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Observer-Based Adaptive Fuzzy Control for Time-Varying State Constrained Strict-Feedback Nonlinear Systems with Dead-Zone

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Abstract This paper focuses on the problem of adaptive fuzzy control for a class of time-varying state constrained strict-feedback nonlinear systems with dead-zone. Based on the arbitrary approximation of fuzzy logic systems (FLSs), the unknown nonlinear functions in the system are approximated by FLSs. Time-varying barrier Lyapunov functions and a fuzzy observer are designed to dispose the unmeasured time-varying constrained states in the system. Furthermore, combining with the adaptive backstepping method and Lyapunov stability theory, it is testified that the proposed control strategy can ensure system stability and all the signals in the closed-loop system are semi-global uniformly ultimately bounded. Finally, the simulation results are given to demonstrate the effectiveness of the proposed method.

Keywords Nonlinear systems · Adaptive fuzzy control · Fuzzy state observer · Time-varying state constraints · Dead-zone

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1 Introduction

In recent years, much attention has been raised in developed linear and nonlinear control approaches for dealing with stability and control design of nonlinear systems, such as adaptive control [1–7], fault-tolerant control [8], optimal control [9], dynamic surface control [10], iterative learning control [11]. In partcular, adaptive fuzzy control [12–15] and adaptive neural network control [16–19] have become two very popular control methods for nonlinear systems, from which fuzzy logic systems (FLSs) or neural networks (NNs) are frequently employed to model unknown nonlinearities of the systems because of the arbitrary approximation of FLSs and NNs. To mention a few, an adaptive fuzzy control method was presented for the strict-feedback nonlinear systems in [1]. The optimal adaptive fuzzy control scheme was designed in [20] for a class of unknown nonlinear discrete-time systems with dead-zone. In [16], the problem of adaptive NN output feedback control was addressed for the nonlinear discrete-time systems with unknown control directions. The authors in [9] developed an adaptive fuzzy decentralized optimal control scheme for a class of nonlinear strict-feedback large-scale systems. In [21], an adaptive fuzzy tracking control design problem was addressed for single-input single-output (SISO) uncertain nonlinear systems in nonstrict-feedback form. The problem of adaptive NN tracking control for robotic manipulators with dead-zone was investigated in [22] . The authors in [17] devised an adaptive NN output feedback controller for a class of discrete-time nonlinear systems. In [23], FLS was utilized to estimate an unknown nonlinear function, and the control problem of uncertain fractional-order nonlinear systems was addressed. The authors in [24] presented an adaptive fuzzy backstepping decentralized control method for nonlinear large-scale systems.

On the other hand, the state and output constrained problems frequently appear in many practical systems [25–28]. Thus, it's necessary to design an appropriate control method to deal with the constrained problems. For example, the authors in [29] first introduced the BLFs for a nonlinear strict-feedback system with output constraints. By utilizing BLFs techniques, the authors in [30] presented an extremum-seeking control scheme for the constrained states. The control problem of nonlinear systems with partial state constraints was addressed by using BLFs in [31]. An adaptive NN tracking control strategy was constructed in [27] to deal with state constraints in a strict-feedback nonlinear system with Nussbaum gain. For nonlinear pure-feedback systems with the full state constraints, an adaptive control scheme was developed in [32]. Particularly, the time-varying state constraints are more complicated and common than aforementioned constants constraints. Tee et al. in [28] introuduced an asymmetric time-varying barrier Lyapunov function (TVBLF), which is utilized in the process of controller design for strict-feedback nonlinear systems. By utilizing NNs and TVBLF technique in [33], an adaptive NN control problem was handled for uncertain time-varying state constrained robotics systems. The tracking control problem in [34] was addressed for a class of nonlinear multi-input multi-output (MIMO) unknown time-varying delay systems with full state constraints.

In many control systems, only partial information of the states is available and some of them can be unmeasured directly, which requires us to design a state observer to estimate these unmeasured states. Initially, under the condition of using H_∞ control technique and Takagi–Sugeno (T–S) fuzzy model, many researchers have dedicated much effort to develop the observer-based adaptive fuzzy control methods for uncertain nonlinear systems subject to unmeasured state variables. Then, the control method had been further developed and widely used to stabilize nonlinear systems, like T–S fuzzy systems [35–37] , discrete-time fuzzy systems [38] and nonlinear systems in diverse forms [39–42] . Some main results are listed below. In [38], recent results on multi-instant observer design for discrete-time T–S fuzzy systems were generalized by designing a ranking-based switching approach. In [43], the problem of adaptive fuzzy control was disposed by designing an observer for uncertain nonlinear systems with unmodeled dynamics. In [40], an adaptive fuzzy output feedback control scheme was developed by utilizing a fuzzy filter state observer for SISO strict-feedback nonlinear systems, unknown dead-zone and unmeasured states were considered in the system. The authors in [44] dealt with the problem of observer design for T–S fuzzy models and studied continuous-time and discrete-time two cases simultaneously. An adaptive robust fuzzy output feedback controller was developed for nonlinear strict-feedback SISO systems with unknown dead-zone and the dynamics uncertainties in [45]. Wang et al. in [41] proposed an adaptive fuzzy control design method for nonlinear nonstrict-feedback system with input delay. In [46], the fault-tolerant control problem was addressed for a stochastic nonstrict-feedback nonlinear system with input quantization and unmeasured states. A valid data-based NCC (VDNCC) algorithm is proposed in [47] for eliminating the effect of the void area. In [48], the problem of a nonlinear switched T–S fuzzy system with actuator saturation and time delay was handled by proposing a state observer-based output feedback controller. However, how to integrate an observer and the TVBLF into the designing process simultaneously to achieve the desired system performance is still an interesting yet challenging problem, which motivates this paper.

On the basis of the aforementioned descriptions, this paper proposes an observer-based adaptive fuzzy control method for a class of time-varying state constrained strict-feedback nonlinear systems with dead-zone. The main contributions of this paper are summarized as follows: (1) A fuzzy state observer is constructed to counteract the effect of unmeasurable states. Then, an observer-based fuzzy controller is designed to guarantee the stability of closed-loop system. (2) The TVBLF is introduced to restrict all state variables to the specified time-varying regions and ensures all the signals in the system are bounded. (3) Dead-zone input, which frequently appears in the practical systems and leads to undesirable effect, is handled in the design process. Finally, some stimulation results are provided to indicate the effectiveness of the proposed control scheme.

Notation For the readability, the following notations will be used throughout this paper. R^n represents the real n-dimensional space, $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ are the largest and smallest eigenvalues of positive-definite matrix Q, respectively. $|\cdot|$ denotes the absolute value for a scalar. $||\cdot||$ denotes the 2-norm for a matrix or a vector. A^{T} stands for the transpose of Hurwitz matrix A.

The remainder of this paper is organized as follows. Section 2 presents system description and preliminaries. An adaptive fuzzy observer is designed in Sect. 3. In Sect. 4, the main results are provided. The simulation results are given in Sect. 5 to depict the effectiveness of the proposed control strategy. Finally, Sect. 6 concludes this paper.

2 System Description and Preliminaries

Consider the following SISO strict-feedback nonlinear system:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(X_i) + d_i, \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_n = u + f_n(X) + d_n \\ y = x_1 \end{cases} \tag{1}$$

where $X = [x_1, \ldots, x_n]^{\mathrm{T}} \in R^n$ is the state vector, $y \in R$ is the system output. $f_i(X_i)(i = 1, 2, \ldots, n)$ are the unknown nonlinear smooth functions, $d_i\ (i = 1, 2, \ldots, n)$ are external disturbances of the system bounded by $|d_i(t)| \le d_i^*$ $(i = 1, \ldots, n)$, where d_i^* are positive constants. Let $X_i = [x_1 \ldots x_i]^{\mathrm{T}}$ and assume that $x_i\ (i \ge 2)$ are unmeasured. The states in this study are constrained in predefined compact sets, i.e., $|x_i(t)| < k_{ci}(t), \forall t \ge 0$, where $k_{ci}(t)\ (i = 1, 2, \ldots, n)$ are positive-valued time-varying constraints. $u(t) = D(v(t))$ is the system dead-zone input, which can be written as

$$u(t) = D(v(t)) = \begin{cases} h_r[v(t) - m_r], & v(t) \ge m_r \\ 0, & -m_l < v(t) < m_r \\ h_l[v(t) + m_l], & v(t) \le -m_l \end{cases} \tag{2}$$

where h_r and h_l are right and left slopes, $h_r = h_l = h$. $m_r > 0$ and $m_l > 0$ are the break points, respectively. Then, the dead-zone model (2) can be represented as

$$D(v(t)) = hv(t) + m(t) \tag{3}$$

where

$$m(t) = \begin{cases} -hm_r, & v(t) \ge m_r \\ -hv(t), & -m_l < v(t) < m_r \\ hm_l, & v(t) \le -m_l \end{cases}$$

and it's obvious that $|m(t)| \le \bar{m} = \max\{hm_r, hm_l\}$.

Remark 1 It should be mentioned that many practical systems can be transformed or expressed as the model (1), such as hydraulic servo-system [49], the robotics system [33] and crane system [50].

Rewriting (1) in state space form as

$$\dot{X} = AX + Ky + \sum_{i=1}^{n} B_i[f_i(X_i) + d_i] + Bu \tag{4}$$

where

$$A = \begin{bmatrix} -k_1 & & & \\ \vdots & & I & \\ -k_n & 0 & \ldots & 0 \end{bmatrix}, K = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}, B_i = [0 \ldots 1 \ldots 0]^{\mathrm{T}}$$

and A is a strict Hurwitz matrix. Then, there exists a positive-definite matrix satisfying

$$A^{\mathrm{T}}P + PA = -2Q \tag{5}$$

FLS [51] in this study will be applied to approximate the unknown smooth functions, and it is described as follows.

R^l: If x_1 is F_1^l, x_2 is $F_2^l, \ldots, x_n$ is F_n^l, then y is G^l, $l = 1, 2, \ldots, N$, where $x = [x_1, \ldots, x_n]^{\mathrm{T}} \in R^n$ represents the input of the system, y denotes the output of the system. F_i^l and G^l stand for fuzzy sets in R. The number of fuzzy rules is N.

According to [51], the FLS can be described as

$$y(x) = \frac{\sum_{l=1}^{N} \bar{y}_l \prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} \left[\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right]} \tag{6}$$

where $\bar{y}_l = \max_{y \in R} \mu_{G^l}(y)$ denotes the point which makes membership function $\mu_{G^l}(y)$ achieve its maximum value and suppose $\mu_{G^l}(y) = 1$. The fuzzy basis functions are

$$\xi_l = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} \left[\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right]}$$

Then, the FLS (6) can be rewritten as

$$y(x) = \Phi^{\mathrm{T}} \xi(x) \tag{7}$$

where $\Phi^{\mathrm{T}} = (\bar{y}_1, \ldots, \bar{y}_N) = (\Phi_1, \ldots, \Phi_N)$ and $\xi(x) = [\xi_1(x), \ldots, \xi_N(x)]^{\mathrm{T}}$.

The control goal of this paper is to design an adaptive fuzzy controller and parameter adaptive functions such that all the signals in the closed-loop system are semi-global uniformly ultimately bounded (SGUUB), the tracking error of the system can reach a small enough neighborhood of zero concurrently and the state constraints are never violated.

The following lemmas and assumptions are needed in this paper.

Lemma 1 [52] *There exists a continuous function $F(x)$ defined on compact set Ω. Then, for any positive constant ε,* a FLS (7) *is designed such that*

$$\sup_{x \in \Omega} |F(x) - \Phi^{\mathrm{T}} \xi(x)| \le \varepsilon \tag{8}$$

Based on Lemma 1, the unknown nonlinear smooth functions in (1) can be approximated by the following FLS as

$$f_i(X_i|\theta_i) = \theta_i^{\mathrm{T}}\varphi_i(X_i), \quad 1 \le i \le n \tag{9}$$

Denote $\hat{X}_i = (\hat{x}_1, \hat{x}_2, \ldots \hat{x}_n)^{\mathrm{T}}$ as the estimation of $X_i = (x_1, x_2, \ldots x_n)^{\mathrm{T}}$, then one has

$$\hat{f}_i(\hat{X}_i|\theta_i) = \theta_i^{\mathrm{T}}\varphi_i(\hat{X}_i), \quad 1 \le i \le n \tag{10}$$

Define the optimal parameter vector θ_i^* as

$$\theta_i^* = \arg\min_{\theta_i \in \Psi_i}\left[\sup_{X_i \in \Theta_{i1},\, \hat{X}_i \in \Theta_{i2}} |\hat{f}_i(\hat{X}_i|\theta_i) - f_i(X_i)|\right] \tag{11}$$

where Ψ_i, Θ_{i1} and Θ_{i2} are compact sets for θ_i, X_i and $\hat{X}_i$, respectively. Let ε_i and δ_i be the minimum fuzzy approximation error and fuzzy approximation error, respectively. The expressions are as follows:

$$\varepsilon_i = f_i(X_i) - \hat{f}_i(\hat{X}_i|\theta_i^*), \ \delta_i = f_i(X_i) - \hat{f}_i(\hat{X}_i|\theta_i)$$

Lemma 2 [33] *For the existing bounded smooth function $k_b(t)$, if $z(t)$ satisfies $|z(t)| < k_b(t)$, the inequality $\log\left(\frac{k_b^2}{k_b^2 - z^2}\right) < \frac{z^2}{k_b^2 - z^2}$ can be proved.*

Assumption 1 [1] There are unknown positive constants ε_i^* and δ_i^* $(i = 1, 2, \ldots, n)$, such that $||\varepsilon_i|| \le \varepsilon_i^*$, $||\delta_i|| \le \delta_i^*$. Denote $\omega_i = \varepsilon_i + d_i$ and $\delta_i^{'} = \delta_i + d_i$. Furthermore, $|\omega_i| \le \omega_i^*$ and $|\delta_i^{'}| \le \delta_i^{'*}$ hold.

Assumption 2 [28] $y_r(t)$ and its jth derivatives $y_r^{(j)}(t)$ $(j = 1, \ldots, n)$ are assumed to satisfy $|y_r^{(j)}(t)| \le B_j$, $\underline{y}_r < y_r(t) \le \bar{y}_r \le k_{c1}(t)$, where $\underline{y}_r, \bar{y}_r, B_0, B_1, \ldots, B_n$ are positive constants and $\underline{y}_r \le B_0 < k_{c1}(t)$, $\bar{y}_r \le B_0 < k_{c1}(t)$.

Assumption 3 [31] There are constants $K_{ci} > 0$ $(i = 0, 1, \ldots, n)$ such that the time-varying constraint $k_{c1}(t)$ and its time derivatives satisfy $0 < k_{c1}(t) \le K_{c0}$ and $|k_{c1}^{(j)}(t)| \le K_{cj}$ $(j = 1, \ldots, n)$, $\forall t \ge 0$.

3 Fuzzy State Observer Design

Considering that system states $x_2, \ldots, x_n$ cannot be measured, a state observer should be designed to estimate the unmeasured states. A nonlinear fuzzy state observer is designed for (1) as

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \hat{f}_1(\hat{X}_1|\theta_1) - k_1(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_i = \hat{x}_{i+1} + \hat{f}_i(\hat{X}_i|\theta_i) - k_i(x_1 - \hat{x}_1) \\ \quad\vdots \\ \dot{\hat{x}}_n = u + \hat{f}_n(\hat{X}_n|\theta_n) - k_n(x_1 - \hat{x}_1) \end{cases} \tag{12}$$

Transform (12) into state space form as

$$\begin{aligned} &\dot{\hat{X}} = A\hat{X} + Ky + \sum_{i=1}^{n} B_i\left[\hat{f}_i(\hat{X}_i|\theta_i)\right] + Bu \\ &\hat{y} = C\hat{X} \end{aligned} \tag{13}$$

where $C = [1 \ldots 0 \ldots 0]$.

Define $e = X - \hat{X}$ as the observer error, where $X = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$, $\hat{X} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n]$. From (4) and (13), the observer errors equation can be obtained as

$$\begin{aligned} \dot{e} &= Ae + \sum_{i=1}^{n} B_i\left[f_i(X_i) - \hat{f}_i(\hat{X}_i|\theta_i) + d_i\right] \\ &= Ae + \delta \end{aligned} \tag{14}$$

where $e = [e_1, \ldots, e_n]^{\mathrm{T}}$, $\delta = \left[\delta_1^{'}, \ldots, \delta_n^{'}\right]^{\mathrm{T}}$.

4 Controller Design and Stability Analysis

4.1 Controller Design

For the design of the controller, we adopt the backstepping method and divide the design procedure into n steps. Firstly, take the following coordinate transformation

$$z_i = \hat{x}_i - \alpha_{i-1}, \quad i = 2, \ldots, n \tag{15}$$

where α_{i-1} is an intermediate control signal, $v(t)$ is the actual controller. The detailed structural steps are emerged in the following contents.

Step 1 Define the tracking error as

$$z_1 = y - y_r \tag{16}$$

Based on $x_2 = \hat{x}_2 + e_2$ and $z_2 = \hat{x}_2 - \alpha_1$, we obtain

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 - \dot{y}_r \\ &= z_2 + \alpha_1 + e_2 + \theta_1^{\mathrm{T}}\varphi_1(\hat{X}_1) + \tilde{\theta}_1^{\mathrm{T}}\varphi_1(\hat{X}_1) + \omega_1 - \dot{y}_r \end{aligned} \tag{17}$$

where $\tilde{\theta}_1 = \theta_1^* - \theta_1$.

Choose the following TVBLF as

$$V_1 = \frac{1}{2}e^{\mathrm{T}}Pe + \frac{1}{2}\log\frac{k_{b1}^2(t)}{k_{b1}^2(t) - z_1^2(t)} + \frac{1}{2\gamma_1}\tilde{\theta}_1^{\mathrm{T}}\tilde{\theta}_1 \tag{18}$$

where $\gamma_1 > 0$ is a design constant. Note that a TVBLF is integrated into the designed Lyapunov function. From (18), V_1 is continuous in the set $\Omega_{z_1} = \{z_1 : |z_1| < k_{b1}(t)\}$. In this paper, the states are confined to the specified regions. In the subsequent design steps, the same technique is adopted to ensure the time-varying state constraints can be satisfied.

From (12), (14) and (17), differentiating V_1 yields

$$\begin{aligned}\dot{V}_1 =& \frac{1}{2}\dot{e}^{\mathrm{T}}Pe + \frac{1}{2}e^{\mathrm{T}}P\dot{e} + \frac{z_1(t)}{k_{b1}^2(t) - z_1^2(t)}\dot{z}_1 \\ & - \frac{1}{\gamma_1}\tilde{\theta}_1^{\mathrm{T}}\dot{\theta} - \frac{z_1^2(t)\dot{k}_{b1}(t)}{k_{b1}(t)\left(k_{b1}^2(t) - z_1^2(t)\right)} \\ =& -e^{\mathrm{T}}Qe + e^{\mathrm{T}}P\delta + \frac{z_1(t)}{k_{b1}^2(t) - z_1^2(t)}[z_2(t) + e_2] \\ & + \frac{z_1(t)}{k_{b1}^2(t) - z_1^2(t)}\left[\alpha_1 + \theta_1^{\mathrm{T}}\varphi_1(\hat{X}_1) - \dot{y}_r\right] \\ & + \frac{1}{\gamma_1}\tilde{\theta}_1^{\mathrm{T}}\left[\gamma_1\frac{z_1(t)}{k_{b1}^2(t) - z_1^2(t)}\varphi_1(\hat{X}_1) - \dot{\theta}_1\right] \\ & + \frac{z_1(t)\omega_1}{k_{b1}^2(t) - z_1^2(t)} - \frac{z_1^2(t)\dot{k}_{b1}(t)}{k_{b1}(t)\left(k_{b1}^2(t) - z_1^2(t)\right)}\end{aligned} \tag{19}$$

Design the intermediate control signal α_1 and the adaptive function $\dot{\theta}_1$ as

$$\begin{aligned}\alpha_1 =& -c_1z_1 - \theta_1^{\mathrm{T}}\varphi_1(\hat{X}_1) + \dot{y}_r + \frac{z_1(t)\dot{k}_{b1}(t)}{k_{b1}(t)} \\ & - \frac{3}{2}\frac{z_1(t)}{k_{b1}^2(t) - z_1^2(t)}\end{aligned} \tag{20}$$

$$\dot{\theta}_1 = \gamma_1\frac{z_1(t)}{k_{b1}^2(t) - z_1^2(t)}\varphi_1(\hat{X}_1) - \sigma_1\theta_1 \tag{21}$$

Using (20) and (20), (19) can be rewritten as

$$\begin{aligned}\dot{V}_1 =& -e^{\mathrm{T}}Qe + e^{\mathrm{T}}P\delta - c_1\frac{z_1^2(t)}{k_{b1}^2(t) - z_1^2(t)} \\ & + \frac{z_1(t)}{k_{b1}^2(t) - z_1^2(t)}[z_2(t) + e_2] + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_1^{\mathrm{T}}\theta_1 \\ & + \frac{z_1(t)\omega_1}{k_{b1}^2(t) - z_1^2(t)} - \frac{3}{2}\frac{z_1^2(t)}{\left(k_{b1}^2(t) - z_1^2(t)\right)^2}\end{aligned} \tag{22}$$

Based on Young's inequality, one can obtain

$$\begin{aligned}e^{\mathrm{T}}P\delta + \frac{z_1(t)e_2}{k_{b1}^2(t) - z_1^2(t)} \le& \frac{1}{2}\frac{z_1^2(t)}{\left(k_{b1}^2(t) - z_1^2(t)\right)^2} \\ & + ||e||^2 + \frac{1}{2}||P\delta||^2\end{aligned} \tag{23}$$

$$\begin{aligned}\frac{z_1(t)}{k_{b1}^2(t) - z_1^2(t)}[z_2(t) + \omega_1] \le& \frac{z_1^2(t)}{\left(k_{b1}^2(t) - z_1^2(t)\right)^2} \\ & + \frac{1}{2}\omega_1^{*2} + \frac{1}{2}z_2^2(t)\end{aligned} \tag{24}$$

$$\frac{\sigma}{\gamma_1}\tilde{\theta}_1^{\mathrm{T}}\theta_1 \le -\frac{\sigma_1}{2\gamma_1}||\tilde{\theta}_1||^2 + \frac{\sigma_1}{2\gamma_1}||\theta_1^*||^2 \tag{25}$$

Substituting (23), (24) and (25) into (22), (22) becomes

$$\begin{aligned}\dot{V}_1 \le& -(\lambda_{\min}(Q) - 1)||e||^2 - c_1\frac{z_1^2(t)}{k_{b1}^2(t) - z_1^2(t)} \\ & + \frac{\sigma_1}{2\gamma_1}||\theta_1^*||^2 + \frac{1}{2}||P\delta||^2 - \frac{\sigma_1}{2\gamma_1}||\tilde{\theta}_1||^2 \\ & + \frac{1}{2}z_2^2(t) + \frac{1}{2}\omega_1^{*2}\end{aligned} \tag{26}$$

Step i $(2 \le i \le n-1)$: Define the error variable $z_i = \hat{x}_i - \alpha_{i-1}$ and based on (12), one has

$$\begin{aligned}\dot{z}_i =& \dot{\hat{x}}_i - \dot{\alpha}_{i-1} \\ =& \hat{x}_{i+1} + H_i + \tilde{\theta}_i^{\mathrm{T}}\varphi_i(\hat{X}_i) + \omega_i - \delta_i' \\ & - \frac{\partial\alpha_{i-1}}{\partial y}e_2 - \frac{\partial\alpha_{i-1}}{\partial y}\delta_1'\end{aligned} \tag{27}$$

where $H_i = -k_ie_1 + \theta_i^{\mathrm{T}}\varphi_i(\hat{X}_i) - \frac{\partial\alpha_{i-1}}{\partial k_{bi}}\dot{k}_{bi} - \frac{\partial\alpha_{i-1}}{\partial \dot{k}_{bi}}\ddot{k}_{bi} - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\theta_k}\dot{\theta}_k - \sum_{k=1}^{i}\frac{\partial\alpha_{i-1}}{\partial y_r^{(k-1)}}y_r^{(k)} - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_k}\dot{\hat{x}}_k - \frac{\partial\alpha_{i-1}}{\partial y}[\hat{x}_2 + \theta_1^{\mathrm{T}}\varphi_1(\hat{X}_1)]$.

Choose the following TVBLF as

$$V_i = \frac{1}{2}\log\frac{k_{bi}^2(t)}{k_{bi}^2(t) - z_i^2(t)} + \frac{1}{2\gamma_i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i \tag{28}$$

where $\gamma_i > 0$ is a design constant.

From (15) and (27), differentiating V_i yields

$$\begin{aligned}\dot{V}_i =& \frac{z_i(t)}{k_{bi}^2(t) - z_i^2(t)}\dot{z}_i(t) - \frac{z_i^2(t)\dot{k}_{bi}(t)}{k_{bi}(t)\left(k_{bi}^2(t) - z_i^2(t)\right)} \\ & - \frac{1}{\gamma_i}\tilde{\theta}_i^{\mathrm{T}}\dot{\theta}_i \\ =& \frac{z_i(t)}{k_{bi}^2(t) - z_i^2(t)}\Big[z_{i+1} + \alpha_i + H_i + \omega_i - \delta_i' \\ & - \frac{\partial\alpha_{i-1}}{\partial y}\left(\delta_1' + e_2\right)\Big] - \frac{z_i^2(t)\dot{k}_{bi}(t)}{k_{bi}(t)\left(k_{bi}^2(t) - z_i^2(t)\right)} \\ & + \frac{1}{\gamma_i}\tilde{\theta}_i^{\mathrm{T}}\left[\gamma_i\frac{z_i(t)}{k_{bi}^2(t) - z_i^2(t)}\varphi_i(\hat{X}_i) - \dot{\theta}_i\right]\end{aligned} \tag{29}$$

Construct the intermediate control signal α_i and the adaptive function $\dot{\theta}_i$ as

$$\begin{aligned}\alpha_i =& -c_iz_i - H_i - \frac{1}{2}z_i(t)\left(k_{bi}^2(t) - z_i^2(t)\right) \\ & - \frac{3}{2}\frac{z_i(t)}{k_{bi}^2(t) - z_i^2(t)} + \frac{z_i(t)\dot{k}_{bi}(t)}{k_{bi}(t)} \\ & - \left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2\frac{z_i(t)}{k_{bi}^2(t) - z_i^2(t)}\end{aligned} \tag{30}$$

$$\dot{\theta}_i = \gamma_i\frac{z_i(t)}{k_{bi}^2(t) - z_i^2(t)}\varphi_i(\hat{X}_i) - \sigma_i\theta_i \tag{31}$$

Using (30) and (31), (29) can be rewritten as

$$\begin{aligned}\dot{V}_i =& -c_i\frac{z_i^2(t)}{k_{bi}^2(t)-z_i^2(t)}-\frac{1}{2}z_i^2(t)+\frac{z_i(t)z_{i+1}(t)}{k_{bi}^2(t)-z_i^2(t)}\\&+\frac{z_i(t)\omega_i}{k_{bi}^2(t)-z_i^2(t)}-\frac{z_i(t)}{k_{bi}^2(t)-z_i^2(t)}\frac{\partial\alpha_{i-1}}{\partial y}\delta_1^{'}\\&-\frac{z_i(t)\delta_i^{'}}{k_{bi}^2(t)-z_i^2(t)}-\frac{z_i(t)}{k_{bi}^2(t)-z_i^2(t)}\frac{\partial\alpha_{i-1}}{\partial y}e_2\\&-\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2\frac{z_i^2(t)}{\left(k_{bi}^2(t)-z_i^2(t)\right)^2}\\&-\frac{3}{2}\frac{z_i^2(t)}{\left(k_{bi}^2(t)-z_i^2(t)\right)^2}+\frac{\sigma_i}{\gamma_i}\tilde{\theta}_i^{\mathrm{T}}\theta_i\end{aligned}\tag{32}$$

Similar to (23)–(25) in Step 1, one can obtain

$$\begin{aligned}&-\frac{z_i(t)}{k_{bi}^2(t)-z_i^2(t)}\left(\delta_i^{'}+\frac{\partial\alpha_{i-1}}{\partial y}e_2+\frac{\partial\alpha_{i-1}}{\partial y}\delta_1^{'}\right)\\&\le\frac{1}{2}\frac{z_i^2(t)}{\left(k_{bi}^2(t)-z_i^2(t)\right)^2}+||e||^2+\frac{1}{2}\delta_1^{'*2}+\frac{1}{2}\delta_i^{'*2}\\&+\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2\frac{z_i^2(t)}{\left(k_{bi}^2(t)-z_i^2(t)\right)^2}\end{aligned}\tag{33}$$

$$\begin{aligned}&\frac{z_i(t)z_{i+1}(t)}{k_{bi}^2(t)-z_i^2(t)}+\frac{z_i(t)\omega_i}{k_{bi}^2(t)-z_i^2(t)}\\&\le\frac{1}{2}z_{i+1}^2(t)+\frac{1}{2}\omega_i^{*2}+\frac{z_i^2(t)}{\left(k_{bi}^2(t)-z_i^2(t)\right)^2}\end{aligned}\tag{34}$$

$$\frac{\sigma_i}{\gamma i}\tilde{\theta}_i^{\mathrm{T}}\theta_i\le-\frac{\sigma_i}{2\gamma i}||\tilde{\theta}_i||^2+\frac{\sigma_i}{2\gamma i}||\theta_i^*||^2\tag{35}$$

Substituting (33), (34) and (35) into (32), one has

$$\begin{aligned}\dot{V}_i\le& -c_i\frac{z_i^2(t)}{k_{bi}^2(t)-z_i^2(t)}-\frac{1}{2}z_i^2(t)+\frac{1}{2}z_{i+1}^2(t)\\&+\frac{1}{2}\omega_i^{*2}+||e||^2+\frac{1}{2}\delta_1^{'*2}+\frac{1}{2}\delta_i^{'*2}\\&-\frac{\sigma_i}{2\gamma_i}||\tilde{\theta}_i||^2+\frac{\sigma_i}{2\gamma_i}||\theta_i^*||^2\end{aligned}\tag{36}$$

Step n Define the error variable $z_n=\hat{x}_n-\alpha_{n-1}$ and based on (12), one has

$$\begin{aligned}\dot{z}_n&=\dot{\hat{x}}_n-\dot{\alpha}_{n-1}\\&=hv(t)+m(t)+H_n+\tilde{\theta}_n^{\mathrm{T}}\varphi_n(\hat{X}_n)+\omega_n\\&-\delta_n^{'}-\frac{\partial\alpha_{n-1}}{\partial y}e_2-\frac{\partial\alpha_{n-1}}{\partial y}\delta_1^{'}\end{aligned}\tag{37}$$

where $H_n=-k_ne_1+\theta_n^{\mathrm{T}}\varphi_n(\hat{X}_n)-\frac{\partial\alpha_{i-1}}{\partial k_{bn}}\dot{k}_{bn}-\frac{\partial\alpha_{i-1}}{\partial\dot{k}_{bi}}\ddot{k}_{bn}-\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_k}\dot{\hat{x}}_k-\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\theta_k}\dot{\theta}_k-\sum_{k=1}^{n}\frac{\partial\alpha_{n-1}}{\partial y_r^{(k-1)}}y_r^{(k)}-\frac{\partial\alpha_{n-1}}{\partial y}[\hat{x}_2+\theta_1^{\mathrm{T}}\varphi_1(\hat{X}_1)]$.

Choose the following TVBLF as

$$V_n=\frac{1}{2}\log\frac{k_{bn}^2(t)}{k_{bn}^2(t)-z_n^2(t)}+\frac{1}{2\gamma_n}\tilde{\theta}_n^{\mathrm{T}}\tilde{\theta}_n\tag{38}$$

where $\gamma_n>0$ is a design constant.

Combining (15) with (37), differentiating V_n yields

$$\begin{aligned}\dot{V}_n&=\frac{z_n(t)}{k_{bn}^2(t)-z_n^2(t)}\dot{z}_n(t)-\frac{z_n^2(t)\dot{k}_{bn}(t)}{k_{bn}(t)\left(k_{bn}^2(t)-z_n^2(t)\right)}\\&-\frac{1}{\gamma n}\tilde{\theta}_n^{\mathrm{T}}\dot{\theta}_n\\&=\frac{z_n(t)}{k_{bn}^2(t)-z_n^2(t)}\Big[hv(t)+m(t)+\omega_n-\delta_n^{'}\\&+H_n-\frac{\partial\alpha_{n-1}}{\partial y}\left(e_2+\delta_1^{'}\right)-\frac{z_n(t)\dot{k}_{bn}(t)}{k_{bn}(t)}\Big]\\&+\frac{1}{\gamma_n}\tilde{\theta}_n^{\mathrm{T}}\left[\gamma_n\frac{z_n(t)}{k_{bn}^2(t)-z_n^2(t)}\varphi_n(\hat{X}_n)-\dot{\theta}_n\right]\end{aligned}\tag{39}$$

Establish the actual controller v and the adaptive function $\dot{\theta}_n$ as

$$\begin{aligned}v=&\frac{1}{h}\Big[-c_nz_n-m(t)-\frac{1}{2}z_n(t)\left(k_{bn}^2(t)-z_n^2(t)\right)\\&-H_n-\frac{z_n(t)}{k_{bn}^2(t)-z_n^2(t)}+\frac{z_n(t)\dot{k}_{bn}(t)}{k_{bn}(t)}\\&-\left(\frac{\partial\alpha_{n-1}}{\partial y}\right)^2\frac{z_n(t)}{k_{bn}^2(t)-z_n^2(t)}\Big]\end{aligned}\tag{40}$$

$$\dot{\theta}_n=\gamma_n\frac{z_n(t)}{k_{bn}^2(t)-z_n^2(t)}\varphi_n(\hat{X}_n)-\sigma_n\theta_n\tag{41}$$

On the basis of (40) and (41), (39) can be rewritten as

$$\begin{aligned}\dot{V}_n=&-c_n\frac{z_n^2(t)}{k_{bn}^2(t)-z_n^2(t)}-\frac{1}{2}z_n^2(t)+\frac{z_n(t)\omega_n}{k_{bn}^2(t)-z_n^2(t)}\\&-\frac{z_n(t)\delta_n^{'}}{k_{bn}^2(t)-z_n^2(t)}-\frac{z_n(t)}{k_{bn}^2(t)-z_n^2(t)}\frac{\partial\alpha_{n-1}}{\partial y}e_2\\&-\frac{z_n^2(t)}{\left(k_{bn}^2(t)-z_n^2(t)\right)^2}-\frac{z_n(t)}{k_{bn}^2(t)-z_n^2(t)}\frac{\partial\alpha_{n-1}}{\partial y}\delta_1^{'}\\&+\frac{\sigma_n}{\gamma_n}\tilde{\theta}_n^{\mathrm{T}}\theta_n-\left(\frac{\partial\alpha_{n-1}}{\partial y}\right)^2\frac{z_n^2(t)}{\left(k_{bn}^2(t)-z_n^2(t)\right)^2}\end{aligned}\tag{42}$$

Similar to (33)–(35) in the above steps, one can obtain

$$\begin{aligned}&-\frac{z_n(t)}{k_{bn}^2(t)-z_n^2(t)}\left(\delta_n^{'}+\frac{\partial\alpha_{n-1}}{\partial y}e_2+\frac{\partial\alpha_{n-1}}{\partial y}\delta_1^{'}\right)\\&\le\frac{1}{2}\frac{z_n^2(t)}{\left(k_{bn}^2(t)-z_n^2(t)\right)^2}+||e||^2+\frac{1}{2}\delta_1^{'*2}+\frac{1}{2}\delta_n^{'*2}\\&+\left(\frac{\partial\alpha_{n-1}}{\partial y}\right)^2\frac{z_n^2(t)}{\left(k_{bn}^2(t)-z_n^2(t)\right)^2}\end{aligned}\tag{43}$$

$$\frac{z_n(t)\omega_n}{k_{bn}^2(t)-z_n^2(t)}\le\frac{1}{2}\omega_n^{*2}+\frac{1}{2}\frac{z_n^2(t)}{\left(k_{bn}^2(t)-z_n^2(t)\right)^2}\tag{44}$$

$$\frac{\sigma_n}{\gamma_n}\tilde{\theta}_n^{\mathrm{T}}\theta_n \le -\frac{\sigma_n}{2\gamma_n}||\tilde{\theta}_n||^2 + \frac{\sigma_n}{2\gamma_n}||\theta_n^*||^2 \tag{45}$$

Substituting (43), (44) and (45) into (42) yields

$$\begin{aligned}\dot{V}_n \le & -c_n\frac{z_n^2(t)}{k_{bn}^2(t)-z_n^2(t)} - \frac{1}{2}z_n^2(t) + \frac{1}{2}\omega_n^{*2} + ||e||^2 \\ & + \frac{1}{2}\delta_1^{\prime *2} + \frac{1}{2}\delta_n^{\prime *2} - \frac{\sigma_n}{2\gamma_n}||\tilde{\theta}_n||^2 + \frac{\sigma_n}{2\gamma_n}||\theta_n^*||^2\end{aligned} \tag{46}$$

Choose the whole Lyapunov function candidate as

$$V = \sum_{j=1}^{n} V_j \tag{47}$$

From (26), (36), (46) and Lemma 2, we can get

$$\begin{aligned}\dot{V} \le & -(\lambda\min(Q)-n)||e||^2 - \sum_{j=1}^{n} c_j \log\frac{k_{bj}^2(t)}{k_{bj}^2(t)-z_j^2(t)} \\ & - \sum_{j=1}^{n}\frac{\sigma}{2\gamma_j}||\tilde{\theta}_j||^2 + \sum_{j=1}^{n}\frac{\sigma_j}{2\gamma_j}||\theta_j^*||^2|| + \sum_{j=1}^{n}\frac{1}{2}||\omega_j^{*2} \\ & + \sum_{j=2}^{n}\frac{1}{2}\delta_j^{\prime *2} + \frac{1}{2}||P\delta||^2 + \frac{(n-1)}{2}\delta_1^{\prime *2}\end{aligned} \tag{48}$$

Thus, the following inequality can be obtained as

$$\dot{V} \le -aV + b \tag{49}$$

where

$$a = \min\{2(\lambda\min(Q)-n)/\lambda\min(P), 2c_i, \sigma_i\},$$

$$\begin{aligned}b = & \sum_{j=1}^{n}\frac{\sigma}{2\gamma_j}||\theta_j^*||^2 + \sum_{j=1}^{n}\frac{1}{2}||\omega_j^{*2}|| + \sum_{j=2}^{n}\frac{1}{2}\delta_j^{\prime 2} \\ & + \frac{1}{2}||P\delta||^2 + \frac{(n-1)}{2}\delta_1^{\prime *2}.\end{aligned}$$

4.2 Stability Analysis

Theorem 1 *For the nonlinear system (1) with time-varying state constraints and dead-zone, if its initial condition satisfies $x_i(0)$ functions (20), (31) and (41), the proposed control approach can ensure that the tracking error to be small enough, the full state constraints don't violate the predefined limits and all the closed-loop signals are SGUUB. The system error variables $z_i(t)$ and the observer error e will be in the compact sets $\Omega_z = \{z_i|\, |z_i(t)| \le \Delta_1,\ i = 1,\ldots,n\}$ and $\{|e| \le \Delta_2\}$, respectively, where Δ_1 and Δ_2 will be introduced later.*

Proof Multiplying e^{-at} on both sides of the inequality (49), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}(V\mathrm{e}^{at}) \le b\mathrm{e}^{at} \tag{50}$$

Integrating $d(V\mathrm{e}^{at}) \le b\mathrm{e}^{at}$ over $[0, t]$ yields

$$0 \le V(t) \le \left[V(0) - \frac{b}{a}\right]\mathrm{e}^{-at} + \frac{b}{a} \tag{51}$$

Since $a > 0$ and $b > 0$, (51) becomes

$$0 \le V(t) \le V(0)\mathrm{e}^{-at} + \frac{b}{a} \tag{52}$$

Then, we have

$$\frac{1}{2}\log\frac{k_{bi}^2(t)}{k_{bi}^2(t)-z_i^2(t)} \le \left[V(0) - \frac{b}{a}\right]\mathrm{e}^{-at} + \frac{b}{a} \tag{53}$$

$$\frac{1}{2}e^2 \le V(0)\mathrm{e}^{-at} + \frac{b}{a} \tag{54}$$

Further, the following inequalities hold

$$|z_i(t)| \le k_{bi}(t)\sqrt{1 - e^{-2\left[V(0)-\frac{b}{a}\right]}e^{-2\frac{b}{a}}} = \Delta_1 \tag{55}$$

$$|e| \le \sqrt{2\left(V(0)\mathrm{e}^{-at} + \frac{b}{a}\right)} = \Delta_2 \tag{56}$$

If $V(0) = \frac{b}{a}$, then, $|z_i(t)| \le k_{bi}(t)\sqrt{1 - e^{-2\frac{b}{a}}}$ holds. If $V(0) \ne \frac{b}{a}$, it can be concluded that given any $\Delta_1 > k_{bi}(t)\sqrt{1 - e^{-2\frac{b}{a}}}$, there exists T^* such that for $\forall t > T^*$, $|z_i(t)| \le \Delta_1$ holds. As $t \to \infty$, $|z_i(t)| \le k_{bi}(t)\sqrt{1 - e^{-2\frac{b}{a}}}$. This implies that $|z_i(t)| \le k_{bi}(t)\sqrt{1 - e^{-2\frac{b}{a}}}$ $(i = 1,\ldots,n)$. We can see that $z_i(t)$ can be made arbitrarily small by selecting the design parameters appropriately.

Since $|z_1(t)| \le \Delta_1 \le k_{b1}(t)$ and $|y_r(t)| \le B_0$, it's obvious that $|x_1(t)| \le |z_1(t)| + |y_r(t)| \le k_{b1}(t) + B_0$. Define $k_{b1}(t) + B_0 = k_{c1}(t)$, it has $|x_1(t)| \le k_{c1}(t)$. Apparently, there is a constant $\bar{\alpha}_i > 0$ so that $|\alpha_i| \le \bar{\alpha}_i$. Furthermore, $|x_i(t)| \le |e| + |z_i(t)| + |\alpha_i| \le \Delta_2 + k_{bi}(t) + \bar{\alpha}_i = k_{ci}(t)$ $(i = 2,\ldots,n)$ holds. Thus, the time-varying state constraints cannot violate the predefined limits.

According to the above proof, all the closed-loop signals are SGUUB. This completes the proof. □

Remark 2 Compared with the previous results in [18] and [21], the designed control method is more suitable for the practical application. Inspired by the Ref. [1], the state observer is constructed, from which we can see that this method is simple and convenient to apply.

Remark 3 In this research, we have presented an observer-based adaptive fuzzy control strategy for time-varying state constrained strict-feedback nonlinear systems with dead-zone. Apparently, the developed control method can also be extended to MIMO nonlinear systems. In that case, we selected the TVBLFs as $\frac{1}{2}\log\frac{k_{bi,j}^2(t)}{k_{bi,j}^2(t)-z_{i,j}^2(t)}$ $(i=1,\ldots,m, j=1,\ldots,n_i)$. Then with minor changes of lemmas and assumptions, the similar result can be achieved.

5 Simulation Results

Consider the following nonlinear system

$$\begin{cases} \dot{x}_1 = 0.1x_1^2 - x_2 \\ \dot{x}_2 = 0.2\sin(x_1)x_2 + x_1 + 2u \\ y = x_1 \end{cases} \tag{57}$$

where x_1, x_2 are the system states and y is the system output, $u(t)$ represents system dead-zone input which is expressed as

$$u(t) = D(v(t)) = \begin{cases} 0.4(v(t)-0.05), & v(t)\geq 0.05 \\ 0, & -0.06 < v(t) < 0.05 \\ 0.4(v(t)+0.06), & v(t) \leq -0.06 \end{cases}$$

Choose the following fuzzy membership functions as

$$\mu_{F_1}^l(\hat{x}_1) = \exp\left[-\frac{(\hat{x}_1+l)^2}{2}\right],$$

$$\mu_{F_2}^l(\hat{x}_1,\hat{x}_2) = \exp\left[-\frac{(\hat{x}_1+l)^2}{2}\right] * \exp\left[-\frac{(\hat{x}_2+l)^2}{2}\right].$$

Define fuzzy basis functions as

$$\varphi_{1j}(\hat{x}_1) = \frac{\mu_{F_1}^l(\hat{x}_1)}{\sum_{j=-5}^{4}\mu_{F_1}^{2j+1}(\hat{x}_1) + \mu_{F_1}^0(\hat{x}_1)},$$

$$\varphi_{2j}(\hat{x}_1,\hat{x}_2) = \frac{\mu_{F_2}^l(\hat{x}_1,\hat{x}_2)}{\sum_{j=-5}^{4}\mu_{F_2}^{2j+1}(\hat{x}_1,\hat{x}_2) + \mu_{F_2}^0(\hat{x}_1,\hat{x}_2)}$$

where $l = 0, \pm1, \pm3, \pm5, \pm7, \pm9$.

To carry out this simulation, the desired reference signal is given as $y_r(t) = 0.07\cos(0.5(t-1))$. The design parameters in this simulation are selected as $k_1 = k_2 = 1$, $c_1 = c_2 = 1$, $\gamma_1 = 2$, $\gamma_2 = [2,2,2,2]^{\mathrm{T}}$, $\sigma_1 = 2$, $\sigma_2 = 6$. The initial values for the system states and adaptive law are given as $x_1(0) = 0.13$, $x_2(0) = 1.43$, $\hat{x}_1(0) = 0.13$, $\hat{x}_2(0) = 1.43$ and $\theta_1(0) = 0.1$, $\theta_2(0) = [0.1, 0.1, 0.1, 0.1]^{\mathrm{T}}$. The time-varying constrained functions $k_{ci}\,(i=1,2)$ are defined as $k_{c1} = 0.07\cos(0.5(t-1)) + 0.2$, $k_{c2} = 0.07\cos(0.5(t-1)) + 1.55$.

With the above given parameters, the relevant simulation results are shown in Figs. 1, 2, 3, 4, 5, 6, 7, 8 and 9. The trajectories of system output y and given reference signal $y_r(t)$ are plotted in Fig. 1. Figure 2 describes the trajectory of system state x_2. From Figs. 1 and 2, it can be seen that the states are restricted to the predefined regions.

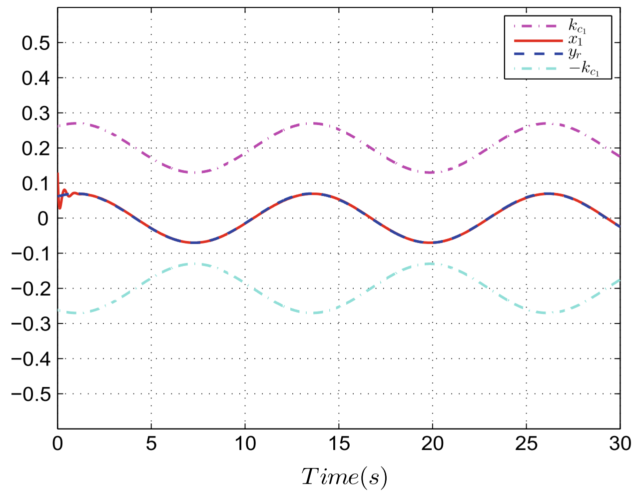

Fig. 1 Trajectories of state x_1 and reference signal y_r

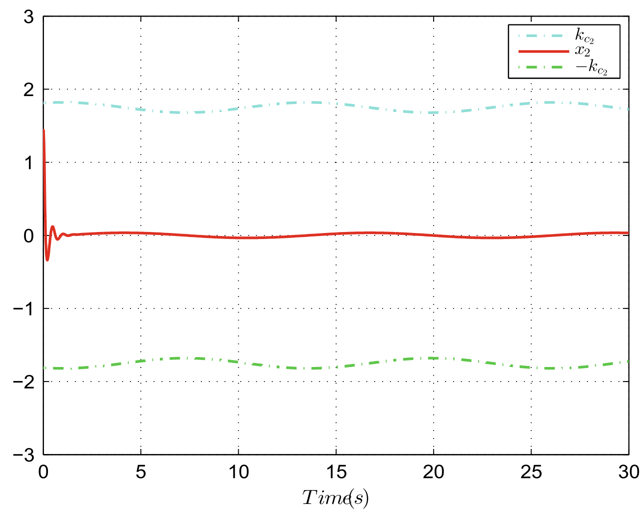

Fig. 2 Trajectory of state x_2

Figures 3 and 4 show the trajectories of the errors z_1 and z_2, respectively, which show that the errors are bounded. Figure 5 displays the trajectories of adaptive parameters θ_1, θ_2. The trajectories of the real controller v and dead-zone input u are shown in Figs. 6 and 7, respectively. The responses of states x_i and their estimations $\hat{x}_i\,(i=1,2)$ are depicted in Figs. 8 and 9, which reveals that system states x_1, x_2 can be estimated effectively by the designed state observer. Apparently, all the simulation results indicate that the proposed control method is effective.

Remark 4 It is noticed that the constrained problems are considered in [25–34] and these results require the states to be measurable; thus, they cannot be directly adopted to control the SISO strict-feedback nonlinear systems with time-varying states constraints. And also, although the authors in [40, 41, 43] designed the observer to estimate unmeasured states, they ignored the negative effect of

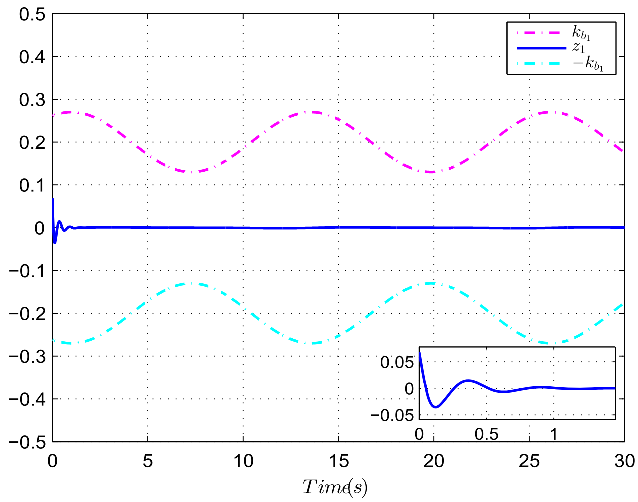

Fig. 3 Trajectory of error z_1

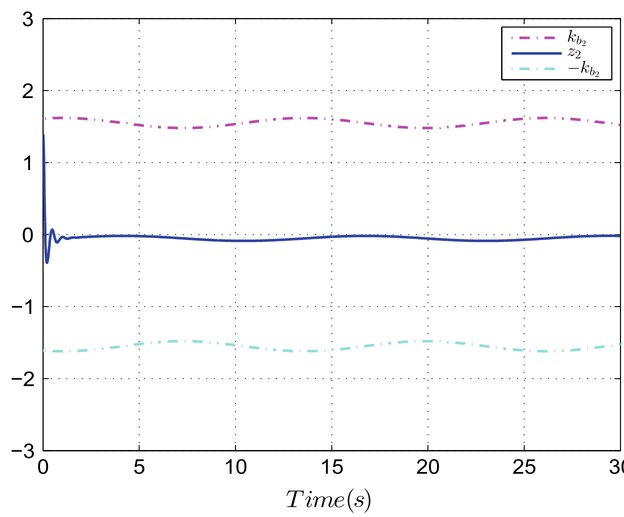

Fig. 4 Trajectory of error z_2

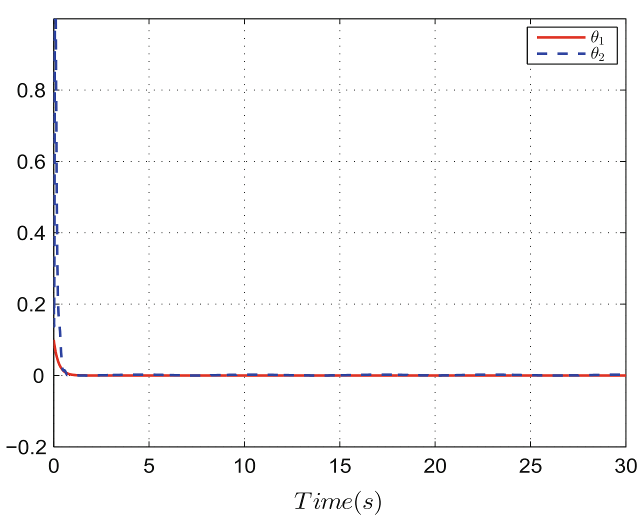

Fig. 5 Trajectories of adaptive parameters θ_1 and θ_2

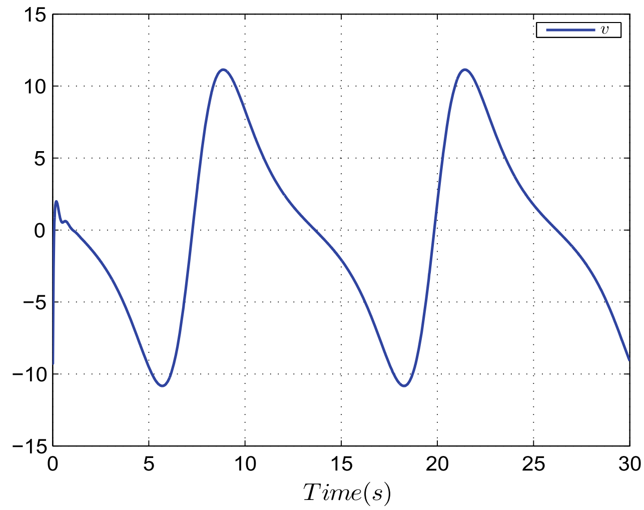

Fig. 6 Trajectory of real controller v

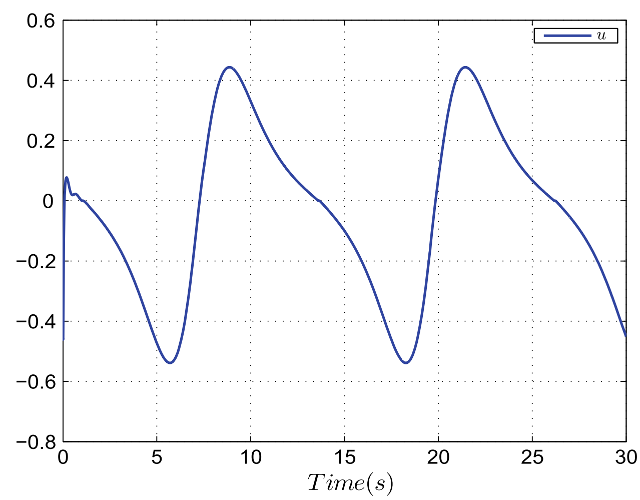

Fig. 7 Trajectory of dead-zone u

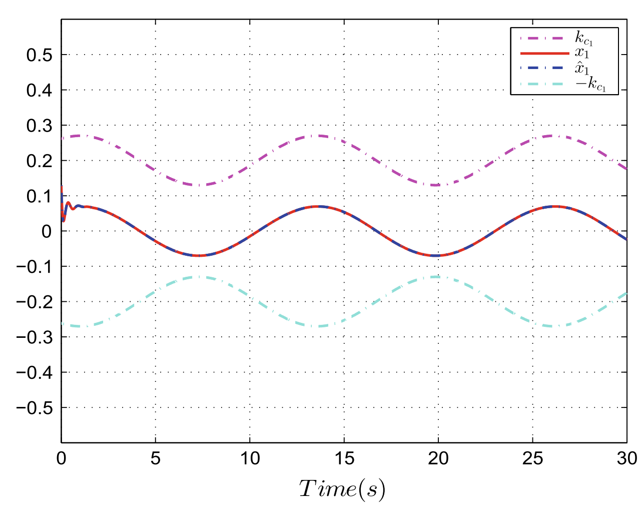

Fig. 8 Trajectories of state x_1 and its estimation $\hat{x}_1$

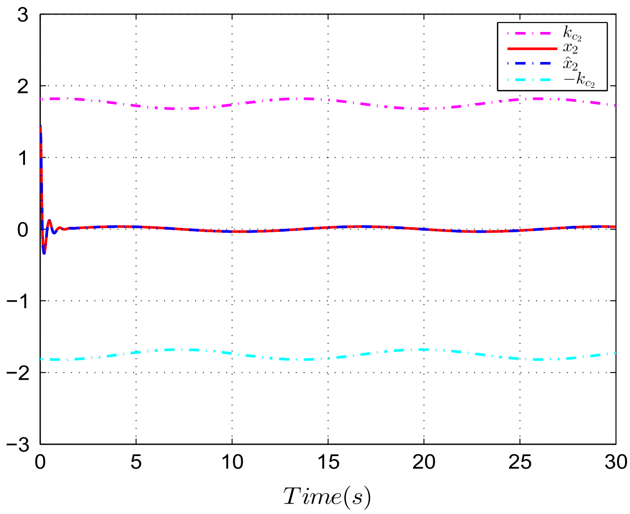

Fig. 9 Trajectories of state x_2 and its estimation $\hat{x}_2$

dead-zone and time-varying constraints. Thus it is difficult to consider two cases concurrently. However, in this paper, we propose an observer-based adaptive fuzzy control scheme, which can be effectively applied to the simulation example.

Remark 5 External disturbances and sensor failures also occur in practical application. There exist some good results to address these issues, for example, the authors in [54] proposed a robust data-driven fault detection scheme. A robust sensor fault detection observer design method was presented in [55] for discrete-time T–S systems using H_-/H_∞ criterion. To further improve the system performance, we will consider these problems in our future work.

6 Conclusion

An observer-based adaptive fuzzy control strategy for SISO time-varying states constrained strict-feedback nonlinear systems subject to dead-zone has been proposed in this paper. The FLSs are used to approximate the unknown nonlinear functions in the observer design procedure. Meanwhile, the TVBLFs are introduced to ensure that all states are confined to the predefined compact sets. Furthermore, according to the adaptive backstepping control technique, the derived controller can effectively guarantee all the signals in the closed-loop system are bounded and the tracking errors can converge to a small neighborhood of zero. Simulation results have been given to illustrate the effectiveness of the proposed control approach.

In our future research, on the basis of arbitrary approximation property of the FLSs and NNs, we will extend the results of this paper to address adaptive finite-time output feedback control problem for stochastic nonlinear large-scale systems subject to more other external factors.

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