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Evaluation Model of Industrial Operation Quality Under Multi-source Heterogeneous Data Information

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Abstract Constructing scientific evaluation system and evaluation methods to make timely quantitative evaluation for regional industrial operation quality is of great practical significance for expediting the new industrialization process and promoting the improvement of national economic operation quality. Aiming at the problem of evaluating the industrial operation quality, this paper constructs a new evaluation system from the perspective of industrial operation performance and industrial development potential, and then proposes a multi-source heterogeneous multi-attribute decision-making method based on the linguistic 2-tuple model to evaluate the industrial operation quality. In this method, the original multi-source heterogeneous data whereby real numbers, interval numbers, and linguistic fuzzy numbers coexist are all transformed into linguistic 2-tuples, then a new ranking method based on grey relational degree of linguistic 2-tuple matrix is presented to rank the level of industrial operation quality for the given cities. Further, a decision-making example of evaluating the industrial operation quality for 14 cities in Hunan Province of China is provided to highlight the implementation, availability, and feasibility of the proposed evaluation model.

Keywords Industrial operation · Quality evaluation · Multi-source heterogeneous data · Multi-source heterogeneous multi-attribute decision-making · Grey relational degree of linguistic 2-tuple matrix

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1 Introduction

Economic globalization is one of the important features of the contemporary world economy and an important trend of world economic development [1–5]. Under the background of economic globalization, China's economy has shifted from a stage of high-speed growth to a stage of high-quality development. Promoting high-quality development is the fundamental requirement for determining development ideas, formulating economic policies and implementing macro-control in the current and future period. China is a major industrial country in the world, and industry plays a dominant role in the national economy. For a long time, China's industrial development attaches importance to speed and scale, while ignoring quality and efficiency. Therefore, accelerating the transformation of the development model towards quality and efficiency provides a new perspective for measuring industrial economy. If we want to achieve high-quality economic development, it is the key to vigorously improve the quality level of industrial economic operation. In order to check the actual operation level of industrial economy and expedite the new industrialization process, constructing scientific evaluation system and evaluation methods to make timely quantitative evaluation of regional industrial economic operation quality has great practical significance.

The scientific and reasonable evaluation index system and comprehensive evaluation methods will provide an important theoretical basis for the industrial management departments to strengthen and formulate various industrial economic policies and improve the scientificity and feasibility of decisions and policies. Industrial operation quality is a comprehensive concept, which needs to be evaluated from multiple dimensions to objectively and comprehensively reflect the actual quality level of industrial operation. In the actual evaluation, the growth rate of industrial added value and growth rate of total investment of industrial enterprises need to be considered, more attention should be paid to the improvement of efficiency, which involves structural optimization, technological progress, environmental improvement, integration of informatization and industrialization, benefits people's livelihood, and so on. To strengthen the quality evaluation of industrial operation, is helpful for the industrial management departments to understand the long-term characteristics, trends, and influencing factors of the industrial economic operation, and deeply analyze the deep problems and contradiction in the industrial economic development, and accurately grasp the objective laws of economic operation. Therefore, the industrial management departments can accurately put forward regional development orientation and target tasks.

The rest of the paper is organized as follows. Section 2 gives the literature review. Section 3 constructs an evaluation index system for evaluating the industrial operation quality from the perspective of industrial operation performance and industrial development potential. Section 4 describes the decision problem. Section 5 proposes a multi-source heterogeneous multi-attribute decision-making (MSHMADM) method based on grey relational degree of linguistic 2-tuple matrix (L2TM-GRD). Section 6 provides a decision-making example of evaluating the industrial operation quality for 14 cities in Hunan Province of China to demonstrate the feasibility and effectiveness of the presented model. Section 7 concludes the paper.

2 Literature Reviews

The extant literature has studied the problem of evaluating industrial operation quality mainly from two aspects, i.e., the construction of evaluation index system, and the design of evaluation methods and models [6–8]. Many classical theories play an important role in the quality evaluation of industrial development, for example, industrial classical theory [9], index system for sustainable development, and index system of industrial international competitiveness [10]. Specially, the United Nations Commission on Sustainable Development has established a well-known index system for sustainable development, which has strong operability and guidance from the four aspects of economy, society, environment, and system, and is a commonly used index system for the later evaluation of sustainable development. In addition, the United Nations Industrial Development Organization [11] issued the 2011 industrial development report in October 2011, which focused on the industrial energy efficiency level, constructed the industrial competitiveness index (CIP) from six aspects, including industrial capacity and manufacturing export capacity, and calculated the industrial competitiveness index of 118 countries and regions from 2005 to 2009.

Since the evaluation problem of industrial operation quality involves multiple evaluation indexes, this problem belongs to a multi-attribute decision-making problem. Moreover, except for some indexes which values are real numbers, there are maybe some indexes with uncertain index values, that is, the index value is an interval number, or a linguistic fuzzy variable, or a hesitant fuzzy number, and so on. Thus, the evaluation problem of industrial operation quality is a problem of uncertain multi-attribute decision-making. For the kind of uncertain multi-attribute decision-making problem, there are many methods and models presented to solve it. For example, Factor Analysis (FA) [12], Principal Component Analysis (PCA) [13], Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) [14, 15], Analytical Hierarchy Process (AHP) [7, 12, 16], Data Envelopment Analysis (DEA) [8], Mahalanobis-Taguchi System (MTS) [17], Strengths Weaknesses Opportunities Threats (SWOT) analysis [6], VlseKriterijuska Optimizacija I Komoromisno Resenje (VIKOR) method [18, 19], Grey Relational Analysis (GRA) method [7, 20, 21], Hesitant fuzzy method [22–26], Double Normalization-based Multiple Aggregation (DNMA) method [27], and so on. In particular, to solve the uncertain multi-attribute decision-making problem of evaluating the industrial operation quality, Yang et al. [16] established an index system of China's industrial transformation, and evaluated the transformation from 2000 to 2009 based on the AHP and the method of weighted average. Wang and Wang [14] presented a new improved TOPSIS method and applied it to evaluate the competitiveness of Chinese high-tech industry using the data from 2011. Wang [7] constructed a multi-level grey relational model by combining the AHP with the grey relational method, and applied it to the quality evaluation of industrial economic operation of Yunnan Province of China. Bakhshi et al. [28] considered the arguments for applying experimental methods to industrial policy measures, and proposed an experimental policy evaluation approach which is called RCT+ to industrial policy evaluation. Bian et al. [8] proposed a two-stage DEA model based on slacks-based measure to evaluate the efficiency of Chinese regional industrial systems, which can estimate the efficiencies of the whole regional industrial system, its production stage and abatement stage simultaneously. Luthra and Mangla [12] recognized the key challenges to Industry 4.0 initiatives and analyzed the identified key challenges to prioritize them for effective Industry 4.0 concepts for supply chain sustainability by using the methods of Explanatory Factor Analysis (EFA) and AHP. Li et al. [29] presented an improved TOPSIS-based approach called the TIWR approach in water quality evaluation. Ren et al. [30] proposed a new fuzzy information form called Normal Wiggly Hesitant Fuzzy Set (NWHFS) for evaluating the environmental quality. Govindan et al. [31] presented a fuzzy analytic network process method to make a barrier evaluation in automotive parts remanufacturing towards cleaner production. Liu et al. [32] used a combined method of grey incidence analysis and grey clustering to evaluate the operation quality of the remanufacturing industry in China. Subsequently, Liu et al. [33] evaluated the remanufacturing industry of China using an improved grey fixed weight clustering method with a case of Jiangsu Province. Arbolino et al. [6] proposed a new approach based on a reappraisal of the classical SWOT analysis to evaluate industrial sustainability achieved at regional level.

With the continuous improvement of China's economic policies, the relevant theories of quality and benefit are becoming more and more mature, and the research on the evaluation of economic operation quality is also constantly improved, and the empirical research is also constantly strengthened. However, with the gradual transformation of the current economic growth stage, there are misunderstanding and deviation for the recognition of the industrial operation quality. For the existing research on quality evaluation of industrial operation, there are some aspects that can be further considered, for example, we can pay more attention to the factors of growth potential, and the indexes on the international competitiveness. At present, there is no unified model and standard for the evaluation index system of industrial economic operation quality. In terms of the evaluation object, most of researches take the country as the main research object, and some of the evaluation indexes are not applicable to some provinces and cities with regional differences, so the evaluation results cannot provide a strong basis for the development policies of the provinces and cities. Under this background, aiming at the problem of evaluating the industrial operation quality, this paper constructs a new evaluation system from the perspective of industrial operation performance and industrial development potential.

In addition, the existing models of evaluating the industrial operation quality in many literatures only consider the evaluation indexes which index values only in the form of real numbers, that is, these methods only can be used to deal with the decision problems with the evaluation criteria values in the forms of real numbers. However, due to the complexity of the decision-making environment, the ambiguity of the human mind and the difficulty of data statistics, it is difficult to obtain complete and accurate data for all evaluation criteria. Thus, many evaluation criteria values are difficult to obtain precisely. In some cases, the values of some evaluation criteria can be given only by a range of values, or an evaluation grade, that is, the criteria values given in the form of interval numbers or linguistic fuzzy numbers such as Very low, Low, Medium, High, and Very High [34, 35]. Thus, in the evaluation criteria values, the real numbers, interval numbers and linguistic fuzzy numbers coexist. We called this kind of data as multi-source heterogeneous data. To deal with the decision problem of evaluating the industrial operation quality under the environment of multi-source heterogeneous data is our main research purpose. This is just the main contribution of the presented decision-making model in this paper comparing with many existing evaluation methods.

3 Evaluation Index System Design of Industrial Operation Quality

In this section, an evaluation index system for evaluating the industrial operation quality was constructed from the perspective of industrial operation performance and industrial development potential.

In 2013, "Implementation Measures for Industrial Operation Quality Evaluation" was enacted and implemented by the Ministry of Industry and Information Technology of the People's Republic of China. In this implementation measures, seven first-level evaluation criteria are proposed for evaluating the industrial operation quality of all provinces and cities, i.e., Steady growth, Running performance, Open competitiveness, Structure optimization, Technology innovation, Integration of informatization and industrialization, and Green promotion. Under these seven first-level evaluation criteria, and drawn from the existing related research [7, 12, 36–40], we take into account fifteen second-level criteria which constitute the evaluation index system of industrial operation quality listed in Table 1.

For the seven first-level criteria, the first three (Steady growth, Running performance, and Open competitiveness) are the evaluation criteria from the perspective of industrial operation performance, and the remaining four (Structure optimization, Technology innovation, Integration of informatization and industrialization, and Green promotion) are the evaluation criteria from the perspective of industrial development potential. Each second-level attribute in Table 1 is briefly described as follows.

B_1 Growth rate of industrial added value (%). Industrial added value above designated scale (annual sales revenue is above ten thousand RMB) refers to the final results of industrial production of industrial enterprises in the form of money during the reporting period, that is, the newly added

Table 1 Evaluation index system of industrial operation quality

First-level criteria	Second-level criteria
Steady growth	B_1 Growth rate of industrial added value (%)
	B_2 Growth rate of total investment of industrial enterprises above a designated scale (%)
	B_3 Growth rate of total profit of industrial enterprises above a designated scale (%)
Running performance	B_4 Industrial labor productivity (Yuan/person)
	B_5 Rate of industrial added value (%)
	B_6 Profit rate of sales (%)
Open competitiveness	B_7 The proportion of industrial export value in sales value (%)
Structure optimization	B_8 Proportion of added value of all industrial parks in industrial added value of one region (%)
	B_9 Proportion of value added of strategic emerging industries in GDP (%)
Technology innovation	B_{10} Investment intensity of industrial R&D (%)
	B_{11} Increment of invention patent authorization of industrial enterprises (piece)
	B_{12} Growth rate of output value of new products (%)
Integration of informatization and industrialization	B_{13} Overall development index for integration of informatization and industrialization
Green promotion	B_{14} Comprehensive utilization rate of industrial solid waste (%)
	B_{15} Degree of industrial environmental pollution

value in the production process of industrial enterprises, which is equal to the total outcome of all production activities minus the value of consumption during production, transferred material goods, and labor value. So we have Growth rate of industrial added value = (The industrial added value in this year − The industrial added value in last year)/The industrial added value in last year) × 100%. The growth rate of industrial added value reflects the growth level of industrial production capacity of a country or a region within a certain period, and is the core standard to measure the scale of industrial operation.

B_2 Growth rate of total investment of industrial enterprises above a designated scale (%). Industrial investment above the designated scale refers to the planned total investment (or actual total investment required) above ten thousand RMB for the project construction, the purchase of fixed assets, and so on. Growth rate of total investment of industrial enterprises above a certain scale = (The industrial total investment in this year − The industrial total investment in last year)/The industrial total investment in last year) × 100%. This index reflects the investment intensity of a country and region to key industrial enterprises in a certain period, and shows the operation and development vitality of industrial enterprises. It is the core index to show the growth rate of expanding production for industrial enterprises.

B_3 Growth rate of total profit of industrial enterprises above a designated scale (%). Total profit of industrial enterprises above a designated scale refers to the surplus of all kinds of incomes after deducting all kinds of consumption in the process of production and operation. It reflects the total profit and loss realized by industrial enterprises above a designated scale in the reporting period. Growth rate of total profit of industrial enterprises above a designated scale = (The total profit in this year − The total profit in last year)/The total profit in last year) × 100%. It is an important index to show the overall strength of industrial enterprises and to reflect the actual profit level of industrial enterprises in the national economy.

B_4 Industrial labor productivity (Yuan/person). Industrial labor productivity refers to the amount of products produced per unit of time by each industrial employee. This index reflects the comprehensive expression of industrial enterprise on production technology level, management level, worker technical proficiency level, and labor enthusiasm. Industrial labor productivity (Yuan/person) = Industrial added value/Total number of employees.

B_5 Rate of industrial added value (%). The rate of industrial added value is a comprehensive embodiment of the profitability and development level of a regional industrial enterprise, and directly reflects the economic benefits of reducing intermediate consumption of industrial enterprises and the effect of input and output, which determines the development level and efficiency of a region. The rate of industrial added value (%) = (The current industrial added value at present price/The total industrial output value at present price) × 100%.

B_6 Profit rate of sales (%). The profit rate of sales refers to the profit gained by unit sales revenue, which is an index to measure the income level of the sales revenue of industrial enterprises, and is a main index to evaluate the final results of the operation activities of industrial enterprises, and a comprehensive embodiment on the final results of all the production processes of industrial enterprises. Profit rate of sales (%) = (The total profit/The total sales revenue) × 100%.

B_7 The proportion of industrial export value in sales value (%) refers to the total value of exported products of industrial enterprises. The proportion of industrial export value in sales value (%) = (The total value of exported products/The total industrial sales value) × 100%. It reflects the scale and level of industrial exports in a region or a country.

B_8 Proportion of added value of all industrial parks in industrial added value of one region (%). The added value of all industrial parks refers to the total value added achieved by industrial enterprises within the scope of all industrial parks. The proportion of added value of all industrial parks in industrial added value of one region = (The total added value of all industrial parks/The total industrial added value of one region) × 100%, which reflects the economic scale and industrial agglomeration level of a region or a country.

B_9 Proportion of value added of strategic emerging industries in GDP (%). The strategic emerging industries are based on important frontier technologies, which represent the future development trend and direction of science and technology as well as the industry. Currently, they are in the growth stage, but have huge development potential, and have a huge leading and driving role for the economy and society. Proportion of value added of strategic emerging industries in GDP (%) = (The total value added of strategic emerging industries in one region/The GDP of one region) × 100%, which reflects the industrial scale and industrial structure optimization level of a region or a country.

B_{10} Investment intensity of industrial R&D (%), refers to the ratio of the total investment of industrial R&D and the total GDP in a region or a country. It is a core index that reflects the investment level of science and technology.

B_{11} Increment of invention patent authorization of industrial enterprises (piece) reflects the actual R&D capability level and investment effectiveness of industrial enterprises in a country or a region within a certain period.

B_{12} Growth rate of output value of new products (%) refers to the growth rate of output value created by newly developed products. Growth rate of output value of new products (%) = (The output value of all new products in the next year − The output value of all new products in the previous year)/The output value of all new products in the previous year × 100%. The higher the growth rate, the stronger the technological innovation ability of enterprises is.

B_{13} Overall development index for integration of informatization and industrialization, comprehensively reflects the integration environment, integration level, and integration benefits of informatization and industrialization, which comes from the weighted sum of three indexes, that is, basic environment index, industrial application index, and application benefit index. In this paper, we use the proportion of added value of information industry in GDP (%) to quantify the overall development index for integration of informatization and industrialization.

B_{14} Comprehensive utilization rate of industrial solid waste (%) is equal to the total comprehensive utilization quantity/(The production quantity of industrial solid waste + The previous storage quantity of industrial solid waste) × 100%, which is the core index to reflect the utilization level of industrial solid waste.

B_{15} Degree of industrial environmental pollution. Industrial pollution refers to the environmental pollution caused by waste gas, waste water, solid emissions, and industrial noise formed in the process of industrial production. Industrial pollution can be divided into waste water pollution, waste gas pollution, waste residue pollution, and noise pollution. Due to the variety and ways of industrial pollution in practice, it is impossible to accurately measure the level of pollution in a city. The level of industrial pollution in a city is generally measured by the regional environmental composite index (ECI). In this paper, according to the value range of ECI, the industrial pollution degree of each city was divided into five grades, i.e., Very low, low, Medium, High, and Very high. The specific rules are given in Table 2 as follows.

The above fifteen second-level criteria form the detailed evaluation criteria to evaluate the level of industrial operation quality for each city, as listed in Table 1. In practical decision-making process, due to the complexity of the decision environment and the difficulty of data statistics, when determining the evaluation values of evaluation criteria, it is difficult to obtain complete and accurate data for all evaluation criteria. That is, a decision maker may provide the evaluation values as the form of real numbers, or interval numbers, or linguistic fuzzy numbers for different evaluation criteria [35, 41–43]. For the fifteen evaluation criteria in Table 1, the scale for performance values is given in the Table 3.

Table 2 The grading standards of industrial pollution degree

Value of ECI	Grades
1–1.9	Very high
2–2.9	High
3–4.9	Medium
5–5.9	Low
$\geq$ 6	Very low

Table 3 Scale and type for evaluation criteria

Criteria	Evaluation scale for performance values	Criteria type
B_1 Growth rate of industrial added value (%)	Real number, or interval number	Benefit
B_2 Growth rate of total investment of industrial enterprises above a designated scale (%)	Real number, or interval number	Benefit
B_3 Growth rate of total profit of industrial enterprises above a designated scale (%)	Real number, or interval number	Benefit
B_4 Industrial labor productivity (Yuan/person)	Real number, or interval number	Benefit
B_5 Rate of industrial added value (%)	Real number, or interval number	Benefit
B_6 Profit rate of sales (%)	Real number, or interval number	Benefit
B_7 The proportion of industrial export value in sales value (%)	Real number, or interval number	Benefit
B_8 Proportion of added value of all industrial parks in industrial added value of one region (%)	Real number, or interval number	Benefit
B_9 Proportion of value added of strategic emerging industries in GDP (%)	Real number, or interval number	Benefit
B_{10} Investment intensity of industrial R&D (%)	Real number, or interval number	Benefit
B_{11} Increment of invention patent authorization of industrial enterprises (piece)	Real number, or interval number	Benefit
B_{12} Growth rate of output value of new products (%)	Real number, or interval number	Benefit
B_{13} Overall development index for integration of informatization and industrialization	Real number, or interval number	Benefit
B_{14} Comprehensive utilization rate of industrial solid waste (%)	Real number, or interval number	Benefit
B_{15} Degree of industrial environmental pollution	Very low, Low, Medium, High, Very high	Cost

Moreover, the above fifteen criteria can be classified into two types, benefit type and cost type (see Table 3). Criteria B_1 to B_{14} are benefit-type criteria, i.e., the higher the criteria value, the better is the corresponding city's industrial operation quality. The rest criterion B_{15} is a cost-type criteria, i.e., the smaller the criteria value, the better is the corresponding city's industrial operation quality.

4 Problem Description

Based on the evaluation index system for evaluating the industrial operation quality constructed in Sect. 3, the problem of evaluating the industrial operation quality is described as follows.

Suppose that we want to evaluate the industrial operation quality for m cities. The set of the cities to be evaluated is denoted as $C = \{C_1, C_2, \ldots, C_m\}$. The evaluation index system is proposed in Sect. 2, and the set of evaluation criteria used to evaluate the industrial operation quality is denoted as $B = \{B_1, B_2, \ldots, B_{15}\}$, and the set of weights for these criteria is denoted as $W = \{w_1, w_2, \ldots, w_{15}\}$, which satisfies $0 \le w_k \le 1$ and $\sum_{k=1}^{15} w_j = 1$.

We can use index data of q years for evaluating the industrial operation quality of all cities, and the ith year is denoted as T_i, $i = 1,2, \ldots, q$. The weight vector of these q years is $V = \{v_1, v_2, \ldots, v_q\}$, which satisfies $0 \le v_i \le 1$ and $\sum_{i=1}^{q} v_i = 1$. For the C_j, $j = 1,2, \ldots, m$, the evaluation value of the evaluation criteria $B_k (k = 1,2, \ldots, 15)$ in the ith year is denoted as $b_{ki}^{(j)}$, which forms the original evaluation matrix R_j of the C_j, $j = 1,2, \ldots, m$, i.e., $R_j = (b_{ki}^{(j)})_{15 \times q}$. From Sect. 2, we know that the values of $b_{ki}^{(j)}$ are either a real number, or an interval number, or a linguistic fuzzy number (see the detailed scale and type for evaluation criteria in Table 3). Thus, the matrix R_j ($j = 1, 2, \ldots, m$) becomes a multi-source heterogeneous data matrix.

Now our goal is to evaluate the industrial operation quality and to give the ranking order for all m cities according to the multi-source heterogeneous data matrices $R_1, R_2, \ldots, R_m$.

5 Evaluation Method of Industrial Operation Quality

From Table 3, as the evaluation values of the evaluation criteria given by the decision maker are the multi-source heterogeneous data information (the real numbers, interval numbers, and linguistic fuzzy numbers coexist), in this section, we now present a multi-source heterogeneous multi-attribute decision-making (MSHMADM) method based on the linguistic 2-tuple model to evaluate the industrial operation quality for the given cities. The linguistic 2-tuple model is first presented by Herrera and Martinez in 2000 [44], which is ideal for the decision-making problems with linguistic assessment information.

In this method, a linguistic fuzzy variable is regarded as a continuous variable within its definitional domain, and a dual combination formed by a linguistic fuzzy variable and a real number is used to express the linguistic assessment information [44, 45].

5.1 Linguistic 2-Tuple

Now we give the relative definitions, operations, and properties of linguistic 2-tuple [20, 35, 44–46], as follows.

Definition 1 [44, 45] A linguistic 2-tuple is denoted as a dual combination (s_k, a_k), which is used to express the linguistic assessment information, where s_k and a_k are defined as follows.

(1) s_k is one element in a predefined linguistic evaluation set $S = \{s_0, s_1, \ldots, s_t\}$, which is formed by $t + 1$ linguistic fuzzy variables $s_0, s_1, \ldots, s_t$, and s_k satisfies the following characteristics.

   (i) Property of ordering, i.e., if $k \geq l$, then $s_k \geq s_l$.
   (ii) Negation operator. $Neg(s_k) = s_l$, where $l = t - k$.

(2) a_k is a numerical value expressing the value of the symbolic translation, such that $a_k \in [-0.5, 0.5)$, which means the deviation between the evaluation result and s_k.

By Definition 1, the five linguistic fuzzy numbers in Table 3, i.e., Very low, Low, Medium, High, and Very high can be regarded as a linguistic evaluation set $S = \{s_0, s_1, s_2, s_3, s_4\}$, where

$$s_0 = \text{Very high}, \quad s_1 = \text{High}, \quad s_2 = \text{Medium}, \quad s_3 = \text{Low}, \quad \text{and } s_4 = \text{Very low}. \tag{1}$$

In fact, we note that these five linguistic fuzzy numbers are the evaluation values for the evaluation criterion B_{15} (Degree of environmental pollution), which is a cost-type criterion, i.e., the smaller the criteria value, the better is the corresponding city's industrial operation quality. Thus, these five linguistic fuzzy numbers are defined by (1).

Based on Definition 1, the comparison operations of linguistic 2-tuples are defined in the following Definition 2.

Definition 2 [44, 45] Suppose that (s_k, a_k) and (s_l, a_l) are any two linguistic 2-tuples, then the comparison operations are defined as follows.

(1) If $k > l$, then $(s_k, a_k) > (s_l, a_l)$.
(2) If $k = l$, then $(s_k, a_k) = (s_l, a_l)$.
(3) If $s_k = s_l$ and $a_k > a_l$, then $(s_k, a_k) > (s_l, a_l)$.
(4) If $s_k = s_l$ and $a_k < a_l$, then $(s_k, a_k) < (s_l, a_l)$.
(5) If $(s_k, a_k) \geq (s_l, a_l)$, then

$$\max\{(s_k, a_k), (s_l, a_l)\} = (s_k, a_k),$$
$$\min\{(s_k, a_k), (s_l, a_l)\} = (s_l, a_l).$$

Further, we give the following definition of distance operator of linguistic 2-tuples.

Definition 3 For any two linguistic 2-tuples A: (s_k, a_k) and B: (s_l, a_l), the distance between A and B is defined as

$$d(A, B) = \frac{|(k + a_k) - (l + a_l)|}{t + 1}, \tag{2}$$

where is $t + 1$ is numbers of the elements in a predefined linguistic evaluation set $S = \{s_0, s_1, \ldots, s_t\}$ defined in Definition 1.

Theorem 1 *For any three linguistic 2-tuples A: (s_k, a_k), B: (s_l, a_l) , and C: (s_g, a_g), the distance operator defined by Eq. (2) satisfies the following three Distance Axioms.*

(i) $d(A, B) \geq 0$, and $d(A, B) = 0 \Leftrightarrow A = B$.
(ii) $d(A, B) = d(B, A)$.
(iii) $d(A, C) \leq d(A, B) + d(B, C)$.

Proof

(i) From Eq. (2), $d(A, B) \geq 0$ is obviously true. Next we prove that $d(A, B) = 0$ is equivalent to $A = B$. On the one hand, if $A = B$, then $k = l$ and $a_k = a_l$, together with

$$d(A, B) = \frac{|(k + a_k) - (l + a_l)|}{t + 1} = \frac{|(k - l) + (a_k - a_l)|}{t + 1},$$

thus we have $d(A, B) = 0$. On the other hand, if $d(A, B) = 0$, then we have

$|(k - l) + (a_k - a_l)| = 0.$

From $k = 0, 1, 2, \ldots, t,$ $l = 0, 1, 2, \ldots, t,$ $-0.5 \leq a_k < 0.5,$ $-0.5 \leq a_l < 0.5,$ so we obtain

$k - l = 0, 1, 2, \ldots, t, \quad -1 < a_k - a_l < 1$

thus $|(k - l) + (a_k - a_l)| = 0$ if and only if $k - l = 0$ and $a_k - a_l = 0$, that is, $A = B$. Otherwise, if $k - l \neq 0$, i.e., $k - l = 1, 2, \ldots, t$, together with $-1 < a_k - a_l < 1$, so we have $|(k - l) + (a_k - a_l)| \neq 0$, which contradicts $d(A, B) = 0$.

(ii) From $\frac{|(k+a_k)-(l+a_l)|}{t+1} = \frac{|(l+a_l)-(k+a_k)|}{t+1}$, so we have $d(A, B) = d(B, A)$.

(iii) Since

$$\begin{aligned} & d(A,B)+d(B,C) \\ & =\frac{|(k+a_k)-(l+a_l)|}{t+1}+\frac{|(l+a_l)0(g+a_g)|}{t+1} \\ & \geq \frac{|[(k+a_k)-(l+a_l)]+[(l+a_l)0(g+a_g)]|}{t+1} \\ & =\frac{|(k+a_k)0(g+a_g)|}{t+1}=d(A,C), \end{aligned}$$

which means (iii) is satisfied. □

5.2 MSHMADM Method Based on Linguistic 2-Tuple

In this section, we present a new MSHMADM method based on the Linguistic 2-tuple given by Sect. 5.1 to evaluate the industrial operation quality for the given cities.

5.2.1 Data Processing

From Sect. 3, we know that R_j (j = 1, 2, …, m) is a multi-source heterogeneous data matrix, that is, the elements in the matrix of $R_j=(b_{ki}^{(j)})_{15\times q}$ are either a real number, or an interval number, or a linguistic fuzzy number. So we can make a normalization processing for the real numbers and the interval numbers first, then transform all multi-source heterogeneous data into linguistic 2-tuples.

(1) *Normalization of data processing* From Table 3, the criteria B_1 to B_{14} are benefit-type criteria. The following two cases are discussed.

(i) When $b_{ki}^{(j)}$ in matrix R_j is a real number, the normalization processing formula is given as follows.

$$c_{ki}^{(j)}=\frac{b_{ki}^{(j)}-\min_j b_{ki}^{(j)}}{\max_j b_{ki}^{(j)}-\min_j b_{ki}^{(j)}} \quad j=1,2,\ldots,m, \quad i=1,2,\ldots,q, \quad k=1,2,\ldots,15 \tag{3}$$

(ii) When $b_{ki}^{(j)}$ in matrix R_j is an interval number denoted as $b_{ki}^{(j)}=[b_{ki}^{(j)L},b_{ki}^{(j)U}]$, by using the following normalization processing formula, $b_{ki}^{(j)}$ becomes $c_{ki}^{(j)}=[c_{ki}^{(j)L},c_{ki}^{(j)U}]$, where

$$\begin{aligned} c_{ki}^{(j)L} &= \frac{b_{ki}^{(j)L}}{\sqrt{\sum_{j=1}^{m}(b_{ki}^{(j)L})^2}}, \\ c_{ki}^{(j)U} &= \frac{b_{ki}^{(j)U}}{\sqrt{\sum_{j=1}^{m}(b_{ki}^{(j)U})^2}} \end{aligned} \tag{4}$$

$$j=1,2,\ldots,m, \quad i=1,2,\ldots,q, \quad k=1,2,\ldots,15$$

By using Eqs. (3) and (4) to process the multi-source heterogeneous data of matrix $R_j=(b_{ki}^{(j)})_{15\times q}$, we can get the normalized matrix $Z_j=(c_{ki}^{(j)})_{15\times q}$. Obviously, when $k=1,2,\ldots,14$, $c_{ki}^{(j)}$ in matrix Z_j is a real number or an interval number; when $k=15$, $c_{ki}^{(j)}$ in matrix Z_j is a linguistic fuzzy variable $s_l\,(l=0,1,2,3,4)$ defined by Eq. (1).

(2) *Data transformation methods* From the normalized matrix $Z_j=(c_{ki}^{(j)})_{15\times q}$, now the methods of transforming the real numbers, interval numbers, and linguistic fuzzy variables into the linguistic 2-tuples are given, respectively, one by one.

(i) Data transformation between a real number and a linguistic 2-tuple

For a real number, the methods given by the following Definitions 4 and 5 can be used to transform it to a linguistic 2-tuple.

Definition 4 [44–46] Suppose that $S=\{s_0,s_1,\ldots,s_t\}$ is a known linguistic evaluation set, and $\beta\in[0,1]$ is a real number which is a value supporting the result of a symbolic aggregation operation, then $\beta\in[0,1]$ can be transformed into an equivalent linguistic 2-tuple by the following function Δ:

$$\begin{aligned} &\Delta:[0,1]\rightarrow S\times[-0.5,0.5), \\ &\Delta(\beta)=(s_k,a_k), \end{aligned}$$

where

$$\begin{cases} k=rnd(\beta\cdot t) \\ a_k=\beta\cdot t-k, \quad a_k\in[-0.5,0.5) \end{cases}$$

and “rnd” is the usual rounding operation. Conversely, from Definition 4 we can conclude that there is an inverse function Δ^{-1} of Δ such that from a 2-tuple (s_k,a_k) it returns its equivalent numerical value $\beta\in[0,1]$, i.e.,

$$\begin{aligned} &\Delta^{-1}:S\times[-0.5,0.5)\rightarrow[0,1], \\ &\Delta^{-1}(s_k,a_k)=\frac{k+a_k}{t}=\beta. \end{aligned}$$

Generally, for a real number $\beta \in [0, t]$, the following Definition 5 gives the transformation method between $\beta \in [0, t]$ and its corresponding linguistic 2-tuple.

Definition 5 [44–46] Suppose that $S = \{s_0, s_1, \ldots, s_t\}$ is a known linguistic evaluation set, and $\beta \in [0, t]$ is a real number which is a value supporting the result of a symbolic aggregation operation, then β can be transformed into an equivalent linguistic 2-tuple by the following function Δ:

$$\Delta : [0, t] \to S \times [-0.5, 0.5), \Delta(\beta) = (s_k, a_k),$$

where

$$\begin{cases} k = rnd(\beta) \\ a_k = \beta - k, \quad a_k \in [-0.5, 0.5) \end{cases}$$

and "rnd" is the usual rounding operation. Conversely, for a known linguistic 2-tuple (s_k, a_k), there is an inverse function Δ^{-1} such that from a 2-tuple (s_k, a_k) it returns its equivalent numerical value $\beta \in [0, t]$, i.e.,

$$\Delta^{-1} : S \times [-0.5, 0.5) \to [0, t],$$
$$\Delta^{-1}(s_k, a_k) = k + a_k = \beta.$$

(ii) Data transformation between an interval number and a linguistic 2-tuple

For an interval number $I = [a, b]$, the method given by the following Definition 6 can be used to transform it to a linguistic 2-tuple.

Definition 6 [45, 47] Suppose that $S = \{s_0, s_1, \ldots, s_t\}$ is a known linguistic evaluation set, and $I = [a, b]$ is an interval number, the intersection of interval number $I = [a, b]$ and each linguistic fuzzy variable s_k in S is denoted as

$$r_k = \max_x \min\{\mu_I(x), \mu_{s_k}(x)\}, \quad k \in [0, 1, \ldots, t]$$

where $\mu_I(x)$ and $\mu_{s_k}(x)$ are the membership function of I and s_k, respectively, thus the value of $\beta = \frac{\sum_{k=0}^{t-1} k \cdot r_k}{\sum_{k=0}^{t-1} r_k}$ is the equivalent numerical value of a 2-tuple (s_k, a_k). Then we can use the method given by Definition 5 to transform β into the corresponding 2-tuple (s_k, a_k).

In Definition 6, the membership function of interval number $I = [a, b]$ can be expressed by

$$\mu_I(x) = \begin{cases} 1 & x \in [a, b] \\ 0 & \text{else} \end{cases},$$

and the membership function $\mu_{s_k}(x)$ of s_k can be determined as follows. When $t = 4$, $S = \{s_0, s_1, \ldots, s_4\} = \{(0, 0, 0.25), (0, 0.25, 0.5), (0.25, 0.5, 0.75), (0.5, 0.75, 1), (0.75, 1, 1)\}$, and the membership function $\mu_{s_k}(x)$ of a triangular fuzzy number $s_k = (b_k, c_k, d_k)$ can be expressed by

$$\mu_{s_k}(x) = \begin{cases} \frac{x - b_k}{c_k - b_k} & x \in [b_k, c_k] \\ \frac{x - d_k}{c_k - d_k} & x \in [c_k, d_k] \\ 0 & \text{else} \end{cases}.$$

(iii) Data transformation between a linguistic fuzzy variable and a linguistic 2-tuple

For a linguistic fuzzy variable, the methods given by the following Definition 7 can be used to transform it to a linguistic 2-tuple.

Definition 7 [44] Let $s_k \in S$ be a linguistic fuzzy variable, then its corresponding linguistic 2-tuple can be determined by the following function θ.

$$\theta : S \to S \times [-0.5, 0.5), \quad \theta(s_k) = (s_k, 0), s_k \in S.$$

Definition 7 shows that the corresponding linguistic 2-tuple for a linguistic fuzzy variable $s_k \in S$ is just $(s_k, 0)$.

By using Definitions 4, 5, 6, and 7, the normalized matrix $Z_j = (c_{ki}^{(j)})_{15 \times q}$ can be transformed into a linguistic 2-tuple matrix, which is denoted by $E_j = [(s_{ki}^{(j)}, a_{ki}^{(j)})]_{15 \times q}$, $j = 1, 2, \ldots, m$.

5.2.2 Ranking Method Based on L2TM-GRD

5.2.2.1 L2TM-GRD From the linguistic 2-tuple matrices $E_1, E_2, \ldots, E_m$, we present a new grey relational degree of linguistic 2-tuple matrix (L2TM-GRD) based on the traditional grey relational analysis (GRA) method [48, 49] to evaluate and rank the industrial operation quality for all m cities. Next we give the definition of L2TM-GRD.

Definition 8 Let E be a grey relational factor set, $E_0 \in E$ be a reference factor matrix, $E_j \in E$ be a comparison factor matrix, $j = 1, 2, \ldots, m$, where E_0 and E_j are all formed by linguistic 2-tuples, i.e.,

$$E_0 = \begin{bmatrix} (s_{11}^{(0)}, a_{11}^{(0)}) & (s_{12}^{(0)}, a_{12}^{(0)}) & \cdots & (s_{1q}^{(0)}, a_{1q}^{(0)}) \\ (s_{21}^{(0)}, a_{21}^{(0)}) & (s_{22}^{(0)}, a_{22}^{(0)}) & \cdots & (s_{2q}^{(0)}, a_{2q}^{(0)}) \\ \vdots & \vdots & & \vdots \\ (s_{p1}^{(0)}, a_{p1}^{(0)}) & (s_{p2}^{(0)}, a_{p2}^{(0)}) & \cdots & (s_{pq}^{(0)}, a_{pq}^{(0)}) \end{bmatrix},$$

$$E_1 = \begin{bmatrix} (s_{11}^{(1)}, a_{11}^{(1)}) & (s_{12}^{(1)}, a_{12}^{(1)}) & \cdots & (s_{1q}^{(1)}, a_{1q}^{(1)}) \\ (s_{21}^{(1)}, a_{21}^{(1)}) & (s_{22}^{(1)}, a_{22}^{(1)}) & \cdots & (s_{2q}^{(1)}, a_{2q}^{(1)}) \\ \vdots & \vdots & & \vdots \\ (s_{p1}^{(1)}, a_{p1}^{(1)}) & (s_{p2}^{(1)}, a_{p2}^{(1)}) & \cdots & (s_{pq}^{(1)}, a_{pq}^{(1)}) \end{bmatrix},$$

$\cdots\cdots$

$$E_m = \begin{bmatrix} (s_{11}^{(m)}, a_{11}^{(m)}) & (s_{12}^{(m)}, a_{12}^{(m)}) & \cdots & (s_{1q}^{(m)}, a_{1q}^{(m)}) \\ (s_{21}^{(m)}, a_{21}^{(m)}) & (s_{22}^{(m)}, a_{22}^{(m)}) & \cdots & (s_{2q}^{(m)}, a_{2q}^{(m)}) \\ \vdots & \vdots & & \vdots \\ (s_{p1}^{(m)}, a_{p1}^{(m)}) & (s_{p2}^{(m)}, a_{p2}^{(m)}) & \cdots & (s_{pq}^{(m)}, a_{pq}^{(m)}) \end{bmatrix},$$

and suppose that $r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)}))$ is a real number, w_k and v_i are the weights of the kth row vector and the ith column vector, respectively, which satisfy

$$0 \le w_k \le 1, \quad \sum_{k=1}^{15} w_k = 1, \quad 0 \le v_i \le 1 \quad \text{and} \quad \sum_{i=1}^{q} v_i = 1.$$

and let $\Delta_{ki}^{(0j)}$ be the difference information between $(s_{ki}^{(0)}, a_{ki}^{(0)})$ and $(s_{ki}^{(j)}, a_{ki}^{(j)})$, where $\Delta_{ki}^{(0j)} = d((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)}))$, which is defined by Eq. (1). If

$$r(E_0, E_j) = \sum_{k=1}^{p} \sum_{i=1}^{q} w_k v_i r\left(\left(s_{ki}^{(0)}, a_{ki}^{(0)}\right), \left(s_{ki}^{(j)}, a_{ki}^{(j)}\right)\right)$$

satisfies the following four conditions (Four Axioms of GRA [48, 49]), then $r(E_0, E_j)$ is called a grey relational degree of linguistic 2-tuple matrix (L2TM-GRD) of E_0 to E_j, and the $r\left(\left(s_{ki}^{(0)}, a_{ki}^{(0)}\right), \left(s_{ki}^{(j)}, a_{ki}^{(j)}\right)\right)$ is called a grey relational coefficient of linguistic 2-tuple matrix (L2TM-GRC).

(i) Normality: $0 \le r(E_0, E_j) \le 1$, $r(E_0, E_j) = 0 \Leftrightarrow E_0, E_j \in \varphi$(empty set), $r(E_0, E_j) = 1 \Leftrightarrow E_0 = E_j$.

(ii) Symplectic symmetry: $E_i, E_j \in E$, $r(E_i, E_j) = r(E_j, E_i) \Leftrightarrow E = \{E_i, E_j\}$.

(iii) Wholeness: $E_i, E_j \in E = \{E_\sigma | \sigma = 1, 2, \cdots, m\}$, $m \ge 2$, $r(E_i, E_j) \overset{often}{\neq} r(E_j, E_i)$

(iv) Approachability: The smaller the difference information $\Delta_{ki}^{(0j)}$ is, the greater the value of $r\left(\left(s_{ki}^{(0)}, a_{ki}^{(0)}\right), \left(s_{ki}^{(j)}, a_{ki}^{(j)}\right)\right)$ is.

Based on Definition 8, the following Theorem 1 can be deduced.

Theorem 2 *For the given linguistic 2-tuple matrices E_0 and E_j, and the given weights w_k and v_i, if*

$$r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)})) = \frac{\Delta_{\min} + \rho\Delta_{\max}}{\Delta_{ki}^{(0j)} + \rho\Delta_{\max}}, \tag{5}$$

$$r(E_0, E_j) = \sum_{k=1}^{p} \sum_{i=1}^{q} w_k v_i r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)})), \tag{6}$$

where $\Delta_{\min} = \min_j \min_k \min_i \Delta_{ki}^{(0j)}$ and $\Delta_{\max} = \max_j \max_k \max_i \Delta_{ki}^{(0j)}$ are the three-level minimum difference and three-level maximum difference, respectively, ρ is a distinguishing coefficient, and $\rho \in (0, 1)$ (Generally $\rho = 0.5$), then $r(E_0, E_j)$ satisfies the Four Axioms of GRA, that is, the $r(E_0, E_j)$ defined by Eqs. (5) and (6) is a L2TM-GRD.

Proof

(i) From $\Delta_{\min} = \min_j \min_k \min_i \Delta_{ki}^{(0j)}$, we have $0 \le \Delta_{\min} \le \Delta_{ki}^{(0j)}$, thus

$$0 \le r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)})) = \frac{\Delta_{\min} + \rho\Delta_{\max}}{\Delta_{ki}^{(0j)} + \rho\Delta_{\max}} \le 1,$$

together with $0 \le w_k \le 1$, $\sum_{k=1}^{15} w_k = 1$, $0 \le v_i \le 1$, and $\sum_{i=1}^{q} v_i = 1$ we can obtain

$$0 \le r(E_0, E_j) = \sum_{k=1}^{p} \sum_{i=1}^{q} w_k v_i r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)})) \le 1.$$

When $r(E_0, E_j) = 0$, if and only if $r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)})) = 0$ for any i, j and k, together with $\rho = 0.5$, $\Delta_{\max} \ge 0$, and $\Delta_{\min} \ge 0$, thus $\Delta_{\max} = \Delta_{\min} = 0$, that is, $\Delta_{ki}^{(0j)} = d((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)})) = 0$, so E_0 is just E_j, which contradicts the condition $r(E_0, E_j) = 0$. In fact, $r(E_0, E_j) = 0$ means E_0 is completely irrelevant with E_j. In conclusion, $r(E_0, E_j) = 0$ if and only if $E_0, E_j \in \phi$

If $\Delta_{ki}^{(0j)} = \Delta_{\min}$, that is $E_0 = E_j$, then $r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)})) = 1$, together with $0 \le w_k \le 1$, $\sum_{k=1}^{15} w_k = 1$, $0 \le v_i \le 1$, and $\sum_{i=1}^{q} v_i = 1$, thus $r(E_0, E_j) = 1$. Moreover, if $\Delta_{ki}^{(0j)} \ne \Delta_{\min}$, then $\Delta_{ki}^{(0j)} > \Delta_{\min}$, thus $\Delta_{\min} + \rho\Delta_{\max} < \Delta_{ki}^{(0j)} + \rho\Delta_{\max}$, which means $0 \le r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)})) = \frac{\Delta_{\min} + \rho\Delta_{\max}}{\Delta_{ki}^{(0j)} + \rho\Delta_{\max}} < 1$. In a word, $r(E_0, E_j) = 1 \Leftrightarrow E_0 = E_j$.

(ii) If $E = \{E_0, E_1\}$, then

$$\Delta_{ki}^{(01)} = d((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(1)}, a_{ki}^{(1)})) = d((s_{ki}^{(1)}, a_{ki}^{(1)}), (s_{ki}^{(0)}, a_{ki}^{(0)})) = \Delta_{ki}^{(10)},$$

and

$$\max_l \max_k \max_i \Delta_{ki}^{(0l)} = \max_l \max_k \max_i \Delta_{ki}^{(1l)} \quad (7)$$

In (7), we set $l = 1$ at left and $l = 0$ at right, then we can obtain $r(E_i, E_j) = r(E_j, E_i)$. That is, the necessity has been proved. Moreover, the sufficiency is obvious.

(iii) If $E = \{E_\sigma | \sigma = 1, 2, \cdots, m\}$, $m \geq 2$, then for any $E_l, E_j \in E$, we have

$$\max_r \max_k \max_i \Delta_{ki}^{(lr)} \neq \max_r \max_k \max_i \Delta_{ki}^{(jr)}.$$

Thus in general $r(E_i, E_j) \overset{often}{\neq} r(E_j, E_i)$ is satisfied.

(iv) From $r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)})) = \frac{\Delta_{\min} + \rho\Delta_{\max}}{\Delta_{ki}^{(0j)} + \rho\Delta_{\max}}$, we can easily see that the smaller the difference information $\Delta_{ki}^{(0j)}$ is, the greater the value of $r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)}))$ is. □

5.2.2.2 Ranking Algorithm Based on the L2TM-GRD given by Definition 8 and Theorem 1, together with the idea of TOPSIS, we present a ranking algorithm to evaluate and rank the industrial operation quality for all m cities. The steps of ranking algorithm are given as follows.

Step 1: From the evaluation criteria $B = \{B_1, B_2, \ldots, B_{15}\}$ of evaluating the industrial operation quality and the methods of determining the criteria values given by Sect. 2, the original multi-source heterogeneous data matrix $R_j = (b_{ki}^{(j)})_{15\times q}$ (j = 1, 2, …, m) formed by the values of 15 evaluation criteria in q years for the m given cities can be obtained.

Step 2: Using Eqs. (3) and (4) to process the multi-source heterogeneous data of matrix $R_j = (b_{ki}^{(j)})_{15\times q}$, the normalized matrix $Z_j = (c_{ki}^{(j)})_{15\times q}$ (j = 1, 2, …, m) is obtained.

Step 3: Using Definitions 4, 5, 6, and 7, the normalized matrix $Z_j = (c_{ki}^{(j)})_{15\times q}$ is transformed into a linguistic 2-tuple matrix $E_j = [(s_{ki}^{(j)}, a_{ki}^{(j)})]_{15\times q}$, j = 1, 2, …, m.

Step 4: From linguistic 2-tuple matrix $E_j = [(s_{ki}^{(j)}, a_{ki}^{(j)})]_{15\times q}$, j = 1, 2, …, m, construct a positive ideal matrix E^+, and a negative ideal matrix E^-, where

$$E^+ = \begin{bmatrix} \max\limits_{1\leq j\leq m}(s_{11}^{(j)}, a_{11}^{(j)}) & \max\limits_{1\leq j\leq m}(s_{12}^{(j)}, a_{12}^{(j)}) & \cdots & \max\limits_{1\leq j\leq m}(s_{1q}^{(j)}, a_{1q}^{(j)}) \\ \max\limits_{1\leq j\leq m}(s_{21}^{(j)}, a_{21}^{(j)}) & \max\limits_{1\leq j\leq m}(s_{22}^{(j)}, a_{22}^{(j)}) & \cdots & \max\limits_{1\leq j\leq m}(s_{2q}^{(j)}, a_{2q}^{(j)}) \\ \vdots & \vdots & & \vdots \\ \max\limits_{1\leq j\leq m}(s_{p1}^{(j)}, a_{p1}^{(j)}) & \max\limits_{1\leq j\leq m}(s_{p2}^{(j)}, a_{p2}^{(j)}) & \cdots & \max\limits_{1\leq j\leq m}(s_{pq}^{(j)}, a_{pq}^{(j)}) \end{bmatrix}, \quad (8)$$

$$E^- = \begin{bmatrix} \min\limits_{1\leq j\leq m}(s_{11}^{(j)}, a_{11}^{(j)}) & \min\limits_{1\leq j\leq m}(s_{12}^{(j)}, a_{12}^{(j)}) & \cdots & \min\limits_{1\leq j\leq m}(s_{1q}^{(j)}, a_{1q}^{(j)}) \\ \min\limits_{1\leq j\leq m}(s_{21}^{(j)}, a_{21}^{(j)}) & \min\limits_{1\leq j\leq m}(s_{22}^{(j)}, a_{22}^{(j)}) & \cdots & \min\limits_{1\leq j\leq m}(s_{2q}^{(j)}, a_{2q}^{(j)}) \\ \vdots & \vdots & & \vdots \\ \min\limits_{1\leq j\leq m}(s_{p1}^{(j)}, a_{p1}^{(j)}) & \min\limits_{1\leq j\leq m}(s_{p2}^{(j)}, a_{p2}^{(j)}) & \cdots & \min\limits_{1\leq j\leq m}(s_{pq}^{(j)}, a_{pq}^{(j)}) \end{bmatrix}, \quad (9)$$

where "max" and "min" mean the maximizing operation and minimizing operation of linguistic 2-tuples, respectively, given by Definition 2.

Step 5: Regard positive ideal matrix E^+ as the reference factor matrix, and linguistic 2-tuple matrix $E_j = [(s_{ki}^{(j)}, a_{ki}^{(j)})]_{15\times q}$, j = 1, 2, …, m, as the comparison factor matrix, and then use Eq. (5) to calculate the L2TM-GRC $r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)}))$ and use Eq. (6) to calculate the L2TM-GRD $r(E^+, E_j)$ of E^+ to E_j.

Step 6: Regard positive ideal matrix E^- as the reference factor matrix, and linguistic 2-tuple matrix $E_j = [(s_{ki}^{(j)}, a_{ki}^{(j)})]_{15\times q}$, j = 1, 2, …, m, as the comparison factor matrix, then use Eq. (5) to calculate the L2TM-GRC $r((s_{ki}^{(0)}, a_{ki}^{(0)}), (s_{ki}^{(j)}, a_{ki}^{(j)}))$ and use Eq. (6) to calculate the L2TM-GRD $r(E^-, E_j)$ of E^- to E_j.

Step 7: Use the following formula (10) to calculate the approach degree u_j, $j = 1, 2, \ldots, m$.

$$u_j = \frac{r^2(E^+, E_j)}{r^2(E^+, E_j) + r^2(E^-, E_j)} \quad (10)$$

Step 8: Rank the quality level of industrial operation for all m cities according to the value of u_j, $j = 1, 2, \ldots, m$. The greater the value of u_j, the higher the quality level of industrial operation of city j is.

6 Numerical Illustration

6.1 The Problem

In this section, we provide a decision-making example of evaluating the industrial operation quality to show how to implement the decision method presented in this paper, and to demonstrate the feasibility and effectiveness of this method.

Here, China's Hunan Province was taken as an example to make an empirical analysis. Concretely, we will evaluate the quality level of industrial operation for 14 cities in Hunan Province based on data from 2013 to 2016.

The 14 cities are Changsha, Zhuzhou, Xiangtan, Hengyang, Shaoyang, Yueyang, Changde, Zhangjiajie, Yiyang, Chenzhou, Yongzhou, Huaihua, Loudi, and West Hunan. The evaluation criteria are given in Table 3, i.e., B_1 Growth rate of industrial added value (%), B_2 Growth rate of total investment of industrial enterprises above a designated scale (%), B_3 Growth rate of total profit of industrial enterprises above a designated scale (%), B_4 Industrial labor productivity (Yuan/person), B_5 Rate of industrial added value (%), B_6 Profit rate of sales (%), B_7 The proportion of industrial export value in sales value (%), B_8 Proportion of added value of all industrial parks in industrial added value of one region (%), B_9 Proportion of value added of strategic emerging industries in GDP (%), B_{10} Investment intensity of industrial R&D (%), B_{11} Increment of invention patent authorization of industrial enterprises (piece), B_{12} Growth rate of output value of new products (%), B_{13} Overall development index for integration of informatization and industrialization, B_{14} Comprehensive utilization rate of industrial solid waste (%), and B_{15} Degree of Industrial environmental pollution. Let the weight set of these 15 evaluation criteria be $W = (w_1, w_2, \ldots, w_{15}) = (0.08, 0.07, 0.06, 0.06, 0.07, 0.05, 0.09, 0.08, 0.07, 0.03, 0.06, 0.07, 0.10, 0.05, 0.06)$, and the weight vector of 4 years (2013–2016) be $V = (v_1, v_2, v_3, v_4) = (0.1, 0.2, 0.3, 0.4)$.

The original multi-source heterogeneous data matrices $R_1, R_2, \ldots, R_{14}$ ($j = 1, 2, \ldots, 14$) formed by the values of the above 15 evaluation criteria in 4 years for the 14 given cities given by the committee of experts are listed in Tables 4, 5, 6, 7, and 8. For the data in Tables 4, 5, 6, 7, and 8, the real numbers all comes from Economic and Information Technology Commission of Hunan Province. For the evaluation criteria B_9 (Proportion of value added of strategic emerging industries in GDP (%)), B_{11} (Increment of invention patent authorization of industrial enterprises (piece)) and B_{12} (Growth rate of output value of new products (%)), due to special reasons, statistics data of relevant management departments in 2013 are missing. Thus, the precise real numbers cannot be obtained. Here, we give the range values (i.e., interval numbers) for these three evaluation criteria in 2013 according to the investigation and survey from each city. For the evaluation criteria B_{15} (Degree of industrial environmental pollution), due to many types of industrial pollution, so we also cannot obtain the precise real numbers for this criteria. Here, we give the evaluation scale in the form of linguistic fuzzy variables, i.e., Very low (s_0), Low (s_1), Medium (s_2), High (s_3), and Very high (s_4), which are defined by Eq. (1).

Table 4 The original evaluation data information of Changsha, Zhuzhou, and Xiangtan

Criteria	Changsha				Zhuzhou				Xiangtan			
	2013	2014	2015	2016	2013	2014	2015	2016	2013	2014	2015	2016
B_1	14.0	12.00	9.2	6.9	12.6	11.60	8.2	7.1	11.2	11.00	8.2	6.90
B_2	26.2	18.90	23.4	10.7	27.3	21.80	17.4	− 9.8	36.1	17.50	19.4	18.80
B_3	12.1	− 1.10	5.0	− 2.2	32.5	17.10	6.6	14.9	82.8	3.50	− 15.8	40.90
B_4	355,559	392,499	511,268	468,261	242,786	209,747	216,156	241,421	390,603	448,198	455,955	520,111
B_5	26.6	44.20	30.5	28.1	32.77	36.10	32.4	30.7	28.59	29.83	26.9	28.10
B_6	6.80	6.09	5.84	5.18	4.63	4.90	4.7	5.28	2.54	2.41	1.95	2.50
B_7	32.9	40.35	0.6	0.671	11.3	11.79	0.18	0.163	5.6	6.92	0.06	0.06
B_8	65.1	73.71	73.6	73.6	55	61.40	67.8	71.9	58.5	66.39	71.9	77.80
B_9	[13, 15]	15.50	15.1	16.7	[14, 16]	16.50	17	15.9	[17, 19]	19.30	20.5	17.70
B_{10}	2.04	1.49	1.5	1.41	2.06	1.37	1.19	1.34	2.5	1.06	0.86	0.74
B_{11}	[50, 60]	18.60	2229.0	326	[30, 50]	29.14	692	− 290	[20, 30]	19.77	215	311.00
B_{12}	[18, 22]	23.60	16.0	6.2	[12, 15]	13.40	18.9	− 14.9	[9, 12]	14.00	18.1	9.00
B_{13}	0.52	5.43	5.67	5.71	0.48	4.43	4.48	4.63	0.47	5.98	4.97	4.40
B_{14}	90.6	85.70	85.00	85.6	85.1	88.6	88.8	88.6	95.7	96.5	94.9	95.3
B_{15}	s_4	s_4	s_4	s_4	s_4	s_4	s_3	s_3	s_4	s_3	s_2	s_2

Table 5 The original evaluation data information of Hengyang, Shaoyang, and Yueyang

Criteria	Hengyang				Shaoyang				Yueyang			
	2013	2014	2015	2016	2013	2014	2015	2016	2013	2014	2015	2016
B_1	11.8	9.30	7.1	6.9	12.4	11.60	9.7	6.5	11.6	9.20	8.3	7.1
B_2	35.4	13.10	11.0	1.6	43.3	8.50	13.6	− 5.4	18.1	3.20	15.8	13.2
B_3	2.8	− 31.40	− 1.0	− 3	26.9	11.20	4.6	− 3.4	26.4	− 13.80	23.2	12.2
B_4	325,527	278,882	207,812	395,174	229,208	238,643	235,553	209,771	356,271	346,896	334,195	392,190
B_5	29.26	45.94	10.7	24.6	28.13	28.15	26.1	21.5	25.41	26.40	56.3	20.6
B_6	4.67	3.82	5.3	4.99	4.02	4.00	3.8	3.45	2.94	2.50	2.94	3.12
B_7	20.1	9.43	0.05	0.043	7.3	8.62	0.11	0.129	1.9	1.63	0.02	0.027
B_8	41.1	57.06	61.5	65.7	39.3	63.76	70	74.3	39	50.66	60.1	69.8
B_9	[10, 13]	10.20	8.4	6.9	[7, 10]	6.50	6	5	[10, 12]	10.30	9.6	8.7
B_{10}	0.83	0.69	1.03	0.89	1.61	0.75	0.75	0.53	0.91	0.95	1.01	0.94
B_{11}	[20, 30]	6.67	122	238	[15, 18]	− 22.22	19	4	[25, 30]	13.95	141	27
B_{12}	[3, 8]	− 4.30	17.8	− 2.7	[20, 30]	25.10	21.4	− 5.5	[18, 25]	18.30	27.7	19.7
B_{13}	0.33	2.15	2.57	2.26	0.42	2.01	1.93	1.76	2.41	2.57	2.24	2.26
B_{14}	81.1	80.2	87.3	86.5	64.5	63.9	65.8	74.5	89.2	76.7	79.2	81.9
B_{15}	s_4	s_2	s_1	s_0	s_4	s_2	s_1	s_1	s_3	s_3	s_2	s_1

Table 6 The original evaluation data information of Changde, Zhangjiajie, and Yiyang

Criteria	Changde				Zhangjiajie				Yiyang			
	2013	2014	2015	2016	2013	2014	2015	2016	2013	2014	2015	2016
B_1	11.2	9.50	7.1	6.6	11	8.50	6.6	6.7	12.6	11.60	7.1	6.7
B_2	34.3	13.70	18.7	4.6	27.6	25.00	18.9	16.6	31.9	24.50	20.6	9.9
B_3	25.5	− 13.10	0.8	− 11.9	24.3	13.00	0.5	− 2.7	20	0.80	12.6	5.2
B_4	478,436	524,187	542,016	641,542	261,118	284,370	283,176	487,944	284,539	314,560	323,114	308,868
B_5	45.82	42.91	39	36.9	37.57	58.82	37.6	31.9	30.57	28.96	26.3	22.6
B_6	9.34	7.96	7.18	3.12	2.69	4.37	5.11	5	3.86	3.43	3.45	3.37
B_7	2.1	2.14	0.03	0.036	0.4	0.60	0.01	0.005	3.3	3.79	0.04	0.049
B_8	27.4	31.27	86	87.2	23.5	23.70	26.4	8.9	50.5	54.62	63.3	68.3
B_9	[2, 3]	3.30	3.2	3.3	[1.2, 2]	1.60	1.5	1.7	[7, 10]	10.20	9.5	7.8
B_{10}	0.63	1.52	1.19	1.27	0.16	0.63	0.75	0.5	1.56	0.35	0.41	0.58
B_{11}	[25, 35]	68.09	228	− 257	[8, 12]	− 6.67	9	− 46	[15, 20]	2.00	88	17
B_{12}	[15, 18]	− 14.50	26.5	0.7	[16, 20]	85.30	17.7	16.3	[8, 10]	7.00	16.5	19.7
B_{13}	0.4	1.62	1.53	1.34	0.33	1.34	1.08	1.12	0.4	4.27	4.08	3.75
B_{14}	96.5	97.9	97.4	95.3	96.5	97.2	97.3	99.3	94.5	88.9	85	85.3
B_{15}	s_4	s_3	s_2	s_2	s_1	s_0	s_0	s_0	s_2	s_2	s_1	s_1

Now, our decision goal is to evaluate the quality level of industrial operation and give the ranking order for all 14 cities according to the multi-source heterogeneous data matrices R_1, R_2, …, R_{14}.

6.2 Decision-Making Process

Using the ranking algorithm given by Sect. 5.2.2.2, we give the detailed decision-making process as follows.

(1) Data processing and transformation. Use Eqs. (3) and (4) to normalize the multi-source heterogeneous data of matrices R_1, R_2, …, R_{14}, and then use

Table 7 The original evaluation data information of Chenzhou, Yongzhou, and Huaihua

Criteria	Chenzhou				Yongzhou				Huaihua			
	2013	2014	2015	2016	2013	2014	2015	2016	2013	2014	2015	2016
B_1	12.5	12.00	7.2	6.7	11.2	12.20	10.1	6.6	11	7.10	7.0	6.5
B_2	34.4	21.40	3.9	12.4	34.5	21.30	23.6	15	18.5	3.20	19.8	4.5
B_3	10.4	0.50	− 7.1	0.9	14.3	17.00	− 6.5	18	− 3	− 17.20	20.2	19.1
B_4	428,099	477,105	509,136	529,307	194,258	207,041	213,006	283,043	292,213	319,473	330,458	505,884
B_5	35.73	31.67	34.6	33.6	31.49	41.80	29.2	25.2	34.1	51.30	34	30.2
B_6	6.98	6.33	5.54	5.29	4.03	4.25	3.46	3.62	1.87	1.57	1.6	1.78
B_7	10.9	10.54	0.1	0.094	1.6	1.71	0.02	0.033	0.7	0.36	0.0002	0.0003
B_8	57	71.48	76.2	77	49	66.26	81.7	87.7	18.8	24.61	34.4	43.3
B_9	[14, 18]	16.20	17	15.4	[2, 5]	3.40	4.1	5.1	[3, 4]	3.70	4.1	4
B_{10}	0.38	0.56	0.62	0.47	0.54	0.58	0.59	0.36	0.2	0.17	0.54	0.48
B_{11}	[16, 20]	6.15	98	− 129	[40, 45]	2.22	53	51	[10, 15]	− 52.38	20	− 142
B_{12}	[20, 25]	− 2.10	28.2	2.1	[23, 26]	29.50	40.1	− 39.6	[8, 12]	11.70	14.9	19.6
B_{13}	0.36	5.64	5.51	5.05	0.42	1.90	1.92	1.95	0.29	1.36	1.3	1.33
B_{14}	49.1	48	50.2	56.8	82.2	83	82.2	82.8	30.6	30.2	29.4	29.8
B_{15}	s_3	s_3	s_2	s_2	s_4	s_3	s_1	s_0	s_3	s_3	s_1	s_0

Table 8 The original evaluation data information of Loudi and West Hunan

Criteria	Loudi				Xiangxi			
	2013	2014	2015	2016	2013	2014	2015	2016
B_1	11	9.50	5.7	6.7	− 6	6.80	5.2	5.5
B_2	26.6	13.10	9.2	17.8	8.3	12.20	− 1.6	9.4
B_3	62.7	− 8.90	− 43.8	66.8	− 47.6	2.60	− 14.7	27.2
B_4	222,903	235,003	229,542	387,696	223,054	180,172	170,130	307,162
B_5	32.02	32.48	24.6	23.3	33.88	49.99	30	30
B_6	5.01	4.44	2.6	4.11	3.52	3.48	3.46	4.53
B_7	1.3	1.55	0.01	0.014	0.6	0.57	0.01	0.003
B_8	36.1	38.29	42.5	60.7	46	54.01	50.9	51.6
B_9	[5, 7]	6.70	6.8	6.5	[4, 6]	3.50	2.8	2.6
B_{10}	1.38	0.93	0.97	0.65	0.13	0.13	0.33	0.27
B_{11}	[35, 40]	− 3.03	39	91	[45, 50]	66.67	21	− 232
B_{12}	[25, 30]	− 1.10	− 5.6	42.4	[15, 18]	− 100.00	17.9	− 15.6
B_{13}	0.38	2.08	1.36	1.46	0.37	1.33	1.31	1.38
B_{14}	92	98.5	97.7	96.8	4	6.1	6.8	6.1
B_{15}	s_0	s_1	s_0	s_0	s_2	s_1	s_0	s_1

Definitions 4, 5, 6, and 7 to transform all data of the normalized matrix into linguistic 2-tuples, so the linguistic 2-tuple matrices E_1, E_2, …, E_{14} are obtained, which are listed in Tables 9, 10, 11, 12, and 13.

(2) Determine the positive ideal matrix and the negative ideal matrix. From the linguistic 2-tuple matrices E_1, E_2, …, E_{14} listed in Tables 9, 10, 11, 12, and 13, by using the methods given by Eqs. (8) and (9), construct the positive ideal matrix E^+ and the negative ideal matrix E^- as follows

Table 9 The linguistic 2-tuple matrix data of Changsha, Zhuzhou, and Xiangtan

Criteria	Changsha				Zhuzhou				Xiangtan			
	2013	2014	2015	2016	2013	2014	2015	2016	2013	2014	2015	2016
B_1	$(s_4, 0)$	$(s_4, -0.15)$	$(s_3, 0.27)$	$(s_4, -0.5)$	$(s_4, -0.28)$	$(s_4, -0.44)$	$(s_2, 0.45)$	$(s_4, 0)$	$(s_3, 0.44)$	$(s_3, 0.11)$	$(s_2,0.45)$	$(s_4, -0.5)$
B_2	$(s_2, 0.05)$	$(s_3, -0.12)$	$(s_4, -0.03)$	$(s_3, -0.13)$	$(s_2, 0.17)$	$(s_3, 0.41)$	$(s_3, 0.02)$	$(s_0, 0)$	$(s_3, 0.17)$	$(s_3, -0.38)$	$(s_3, 0.33)$	$(s_4, 0)$
B_3	$(s_2, -0.17)$	$(s_2, 0.50)$	$(s_3, -0.09)$	$(s_0, 0.49)$	$(s_2, 0.46)$	$(s_4, 0)$	$(s_3, 0.01)$	$(s_1, 0.36)$	$(s_4, 0)$	$(s_3, -0.12)$	$(s_2, -0.33)$	$(s_3, -0.32)$
B_4	$(s_2, 0.27)$	$(s_2, 0.47)$	$(s_4, -0.33)$	$(s_2, 0.40)$	$(s_1, -0.32)$	$(s_0, 0.34)$	$(s_0, 0.50)$	$(s_0, 0.29)$	$(s_3, -0.24)$	$(s_3, 0.12)$	$(s_3, 0.07)$	$(s_3, -0.13)$
B_5	$(s_0, 0.23)$	$(s_2, 0.20)$	$(s_2, -0.26)$	$(s_2, -0.16)$	$(s_1, 0.44)$	$(s_1, 0.20)$	$(s_2, -0.10)$	$(s_2, 0.48)$	$(s_1, -0.38)$	$(s_0, 0.42)$	$(s_1, 0.42)$	$(s_2, -0.16)$
B_6	$(s_3, -0.36)$	$(s_3, -0.17)$	$(s_3, -0.04)$	$(s_4, -0.12)$	$(s_1, 0.48)$	$(s_2, 0.09)$	$(s_2, 0.22)$	$(s_4, -0.01)$	$(s_0, 0.36)$	$(s_1, -0.47)$	$(s_0, 0.25)$	$(s_1, -0.18)$
B_7	$(s_4, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_1, 0.34)$	$(s_1, 0.14)$	$(s_1, 0.20)$	$(s_1, -0.03)$	$(s_1, -0.36)$	$(s_1, -0.34)$	$(s_0, 0.40)$	$(s_0, 0.36)$
B_8	$(s_4, 0)$	$(s_4, 0)$	$(s_3, 0.17)$	$(s_3, 0.28)$	$(s_3, 0.13)$	$(s_3, 0.02)$	$(s_3, -0.22)$	$(s_3, 0.20)$	$(s_3, 0.43)$	$(s_3, 0.41)$	$(s_3, 0.05)$	$(s_3, 0.49)$
B_9	$(s_2, -0.49)$	$(s_3, 0.14)$	$(s_3, -0.14)$	$(s_4, -0.25)$	$(s_2, -0.38)$	$(s_3, 0.37)$	$(s_3, 0.26)$	$(s_4, -0.45)$	$(s_2, -0.06)$	$(s_4, 0)$	$(s_4, 0)$	$(s_4, 0)$
B_{10}	$(s_3, 0.22)$	$(s_4, -0.09)$	$(s_4, 0)$	$(s_4, 0)$	$(s_3, 0.26)$	$(s_4, -0.43)$	$(s_3, -0.06)$	$(s_4, -0.25)$	$(s_4, 0)$	$(s_3, -0.32)$	$(s_2, -0.19)$	$(s_2, -0.35)$
B_{11}	$(s_2, -0.15)$	$(s_2, 0.36)$	$(s_4, 0)$	$(s_4, 0)$	$(s_1, 0.32)$	$(s_3, -0.29)$	$(s_1, 0.23)$	$(s_0, 0)$	$(s_1, -0.17)$	$(s_2, 0.40)$	$(s_0, 0.37)$	$(s_4, -0.10)$
B_{12}	$(s_1, 0.17)$	$(s_3, -0.33)$	$(s_2, -0.11)$	$(s_2, 0.23)$	$(s_1, -0.21)$	$(s_2, 0.45)$	$(s_2, 0.14)$	$(s_1, 0.21)$	$(s_1, -0.39)$	$(s_2, 0.46)$	$(s_2, 0.07)$	$(s_2, 0.37)$
B_{13}	$(s_0, 0.43)$	$(s_4, -0.47)$	$(s_4, 0)$	$(s_4, 0)$	$(s_0, 0.36)$	$(s_3, -0.33)$	$(s_3, -0.04)$	$(s_3, 0.06)$	$(s_0, 0.34)$	$(s_4, 0)$	$(s_3, 0.39)$	$(s_3, -0.14)$
B_{14}	$(s_4, -0.25)$	$(s_3, 0.45)$	$(s_3, 0.44)$	$(s_3, 0.41)$	$(s_4, -0.49)$	$(s_4, -0.43)$	$(s_4, -0.39)$	$(s_4, -0.46)$	$(s_4, -0.03)$	$(s_4, -0.09)$	$(s_4, -0.12)$	$(s_4, -0.17)$
B_{15}	$(s_4, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_3, 0)$	$(s_3, 0)$	$(s_4, 0)$	$(s_3, 0)$	$(s_2, 0)$	$(s_2, 0)$

Table 10 The linguistic 2-tuple matrix data of Hengyang, Shaoyang, and Yueyang

Criteria	Hengyang				Shaoyang				Yueyang			
	2013	2014	2015	2016	2013	2014	2015	2016	2013	2014	2015	2016
B_1	$(s_4, -0.44)$	$(s_2, -0.15)$	$(s_2, -0.45)$	$(s_4, -0.5)$	$(s_4, -0.32)$	$(s_4, -0.44)$	$(s_4, -0.33)$	$(s_3, -0.5)$	$(s_4, -0.48)$	$(s_2, -0.22)$	$(s_3, -0.47)$	$(s_4, 0)$
B_2	$(s_3, 0.10)$	$(s_2, -0.18)$	$(s_2, 0)$	$(s_2, -0.41)$	$(s_4, 0)$	$(s_1, -0.03)$	$(s_2, 0.41)$	$(s_1, -0.39)$	$(s_1, 0.12)$	$(s_0, 0)$	$(s_3, -0.24)$	$(s_3, 0.22)$
B_3	$(s_2, -0.45)$	$(s_0, 0)$	$(s_3, -0.45)$	$(s_0, 0.45)$	$(s_2, 0.29)$	$(s_4, -0.49)$	$(s_3, -0.11)$	$(s_0, 0.43)$	$(s_2, 0.27)$	$(s_1, 0.45)$	$(s_4, 0)$	$(s_1, 0.23)$
B_4	$(s_2, -0.15)$	$(s_1, 0.15)$	$(s_0, 0.41)$	$(s_2, -0.23)$	$(s_0, 0.49)$	$(s_1, -0.32)$	$(s_1, -0.30)$	$(s_0, 0)$	$(s_2, 0.28)$	$(s_2, -0.06)$	$(s_2, -0.24)$	$(s_2, -0.31)$
B_5	$(s_1, -0.25)$	$(s_2, 0.41)$	$(s_0, 0)$	$(s_1, -0.02)$	$(s_1, -0.47)$	$(s_0, 0.22)$	$(s_1, 0.35)$	$(s_0, 0.22)$	$(s_0, 0)$	$(s_0, 0)$	$(s_4, 0)$	$(s_0, 0)$
B_6	$(s_2, -0.50)$	$(s_1, 0.41)$	$(s_3, -0.35)$	$(s_4, -0.34)$	$(s_1, 0.15)$	$(s_2, -0.48)$	$(s_2, -0.42)$	$(s_2, -0.10)$	$(s_1, -0.43)$	$(s_1, -0.42)$	$(s_1, -0.04)$	$(s_2, -0.47)$
B_7	$(s_2, 0.43)$	$(s_1, -0.09)$	$(s_0, 0.33)$	$(s_0, 0.26)$	$(s_1, -0.15)$	$(s_1, -0.17)$	$(s_1, -0.27)$	$(s_1, -0.23)$	$(s_0, 0.19)$	$(s_0, 0.13)$	$(s_0, 0.13)$	$(s_0, 0.16)$
B_8	$(s_2, -0.07)$	$(s_3, -0.33)$	$(s_2, 0.36)$	$(s_3, -0.12)$	$(s_2, -0.23)$	$(s_3, 0.20)$	$(s_3, -0.07)$	$(s_3, 0.32)$	$(s_2, -0.25)$	$(s_2, 0.16)$	$(s_2, 0.26)$	$(s_3, 0.09)$
B_9	$(s_1, 0.23)$	$(s_2, -0.06)$	$(s_1, 0.45)$	$(s_1, 0.3)$	$(s_1, -0.09)$	$(s_1, 0.11)$	$(s_1, -0.05)$	$(s_1, -0.17)$	$(s_1, 0.18)$	$(s_2, -0.03)$	$(s_2, -0.29)$	$(s_2, -0.25)$
B_{10}	$(s_1, 0.18)$	$(s_2, -0.39)$	$(s_2, 0.39)$	$(s_2, 0.18)$	$(s_2, 0.49)$	$(s_2, -0.22)$	$(s_1, 0.44)$	$(s_1, -0.09)$	$(s_1, 0.32)$	$(s_2, 0.36)$	$(s_2, 0.33)$	$(s_2, 0.35)$
B_{11}	$(s_1, -0.17)$	$(s_2, -0.04)$	$(s_0, 0.20)$	$(s_3, 0.43)$	$(s_1, -0.45)$	$(s_1, 0)$	$(s_0, 0.02)$	$(s_2, -0.09)$	$(s_1, -0.08)$	$(s_2, 0.20)$	$(s_0, 0.24)$	$(s_2, 0.06)$
B_{12}	$(s_0, 0.31)$	$(s_2, 0.07)$	$(s_2, 0.05)$	$(s_2, -0.2)$	$(s_1, 0.24)$	$(s_3, -0.3)$	$(s_2, 0.36)$	$(s_2, -0.34)$	$(s_1, 0.44)$	$(s_3, -0.45)$	$(s_3, -0.08)$	$(s_3, -0.11)$
B_{13}	$(s_0, 0.08)$	$(s_1, -0.29)$	$(s_1, 0.30)$	$(s_1, -0.01)$	$(s_0, 0.25)$	$(s_1, -0.41)$	$(s_1, -0.26)$	$(s_1, -0.44)$	$(s_4, 0)$	$(s_1, 0.07)$	$(s_1, 0.01)$	$(s_1, -0.01)$
B_{14}	$(s_3, 0.33)$	$(s_3, 0.21)$	$(s_4, -0.46)$	$(s_3, 0.45)$	$(s_3, -0.38)$	$(s_3, -0.50)$	$(s_3, -0.4)$	$(s_3, -0.06)$	$(s_4, -0.32)$	$(s_3, 0.06)$	$(s_3, 0.19)$	$(s_3, 0.25)$
B_{15}	$(s_4, 0)$	$(s_2, 0)$	$(s_1, 0)$	$(s_0, 0)$	$(s_4, 0)$	$(s_2, 0)$	$(s_1, 0)$	$(s_1, 0)$	$(s_3, 0)$	$(s_3, 0)$	$(s_2, 0)$	$(s_1, 0)$

Table 11 The linguistic 2-tuple matrix data of Changde, Zhangjiajie, and Yiyang

Criteria	Changde				Zhangjiajie				Yiyang			
	2013	2014	2015	2016	2013	2014	2015	2016	2013	2014	2015	2016
B_1	$(s_3, 0.44)$	$(s_2, 0)$	$(s_2, 0.45)$	$(s_3, -0.25)$	$(s_3, 0.40)$	$(s_1, 0.26)$	$(s_1, 0.14)$	$(s_3, 0)$	$(s_4, -0.28)$	$(s_4, -0.44)$	$(s_2, -0.45)$	$(s_3, 0)$
B_2	$(s_3, -0.03)$	$(s_2, -0.07)$	$(s_3, 0.22)$	$(s_2, 0.01)$	$(s_2, 0.21)$	$(s_4, 0)$	$(s_3, 0.25)$	$(s_4, -0.31)$	$(s_3, -0.30)$	$(s_4, -0.09)$	$(s_4, -0.47)$	$(s_3, -0.25)$
B_3	$(s_2, 0.24)$	$(s_2, -0.49)$	$(s_3, -0.34)$	$(s_0, 0)$	$(s_2, 0.21)$	$(s_4, -0.34)$	$(s_3, -0.36)$	$(s_0, 0.47)$	$(s_2, 0.07)$	$(s_3, -0.34)$	$(s_3, 0.37)$	$(s_1, -0.13)$
B_4	$(s_4, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_1, -0.06)$	$(s_1, 0.21)$	$(s_1, 0.22)$	$(s_3, -0.42)$	$(s_1, 0.27)$	$(s_2, -0.44)$	$(s_2, -0.36)$	$(s_1, -0.08)$
B_5	$(s_4, 0)$	$(s_2, 0.04)$	$(s_2, 0.48)$	$(s_4, 0)$	$(s_2, 0.38)$	$(s_4, 0)$	$(s_2, 0.36)$	$(s_3, -0.23)$	$(s_1, 0.01)$	$(s_0, 0.32)$	$(s_1, 0.37)$	$(s_0, 0.49)$
B_6	$(s_4, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_2, -0.47)$	$(s_0, 0.44)$	$(s_2, -0.25)$	$(s_3, -0.48)$	$(s_4, -0.33)$	$(s_1, 0.07)$	$(s_1, 0.16)$	$(s_1, 0.33)$	$(s_2, -0.19)$
B_7	$(s_0, 0.21)$	$(s_0, 0.18)$	$(s_0, 0.20)$	$(s_0, 0.21)$	$(s_0, 0)$	$(s_0, 0.02)$	$(s_0, 0.07)$	$(s_0, 0.30)$	$(s_0, 0.36)$	$(s_0, 0.34)$	$(s_0, 0.27)$	$(s_0, 0.29)$
B_8	$(s_1, -0.26)$	$(s_1, -0.39)$	$(s_4, 0)$	$(s_4, -0.02)$	$(s_0, 0.41)$	$(s_0, 0)$	$(s_0, 0)$	$(s_0, 0)$	$(s_3, -0.26)$	$(s_2, 0.47)$	$(s_2, 0.48)$	$(s_3, 0.02)$
B_9	$(s_2, 0.27)$	$(s_0, 0.38)$	$(s_0, 0.36)$	$(s_0, 0.4)$	$(s_0, 0.17)$	$(s_0, 0)$	$(s_0, 0)$	$(s_0, 0)$	$(s_1, -0.09)$	$(s_2, -0.06)$	$(s_2, -0.32)$	$(s_2, -0.47)$
B_{10}	$(s_1, -0.16)$	$(s_4, 0)$	$(s_3, -0.06)$	$(s_4, -0.49)$	$(s_0, 0.05)$	$(s_1, 0.44)$	$(s_1, 0.44)$	$(s_1, -0.19)$	$(s_2, 0.41)$	$(s_1, -0.37)$	$(s_0, 0.27)$	$(s_1, 0.09)$
B_{11}	$(s_1, 0)$	$(s_4, 0)$	$(s_0, 0.40)$	$(s_0, 0.21)$	$(s_2, 0.33)$	$(s_2, -0.48)$	$(s_0, 0)$	$(s_2, -0.42)$	$(s_2, 0.58)$	$(s_2, -0.19)$	$(s_0, 0.14)$	$(s_2,$
B_{12}	$(s_1, -0.04)$	$(s_2, -0.15)$	$(s_3, -0.19)$	$(s_2, -0.03)$	$(s_1, 0.05)$	$(s_4, 0)$	$(s_2, 0.04)$	$(s_3, -0.27)$	$(s_2, 0.52)$	$(s_2, 0.31)$	$(s_2, -0.07)$	$(s_3, -0.11)$
B_{13}	$(s_0, 0.21)$	$(s_0, 0.25)$	$(s_0, 0.39)$	$(s_0, 0.19)$	$(s_0, 0.08)$	$(s_0, 0.01)$	$(s_0, 0)$	$(s_0, 0)$	$(s_0, 0.21)$	$(s_3, -0.47)$	$(s_3, -0.39)$	$(s_2, 0.29)$
B_{14}	$(s_4, 0)$	$(s_4, -0.03)$	$(s_4, -0.01)$	$(s_4, -0.17)$	$(s_4, 0)$	$(s_4, -0.06$	$(s_4, -0.02)$	$(s_4, 0)$	$(s_4, -0.09)$	$(s_4, -0.42)$	$(s_3, 0.44)$	$(s_3, 0.40)$
B_{15}	$(s_4, 0)$	$(s_3, 0)$	$(s_2, 0)$	$(s_2, 0)$	$(s_1, 0)$	$(s_0, 0)$	$(s_0, 0)$	$(s_0, 0)$	$(s_2, 0)$	$(s_2, 0)$	$(s_1, 0)$	$(s_1, 0)$

Table 12 The linguistic 2-tuple matrix data of Chenzhou, Yongzhou, and Huaihua

Criteria	Chenzhou				Yongzhou				Huaihua			
	2013	2014	2015	2016	2013	2014	2015	2016	2013	2014	2015	2016
B_1	$(s_4, -0.3)$	$(s_4, -0.15)$	$(s_2, -0.37)$	$(s_3, 0)$	$(s_3, 0.44)$	$(s_4, 0)$	$(s_4, 0)$	$(s_3, -0.25)$	$(s_3, 0.4)$	$(s_0, 0.22)$	$(s_1, 0.47)$	$(s_3, -0.5)$
B_2	$(s_3, -0.02)$	$(s_3, 0.34)$	$(s_1, -0.13)$	$(s_3, 0.11)$	$(s_3, -0.01)$	$(s_3, 0.32)$	$(s_4, 0)$	$(s_3, 0.47)$	$(s_1, 0.17)$	$(s_0, 0)$	$(s_3, 0.40)$	$(s_2, 0)$
B_3	$(s_2, -0.22)$	$(s_3, -0.37)$	$(s_2, 0.19)$	$(s_1, -0.35)$	$(s_2, -0.10)$	$(s_4, -0.01)$	$(s_2, 0.23)$	$(s_2, -0.48)$	$(s_1, 0.37)$	$(s_1, 0.17)$	$(s_4, -0.18)$	$(s_2, -0.42)$
B_4	$(s_3, 0.29)$	$(s_3, 0.45)$	$(s_4, -0.35)$	$(s_3, -0.04)$	$(s_0, 0)$	$(s_0, 0.31)$	$(s_0, 0.46)$	$(s_1, -0.32)$	$(s_1, 0.38)$	$(s_2, -0.38)$	$(s_2, -0.28)$	$(s_3, -0.26)$
B_5	$(s_2, 0.02)$	$(s_1, -0.35)$	$(s_2, 0.10)$	$(s_3, 0.19)$	$(s_1, 0.19)$	$(s_2, -0.1)$	$(s_2, -0.38)$	$(s_1, 0.13)$	$(s_2, -0.30)$	$(s_3, 0.07)$	$(s_2, 0.04)$	$(s_2, 0.36)$
B_6	$(s_3, -0.26)$	$(s_3, -0.02)$	$(s_3, -0.18)$	$(s_4, 0)$	$(s_1, 0.16)$	$(s_2, -0.32)$	$(s_1, 0.33)$	$(s_2, 0.10)$	$(s_0, 0)$	$(s_0, 0)$	$(s_0, 0)$	$(s_0, 0)$
B_7	$(s_1, 0.29)$	$(s_1, 0.02)$	$(s_1, -0.33)$	$(s_1, -0.44)$	$(s_0, 0.15)$	$(s_0, 0.14)$	$(s_0, 0.13)$	$(s_0, 0.20)$	$(s_0, 0.04)$	$(s_0, 0)$	$(s_0, 0)$	$(s_0, 0)$
B_8	$(s_3, 0.30)$	$(s_4, -0.18)$	$(s_3, 0.34)$	$(s_4, -0.54)$	$(s_3, -0.39)$	$(s_3, 0.40)$	$(s_4, -0.29)$	$(s_4, 0)$	$(s_0, 0)$	$(s_0, 0.07)$	$(s_1, -0.46)$	$(s_2, -0.25)$
B_9	$(s_1, 0.21)$	$(s_3, 0.30)$	$(s_3, 0.26)$	$(s_3, 0.43)$	$(s_0, 0.37)$	$(s_0, 0.41)$	$(s_1, -0.45)$	$(s_1, -0.15)$	$(s_0, 0.38)$	$(s_0, 0.48)$	$(s_1, -0.45)$	$(s_1, -0.42)$
B_{10}	$(s_0, 0.42)$	$(s_1, 0.24)$	$(s_1, -0.01)$	$(s_1, -0.30)$	$(s_1, -0.31)$	$(s_1, 0.30)$	$(s_1, -0.11)$	$(s_0, 0.32)$	$(s_0, 0.12)$	$(s_2, 0.12)$	$(s_1, -0.28)$	$(s_1, -0.26)$
B_{11}	$(s_1, -0.4)$	$(s_2, -0.06)$	$(s_0, 0.16)$	$(s_2, -0.12)$	$(s_1, 0.43)$	$(s_2, -0.19)$	$(s_0, 0.08)$	$(s_2, 0.21)$	$(s_0, 0.42)$	$(s_0, 0)$	$(s_0, 0.02)$	$(s_1, -0.04)$
B_{12}	$(s_1, 0.31)$	$(s_2, 0.11)$	$(s_3, -0.04)$	$(s_2, 0.03)$	$(s_1, 0.43)$	$(s_3, -0.2)$	$(s_4, 0)$	$(s_0, 0)$	$(s_1, -0.42)$	$(s_2, 0.41)$	$(s_2, -0.21)$	$(s_3, -0.11)$
B_{13}	$(s_0, 0.13)$	$(s_4, -0.29)$	$(s_3, -0.14)$	$(s_3, 0.43)$	$(s_0, 0.25)$	$(s_0, 0.49)$	$(s_1, -0.27)$	$(s_1, -0.28)$	$(s_0, 0)$	$(s_0, 0.03)$	$(s_0, 0.19)$	$(s_0, 0.18)$
B_{14}	$(s_2, -0.05)$	$(s_2, -0.19)$	$(s_2, -0.19)$	$(s_2, 0.18)$	$(s_3, 0.38)$	$(s_3, 0.33)$	$(s_3, 0.32)$	$(s_3, 0.29)$	$(s_1, 0.15)$	$(s_1, 0.04)$	$(s_1, -0.01)$	$(s_1,0.02)$
B_{15}	$(s_3, 0)$	$(s_3, 0)$	$(s_2, 0)$	$(s_2, 0)$	$(s_4, 0)$	$(s_3, 0)$	$(s_1, 0)$	$(s_0, 0)$	$(s_3, 0)$	$(s_3, 0)$	$(s_1, 0)$	$(s_0, 0)$

Table 13 The linguistic 2-tuple matrix data of Loudi and West Hunan

Criteria	Loudi				West Hunan			
	2013	2014	2015	2016	2013	2014	2015	2016
B_1	$(s_3, 0.4)$	$(s_2, 0)$	$(s_0, 0.41)$	$(s_3, 0)$	$(s_0, 0)$	$(s_0, 0)$	$(s_0, 0)$	$(s_0, 0)$
B_2	$(s_2, 0.09)$	$(s_2, -0.18)$	$(s_2, -0.29)$	$(s_4, -0.14)$	$(s_0, 0)$	$(s_2, -0.35)$	$(s_0, 0)$	$(s_3, -0.32)$
B_3	$(s_3, 0.38)$	$(s_2, -0.14)$	$(s_0, 0)$	$(s_4, 0)$	$(s_0, 0)$	$(s_3, -0.20)$	$(s_2, -0.26)$	$(s_2, -0.01)$
B_4	$(s_0, 0.40)$	$(s_1, -0.36)$	$(s_1, -0.36)$	$(s_2, -0.35)$	$(s_0, 0.41)$	$(s_0, 0)$	$(s_0, 0)$	$(s_1, 0.10)$
B_5	$(s_1, 0.30)$	$(s_1, -0.25)$	$(s_1, 0.22)$	$(s_1, -0.34)$	$(s_2, -0.34)$	$(s_3, -0.09)$	$(s_2, -0.31)$	$(s_2, 0.31)$
B_6	$(s_2, -032)$	$(s_2, -0.20)$	$(s_1, -0.28)$	$(s_3, -0.34)$	$(s_1, -0.12)$	$(s_1, 0.20)$	$(s_0, 0.33)$	$(s_3, 0.1)$
B_7	$(s_0, 0.11)$	$(s_0, 0.12)$	$(s_0, 0.07)$	$(s_0, 0.08)$	$(s_0, 0.03)$	$(s_0, 0.02)$	$(s_0, 0.07)$	$(s_0, 0.2)$
B_8	$(s_2, -0.5)$	$(s_1, 0.17)$	$(s_1, 0.08)$	$(s_3, -0.37)$	$(s_2, 0.35)$	$(s_0, 0.42)$	$(s_0, -0.35)$	$(s_2, 0.17)$
B_9	$(s_1, -0.36)$	$(s_1, 0.15)$	$(s_1, 0.12)$	$(s_1, 0.2)$	$(s_1, 0.47)$	$(s_0, 0.43)$	$(s_0, 0.27)$	$(s_0, 0.23)$
B_{10}	$(s_2, 0.11)$	$(s_2, 0.30)$	$(s_2, 0.19)$	$(s_1, 0.33)$	$(s_0, 0)$	$(s_0, 0))$	$(s_0, 0)$	$(s_0, 0)$
B_{11}	$(s_1, 0.26)$	$(s_2, -0.36)$	$(s_0, 0.05)$	$(s_2, 0.47)$	$(s_2, -0.40)$	$(s_4, -0.05)$	$(s_0, 0.02)$	$(s_0, 0.38)$
B_{12}	$(s_2, -0.4)$	$(s_2, 0.14)$	$(s_0, 0)$	$(s_4, 0)$	$(s_1, -0.04)$	$(s_0, 0)$	$(s_2, 0.06)$	$(s_1, 0.17)$
B_{13}	$(s_0, 0.17)$	$(s_1, -0.35)$	$(s_0, 0.24)$	$(s_0, 0.30)$	$(s_0, 0.15)$	$(s_0, 0)$	$(s_0, 0.2)$	$(s_0, 0.23)$
B_{14}	$(s_4, -0.19)$	$(s4, 0)$	$(s_4, 0)$	$(s_4, -0.11)$	$(s0, 0)$	$(s_0, 0)$	$(s0, 0)$	$(s0, 0)$
B_{15}	$(s_0, 0)$	$(s_1, 0)$	$(s_0, 0)$	$(s_0, 0)$	$(s_2, 0)$	$(s_1, 0)$	$(s_0, 0)$	$(s_1, 0)$

$$E^+ = \begin{bmatrix} (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_2,-0.06) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_2,-0.15) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_2,-0.40) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \\ (s_4,0) & (s_4,0) & (s_4,0) & (s_4,0) \end{bmatrix},$$

$$E^- = \begin{bmatrix} (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0.17) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0.33) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0.31) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \\ (s_0,0) & (s_0,0) & (s_0,0) & (s_0,0) \end{bmatrix}$$

(3) Calculate L2TM-GRDR. Regard positive ideal matrix E^+(negative ideal matrix E^-) as the reference factor matrix, and linguistic 2-tuple matrix E_j, $j = 1, 2, \ldots, 14$, as the comparison factor matrix, then use Eqs. (5) and (6) to calculate the L2TM-GRD $r(E^+, E_j)$ of E^+ to E_j (L2TM-GRD $r(E^-, E_j)$ of E^- to E_j). The calculation results are as follows.

$$
\begin{aligned}
&r(E^+,E_1)=0.778, \quad r(E^+,E_2)=0.615,\\
&r(E^+,E_3)=0.650, \quad r(E^+,E_4)=0.504,\\
&r(E^+,E_5)=0.536, \quad r(E^+,E_6)=0.539,\\
&r(E^+,E_7)=0.598, \quad r(E^+,E_8)=0.513,\\
&r(E^+,E_9)=0.446, \quad r(E^+,E_{10})=0.618,\\
&r(E^+,E_{11})=0.554, \quad r(E^+,E_{12})=0.445,\\
&r(E^+,E_{13})=0.505, \quad r(E^+,E_{14})=0.428;\\
&r(E^-,E_1)=0.407, \quad r(E^-,E_2)=0.509,\\
&r(E^-,E_3)=0.492, \quad r(E^-,E_4)=0.597,\\
&r(E^-,E_5)=0.589, \quad r(E^-,E_6)=0.578,\\
&r(E^-,E_7)=0.585, \quad r(E^-,E_8)=0.690,\\
&r(E^-,E_9)=0.406, \quad r(E^-,E_{10})=0.493,\\
&r(E^-,E_{11})=0.612, \quad r(E^-,E_{12})=0.715,\\
&r(E^-,E_{13})=0.657, \quad r(E^-,E_{14})=0.763.
\end{aligned}
$$

(4) Calculate the approach degree. Using Eq. (10) to calculate the approach degree u_j, j = 1, 2, …, 14, we obtain

$$
\begin{aligned}
&u_1=0.785, \quad u_2=0.594, \quad u_3=0.636,\\
&u_4=0.416, \quad u_5=0.453, \quad u_6=0.466,\\
&u_7=0.511, \quad u_8=0.356, \quad u_9=0.547,\\
&u_{10}=0.611, u_{11}=0.450, \quad u_{12}=0.280,\\
&u_{13}=0.371, \quad u_{14}=0.240.
\end{aligned}
$$

(5) Rank the preference order. Rank the quality level of industrial operation for all 14 cities according to the value of u_j, j = 1, 2, …, 14 listed in the above step (4).

Since $u_1 > u_3 > u_{10} > u_2 > u_9 > u_7 > u_6 > u_5 > u_{11} > u_4 > u_{13} > u_8 > u_{12} > u_{14}$, the ranking is

Changsha ≻ Xiangtan ≻ Chenzhou ≻ Zhuzhou ≻ Yiyang ≻ Changde ≻ Yueyang ≻ Shaoyang ≻ Yongzhou ≻ Hengyang ≻ Loudi ≻ Zhangjiajie ≻ Huaihua ≻ West Hunan.

6.3 Discussion

From the calculation result in the step (5) of Sect. 5.2, we can get the ranking preference order of industrial operation quality for 14 cities in Hunan Province in recent 4 years. The result shows that the comprehensive industrial operation quality of Changsha, Xiangtan, and Chenzhou was ranked in the top three, while the industrial operation quality of Zhangjiajie, Huaihua, and West Hunan was ranked in the bottom three. The rest cities are in the middle developmental level. This analysis result is consistent with the results published in "Evaluation Report of Industrial Operation Quality of Hunan Province in 2016" given by the Economic and Information Technology Commission of Hunan Province.

In addition, the above analysis result is the comparison result among all given cities. We call this comparison as a macro comparison. In fact, we can make another type of comparison, i.e., microscopic comparison, which can help us to compare the advantages and disadvantages of all 15 evaluation criteria for a given city. If the industrial administration departments definitely know the advantageous criteria and disadvantageous criteria for one city, then they will develop some effective countermeasures and measures to improve the quality level of industrial operation for this city.

In practical decision making, we can make a microscopic comparison by calculating the grey relational coefficient of linguistic 2-tuple matrix (L2TM-GRC) given by step 5 in Sect. 5.2.2.2. Concretely, we can calculate the L2TM-GRC between the positive ideal matrix E^+ and the comparison factor matrix E_j, j = 1, 2, …, 14, and the L2TM-GRC between the negative ideal matrix E^- and the comparison factor matrix E_j, j = 1, 2, …, 14. As it was restricted by the length, we only give one example, i.e., the L2TM-GRC between E^+ and E_1. By using Eq. (5), we can obtain the following L2TM-GRC matrix.

$$
H_1=\begin{bmatrix}
1 & 0.93 & 0.73 & 0.8\\
0.51 & 0.64 & 0.98 & 0.64\\
0.48 & 0.57 & 0.65 & 0.36\\
0.54 & 0.57 & 0.86 & 0.55\\
0.34 & 0.53 & 0.47 & 0.48\\
0.60 & 0.65 & 0.68 & 0.94\\
1 & 1 & 1 & 1\\
1 & 1 & 0.7 & 0.73\\
0.82 & 0.7 & 0.64 & 0.89\\
0.72 & 0.96 & 1 & 1\\
1 & 0.55 & 1 & 1\\
0.82 & 0.60 & 0.49 & 0.53\\
0.36 & 0.81 & 1 & 1\\
0.89 & 0.78 & 0.78 & 0.77\\
1 & 1 & 1 & 1
\end{bmatrix}.
$$

From the L2TM-GRC data in the above matrix H_1, we can clearly observe the closeness degree between the city of Changsha and the positive ideal (the fictitious city with the highest quality level of industrial operation) under each evaluation criteria. That is, we can make the advantage analysis based on H_1. Concretely, by observing the values in H_1, we can directly see that the closeness degree of Changsha and the positive ideal under the evaluation criteria B_7 (The proportion of industrial export value in sales value) and B_{14} (Comprehensive utilization rate of industrial solid waste) are all 1 in the given four consecutive years, which means that Changsha has the highest quality level of industrial operation under these two evaluation criteria, that is, Changsha has the absolute advantages in the evaluation criteria B_7 and B_{14}.

In addition, we also can find that Changsha has made great progress in some aspects, for example, in the aspects of B_6 (Profit rate of sales), B_{10} (Investment intensity of industrial R&D), and B_{13} (Overall development index for integration of informatization and industrialization). From the data in the rows 6, 10, and 13 of H_1, the values in each row are all increasing, which means the profit rate of sales in Changsha has gradually developed to a high level, and the investment intensity of industrial and the overall development for integration of informatization and industrialization of Changsha have all gradually developed to the highest level in Hunan Province.

However, in addition to seeing the advantages and developing the advantages, we should pay more attention to the disadvantages, which can help a city to have a clear aim to improve the deficiency and further improve the industrial operation quality. Especially, from the data in rows 3 and 5 of H_1, the closeness degree between the city of Changsha and the positive ideal on the evaluation criteria B_3 (Growth rate of total profit of industrial enterprises above a designated scale) and B_5 (Rate of industrial added value) are all very low, which means the comprehensive performance on B_3 and B_5 for Changsha are not good, and there is still a lot of room for improvement. Moreover, from the data in rows 8, 12, and 14, the comprehensive performance on B_8 (Proportion of added value of all industrial parks in industrial added value of one region), B_{12} (Growth rate of output value of new products), and B_{14} (Comprehensive utilization rate of industrial solid waste) has continuous downward trend. This analysis result has given warning to relevant management departments, and corresponding countermeasures and measures must be strengthened to curb the trend of decline. By using the above similar method, we can make the advantage analysis for other cities.

6.4 Comparison with Some Traditional Methods

In order to show the advantages of the MSHMADM method based on the linguistic 2-tuple presented in this paper, here we compare it with several traditional methods, i.e., TOPSIS method with interval numbers [50], Multi-hierarchy Grey Relative Analysis Method [7] and MTS [17], and so on. First of all, we give the decision-making process of TOPSIS method with interval numbers for the same application example given by Sect. 5.1. The detailed decision-making steps are listed as follows.

Step 1: Transform each original multi-source heterogeneous data matrix $R_j = \left(b_{ki}^{(j)}\right)_{15\times 4}$, into a interval number matrix $Z_j = \left(c_{ki}^{(j)}\right)_{15\times 4}$, where

$$c_{ki}^{(j)} = \left[c_{ki}^{(j)L}, c_{ki}^{(j)U}\right], \quad j = 1, 2, \ldots, 14.$$

In the multi-source heterogeneous data matrix R_j, if $b_{ki}^{(j)}$ is a real number b, then the corresponding transformed interval number is $b = [b, b]$. If $b_{ki}^{(j)}$ is a linguistic fuzzy number s_0, s_1, s_2, s_3 and s_4, then the corresponding transformed interval numbers are as follows.

$$s_0 = [0, 0.2], \quad s_1 = [0.2, 0.4], \quad s_2 = [0.4, 0.6], \quad s_3 = [0.6, 0.8], \quad s_4 = [0.8, 1].$$

Step 2: Normalize the interval number matrix $Z_j = \left(c_{ki}^{(j)}\right)_{15\times 4}$, and the normalized interval number matrix is denoted as $\tilde{G}_j = \left(\tilde{g}_{ki}^{(j)}\right)_{15\times 4} = \left(\left[\tilde{g}_{ki}^{(j)L}, \tilde{g}_{ki}^{(j)U}\right]\right)_{15\times 4}$, where

$$\tilde{g}_{ki}^{(j)L} = \frac{c_{ki}^{(j)L}}{\sqrt{\sum_{j=1}^{14}\left(c_{ki}^{(j)L}\right)^2}}, \quad \tilde{g}_{ki}^{(j)U} = \frac{c_{ki}^{(j)U}}{\sqrt{\sum_{j=1}^{14}\left(c_{ki}^{(j)U}\right)^2}}$$

$$j = 1, 2, \ldots, 14, \quad i = 1, 2, 3, 4, \quad k = 1, 2, \ldots, 15.$$

Step 3: Make the weighted process for the normalized interval number matrix $\tilde{G}_j = (\tilde{g}_{ki}^{(j)})_{15\times 4}$, j = 1, 2, …, 14. The weighted normalized interval number matrix is denoted as $G_j = \left(g_{ki}^{(j)}\right)_{15\times 4} = \left(\left[g_{ki}^{(j)L}, g_{ki}^{(j)U}\right]\right)_{15\times 4}$, j = 1, 2, …, 14, where

$$\left[g_{ki}^{(j)L}, g_{ki}^{(j)U}\right] = \left[\tilde{g}_{ki}^{(j)L}\cdot v_i \cdot w_k,\ \tilde{g}_{ki}^{(j)U}\cdot v_i \cdot w_k\right],$$
$$i = 1, 2, 3, 4, \quad k = 1, 2, \ldots, 15.$$

Step 4: Determine the positive ideal matrix (PIM) H^+ and the negative ideal matrix (NIM) H^- from the weighted normalized interval number matrices G_1, G_2, …, G_{14} obtained by Step 3, where

$$H^+ = (h_{ki})_{15\times 4} = \left(\left[\max_j g_{ki}^{(j)L}, \max_j g_{ki}^{(j)U}\right]\right)_{15\times 4}, \quad j = 1, 2, \ldots, 14,$$

$$H^- = (\bar{h}_{ki})_{15\times 4} = \left(\left[\min_j g_{ki}^{(j)L}, \min_j g_{ki}^{(j)U}\right]\right)_{15\times 4}, \quad j = 1, 2, \ldots, 14.$$

Step 5: Calculate the distance of each city and PIM (NIM), i.e.,

$$d_j^+ = \sum_{k=1}^{15}\sum_{i=1}^{4} d\left(g_{ki}^{(j)}, h_{ki}\right), \quad j = 1, 2, \ldots, 14.$$

$$d_j^- = \sum_{k=1}^{15}\sum_{i=1}^{4} d(g_{ki}^{(j)}, \bar{h}_{ki}), \quad j = 1, 2, \ldots, 14.$$

where $d(\cdot)$ is the distance operator between two interval numbers, i.e.,

$$d\left(g_{ki}^{(j)}, h_{ki}\right) = \sqrt{\left(g_{ki}^{(j)L} - \max_j g_{ki}^{(j)L}\right)^2 + \left(g_{ki}^{(j)U} - \max_j g_{ki}^{(j)U}\right)^2},$$

$$d\left(g_{ki}^{(j)}, \bar{h}_{ki}\right) = \sqrt{\left(g_{ki}^{(j)L} - \min_j g_{ki}^{(j)L}\right)^2 + \left(g_{ki}^{(j)U} - \min_j g_{ki}^{(j)U}\right)^2}.$$

Thus we obtain

$$d_1^+ = 0.152, \quad d_2^+ = 0.339, \quad d_3^+ = 0.300, \quad d_4^+ = 0.442, \quad d_5^+ = 0.449, \quad d_6^+ = 0.386, \quad d_7^+ = 0.415, d_8^+ = 0.460, \quad d_9^+ = 0.397, \quad d_{10}^+ = 0.344, \quad d_{11}^+ = 0.435, \quad d_{12}^+ = 0.504, \quad d_{13}^+ = 0.442, \quad d_{14}^+ = 0.545;$$
$$d_1^- = 0.551, \quad d_2^- = 0.365, \quad d_3^- = 0.403, \quad d_4^- = 0.262, \quad d_5^- = 0.254, \quad d_6^- = 0.318, \quad d_7^- = 0.289, d_8^- = 0.244, \quad d_9^- = 0.306, \quad d_{10}^- = 0.360, \quad d_{11}^- = 0.269, \quad d_{12}^- = 0.199, \quad d_{13}^- = 0.261, \quad d_{14}^- = 0.158.$$

.

Step 6: Calculate the approach degree P_j of each city by the following method.

$$P_j = \frac{d_j^-}{d_j^+ + d_j^-}, \quad j = 1, 2, \ldots, 14.$$

Then we get

$$P_1 = 0.784, \quad P_2 = 0.518, \quad P_3 = 0.573, \quad P_4 = 0.372, \quad P_5 = 0.362, \quad P_6 = 0.452, \quad P_7 = 0.411, \quad P_8 = 0.347, \quad P_9 = 0.436, \quad P_{10} = 0.511, \quad P_{11} = 0.382, \quad P_{12} = 0.283, \quad P_{13} = 0.371, \quad P_{14} = 0.225.$$

Step 7: Rank the quality level of industrial operation for all 14 cities according to the value of P_j, $j = 1, 2, \ldots, 14$ given above.
Since
$P_1 > P_3 > P_2 > P_{10} > P_6 > P_9 > P_7 > P_{11} > P_4 > P_{13} > P_5 > P_8 > P_{12} > u_{14}$,
the ranking is
Changsha $\succ$ Xiangtan $\succ$ Zhuzhou $\succ$ Chenzhou $\succ$ Yueyang $\succ$ Yiyang $\succ$ Changde $\succ$ Yongzhou $\succ$ Hengyang $\succ$ Loudi $\succ$ Shaoyang $\succ$ Zhangjiajie $\succ$ Huaihua $\succ$ West Hunan.

The ranking preference order of the above TOPSIS method with interval numbers and the ranking method based on L2TM-GRD given in this paper are listed in Table 14.

In addition, by using the Multi-hierarchy Grey Relative Analysis Method and MTS to evaluate the quality level of industrial operation for 14 cities in Hunan Province, we can also obtain the ranking results which are listed in Table 14.

From the evaluation results listed in Table 14, it can be seen that all the four comprehensive evaluation methods can provide the comprehensive ranking of the evaluated cities. Due to the principles of comprehensive evaluation methods are different, the ranking preference order has certain difference, but most of the distribution of advantages and disadvantages is consistent, namely, the industrial operation quality of Changsha, Xiangtan, Zhuzhou, and Chenzhou is high, and the industrial operation quality of Loudi, Zhangjiajie, Huaihua, and West Hunan is low. This result shows that the evaluation method (L2TM-GRD) of evaluating the industrial operation quality presented in this paper is feasible.

Table 14 Rankings of different decision methods

Cities	Rankings by L2TM-GRD	Rankings by TOPSIS method with interval numbers	Rankings by multi-hierarchy grey relative analysis method	Rankings by MTS
Changsha	1	1	1	2
Zhuzhou	4	3	4	3
Xiangtan	2	2	2	1
Hengyang	10	9	8	4
Shaoyang	8	11	9	9
Yueyang	7	5	5	6
Changde	6	7	7	7
Zhangjiajie	12	12	10	13
Yiyang	5	6	6	10
Chenzhou	3	4	3	5
Yongzhou	9	8	13	8
Huaihua	13	13	12	14
Loudi	11	10	11	12
West Hunan	14	14	14	11

However, for the four methods listed in Table 14, the MTS and the Multi-hierarchy Grey Relative Analysis Method give the evaluation results based on the data of only 1 year. The data of 1 year are often random and cannot represent the comprehensive level of the evaluation cities, which will lead to incomplete or inaccurate decision results. However, the evaluation method (L2TM-GRD) proposed in this paper and the TOPSIS method with interval numbers can just make up for this deficiency. This is because these two methods give the ranking results of industrial operation quality for 14 cities based on the data of four consecutive years (2013–2016), so the evaluation results of these two methods are more comprehensive and credible.

Moreover, from the ranking results listed in Table 13, we can obviously see that most of the rankings in the TOPSIS method with interval numbers are consistent with the rankings in the method of L2TM-GRD, but where the rankings between Hengyang and Loudi are great different in these two methods. By observing the ranking result in all methods listed in Table 14, we can find that the industrial operation quality in Shaoyang is better than that in Loudi given by three methods, i.e., TOPSIS method with interval numbers, Multi-hierarchy Grey Relative Analysis Method, and MTS. Thus, the ranking results in the method of L2TM-GRD are more reasonable than that in the TOPSIS method with interval numbers. In fact, in the TOPSIS method with interval numbers, all multi-source heterogeneous data are transformed into interval numbers. This data process method is easy to cause information loss and information distortion. However, in the evaluation method presented in this paper, all decision information is expressed by the linguistic 2-tuple, which can effectively reduce the information loss and information distortion [41, 45, 51].

Similar to the problem discussed in this paper, Li et al. [52] and Tang et al. [53] also studied the problem of decision making with heterogeneous data. In the proposed method in [52], four kinds of information: real numbers, interval numbers, triangular fuzzy numbers, and trapezoidal fuzzy numbers, were considered. The heterogeneous data were not transformed into a single form, but are directly integrated by a weighted-power average operator (WPA) operator, and a ranking formula with heterogeneous TOPSIS is adopted to select the best alternative. In [53], the ordinal consensus process in the environment of heterogeneous large-scale group decision making (LSGDM) was studied, and a k-means clustering algorithm considering preference orderings is extended. These two literatures provided us with other two new effective methods to deal with the problems of multi-source heterogeneous decision making. Drawing on their advantages of [52, 53], and combining with the research results of this paper, we can propose some more effective heterogeneous decision-making methods to deal with the evaluation problem of industrial operation quality with large-scale dynamic heterogeneous data (continuous time series heterogeneous data in many years).

7 Conclusion

In this paper, we present an evaluation model of industrial operation quality under the information environment of multi-source heterogeneous data. In this model, we present a new ranking method called L2TM-GRD, and propose a MSHMADM method based on L2TM-GRD. Comparing with the existing evaluation models of industrial operation quality such as Factor Analysis, Principal Component Analysis, TOPSIS, GRA method, MTS, AHP, VIKOR method, and so on, the contributions of the evaluation model based on L2TM-GRD presented in this paper are as follows.

(i) In most existing models of evaluating the industrial operation quality, the evaluation criteria values are all real numbers, that is, their methods only can be used to deal with the decision problem with the evaluation criteria values in the form of real numbers. In this paper, the model of evaluating the industrial operation quality is given under the information environment of multi-source heterogeneous data, which gives full consideration to the characterisation of multi-source heterogeneous data, that is, in the evaluation criteria values, the real numbers, interval numbers, and linguistic fuzzy numbers coexist. Thus, our model is more widely applicable, and it provides a new way to solve the problem of evaluating the industrial operation quality under uncertain information environment.

(ii) Many existing evaluation models only use the static data (evaluation criteria data within 1 year) to make evaluation decisions. The data of 1 year are often random and cannot represent the comprehensive level of the evaluation object, which will lead to incomplete or inaccurate decision results. In the presented model in this paper, the dynamic data (continuous time series data in recent several years) are used to make the evaluation and decision, so the decision results are more comprehensive and more reliable.

(iii) The evaluation model proposed in this paper can not only give a macro comparison but also give a microscopic comparison for the industrial operation quality of all cities. By a macro comparison, we can obtain the comprehensive evaluation ranking result for all cities, which can help the relevant management departments to find the difference of industrial operation quality among all cities. By a microscopic comparison, the advantages and disadvantages of all evaluation criteria for each city are fully understood, which can help the city to find out the improvement directions and measures to overcome their deficiencies. If the relevant management departments definitely know the advantageous criteria and disadvantageous criteria for one city, then they will suit the remedy to the case, and develop some corresponding effective countermeasures and measures to improve the quality level of industrial operation for this city.

(iv) In our evaluation model, we transform the problem of evaluating the industrial operation quality into a multi-attribute decision-making problem with multi-source heterogeneous data. In the decision-making process, all multi-source heterogeneous data, i.e., the real numbers, the interval numbers and the linguistic fuzzy numbers, are all transformed into the linguistic 2-tuples, and a new ranking method based on L2TM-GRD is proposed to evaluate and rank the quality level of industrial operation for all cities. This kind of data process method can effectively reduce the information loss and information distortion in the process of information gathering comparing with some existing methods.

In future, we will continue to do some further relative works, for example, we will study on group decision-making methods to deal with the problem of evaluating the industrial operation quality under the information environment of multi-source heterogeneous data, and design an interactive online evaluation system to evaluate the industrial operation quality based on the evaluation model presented in this paper, and so on.

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