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On Stabilization Conditions for T–S Systems with Nonlinear Consequent Parts

Hoda Moodi[1] · Mohammad Farrokhi[2] · Thierry Marie Guerra[3] · Jimmy Lauber[3]

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Abstract This paper deals with T–S fuzzy model with nonlinear consequent parts that has shown to reduce the number of fuzzy rules and decrease modeling error comparing with conventional T–S with linear consequent parts. To further increase the benefits of using such model, many novelties in analyzing and applying it are introduced here. Canceling the nonlinear part of subsystems by fuzzy feedback linearization, using a novel fuzzy non-quadratic Lyapunov function and new relaxation methods for further reduction of conservativeness and maximizing the region of attractions are all discussed in this paper. Numerical examples illustrate the effectiveness of the proposed method.

Keywords Fuzzy control · Takagi–Sugeno model · Nonlinear subsystems · Non-quadratic Lyapunov function

✉ Hoda Moodi
h.moodi@qiet.ac.ir

Mohammad Farrokhi
farrokhi@iust.ac.ir

Thierry Marie Guerra
guerra@univ-valenciennes.fr

Jimmy Lauber
lauber@univ-valenciennes.fr

[1] Department of Electrical Engineering, Quchan University of Technology, 94771-67335, Quchan, Iran

[2] Department of Electrical Engineering, Iran University of Science and Technology, 16846-13114, Narmak, Tehran, Iran

[3] LAMIH UMR CNRS 8201, University of Valenciennes and Hainaut-Cambresis, Le Mont Houy, 59313 Valenciennes, France

1 Introduction

Takagi–Sugeno (T–S) fuzzy model is a universal approximator which can model any smooth nonlinear system with any degree of accuracy. Moreover, the convex structure and linear subsystems of this model allow one to use powerful linear systems tools, such as linear matrix inequalities (LMIs), to analyze and synthesize the T–S fuzzy systems.

Recently, the use of nonlinear local subsystems for T–S model has gained more attention as it decreases the number of rules and, hence, the number and dimensions of LMIs [22, 28]. Therefore, a better accuracy can be obtained using less local models. The key idea of using nonlinear terms in the consequent parts is, in a sense, to decouple the nonlinearities into two parts: a sector nonlinearity approach for the premises and a "local" characterization for the consequences.

Many different types of T–S with nonlinear consequences are introduced in the literature. The most popular one is the polynomial fuzzy system [23]. This model can be built using expansion of the Taylor series [19, 20]. Due to the nature of its consequence parts, a natural choice that appeared to study the stability/stabilization problems was the use of the sum-of-square (SOS) tools [10, 24]. Moreover, combining the polynomial and the fuzzy nature of these models, polynomial fuzzy Lyapunov functions were also proved to be of some interests [2, 10]. Effectively, the reduction of the number of rules associated with a Lyapunov function that exploits the polynomial nature of the consequence parts reduced the conservativeness of the previous studies on the LMI-based conditions. With respect to polynomial T–S fuzzy models, a fuzzy switching control mechanism instead of fuzzy blending mechanism is developed in [5]. It should be noted that in a switched T–S fuzzy model, the consequence could be any function [6]. In [6], stable controllers are designed for a switched T–S fuzzy model, where the consequents are affine nonlinear state dynamic equations. The T–S models with bilinear consequent parts are also discussed in [4].

Another way to construct the T–S systems with nonlinear consequent parts is simply adding a nonlinear term to the linear consequents, as in [18], where the term added is a simple sin function. More advanced works propose the use of sector-bounded functions in the consequent part [7–9, 28]. This type of nonlinear Sugeno model has gained an increasing interest in recent years [14–17, 22].

The work presented in this manuscript follows the same idea, i.e., adding a sector-bounded nonlinear term to the linear consequent part of T–S models. Sector constraint on the nonlinear terms allows designer to use LMIs for analyzing such systems. In order to avoid classical conservativeness of the results for such T–S nonlinear models several new approaches are proposed here. The first one tries to remove the added nonlinear terms realizing a so-called *fuzzy feedback linearization* that can reduce both the computational burden and the conservativeness. Other improvements are made trying to mix several approaches. For example, restricting the family of studied T–S nonlinear models—as for its equivalent T–S linear models—path independency on the consequent part can be used for the Lyapunov function. Hence, a novel non-quadratic fuzzy Lyapunov function is introduced. The so-called domain of attraction and its maximization conditions are derived for the proposed Lyapunov function. Finally, following the results in [3], some transformation properties from [21] can be used in order to introduce new control laws and a Lyapunov function with more degrees of freedom.

The paper is organized as follows. In Sect. 2, the nonlinear Takagi–Sugeno model and its stability conditions are described. In Sect. 3, the proposed methods are introduced. Simulation results are given in Sect. 4. Section 5 concludes the paper.

2 Preliminaries and Notations

Consider a class of nonlinear systems described by

$$\dot{x}(t) = f_a\big(x(t)\big) + f_b\big(x(t)\big)\varphi\big(x(t)\big) + g\big(x(t)\big)u(t) \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state, $u(t) \in \mathbb{R}^{n_u}$ is the control input, $f_n\big(x(t)\big) : n \in \{a, b\}$ and $g\big(x(t)\big) \in \mathbb{R}^{(n_x \times n_u)}$ are nonlinear functions and $\varphi\big(x(t)\big)$ is a vector of sector-bounded continuous nonlinear functions satisfying the following cone condition:

$$\varphi_i\big(x(t)\big) \in \mathrm{co}\{E_{Li}x(t), E_{Ui}x(t)\}, \quad 1 \le i \le n_\varphi \tag{2}$$

where $E_{Li} \in \mathbb{R}^{1\times n_x}$ and $E_{Ui} \in \mathbb{R}^{1\times n_x}$. This implies that the nonlinear term $\varphi_i\big(x(t)\big)$ is bounded inside cone (2). It is always possible to find a basis transformation such that

$$\varphi_i\big(x(t)\big) \in co\{0, E_i x(t)\}, \quad 1 \le i \le n_\varphi \tag{3}$$

This results [9]

$$\varphi_i\big(x(t)\big)\Big(E_i x(t) - \varphi_i\big(x(t)\big)\Big) \ge 0. \tag{4}$$

Defining $E = \begin{bmatrix} E_1^{\mathrm{T}} & E_2^{\mathrm{T}} & \dots & E_{n_\varphi}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ and considering a diagonal matrix $\Gamma > 0$, it immediately follows that

$$\varphi^{\mathrm{T}}\big(x(t)\big)\Gamma^{-1}Ex(t) - \varphi^{\mathrm{T}}\big(x(t)\big)\Gamma^{-1}\varphi\big(x(t)\big) \ge 0. \tag{5}$$

Note that to conclude (5) from (4), without extra properties on $\varphi\big(x(t)\big), \Gamma$ has to be diagonal, although it might be a source of conservativeness.

2.1 Nonlinear Sugeno Model

System (1) can be represented by the following T–S fuzzy model with nonlinear subsequents:

Plant Rule i :
IF $z_1(t)$ is $M_{i1}(z), \dots,$ and $z_p(t)$ is $M_{ip}(z)$ THEN :

$$\dot{x}(t) = A_i x(t) + G_{xi}\varphi\big(x(t)\big) + B_i u(t) \tag{6}$$

where $A_i \in \mathbb{R}^{(n_x\times n_x)}$, $B_i \in \mathbb{R}^{(n_x\times n_u)}$ and $G_{xi} \in \mathbb{R}^{(n_x\times n_\varphi)}$ $(i = 1, \dots, r)$ are constant matrices, in which r is the number of rules, n_x is the number of states, n_u is the number of inputs and n_φ is the number of nonlinear functions in vector $\varphi(x)$. Moreover, $z_1(t), \dots, z_p(t)$ are the so-called premise variables and $M_{ij}(j = 1, \dots, p)$ denote the fuzzy sets. Therefore, a more compact representation of the fuzzy system is

$$\dot{x}(t) = \sum_{i=1}^{r} \omega_i(z)\big[A_i x(t) + G_{xi}\varphi\big(x(t)\big) + B_i u(t)\big] \tag{7}$$

where

$$\omega_i(z) = \frac{h_i(z)}{\sum_{k=1}^{r} h_k(z)}, \quad h_i(z) = \Pi_{j=1}^{p}\mu_{ij}(z). \tag{8}$$

and $\mu_{ij}(z)$ is the grade of membership of z_j in M_{ij}.

Note that as $\varphi\big(x(t)\big)$ is a vector of known functions, instead of a vector of uncertainties, therefore it can be used in controller (9) or in the Lyapunov function. Introducing this “extra” information into the design of the control, it is expected that the quality of the results is improved. This is shown in [9] and in the examples provided thereinafter.

2.2 Controller Structure

For system (6) the following controller is suggested [9]:

Controller Rule i:

IF $z_1(t)$ is $M_{i1}(z), \ldots,$ and $z_p(t)$ is $M_{ip}(z)$ THEN :

$$u(t) = K_{ai}x(t) + K_{bi}\varphi\big(x(t)\big). \tag{9}$$

where K_{ai} and $K_{bi}i = (1, \ldots, r)$ are controller gains with proper dimensions. From (6) and (9), the closed-loop fuzzy system is obtained in the following form:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i(z)\omega_j(z)\big[(A_i + B_iK_{aj})x(t) + (G_{xi} + B_iK_{bj})\varphi\big(x(t)\big)\big]. \tag{10}$$

The following lemmas will be useful to derive the main results.

Lemma 1 [26] *If the following conditions hold:*

$$\begin{aligned} &\Xi_{ii} < 0, \quad 1 < i < r \\ &\frac{1}{r-1}\Xi_{ii} + \frac{1}{2}(\Xi_{ij} + \Xi_{ji}) < 0, \quad 1 < i \neq j < r \end{aligned} \tag{11}$$

then, the double convex sum negativity problem holds:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_i\alpha_j\Xi_{ij} < 0 \tag{12}$$

where $0 \leq \alpha_i \leq 1$ *and* $\sum_{i=1}^{r}\alpha_i = 1$.

Next, let us concern writing LMI conditions for those that are expressed as multiple-summations negativity problems. In this case a more general relaxation will be used.

Lemma 2 [13] *Let* $P(i_1, i_2, i_3, i_4, i_5)$ *be the set of all permutations of indexes* i_1, i_2, i_3, i_4 *and* i_5*; thus, the following conditions*

$$\begin{aligned} &\Xi_{i_1i_1i_1i_1i_1} < 0 \\ &\sum_{s\in P(i_1i_2i_2i_2i_2)}\Xi_s < 0, i_1 \neq i_2 \\ &\sum_{s\in P(i_1i_2i_3i_3i_3)}\Xi_s < 0, i_1 \neq i_2, i_1 \neq i_3, i_2 \neq i_3 \\ &\sum_{s\in P(i_1i_2i_3i_4i_4)}\Xi_s < 0, \; i_1 \neq i_2, i_1 \neq i_3, i_1 \neq i_4, i_2 \neq i_3, i_2 \neq i_4, i_3 \neq i_4 \\ &\sum_{s\in P(i_1i_2i_3i_4i_5)}\Xi_s < 0, \; i_1 \neq i_2, i_1 \neq i_3, i_1 \neq i_4, i_1 \neq i_5, i_1 \neq i_5, i_3 \neq i_4, i_3 \neq i_5, i_4 \neq i_5, i_2 \neq i_3, i_2 \neq i_4, \end{aligned} \tag{13}$$

imply

$$\sum_{i_1=1}^{r}\sum_{i_2=1}^{r}\sum_{i_3=1}^{r}\sum_{i_4=1}^{r}\sum_{i_5=1}^{r}\alpha_{i_1}\alpha_{i_2}\alpha_{i_3}\alpha_{i_4}\alpha_{i_5}\Xi_{i_1i_2i_3i_4i_5} < 0 \tag{14}$$

where $0 \leq \alpha_i \leq 1$ *and* $\sum_{i=1}^{r}\alpha_i = 1$.

In the following lemma, sufficient conditions for stability of the closed-loop dynamical system (10) are recalled.

Lemma 3 [9] *If there exist matrices* $P = P^{\mathrm{T}} > 0, X_{ai}$ *and* $X_{bi}, 1 \leq i \leq r$*, and diagonal matrix* $\Gamma > 0$ *such that* (11) *is satisfied with* Ξ_{ij} *defined as*

$$\Xi_{ij} = \begin{bmatrix} \mathrm{He}(A_iP + B_iX_{aj}) & * \\ \Gamma G_{xi}^{\mathrm{T}} + X_{bj}^{\mathrm{T}}B_i^{\mathrm{T}} + EP & -2\Gamma \end{bmatrix} \tag{15}$$

then, fuzzy system (6) is asymptotically stable using controller (9) with

$$K_{ai} = X_{ai}P^{-1} \quad K_{bi} = X_{bi}\Gamma^{-1} \quad 1 \leq i \leq r. \tag{16}$$

In (15), $\mathrm{He}(A)$ denotes the Hermite form of matrix A and $*$ indicates the symmetric term.

Remark 1 To introduce a decay rate β, just replace the first block of (15) with: He $(A_iP + B_iX_{aj}) + \beta P$. This results that if $V(x(t))$ is a Lyapunov function for system (10), then $\dot{V}(x(t)) < -\beta V(x(t)))$.

3 Main Results

To further improve the results obtained by controller (9) and to reach the results on more general types of nonlinear terms in the consequents, some novel ideas are introduced in this section. First, an idea based on feedback linearization approach is introduced. Second the vector of nonlinearities, $\varphi\big(x(t)\big)$, is introduced into an extended Lyapunov function keeping constant matrices. It is shown that both of the mentioned ideas can omit the necessity of sector constraint (2). Then, the extra term added in the Lyapunov function is fuzzified and the outermost Lyapunov level for such fuzzy Lyapunov function is obtained. At last the concept of multiple indexes in Lemma 2 is utilized to further reduce the conservativeness in the design.

3.1 Feedback Linearization

The numerical complexity of LMI conditions is closely related to the number of lines ($\mathcal{L}$) and decision variables ($\mathcal{D}$). I.e., the LMI can be solved in polynomial time with complexity proportional to $\mathcal{D}^3\mathcal{L}$ [9]. Since T–S models with nonlinear consequent parts have fewer numbers of fuzzy IF-THEN rules, the computational burden is lowered according to ($\mathcal{L}$). However, for $\mathcal{D}$, as controller (9) introduces the extra decision variables K_{bi}, it also increases the complexity. A way to face the last complexity problem is to use the gains K_{bi} to compensate, if possible, the nonlinear terms in order that they "disappear" from the future LMI constraints problem. In this case, the output of such system is similar, but not exactly equal to a T–S model with linear consequent (linear T–S) without the nonlinear term. The error is due to the cross-effects of subsystems on each other in a fuzzy system. To explain it in another way, the "classical" T–S model, i.e., with linear consequent part, would have split the terms $\varphi_i(x(t))$ via sector nonlinearity approach. In comparison, the linear consequent T–S, obtained after local linearization, cancels the terms $\varphi_i(x(t))$ and therefore gives a simplified T–S model that is expected to give less conservative results. A way to cancel G_{xi} is to solve

$$G_{xi} + B_i K_{bi} = 0 \tag{17}$$

Pseudo-inverse gives

$$K_{bi} = -(B_i^{\mathrm{T}} B_i)^{-1} B_i^{\mathrm{T}} G_{xi} \tag{18}$$

which is the least-square estimation of the solution. Note that as $B_i K_{bj}\varphi(x(t))$ terms also exist in the model and no constraints exist on their value, the nonlinearities do not vanish in the whole fuzzy system. Then, the following theorem can be derived.

Theorem 1 *If there exist matrices $P = P^{\mathrm{T}} > 0, X_{ai}, 1 \le i \le r$, and diagonal matrix $\Gamma > 0$ such that (11) is satisfied with*

$$\Xi_{ij} = \begin{bmatrix} \mathrm{He}(A_i P + B_i X_{aj}) & * \\ \Gamma(G_{xi} + B_i K_{bj})^{\mathrm{T}} + EP & -2\Gamma \end{bmatrix} \tag{19}$$

then, fuzzy system (6) is asymptotically stable via controller (9) with $K_{ai} = X_{ai}P^{-1}$ and K_{bi} defined in (18).

Proof Proof is easily obtained by replacing X_{bj} from (16) into (15). □

Remark 2 If (17) holds for all $i \in (1, \ldots, r)$, then

$$G_{xi} + B_i K_{bj} + G_{xj} + B_j K_{bi} = (B_i - B_j)(K_{bj} - K_{bi}) \tag{20}$$

In this case, if $B_i = B_j$ or $K_{bi} = K_{bj}$, the term in (20) will be null and hence, if $\varphi(x(t))$ "disappeared" from the closed-loop model or if no disturbance exists in the model, then no sector constraints on $\varphi(x(t))$ are required and $\Xi_{ij} = \mathrm{He}(A_i P + B_i X_{aj})$ can replace Ξ_{ij} definition in Theorem 1.

Remark 3 Another way to reduce the conservatism of conditions appears in case of trying to write as for a perfect cancelation of the term in $\varphi(x(t))$, i.e.,

$$G_{xz} + B_z K_{bz} = 0 \tag{21}$$

where $\mathcal{X}_z = \sum_{i=1}^{r} \omega_i(z)\mathcal{X}_i, \mathcal{X} \in \{B, K_b, G_x\}$. For (3), the pseudo-inverse will give the best result in the sense of least square:

$$K_b(z) = -(B_z^{\mathrm{T}} B_z)^{-1} B_z^{\mathrm{T}} G_{xz}. \tag{22}$$

Notice that the solution is denoted with $K_b(z)$ and not K_{bz}, because (22) is not a simple sum depending on $h_i(z)$. However, (3) can be reinforced by minimizing $||G_{xz} + B_z K_{bz}||$. With the help of Schur's complement, it can be written as

$$\begin{aligned} &\min \quad \varepsilon \\ &\text{subject to}: \quad \begin{bmatrix} \varepsilon I & (G_{xz} + B_z K_{bz}) \\ * & I \end{bmatrix} > 0 \end{aligned} \tag{23}$$

Lemma 1 can be used for relaxation with $\Xi_{ij} = \begin{bmatrix} \varepsilon I & (G_{xi} + B_i K_{bj}) \\ * & I \end{bmatrix}$.

3.2 Non-quadratic Stabilization

Following works for the classical T–S systems or recurrent neural networks [12], a way to reduce conservatism of Theorem 1 is to introduce non-quadratic (NQ) Lyapunov functions via added extra positive terms. The choice made in our work is to use a path-independent Lyapunov function on the nonlinear part of the consequences:

$$V(x, \varphi(x)) := x^{\mathrm{T}}(t)P^{-1}x(t) + 2\alpha \int_0^x \varphi^{\mathrm{T}}(y)\Gamma^{-1}E\,\mathrm{d}y. \tag{24}$$

where α is an arbitrary value. To use this Lyapunov function, the family of nonlinear T–S is reduced to the one fulfilling the so-called path-independent conditions., i.e., the following assumption is satisfied.

Assumption 1 *$\varphi(x(t))$ and E are such that*

$$\frac{d}{dt}\left(\int_0^x \varphi^{\mathrm{T}}(y)\Gamma^{-1}E\,dy\right) = \varphi^{\mathrm{T}}(x)\Gamma^{-1}E\dot{x}. \tag{25}$$

Using Lyapunov function (24) the following lemma is stated.

Lemma 4 [17] *Considering system (6) under Assumption (1) and control (9), define the matrices $P = P^{\mathrm{T}} > 0, X_{ai}, X_{bi}, 1 \le i \le r$, diagonal matrix $\Gamma > 0$ and a scalar α and:*

$$\Xi_{ij} = \begin{bmatrix} \mathrm{He}(A_i P + B_i X_{aj}) & * \\ \Xi_{ij}^{21} & \Xi_{ij}^{22} \end{bmatrix} \tag{26}$$

where

$$\begin{aligned}\Xi_{ij}^{21} &= \Gamma G_{xi}^{\mathrm{T}} + X_{bj}^{\mathrm{T}}B_i^{\mathrm{T}} + EP + \alpha E(A_iP + B_iX_{aj})\\ \Xi_{ij}^{22} &= \mathrm{He}(-\Gamma + \alpha E(G_{xi}\Gamma + B_iX_{bj}))\end{aligned} \tag{27}$$

If one of the following problems:

1. $\alpha > 0$ *and* (11) *with* Ξ_{ij} *defined in* (26)
2. $\begin{bmatrix} P & * \\ EP & -\frac{1}{\alpha}\Gamma \end{bmatrix} > 0$ *with* Ξ_{ij} *defined in* (26)

has a solution, then the closed-loop system is stable and the controller gains are obtained as in (16).

Remark 4 If by changing Ξ_{ij} in (27) as

$$\begin{aligned}\Xi_{ij}^{22} &= \mathrm{He}(\alpha E(G_{xi}\Gamma + B_iX_{bj}))\\ \Xi_{ij}^{21} &= \Gamma G_{xi}^{\mathrm{T}} + X_{bj}^{\mathrm{T}}B_i^{\mathrm{T}} + \alpha E(A_iP + B_iX_{aj})\end{aligned} \tag{28}$$

$\Xi_{ij} < 0$ still holds, then the condition of sector boundedness of $\varphi(x(t))$ is not necessary for making $\dot{V}(x, \varphi(x)) < 0$. This can be interesting to omit this condition as it is a source of conservativeness. However, to guarantee positiveness of the Lyapunov function, a milder condition on $\varphi(x(t))$ must be satisfied. That is, $\varphi^{\mathrm{T}}(x(t))Ex(t) > 0$. This new condition is obviously less conservative than the sector boundedness condition (5).

3.3 Extension to Fuzzy Decision Variables

Previous sections showed improvements considering a non-quadratic Lyapunov function. Nevertheless, in that case a unique matrix Γ remains for all subsystems. A way to introduce extra degrees of freedom on the Lyapunov function and in the control law, is employing Γ_z instead of Γ. In order to succeed, some transformations and a new control law have to be considered:

$$u = F_{az}H_z^{-1}x + F_{bz}\tilde{H}_z^{-1}\varphi(x). \tag{29}$$

where F_{az}, F_{bz}, H_z and $\tilde{H}_z$ are gain matrices with proper dimensions. Then, the equation of the closed-loop system will be

$$\begin{aligned}\dot{x}(t) &= (A_z + B_zF_{az}H_z^{-1})x(t) + (G_{xz} + B_zF_{bz}\tilde{H}_z^{-1})\varphi(x)\\ &= \bar{A}_zx(t) + \bar{G}_{xz}\varphi(x)\end{aligned} \tag{30}$$

Next, consider the following fuzzy Lyapunov function:

$$V(x, \varphi(x)) = x(t)^{\mathrm{T}}P^{-1}x(t) + 2\alpha\int_0^x \varphi^{\mathrm{T}}(y)\Gamma_z^{-1}Edy \tag{31}$$

where Γ_z is a diagonal matrix. In order to guarantee that the integrand in (31) has a symmetric Jacobian matrix and to use the property of path independency, Γ_z is defined as follows:

$$\Gamma_z = \sum_{i=1}^{r}\omega_i(z)\mathrm{diag}([\Gamma_1^{\alpha_{i1}}, \ldots, \Gamma_{n_\varphi}^{\alpha_{in_\phi}}]) \tag{32}$$

where $\alpha_{ij} \in [1, \ldots, r_j]$ specifies which x_k-based fuzzy set is used in the ith fuzzy rule, while x_k is the state that $\varphi_j(x)$ depends on and r_j is the number of x_k-based fuzzy sets. It should be noted that according to (32), Γ_j varies based on the fuzzy rules only if x_k is used as a premise variable. Hence, by choosing a proper order of the nonlinear terms in the vector $\varphi(x(t))$ it is possible to have symmetric Jacobian matrix for the integrand in (31).

As a result, the derivative of Lyapunov function (31) can be written as

$$\begin{aligned}\dot{V}(x, \varphi(x)) &= 2x(t)^{\mathrm{T}}P^{-1}\dot{x}(t) + 2\alpha\varphi^{\mathrm{T}}(x(t))\Gamma_z^{-1}E\dot{x}(t)\\ &= \begin{bmatrix} x(t) \\ \varphi(x(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathrm{He}(P^{-1}\bar{A}_z) & P^{-1}\bar{G}_{xz} + \alpha\bar{A}_z^{\mathrm{T}}E^{\mathrm{T}}\Gamma_z^{-1} \\ * & \alpha(\Gamma_z^{-1}E\bar{G}_{xz} + \bar{G}_z^{\mathrm{T}}E^{\mathrm{T}}\Gamma_z^{-T}) \end{bmatrix}\\ &\quad \times \begin{bmatrix} x(t) \\ \varphi(x(t)) \end{bmatrix}\end{aligned} \tag{33}$$

For the nonlinear term $\varphi(x)$, sector condition (5) can be rewritten as

$$\begin{bmatrix} x(t) & \varphi(x(t)) \end{bmatrix} \begin{bmatrix} 0 & * \\ \Gamma_z^{-1}E & -2\Gamma_z^{-1} \end{bmatrix} \begin{bmatrix} x(t) \\ \varphi(x(t)) \end{bmatrix} > 0. \tag{34}$$

Based on the S-procedure [27], $\dot{V}(x, \varphi(x)) < 0$ if the following condition is satisfied:

$$\begin{aligned}&\begin{bmatrix} x(t) \\ \varphi(x(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathrm{He}(P^{-1}\bar{A}_z) & P^{-1}\bar{G}_{xz} + \alpha\bar{A}_z^{\mathrm{T}}E^{\mathrm{T}}\Gamma_z^{-1} + E^{\mathrm{T}}\Gamma_z^{-1} \\ * & \alpha(\Gamma^{-1}E\bar{G}_{xz} + \bar{G}_{xz}^{\mathrm{T}}E^{\mathrm{T}}\Gamma_z^{-1}) - 2\Gamma_z^{-1} \end{bmatrix}\\ &\times \begin{bmatrix} x(t) \\ \varphi(x(t)) \end{bmatrix} < 0.\end{aligned} \tag{35}$$

Or equivalently

$$\begin{aligned}&\begin{bmatrix} P^{-1} & 0 \\ 0 & \Gamma_z^{-1} \end{bmatrix} \begin{bmatrix} \bar{A}_z & \bar{G}_{xz} \\ \alpha E\bar{A}_z + E & \alpha E\bar{G}_{xz} - I \end{bmatrix} + * < 0\\ &\equiv \tilde{P}_z^{-1}\tilde{A}_z + * < 0\end{aligned} \tag{36}$$

Now, following the works of [13, 21], let us multiply the left- and right-hand sides by $\tilde{P}_z$ and considering a small enough ε, (36) holds if

$$\mathrm{He}(\tilde{A}_z\tilde{P}_z) + \varepsilon\tilde{A}_z\tilde{P}_z\tilde{A}_z^{\mathrm{T}} < 0 \tag{37}$$

which is equivalent to

$$\mathrm{He}(\varepsilon\tilde{A}_z\tilde{P}_z) + \varepsilon^2\tilde{A}_z\tilde{P}_z\tilde{A}_z^{\mathrm{T}} + \tilde{P}_z - \tilde{P}_z < 0 \tag{38}$$

or

$$(I + \varepsilon\tilde{A}_z)\tilde{P}_z(*) - \tilde{P}_z < 0. \tag{39}$$

Using the Schur complement, (39) is equivalent to

$$\begin{bmatrix} \tilde{P}_z & I + \varepsilon\tilde{A}_z \\ * & \tilde{P}_z^{-1} \end{bmatrix} > 0. \tag{40}$$

Multiplying both sides of (40) by the full-rank matrix $\begin{bmatrix} I & 0 \\ 0 & \bar{H}_z^{\mathrm{T}} \end{bmatrix}$ gives

$$\begin{bmatrix} \tilde{P}_z & \bar{H}_z + \varepsilon(\tilde{A}_z\bar{H}_z) \\ * & \bar{H}_z^{\mathrm{T}}\tilde{P}_z^{-1}\bar{H}_z \end{bmatrix} > 0. \tag{41}$$

Using inequality $(\bar{H}_z - \tilde{P}_z)^{\mathrm{T}}\tilde{P}_z^{-1}(\bar{H}_z - \tilde{P}_z) \geq 0$, it yields $\bar{H}_z^{\mathrm{T}}\tilde{P}_z^{-1}\bar{H}_z + \tilde{P}_z - \bar{H}_z^{\mathrm{T}} - \bar{H}_z \geq 0$. Therefore, (41) holds if

$$\begin{bmatrix} \tilde{P}_z & \bar{H}_z + \varepsilon(\tilde{A}_z\bar{H}_z) \\ * & \bar{H}_z + \bar{H}_z^{\mathrm{T}} - \tilde{P}_z \end{bmatrix} > 0 \tag{42}$$

where

$$\begin{aligned} \tilde{A}_z\bar{H}_z &= \begin{bmatrix} \bar{A}_z & \bar{G}_{xz} \\ \alpha E\bar{A}_z + E & \alpha E\bar{G}_{xz} - I \end{bmatrix}\begin{bmatrix} H_z & 0 \\ 0 & \tilde{H}_z \end{bmatrix} \\ &= \begin{bmatrix} (A_zH_z + B_zF_{az}) & (G_{xz}\tilde{H}_z + B_zF_{bz}) \\ \alpha E(A_zH_z + B_zF_{az}) + EH_z & \alpha E(G_{xz}\tilde{H}_z + B_zF_{bz}) - \tilde{H}_z \end{bmatrix} \end{aligned} \tag{43}$$

Therefore, $\dot{V}(x, \varphi(x)) < 0$ if the following inequality is satisfied:

$$\begin{bmatrix} \begin{bmatrix} P & 0 \\ 0 & \Gamma_z \end{bmatrix} & \begin{bmatrix} H_z & 0 \\ 0 & \tilde{H}_z \end{bmatrix} + \varepsilon\tilde{A}_z\bar{H}_z \\ * & \begin{bmatrix} H_z + H_z^{\mathrm{T}} - P & 0 \\ 0 & \tilde{H}_z + \tilde{H}_z^{\mathrm{T}} - \Gamma_z \end{bmatrix} \end{bmatrix} > 0 \tag{44}$$

Then, the following theorem can be stated.

Theorem 2 *If there exist matrix $P = P^{\mathrm{T}} > 0$, diagonal matrices $\Gamma_i > 0$ and matrices F_{ai} and $F_{bi}, 1 \leq i \leq r$, and $\varepsilon > 0$ such that (11) holds with*

$$\Xi_{ij} = \begin{bmatrix} \begin{bmatrix} P & 0 \\ 0 & \Gamma_i \end{bmatrix} & \begin{bmatrix} H_j & 0 \\ 0 & \tilde{H}_j \end{bmatrix} + \varepsilon\tilde{A}_{ij}\bar{H}_j \\ * & \begin{bmatrix} H_j + H_j^{\mathrm{T}} - P & 0 \\ 0 & \tilde{H}_j + \tilde{H}_j^{\mathrm{T}} - \Gamma_i \end{bmatrix} \end{bmatrix} > 0 \tag{45}$$

where

$$\begin{aligned} &\tilde{A}_{ij}\bar{H}_j \\ &= \begin{bmatrix} (A_iH_j + B_iF_{aj}) & (G_{xi}\tilde{H}_j + B_iF_{bj}) \\ \alpha E(A_iH_j + B_iF_{aj}) + EH_j & \alpha E(G_{xi}\tilde{H}_j + B_iF_{bj}) - \tilde{H}_j \end{bmatrix} \end{aligned} \tag{46}$$

then, fuzzy system (6) is asymptotically stable when using controller (29).

Proof Applying Lemma 1 to (44) concludes the proof.□

Remark 5 Note that (44) is not strictly LMI in parameters ε and α. To solve this problem, methods such as grid and/or line search can be used. Nevertheless, for the first scalar (ε), it is possible to fix a finite number of choices [13]. This finite number is generally chosen in a logarithmically spaced set: $\begin{bmatrix} 10^{-6} & 10^{-5}\ldots & 10^{6} \end{bmatrix}$. For the second scalar (α), although line search could be used, it appears that due to the two possible LMI problems concerned with its sign, it is usually enough to check for just two values, 1 and − 1.

Remark 6 When the compact set of the definition of the T–S model is not $\mathbb{R}^n$, only the local stability comes at hand. Hence, a very crucial point is to find the largest so-called region of attraction. In this case, the following lemma is useful.

Lemma 5 *In Theorem 2, a lower and an upper estimation of the maximum region of attraction in the polytope $\mathbb{P} = \{x \in \mathbb{R}^n,\ a_k^{\mathrm{T}}x \leq 1,\ k = 1, \ldots, q\}$ can be obtained via the following optimization problem:*

$$\begin{aligned} &\max\ \lambda \\ &\text{subject to}: \begin{bmatrix} P - \lambda I & PE^{\mathrm{T}} \\ EP & (1/\alpha)\Gamma_i + EPE^{\mathrm{T}} \end{bmatrix} > 0 \\ &\text{and} \begin{bmatrix} 1 & a_k^{\mathrm{T}} \\ a_k & P \end{bmatrix} > 0 \end{aligned} \tag{47}$$

Proof The proof is similar to the proof of Theorem 3 in [17]. Note that in contrast to [17] the maximum region of attraction is exhibited by a shaded area resulted from the multiple Lyapunov functions used.

Remark 7 In order to further reduce the conservativeness, Theorem 2 can be easily expanded for a PDC law F_z to F_{zzzz} using multiple indexes, and the new control law will be:

$$u = F_{az\cdots z}H_{z\cdots z}^{-1}x + F_{bz\cdots z}\tilde{H}_{z\cdots z}^{-1}\varphi(x). \tag{48}$$

where

$$F_{zzzz} = \sum_{i_1=1}^{r}\sum_{i_2=1}^{r}\sum_{i_3=1}^{r}\sum_{i_4=1}^{r} \omega_{i_1}(z)\omega_{i_2}(z)\omega_{i_3}(z)\omega_{i_4}(z)F_{i_1 i_2 i_3 i_4} \tag{49}$$

In this case Eq. (13) is used with $\Xi_{i_1 i_2 i_3 i_4 i_5}$ defined in (50).

$$\Xi_{i_1 i_2 i_3 i_4 i_5} = \begin{bmatrix} \begin{bmatrix} P & 0 \\ 0 & \Gamma_{i_1} \end{bmatrix} & \begin{bmatrix} H_{i_2 i_3 i_4 i_5} & 0 \\ 0 & \tilde{H}_{i_2 i_3 i_4 i_5} \end{bmatrix} + \varepsilon \tilde{A}_{i_1 i_2 i_3 i_4 i_5}\bar{H}_{i_2 i_3 i_4 i_5} \\ * & \begin{bmatrix} H_{i_2 i_3 i_4 i_5} + H^{\mathrm{T}}_{i_2 i_3 i_4 i_5} - P & 0 \\ 0 & \tilde{H}_{i_2 i_3 i_4 i_5} + \tilde{H}^{\mathrm{T}}_{i_2 i_3 i_4 i_5} - \Gamma_{i_1} \end{bmatrix} \end{bmatrix} > 0 \tag{50}$$

where

$$\tilde{A}_{i_1 i_2 i_3 i_4 i_5}\bar{H}_{i_2 i_3 i_4 i_5} = \begin{bmatrix} (A_{i_1}H_{i_2 i_3 i_4 i_5} + B_{i_1}F_{ai_2 i_3 i_4 i_5}) & (G_{xi_1}\tilde{H}_{i_2 i_3 i_4 i_5} + B_{i_1}F_{bi_2 i_3 i_4 i_5}) \\ \alpha E(A_{i_1}H_{i_2 i_3 i_4 i_5} + B_{i_1}F_{ai_2 i_3 i_4 i_5}) + EH_{i_2 i_3 i_4 i_5} & \alpha E(G_{xi_1}\tilde{H}_{i_2 i_3 i_4 i_5} + B_{i_1}F_{bi_2 i_3 i_4 i_5}) - \tilde{H}_{i_2 i_3 i_4 i_5} \end{bmatrix} \tag{51}$$

Proof It follows directly from (29) to (44) using Lemma 2.

4 Simulations

In this section, four examples are given to show the effectiveness of the new techniques proposed in this paper. Example 1 is presented to illustrate that the feedback linearization technique can effectively lighten the computational burden and in some cases can relax the requirement for any sector condition. Example 2 also shows the effectiveness of the proposed feedback linearization technique in a practical situation. In Example 3, the effectiveness of the new fuzzy non-quadratic Lyapunov function is illustrated. It is shown that the proposed method can design fuzzy controllers with less conservatism than the conventional methods. In Example 4, the effects of the proposed fuzzy Lyapunov function with the multiple indexes are shown.

All LMIs are solved using YALMIP [11] as a parser and SeDuMi as a solver.

Example 1 Consider system (6) with the following matrices [9]:

$$A_1 = \begin{pmatrix} -5 & -3 \\ 0 & 4 \end{pmatrix}, A_2 = \begin{pmatrix} -2.6 & -2 \\ 1 & -4 \end{pmatrix}$$
$$B_1 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, B_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \ \varphi(x) = (1 - \cos(x_1))\sin(x_1)$$
$$G_{x1} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \ G_{x2} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}, \ E = (\,2/\pi 0\,). \tag{52}$$

For this system, the feedback gains K_{bi} can be calculated using (18), which results $K_{b1} = K_{b2} = -2$. Theorem 1 results $\Gamma = 0.0718$. It is obvious that (17) also holds for this system. Hence, based on Remark 2, no sector constraint on the nonlinear term is needed here. If $\varphi(x)$ changes, by increasing the sector limit E, Lemma 3 becomes infeasible, while Remark 2 is always feasible.

In [9], it is shown how the nonlinear T–S decreases the computational burden of the CPU time and the corresponding numerical complexity index $\mathcal{C} = \mathcal{D}^3\mathcal{L}$ in comparison with the linear T–S for this example. Yet, more reduction is possible using the proposed approach in this paper. For system (52), the CPU time and the numerical complexity index using Lemma 3, Theorem 1 and Remark 2 are shown in Table 1, while the feedback gains are almost the same for all methods.

Example 2 Consider the problem of stabilizing an inverted pendulum with the following model [25]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{-f(M+m)x_2 - (mlx_2)^2\sin(x_1)\cos(x_1)}{(M+m)(J+ml^2) - (ml\cos(x_1)^2)} \\ &\quad + \frac{(M+m)mgl\sin(x_1) - ml\cos(x_1)u}{(M+m)(J+ml^2) - (ml\cos(x_1)^2)} \\ y &= x_1 \end{aligned} \tag{53}$$

where x_1 is the pendulum angle (rad), x_2 is its rotational speed (rad/s), $m = 3$ (kg) is the pendulum mass, $M = 15$ (kg) is the cart mass, $l = 0.3$ (m) is the pendulum length, $J = 0.005$ (kg m^2) is the moment of inertia, $f = 0.007$ (N/rad/s) is the friction constant, $g = 9.8$ (m/s^2) is the gravitational acceleration, and u is the force (N) applied to the cart. Supposing $\varphi(x) = \sin(x_1) - \frac{2}{\pi}x_1$ and choosing $\cos(x_1)$ as the premise variable with three set points $[1, 0, -1]$, fuzzy model (6) is obtained with the following matrices:

$$A_1 = \begin{pmatrix} 0 & 1 \\ 24.9555 & -0.0311 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 1 \\ 20.4165 & -0.0255 \end{pmatrix}$$
$$A_3 = \begin{pmatrix} 0 & 1 \\ 24.9555 & -0.0311 \end{pmatrix}$$
$$B_1 = \begin{pmatrix} 0 \\ -0.2222 \end{pmatrix}, B_2 = \begin{pmatrix} 0 \\ -0.0018 \end{pmatrix}, B_3 = \begin{pmatrix} 0 \\ 0.2222 \end{pmatrix}$$

$$G_{x1} = (\,0 \quad 39.2\,), G_{x2} = (\,0 \quad 32.0733\,), G_{x3} = (\,0 \quad 39.2\,),$$
$$E = (\,1 - 2/\pi \quad 0\,), C = (\,1 \quad 0\,)\ \varphi(x) = \sin(x_1) - \frac{2}{\pi}x_1 \tag{54}$$

Membership functions are Gaussian with $\sigma = 0.5$.

Case (a): In this case, the pendulum is controlled in the range $[-\pi/2, \ \pi/2]$; hence, only the first two rules are required. Two controllers are designed based on Lemma 3

Table 1 Comparison of different algorithms in example 1

	Lemma 3	Theorem 1	Remark 2
CPU time (s)	0.37	0.36	0.33
Numerical complexity $\mathcal{C} = \mathcal{D}^3\mathcal{L}^*$	$10^3 \times 15$	$8^3 \times 15$	$7^3 \times 10$

*For Lemmas 3 and 1 $\mathcal{L} = n_p + n_\Gamma + r^2 n_\Xi$. For Remark 2 $\mathcal{L} = n_p + r^2 n_\Xi$, where n_P, n_Γ and n_Ξ are the dimensions of P, Γ and Ξ, respectively

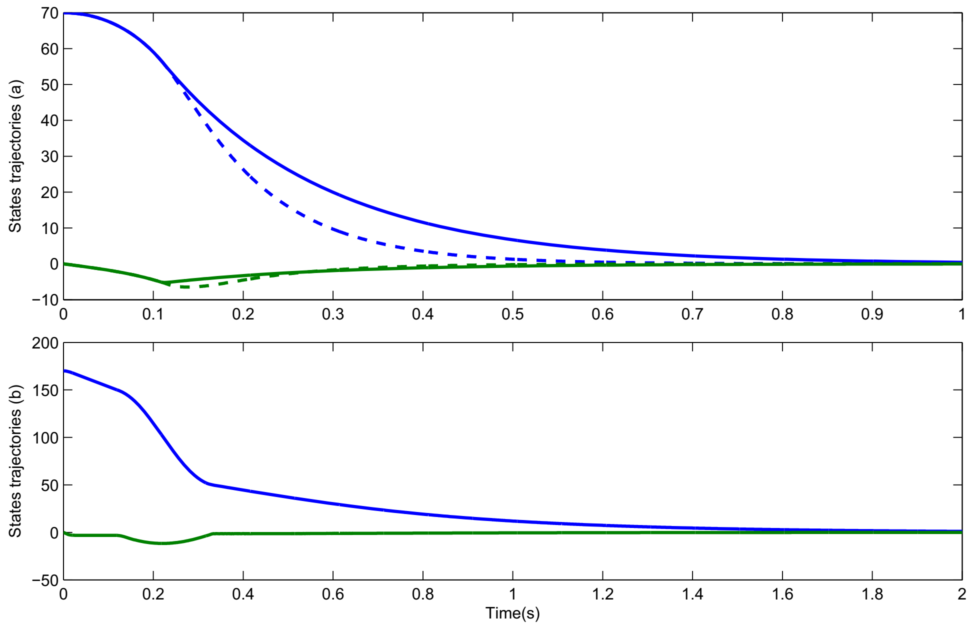

Fig. 1 State trajectories of inverted pendulum in Example 2 (**a**) based on Lemma 3 (solid line) and Theorem 1 (dash line) (**b**) based on Remark 3.3

and Theorem 1. Feedback linearization method results the following control gains:

$$K_{b1} = 98,\ K_{b2} = 9800,$$
$$K_{a1} = (\,2179.6 \quad 384.3\,),\ K_{a2} = (\,3908.5 \quad 772.4\,), \tag{55}$$

Lemma 3 results the following control gains:

$$K_{b1} = -33.3034,\ K_{b2} = -303.3980,$$
$$K_{a1} = (\,5402.2 \quad 1302.8\,),\ K_{a2} = (\,66169 \quad 16099\,), \tag{56}$$

Simulation results are compared in Fig. 1. It is shown that feedback linearization method results in a faster response, while it uses two fewer LMI variables for the analysis.

Case (b): In this case, the pendulum is controlled in the range $[-\pi,\ \pi]$; hence, all three rules are required. None of the past two controllers are capable of controlling the pendulum in this range. Instead, Remark 3.3 is used. Simulation result for a sample initial condition is shown in Fig. 1.

Example 3 Consider system (6) by the following set of matrices:

$$A_1 = \begin{pmatrix} 2 & -10 \\ 2 & 0 \end{pmatrix},\quad A_2 = \begin{pmatrix} a & -5 \\ 1 & 2 \end{pmatrix}$$
$$B_1 = (\,1 \quad 1\,)^{\mathrm{T}},\quad B_2 = (\,b \quad 2\,)^{\mathrm{T}} \tag{57}$$
$$G_{x1} = G_{x2} = \begin{pmatrix} 0.1b & 0 \\ 0 & 0.1a \end{pmatrix},\quad E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Supposing $\varphi(x(t))$ has a symmetric Jacobian and the premise variable is x_2, Fig. 2 compares the feasible area for Lemma 3 (stars) and Theorem 2 (circles) using different values for parameters a and b for this system. Obviously, introducing fuzziness to the Lyapunov function has

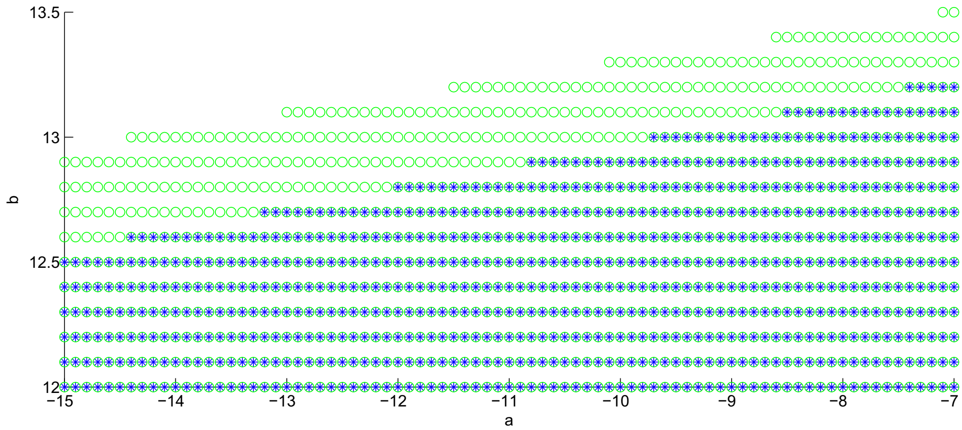

Fig. 2 Feasible area based on Lemma 3 (stars) and Theorem 2 (circles) in Example 3

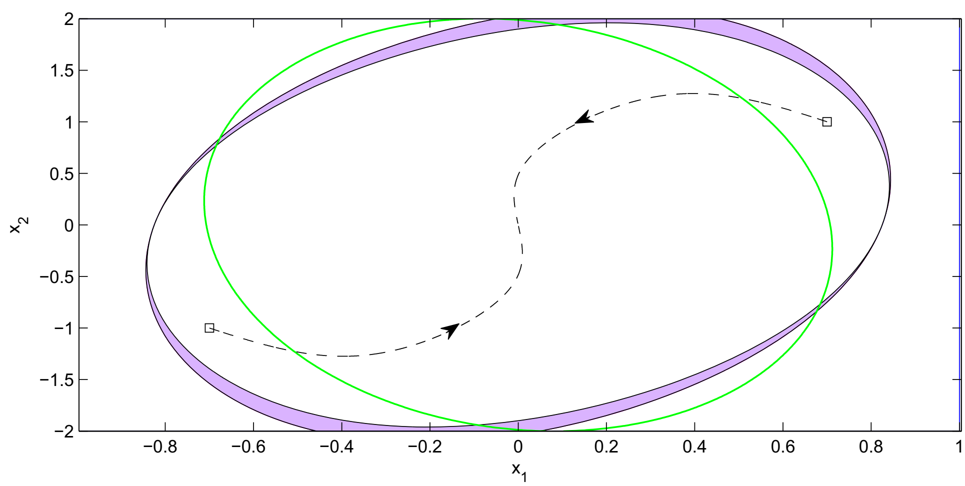

Fig. 3 Feasible area of Example 4 for parameters b and c based on nonlinear T–S (Lemma 3) (dots) and nonlinear T–S with non-quadratic Lyapunov function (Lemma 4) (dots and stars) and with the fuzzy Lyapunov function (Theorem 2) (dots, stars and squares) and with the multiple indexes (Remark 3.3) (dots, stars, squares and circles)

increased the feasible area.

Example 4 Consider the following system (Example 1 in [1] with minor modifications):

$$\dot{x}(t) = \begin{bmatrix} a + bx_2^2 & -1 \\ 2 & c + dx_2^2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ -x_2^2 \end{bmatrix} u(t) + \begin{bmatrix} b & 0 \\ 1 & d \end{bmatrix} \begin{bmatrix} x_1^3 \\ \sin(x_2) \end{bmatrix} \tag{58}$$

with $a = 10$, $b = -3$, $c = -0.25$ and $d = 0.3427$. Suppose $x_1 \in [-1 \quad 1]$ and $x_2 \in [-2 \quad 2]$. By selecting x_2^2 as the premise variable and $\varphi(x(t)) = \begin{bmatrix} x_1^3 & \sin(x_2) \end{bmatrix}^T$, this system can be modeled using the sector nonlinearity approach as model (6) with the following matrices:

$$\begin{aligned} &A_1 = \begin{pmatrix} a & -1 \\ 2 & c \end{pmatrix}, \quad A_2 = \begin{pmatrix} a + 4b & -1 \\ 2 & c + 4d \end{pmatrix} \\ &B_1 = \begin{pmatrix} 1 \\ -4 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &G_{x1} = G_{x2} = \begin{pmatrix} b & 0 \\ 1 & d \end{pmatrix}, \quad E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \\ &\mu_2 = x_2^2/4, \quad \mu_1 = 1 - \mu_2 \end{aligned} \tag{59}$$

For this system performance of a T–S system with nonlinear consequent part is compared with T–S system with linear consequents in [17]. Moreover, it is shown that the non-quadratic Lyapunov function can improve the decay rate and the convergence region comparing to the quadratic one. Now, feasibility area for the different methods introduced in this paper is shown in Fig. 3. Figure 4 compares the convergence region for Lemma 3 and Theorem 2 ($\alpha = 0.5$). For the second one, fuzziness of the Lyapunov

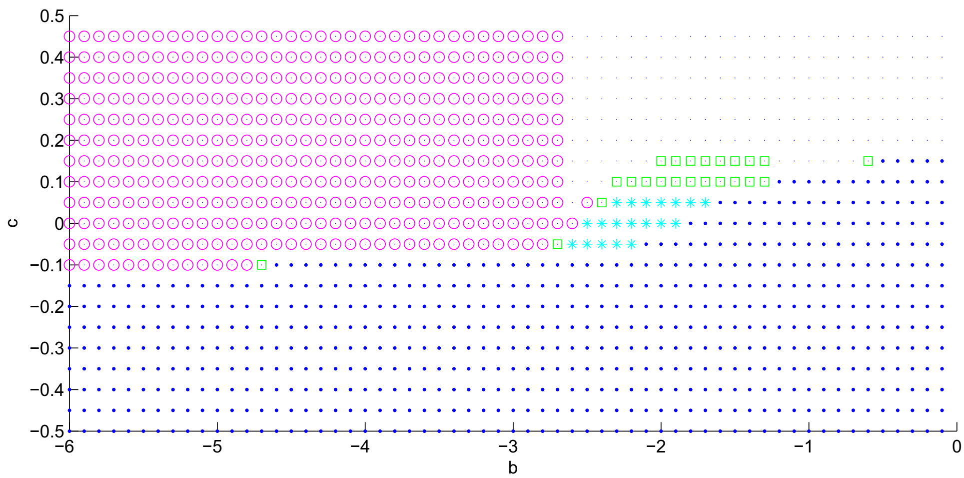

Fig. 4 Stability area of Lyapunov function for Lemma 3 (green line) and Theorem 2 (shaded area) in Example 4

function is resulted in a shaded area for convergence bound. It should be mentioned that conditions of Lemma 5 have also been applied to maximize the convergence region. The dash line shows the state trajectories for some initial conditions for Theorem 2.

5 Conclusion

In this paper, the nonlinear Takagi–Sugeno model is considered for which in each subsystem a linear part plus a sector-bounded nonlinearity is used. This may seem similar to traditional linear T–S plus disturbance, but the major difference is that the nonlinear term is known and appears in the controller too. This resulted in introducing a sort of feedback linearization in the fuzzy systems. It is shown that this feedback linearization can reduce conservativeness in the sector bounds of the nonlinearities. Moreover, a novel non-quadratic fuzzy Lyapunov function including the nonlinear term is proposed, which can help for further reduction in conservativeness.

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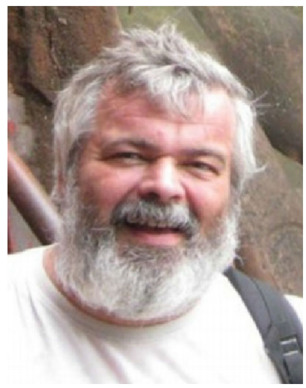
Hoda Moodi received her B.S. degree from the Ferdowsi University, Mashhad, Iran, in 2005, her M.Sc. degree from Sharif University of Technology, Tehran, Iran, in 2008 and her Ph.D. from Iran University of Science and Technology, Tehran, Iran, in 2013, all in electrical engineering. She joined Quchan University of Technology in 2016, where she is currently an assistant professor of electrical engineering. Her research interests include automatic control, fuzzy systems and robotics.

Mohammad Farrokhi received his B.S. degree from K.N. Toosi University, Tehran, Iran, in 1985, and his M.Sc. and Ph.D. degrees from Syracuse University, Syracuse, New York, in 1989 and 1996, respectively, all in electrical engineering. He joined Iran University of Science and Technology in 1996, where he is currently a Professor of electrical engineering. His research interests include intelligent control, fuzzy systems and neural networks.



Thierry Marie Guerra was born in Mulhouse, France, in 1963. He is currently full professor at the University of Valenciennes and Hainaut-Cambresis (UVHC), France. He received his Ph.D. degree in automatic control from the UVHC in 1991 and the HDR in 1999. He is head of the Laboratory of Industrial and Human Automation, Mechanics and Computer Science. Since 2014, he is chair of the Technical Committee 3.2 "Computational Intelligence in Control" for IFAC (International Federation of Automatic Control). He is also actually member of the IFAC TC 7.1 "Automotive Control," Area Editor of the international journals Fuzzy Sets and Systems and IEEE Transactions on Vehicular Technology. His major research fields and topics of interest are nonlinear control, LPV, quasi-LPV (Takagi–Sugeno) models control and observation, LMI constraints, non-quadratic Lyapunov functions, with applications to powertrain systems (IC engine, hybrid vehicles) and to disabled people.

Jimmy Lauber was born in Tours, France, in 1976. He is currently full professor at the University of Valenciennes and Hainaut-Cambresis (UVHC), France. He received his Ph.D. degree in Automatic Control from the UVHC in 2003 and the HDR in 2011. His major research fields are quasi-LPV (Takagi–Sugeno) models control and observation, non-quadratic Lyapunov functions and applications to automotive powertrain management (IC engine and high downsizing, single and dual clutch, etc.).