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A Solving Method for Fuzzy Linear Programming Problem with Interval Type-2 Fuzzy Numbers

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Abstract Thus far, many methods have been suggested to solve the fuzzy linear programming (FLP) problems with interval type-2 fuzzy numbers (IT2FNs) ambiguous of kind *Vagueness* (uncertainty at the satisfaction level of the objective function and constraints), while studies on models of the interval type-2 FLP problems with uncertainty of kind *Ambiguity* in which all or part of the parameters are ambiguous (all or part of the coefficients in FLP problem are IT2FNs) are very limited. In this paper, first, an interval type-2 FLP problem with uncertainty of kind *Ambiguity* was considered generally and all the coefficients in the problem were interval type-2 triangular fuzzy numbers. Next, a method for solving it based on the nearest interval approximation was proposed. Finally, the method was illustrated using some numerical examples.

Keywords Best–worst cases (BWC) · Fuzzy linear programming (FLP) · Interval linear programming (ILP) · Interval type-2 fuzzy number (IT2FN) · Membership function (MF)

1 Introduction

Many real-world problems are modeled as optimization problems, which are often ambiguous in all or part of their parameters. These uncertainties can occur in different parts of the optimization model ambiguous of kinds of *Vagueness* and *Ambiguity*. The modeling of *Vagueness* represents uncertainty at the satisfaction level of the objective function and constraints and that of *Ambiguity* represents uncertainty at the coefficients in the problem.

After expression of the theory of type-1 fuzzy sets in 1965 and type-2 fuzzy sets in 1975 by Zadeh [23, 24], another kind namely: the interval type-2 fuzzy set, was defined [14, 18]. The type-2 fuzzy sets are complex, and computational operations are more difficult than the interval type-2 fuzzy sets. Also, the interval type-2 fuzzy sets present more information and uncertainties than the type-1 fuzzy sets. Hence, the interval type-2 fuzzy sets are preferable to other types of fuzzy sets for modeling optimization problems.

Thus far, the idea of using the interval type-2 fuzzy sets in modeling FLP problems has been more considered [5–12, 17, 25]. For example, Figueroa [11] enhanced the Zimmermann method [27] to solve the interval type-2 FLP problem in which right-hand side values in it are presented by membership functions (MFs). Moreover, he proposed many different methods for modeling and solving the interval type-2 FLP problem obscure of kind *Vagueness* by displaying uncertainties by the MFs [6–12]. In these works, the MFs are applied to represent uncertainty at the satisfaction level of the objective function and constraints. Also, Golpayegani et al. [12] presented a new way in two special cases of interval type-2 FLP problem.

Recently, some ranking functions for solving interval type-2 FLP problem with uncertainty of kind *Ambiguity* were introduced [15–17, 19, 20, 25] in which, by replacing the crisp numbers by the IT2FNs in problem, they are converted to the linear programming problem. In this paper, one new technique was proposed to solve a FLP problem with uncertainty of kind *Ambiguity* and all the

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coefficients in the problem are interval type-2 triangular fuzzy numbers.

2 Preliminaries

In this section, some concepts and prerequisites are introduced to express the proposed method. It should be noted that the symbols defined here are used to describe the method and solve numerical examples.

Definition 1 A type-1 fuzzy set, A, which is concerning with a unique variable, $x \in X$ is a generalization of a crisp set. It is defined in the source X and characterized by the MF $\mu_A(x)$ that replaces values in the range [0,1]. Such a set can be showed as:

$$A = \{(x, \mu_A(x)) | \ \forall \ x \in X\},$$

type-1 MF, $\mu_A(x)$ is bound to be among 0 and 1 for all $x \in X$, and is a two-dimensional function [15–17].

Definition 2 A type-1 fuzzy set, A by the MF $\mu_A(x)$, defined on the set of the real numbers is called a type-1 fuzzy number when, $\sup_{x\in\mathbb{R}} \mu_A(x) = 1$, that is, the type-1 fuzzy set A is normal, $\mu_A\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$, for all $\lambda \in [0,1]$, that is, the set A is convex and $\mu_A(x)$ is a continuous function by intervals [15–17].

Definition 3 The α-cut of a type-1 fuzzy number A is a set defined as $A_\alpha = \{x \in \mathbb{R} | \mu_A(x) \geq \alpha\} = [\underline{A}(\alpha), \overline{A}(\alpha)]$, where $\underline{A}(\alpha) = \mathit{inf}\{x \in \mathbb{R} | \mu_A(x) \geq \alpha\}$, $\overline{A}(\alpha) = \mathit{sup}\{x \in \mathbb{R} | \mu_A(x) \ \geq \alpha\}$, where *inf* and *sup* are the greatest lower bound and least upper bound, respectively. For example the α-cut of a type-1 triangular fuzzy number A is shown in Fig. 1.

Definition 4 The core of a type-1 fuzzy number, A is a set defined as $Core(A) = \{x \in \mathbb{R} | \mu_A(x) = 1\}$ [13].

Definition 5 A type-2 fuzzy set, $\tilde{A}$, is defined as $\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\}$, and $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$, where J_x is the primary membership of x. When the elements of a type-2 fuzzy set $\tilde{A}$ are continuous, $\tilde{A}$ can be represented as:

$$\tilde{A} = \int_{x\in X} \int_{u \in J_x} \frac{\mu_{\tilde{A}}(x,u)}{(x,u)} = \int_{x \in X} \frac{\left(\int_{u\in J_x} \frac{\mu_{\tilde{A}}(x,u)}{u}\right)}{x},$$

where $\int\int$ denotes the union for all x and u in feasible region. For discrete spaces, $\int$ is replaced by $\sum$ [18, 20, 25].

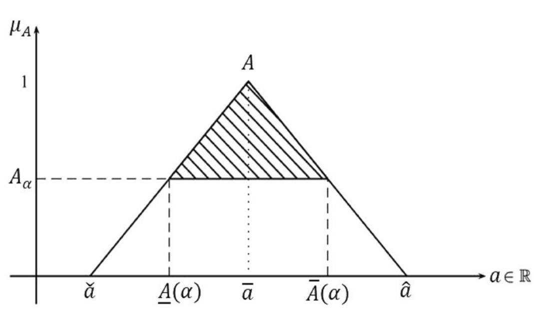

Fig. 1 The α-cut of a type-1 triangular fuzzy number A

Definition 6 In definition 5, when $\mu_{\tilde{A}}(x,u) = 1, \forall x \in X, u \in J_x \subseteq [0,1]$, $\tilde{A}$ is called interval type-2 fuzzy set.

Definition 7 Uncertainty in the primary memberships of a type-2 fuzzy set $\tilde{A}$ consists of a bounded region called the footprint of uncertainty, that is, FOU=$\cup_{x\in X} J_x$. The footprint of uncertainty of $\tilde{A}$ is bounded by two MFs: a lower MF (LMF) $\underline{\mu}_{\tilde{A}}$ and an upper MF (UMF) $\overline{\mu}_{\tilde{A}}$. Thus, an interval type-2 fuzzy set is bounded by two type-1 fuzzy sets. Moreover, the interval type-2 fuzzy set is called the IT2FN when its UMF and LMF are type-1 fuzzy numbers. It is noteworthy that $\tilde{A}$ has embedded type-1 fuzzy sets (A_e) as well; therefore, there is an infinite amount of A_e enclosed into the footprint of uncertainty of $\tilde{A}$ [15–20].

Definition 8 The core of the IT2FN $\tilde{A} = [\underline{A}, \overline{A}]$ is $Core(\tilde{A}) = [min\, Core(\underline{A}), \max Core(\underline{A})]$ when $\phi \neq Core(\underline{A}) \subseteq Core(\overline{A})$, where ϕ is the empty set and $\underline{\mu}_{\tilde{A}}$ and $\overline{\mu}_{\tilde{A}}$ are lower and upper MFs of $\tilde{A}$, respectively.

Definition 9 Interval type-2 triangular fuzzy number, $\tilde{A}$ is defined on interval $[\bar{\breve{a}}, \bar{\hat{a}}]$, its lower MF and upper MF take the value equal to $\underline{h} \in [0,1]$ in $\underline{a}$ and $\bar{h} \in [0,1]$ in $\bar{a}$, respectively, where $\bar{\breve{a}} \leq \underline{\breve{a}} \leq \underline{a} \leq \underline{\hat{a}} \leq \bar{\hat{a}}$ and $\bar{\breve{a}} \leq \bar{a} \leq \bar{\hat{a}}$. Thus, the interval type-2 triangular fuzzy number, $\tilde{A}$ is notated as $\tilde{A} = (\underline{A}, \overline{A}) = \left((\underline{\breve{a}}, \underline{a}, \underline{\hat{a}}, \underline{h}), (\bar{\breve{a}}, \bar{a}, \bar{\hat{a}}, \bar{h})\right)$ and its MFs are:

$$\underline{\mu}_{\tilde{A}}(x) = \begin{cases} \underline{h}\dfrac{x - \underline{\breve{a}}}{\underline{a} - \underline{\breve{a}}} & \underline{\breve{a}} \leq x \leq \underline{a} \\ \underline{h}\dfrac{\underline{\hat{a}} - x}{\underline{\hat{a}} - \underline{a}} & \underline{a} \leq x \leq \underline{\hat{a}} \\ 0 & x \leq \underline{\breve{a}}, x \geq \underline{\hat{a}}, \end{cases}$$

and

$$\overline{\mu}_{\tilde{A}}(x) = \begin{cases} \bar{h}\dfrac{x - \bar{\breve{a}}}{\bar{a} - \bar{\breve{a}}} & \bar{\breve{a}} \leq x \leq \bar{a} \\ \bar{h}\dfrac{\bar{\hat{a}} - x}{\bar{\hat{a}} - \bar{a}} & \bar{a} \leq x \leq \bar{\hat{a}} \\ 0 & x \leq \bar{\breve{a}}, x \geq \bar{\hat{a}}. \end{cases}$$

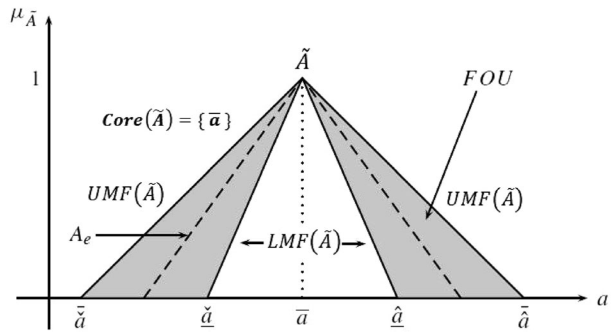

Fig. 2 The interval type-2 triangular fuzzy number $\tilde{A} = (\underline{A}, \overline{A})$

A graphical representation of the interval type-2 triangular fuzzy number, $\tilde{A}$ defined over $a \in \mathbb{R}$, its footprint of uncertainty, $Core(\tilde{A})$ and A_e, is shown in Fig. 2 [16, 17].

In the following, the concept of the nearest interval approximation of the type-1 fuzzy numbers was introduced, that is, a type-1 fuzzy number approximated with a closed interval. Then, in particular, this concept was used for a type-1 triangular fuzzy number and the closed interval was found for it.

One of the applications of every existing defuzzification methods is to solve single- or multi-objective type-1 and interval type-2 FLP problems. In the general interval type-2 FLP problems, all coefficients and numbers used are interval type-2 fuzzy numbers. So far, many methods, such as ranking or ordering methods and approximation techniques are proposed to solve the FLP problems, and each method has its own advantages and disadvantages; hence, it would be a difficult task to decide which one is the best. Thus, the decision maker should choose the defuzzification method for any particular problem, each time. In fact, well-defined methods that are more preferable than many existing methods are sought. Fuzzy set theory allows one to effectively model and transform imprecise information. However, sometimes a given interval type-2 fuzzy set has to be approximated by a crisp one. If a defuzzification operator such as ranking or ordering methods which replaces an interval type-2 fuzzy set by a single number is used, too many important information are generally lost. Since an interval approximation is an extension of approximation as a crisp number and it is more acceptable, if an interval type-2 fuzzy set is replaced by a closed interval, less significant information is lost. The distinction among defuzzification methods is only possible by counting the number of proven well-defined properties for them. In this paper, the proposed technique is based on the nearest interval approximation which possesses many desired properties like continuity, monotonic and linearity [13].

Suppose A is a type-1 fuzzy number and $[\underline{A}(\alpha), \overline{A}(\alpha)]$ is its α-cut. The closed interval $C(A) = [\underline{C}, \overline{C}]$ is the nearest interval approximation to A, where $\underline{C} = \int_0^1 \underline{A}(\alpha)\mathrm{d}\alpha$, $\overline{C} = \int_0^1 \overline{A}(\alpha)\mathrm{d}\alpha$ [13].

Theorem 1 *The nearest interval approximation for the type-1 triangular fuzzy number* $A = (\check{a}, \bar{a}, \hat{a})$ *is* $\left[\frac{\check{a}+\bar{a}}{2}, \frac{\hat{a}+\bar{a}}{2}\right]$.

Proof The MF of the type-1 triangular fuzzy number A can be described in a following manner:

$$\mu_A(x) = \begin{cases} \underline{\mu}_A(x) = \dfrac{x - \check{a}}{\bar{a} - \check{a}} & \check{a} \le x < \bar{a} \\ 1 & x = \bar{a} \\ \overline{\mu}_A(x) = \dfrac{\hat{a} - x}{\hat{a} - \bar{a}} & \bar{a} < x \le \hat{a} \\ 0 & x \le \check{a}, x \ge \hat{a} \end{cases}$$

The set of the α-cuts of a type-1 triangular fuzzy number A is obtained as $[\underline{A}(\alpha), \overline{A}(\alpha)] = [\check{a} + \alpha(\bar{a} - \check{a}), \hat{a} + \alpha(\bar{a} - \hat{a})]$; thus, with the help of the nearest interval approximation concept for it, the proof can be completed as follows:

$$\begin{aligned} C(A) &= \left[\int_0^1 \underline{A}(\alpha)\mathrm{d}\alpha, \int_0^1 \overline{A}(\alpha)\mathrm{d}\alpha\right] \\ &= \left[\int_0^1 (\check{a} + \alpha(\bar{a} - \check{a}))\mathrm{d}\alpha, \int_0^1 (\hat{a} + \alpha(\bar{a} - \hat{a}))\mathrm{d}\alpha\right] \\ &= \left[\frac{\check{a} + \bar{a}}{2}, \frac{\hat{a} + \bar{a}}{2}\right] \end{aligned}$$

□

In the general ILP problem, all the parameters (the objective function, technological coefficients and the resource values) are closed intervals. Therefore, the ILP problem can be modeled as follows:

$$\begin{aligned} \max \quad & z = \Sigma[\underline{c}_j, \overline{c}_j]x_j \\ \text{s.t.} \quad & \Sigma[\underline{a}_{ij}, \overline{a}_{ij}]x_j \le [\underline{b}_i, \overline{b}_i], \quad i = 1, 2, \ldots, m \\ & x_j \ge 0, \quad j = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

where $\underline{c}_j, \overline{c}_j, \underline{b}_i, \overline{b}_i, \underline{a}_{ij}$ and $\overline{a}_{ij}$ are real numbers.

Many methods have been introduced to solve the ILP problem [1–4, 21, 22, 26]. As a basic method, we introduce the best–worst cases (BWC) method [21] which solves it with the conversion of the ILP problem into the two sub-models, namely the best and the worst sub-models. Infact, the BWC method was proposed to solve ILP model (1) by formulating two sub-models as the best sub-model: $\max\{\bar{z} | \Sigma\underline{a}_{ij}x_j \le \overline{b}_i, \forall x_j \ge 0\}$ and the worst sub-model: $\max\{\underline{z} | \Sigma\overline{a}_{ij}x_j \le \underline{b}_i, \forall x_j \ge 0\}$.

Theorem 2 *The objective function value of any arbitrary characteristic model of ILP model* (1) *belongs to the interval of the objective function values of the BWC method* [21].

Next, an IT2FN becomes an interval number with the proposed technique based on the nearest interval approximation, and the interval type-2 FLP problem becomes an equivalent problem, that is, the ILP problem. Hence, to solve the interval type-2 FLP problem, the equivalent problem obtained by the proposed technique was constructed and solved, and can easily be solved with BWC method for ILP problems.

3 The FLP Problem with IT2FNs

In this section, an interval type-2 FLP problem with uncertainty of kind *Ambiguity* was first considered which all the coefficients in it were of interval type-2 triangular fuzzy numbers. Then, the middle model was introduced and solved by replacing the *Core* of the interval type-2 triangular fuzzy numbers instead of themselves in the problem. Moreover, it was transformed into the FLP problem with characteristic type-1 fuzzy numbers and with the help of the nearest interval approximation, it was converted into the ILP problem. Finally, the optimal solution and the optimal value were obtained by using the BWC method twice. In the following, the proposed method is described step by step in details:

Consider the interval type-2 triangular FLP problem with uncertainties in right-hand sides of the constraints and the coefficients of both objective function and constraints as follows:

$$\begin{aligned} &\max\ \tilde{z} = \Sigma \tilde{c}_j x_j \\ &\text{s.t.}\ \ \Sigma \tilde{a}_{ij} x_j \le \tilde{b}_i, \qquad i = 1,2,\ldots,m \\ &\qquad\quad x_j \ge 0, \qquad j = 1,2,\ldots,n, \end{aligned} \tag{2}$$

where $\tilde{c}_j$, $\tilde{b}_i$ and $\tilde{a}_{ij}$ are IT2FNs.

The minimized and maximized point of the *Core* of the above interval type-2 triangular fuzzy numbers in problem 2 are equal; thus one middle model is solved as:

$$\begin{aligned} &\max\ \bar{z} = \Sigma \overline{c}_j x_j \\ &\text{s.t.}\ \ \Sigma \overline{a}_{ij} x_j \le \overline{b}_i, \qquad i = 1,2,\ldots,m \\ &\qquad\quad x_j \ge 0, \qquad j = 1,2,\ldots,n, \end{aligned} \tag{3}$$

by solving linear programming problem 3, $\bar{x}_{jopt}$ and $\bar{z}_{opt}$ are obtained. Since an interval type-2 triangular fuzzy number is composed of the infinite union of the characteristic type-1 triangular fuzzy numbers, the traditional embedded type-1 triangular fuzzy numbers are used for all the interval type-2 triangular fuzzy numbers of problem 2. Therefore, it is converted into the following characteristic type-1 triangular FLP problem,

$$\begin{aligned} &\max\ z^e = \Sigma c_j^e x_j \\ &\text{s.t.}\ \ \Sigma a_{ij}^e x_j \le b_i^e, \qquad i = 1,2,\ldots,m \\ &\qquad\quad x_j \ge 0, \qquad j = 1,2,\ldots,n, \end{aligned} \tag{4}$$

where $c_j^e \in FOU(\tilde{c}_j)$, $b_i^e \in FOU(\tilde{b}_i)$ and $a_{ij}^e \in FOU(\tilde{a}_{ij})$ are the embedded type-1 triangular fuzzy numbers.

Now, by using the nearest interval approximation of the type-1 triangular fuzzy numbers of problem 4, it becomes equivalent to the following ILP problem,

$$\begin{aligned} &\max\ z^e = \Sigma [\check{c}_j^e, \hat{c}_j^e] x_j \\ &\text{s.t.}\ \ \Sigma [\check{a}_{ij}^e, \hat{a}_{ij}^e] x_j \le [\check{b}_i^e, \hat{b}_i^e], \qquad i = 1,2,\ldots,m \\ &\qquad\quad x_j \ge 0, \qquad j = 1,2,\ldots,n, \end{aligned} \tag{5}$$

where $\check{c}_j^e \in [\underline{\check{c}}_j^e, \overline{\check{c}}_j^e]$, $\hat{c}_j^e \in [\underline{\hat{c}}_j^e, \overline{\hat{c}}_j^e]$, $\check{a}_{ij}^e \in [\underline{\check{a}}_{ij}^e, \overline{\check{a}}_{ij}^e]$, $\hat{a}_{ij}^e \in [\underline{\hat{a}}_{ij}^e, \overline{\hat{a}}_{ij}^e]$, $\check{b}_i^e \in [\underline{\check{b}}_i^e, \overline{\check{b}}_i^e]$, $\hat{b}_i^e \in [\underline{\hat{b}}_i^e, \overline{\hat{b}}_i^e]$.

Also, with Theorem 2, the best and worst sub-models of the ILP problem 5 are obtained as follows:

$$\begin{aligned} &\max\ \hat{z} = \Sigma \hat{c}_j^e x_j \\ &\text{s.t.}\ \ \Sigma \check{a}_{ij}^e x_j \le \hat{b}_i^e, \qquad i = 1,2,\ldots,m \\ &\qquad\quad x_j \ge 0, \qquad j = 1,2,\ldots,n, \end{aligned} \tag{6}$$

and

$$\begin{aligned} &\max\ \check{z} = \Sigma \check{c}_j^e x_j \\ &\text{s.t.}\ \ \Sigma \hat{a}_{ij}^e x_j \le \check{b}_i^e, \qquad i = 1,2,\ldots,m \\ &\qquad\quad x_j \ge 0, \qquad j = 1,2,\ldots,n, \end{aligned} \tag{7}$$

respectively.

It is clear that the problems 6 and 7 are ILP models. Hence, their sub-models are obtained by reusing Theorem 2 for them. The best–best and the best–worst models obtained for 6 are:

1. The best–best model

   $$\begin{aligned} &\max\ \overline{\hat{z}} = \Sigma \overline{\hat{c}}_j^e x_j \\ &\text{s.t.}\ \ \Sigma \underline{\check{a}}_{ij}^e x_j \le \overline{\hat{b}}_i^e, \qquad i = 1,2,\ldots,m \\ &\qquad\quad x_j \ge 0, \qquad j = 1,2,\ldots,n, \end{aligned} \tag{8}$$

   by solving linear programming problem 8, $\overline{\hat{x}}_{jopt}$ and $\overline{\hat{z}}_{opt}$ are obtained.
2. The best–worst model

   $$\begin{aligned} &\max\ \underline{\hat{z}} = \Sigma \underline{\hat{c}}_j^e x_j \\ &\text{s.t.}\ \ \Sigma \overline{\check{a}}_{ij}^e x_j \le \underline{\hat{b}}_i^e, \qquad i = 1,2,\ldots,m \\ &\qquad\quad x_j \ge 0, \qquad j = 1,2,\ldots,n, \end{aligned} \tag{9}$$

   by solving linear programming problem 9, $\underline{\hat{x}}_{jopt}$ and $\underline{\hat{z}}_{opt}$ are obtained. In addition, the worst-best and the worst-worst models obtained for 7 are:

3. The worst-best model

$$\begin{aligned} &\max\ \underline{\check{z}} = \Sigma \overline{\check{c}}_j^e x_j \\ &\text{s.t.}\ \ \Sigma \underline{\hat{a}}_{ij}^e x_j \leq \overline{\check{b}}_i^e, \qquad i = 1, 2, \ldots, m \\ &\qquad\quad x_j \geq 0, \qquad j = 1, 2, \ldots, n, \end{aligned} \tag{10}$$

by solving linear programming problem 10, $\underline{\check{x}}_{jopt}$ and $\underline{\check{z}}_{opt}$ are obtained.

4. The worst-worst model

$$\begin{aligned} &\max\ \overline{\check{z}} = \Sigma \underline{\check{c}}_j^e x_j \\ &\text{s.t.}\ \ \Sigma \overline{\hat{a}}_{ij}^e x_j \leq \underline{\check{b}}_i^e, \qquad i = 1, 2, \ldots, m \\ &\qquad\quad x_j \geq 0, \qquad j = 1, 2, \ldots, n, \end{aligned} \tag{11}$$

also, by solving linear programming problem 11, $\overline{\check{x}}_{jopt}$ and $\overline{\check{z}}_{opt}$ are obtained.

Therefore, the interval type-2 triangular fuzzy numbers $\tilde{x}_{jopt} = \Big((\overline{\check{x}}_{jopt}, \overline{x}_{jopt}, \underline{\check{x}}_{jopt}), (\underline{\hat{x}}_{jopt}, \overline{x}_{jopt}, \overline{\hat{x}}_{jopt}) \Big)$ and $\tilde{z}_{opt} = \Big((\overline{\check{z}}_{opt}, \overline{z}_{opt}, \underline{\check{z}}_{opt}), (\underline{\hat{z}}_{opt}, \overline{z}_{opt}, \overline{\hat{z}}_{opt}) \Big)$ are optimal solution and optimal value for problem 2, respectively.

In recent decades, many ranking methods are defined to solve FLP problems and all existing coefficients and numbers in problem are of type-1 fuzzy numbers or IT2FNs. In these ranking methods for each fuzzy number in the problem, a crisp number is assigned, and the problem becomes a crisp problem, which is simply solved, although much information is lost from the main problem. Moreover, some approximation methods to solve FLP problems are defined and all existing coefficients and numbers in the problem are type-1 fuzzy numbers. These approximation methods are the generalization of ranking methods, because each type-1 fuzzy number is assigned by a closed interval and the FLP problem is converted to an ILP problem. So far, no approximation method for solving the FLP problems in which each interval type-2 fuzzy number in the problem is assigned to a closed interval, has been presented; so, the proposed technique has a new and different nature. Therefore, it can be regarded as a generalization of previous modes.

4 Numerical Examples

In this section, two examples are presented for illustration and checking the efficiency of the proposed method. In the first example, the FLP problem with interval type-2 symmetric triangular fuzzy numbers (which is the simplest problem of its kind) is considered and then, it is solved with full details, step by step. In the next example, the assumption of symmetry was excluded and the new FLP problem is completely analyzed. Finally, in both examples, the optimal solution and optimal values are obtained as interval type-2 triangular fuzzy numbers.

Example 1 Consider the FLP problem with the interval type-2 symmetric triangular fuzzy numbers as:

$$\begin{aligned} &\max\ \tilde{z} = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 \\ &\text{s.t.}\ \ \tilde{a}_{11} x_1 + \tilde{a}_{12} x_2 \leq \tilde{b}_1 \\ &\qquad \tilde{a}_{21} x_1 + \tilde{a}_{22} x_2 \leq \tilde{b}_2 \\ &\qquad\qquad x_1, x_2 \geq 0, \end{aligned} \tag{12}$$

where

$$\begin{aligned} \tilde{c}_1 &= \Big((1, 1.5, 2), (0.5, 1.5, 2.5) \Big) \\ &= \Big((\underline{\check{c}}_1, \overline{c}_1, \underline{\hat{c}}_1), (\overline{\check{c}}_1, \overline{c}_1, \overline{\hat{c}}_1) \Big), \\ \tilde{c}_2 &= \Big((2, 3, 4), (1, 3, 5) \Big) \\ &= \Big((\underline{\check{c}}_2, \overline{c}_2, \underline{\hat{c}}_2), (\overline{\check{c}}_2, \overline{c}_2, \overline{\hat{c}}_2) \Big), \\ \tilde{a}_{11} &= \Big((0.5, 1, 1.5), (0, 1, 2) \Big) \\ &= \Big((\underline{\check{a}}_{11}, \overline{a}_{11}, \underline{\hat{a}}_{11}), (\overline{\check{a}}_{11}, \overline{a}_{11}, \overline{\hat{a}}_{11}) \Big), \\ \tilde{a}_{12} &= \Big((3, 4, 5), (2, 4, 6) \Big) \\ &= \Big((\underline{\check{a}}_{12}, \overline{a}_{12}, \underline{\hat{a}}_{12}), (\overline{\check{a}}_{12}, \overline{a}_{12}, \overline{\hat{a}}_{12}) \Big), \\ \tilde{b}_1 &= \Big((9, 11, 13), (7, 11, 15) \Big) \\ &= \Big((\underline{\check{b}}_1, \overline{b}_1, \underline{\hat{b}}_1), (\overline{\check{b}}_1, \overline{b}_1, \overline{\hat{b}}_1) \Big) \\ , \tilde{a}_{21} &= \Big((3, 4, 5), (2, 4, 6) \Big) \\ &= \Big((\underline{\check{a}}_{21}, \overline{a}_{21}, \underline{\hat{a}}_{21}), (\overline{\check{a}}_{21}, \overline{a}_{21}, \overline{\hat{a}}_{21}) \Big), \\ \tilde{a}_{22} &= \Big((1.5, 2.5, 3.5), (0.5, 2.5, 4.5) \Big) \\ &= \Big((\underline{\check{a}}_{22}, \overline{a}_{22}, \underline{\hat{a}}_{22}), (\overline{\check{a}}_{22}, \overline{a}_{22}, \overline{\hat{a}}_{22}) \Big), \\ \tilde{b}_2 &= \Big((8, 12, 16), (4, 12, 20) \Big) \\ &= \Big((\underline{\check{b}}_2, \overline{b}_2, \underline{\hat{b}}_2), (\overline{\check{b}}_2, \overline{b}_2, \overline{\hat{b}}_2) \Big). \end{aligned}$$

All the IT2FNs in problem 12 were considered as the characteristic type-1 fuzzy numbers, and with the help of their nearest interval approximation, problem 12 was converted into the following ILP problem:

$$
\begin{aligned}
\max\ z^e &= \left[\frac{\check{c}_1+\bar{c}_1}{2},\frac{\hat{c}_1+\bar{c}_1}{2}\right]x_1+\left[\frac{\check{c}_2+\bar{c}_2}{2},\frac{\hat{c}_2+\bar{c}_2}{2}\right]x_2\\
\text{s.t.}\ &\left[\frac{\check{a}_{11}+\bar{a}_{11}}{2},\frac{\hat{a}_{11}+\bar{a}_{11}}{2}\right]x_1+\left[\frac{\check{a}_{12}+\bar{a}_{12}}{2},\frac{\hat{a}_{12}+\bar{a}_{12}}{2}\right]x_2\le\left[\frac{\check{b}_1+\bar{b}_1}{2},\frac{\hat{b}_1+\bar{b}_1}{2}\right]\\
&\left[\frac{\check{a}_{21}+\bar{a}_{21}}{2},\frac{\hat{a}_{21}+\bar{a}_{21}}{2}\right]x_1+\left[\frac{\check{a}_{22}+\bar{a}_{22}}{2},\frac{\hat{a}_{22}+\bar{a}_{22}}{2}\right]x_2\le\left[\frac{\check{b}_2+\bar{b}_2}{2},\frac{\hat{b}_2+\bar{b}_2}{2}\right]\\
&x_1,x_2\ge 0,
\end{aligned}
\tag{13}
$$

where $\hat{c}_j\in[\underline{\hat{c}}_j,\overline{\hat{c}}_j]$, $\check{a}_{ij}\in[\overline{\check{a}}_{ij},\underline{\check{a}}_{ij}]$, $\hat{b}_i\in[\underline{\hat{b}}_i,\overline{\hat{b}}_i]$, $\check{c}_j\in[\overline{\check{c}}_j,\underline{\check{c}}_j]$, $\hat{a}_{ij}\in[\underline{\hat{a}}_{ij},\overline{\hat{a}}_{ij}]$, $\check{b}_i\in[\overline{\check{b}}_i,\underline{\check{b}}_i]$.

With Theorem 2, the best sub-model will be:

$$
\begin{aligned}
\max\ \hat{z} &= \left(\frac{\hat{c}_1+\bar{c}_1}{2}\right)x_1+\left(\frac{\hat{c}_2+\bar{c}_2}{2}\right)x_2\\
\text{s.t.}\ &\left(\frac{\check{a}_{11}+\bar{a}_{11}}{2}\right)x_1+\left(\frac{\check{a}_{12}+\bar{a}_{12}}{2}\right)x_2\le\frac{\hat{b}_1+\bar{b}_1}{2}\\
&\left(\frac{\check{a}_{21}+\bar{a}_{21}}{2}\right)x_1+\left(\frac{\check{a}_{22}+\bar{a}_{22}}{2}\right)x_2\le\frac{\hat{b}_2+\bar{b}_2}{2}\\
&x_1,x_2\ge 0,
\end{aligned}
\tag{14}
$$

where $\hat{c}_j\in[\underline{\hat{c}}_j,\overline{\hat{c}}_j]$, $\check{a}_{ij}\in[\overline{\check{a}}_{ij},\underline{\check{a}}_{ij}]$, $\hat{b}_i\in[\underline{\hat{b}}_i,\overline{\hat{b}}_i]$,
and the worst sub-model is as follows:

$$
\begin{aligned}
\max\ \check{z} &= \left(\frac{\check{c}_1+\bar{c}_1}{2}\right)x_1+\left(\frac{\check{c}_2+\bar{c}_2}{2}\right)x_2\\
\text{s.t.}\ &\left(\frac{\hat{a}_{11}+\bar{a}_{11}}{2}\right)x_1+\left(\frac{\hat{a}_{12}+\bar{a}_{12}}{2}\right)x_2\le\frac{\check{b}_1+\bar{b}_1}{2}\\
&\left(\frac{\hat{a}_{21}+\bar{a}_{21}}{2}\right)x_1+\left(\frac{\hat{a}_{22}+\bar{a}_{22}}{2}\right)x_2\le\frac{\check{b}_2+\bar{b}_2}{2}\\
&x_1,x_2\ge 0,
\end{aligned}
\tag{15}
$$

where $\check{c}_j\in[\overline{\check{c}}_j,\underline{\check{c}}_j]$, $\hat{a}_{ij}\in[\underline{\hat{a}}_{ij},\overline{\hat{a}}_{ij}]$, $\check{b}_i\in[\overline{\check{b}}_i,\underline{\check{b}}_i]$.

Since minimized and maximized points of the *Core* of the above interval type-2 triangular fuzzy numbers in problem 12 are equal, one middle model is solved as:

$$
\begin{aligned}
\max\ \bar{z} &= \bar{c}_1x_1+\bar{c}_2x_2\\
\text{s.t.}\ &\bar{a}_{11}x_1+\bar{a}_{12}x_2\le\bar{b}_1\\
&\bar{a}_{21}x_1+\bar{a}_{22}x_2\le\bar{b}_2\\
&x_1,x_2\ge 0,
\end{aligned}
$$

That is,

$$
\begin{aligned}
\max\ \bar{z} &= 1.5x_1+3x_2\\
\text{s.t.}\ &x_1+4x_2\le 11\\
&4x_1+2.5x_2\le 12\\
&x_1,x_2\ge 0,
\end{aligned}
\tag{16}
$$

The optimal solution rounded in three decimal places was obtained as: $\bar{x}_{1opt}=1.519,\bar{x}_{2opt}=2.370$ and $\bar{z}_{opt}=9.389$.

It is clear that all of the coefficients in problems 14 and 15 are intervals. Hence, the best and worst models, 14 and 15, are ILP problems. Again, Theorem 2 was applied for them, and the sub-models of the best and worst models are obtained as:

1. The best–best model

$$
\begin{aligned}
\max\ \overline{\hat{z}} &= \left(\frac{\overline{\hat{c}}_1+\bar{c}_1}{2}\right)x_1+\left(\frac{\overline{\hat{c}}_2+\bar{c}_2}{2}\right)x_2\\
\text{s.t.}\ &\left(\frac{\overline{\check{a}}_{11}+\bar{a}_{11}}{2}\right)x_1+\left(\frac{\overline{\check{a}}_{12}+\bar{a}_{12}}{2}\right)x_2\le\frac{\overline{\hat{b}}_1+\bar{b}_1}{2}\\
&\left(\frac{\overline{\check{a}}_{21}+\bar{a}_{21}}{2}\right)x_1+\left(\frac{\overline{\check{a}}_{22}+\bar{a}_{22}}{2}\right)x_2\le\frac{\overline{\hat{b}}_2+\bar{b}_2}{2}\\
&x_1,x_2\ge 0,
\end{aligned}
$$

That is,

$$
\begin{aligned}
\max\ \overline{\hat{z}} &= 2x_1+4x_2\\
\text{s.t.}\ &0.5x_1+3x_2\le 13\\
&3x_1+1.5x_2\le 16\\
&x_1,x_2\ge 0,
\end{aligned}
\tag{17}
$$

2. The best–worst model

$$
\begin{aligned}
\max\ \underline{\hat{z}} &= \left(\frac{\underline{\hat{c}}_1+\bar{c}_1}{2}\right)x_1+\left(\frac{\underline{\hat{c}}_2+\bar{c}_2}{2}\right)x_2\\
\text{s.t.}\ &\left(\frac{\underline{\check{a}}_{11}+\bar{a}_{11}}{2}\right)x_1+\left(\frac{\underline{\check{a}}_{12}+\bar{a}_{12}}{2}\right)x_2\le\frac{\underline{\hat{b}}_1+\bar{b}_1}{2}\\
&\left(\frac{\underline{\check{a}}_{21}+\bar{a}_{21}}{2}\right)x_1+\left(\frac{\underline{\check{a}}_{22}+\bar{a}_{22}}{2}\right)x_2\le\frac{\underline{\hat{b}}_2+\bar{b}_2}{2}\\
&x_1,x_2\ge 0,
\end{aligned}
$$

That is,

$$
\begin{aligned}
\max\ \underline{\hat{z}} &= 1.75x_1+3.5x_2\\
\text{s.t.}\ &0.75x_1+3.5x_2\le 12\\
&3.5x_1+2x_2\le 14\\
&x_1,x_2\ge 0,
\end{aligned}
\tag{18}
$$

3. The worst-best model

$$
\begin{aligned}
\max\ \underline{\check{z}} &= \left(\frac{\underline{\check{c}}_1+\bar{c}_1}{2}\right)x_1+\left(\frac{\underline{\check{c}}_2+\bar{c}_2}{2}\right)x_2\\
\text{s.t.}\ &\left(\frac{\underline{\hat{a}}_{11}+\bar{a}_{11}}{2}\right)x_1+\left(\frac{\underline{\hat{a}}_{12}+\bar{a}_{12}}{2}\right)x_2\le\frac{\underline{\check{b}}_1+\bar{b}_1}{2}\\
&\left(\frac{\underline{\hat{a}}_{21}+\bar{a}_{21}}{2}\right)x_1+\left(\frac{\underline{\hat{a}}_{22}+\bar{a}_{22}}{2}\right)x_2\le\frac{\underline{\check{b}}_2+\bar{b}_2}{2}\\
&x_1,x_2\ge 0,
\end{aligned}
$$

That is,

$$\begin{aligned} \max\ & \underline{\check{z}} = 1.25x_1 + 2.5x_2 \\ \text{s.t.}\ & 1.25x_1 + 4.5x_2 \leq 10 \\ & 4.5x_1 + 3x_2 \leq 10 \\ & x_1, x_2 \geq 0, \end{aligned} \tag{19}$$

4. The worst-worst model

$$\begin{aligned} \max\ & \bar{\check{z}} = \left(\frac{\bar{\check{c}}_1 + \bar{c}_1}{2}\right)x_1 + \left(\frac{\bar{\check{c}}_2 + \bar{c}_2}{2}\right)x_2 \\ \text{s.t.}\ & \left(\frac{\bar{\check{a}}_{11} + \bar{a}_{11}}{2}\right)x_1 + \left(\frac{\bar{\check{a}}_{12} + \bar{a}_{12}}{2}\right)x_2 \leq \frac{\bar{\check{b}}_1 + \bar{b}_1}{2} \\ & \left(\frac{\bar{\check{a}}_{21} + \bar{a}_{21}}{2}\right)x_1 + \left(\frac{\bar{\check{a}}_{22} + \bar{a}_{22}}{2}\right)x_2 \leq \frac{\bar{\check{b}}_2 + \bar{b}_2}{2} \\ & x_1, x_2 \geq 0, \end{aligned}$$

That is,

$$\begin{aligned} \max\ & \bar{\check{z}} = x_1 + 2x_2 \\ \text{s.t.}\ & 1.5x_1 + 5x_2 \leq 9 \\ & 5x_1 + 3.5x_2 \leq 8 \\ & x_1, x_2 \geq 0, \end{aligned} \tag{20}$$

by solving the four above linear programming problem, their optimal solution and optimal value rounded in three decimal places are:
$\bar{\hat{x}}_{1opt} = 3.454, \bar{\hat{x}}_{2opt} = 3.758, \bar{\hat{z}}_{opt} = 21.939, \underline{\hat{x}}_{1opt} = 2.326, \underline{\hat{x}}_{2opt} = 2.930, \underline{\hat{z}}_{opt} = 14.326,$
$\underline{\check{x}}_{1opt} = 0.909, \underline{\check{x}}_{2opt} = 1.970, \underline{\check{z}}_{opt} = 6.061, \bar{\check{x}}_{1opt} = 0.430, \bar{\check{x}}_{2opt} = 1.671, \bar{\check{z}}_{opt} = 3.772,$

respectively. Therefore, by applying the defined method in the present paper, optimal solution and optimal value, rounded in three decimal places, for problem 12 are obtained as the following interval type-2 triangular fuzzy numbers:

$$\begin{aligned} \tilde{x}_{1opt} &= \Big((0.909, 1.519, 2.326), (0.430, 1.519, 3.454)\Big), \\ \tilde{x}_{2opt} &= \Big((1.970, 2.370, 2.930), (1.671, 2.370, 3.758)\Big), \\ \tilde{z}_{opt} &= \Big((6.061, 9.389, 14.326), (3.772, 9.389, 21.939)\Big). \end{aligned}$$

Example 2 Consider the following FLP problem with the interval type-2 triangular fuzzy numbers:

$$\begin{aligned} \max\ & \tilde{z} = \tilde{c}_1x_1 + \tilde{c}_2x_2 \\ \text{s.t.}\ & \tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 \leq \tilde{b}_1 \\ & \tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 \leq \tilde{b}_2 \\ & x_1, x_2 \geq 0, \end{aligned} \tag{21}$$

where

$$\begin{aligned} \tilde{c}_1 &= \Big((0.5, 1.2, 3), (0.2, 1.2, 3.2)\Big) \\ &= \Big((\underline{\check{c}}_1, \overline{c}_1, \underline{\hat{c}}_1), (\bar{\check{c}}_1, \overline{c}_1, \bar{\hat{c}}_1)\Big), \\ \tilde{c}_2 &= \Big((1, 3, 4.5), (0.5, 3, 5)\Big) \\ &= \Big((\underline{\check{c}}_2, \overline{c}_2, \underline{\hat{c}}_2), (\bar{\check{c}}_2, \overline{c}_2, \bar{\hat{c}}_2)\Big), \\ \tilde{a}_{11} &= \Big((0.5, 0.95, 1.6), (0.45, 0.95, 2)\Big) \\ &= \Big((\underline{\check{a}}_{11}, \overline{a}_{11}, \underline{\hat{a}}_{11}), (\bar{\check{a}}_{11}, \overline{a}_{11}, \bar{\hat{a}}_{11})\Big), \\ \tilde{a}_{12} &= \Big((1, 3.65, 3.7), (0.95, 3.65, 3.8)\Big) \\ &= \Big((\underline{\check{a}}_{12}, \overline{a}_{12}, \underline{\hat{a}}_{12}), (\bar{\check{a}}_{12}, \overline{a}_{12}, \bar{\hat{a}}_{12})\Big), \\ \tilde{b}_1 &= \Big((2, 3, 6), (1, 3, 7)\Big) \\ &= \Big((\underline{\check{b}}_1, \overline{b}_1, \underline{\hat{b}}_1), (\bar{\check{b}}_1, \overline{b}_1, \bar{\hat{b}}_1)\Big), \\ \tilde{a}_{21} &= \Big((0.9, 2.7, 2.9), (0.7, 2.7, 4)\Big) \\ &= \Big((\underline{\check{a}}_{21}, \overline{a}_{21}, \underline{\hat{a}}_{21}), (\bar{\check{a}}_{21}, \overline{a}_{21}, \bar{\hat{a}}_{21})\Big), \\ \tilde{a_{22}} &= \Big((0.6, 1.85, 2), (0.55, 1.85, 2.5)\Big) \\ &= \Big((\underline{\check{a}}_{22}, \overline{a}_{22}, \underline{\hat{a}}_{22}), (\bar{\check{a}}_{22}, \overline{a}_{22}, \bar{\hat{a}}_{22})\Big), \\ \tilde{b}_2 &= \Big((1.5, 4.75, 5.75), (1.25, 4.75, 6)\Big) \\ &= \Big((\underline{\check{b}}_2, \overline{b}_2, \underline{\hat{b}}_2), (\bar{\check{b}}_2, \overline{b}_2, \bar{\hat{b}}_2)\Big). \end{aligned}$$

The traditional embedded type-1 triangular fuzzy numbers were used for all the interval type-2 triangular fuzzy numbers of problem 21. Thus, problem 21 can be converted into the following characteristic type-1 triangular FLP problem:

$$\begin{aligned} \max\ & z^e = \Sigma c_j^e x_j \\ \text{s.t.}\ & \Sigma a_{ij}^e x_j \leq b_i^e, \qquad i = 1, 2, \\ & x_j \geq 0, \qquad j = 1, 2, \end{aligned} \tag{22}$$

where $c_j^e = (c_j^{\check{e}}, c_j^{\bar{e}}, c_j^{\hat{e}})$, $b_i^e = (b_i^{\check{e}}, b_i^{\bar{e}}, b_i^{\hat{e}})$ and $a_{ij}^e = (a_{ij}^{\check{e}}, a_{ij}^{\bar{e}}, a_{ij}^{\hat{e}})$ are the embedded type-1 triangular fuzzy numbers.

Now, the nearest interval approximation of the above type-1 fuzzy numbers was applied. Then, problem 21 was reduced to the following ILP problem:

$$\begin{aligned}
\max\ z^e &= \left[\frac{\check{c}_1+\bar{c}_1}{2}, \frac{\hat{c}_1+\bar{c}_1}{2}\right]x_1 + \left[\frac{\check{c}_2+\bar{c}_2}{2}, \frac{\hat{c}_2+\bar{c}_2}{2}\right]x_2 \\
\text{s.t.}\ & \left[\frac{\check{a}_{11}+\bar{a}_{11}}{2}, \frac{\hat{a}_{11}+\bar{a}_{11}}{2}\right]x_1 + \left[\frac{\check{a}_{12}+\bar{a}_{12}}{2}, \frac{\hat{a}_{12}+\bar{a}_{12}}{2}\right]x_2 \\
& \le \left[\frac{\check{b}_1+\bar{b}_1}{2}, \frac{\hat{b}_1+\bar{b}_1}{2}\right] \quad \left[\frac{\check{a}_{21}+\bar{a}_{21}}{2}, \frac{\hat{a}_{21}+\bar{a}_{21}}{2}\right]x_1 \\
& + \left[\frac{\check{a}_{22}+\bar{a}_{22}}{2}, \frac{\hat{a}_{22}+\bar{a}_{22}}{2}\right]x_2 \le \left[\frac{\check{b}_2+\bar{b}_2}{2}, \frac{\hat{b}_2+\bar{b}_2}{2}\right] \\
& x_1, x_2 \ge 0,
\end{aligned} \tag{23}$$

where $\hat{c}_j \in [\underline{\hat{c}}_j, \bar{\hat{c}}_j]$, $\check{a}_{ij} \in [\bar{\check{a}}_{ij}, \underline{\check{a}}_{ij}]$, $\hat{b}_i \in [\underline{\hat{b}}_i, \bar{\hat{b}}_i]$, $\check{c}_j \in [\bar{\check{c}}_j, \underline{\check{c}}_j]$, $\hat{a}_{ij} \in [\underline{\hat{a}}_{ij}, \bar{\hat{a}}_{ij}]$, $\check{b}_i \in [\bar{\check{b}}_i, \underline{\check{b}}_i]$.

With Theorem 2 the best sub-model is as follows:

$$\begin{aligned}
\max\ \hat{z} &= \left(\frac{\hat{c}_1+\bar{c}_1}{2}\right)x_1 + \left(\frac{\hat{c}_2+\bar{c}_2}{2}\right)x_2 \\
\text{s.t.}\ & \left(\frac{\check{a}_{11}+\bar{a}_{11}}{2}\right)x_1 + \left(\frac{\check{a}_{12}+\bar{a}_{12}}{2}\right)x_2 \le \frac{\hat{b}_1+\bar{b}_1}{2} \\
& \left(\frac{\check{a}_{21}+\bar{a}_{21}}{2}\right)x_1 + \left(\frac{\check{a}_{22}+\bar{a}_{22}}{2}\right)x_2 \le \frac{\hat{b}_2+\bar{b}_2}{2} \\
& x_1, x_2 \ge 0,
\end{aligned} \tag{24}$$

where $\hat{c}_j \in [\underline{\hat{c}}_j, \bar{\hat{c}}_j]$, $\check{a}_{ij} \in [\bar{\check{a}}_{ij}, \underline{\check{a}}_{ij}]$, $\hat{b}_i \in [\underline{\hat{b}}_i, \bar{\hat{b}}_i]$,
and the worst sub-model is as follows:

$$\begin{aligned}
\max\ \check{z} &= \left(\frac{\check{c}_1+\bar{c}_1}{2}\right)x_1 + \left(\frac{\check{c}_2+\bar{c}_2}{2}\right)x_2 \\
\text{s.t.}\ & \left(\frac{\hat{a}_{11}+\bar{a}_{11}}{2}\right)x_1 + \left(\frac{\hat{a}_{12}+\bar{a}_{12}}{2}\right)x_2 \le \frac{\check{b}_1+\bar{b}_1}{2} \\
& \left(\frac{\hat{a}_{21}+\bar{a}_{21}}{2}\right)x_1 + \left(\frac{\hat{a}_{22}+\bar{a}_{22}}{2}\right)x_2 \le \frac{\check{b}_2+\bar{b}_2}{2} \\
& x_1, x_2 \ge 0,
\end{aligned} \tag{25}$$

where $\check{c}_j \in [\bar{\check{c}}_j, \underline{\check{c}}_j]$, $\hat{a}_{ij} \in [\underline{\hat{a}}_{ij}, \bar{\hat{a}}_{ij}]$, $\check{b}_i \in [\bar{\check{b}}_i, \underline{\check{b}}_i]$.

Since minimized and maximized points of the *Core* of the above interval type-2 triangular fuzzy numbers in problem 21 are equal, one middle model can be solved as:

$$\begin{aligned}
\max\ \bar{z} &= 1.2x_1 + 3x_2 \\
\text{s.t.}\ & 0.95x_1 + 3.65x_2 \le 3 \\
& 2.7x_1 + 1.85x_2 \le 4.75 \\
& x_1, x_2 \ge 0,
\end{aligned} \tag{26}$$

The optimal solution rounded in three decimal places was obtained as $\bar{x}_{1opt} = 1.456, \bar{x}_{2opt} = 0.443$ and $\bar{z}_{opt} = 3.076$.

Moreover, the best and the worst models, 24 and 25, are ILP problems. Again, Theorem 2 was applied. Thus, the sub-models of best and worst models are:

1. The best–best model

$$\begin{aligned}
\max\ \hat{\bar{z}} &= 2.2x_1 + 4x_2 \\
\text{s.t.}\ & 0.7x_1 + 2.3x_2 \le 5 \\
& 1.7x_1 + 1.2x_2 \le 5.375 \\
& x_1, x_2 \ge 0,
\end{aligned} \tag{27}$$

2. The best–worst model

$$\begin{aligned}
\max\ \underline{\hat{z}} &= 2.1x_1 + 3.75x_2 \\
\text{s.t.}\ & 0.725x_1 + 2.325x_2 \le 4.5 \\
& 1.8x_1 + 1.225x_2 \le 5.25 \\
& x_1, x_2 \ge 0,
\end{aligned} \tag{28}$$

3. The worst-best model

$$\begin{aligned}
\max\ \underline{\check{z}} &= 0.85x_1 + 2x_2 \\
\text{s.t.}\ & 1.275x_1 + 3.675x_2 \le 2.5 \\
& 2.8x_1 + 1.925x_2 \le 3.125 \\
& x_1, x_2 \ge 0,
\end{aligned} \tag{29}$$

4. The worst-worst model

$$\begin{aligned}
\max\ \check{\bar{z}} &= 0.7x_1 + 1.75x_2 \\
\text{s.t.}\ & 1.475x_1 + 3.725x_2 \le 2 \\
& 3.35x_1 + 2.175x_2 \le 3 \\
& x_1, x_2 \ge 0,
\end{aligned} \tag{30}$$

by solving the above four linear programming problems, their optimal solution and optimal value, rounded in three decimal places, are:

$\hat{\bar{x}}_{1opt} = 2.072, \hat{\bar{x}}_{2opt} = 1.543, \hat{\bar{z}}_{opt} = 10.732, \underline{\hat{x}}_{1opt} = 2.030, \underline{\hat{x}}_{2opt} = 1.302, \underline{\hat{z}}_{opt} = 9.148,$

$\underline{\check{x}}_{1opt} = 0.851, \underline{\check{x}}_{2opt} = 0.385, \underline{\check{z}}_{opt} = 1.493, \check{\bar{x}}_{1opt} = 0.405, \check{\bar{x}}_{2opt} = 0.377, \check{\bar{z}}_{opt} = 0.942,$

respectively. Therefore, by applying the defined method in the present paper, optimal solution and optimal value, rounded in three decimal places, for problem 21 are obtained as the following interval type-2 triangular fuzzy numbers:

$$\begin{aligned}
\tilde{x}_{1opt} &= \big((0.851, 1.456, 2.030), (0.405, 1.456, 2.072)\big), \\
\tilde{x}_{2opt} &= \big((0.385, 0.443, 1.302), (0.377, 0.443, 1.543)\big), \\
\tilde{z}_{opt} &= \big((1.493, 3.076, 9.148), (0.942, 3.076, 10.732)\big).
\end{aligned}$$

5 Conclusion

In this paper, the FLP problem with interval type-2 triangular fuzzy numbers obscure of kind *Ambiguity* is considered, which all coefficients in it are interval type-2 triangular fuzzy numbers. Besides that, the nearest interval approximation of the type-1 triangular fuzzy number is used to solve the FLP problem with interval type-2 triangular fuzzy numbers. Each interval type-2 triangular fuzzy number is composed of the infinite union of the characteristic type-1 triangular fuzzy numbers. Therefore, with the help of the nearest interval approximation, each interval type-2 triangular fuzzy number in the problem is attributed to an interval, and the FLP problem with interval type-2 triangular fuzzy numbers turns into the ILP problem, in it all the coefficients are intervals. Then, with the BWC method, the optimal value and optimal solution are obtained. Here, an easy and useful way to solve the FLP problem with interval type-2 triangular fuzzy numbers was provided. This method can be generalized to the FLP problem with interval type-2 non triangular fuzzy numbers. Moreover, by replacing other solving methods of the ILP problem with the BWC method, the proposed technique in this paper to solve the FLP problem, can be examined with IT2FNs obscure of kind *Ambiguity*, *Vagueness* and even both of them. They may be considered as some new investigation ideas for researchers and enthusiasts of this topic.

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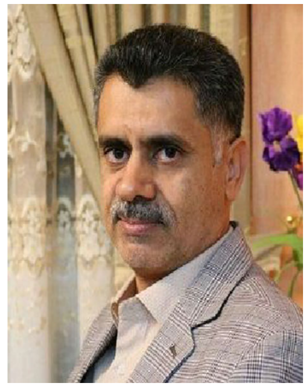

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