimens. Syst. Signal Process. **30**(2), 543–560 (2019). https://doi.org/10.1007/s11045-018-0569-1
18. Zhang, L., Cui, N., Liu, M., Zhao, Y.: Asynchronous filtering of discrete-time switched linear systems with average dwell time. IEEE Trans. Circuits Syst. I Regul. Pap. **58**(5), 1109–1118 (2011). https://doi.org/10.1109/TCSI.2010.2092151
19. Lian, J., Mu, C., Shi, P.: Asynchronous H_∞ filtering for switched stochastic systems with time-varying delay. Inf. Sci. **224**, 200–212 (2013). https://doi.org/10.1016/j.ins.2012.10.009
20. Xiang, W., Xiao, J.: H_∞ filtering for switched nonlinear systems under asynchronous switching. Int. J. Syst. Sci. **42**(5), 751–765 (2011). https://doi.org/10.1080/00207721.2010.488763
21. Xiang, W., Xiao, J., Mahmoud, M.S.: H_∞ filtering for switched discrete-time systems under asynchronous switching: a dwell-time dependent Lyapunov functional method. Int. J. Adapt. Control Signal Process. **29**(8), 971–990 (2015). https://doi.org/10.1002/acs.2513
22. Xiao, J., Xiang, W.: Convex sufficient conditions on asymptotic stability and L_2 gain performance for uncertain discrete-time switched linear systems. IET Control Theory Appl. **8**(3), 211–218 (2014a). https://doi.org/10.1049/iet-cta.2013.0409
23. Xiao, J., Xiang, W.: New results on asynchronous H_∞ control for switched discrete-time linear systems under dwell time constraint. Appl. Math. Comput. **242**, 601–611 (2014b). https://doi.org/10.1016/j.amc.2014.05.097
24. Zhang, L., Zhuang, S., Shi, P.: Non-weighted quasi-time-dependent H_∞ filtering for switched linear systems with persistent dwell-time. Automatica **54**, 201–209 (2015). https://doi.org/10.1016/j.automatica.2015.02.010
25. Yuan, S., Zhang, L., De Schutter, B., Baldi, S.: A novel Lyapunov function for a non-weighted L_2 gain of asynchronously switched linear system. Automatica **87**, 310–317 (2018). https://doi.org/10.1016/j.automatica.2017.10.018
26. Li, Y., Du, W., Xu, X., Zhang, H., Xia, J.: A novel approach to L_1 filter design for asynchronously switched positive linear systems with dwell time. Int. J. Robust Nonlinear Control **29**(17), 5957–5978 (2019). https://doi.org/10.1002/rnc.4702
27. Mahmoud, M.S., Shi, P.: Asynchronous H_∞ filtering of discrete-time switched systems. Signal Process. **92**(10), 2356–2364 (2012). https://doi.org/10.1016/j.sigpro.2012.02.007
28. Wang, B., Zhang, H., Wang, G., Dang, C.: Asynchronous H_∞ filtering for linear switched systems with average dwell time. Int. J. Syst. Sci. **47**(12), 2783–2791 (2016). https://doi.org/10.1080/00207721.2015.1023758
29. Li, Y., Xiang, W., Zhang, H., Xia, J., Zheng, Q.: New stability conditions for switched linear systems: a reverse-timer-dependent multiple discontinuous Lyapunov function approach. IEEE Trans. Syst. Man Cybern. Syst. pp. 1–12, (2020), https://doi.org/10.1109/TSMC.2019.2963142
30. Mao, Y., Zhang, H.: Exponential stability and robust H_∞ control of a class of discrete-time switched non-linear systems with time-varying delays via T-S fuzzy model. Int. J. Syst. Sci. **45**(5), 1112–1127 (2014). https://doi.org/10.1080/00207721.2012.745025
31. Zheng, Q., Zhang, H.: Asynchronous H_∞ fuzzy control for a class of switched nonlinear systems via switching fuzzy Lyapunov function approach. Neurocomputing **182**, 178–186 (2016). https://doi.org/10.1016/j.neucom.2015.12.037
32. Zheng, Q., Zhang, H., Zheng, D.: Stability and asynchronous stabilization for a class of discrete-time switched nonlinear systems with stable and unstable subsystems. Int. J. Control Autom.

Syst. **15**(3), 986–994 (2017). https://doi.org/10.1007/s12555-016-0301-6
33. Liberzon, D.: Switching in systems and control. Systems & control: foundations & applications. Birkhäuser, Boston (2003). https://doi.org/10.1007/978-1-4612-0017-8
34. Zhang, L., Xiang, W.: Mode-identifying time estimation and switching-delay tolerant control for switched systems: An elementary time unit approach. Automatica **64**, 174–181 (2016). https://doi.org/10.1016/j.automatica.2015.11.010
35. Xiang, Z., Wang, R.: Robust control for uncertain switched nonlinear systems with time delay under asynchronous switching. IET Control Theory Appl. **3**(8), 1041–1050 (2009). https://doi.org/10.1049/iet-cta.2008.0150
36. Tanaka, K., Hori, T., Wang, H.O.: A multiple Lyapunov function approach to stabilization of fuzzy control systems. IEEE Trans. Fuzzy Syst. **11**(4), 582–589 (2003). https://doi.org/10.1109/TFUZZ.2003.814861
37. Tanaka, K., Hori, T., Wang, H.: A fuzzy lyapunov approach to fuzzy control system design. In: Proceedings of American Control Conference, vol. 6, pp. 4790–4795 (2001). https://doi.org/10.1109/ACC.2001.945740



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