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Multicriteria Group Decision-Making for Supplier Selection Based on Intuitionistic Cubic Fuzzy Aggregation Operators

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Abstract This article is an advanced approach to intuitionistic fuzzy set through application of cubic set theory. For instance, we establish the idea of the intuitionistic cubic fuzzy set (ICFS) theory and define several operations for ICFS ; also establish a series of weighted aggregation operators under intuitionistic cubic fuzzy information, so-called intuitionistic cubic fuzzy weighted averaging (ICFWA) operator, intuitionistic cubic fuzzy order weighted averaging (ICFOWA) operator, intuitionistic cubic fuzzy weighted geometric (ICFWG) operator, intuitionistic cubic fuzzy order weighted geometric (ICFOWG) operator, intuitionistic cubic fuzzy hybrid averaging (ICFHA) operator, and intuitionistic cubic fuzzy hybrid geometric (ICFHG) operator; and further study their fundamental properties and showed the relationship among these aggregation operators. In order to demonstrate the feasibility and practicality of the mentioned new technique, we develop multicriteria group decision-making algorithm under intuitionistic cubic fuzzy environment. Further, the proposed method applied to supply chain management and for implementation, consider numerical application of supply chain management. Also the selected supplier by ICFD aggregation operators is verified by VIKOR method. Comparing the proposed techniques with other pre-existing aggregation operators, we concluded that the proposed technique is better, reliable, and effective.

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1 Introduction

Multicriteria group decision-making (MCGDM) has played an important role in daily activities, such as economic, engineering, education, medical, and so on. In MCGDM, one of the problems involves gathering many sources of information, i.e., finite alternatives giving the final result according to the attribute values of different alternatives via aggregating process [6, 17, 28, 32, 42, 50]. In the decision process, an important problem is how to express the attribute value. Because of the complexity of decision-making problems, sometimes it is complicated to represent the attributes by crisp numbers. As decision-makers can make decisions at a certain level, because of the complications of such decision-making issues themselves and management environments, it is possible that they have doubts for their interpretations. For dealing with such uncertain situation Zadeh [47] in 1965, presented the idea of the fuzzy set (FS), Zadeh assigned membership grades to elements of a set in the interval [0,1] by offering the idea of fuzzy sets. Zadeh's work in this direction is remarkable as many of the set theoretic properties of crisp cases were defined for fuzzy sets. After many applications of fuzzy set theory, Atanassov observed that there are many shortcomings in this theory. Namely, hesitancy degree may exist there, which is too essential factor to be focused on, while establishing perfectly beneficent models and results of the problems. This kind of hesitancy degrees is appropriately presented by intuitionistic fuzzy values instead of accurate numbers. Zadeh fuzzy sets [47] are generalized in the form of the Atanassov's intuitionistic fuzzy sets (AIFSs) [1]. An element of an AIFS is represented by an ordered pair consists of function of membership and function of non-membership, where the sum of both mentioned functions is less than or equal to one, hence sketching the fuzzy characteristic of information in more detailed and comprehensive manner as compared to fuzzy set, which is distinguished by a membership function only. Several researchers have done quite valuable contributions in the expansion of AIFS generalization and its approach to different fields; the result of which is in the form of great success of AIFSs in theoretical and technical aspects.

A major part of MCGDM with IFSs is the aggregation of intuitionistic fuzzy information [15, 20, 24–27, 34, 37, 39, 45, 51, 52, 54]. The decision-making process under undetermined or firm circumstances, IFNs are too much convenient to disclose sensitive information of a decision-maker over objects. The aggregation of IFNs is an essential step to get a decision problem's outcome. For this purpose, a number of operators have been introduced recently to aggregate IFNs which are known as intuitionistic fuzzy hybrid aggregation (IFHA) operator, intuitionistic fuzzy hybrid geometric (IFHG) operator, intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, intuitionistic fuzzy ordered weighted geometric (IFOWG) operator, intuitionistic fuzzy weighted averaging (IFWA) operator, and intuitionistic fuzzy weighted geometric (IFWG) operator [5, 21, 29, 30, 33, 36, 38, 40, 41, 43, 44, 46].

However, intuitionistic fuzzy set does not explain the uncertainty problems. To overcome this difficulty, Jun [18] introduced cubic fuzzy set (CFS). This theory made it possible to deal with uncertainty problems. Cubic set theory also explains the satisfied, unsatisfied, and unpredictable information, which was not explained by fuzzy set theory and intuitionistic fuzzy set theory [7–14, 19, 31, 35, 38, 48]. Cubic set has more desirable information than FS and AIFS [22, 23]. It is one of the generalized forms of fuzzy set and AIFS, just like AIFS, every element of a cubic fuzzy set is represented as a structure of an ordered pair, which is characterized by function of membership and function of non-membership. The non-membership is just like the normal fuzzy set whereas function of membership is grip in the form of an interval.

In today's competitive global markets, supplier selection plays an important role in reducing the company's production and material costs [2–4, 49, 55]. This leads to success in survival and sustainability in competitive market. For this reason, the evaluation and selection of suitable suppliers have become an important part of supply chain management. The nature of supplier selection process is a complex multicriteria group decision-making (MCGDM) problem that deals with qualitative and quantitative factors that may conflict with incomplete and uncertain information. To solve this problem, it is desirable to develop an MCGDM effective supplier selection model. In this study, an application of ICF aggregation operators under ICF environment has been used to solve supplier selection problems with conflicting and incommensurable (different units) criteria, assuming that compromising is acceptable for conflict resolution. The decision-maker wants a solution, which must be closest to the ideal, and the alternatives are evaluated according to all established criteria. For this purpose, MAGDM model based on ICFS theory has been proposed to deal with the supplier selection problems in the supply chain management. A case study has been illustrated as an application of the proposed model. VIKOR method [16] focuses on ranking and sorting a set of alternatives against various, or possibly conflicting and non-commensurable, decision criteria assuming that compromising is acceptable to resolve conflicts. To verify the results of the proposed methods, the same case is solved by fuzzy VIKOR method.

Due to the motivation and inspiration of the above discussion in this study, we have given a new approach to AIFS through application of cubic set theory. For instance, the concept of intuitionistic cubic fuzzy set (ICFS) is introduced . Each element of which consists function of membership and function of non-membership. Membership function is cubic fuzzy set and non-membership function is also cubic fuzzy set. ICFS is the hybrid set which can contain much more information to express a CFS and an AIFS simultaneously for handling the uncertainties in the data.

In this article firstly given the conceptual information of intuitionistic cubic fuzzy numbers (ICFNs), and initiate some fundamental laws of ICFNs. We also establish the concepts of accuracy function and score function of ICFNs, on the basis of these functions a simple procedure for ranking of ICFNs is introduced. Since an aggregation operator is an important mathematical tool in decision-making problems, we introduce the aggregation proficiency for intuitionistic cubic fuzzy information and establish several aggregation operators, such as the intuitionistic cubic fuzzy hybrid geometric (ICFHG) operator, intuitionistic cubic fuzzy hybrid averaging (ICFHA) operator, intuitionistic cubic fuzzy order weighted averaging (ICFOWA) operator, intuitionistic cubic fuzzy order weighted geometric (ICFOWG) operator, intuitionistic cubic fuzzy weighted averaging (ICFWA) operator, and intuitionistic cubic fuzzy weighted geometric (ICFWG) operator and present a number of properties of the mentioned operators.

To complete the said task, the remaining study is organized accordingly. In Sect. 2, firstly we review some fundamental concepts of intuitionistic fuzzy set. Then, we define a number of intuitionistic fuzzy aggregation operators. In Sect. 3, the concept of intuitionistic cubic fuzzy set is presented and some valuable fundamental properties are studied. In Sect. 4, a number of ICF aggregation operators are introduced such as ICFWA operator, ICFOWA operator, ICFWG operator, ICFOWG operator, ICFHA operator, and ICFHG operator and discussed their few properties. In Sect. 5, the mentioned operators are used to resolve a decision-making problem under ICF environment. Also, a numerical application related to the selection of suitable supplier for the purchase of components by a company is given to illustrate the feasibility and practicality of the mentioned new techniques. In Sect. 6, the verification of the proposed ICFD averaging operators by VIKOR method is given. In Sect. 7, the comparison of suggested ICF averaging operators to the pre-existing averaging operators are discussed, and finally in the last section, the conclusions are presented.

2 Preliminaries

We introduce some basic definitions and their valuable properties in this section.

Definition 1 [1] Suppose non-empty set G. An intuitionistic fuzzy set A of G is defined as:

$$A = \{(g, \ddot{a}_A(g), \ddot{u}_A(g) | g \in G\},$$

where for each element $g \in G$ to the set G, the function of membership is denoted by the functions $\ddot{a}_A(g) : G \to [0, 1]$ and the function $\ddot{u}_A(g) : G \to [0, 1]$, denotes the function of non-membership, respectively, with $0 \leq \ddot{u}_A(g) + \ddot{a}_A(g) \leq 1$ for every $g \in G$. Furthermore, the intuitionistic fuzzy hesitation margin of g to A or fuzzy index is given by $\pi_A(g) = 1 - \ddot{a}_A(g) - \ddot{u}_A(g)$, [1]. $\pi_A(g)$ is also known as the indeterminacy degree of $g \in G$ to the IFS A [1].

Definition 2 [18] A cubic fuzzy set A in G is given as follows:

$$A = \left\{(g, [\ddot{a}_A^-(g), \ddot{a}_A^+(g)], \ddot{u}_A(g)) | g \in G\right\},$$

where the first element is an interval-valued fuzzy (IVF) number, which denotes the membership degree, and second is a simple fuzzy number, which denotes the non-membership degree.

Definition 3 [40] Let $A_p = \langle \ddot{a}_{A_p}(g), \ddot{u}_{A_p}(g) \rangle$ $(p = 1, 2, \ldots, l)$, is a collection of AFNs in G. An IFWA operator of dimension l is given by;

$$\begin{aligned}&\mathrm{IFWA}_{\varpi}(A_1, A_2, A_3, \ldots, A_l)\\&= \{1 - \Pi_{p=1}^{l}(1 - \ddot{a}_{A_p}(g))^{\varpi_p}, \Pi_{p=1}^{l}\ddot{u}_{A_p}^{\varpi_p}(g)\}.\end{aligned}$$

where the weight vector of A_p $(p = 1, 2, \ldots l)$, be $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$, with $\Sigma_{p=1}^{l}\varpi_p = 1$ and $\varpi_p \in [0, 1]$.

Definition 4 [41] Suppose $A_p = \langle \ddot{a}_{A_p}(g), \ddot{u}_{A_p}(g) \rangle$ $(p = 1, 2, \ldots, l)$, is a collection of IFVs in G, then an IFWG operator is given by;

$$\begin{aligned}&\mathrm{IFWG}_{\varpi}(A_1, A_2, A_3, \ldots, A_l)\\&= \{\Pi_{p=1}^{l}\ddot{a}_{A_p}^{\varpi_p}(g), 1 - \Pi_{p=1}^{l}(1 - \ddot{u}_{A_p}(g))^{\varpi_p}\}.\end{aligned}$$

where the weight vector of $A_p (p = 1, 2, \ldots l)$, is $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$ with $\Sigma_{p=1}^{l}\varpi_p = 1$ and $\varpi_p \in [0, 1]$

Definition 5 [40] Suppose $A_p = \langle \ddot{a}_{A_p}(g), \ddot{u}_{A_p}(g) \rangle$ $(p = 1, 2, \ldots, l)$, be a collection of IFVs in G. An IFHA operator of dimension n is given as a mapping IFHA : $\Omega^l \to \Omega$, having an associated weight vector $\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$ and $\varpi_p \in [0, 1]$ such that $\Sigma_{p=1}^{l}\varpi_p = 1$

$$\begin{aligned}&\mathrm{IFHA}_{w,\varpi}(A_1, A_2, \ldots, A_l)\\&= \varpi_1\tilde{A}_{\sigma_{(1)}} \oplus \varpi_2\tilde{A}_{\sigma_{(2)}} \oplus \varpi_3\tilde{A}_{\sigma_{(3)}} \oplus, \ldots, \oplus\varpi_l\tilde{A}_{\sigma_{(l)}},\end{aligned}$$

where the jth biggest of the weighted IFVs $\tilde{A}_p$ is $\tilde{A}_{\sigma_{(p)}}$, i.e.,$\tilde{A} = lw_pA_p$. Also $w = (w_1, \ldots, w_l)^{\mathrm{T}}$ is the weight of A_p and $w_p \in [0, 1]$ such that $\Sigma_{p=1}^{l}w_p = 1$, where l is the balancing coefficient, to maintain the balance. By using the IFHA operator, the aggregated value of A_p is again an IFV, as expressed here:

$$\begin{aligned}&\mathrm{IFHA}_{w,\varpi}(A_1, A_2, \ldots, A_{l-1}, A_l)\\&= \{1 - \Pi_{p=1}^{l}(1 - \ddot{a}_{\tilde{A}_{\sigma_p}}(g))^{\varpi_p}, \Pi_{p=1}^{l}\ddot{u}_{\tilde{A}_{\sigma_p}}^{\varpi_p}(g)\}.\end{aligned}$$

Definition 6 [40] Suppose $A_p = \langle \ddot{a}_{A_p}(g), \ddot{u}_{A_p}(g) \rangle$ $(p = 1, 2, \ldots, l)$, is a collection of IFVs in G. An IFHG operator of dimension l is given by:

$$\begin{aligned}&\mathrm{IFHG}_{w,\varpi}(A_1, A_2, \ldots A_{l-1}, A_l)\\&= \{\Pi_{p=1}^{l}\ddot{a}_{\tilde{A}_{\sigma_p}}^{\varpi_p}(g), 1 - \Pi_{p=1}^{l}(1 - \ddot{u}_{\tilde{A}_{\sigma_p}}^{\varpi_p}(g))^{\varpi_p}\},\end{aligned}$$

where the jth biggest of the weighted intuitionistic fuzzy values $\tilde{A}_p$ is $\tilde{A}_{\sigma_{(p)}}$. *A.e.* $\tilde{A}_{\sigma_{(p)}} = lw_pA_p, (p = 1, 2, \ldots l)$. Also, the associated weight vector of A_p $(p = 1, 2, \ldots l)$, be $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$, with $\Sigma_{p=1}^{l}\varpi_p = 1$ and $\varpi_p \in [0, 1]$.and l is the balancing coefficient, which maintains the balance.

3 Intuitionistic Cubic Fuzzy Set and its Basic Relations and Operations

In this section, we define the intuitionistic cubic fuzzy set and also describe its fundamental relations and different operations.

Definition 7 Let G be a non-empty set. Then, the intuitionistic cubic fuzzy set (ICFS) I in G, is given as follows:

$$I = \{g, \langle c_I, c'_I \rangle | g \in G\},$$

or

$$I = \{(g, \langle [e^-, e^+], \lambda \rangle, \langle [r^-, r^+], \delta \rangle) | g \in G\},$$

where $\langle [e^-, e^+], \lambda \rangle$ indicates the exact degree of membership and $\langle [r^-, r^+], \delta \rangle$ indicates the exact degree of non-membership of I. Here, $[r^-, r^+] \subset [0,1]$, $[e^-, e^+] \subset [0,1]$, $\delta : G \to [0,1]$, and $\lambda : G \to [0,1]$. Subject to $\sup[r^-, r^+] + \sup[e^-, e^+] \leq 1$ and $\lambda + \delta \leq 1$. For simplicity, we call $(\langle [e^-, e^+], \lambda \rangle, \langle [r^-, r^+], \delta \rangle)$ is an intuitionistic cubic fuzzy number (ICFN) denoted by $I_1 = \langle c_I, c'_I \rangle$, Furthermore, we have the hesitation margin defined as:

$$\pi(I) = \{\langle [1,1] - [[e^-, e^+] + [r^-, r^+]] \rangle, \langle 1 - (\lambda + \delta) \rangle\},$$
$$\pi(I) = \{[1 - (e^- + r^-), 1 - (e^+ + r^+)], 1 - (\lambda + \delta)\},$$

which is also called ICFS index or the indeterminacy degree of $g \in G$ for ICFS.

On the basis of intuitionistic cubic fuzzy values, we established a score function $S(I)$ which estimates the compliance degree that an alternative satisfies the need of a decision-maker.

Definition 8 Let $I_1 = \langle c_I, c'_I \rangle$ be the intuitionistic cubic fuzzy value (ICFV). Then, score function $S(I)$ is expressed below as:

$$S(I) = [(e^- + e^+ + \lambda - r^- - r^+ - \delta)/3],$$

such that $S(I) \in [-1, 1]$. The function $S(I)$ calculates the score of the intuitionistic cubic fuzzy value I. From this definition, it is quite clear that the score of I straight indicates the difference of $\langle [e^-, e^+], \lambda \rangle$ and $\langle [r^-, r^+], \delta \rangle$. , i.e., the higher difference will show the higher score of I and hence means the bigger intuitionistic cubic fuzzy value I.

Definition 9 For the evaluation of the accuracy degree of the ICFV I, an accuracy function $H(I)$ is defined as:

$$H(I) = [(e^- + e^+ + \lambda + r^- + r^+ + \delta)/3],$$

where $H(I) \in [0,1]$. If $H(I)$ has a bigger value, it will indicate the higher level of accuracy of the membership degree of the ICFV.

Definition 10 Let $I_1 = \left\langle c_{I_1}, c'_{I_1} \right\rangle$ and $I_2 = \left\langle c_{I_2}, c'_{I_2} \right\rangle$ be two ICFNs and the score functions of I_2 and I_1 are represented as $S(I_2)$ and $S(I_1)$ and the accuracy functions be given as $H(I_2)$ and $H(I_1)$, respectively. Then,

Definition 11

i) $S(I_2) < S(I_1) \Longrightarrow I_2 < I_1$.
ii) $S(I_2) = S(I_1)$, and

a) $H(I_2) < H(I_1) \Longrightarrow I_2 < I_1$
b) $H(I_2) = H(I_1) \Longrightarrow I_2 = I_1$.

Proposition 1 *Let* $I_1 = \left\langle c_{I_1}, c'_{I_1} \right\rangle$ *and* $I_2 = \left\langle c_{I_2}, c'_{I_2} \right\rangle$ *be two ICFSs in G, and* $k > 0$, *we have the below operations:*

(1) *Inclusion:* $I_1 \subset I_2 \Longleftrightarrow c_{I_1} \leq c_{I_2}$ *and* $c'_{I_1} \geq c'_{I_2}$ *for all* $g \in G$
(2) *Equality:* $I_1 = I_2 \Longleftrightarrow c_{I_1} = c_{I_2}$ *and* $c'_{I_1} = c'_{I_2}$ *for all* $g \in G$
(3) $I^c = \{g, \langle c'_I, c_I \rangle | g \in G\}$ *or* $I^c = \{(g, \langle [r^-, r^+], \delta \rangle, \langle [e^-, e^+], \lambda \rangle) | g \in G\}$.
(4) $I_1 \cup I_2 = \left\{ \begin{array}{c} g, (r\max\{[e_1^-, e_1^+], [e_2^-, e_2^+]\}, \min\{\lambda_1, \lambda_2\}), \\ (r\min\{[r_1^-, r_1^+], [r_2^-, r_2^+]\}, \max\{\delta_1, \delta_2\}) \end{array} \right\}$.
(5) $I_1 \cap I_2 = \left\{ \begin{array}{c} g, (r\min\{[e_1^-, e_1^+], [e_2^-, e_2^+]\}, \max\{\lambda_1, \lambda_2\}), \\ (r\max\{[r_1^-, r_1^+], [r_2^-, r_2^+]\}, \min\{\delta_1, \delta_2\}) \end{array} \right\}$.
(6) $I_1 \oplus I_2 = \left\{ \begin{array}{l} ([e_1^- + e_2^- - e_1^- e_2^-, e_1^+ + e_2^+ - e_1^+ e_2^+], \lambda_1 \\ \quad + \lambda_2 - \lambda_1 \lambda_2), \\ ([r_1^- r_2^-, r_1^+ r_2^+], \delta_1 \delta_2) \end{array} \right\}$.
(7) $I_1 - I_2 = \left\{ \begin{array}{l} (\min\{[e_1^-, e_1^+], [r_2^-, r_2^+]\}, \max\{\delta_1, \lambda_2\}), \\ (\max\{[r_1^-, r_1^+], [e_2^-, e_2^+]\}, \min\{\delta_2, \lambda_1\}) \end{array} \right\}$.
(8) $I_1 \otimes I_2 = \left\{ \begin{array}{l} ([e_1^- e_2^-, e_1^+ e_2^+], \lambda_1 \lambda_2), ([r_1^- + r_2^- - r_1^- r_2^-, \\ \quad r_1^+ r_2^+ - r_1^+ r_2^+], \\ \delta_1 + \delta_2 - \delta_1 \delta_2) \end{array} \right\}$.
(9) $kI = \{([1 - (1 - e^-)^k, 1 - (1 - e^+)^k], 1 - (1 - \lambda)^k), ([r^{-k}, r^{+k}], \delta^k)\}$.
(10) $I^k = \{[([e^{-k}, e^{+k}], \lambda^k), ([1 - (1 - r^-)^k, 1 - (1 - r^+)^k], 1 - (1 - \delta)^k)\}$.

Some properties of intuitionistic cubic fuzzy sets is given in the following proposition, which can easily be proved.

Proposition 2 *Let* $I_1 = \left\langle c_{I_1}, c'_{I_1} \right\rangle$ *and* $I_2 = \left\langle c_{I_2}, c'_{I_2} \right\rangle$ *be ICFSs in G,* and $k_1, k_2 > 0$; *then we have,*

(1) $I_2 \oplus I_1 = I_1 \oplus I_2$
(2) $I_2 \otimes I_1 = I_1 \otimes I_2$
(3) $k(I_1 \oplus I_2) = kI_1 \oplus kI_2$
(4) $k(I_1 \otimes I_2) = kI_1 \otimes kI_2$
(5) $(k_1 \oplus k_2)I = k_1 I \oplus k_2 I$
(6) $(k_1 \otimes k_2)I = k_1 I \otimes k_2 I$
(7) $(I_1 \oplus I_2)^k = I_1^k \oplus I_2^k$

(8) $(I_1 \otimes I_2)^k = I_1^k \otimes I_2^k$
(9) $I^{k_1+k_2} = I^{k_1} \otimes I^{k_2}$
(10) $(I^k)^m = (I^m)^k$

Definition 12 Let $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle (p = 1, 2)$ be any two ICFNs on a set G. The distance measure between I_1 and I_2 can be defined as follows:

$$d(I_1, I_2) = \frac{1}{6}[|e_1^- - e_2^-| + |e_1^+ - e_2^+| + |\lambda_1 - \lambda_2| + |r_1^- - r_2^-| + |r_1^- - r_2^-| + |\delta_1 - \delta_2|].$$

4 Intuitionistic Cubic Fuzzy Aggregation Operators

We introduced a number of intuitionistic cubic fuzzy aggregation operators and discussed some of their characteristics in this section.

4.1 Intuitionistic Cubic Fuzzy Weighted Averaging Operators

This section contains the definitions of ICFWA operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 13 Let $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle (p = 1, 2, \ldots l)$, be a set of ICFVs in G and let ICFWA operator of dimension l is a mapping ICFWA : $\Omega^l \to \Omega$, and is defined as,

$$\mathrm{ICFWA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = \varpi_1 I_1 \oplus \varpi_2 I_2 \oplus \varpi_3 I_3 \oplus, \ldots, \oplus \varpi_l I_l.$$

Here, the weight vector of I_p $(p = 1, \ldots l)$, is $\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$, with $\Sigma_{p=1}^{l} \varpi_p = 1$ and $\varpi_p \in [0, 1]$. The ICFWA operator transforms to an ICFA operator of dimension l when $\varpi = (1/l, 1/l, 1/l, \ldots, 1/l)^{\mathrm{T}}$, which is defined below:

$$\mathrm{ICFWA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = 1/l(I_1 \oplus I_2 \oplus I_3 \oplus, \ldots, \oplus I_l).$$

Based on the intuitionistic cubic laws part (6) and (9) in Proposition (1) of ICFVs, the ICFWA operator can be converted into the below structure by induction on l.

Theorem 1 *Suppose* $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle (p = 1, \ldots l)$, *be a group of ICFVs in G, then their aggregated value by using the ICFWA operator is again an ICFV and is written as:*

$$\mathrm{ICFWA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = \left\{ \begin{array}{c} ([1 - \Pi_{p=1}^{l}(1 - e_p^-)^{\varpi_p}, 1 - \Pi_{p=1}^{l}(1 - e_p^+)^{\varpi_p}], \\ 1 - \Pi_{p=1}^{l}(1 - \lambda_p)^{\varpi_p}), ([\Pi_{p=1}^{l} r_p^{-\varpi_p}, \Pi_{p=1}^{l} r_p^{+\varpi_p}], \Pi_{p=1}^{l} \delta_p^{\varpi_p}) \end{array} \right\},$$

where $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$ *is the weight vector of* $I_p (p = 1, 2, \ldots, l)$, *with* $\Sigma_{p=1}^{l} \varpi_p = 1$ *and* $\varpi_p \in [0, 1]$. *We have established some properties of ICFWA operator on the basis of this theorem.*

Proposition 3 *Let* $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle, (p = 1, 2, \ldots, l)$ *be a group of ICFVs in G and the weight vector of* I_p *be* $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$, *with* $\Sigma_{p=1}^{l} \varpi_p = 1$ *and* $\varpi_p \in [0, 1]$, *then the following properties are established.*

Idempotency: If all $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle, (p = 1, 2, \ldots, l)$ are equal, i.e., $I_p = I$, then

$$\mathrm{ICFWA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = I$$

Proof Straightforward. □

Boundary: For every ϖ, $I^- \leq \mathrm{ICFWA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq I^+$, where

$$I^+ = \{([\min e_p^-, \max e_p^+], \max \lambda_p), ([\max r_p^-, \min r_p^+], \min \delta_p)\}$$
$$I^- = \{([\max e_p^-, \min e_p^+], \min \lambda_p), ([\min r_p^-, \max r_p^+], \max \delta_p)\}.$$

Monotonicity: Let $I_p^* = \left\langle c_{I_p^*}, c'_{I_p^*} \right\rangle (p = 1, 2, \ldots l)$ be a group of ICFVs, if for all p, $I_p \leq I_p^*$. Then,

$$\mathrm{ICFWA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq \mathrm{ICFWA}_{\varpi}(I_1^*, I_2^*, I_3^*, \ldots, I_l^*).$$

4.2 Intuitionistic Cubic Fuzzy Order Weighted Averaging Operators

We introduce ICFOWA operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 14 Let $I_p = \langle c_{I_p}, c'_{I_p} \rangle (p = 1, \ldots l)$, be a set of ICFVs in G. An intuitionistic cubic fuzzy order weighted averaging (ICFOWA) operator is defined as a mapping ICFOWA : $\Omega^l \to \Omega$, and is defined as below:

$$\mathrm{ICFOWA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = \varpi_1 I_{\sigma_{(1)}} \oplus \varpi_2 I_{\sigma_{(2)}} \oplus \varpi_3 I_{\sigma_{(3)}} \oplus, \ldots, \oplus \varpi_l I_{\sigma_{(l)}},$$

then, ICFOWA operator is known as intuitionistic cubic fuzzy order weighted averaging operator of dimension l.

where for all p, $I_{\sigma_{(p-1)}} \geq I_{\sigma_{(p)}}$ $(\sigma_{(1)}, \sigma_{(2)}, \sigma_{(3)}, \ldots, \sigma_{(l)})$ is a permutation of (1, 2, 3, …, l). Also, the weight vector of $I_p(p = 1, 2, \ldots, l)$, is $\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$, with $\Sigma_{p=1}^{l}\varpi_p = 1$ and $\varpi_p \in [0, 1]$. Furthermore, the ICFOWA operator will convert to an ICFA operator of dimension l if $\varpi = (1/l, 1/l, 1/l, \ldots, 1/l)^{\mathrm{T}}$, which is given below:

$$\mathrm{ICFOWA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = 1/l(I_1 \oplus I_2 \oplus I_3 \oplus, \ldots, \oplus I_l).$$

Based on the intuitionistic cubic laws part (6), (9) of Proposition (1) and Theorem 1 of ICFVs, the ICFOWA operator can be converted into the below structure by induction on l.

Theorem 2 *Suppose* $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle (p = 1, \ldots l)$, *be a set of ICFVs in G, as defined in Theorem (1), then their aggregated value by using the ICFOWA operator is again an ICFV, and is given by,*

$$\mathrm{ICFOWA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = \left\{ \begin{array}{l} ([1 - \Pi_{p=1}^{l}(1 - e^{-}_{\sigma_{(p)}})^{\varpi_p}, 1 - \Pi_{p=1}^{l}(1 - e^{+}_{\sigma_{(p)}})^{\varpi_p}], \\ 1 - \Pi_{p=1}^{l}(1 - \lambda_{\sigma(p)})^{\varpi_p}), ([\Pi_{p=1}^{l} r^{-\varpi_p}_{\sigma_{(p)}}, \Pi_{p=1}^{l} r^{+\varpi_p}_{\sigma(p)}], \Pi_{p=1}^{l}\delta^{\varpi_p}_{\sigma_{(p)}}) \end{array} \right\},$$

where the weight vector of $I_p(p = 1, 2, \ldots, l)$, *is* $\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$ *, with* $\Sigma_{p=1}^{l}\varpi_p = 1$ *and* $\varpi_p \in [0, 1]$.

Proposition 4 *Let* $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle, (p = 1, 2, \ldots, l)$ *be a group of ICFVs in G and the weight vector of* I_p *be* $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$, *with* $\Sigma_{p=1}^{l}\varpi_p = 1$ *and* $\varpi_p \in [0, 1]$, *then the following properties are established.*

Idempotency: If all $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle, (p = 1, 2, \ldots, l)$ are equal, i.e., $I_p = I$, then

$$\mathrm{ICFOWA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = I.$$

Boundary: For every ϖ, $I^{-} \leq \mathrm{ICFWOA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq I^{+}$, where

$$I^{+} = \{([\min e^{-}_{\sigma_{(p)}}, \max e^{+}_{\sigma_{(p)}}], \max \lambda_{\sigma(p)}), ([\max r^{-}_{\sigma_{(p)}}, \min r^{+}_{\sigma_{(p)}}], \min \delta_{\sigma_{(p)}})\},$$
$$I^{-} = \{([\max e^{-}_{\sigma_{(p)}}, \min e^{+}_{\sigma_{(p)}}], \min \lambda_{\sigma(p)}), ([\min r^{-}_{\sigma_{(p)}}, \max r^{+}_{\sigma_{(p)}}], \max \delta_{\sigma_{(p)}})\}.$$

Monotonicity: Let $I^{*}_{p} = \left\langle c_{I^{*}_{p}}, c'_{I^{*}_{p}} \right\rangle (p = 1, 2, \ldots l)$ be a group of ICFVs, if for all p, $I_p \leq I^{*}_{p}$. Then,

$$\mathrm{ICFOWA}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq \mathrm{ICFOWA}_{\varpi}(I^{*}_1, I^{*}_2, I^{*}_3, \ldots, I^{*}_l).$$

4.3 Intuitionistic Cubic Fuzzy Hybrid Averaging Operator

Here, we introduce ICFHA operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 15 Let $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle (p = 1, 2, \ldots l)$, is a set of ICFVs in G. An intuitionistic cubic fuzzy hybrid averaging ICFHA operator of dimension l is a mapping ICFHA : $\Omega^{l} \to \Omega$, having an associated weight vector $\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$ with $\Sigma_{p=1}^{l}\varpi_p = 1$ and $\varpi_p \in [0, 1]$, such that,

$$\mathrm{ICFHA}_{w,\varpi}(I_1, I_2, \ldots, I_l) = \varpi_1 \widetilde{I}_{\sigma_{(1)}} \oplus \varpi_2 \widetilde{I}_{\sigma_{(2)}} \oplus, \ldots, \oplus \varpi_l \widetilde{I}_{\sigma_{(l)}},$$

here $\widetilde{I}_{\sigma_{(p)}}$ is the jth largest of the weighted intuitionistic fuzzy values $\tilde{I}_p$. Also, the weight of I_p is $w = (w_1, \ldots, w_l)^{\mathrm{T}}$, with $\Sigma_{p=1}^{l} w_p = 1$ and $w_p \in [0, 1]$, *i.e.*, $\widetilde{I}_p = l w_p I_p (p = 1, 2, \ldots l)$. Here l is the balancing coefficient, which keeps the proper balance, especially the ICFHA operator will obtain the form of an intuitionistic cubic fuzzy averaging (IFA) operator of dimension l if $\varpi = (1/l, 1/l, 1/l, \ldots, 1/l)^{\mathrm{T}}$.

Based on the intuitionistic cubic laws part (6), (9) in Proposition (1) and Theorem 1 of ICFVs, the ICFHA operator can be converted to the below structure by induction on l.

Theorem 3 *Let* $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle (p = 1, 2, \ldots, l)$, *is a class of ICFVs in G, then their aggregated value by using the ICFHA operator is again an ICFV and of the form,*

$$\mathrm{ICFHA}_{w,\varpi}(I_1, I_2, I_3, \ldots, I_l) = \left\{ \begin{array}{c} ([1 - \Pi_{p=1}^{l}(1 - \widetilde{e}^{-}_{\sigma_{(p)}})^{\varpi_p}, 1 - \Pi_{p=1}^{l}(1 - \widetilde{e}^{+}_{\sigma_{(p)}})^{\varpi_p}], \\ 1 - \Pi_{p=1}^{l}(1 - \widetilde{\lambda}_{\sigma_{(p)}})^{\varpi_p}), ([\Pi_{p=1}^{l} \widetilde{r}^{-\varpi_p}_{\sigma_{(p)}}, \Pi_{p=1}^{l} \widetilde{r}^{+\varpi_p}_{\sigma_{(p)}}], \Pi_{p=1}^{l} \widetilde{\delta}^{\varpi_p}_{\sigma_{(p)}}) \end{array} \right\},$$

$\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$ *is the weight vector of* $I_p, (p = 1, 2, \ldots, l)$, *with* $\Sigma_{p=1}^{l}\varpi_p = 1$ *and* $\varpi_p \in [0, 1]$.

Proposition 5 *Let* $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle, (p = 1, 2, \ldots, l)$ *be a group of ICFVs in G and the weight vector of* I_p *be* $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$, *with* $\Sigma_{p=1}^{l}\varpi_p = 1$ *and* $\varpi_p \in [0, 1]$, *then the following properties are established.*

Idempotency: If all $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle, (p = 1, 2, \ldots, l)$ are equal, i.e., $I_p = I$, then

$$\mathrm{ICFHA}_{w,\varpi}(I_1, I_2, I_3, \ldots, I_l) = I.$$

Boundary: For every ϖ, $I^{-} \leq \mathrm{ICFHA}_{w,\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq I^{+}$,where

$$I^{+}=\{([\min\widetilde{e}^{-}_{\sigma_{(p)}},\max\widetilde{e}^{+}_{\sigma_{(p)}}],\max\widetilde{\lambda}_{\sigma_{(p)}}),([\max\widetilde{r}^{-}_{\sigma_{(p)}},\min\widetilde{r}^{+}_{\sigma_{(p)}}],\min\widetilde{\delta}_{\sigma_{(p)}})\}$$
$$I^{-}=\{([\max\widetilde{e}^{-}_{\sigma_{(p)}},\min\widetilde{e}^{+}_{\sigma_{(p)}}],\min\widetilde{\lambda}_{\sigma_{(p)}}),([\min\widetilde{r}^{-}_{\sigma_{(p)}},\max\widetilde{r}^{+}_{\sigma_{(p)}}],\max\widetilde{\delta}_{\sigma_{(p)}})\}.$$

Monotonicity: Let $I_p^* = \left\langle c_{I_p^*}, c'_{I_p^*}\right\rangle (p = 1, 2, \ldots l)$ be a group of ICFVs, if for all p, $I_p \leq I_p^*$. Then,

$$\mathrm{ICFHA}_{w,\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq \mathrm{ICFHA}_{w,\varpi}(I_1^*, I_2^*, I_3^*, \ldots, I_l^*).$$

4.4 Intuitionistic Cubic Fuzzy Weighted Geometric Operator

Here, we introduce ICFWG operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 16 Let $I_p = \left\langle c_{I_p}, c'_{I_p}\right\rangle (p = 1, \ldots l)$,be a set of ICFVs in G, and let ICFWG operator is a mapping $\mathrm{ICFWG} : \Omega^l \to \Omega$ and is defined as follows,

$$\mathrm{ICFWG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = \varpi_1 I_1 \otimes \varpi_2 I_2 \otimes \varpi_3 I_3 \otimes, \ldots, \otimes \varpi_l I_l,$$

then ICFWG is said to be intuitionistic cubic fuzzy weighted geometric operator of dimension l. the weight vector of $I_p (p = 1, 2, \ldots, l)$, is $\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$, with $\Sigma_{p=1}^{l} \varpi_p = 1$ and $\varpi_p \in [0, 1]$. the ICFWG operator will obtain the form of an ICFG operator of dimension l, if $\varpi = (1/l, 1/l, 1/l, \ldots, 1/l)^{\mathrm{T}}$, as written below:

$$\mathrm{ICFG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = 1/l(I_1 \otimes I_2 \otimes, \ldots, \otimes I_l).$$

Based on the intuitionistic cubic laws part (8) and (10) of Proposition (1) of ICFVs, the (ICFWG) operator can be converted to the below structure by induction on l.

Theorem 4 *Suppose* $I_p = \left\langle c_{I_p}, c'_{I_p}\right\rangle (p = 1, \ldots l)$, *be a group of ICFVs in G, then their aggregated value by using the ICFWG operator is again an ICFV, and is given by;*

$$\mathrm{ICFWG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = \left\{ \begin{array}{c} ([\Pi_{p=1}^{l} e_p^{-\varpi_p}, \Pi_{p=1}^{l} e_p^{+\varpi_p}], \Pi_{p=1}^{l} \lambda_p^{\varpi_p}), ([1 - \Pi_{p=1}^{l}(1 - r_p^{-})^{\varpi_p}, \\ 1 - \Pi_{p=1}^{l}(1 - r_p^{+})^{\varpi_p}], 1 - \Pi_{p=1}^{l}(1 - \delta_p)^{\varpi_p}) \end{array} \right\},$$

where the weight vector of $I_p (p = 1, 2, \ldots, l)$, *is* $\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$, *with* $\Sigma_{p=1}^{l} \varpi_p = 1$ *and* $\varpi_p \in [0, 1]$.

Proposition 6 *Let* $I_p = \left\langle c_{I_p}, c'_{I_p}\right\rangle, (p = 1, 2, \ldots, l)$ *be a group of ICFVs in G and the weight vector of* I_p *be* $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$, *with* $\Sigma_{p=1}^{l} \varpi_p = 1$ *and* $\varpi_p \in [0, 1]$, *then the following properties are established.*

Idempotency: If all $I_p = \left\langle c_{I_p}, c'_{I_p}\right\rangle, (p = 1, 2, \ldots, l)$ are equal, i.e., $I_p = I$, then

$$\mathrm{ICFWG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = I$$

Boundary: For every ϖ, $I^- \leq \mathrm{ICFWG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq I^+$, where,

$$I^{+} = \{\left([\min e_p^{-}, \max e_p^{+}], \max \lambda_p\right), \left([\max r_p^{-}, \min r_p^{+}], \min \delta_p\right)\},$$
$$I^{-} = \{\left([\max e_p^{-}, \min e_p^{+}], \min \lambda_p\right), \left([\min r_p^{-}, \max r_p^{+}], \max \delta_p\right)\}.$$

Monotonicity: Let $I_p^* = \left\langle c_{I_p^*}, c'_{I_p^*}\right\rangle (p = 1, 2, \ldots l)$ be a group of ICFVs, if for all p, $I_p \leq I_p^*$. Then,

$$\mathrm{ICFWG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq \mathrm{ICFWG}_{\varpi}(I_1^*, I_2^*, I_3^*, \ldots, I_l^*).$$

4.5 Intuitionistic Cubic Fuzzy Order Weighted Geometric Operator

Here, we introduce ICFOWG operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 17 Let $I_p = \left\langle c_{I_p}, c'_{I_p}\right\rangle (p = 1, 2, \ldots, l)$, be a group of ICFVs in G, as stated on Theorem (1). An ICFOWG operator is a mapping $\mathrm{ICFWG} : \Omega^l \to \Omega$, and is defined as follows,

$$\mathrm{ICFOWG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = \varpi_1 I_{\sigma_{(1)}} \otimes \varpi_2 I_{\sigma_{(2)}} \otimes \varpi_3 I_{\sigma_{(3)}} \otimes, \ldots, \otimes \varpi_l I_{\sigma_{(l)}},$$

then ICFOWG operator is known as intuitionistic cubic fuzzy order weighted geometric operator of dimension l. where for all p, $I_{\sigma_{(p-1)}} \geq I_{\sigma_{(p)}}$, $(\sigma(1), \sigma(2), \ldots, \sigma_{(l)})$ is a permutation of (1, 2, …, l). Also, the weight vector of $I_p (p = 1, 2, \ldots, l)$ is $\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$, with $\Sigma_{p=1}^{l} \varpi_p = 1$ and $\varpi_p \in [0, 1]$. Furthermore, the ICFOWG operator will obtain the form of an intuitionistic cubic fuzzy geometric (ICFG) operator of dimension l, if $\varpi = (1/l, 1/l, 1/l, \ldots, 1/l)^{\mathrm{T}}$, as given below:

$$\mathrm{ICFG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = 1/l(I_1 \otimes I_2 \otimes, \ldots, \otimes I_l).$$

Based on the intuitionistic cubic laws part (8) and (10) of Proposition (1) of ICFVs, the ICFWG operator can be converted into the below structure by induction on l.

Theorem 5 *Let* $I_p = \left\langle c_{I_p}, c'_{I_p}\right\rangle (p = 1, 2, \ldots l)$, *is a group of ICFVs in G, then their aggregated value by using the ICFOWG operator is again an ICFV, as given by,*

$$\mathrm{ICFOWG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = \left\{ \begin{array}{c} ([\Pi_{p=1}^{l} e_{\sigma_p}^{-\varpi_p}, \Pi_{p=1}^{l} e_{\sigma_p}^{+\varpi_p}], \Pi_{p=1}^{l} \lambda_{\sigma_p}^{\varpi_p}), ([1 - \Pi_{p=1}^{l}(1 - r_{\sigma_{(p)}}^{-})^{\varpi_p}, \\ 1 - \Pi_{p=1}^{l}(1 - r_{\sigma_p}^{+})^{\varpi_p}], 1 - \Pi_{p=1}^{l}(1 - \delta_{\sigma_p})^{\varpi_p}) \end{array} \right\},$$

where the weight vector of $I_p (p = 1, 2, \ldots, l)$ *is* $\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$, *with* $\Sigma_{p=1}^{l} \varpi_p = 1$ *and* $\varpi_p \in [0, 1]$.

Proposition 7 *Let* $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle, (p = 1, 2, \ldots, l)$ *be a group of ICFVs in G and the weight vector of* I_p *be* $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$, *with* $\Sigma_{p=1}^{l} \varpi_p = 1$ *and* $\varpi_p \in [0, 1]$, *then the following properties are established.*

Idempotency: If all $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle, (p = 1, 2, \ldots, l)$ are equal, i.e., $I_p = I$, then

$$\mathrm{ICFOWG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) = I.$$

Boundary: For every ϖ, $I^- \leq \mathrm{ICFOWG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq I^+$, where

$$I^+ = \{([\min e^-_{\sigma(p)}, \max e^+_{\sigma(p)}], \max \lambda_{\sigma(p)}), ([\max r^-_{\sigma(p)}, \min r^+_{\sigma(p)}], \min \delta_{\sigma(p)})\},$$

$$I^- = \{([\max e^-_{\sigma(p)}, \min e^+_{\sigma(p)}], \min \lambda_{\sigma(p)}), ([\min r^-_{\sigma(p)}, \max r^+_{\sigma(p)}], \max \delta_{\sigma(p)})\}.$$

Monotonicity: Let $I^*_p = \left\langle c_{I^*_p}, c'_{I^*_p} \right\rangle (p = 1, 2, \ldots l)$ be a group of ICFVs, if for all p, $I_p \leq I^*_p$. Then,

$$\mathrm{ICFOWG}_{\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq \mathrm{ICFOWG}_{\varpi}(I^*_1, I^*_2, I^*_3, \ldots, I^*_l).$$

4.6 Intuitionistic Cubic Fuzzy Hybrid Geometric Operator

Here, we have introduced ICFHG operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 18 Let $I_p = < c_{I_p}, c'_{I_p} >, (p = 1, 2, \ldots l)$ be a collection of ICFVs in G. An ICFHG operator of dimension l is a mapping $\mathrm{ICFHG} : \Omega^l \rightarrow \Omega$, having an associated weight vector $\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$ with $\Sigma_{p=1}^{l} \varpi_p = 1$ and $\varpi_p \in [0, 1]$. such that,

$$\mathrm{ICFHG}_{w,\varpi}(I_1, I_2, I_3, \ldots, I_l) = \varpi_1 \widetilde{I}_{\sigma(1)} \otimes \varpi_2 \widetilde{I}_{\sigma(2)} \otimes \varpi_3 \widetilde{I}_{\sigma(3)} \otimes, \ldots, \otimes \varpi_l \widetilde{I}_{\sigma(l)}.$$

where $\widetilde{I}_{\sigma(p)}$ is the jth largest of the weighted intuitionistic fuzzy values $\widetilde{I}_p$, i.e., $\widetilde{I}_p = l w_p I_p$. Also, the weight of $\widetilde{I}_p, (p = 1, 2, \ldots l)$ is $w = (w_1, \ldots, w_l)^{\mathrm{T}}$, with $\Sigma_{p=1}^{l} w_p = 1$ and $w_p \in [0, 1]$ $\widetilde{I}_p$, and l is the balancing coefficient, which maintains the balance, especially when $\varpi = (1/l, 1/l, 1/l, \ldots, 1/l)^{\mathrm{T}}$, the ICFHG operator will obtain the form of an intuitionistic cubic fuzzy geometric (IFG) operator of dimension l.

Based on the intuitionistic cubic laws part (8) and (10) of Proposition (1) of ICFVs, the ICFWG of ICFVs the ICFHG operator can be converted to the below structure by induction on l.

Theorem 6 *Let* $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle, (p = 1, 2, \ldots l)$ *be a set of ICFVs in G, then by using the ICFHG operator their aggregated value is again an ICFV and of the form,*

$$\mathrm{ICFHG}_{w,\varpi}(I_1, I_2, I_3, \ldots, I_l) = \left\{ \begin{array}{c} ([\Pi_{p=1}^{l} \widetilde{e}^{-\varpi_p}_{\sigma(p)}, \Pi_{p=1}^{l} \widetilde{e}^{+\varpi_p}_{\sigma(p)}], \Pi_{p=1}^{l} \widetilde{\lambda}^{\varpi_p}_{\sigma(p)}), ([1 - \Pi_{p=1}^{l}(1 - \widetilde{r}^{-}_{\sigma(p)})^{\varpi_p}, \\ 1 - \Pi_{p=1}^{l}(1 - \widetilde{r}^{+}_{\sigma(p)})^{\varpi_p}], 1 - \Pi_{p=1}^{l}(1 - \widetilde{\delta}_{\sigma(p)})^{\varpi_p}) \end{array} \right\},$$

where the weight vector of $I_p, (p = 1, 2, \ldots, l)$, *is* $\varpi = (\varpi_1, \ldots, \varpi_l)^{\mathrm{T}}$ *with* $\Sigma_{p=1}^{l} \varpi_p = 1$ *and* $\varpi_p \in [0, 1]$.

Proposition 8 *Let* $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle, (p = 1, 2, \ldots, l)$ *be a group of ICFVs in G and the weight vector of* I_p *be* $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$, *with* $\Sigma_{p=1}^{l} \varpi_p = 1$ *and* $\varpi_p \in [0, 1]$, *then the following properties are established.*

Idempotency: If all $I_p = \left\langle c_{I_p}, c'_{I_p} \right\rangle, (p = 1, 2, \ldots, l)$ are equal, i.e., $I_p = I$, then

$$\mathrm{ICFHG}_{w,\varpi}(I_1, I_2, I_3, \ldots, I_l) = I.$$

Boundary: For every ϖ, $I^- \leq \mathrm{ICFHG}_{w,\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq I^+$, where

$$I^+ = \{([\min \widetilde{e}^-_{\sigma(p)}, \max \widetilde{e}^+_{\sigma(p)}], \max \widetilde{\lambda}_{\sigma(p)}), ([\max \widetilde{r}^-_{\sigma(p)}, \min \widetilde{r}^+_{\sigma(p)}], \min \widetilde{\delta}_{\sigma(p)})\},$$

$$I^- = \{([\max \widetilde{e}^-_{\sigma(p)}, \min \widetilde{e}^+_{\sigma(p)}], \min \widetilde{\lambda}_{\sigma(p)}), ([\min \widetilde{r}^-_{\sigma(p)}, \max \widetilde{r}^+_{\sigma(p)}], \max \widetilde{\delta}_{\sigma(p)})\}.$$

Monotonicity: Let $I^*_p = \left\langle c_{I^*_p}, c'_{I^*_p} \right\rangle (p = 1, 2, \ldots l)$ be a group of ICFVs, if for all p, $I_p \leq I^*_p$. Then,

$$\mathrm{ICFHG}_{w,\varpi}(I_1, I_2, I_3, \ldots, I_l) \leq \mathrm{ICFHG}_{w,\varpi}(I^*_1, I^*_2, I^*_3, \ldots, I^*_l).$$

Theorem 7 *The ICFWA and ICFOWA operators are particular cases of the ICFHA operator.*

Proof Let $\varpi = (1/l, 1/l, \ldots, 1/l)^{\mathrm{T}}$. Then

$$\begin{aligned} &\mathrm{ICFHA}_{w,\varpi}(I_1, I_2, I_3, \ldots, I_l) \\ &= \varpi_1 \widetilde{I}_{\sigma(1)} \oplus \varpi_2 \widetilde{I}_{\sigma(2)} \oplus, \ldots, \oplus \varpi_l \widetilde{I}_{\sigma(l)} \\ &= \frac{1}{l}(I_{\sigma(1)} \oplus I_{\sigma(2)} \oplus, \ldots, \oplus I_{\sigma(l)}) \\ &= \varpi_1 I_1 \oplus \varpi_2 I_2 \oplus, \ldots, \oplus \varpi_l I_l \\ &= \mathrm{ICFWA}_{\varpi}(I_1, I_2, , \ldots, I_l). \end{aligned}$$

For ICFOWA, we let $w = (1/l, 1/l, \ldots, 1/l)^{\mathrm{T}}$. Then proof follows above theorem. □

Theorem 8 *The ICFWG and ICFOWG operators are particular cases of ICFHG operator.*

Proof The proof follows above theorem. □

5 Intuitionistic Cubic Fuzzy Multicriteria Decision-Making

Intuitionistic cubic fuzzy weighted averaging operators are used to MCDM process, in this section. Suppose there are l alternatives $\beta = \{\beta_1, \beta_2, \ldots \beta_l\}$ and m criteria $C = \{C_1, C_2, \ldots, C_m\}$ to be evaluated with weight vector $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$ such that $\Sigma_{p=1}^{l} \varpi_p = 1$ and $\varpi_p \in [0, 1]$. To evaluate the accomplishment on the basis of criteria C_p of the alternative β_i, the decision-makers have to give not only the statistics about the alternative β_i, satisfying the criteria but also about the alternative β_i, not satisfying the criteria C_p. Let the rating of alternatives β_i on criteria C_p, given by decision-makers be ICFNs in $G : I_{ip} = \left\langle c_{ip}, c'_{ip} \right\rangle$ $(i = 1, 2, \ldots l)(p = 1, 2, \ldots m)$. Let c_{ip} shows the degree of alternative β_i satisfying the criteria C_p and c'_{ip} shows the degree of alternative β_i not satisfying the criteria C_p, such that $c_{ip} = \left\langle [e^-_{ip}, e^+_{ip}], \lambda_{ip} \right\rangle$, and $c'_{ip} = \left\langle [r^-_{ip}, r^+_{ip}], \delta_{ip} \right\rangle$ with the condition that $[e^-_{ip}, e^+_{ip}] \subset [0, 1]$, $[r^-_{ip}, r^+_{ip}] \subset [0, 1]$, and $\delta_{ip} : G \to [0, 1]$, $\lambda_{ip} : G \to [0, 1]$. Subject to $\sup[e^-_{ip}, e^+_{ip}] + \sup[r^-_{ip}, r^+_{ip}] \leq 1$ and $\lambda_{ip} + \delta_{ip} \leq 1, (i = 1, 2, \ldots l;$ $p = 1, 2, \ldots m)$.. Thus, a MCDM problem can be briefly represented in an ICF decision matrix

$$D = (I_{ip})_{l\times m} = \left(\left\langle c_{ip}, c'_{ip} \right\rangle\right)_{l\times m}, (i = 1, 2, \ldots, l; \; p = 1, 2, \ldots, m).$$

Step 1. Make intuitionistic cubic fuzzy decision matrix, $D = (I_{ip})_{l\times m} = (\left\langle c_{ip}, c'_{ip} \right\rangle)_{l\times m}, \quad (p = 1, 2, \ldots, m; i = 1, 2, \ldots, l)$. Usually, the criteria can be arranged into two classes: cost criteria and benefit criteria. There is no need of normalizing the rating values, If all the criteria are of same type. Whereas the rating values of the cost type can be converted into the rating values of the benefit type by the below normalization formula, if the decision matrix contains both benefit criteria and cost criteria.

$$s_{ip} = \left\langle v_{ip}, t_{ip} \right\rangle = \begin{cases} d_{ip}, \text{if the criteria is of benefit type} \\ d^c_{ip}, \text{if the criteria is of cost type} \end{cases},$$

d^c_{ip} is the complement of d_{ip}. Thus, we get the normalized intuitionistic cubic fuzzy decision matrix. The normalized ICF decision matrix is denoted by D^l and is given by $D^l = (s_{ip})_{l\times m} = (\left\langle v_{ip}, t_{ip} \right\rangle)_{l\times m}, \; (p = 1, 2, \ldots, m; i = 1, 2, \ldots, l)$.

Next, we will apply the ICFWA, ICFOWA, ICFWG, ICFOWG, ICFHA, and ICFHG operators to MCDM, which further contains the below steps.

Step 2. Use the suggested aggregation operators to compute the ICFNs $I_i (i = 1, 2, \ldots, l)$, for the alternatives $\beta_i (i = 1, 2, \ldots, l)$. , i.e., the developed operators to stem the collective overall preference values $I_i (i = 1, 2, \ldots, l)$ of the alternatives β_i, where $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_l)^{\mathrm{T}}$ is the weight vector of the criteria.

Step 3. Using score function of ICFNs we compute the scores $S(I_i)$ of all the values I_i.

: Select the best one after ranking all the alternatives.

5.1 Application of the Proposed Method

Supplier selection is an important part of the industry and production strategy for industrial organizations. Choosing the best supplier will improve the quality of your company and economic growth, but it is still difficult to choose the appropriate supplier. Thus, the proposed model will be used to evaluate and select the most suitable supplier for a company in the western part of Pakistan. The proposed supplier selection approach has been made in as follows:

The company plans to find suitable supplier for the purchase of components. The internal purchasing manager considers the following four criteria, organizational culture, and plane of action (for example, trust, internal and external group action of suppliers, understanding between levels, and operations of the supplier and buyer) C_1; finance (for example, economy, national stability) C_2; technology (for example, ability to produce, ability to cope with technology changes, ability to design,) C_3 ; presentation (for example, delivery, quality, price) C_4. The set of the four criteria is denoted by $\{C_1, C_2, C_3, C_4\}$, the weight vector of the criteria $\{C_1, C_2, C_3, C_4\}$ is $\varpi = (0.2, 0.3, 0.1, 0.4)^{\mathrm{T}}$. A committee of three decision-makers noted that four suppliers should be further evaluated. The group of the four providers is given by $\{\beta_1, \beta_2, \beta_3, \beta_4\}$. The ranking is required to evaluate the suppliers. These experts described the decision matrices in the form of ICFVs as follows.

Step 1 The decision-makers have given their decisions in Tables 1, 2, 3. Here is no need of normalization as all the measure values are of same type, i.e., benefit type.

Utilizing ICFWA operator, where used $\varpi = (0.4, 0.25, 0.35)^{\mathrm{T}}$ as the decision-maker's weight vector, we aggregated the information given by three decision-

Table 1 First decision-maker's information

	C_1	C_2	C_3	C_4
β_1	$\begin{pmatrix}([0.1,0.5],0.3),\\([0.2,0.4],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.3],0.2),\\([0.2,0.5],0.3)\end{pmatrix}$	$\begin{pmatrix}([0.4,0.7],0.6),\\([0.1,0.3],0.3)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.6],0.7),\\([0.2,0.4],0.2)\end{pmatrix}$
β_2	$\begin{pmatrix}([0.3,0.5],0.2),\\([0.2,0.5],0.4)\end{pmatrix}$	$\begin{pmatrix}([0.4,0.7],0.6),\\([0.1,0.3],0.3)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.4],0.2),\\([0.3,0.5],0.6)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.3],0.2),\\([0.2,0.5],0.3)\end{pmatrix}$
β_3	$\begin{pmatrix}([0.2,0.6],0.7),\\([0.2,0.4],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.5],0.3),\\([0.2,0.4],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.5],0.4),\\([0.2,0.4],0.4)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.4],0.4),\\([0.4,0.6],0.6)\end{pmatrix}$
β_4	$\begin{pmatrix}([0.1,0.5],0.4),\\([0.2,0.4],0.4)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.4],0.4),\\([0.4,0.6],0.6)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.4],0.3),\\([0.3,0.5],0.6)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.5],0.3),\\([0.2,0.5],0.5)\end{pmatrix}$

Table 2 Second decision-maker's information

	C_1	C_2	C_3	C_4
β_1	$\begin{pmatrix}([0.1,0.5],0.4),\\([0.2,0.4],0.4)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.6],0.7),\\([0.2,0.4],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.4],0.2),\\([0.3,0.5],0.6)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.3],0.2),\\([0.2,0.5],0.2)\end{pmatrix}$
β_2	$\begin{pmatrix}([0.2,0.4],0.4),\\([0.4,0.6],0.6)\end{pmatrix}$	$\begin{pmatrix}([0.4,0.7],0.6),\\([0.1,0.3],0.3)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.6],0.7),\\([0.2,0.4],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.3],0.6),\\([0.2,0.5],0.3)\end{pmatrix}$
β_3	$\begin{pmatrix}([0.1,0.3],0.2),\\([0.2,0.5],0.3)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.5],0.4),\\([0.2,0.4],0.4)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.4],0.4),\\([0.4,0.6],0.6)\end{pmatrix}$	$\begin{pmatrix}([0.4,0.7],0.6),\\([0.1,0.3],0.3)\end{pmatrix}$
β_4	$\begin{pmatrix}([0.1,0.3],0.6),\\([0.2,0.5],0.3)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.4],0.3),\\([0.2,0.5],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.5],0.4),\\([0.2,0.4],0.4)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.3],0.2),\\([0.2,0.5],0.3)\end{pmatrix}$

Table 3 Third decision-maker's information

	C_1	C_2	C_3	C_4
β_1	$\begin{pmatrix}([0.2,0.4],0.7),\\([0.1,0.5],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.6],0.7),\\([0.2,0.4],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.4],0.2),\\([0.3,0.5],0.6)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.5],0.3),\\([0.2,0.5],0.5)\end{pmatrix}$
β_2	$\begin{pmatrix}([0.2,0.4],0.4),\\([0.2,0.6],0.6)\end{pmatrix}$	$\begin{pmatrix}([0.3,0.7],0.2),\\([0.1,0.3],0.7)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.5],0.3),\\([0.2,0.5],0.5)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.5],0.6),\\([0.2,0.4],0.2)\end{pmatrix}$
β_3	$\begin{pmatrix}([0.1,0.3],0.2),\\([0.2,0.5],0.3)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.4],0.7),\\([0.1,0.5],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.6],0.6),\\([0.1,0.4],0.4)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.4],0.3),\\([0.2,0.5],0.6)\end{pmatrix}$
β_4	$\begin{pmatrix}([0.1,0.3],0.3),\\([0.4,0.7],0.6)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.3],0.2),\\([0.2,0.5],0.3)\end{pmatrix}$	$\begin{pmatrix}([0.3,0.7],0.2),\\([0.1,0.3],0.7)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.4],0.7),\\([0.1,0.5],0.2)\end{pmatrix}$

Table 4 Aggregated information of decision-makers

	C_1	C_2	C_3	C_4
β_1	$\begin{pmatrix}([0.15,0.45],0.55),\\([0.14,0.45],0.23)\end{pmatrix}$	$\begin{pmatrix}([0.17,0.53],0.6),\\([0.2,0.42],0.23)\end{pmatrix}$	$\begin{pmatrix}([0.27,0.51],0.35),\\([0.22,0.43],0.49)\end{pmatrix}$	$\begin{pmatrix}([0.13,0.5],0.44),\\([0.20,0.47],0.34)\end{pmatrix}$
β_2	$\begin{pmatrix}([0.23,0.43],0.35),\\([0.23,0.57],0.54)\end{pmatrix}$	$\begin{pmatrix}([0.35,0.7],0.43),\\([0.1,0.3],0.46)\end{pmatrix}$	$\begin{pmatrix}([0.15,0.49],0.38),\\([0.23,0.48],0.39)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.41],0.60),\\([0.2,0.45],0.24)\end{pmatrix}$
β_3	$\begin{pmatrix}([0.13,0.41],0.40),\\([0.2,0.47],0.27)\end{pmatrix}$	$\begin{pmatrix}([0.15,0.45],0.56),\\([0.14,0.45],0.23)\end{pmatrix}$	$\begin{pmatrix}([0.13,0.54],0.51),\\([0.16,0.43],0.43)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.48],0.40),\\([0.21,0.48],0.52)\end{pmatrix}$
β_4	$\begin{pmatrix}([0.1,0.35],0.40),\\([0.28,0.36],0.38)\end{pmatrix}$	$\begin{pmatrix}([0.13,0.35],0.29),\\([0.25,0.53],0.34)\end{pmatrix}$	$\begin{pmatrix}([0.23,0.59],0.27),\\([0.14,0.37],0.6)\end{pmatrix}$	$\begin{pmatrix}([0.15,0.41],0.53),\\([0.14,0.5],0.29)\end{pmatrix}$

makers according to the varied importance of all the decision-makers and presented in Table 4.

Utilizing ICFWA operator again, having $\varpi = (0.2, 0.3, 0.1, 0.4)^{\mathrm{T}}$ as the criteria weight vector, we

Table 5 Comparative study and ranking of the alternatives

Operators	$S(\beta_1)$	$S(\beta_2)$	$S(\beta_3)$	$S(\beta_4)$	Ranking
ICFWA	0.09	0.086	0.03	− 0.013	$\beta_1 > \beta_2 > \beta_3 > \beta_4$
ICFOWA	0.076	0.04	0.02	0.027	$\beta_1 > \beta_2 > \beta_3 > \beta_4$
ICFWG	0.07	0.047	0.017	− 0.04	$\beta_1 > \beta_2 > \beta_3 > \beta_4$
ICFOWG	0.05	0.007	0.013	− 0.06	$\beta_1 > \beta_2 > \beta_3 > \beta_4$
ICFHA	0.03	0.11	0.00	− 0.07	$\beta_2 > \beta_1 > \beta_3 > \beta_4$
ICFHG	− 0.15	− 0.05	− 0.14	− 0.22	$\beta_2 > \beta_1 > \beta_3 > \beta_4$

get the collective ICFNs for the alternatives β_i $(i = 1, 2, 3, 4)$.

$\beta_1 = (([0.16, 0.50], 0.51), ([0.18, 0.44], 0.29))$
$\beta_2 = (([0.21, 0.52], 0.48), ([0.17, 0.42], 0.36))$
$\beta_3 = (([0.16, 0.46], 0.46), ([0.18, 0.46], 0.35))$
$\beta_4 = (([0.14, 0.40], 0.42), ([0.19, 0.46], 0.35))$

Step 3 Using Definition (8), compute the scores $S(\beta_i)$ of $\beta_i (i = 1, 2, 3.4)$ as given by;

$S(\beta_1). = 0.09, S(\beta_2). = 0.086, S(\beta_3). = 0.03,$
$S(\beta_4). = -0.013.$

Step 4 Place the ICFNs in descending order of their scores, respectively, and select the best alternative accordingly, i.e.,

$\beta_1 > \beta_2 > \beta_3 > \beta_4.$

Comparative study and ranking of the alternatives by means of their score values is given in Table 5. It is clear from Table 5 that β_1 has comparatively larger score.

Hence β_1 is the most suitable supplier for the purchase of components.

6 Verification by VIKOR Method

Here, we verify the result given by the ICFWA operators by VIKOR method. The aggregated values of all the individual decision-makers' assessment information based on intuitionistic cubic fuzzy weighted averaging operator are given in Table 4. For this purpose using $\varpi = (0.2, 0.3, 0.1, 0.4)^{\mathrm{T}}$ as the criteria weight vector, we will apply VIKOR method on the information given in Table 4.

Solution 1 Now to solve this example using the VIKOR method, the following steps are utilized.

Step 1 Normalize the decision matrix given in Table 4. Here is no need of normalization as all the measure values are of same type, i.e., benefit type.

Step 2 Identifying the PIS $R^{+}$and NIS R^{-}. The PIS $R^{+}$and NIS R^{-} are defined as follows:

$R^{+} = (\zeta_1^{+}, \zeta_2^{+}, \zeta_3^{+}, \zeta_4^{+}), R^{-} = (\zeta_1^{-}, \zeta_2^{-}, \zeta_3^{-}, \zeta_4^{-}),$ where $\zeta_j^{+} = \max\{\zeta_{ij}/1 \leq j \leq 4\}$ and $\zeta_j^{-} = \min\{\zeta_{ij}/1 \leq j \leq 4\}$, which are calculated by using the score function $S(I) = [(e^{-} + e^{+} + \lambda - r^{-} - r^{+} - \delta)/3]$.

Step 3 Calculate the values S_i, R_i and Q_i can be obtained by using equations,

$$S_i = \sum_{j=1}^{m} \frac{w_j d(\zeta_{ij}, \zeta_j^{+})}{d(\zeta_j^{+}, \zeta_j^{-})},$$

$$R_i = \max_{i \leq j \leq m} \frac{w_j d(\zeta_{ij}, \zeta_j^{+})}{d(\zeta_j^{+}, \zeta_j^{-})},$$

and

$$Q_i = \frac{v(S_i - S^{*})}{(S^{-} - S^{*})} + \frac{(1 - v)(R_i - R^{*})}{(R^{-} - R^{*})}.$$

Assume $v = 0.5$, then the calculated results are shown in the Table 6.

Step 4 Rank the alternatives by sorting each S_i, R_i, and Q_i values in an decreasing order. The values of Q_i are ranked as

$Q_1 > Q_2 > Q_3 > Q_4.$

Table 6 Comparison table

Operators	$S(\beta_1)$	$S(\beta_2)$	$S(\beta_3)$	$S(\beta_4)$	Ranking
ICFWA	0.08	0.08	0.075	0.07	$\beta_1 = \beta_2 > \beta_3 > \beta_4$
ICFOWA	0.09	0.08	0.072	0.07	$\beta_1 > \beta_2 > \beta_3 > \beta_4$
ICFWG	0.08	0.07	0.072	0.075	$\beta_1 > \beta_4 > \beta_3 > \beta_2$
ICFOWG	0.08	0.08	0.10	0.0079	$\beta_3 > \beta_1 = \beta_2 > \beta_4$
ICFHA	0.09	0.082	0.07	0.08	$\beta_1 > \beta_2 > \beta_4 > \beta_3$
ICFHG	0.95	0.088	0.09	0.08	$\beta_1 > \beta_3 > \beta_2 > \beta_4$

7 Comparison

In this section, our suggested advance fuzzy aggregation operators are compared to pre-existing fuzzy aggregation operators and the conclusion of our work is stated. Although AIFS theory has a great impact in various fields but there are some real-world problems which were not possible to be solved by AIFS and even not possible to be solved by interval-valued fuzzy sets (IVFS). Just like AIFS, every element of an IVFS is represented as a structure of an ordered pair, which is characterized by degree of membership and degree of non-membership. The function of membership is grip in the form of an interval and the function of non-membership is described in the same fashion [53]. While of ICFS, each element consists of function of membership and function of non-membership. Membership function is cubic fuzzy number and non-membership function is also cubic fuzzy number. If we consider the discussed numerical problem in Sect. 5, as ICFS is the most advance structure, so it is not possible for the existing fuzzy aggregation operators to solve the data contained in the said problem, which shows the existing aggregation operators have limited approach. But if we consider any problem under the interval-valued fuzzy information, we can solve it easily by ICFS by converting data in the form of interval-valued fuzzy number to ICFN by taking the values outside the interval in ICFN equal to zero, in both membership and non-membership degree, i.e., the IVF number ([0.2, 0.4], [0.1, 0.5]) can be converted to ICFN (([0.2, 0.4], 0), ([0.1, 0.5], 0)). Thus, ICFA operators are more powerful to resolve the unpredictable problems.

Moreover, if we consider the discussed numerical problem in Sect. 5, and instead of using the score function of ICFNs, we used the score function of cubic fuzzy numbers by considering the membership and non-membership functions of ICFN as individual cubic fuzzy numbers, i.e., here we take an ICFN $\langle[e^-, e^+], \lambda\rangle, \langle[r^-, r^+], \delta\rangle$ as the collection of two cubic numbers $C_1 = \langle[e^-, e^+], \lambda\rangle$ and $C_2 = \langle[r^-, r^+], \delta\rangle$, calculate the score value of each individually by the score function of CFNs [12], and then by taking the average of both $\frac{1}{2}(S(C_1) + S(C_1)) = \frac{1}{2}(\frac{e^- + e^+ - \lambda}{3} + \frac{r^- + r^+ - \delta}{3})$, we get the ranking results of the alternatives, which are given in Table 6, and find the same result as given in Table 5 by using ICF score function, i.e., again β_1 is the best choice among all alternatives.

It can easily be observed from the existing approaches [24, 25, 27, 37] by various researchers that the algorithms by using IF information for MCGDM problems have some limitations and can not handle the problems under some uncertain situations; hence their suggested algorithms may not produce accurate results. However, ICF aggregation operators do not have such limitations and thus can give more accurate results.

8 Conclusion and Future Work

We have established an advanced approach to AIFS through application of cubic fuzzy set theory and introduced the concept of an intuitionistic cubic fuzzy set. Also, we have defined accuracy degree and score function for the comparison of two intuitionistic cubic fuzzy numbers. We defined some connectivity of two intuitionistic cubic fuzzy numbers, i.e., the operational laws of intuitionistic cubic fuzzy numbers introduced. Some ICF operational laws have been developed and defined ICF distance between intuitionistic cubic fuzzy numbers. We also established a number of intuitionistic cubic fuzzy aggregation operators, i.e., we proposed ICFWA operator, ICFOWA, ICFWG, ICFOWG, ICFHA, and ICFHG operator under ICF environment; discussed some properties of these operators like idempotency, boundary, and monotonicity, and showed relationships among these developed operators. The operator is characterized by considering information about the relationship among the intuitionistic cubic fuzzy numbers ICFNs being aggregated. To demonstrate the performance of these new techniques, we develop a MCGDM based on the proposed operators under the ICF information. Resolving the problem of evaluation and ranking the potential suppliers has become a key strategic element for the company. As the intelligent and automated information systems were developed in the information era, more effective decision-making methods have become necessary. For instance, a numerical application related to the selection of suitable supplier of the proposed operators under the ICF information has been presented, which shows that the suggested operators delivers an alternative way to solve decision-making process in a more actual way. Also the selected supplier by ICFD aggregation operators is verified by VIKOR method. For this purpose, the same case is solved by VIKOR method. Finally, we have provided comparison of the proposed operators to the existence operators to show the validity, practicality, and effectiveness of the proposed methods. Our proposed method is different from all the previous techniques for group decision-making due to the fact that the proposed method uses intuitionistic cubic fuzzy information, which will not cause any loss of information in the process. So it is efficient and feasible for real-world decision-making applications.

In future work, we will further develop more aggregation operators under ICF information, like Dombi aggregation operators and some more. Will also expand TOPSIS, VIKOR, and few other methods under ICF environments and will apply them to expand a number of strategies to resolve MCGDM problems, risk evaluation, and other domains under indeterminate conditions.

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