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Asynchronous H_∞ Filtering for Switched T–S Fuzzy Systems and Its Application to the Continuous Stirred Tank Reactor

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Abstract This paper is concerned with the H_∞ filtering problem for switched Takagi–Sugeno (T–S) fuzzy systems with asynchronous switching, where "asynchronous" means that the switching of the filters has a lag to the switching of system models. In the switched T–S fuzzy systems, every subsystem is represented by the well-known T–S fuzzy model. Using the multiple Lyapunov functions approach and mode-dependent average dwell time technique, a sufficient condition is developed to ensure the filtering error system to be globally uniformly asymptotically stable with a weighted H_∞ performance index. Moreover, the desired asynchronous H_∞ filters can be constructed by solving a set of linear matrix inequalities. Finally, an example about the continuous stirred tank reactor is provided to demonstrate the applicability of the obtained results.

Keywords H_∞ filtering · Switched T–S fuzzy systems · Asynchronous switching · Multiple Lyapunov functions · Mode-dependent average dwell time

1 Introduction

During recent decades, switched systems have been extensively investigated because many systems encountered in practice possess switching features [1]. A general switched system comprises some discrete-time or continuous-time subsystems and a switching signal which orchestrates the switching among these subsystems. Until now, the problems of stability analysis and stabilization for switched systems have received considerable attention. A number of methodologies have been developed to solve the above problems [2–11]. For the stability analysis of switched systems under arbitrary switching, the main method is constructing a common quadratic Lyapunov function for all subsystems [1]. For switched systems under constrained switching, together the multiple Lyapunov functions approach [2] with the average dwell time switching [3] may lead to well analysis results. Furthermore, by fully taking consideration of the characteristics of every subsystem, the mode-dependent average dwell time method is proposed in [12]. The conservativeness of the results obtained by the average dwell time approach can be further reduced by using the mode-dependent average dwell time method. Therefore, it is of practical and theoretical importance for us to study switched systems using the mode-dependent average dwell time method.

Due to the widespread existence of nonlinearlity in real world, the study about the nonlinear switched systems has become a hot spot. However, the existence of nonlinearity makes it difficult to analyze nonlinear switched systems

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directly. For nonlinear control systems, it has been proven that an effective approach is to model the studied nonlinear systems as T–S fuzzy systems [13–28]. The T–S fuzzy model utilizes the local linear system description for every fuzzy rule and then connects these local models. Recently, the T–S fuzzy model has been extended to study the nonlinear switched systems. Using the T–S fuzzy model to represent every nonlinear subsystem, the considered nonlinear switched systems can be modeled as the switched T–S fuzzy systems. Based on the switched T–S fuzzy systems, considerable efforts have been made to study the nonlinear switched systems [29–33].

The state estimation problem of dynamic systems has received considerable attention because of its practical applications in signal processing and control. Among the various methods of state estimation, the H_∞ filtering keeps attracting more and more attention [34–37]. The advantage of H_∞ filtering is that there is no restriction on the statistical properties of disturbances, which is more general than the classical Kalman filtering [38]. Naturally, the H_∞ filtering problem for switched systems has also been extensively investigated [5, 6, 10, 30, 33]. However, all of these works assumed that the switching between the system model and its matched filter is synchronous. In practice, the synchronous switching is a quite ideal case. Due to system identification and other reasons, the matched controller/filter of every subsystem would not be operated immediately at each switching instant. Therefore, the asynchronous switching generally exists in switched systems. In recent years, the effects of asynchronous switching on the filter/controller design have been studied [4, 5, 10, 39, 40].

As mentioned above, the works [5] and [10] have studied the asynchronous H_∞ filtering problem for discrete-time and continuous-time linear switched systems, respectively. However, to the best of our knowledge, the problem of asynchronous H_∞ filtering for continuous-time nonlinear switched systems remains to be unsolved, which motivates the research in this paper. The average dwell time technique was used in [5] and [10] to study the asynchronous H_∞ filtering problem for linear switched systems. It has been pointed out that the conservativeness of the results obtained by the average dwell time approach can be further reduced by using the mode-dependent average dwell time method [12]. Based on the above considerations, using the T–S fuzzy model, our work investigates the asynchronous H_∞ filtering problem for nonlinear switched systems with mode-dependent average dwell time switching.

The main contributions of our work are listed as follows: (1) Using the T–S fuzzy model, the asynchronous H_∞ filtering problem for the continuous-time nonlinear switched systems is studied in this paper, which receives little attention. (2) The obtained results can also be reduced to study the asynchronous H_∞ filtering problem for linear switched systems with mode-dependent average dwell time switching.

The organization of this paper is given as follows. The preliminaries and problem formulation are presented in Sect. 2. The main results are shown in Sect. 3. In Sect. 4, a practical example about the continuous stirred tank reactor is given to demonstrate the applicability of our approach. Finally, some conclusions are drawn in Sect. 5.

Notations The notations used throughout this paper are fairly standard. R^n represents the n-dimensional Euclidean space. The symbol "$*$" in a matrix stands for the transposed elements in the symmetric positions. M^T denotes the transpose of the matrix M. I and 0 represent the identity matrix and zero matrix in the block matrix, respectively. For a vector, $\|\cdot\|$ denotes its Euclidean norm. The space of square-integrable functions is denoted by $L_2[0,\infty)$. For $v(t)\in L_2[0,\infty)$, $\|v(t)\|_2=\sqrt{\int_0^\infty v(t)^T v(t)\mathrm{d}t}$ represents its norm. The space of continuously differentiable function is represented by C^1. A continuous function $\alpha:[0,\infty)\to[0,\infty)$ is said to be of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0)=0$. If α is also unbounded, then it is said to be of class $\mathcal{K}_\infty$. A function $\beta:[0,\infty)\times[0,\infty)\to[0,\infty)$ is said to be of class $\mathcal{KL}$ if $\beta(\cdot,t)$ is of class $\mathcal{K}$ for each fixed $t\geq 0$ and $\beta(s,t)$ decreases to 0 as $t\to\infty$ for each fixed $s\geq 0$. The symbol $M>0$ (≥ 0, <0, ≤ 0) is used to denote a positive definite (semi-positive definite, negative definite, semi-negative definite) matrix M. If not explicitly stated, matrices are assumed to have compatible dimensions.

2 Preliminaries and Problem Formulation

In this paper, let us consider the switched T–S fuzzy system with every subsystem described as

Rule m for a subsystem $\sigma(t)$: IF $v_{\sigma(t)1}(t)$ is $M_{\sigma(t)1m}$ and $\cdots$ and $v_{\sigma(t)p}(t)$ is $M_{\sigma(t)pm}$, THEN

$$\begin{cases}\dot{x}(t)=A_{\sigma(t)m}x(t)+B_{\sigma(t)m}w(t),\\ y(t)=C_{\sigma(t)m}x(t)+D_{\sigma(t)m}w(t),\\ z(t)=E_{\sigma(t)m}x(t),\end{cases}\tag{1}$$

where $x(t)\in R^{n_x}$ is the state vector, $y(t)\in R^{n_y}$ is the measurement vector, $z(t)\in R^{n_z}$ is the output signal to be estimated, and $w(t)\in R^{n_w}$ is the disturbance that belongs to $L_2[0,\infty)$. A piecewise constant function of time $\sigma(t):[0,+\infty)\to\mathcal{S}=\{1,2,\ldots,N\}$ is called the switching signal, where N is the number of subsystems. For a switching sequence $0=t_0<t_1<\cdots<t_k<t_{k+1}<\cdots$, $\sigma(t)$ is continuous from right everywhere. When $t\in[t_k,t_{k+1})$, the $\sigma(t_k)$ subsystem is activated. For $\sigma(t_k)=i$, $i\in\mathcal{S}$,

$A_{im}, B_{im}, C_{im}, D_{im}$ and E_{im} are constant real matrices of the m local model of the i subsystem. $v_i(t) = (v_{i1}(t), v_{i2}(t), \ldots, v_{ip}(t))$ are some measurable premise variables, and $M_{ilm}(l = 1, 2, \ldots, p)$ are fuzzy sets.

By "fuzzy blending," the final output of the i subsystem is inferred as follows

$$\begin{cases} \dot{x}(t) = \sum_{m=1}^{r} h_{im}(t)[A_{im}x(t) + B_{im}w(t)], \\ y(t) = \sum_{m=1}^{r} h_{im}(t)[C_{im}x(t) + D_{im}w(t)], \\ z(t) = \sum_{m=1}^{r} h_{im}(t)E_{im}x(t), \end{cases} \tag{2}$$

where $h_{im}(t) = v_{im}(t)/\sum_{m=1}^{r} v_{im}(t)$, $v_{im}(t) = \prod_{l=1}^{p} M_{ilm}(v_{il}(t))$, r is the number of IF-THEN rules, and $M_{ilm}(v_{il}(t))$ is the grade of the membership function of v_{il} in M_{ilm}. It is assumed that $v_{im}(t) \geq 0$ for all t, $m = 1, 2, \ldots, r$. It is obvious that the normalized membership function $h_{im}(t)$ satisfies

$$h_{im}(t) \geq 0, \quad \sum_{m=1}^{r} h_{im}(t) = 1. \tag{3}$$

The following mode-dependent fuzzy filter is designed for the T–S fuzzy subsystem (2).

Rule m: IF $v_{i1}(t)$ is M_{i1m} and $\cdots$ and $v_{ig}(t)$ is M_{igm}, THEN

$$\begin{cases} \dot{x}_f(t) = A_{fim}x_f(t) + B_{fim}y(t) \\ z_f(t) = E_{fim}x_f(t) \end{cases} \tag{4}$$

where $x_f(t)$ is the state vector of the filter, A_{fim}, B_{fim}, E_{fim} are the filter parameters to be designed. The final output of the filter is inferred as follows

$$\begin{cases} \dot{x}_f(t) = \sum_{m=1}^{r} h_{im}(t)\left[A_{fim}x_f(t) + B_{fim}y(t)\right], \\ z_f(t) = \sum_{m=1}^{r} h_{im}(t)E_{fim}x_f(t). \end{cases} \tag{5}$$

In this paper, in view of the asynchronous switching behavior, we aim to design the more practical asynchronous filter. Thus, combing (2) with (5) and defining $\tilde{x}(t) = [x^T(t), x_f^T(t)]^T$ and $e(t) = z(t) - z_f(t)$, we can obtain the following filtering error subsystem

$$\begin{cases} \begin{cases} \dot{\tilde{x}}(t) = \bar{A}_i(t)\tilde{x}(t) + \bar{B}_i(t)w(t), \\ e(t) = \bar{E}_i(t)\tilde{x}(t), \end{cases} \forall t \in [t_k, \bar{t}_k), \\ \begin{cases} \dot{\tilde{x}}(t) = \hat{A}_i(t)\tilde{x}(t) + \hat{B}_i(t)w(t), \\ e(t) = \hat{E}_i(t)\tilde{x}(t), \end{cases} \forall t \in [\bar{t}_k, t_{k+1}), \end{cases} \tag{6}$$

where the notation $\bar{t}_k$ ($t_k \leq \bar{t}_k < t_{k+1}$) represents the starting-operating instant of the matched filter, and

$$\bar{A}_i(t) = \begin{bmatrix} A_i(t) & 0 \\ B_{fj}(t)C_i(t) & A_{fj}(t) \end{bmatrix} = \sum_{m=1}^{r}\sum_{l=1}^{r} h_{im}(t)h_{jl}(t)\begin{bmatrix} A_{im} & 0 \\ B_{fjl}C_{im} & A_{fjl} \end{bmatrix},$$

$$\bar{B}_i(t) = \begin{bmatrix} B_i(t) \\ B_{fj}(t)D_i(t) \end{bmatrix} = \sum_{m=1}^{r}\sum_{l=1}^{r} h_{im}(t)h_{jl}(t)\begin{bmatrix} B_{im} \\ B_{fjl}D_{im} \end{bmatrix},$$

$$\bar{E}_i(t) = [E_i(t) \quad -E_{fj}(t)] = \sum_{m=1}^{r}\sum_{l=1}^{r} h_{im}(t)h_{jl}(t)[E_{im} \quad -E_{fjl}],$$

$$\hat{A}_i(t) = \begin{bmatrix} A_i(t) & 0 \\ B_{fi}(t)C_i(t) & A_{fi}(t) \end{bmatrix} = \sum_{m=1}^{r}\sum_{l=1}^{r} h_{im}(t)h_{il}(t)\begin{bmatrix} A_{im} & 0 \\ B_{fil}C_{im} & A_{fil} \end{bmatrix},$$

$$\hat{B}_i(t) = \begin{bmatrix} B_i(t) \\ B_{fi}(t)D_i(t) \end{bmatrix} = \sum_{m=1}^{r}\sum_{l=1}^{r} h_{im}(t)h_{il}(t)\begin{bmatrix} B_{im} \\ B_{fil}D_{im} \end{bmatrix},$$

$$\hat{E}_i(t) = [E_i(t) \quad -E_{fi}(t)] = \sum_{m=1}^{r}\sum_{l=1}^{r} h_{im}(t)h_{il}(t)[E_{im} \quad -E_{fil}].$$

To obtain the main results, the following definitions will be needed.

Definition 1 [1] The filtering error system (6) with $w(t) \equiv 0$ is globally uniformly asymptotically stable if there exists a class $\mathcal{KL}$ function β such that for all switching signals $\sigma(t)$ and all initial condition $\tilde{x}(t_0)$, the solutions of the filtering error system (6) satisfy the following inequality

$$\|\tilde{x}(t)\| \leq \beta(\|\tilde{x}(t_0)\|, t), \quad \forall t \geq t_0. \tag{7}$$

Definition 2 [12] For any $T_2 > T_1 \geq 0$, $i \in \mathcal{S}$, let $N_{\sigma i}(T_1, T_2)$ denote the switching numbers such that the i subsystem is activated over the interval $[T_1, T_2]$, $T_i(T_1, T_2)$ represent the total running time of the i subsystem over the interval $[T_1, T_2]$. If there exist two constants $T_{ai} > 0$ and N_{0i} (N_{0i} is the mode-dependent chatter bounds), such that the following inequality holds

$$N_{\sigma i}(T_1, T_2) \leq N_{0i} + \frac{T_i(T_1, T_2)}{T_{ai}}, \tag{8}$$

then, we can say that the switched systems have mode-dependent average dwell time T_{ai}.

Definition 3 For $\alpha > 0$ and $\gamma > 0$, the filtering error system (6) is said to have a weighted H_∞ performance index γ, if under zero initial condition (i.e., $\tilde{x}(t_0) = 0$), the following inequality holds

$$\int_0^\infty \mathrm{e}^{-\alpha s}\mathrm{e}^T(s)e(s)\mathrm{d}s \leq \gamma^2 \int_0^\infty w^T(s)w(s)\mathrm{d}s. \tag{9}$$

3 Main Results

In this section, the asynchronous H_∞ filtering problem for the switched T–S fuzzy system (2) is studied. To begin with, three notations are introduced, i.e., $T(t_k, t_{k+1})$, $T_\uparrow(t_k, t_{k+1})$ and $T_\downarrow(t_k, t_{k+1})$. $T(t_k, t_{k+1})$ represents the running time interval of one subsystem. $T_\uparrow(t_k, t_{k+1})$ represents the running time of the unmatched filter in $T(t_k, t_{k+1})$. $T_\downarrow(t_k, t_{k+1})$ denotes the running time of the matched filter in

$T(t_k, t_{k+1})$. Therefore, during $T_\uparrow(t_k, t_{k+1})$, the Lyapunov function may increase or decrease, while during $T_\downarrow(t_k, t_{k+1})$, the Lyapunov function is strictly decreasing. It can be seen from (6) that $T(t_k, t_{k+1}) = T_\uparrow(t_k, t_{k+1}) \bigcup T_\downarrow(t_k, t_{k+1})$. The conditions for the stability of the filtering error system (6) with a weighted H_∞ performance index can be summarized in the following lemma.

Lemma 1 *For the given filtering error system* (6), *and constants* $\alpha_i > 0$, $\beta_i > 0$, $\gamma > 0$ *and* $\mu_i \geq 1$, $\forall(\sigma(t_k) = i, \sigma(t_k^-) = j) \in S \times S, i \neq j$, *if there exists positive definite* $\mathcal{C}^1$ *function* $V_{\sigma(t_k)}$: $R^n \rightarrow R$ *with* $V_{\sigma(t_0)}(\tilde{x}(t_0)) \equiv 0$ *satisfying*

$$V_i(\tilde{x}(t_k)) \leq \mu_i V_j(\tilde{x}(t_k^-)), \tag{10}$$

and

$$\dot{V}_i(\tilde{x}(t)) \leq \begin{cases} \beta_i V_i(\tilde{x}(t)) - \Gamma(t), & t \in [t_k, \bar{t}_k), \\ -\alpha_i V_i(\tilde{x}(t)) - \Gamma(t), & t \in [\bar{t}_k, t_{k+1}), \end{cases} \tag{11}$$

where $\Gamma(t) = e^T(t)e(t) - \gamma^2 w^T(t)w(t)$, *then for any switching signal satisfying*

$$T_{ai} \geq T_{ai}^* = \frac{\mathcal{T}(\beta_i + \alpha_i) + \ln \mu_i}{\alpha_i}, \tag{12}$$

the filtering error system (6) *is globally uniformly asymptotically stable with a weighted* H_∞ *performance index* $\hat{\gamma} = \sqrt{\exp\{\sum_{i=1}^N (T_{ai}^* \alpha_i) + \theta_{\max}\mathcal{T}\}}\gamma$, *where* $\mathcal{T} \triangleq \max\{T_\uparrow(t_k, t_{k+1}), \forall k = 0, 1, 2, \ldots\}$.

Proof Here, we consider the worst situation, i.e., for an arbitrary running time interval $[t_k, t_{k+1})$ $(k = 0, 1, 2, \cdots)$, let the asynchronous time $T_\uparrow(t_k, t_{k+1})$ of this time interval take its maximal value $\mathcal{T}$. Denote $\theta_i = \alpha_i + \beta_i$ and $\theta_{\max} = \max\{\theta_i\}$. $\forall t \in [t_k, t_{k+1})$, integrating (11), we have

$$\begin{aligned} V(\tilde{x}(t)) &\leq e^{-\alpha_i(t - t_k - \mathcal{T}) + \beta_i \mathcal{T}} V(\tilde{x}(t_k)) - \int_{t_k+\mathcal{T}}^t e^{-\alpha_i(t-s)}\Gamma(s)ds \\ &\quad - e^{-\alpha_i(t-t_k-\mathcal{T})} \int_{t_k}^{t_k+\mathcal{T}} e^{\beta_i(t_k+\mathcal{T}-s)}\Gamma(s)ds \\ &= e^{-\alpha_i(t-t_k)+\theta_i\mathcal{T}} V(\tilde{x}(t_k)) - \int_{t_k+\mathcal{T}}^t e^{-\alpha_i(t-s)}\Gamma(s)ds \\ &\quad - \int_{t_k}^{t_k+\mathcal{T}} e^{-\alpha_i(t-s)+\theta_i(t_k+\mathcal{T}-s)}\Gamma(s)ds. \end{aligned} \tag{13}$$

By (10) and (13), we have

$$\begin{aligned} V(\tilde{x}(t)) &\leq \mu_i e^{-\alpha_i(t-t_k)+\theta_i\mathcal{T}} V(\tilde{x}(t_k^-)) - \int_{t_k+\mathcal{T}}^t e^{-\alpha_i(t-s)}\Gamma(s)ds \\ &\quad - \int_{t_k}^{t_k+\mathcal{T}} e^{-\alpha_i(t-s)+\theta_i(t_k+\mathcal{T}-s)}\Gamma(s)ds \\ &\leq \mu_i e^{-\alpha_i(t-t_k)-\alpha_{i-1}(t_k-t_{k-1})+(\theta_i+\theta_{i-1})\mathcal{T}} V(\tilde{x}(t_{k-1})) \\ &\quad - \mu_i e^{-\alpha_i(t-t_k)+\theta_i\mathcal{T}} \Big[\int_{t_{k-1}+\mathcal{T}}^{t_k} e^{-\alpha_{i-1}(t_k-s)}\Gamma(s)ds \\ &\quad + \int_{t_{k-1}}^{t_{k-1}+\mathcal{T}} e^{-\alpha_{i-1}(t_k-s)+\theta_{i-1}(t_{k-1}+\mathcal{T}-s)}\Gamma(s)ds\Big] \\ &\quad - \int_{t_k}^{t_k+\mathcal{T}} e^{-\alpha_i(t-s)+\theta_i(t_k+\mathcal{T}-s)}\Gamma(s)ds - \int_{t_k+\mathcal{T}}^t e^{-\alpha_i(t-s)}\Gamma(s)ds \\ &\leq \cdots \\ &\leq \mu_i\mu_{i-1}\cdots\mu_1 e^{-\alpha_i(t-t_k)-\alpha_{i-1}(t_k-t_{k-1})-\cdots-\alpha_0(t_1-t_0)+\mathcal{T}(\theta_i+\theta_{i-1}+\cdots+\theta_0)} V(\tilde{x}(t_0)) \\ &\quad - \mu_i\mu_{i-1}\cdots\mu_1 e^{-\alpha_i(t-t_k)-\alpha_{i-1}(t_k-t_{k-1})-\cdots-\alpha_1(t_2-t_1)+\mathcal{T}(\theta_i+\cdots+\theta_1)} \\ &\quad \Big[\int_{t_0+\mathcal{T}}^{t_1} e^{-\alpha_0(t_1-s)}\Gamma(s)ds + \int_{t_0}^{t_0+\mathcal{T}} e^{-\alpha_0(t_1-s)+\theta_0(t_0+\mathcal{T}-s)}\Gamma(s)ds\Big] \\ &\quad - \cdots - \int_{t_k}^{t_k+\mathcal{T}} e^{-\alpha_i(t-s)+\theta_i(t_k+\mathcal{T}-s)}\Gamma(s)ds - \int_{t_k+\mathcal{T}}^t e^{-\alpha_i(t-s)}\Gamma(s)ds \\ &= \Phi(t) - \Lambda(s), \end{aligned} \tag{14}$$

where

$$\Phi(t) = e^{\ln\mu_i + \ln\mu_{i-1} + \cdots + \ln\mu_1 - \alpha_i(t-t_k) - \alpha_{i-1}(t_k-t_{k-1}) - \cdots - \alpha_0(t_1-t_0) + \mathcal{T}(\theta_i+\theta_{i-1}+\cdots+\theta_0)} V(\tilde{x}(t_0)),$$

$$\begin{aligned} \Lambda(s) &= \exp\left\{\sum_{i=1}^N (\ln\mu_i + \theta_i\mathcal{T}) - \alpha_i(t-t_k) - \cdots - \alpha_1(t_2-t_1)\right\} \\ &\quad \Big[\int_{t_0+\mathcal{T}}^{t_1} e^{-\alpha_0(t_1-s)}\Gamma(s)ds + \int_{t_0}^{t_0+\mathcal{T}} e^{-\alpha_0(t_1-s)+\theta_0(t_0+\mathcal{T}-s)}\Gamma(s)ds\Big] \\ &\quad + \cdots + \int_{t_k}^{t_k+\mathcal{T}} e^{-\alpha_i(t-s)+\theta_i(t_k+\mathcal{T}-s)}\Gamma(s)ds + \int_{t_k+\mathcal{T}}^t e^{-\alpha_i(t-s)}\Gamma(s)ds. \end{aligned}$$

Let $N_{\sigma i}$ denote $N_{\sigma i}(t, t_0)$ for simplicity. The following equation can be obtained

$$\Phi(t) = \exp\left\{\sum_{i=1}^N N_{\sigma i}(-\alpha_i T_{ai} + \ln\mu_i + \theta_i\mathcal{T})\right\} V(\tilde{x}(t_0)). \tag{15}$$

If supposing

$$-\alpha_i T_{ai} + \ln\mu_i + \theta_i\mathcal{T} \leq 0, \tag{16}$$

a sufficient condition that guarantees the filtering error system (6) to be globally uniformly asymptotically stable can be obtained. The inequality (16) can be rewritten as

$$T_{ai} \geq T_{ai}^* = \frac{\mathcal{T}(\beta_i + \alpha_i) + \ln\mu_i}{\alpha_i}. \tag{17}$$

Then, it can be concluded that $V_i(\tilde{x}(t))$ converges to zero as $t \rightarrow \infty$ if the inequality (17) holds.

Next, the weighted H_∞ performance index for the filtering error system (6) will be established. Defining

$\alpha_{\max} \overset{\Delta}{=} \max\{\alpha_i\}$ and letting all $\alpha_i = \alpha_{\max}$, the following inequality can be obtained

$$\begin{aligned}\Lambda(s) &\geq \Theta(s)\\ &= \exp\left\{\sum_{i=1}^{N}(\ln\mu_i + \theta_i\mathcal{T})\right\}\left[\int_{t_0+\mathcal{T}}^{t_1} \mathrm{e}^{-\alpha_{\max}(t-s)}\Gamma(s)\mathrm{d}s\right.\\ &\quad\left.+\int_{t_0}^{t_0+\mathcal{T}} \mathrm{e}^{-\alpha_{\max}(t-s)+\theta_0(t_0+\mathcal{T}-s)}\Gamma(s)\mathrm{d}s\right]\\ &\quad+\cdots\\ &\quad+\int_{t_k+\mathcal{T}}^{t} \mathrm{e}^{-\alpha_{\max}(t-s)}\Gamma(s)\mathrm{d}s\\ &\quad+\int_{t_k}^{t_k+\mathcal{T}} \mathrm{e}^{-\alpha_{\max}(t-s)+\theta_i(t_k+\mathcal{T}-s)}\Gamma(s)\mathrm{d}s. \end{aligned} \tag{18}$$

As for $\Gamma(s) = \mathrm{e}^T(s)e(s) - \gamma^2 w^T(s)w(s)$, we define $\bar{\Lambda}(s)$ and $\hat{\Lambda}(s)$ with the $\mathrm{e}^T(s)e(s)$ and $\gamma^2 w^T(s)w(s)$ terms, respectively. The following equality is introduced

$$\Theta(s) = \bar{\Lambda}(s) - \hat{\Lambda}(s), \tag{19}$$

where

$$\begin{aligned}\bar{\Lambda}(s) &= \exp\left\{\sum_{i=1}^{N}(\ln\mu_i + \theta_i\mathcal{T})\right\}\left[\int_{t_0+\mathcal{T}}^{t_1} \mathrm{e}^{-\alpha_{\max}(t-s)}\mathrm{e}^T(s)e(s)\mathrm{d}s\right.\\ &\quad\left.+\int_{t_0}^{t_0+\mathcal{T}} \mathrm{e}^{-\alpha_{\max}(t-s)+\theta_0(t_0+\mathcal{T}-s)}\mathrm{e}^T(s)e(s)\mathrm{d}s\right]\\ &\quad+\cdots\\ &\quad+\int_{t_k}^{t_k+\mathcal{T}} \mathrm{e}^{-\alpha_{\max}(t-s)+\theta_i(t_k+\mathcal{T}-s)}\mathrm{e}^T(s)e(s)\mathrm{d}s\\ &\quad+\int_{t_k+\mathcal{T}}^{t} \mathrm{e}^{-\alpha_{\max}(t-s)}\mathrm{e}^T(s)e(s)\mathrm{d}s, \end{aligned} \tag{20}$$

and

$$\begin{aligned}\hat{\Lambda}(s) &= \exp\left\{\sum_{i=1}^{N}(\ln\mu_i + \theta_i\mathcal{T})\right\}\left[\int_{t_0+\mathcal{T}}^{t_1} \mathrm{e}^{-\alpha_{\max}(t-s)}\gamma^2 w^T(s)w(s)\mathrm{d}s\right.\\ &\quad\left.+\int_{t_0}^{t_0+\mathcal{T}} \mathrm{e}^{-\alpha_{\max}(t-s)+\theta_0(t_0+\mathcal{T}-s)}\gamma^2 w^T(s)w(s)\mathrm{d}s\right]\\ &\quad+\cdots\\ &\quad+\int_{t_k}^{t_k+\mathcal{T}} \mathrm{e}^{-\alpha_{\max}(t-s)+\theta_i(t_k+\mathcal{T}-s)}\gamma^2 w^T(s)w(s)\mathrm{d}s\\ &\quad+\int_{t_k+\mathcal{T}}^{t} \mathrm{e}^{-\alpha_{\max}(t-s)}\gamma^2 w^T(s)w(s)\mathrm{d}s. \end{aligned} \tag{21}$$

Owing to $\exp\{\sum_{i=1}^{N}(\ln\mu_i + \theta_i\mathcal{T})\} \geq 1$ and $\theta_i(t_k + \mathcal{T} - s) \geq 0$, the following inequality can be obtained

$$\begin{aligned}\bar{\Lambda}(s) &\geq \left[\int_{t_0+\mathcal{T}}^{t_1} \mathrm{e}^{-\alpha_{\max}(t-s)}\mathrm{e}^T(s)e(s)\mathrm{d}s\right.\\ &\quad\left.+\int_{t_0}^{t_0+\mathcal{T}} \mathrm{e}^{-\alpha_{\max}(t-s)}\mathrm{e}^T(s)e(s)\mathrm{d}s\right]\\ &\quad+\cdots\\ &\quad+\int_{t_k}^{t_k+\mathcal{T}} \mathrm{e}^{-\alpha_{\max}(t-s)}\mathrm{e}^T(s)e(s)\mathrm{d}s\\ &\quad+\int_{t_k+\mathcal{T}}^{t} \mathrm{e}^{-\alpha_{\max}(t-s)}\mathrm{e}^T(s)e(s)\mathrm{d}s\\ &= \int_{t_0}^{t} \mathrm{e}^{-\alpha_{\max}(t-s)}\mathrm{e}^T(s)e(s)\mathrm{d}s. \end{aligned} \tag{22}$$

Since $\theta_{\max}\mathcal{T} \geq 0$ and $\theta_i(t_k + \mathcal{T} - s) \leq \theta_{\max}\mathcal{T}$, the following inequality can be obtained

$$\begin{aligned}\hat{\Lambda}(s) &\leq \exp\left\{\sum_{i=1}^{N}(\ln\mu_i + \theta_i\mathcal{T})\right\}\left[\int_{t_0+\mathcal{T}}^{t_1} \mathrm{e}^{-\alpha_{\max}(t-s)+\theta_{\max}\mathcal{T}}\gamma^2 w^T(s)w(s)\mathrm{d}s\right.\\ &\quad\left.+\int_{t_0}^{t_0+\mathcal{T}} \mathrm{e}^{-\alpha_{\max}(t-s)+\theta_{\max}\mathcal{T}}\gamma^2 w^T(s)w(s)\mathrm{d}s\right]\\ &\quad+\cdots\\ &\quad+\exp\left\{\sum_{i=1}^{N}(\ln\mu_i + \theta_i\mathcal{T})\right\}\left[\int_{t_k}^{t_k+\mathcal{T}} \mathrm{e}^{-\alpha_{\max}(t-s)+\theta_{\max}\mathcal{T}}\gamma^2 w^T(s)w(s)\mathrm{d}s\right.\\ &\quad\left.+\int_{t_k+\mathcal{T}}^{t} \mathrm{e}^{-\alpha_{\max}(t-s)+\theta_{\max}\mathcal{T}}\gamma^2 w^T(s)w(s)\mathrm{d}s\right]\\ &= \exp\left\{\sum_{i=1}^{N}(\ln\mu_i + \theta_i\mathcal{T})\right\}\int_{t_0}^{t} \mathrm{e}^{-\alpha_{\max}(t-s)+\theta_{\max}\mathcal{T}}\gamma^2 w^T(s)w(s)\mathrm{d}s. \end{aligned} \tag{23}$$

Combining (17) with (23) leads to

$$\hat{\Lambda}(s) \le \exp\left\{\sum_{i=1}^{N}(T_{ai}^{*}\alpha_i)\right\}\int_{t_0}^{t} e^{-\alpha_{\max}(t-s)+\theta_{\max}\mathcal{T}}\gamma^2 w^T(s)w(s)ds. \tag{24}$$

As for $\Lambda(s)$, under zero initial condition, (14) gives

$$\Lambda(s) \le 0. \tag{25}$$

Then, combining (19), (22), (23), (24) with (25), the following inequality can be obtained

$$\int_{t_0}^{t} e^{-\alpha_{\max}t}e^T(s)e(s)ds \le \exp\left\{\sum_{i=1}^{N}(T_{ai}^{*}\alpha_i) + \theta_{\max}\mathcal{T}\right\} \int_{t_0}^{t} e^{-\alpha_{\max}(t-s)}\gamma^2 w^T(s)w(s)ds. \tag{26}$$

Let $t_0 = 0$, integrating both sides of inequality (26) form $t = 0$ to ∞ leads to

$$\int_{0}^{\infty} e^{-\alpha_{\max}s}e^T(s)e(s)ds \le \hat{\gamma}^2\int_{0}^{\infty} w^T(s)w(s)ds, \tag{27}$$

where $\hat{\gamma} = \sqrt{\exp\{\sum_{i=1}^{N}(T_{ai}^{*}\alpha_i) + \theta_{\max}\mathcal{T}\}}\gamma$. The proof is completed. □

Remark 1 In the above proof, the worst situation that is the asynchronous time $T_\uparrow(t_k, t_{k+1})$ takes its maximal value $\mathcal{T}$ is considered. Therefore, the obtained results have more or less conservativeness compared with the case that not all the asynchronous time $T_\uparrow(t_k, t_{k+1})$ takes its maximal value $\mathcal{T}$. In practice, if not all the asynchronous time $T_\uparrow(t_k, t_{k+1})$ takes its maximal value $\mathcal{T}$, the filtering error system (6) can be globally uniformly asymptotically stable with a relatively small T_{ai}^{*} and has a relatively better H_∞ performance.

Remark 2 By the same procedure as the proof of Lemma 1, we can conclude that the filtering error system (6) with $w(t) = 0$ is globally uniformly asymptotically stable for any switching signal satisfying (12).

Lemma 2 *For the given filtering error system (6), and constants $\alpha_i > 0$, $\beta_i > 0$, $\gamma > 0$ and $\mu_i \ge 1$, $\forall(i,j) \in S \times S, i \ne j$, if there exist matrices $P_i > 0$ satisfying*

$$P_i < \mu_i P_j, \tag{28}$$

$$\begin{bmatrix} P_i\bar{A}_i(t) + \bar{A}_i^T(t)P_i - \beta_i P_i & P_i\bar{B}_i(t) & \bar{E}_i^T(t) \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad \forall t \in [t_k, \bar{t}_k), \tag{29}$$

$$\begin{bmatrix} P_i\hat{A}_i(t) + \hat{A}_i^T(t)P_i + \alpha_i P_i & P_i\hat{B}_i(t) & \hat{E}_i^T(t) \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad \forall t \in [\bar{t}_k, t_{k+1}), \tag{30}$$

then the filtering error system (6) is globally uniformly asymptotically stable with a weighted H_∞ performance index $\hat{\gamma}$ for any switching signal satisfying (12).

Proof Choose the following Lyapunov functions for the filtering error system (6)

$$V_i(\tilde{x}(t)) = \tilde{x}^T(t)P_i\tilde{x}(t). \tag{31}$$

First, let us consider $t \in T_\uparrow(t_k, t_{k+1})$. The following equality can be obtained from the filtering error system (6)

$$\dot{V}_i(\tilde{x}(t)) - \beta_i V_i(\tilde{x}(t)) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) = \eta^T(t)\prod_{i1}\eta(t),$$

where

$$\eta^T(t) = \left[\tilde{x}^T(t), w^T(t)\right],$$

$$\prod_{i1} = \begin{bmatrix} P_i\bar{A}_i(t) + \bar{A}_i^T(t)P_i - \beta_i P_i + \bar{E}_i^T(t)\bar{E}_i(t) & P_i\bar{B}_i(t) \\ * & -\gamma^2 I \end{bmatrix}.$$

Similarly, for $t \in T_\downarrow(t_k, t_{k+1})$, the following equality holds

$$\dot{V}_i(\tilde{x}(t)) + \alpha_i V_i(\tilde{x}(t)) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) = \eta^T(t)\prod_{i2}\eta(t),$$

where

$$\prod_{i2} = \begin{bmatrix} P_i\hat{A}_i(t) + \hat{A}_i^T(t)P_i + \alpha_i P_i + \hat{E}_i^T(t)\hat{E}_i(t) & P_i\hat{B}_i(t) \\ * & -\gamma^2 I \end{bmatrix}.$$

Using the Schur's complement, it can be concluded that the inequalities (29) and (30) imply $\Pi_{i1} < 0$ and $\Pi_{i2} < 0$. Let $\Gamma(t) = e^T(t)e(t) - \gamma^2 w^T(t)w(t)$, the following inequality can be obtained

$$\dot{V}_i(\tilde{x}(t)) \le \begin{cases} \beta_i V_i(\tilde{x}(t)) - \Gamma(t), & t \in T_\uparrow(t_k, t_{k+1}), \\ -\alpha_i V_i(\tilde{x}(t)) - \Gamma(t), & t \in T_\downarrow(t_k, t_{k+1}). \end{cases}$$

For $P_i < \mu_i P_j$, it can be obtained

$$V_i(\tilde{x}(t_k)) < \mu_i V_j(\tilde{x}(t_k^-)).$$

By Lemma 1, it can be concluded that the filtering error system (6) is globally uniformly asymptotically stable with a weighted H_∞ performance index $\hat{\gamma}$ for any switching signal satisfying (12). The proof is completed. □

In Lemma 2, the filter parameter matrices are coupled with the matrix variable P_i in (29) and (30). Therefore, it is difficult to use Lemma 2 to design the asynchronous H_∞ filter directly. To overcome this problem, a decoupling technique is introduced in the following lemma.

Lemma 3 *Let $\alpha_i > 0$, $\beta_i > 0$, $\gamma > 0$ and $\mu_i \geq 1$ be given constants. $\forall (i,j) \in S \times S, i \neq j$, if there exist matrices $\bar{P}_{i1}$, $\bar{P}_{i2}$, $\bar{P}_{i3}$, $A_{Fi}(t)$, $B_{Fi}(t)$, $E_{Fi}(t)$, L, R and X satisfying*

$$\bar{P}_i = \begin{bmatrix} \bar{P}_{i1} & \bar{P}_{i2} \\ * & \bar{P}_{i3} \end{bmatrix} > 0, \quad \bar{P}_i \leq \mu_i \bar{P}_j, \tag{32}$$

$$\begin{bmatrix} \Phi_{i11} & \Phi_{i12} & \Phi_{i13} & \Phi_{i14} & \Phi_{i15} & E_i^T(t) \\ * & \Phi_{i22} & \Phi_{i23} & \Phi_{i24} & \Phi_{i25} & -E_{Fj}^T(t) \\ * & * & \Phi_{i33} & \Phi_{i34} & \Phi_{i15} & 0 \\ * & * & * & \Phi_{i44} & \Phi_{i25} & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{33}$$

and

$$\begin{bmatrix} \Psi_{i11} & \Psi_{i12} & \Psi_{i13} & \Psi_{i14} & \Psi_{i15} & E_i^T(t) \\ * & \Psi_{i22} & \Psi_{i23} & \Psi_{i24} & \Psi_{i25} & -E_{Fi}^T(t) \\ * & * & \Psi_{i33} & \Psi_{i34} & \Psi_{i15} & 0 \\ * & * & * & \Psi_{i44} & \Psi_{i25} & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{34}$$

where

$$\Phi_{i11} = A_i^T(t)L + B_{Fj}(t)C_i(t) + L^T A_i(t) + C_i^T(t)B_{Fj}^T(t) - \beta_i \bar{P}_{i1},$$
$$\Phi_{i12} = A_{Fj}(t) + A_i^T(t)R + C_i^T(t)B_{Fj}^T(t) - \beta_i \bar{P}_{i2},$$
$$\Phi_{i13} = \bar{P}_{i1} - L^T + A_i^T(t)L + C_i^T(t)B_{Fj}^T(t),$$
$$\Phi_{i14} = \bar{P}_{i2} - X^T + A_i^T(t)R + C_i^T(t)B_{Fj}^T(t),$$
$$\Phi_{i15} = L^T B_i(t) + B_{Fj}(t)D_i(t), \quad \Phi_{i22} = A_{Fj}(t) + A_{Fj}^T(t) - \beta_i \bar{P}_{i3},$$
$$\Phi_{i23} = \bar{P}_{i2}^T - R^T + A_{Fj}^T(t), \quad \Phi_{i24} = \bar{P}_{i3} - X^T + A_{Fj}^T(t),$$
$$\Phi_{i33} = -L - L^T, \quad \Phi_{i34} = -R - X^T, \quad \Phi_{i44} = -X - X^T,$$
$$\Psi_{i11} = A_i^T(t)L + B_{Fi}(t)C_i(t) + L^T A_i(t) + C_i^T B_{Fi}^T(t) + \alpha_i \bar{P}_{i1},$$
$$\Psi_{i12} = A_{Fi}(t) + A_i^T(t)R + C_i^T(t)B_{Fi}^T(t) + \alpha_i \bar{P}_{i2},$$
$$\Psi_{i13} = \bar{P}_{i1} - L^T + A_i^T(t)L + C_i^T(t)B_{Fi}^T(t),$$
$$\Psi_{i15} = L^T B_i(t) + B_{Fi}(t)D_i(t),$$
$$\Psi_{i23} = \bar{P}_{i2}^T - R^T + A_{Fi}^T(t), \quad \Psi_{i24} = \bar{P}_{i3} - X^T + A_{Fi}^T(t),$$
$$\Psi_{i33} = -L - L^T, \quad \Psi_{i34} = -R - X^T, \quad \Psi_{i44} = -X - X^T.$$

then, we can conclude that the inequalities (28), (29) and (30) hold.

Proof In order to decouple the matrix variables P_i and filter parameter matrices, we introduce a slack matrix Q. Moreover, introducing slack matrices can also reduce the design conservativeness [15]. Similar as in [41], by introducing a slack matrix Q, we introduce the following inequalities

$$\begin{bmatrix} \prod_{i3} & P_i - Q^T + \bar{A}_i^T(t)Q & Q^T \bar{B}_i(t) & \bar{E}_i^T(t) \\ * & -Q - Q^T & Q^T \bar{B}_i(t) & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{35}$$

$$\begin{bmatrix} \prod_{i4} & P_i - Q^T + \hat{A}_i^T(t)Q & Q^T \hat{B}_i(t) & \hat{E}_i^T(t) \\ * & -Q - Q^T & Q^T \hat{B}_i(t) & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{36}$$

where

$$\prod_{i3} = Q^T \bar{A}_i(t) + \bar{A}_i^T(t)Q - \beta_i P_i,$$
$$\prod_{i4} = Q^T \hat{A}_i(t) + \hat{A}_i^T(t)Q + \alpha_i P_i.$$

Multiplying (35) from the left and right, respectively, by $\bar{\Omega}_i(t)$ and its transpose. Similarly, multiplying (36) from the left and right, respectively, by $\hat{\Omega}_i(t)$ and its transpose, where

$$\bar{\Omega}_i(t) = \begin{bmatrix} I & \bar{A}_i^T(t) & 0 & 0 \\ 0 & \bar{B}_i^T(t) & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \hat{\Omega}_i(t) = \begin{bmatrix} I & \hat{A}_i^T(t) & 0 & 0 \\ 0 & \hat{B}_i^T(t) & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}.$$

Then, we can conclude that (29) and (30) hold. If the conditions in (35) and (36) hold, the matrix Q is nonsingular. Partition the matrix Q as

$$Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_4 & Q_3 \end{bmatrix}. \tag{37}$$

Because of our consideration about a full-order filter, Q_2 and Q_4 are both square. Without loss of generality, Q_3 and Q_4 are assumed to be perturbed, respectively, by matrices ΔQ_3 and ΔQ_4. The matrices ΔQ_3 and ΔQ_4 are norms bounded, and the norm bounds are sufficiently small. Then $Q_3 + \Delta Q_3$ and $Q_4 + \Delta Q_4$ are nonsingular and satisfy (35) and (36). Define some matrices as follows

$$q = \begin{bmatrix} I & 0 \\ 0 & Q_3^{-1}Q_4 \end{bmatrix}, \quad \bar{P}_i = \begin{bmatrix} \bar{P}_{i1} & \bar{P}_{i2} \\ * & \bar{P}_{i3} \end{bmatrix} = q^T P_i q > 0,$$
$$L = Q_1, \quad R = Q_2 Q_3^{-1} Q_4, \quad X = Q_4^T Q_3^{-T} Q_4 \tag{38}$$

and

$$\begin{bmatrix} A_{Fi}(t) & B_{Fi}(t) \\ E_{Fi}(t) & 0 \end{bmatrix} = \begin{bmatrix} Q_4^T & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{fi}(t) & B_{fi}(t) \\ E_{fi}(t) & 0 \end{bmatrix} \begin{bmatrix} Q_3^{-1}Q_4 & 0 \\ 0 & I \end{bmatrix}. \tag{39}$$

Performing a congruence transformation to (35) and (36) by the diagonal matrix $diag(q_1, I)$ with $q_1 = diag(q, q, I)$, the following inequalities can be obtained

$$\begin{bmatrix} \prod_{i5} & \prod_{i6} & q^TQ^T\bar{B}_i(t) & q^T\bar{E}_i^T(t) \\ * & -q^T(Q+Q^T)q & q^TQ^T\bar{B}_i(t) & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (40)$$

$$\begin{bmatrix} \prod_{i7} & \prod_{i8} & q^TQ^T\hat{B}_i(t) & q^T\hat{E}_i^T(t) \\ * & -q^T(Q+Q^T)q & q^TQ^T\hat{B}_i(t) & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (41)$$

where

$$\prod_{i5} = q^T\left[Q^T\bar{A}_i(t) + \bar{A}_i^T(t)Q - \beta_i P_i\right]q,$$
$$\prod_{i6} = q^T\left[P_i - Q^T + \bar{A}_i^T(t)Q\right]q,$$
$$\prod_{i7} = q^T\left[Q^T\hat{A}_i(t) + \hat{A}_i^T(t)Q + \alpha_i P_i\right]q,$$
$$\prod_{i8} = q^T\left[P_i - Q^T + \hat{A}_i^T(t)Q\right]q.$$

Considering (35)–(39) and (6), it can be concluded that (40) and (41) imply (33) and (34), respectively. Moreover, performing a congruence transformation to $P_i \le \mu_i P_j$ by the matrix q, it can be obtained

$$q^TP_iq \le \mu_i q^TP_jq \Leftrightarrow \bar{P}_i \le \mu_i\bar{P}_j. \quad (42)$$

Then, it can be concluded that (28)–(30) hold if (32)–(34) hold. The proof is completed. □

Based on the above lemmas, a set of mode-dependent filters will be designed to estimate the output of the switched T–S fuzzy system (2).

Theorem 1 *For the given filtering error system (6), and constants $\alpha_i > 0$, $\beta_i > 0$, $\gamma > 0$ and $\mu_i \ge 1$, $\forall (i,j) \in S \times S, i \neq j$, if there exist matrices $\bar{P}_i$, A_{Fim}, B_{Fim}, E_{Fim}, L, R and X satisfying*

$$\bar{P}_i = \begin{bmatrix} \bar{P}_{i1} & \bar{P}_{i2} \\ * & \bar{P}_{i3} \end{bmatrix} > 0, \quad (43)$$

$$\bar{P}_i \le \mu_i \bar{P}_j, \quad (44)$$

$$\bar{\Theta}_{ijml} < 0, \quad (45)$$

$$\hat{\Theta}_{iiml} + \hat{\Theta}_{iilm} < 0, \quad m \le l, \quad (46)$$

where

$$\bar{\Theta}_{iml} = \begin{bmatrix} \bar{\Phi}_{i11} & \bar{\Phi}_{i12} & \bar{\Phi}_{i13} & \bar{\Phi}_{i14} & \bar{\Phi}_{i15} & E_{im}^T \\ * & \bar{\Phi}_{i22} & \bar{\Phi}_{i23} & \bar{\Phi}_{i24} & \bar{\Phi}_{i25} & -E_{Fjl}^T \\ * & * & \bar{\Phi}_{i33} & \bar{\Phi}_{i34} & \bar{\Phi}_{i15} & 0 \\ * & * & * & \bar{\Phi}_{i44} & \bar{\Phi}_{i25} & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix},$$

$$\hat{\Theta}_{iml} = \begin{bmatrix} \hat{\Psi}_{i11} & \hat{\Psi}_{i12} & \hat{\Psi}_{i13} & \hat{\Psi}_{i14} & \hat{\Psi}_{i15} & E_{im}^T \\ * & \hat{\Psi}_{i22} & \hat{\Psi}_{i23} & \hat{\Psi}_{i24} & \hat{\Psi}_{i25} & -E_{Fil}^T \\ * & * & \hat{\Psi}_{i33} & \hat{\Psi}_{i34} & \hat{\Psi}_{i15} & 0 \\ * & * & * & \hat{\Psi}_{i44} & \hat{\Psi}_{i25} & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix},$$

$$\bar{\Phi}_{i11} = A_{im}^TL + B_{Fjl}C_{im} + L^TA_{im} + C_{im}^TB_{Fjl}^T - \beta_i\bar{P}_{i1},$$
$$\bar{\Phi}_{i12} = A_{Fjl} + A_{im}^TR + C_{im}^TB_{Fjl}^T - \beta_i\bar{P}_{i2},$$
$$\bar{\Phi}_{i13} = \bar{P}_{i1} - L^T + A_{im}^TL + C_{im}^TB_{Fjl}^T,$$
$$\bar{\Phi}_{i14} = \bar{P}_{i2} - X^T + A_{im}^TR + C_{im}^TB_{Fjl}^T,$$
$$\bar{\Phi}_{i15} = L^TB_{im} + B_{Fjl}D_{im}, \quad \bar{\Phi}_{i22} = A_{Fjl} + A_{Fjl}^T - \beta_i\bar{P}_{i3},$$
$$\bar{\Phi}_{i23} = \bar{P}_{i2}^T - R^T + A_{Fjl}^T, \quad \bar{\Phi}_{i24} = \bar{P}_{i3} - X^T + A_{Fjl}^T,$$
$$\bar{\Phi}_{i25} = R^TB_{im} + B_{Fjl}D_{im},$$
$$\bar{\Phi}_{i33} = -L - L^T, \quad \bar{\Phi}_{i34} = -R - X^T, \quad \bar{\Phi}_{i44} = -X - X^T,$$
$$\hat{\Psi}_{i11} = A_{im}^TL + B_{Fil}C_{im} + L^TA_{im} + C_{im}^TB_{Fil}^T + \alpha_i\bar{P}_{i1},$$
$$\hat{\Psi}_{i12} = A_{Fil} + A_{im}^TR + C_{im}^TB_{Fil}^T + \alpha_i\bar{P}_{i2},$$
$$\hat{\Psi}_{i13} = \bar{P}_{i1} - L^T + A_{im}^TL + C_{im}^TB_{Fil}^T,$$
$$\hat{\Psi}_{i14} = \bar{P}_{i2} - X^T + A_{im}^TR + C_{im}^TB_{Fil}^T,$$
$$\hat{\Psi}_{i15} = L^TB_{im} + B_{Fil}D_{im}, \quad \hat{\Psi}_{i22} = A_{Fil} + A_{Fil}^T + \alpha_i\bar{P}_{i3},$$
$$\hat{\Psi}_{i23} = \bar{P}_{i2}^T - R^T + A_{Fil}^T, \quad \hat{\Psi}_{i24} = \bar{P}_{i3} - X^T + A_{Fil}^T,$$
$$\hat{\Psi}_{i25} = R^TB_{im} + B_{Fil}D_{im},$$
$$\hat{\Psi}_{i33} = -L - L^T, \quad \hat{\Psi}_{i34} = -R - X^T, \quad \hat{\Psi}_{i44} = -X - X^T,$$

then the filtering error system (6) is globally uniformly asymptotically stable with a weighted H_∞ performance index $\hat{\gamma}$ for any switching signal satisfying (12). Furthermore, the filter matrices are given as

$$\begin{bmatrix} A_{fim} & B_{fim} \\ E_{fim} & 0 \end{bmatrix} = \begin{bmatrix} X^{-T} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} A_{Fim} & B_{Fim} \\ E_{Fim} & 0 \end{bmatrix}. \quad (47)$$

Proof Denote the left side of (33) and (34) as $\bar{\Theta}_i(t)$, $\hat{\Theta}_i(t)$, respectively. If the conditions in Theorem 1 hold, the following inequalities can be obtained

$$\bar{\Theta}_i(t) = \sum_{m=1}^{r}\sum_{l=1}^{r} h_{im}(t)h_{jl}(t)\bar{\Theta}_{ijml} < 0,$$

and

$$\hat{\Theta}_i(t) = \sum_{m=1}^{r}\sum_{l=1}^{r} h_{im}(t)h_{il}(t)\hat{\Theta}_{iiml}$$
$$= \sum_{m=1}^{r} h_{im}^2(t)\hat{\Theta}_{iimm} + \sum_{m=1}^{r}\sum_{m<l}^{r} h_{im}(t)h_{il}(t)$$
$$(\hat{\Theta}_{iiml} + \hat{\Theta}_{iilm}) < 0.$$

From Lemmas 1, 2 and 3, it can be concluded that the asynchronous H_∞ filter design problem for the switched T–S fuzzy system (2) is solved.

Finally, by (39), the filter matrices can be obtained as follows

$$\begin{bmatrix} A_{fim} & B_{fim} \\ E_{fim} & 0 \end{bmatrix} = \begin{bmatrix} Q_4^{-T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{Fim} & B_{Fim} \\ E_{Fim} & 0 \end{bmatrix} \begin{bmatrix} Q_4^{-1}Q_3 & 0 \\ 0 & I \end{bmatrix}$$
$$= \begin{bmatrix} \left(Q_4^{-1}Q_3\right)^{-1}X^{-T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{Fim} & B_{Fim} \\ E_{Fim} & 0 \end{bmatrix} \begin{bmatrix} Q_4^{-1}Q_3 & 0 \\ 0 & I \end{bmatrix}. \quad (48)$$

Then, the filter matrices A_{fim}, B_{fim} and E_{fim} in (5) can be written as (48). Setting $Q_4^{-1}Q_3 = I$, we can obtain (47). Then, the filter matrices in (5) can be constructed by (47). The proof is completed. □

Remark 3 Theorem 1 provides a sufficient condition for the existence of the asynchronous H_∞ filter for the switched T–S fuzzy system. If (45) in Theorem 1 is removed, Theorem 1 can also be used to study the H_∞ filter design problem for the switched T–S fuzzy system without asynchronous switching. Then, it can be concluded that the filtering error system without asynchronous switching is globally uniformly asymptotically stable with a weighted H_∞ performance index $\hat{\gamma} = \sqrt{\exp\{\sum_{i=1}^{N}(T_{ai}^*\alpha_i)\}}\gamma$ for any switching signal satisfying $T_{ai} \geq T_{ai}^* = \frac{\ln \mu_i}{\alpha_i}$.

Remark 4 The conclusions of Theorem 1 describe how to obtain the filter gains for switched T–S fuzzy systems with asynchronous switching. All of these conditions have a general form for the asynchronous filter design. Therefore, the presented approach in this paper can also be applied to handle the asynchronous H_∞ filtering problem for other switched systems, such as switched time-delay systems, linear switched systems.

Remark 5 All the parameters (α_i, β_i and μ_i) of Theorem 1 have their physical meaning. Specifically speaking, the parameter $\alpha_i > 0$ denotes the decline rate of the Lyapunov function, which corresponds to the convergence rate of the switched system in the synchronous state. The parameter $\beta_i > 0$ denotes the increasing rate of the Lyapunov function when the switched system is running in the asynchronous state. The parameter $\mu_i \geq 1$ represents the increasing rate bound from the j subsystem to the i subsystem. In practice, all these parameters can be designed in a proper range. The design of these parameters increases the flexibility of our approach.

4 Example

In this section, a practical example is used to show the effectiveness of the obtained results. Consider a continuous stirred tank reactor where an exothermic, irreversible reaction of the form $A \rightarrow B$ happens. There are two different feeding streams to feed the reactor, and these two feeding streams are selected by a selector. In other words, the reactor has two modes with respect to the feeding stream. For each mode of operation, the mathematical model for the process has the following differential equations [29].

$$\dot{C_A} = \frac{F_\sigma}{V}(C_{A\sigma} - C_A) - k_0 e^{-E/RT_R}C_A,$$
$$\dot{T_R} = \frac{F_\sigma}{V}(T_{A\sigma} - T_R) + \frac{-\Delta H}{\rho c_p}k_0 e^{-E/RT_R}C_A + \frac{Q_\sigma}{\rho c_p V}, \quad (49)$$

where C_A represents the concentration of the species A, T_R denotes the temperature of the reactor, Q_σ is the heat removed from the reactor, V is the volume of the reactor, $k_0, E, \Delta H$ are the pre-exponential constant, the activation energy, and enthalpy of the reaction, c_p, ρ are the heat capacity and fluid density in the reactor, and $\sigma(t) \in \{1, 2\}$ is the switching signal which is a discrete variable. The values of all process parameters can be found in [29].

The system (49) is a nonlinear switched system. Substituting the process parameters into equation (49), the following two subsystems can be obtained

Subsystem 1: $(\sigma = 1)$

$$\dot{C_A} = -0.0334C_A - 1.2 \times 10^9 e^{-10000T_R}C_A + 0.026386,$$
$$\dot{T_R} = -0.0334T_R + 2.4 \times 10^{11} e^{-10000T_R}C_A + 11.77684 + \frac{Q_\sigma}{23.9},$$

Subsystem 2: $(\sigma = 2)$

$$\dot{C_A} = -0.0167C_A - 1.2 \times 10^9 e^{-10000T_R}C_A + 0.0167,$$
$$\dot{T_R} = -0.0167T_R + 2.4 \times 10^{11} e^{-10000T_R}C_A + 5.177 + \frac{Q_\sigma}{23.9}.$$

When $Q_\sigma = 0$, the two steady states can be easily obtained as $(C_A, T_R)_1 = (0.57, 395.3)$ and $(C_A, T_R)_2 = (0.738, 509.12)$.

Using the T–S fuzzy model [13] and from [29], the nonlinear switched system (49) can be approximated by the following subsystem S_σ:

Subsystem S_σ:

Rule 1: IF the concentration of the species A is $M_{\sigma 1}(x1)$ (i.e.,$x_1(t)$ is 0.57), THEN

$$\delta\dot{x}_\sigma(t) = A^c_{\sigma 1}\delta x(t), \tag{50}$$

Rule 2: IF the concentration of the species A is $M_{\sigma 2}(x1)$ (i.e.,$x_1(t)$ is 0.738),THEN

$$\delta\dot{x}_\sigma(t) = A^c_{\sigma 2}\delta x(t), \tag{51}$$

where $\sigma \in \{1,2\}$ represents the subsystem subscript, $x_\sigma(t) = [x^T_{\sigma 1}(t), x^T_{\sigma 2}(t)]^T = [C^T_A, T^T_R]^T$, $\delta x_\sigma(t) = x_\sigma(t) - x^d_\sigma$, and x^d_σ is the stationary point of the subsystem σ. It was shown in [29] that the $A^c_{\sigma 1}$ and $A^c_{\sigma 2}$ have the following values

$$A^c_{11} = \begin{bmatrix} -4.5803\times 10^{-2} & 6.6748\times 10^{-5} \\ 2.4807 & -3.61\times 10^{-3} \end{bmatrix},$$
$$A^c_{12} = \begin{bmatrix} -3.5728 & 5.1826\times 10^{-5} \\ 707.89 & -0.010268 \end{bmatrix},$$
$$A^c_{21} = \begin{bmatrix} -0.029103 & 5.1833\times 10^{-5} \\ 2.4807 & -0.0036045 \end{bmatrix},$$
$$A^c_{22} = \begin{bmatrix} -3.564 & 5.1826\times 10^{-5} \\ 706.13 & -0.010265 \end{bmatrix}.$$

Note that both of the models in (50) and (51) are unstable. However, in order to use the filtering techniques, all the models of the switched T–S fuzzy system (2) should be stable. Different from [29], we assume that each model is firstly stabilized by some control law and get a closed-loop switched system $\delta\dot{x}_\sigma(t) = A_{\sigma m}\delta x(t)$. Then, the closed-loop switched T–S fuzzy system can be obtained with the following matrices

$$A_{11} = \begin{bmatrix} -0.0458 & -0.002 \\ 2.4752 & -1.436 \end{bmatrix},\quad A_{12} = \begin{bmatrix} -3.5728 & -0.002 \\ 707.8839 & -1.658 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} -0.291 & -0.002 \\ 2.4752 & -1.437 \end{bmatrix},\quad A_{22} = \begin{bmatrix} -3.564 & -0.002 \\ 706.1236 & -1.743 \end{bmatrix}.$$

Suppose other system parameters to be

$$B_{11} = B_{12} = B_{21} = B_{22} = [0.01;\quad 0],$$
$$C_{11} = C_{12} = C_{21} = C_{22} = [0\quad 0.01],$$
$$D_{11} = D_{12} = D_{21} = D_{22} = 0,$$
$$E_{11} = E_{12} = E_{21} = E_{22} = [1\quad 0].$$

So the measurements are $y_1(t) = 0.01x_{12}(t)$, $y_2(t) = 0.01x_{22}(t)$, and the signals to be estimated are $z_1(t) = x_{11}(t)$, $z_2(t) = x_{21}(t)$.

The normalized membership functions for Rule 1 and Rule 2 of the two subsystems are taken as

$$\begin{cases} h_{11}(x_1) = h_{21}(x_1) = \dfrac{\arctan(50*(x_1 - 0.654)) + \pi/2}{\pi}, \\ h_{12}(x_1) = 1 - h_{11}(x_1), h_{22}(x_1) = 1 - h_{21}(x_1). \end{cases}$$

Let $\alpha_1 = 0.02$, $\alpha_2 = 0.01$, $\beta_1 = 0.012$, $\beta_2 = 0.01$, $\mu_1 = 1.5$, $\mu_2 = 1.5$, $\gamma = 2$, $\mathcal{T} = 5$, and $E_{f11} = E_{f12} = E_{f21} = E_{f22} = [1\quad 0]$. By Theorem 1, we can get $T^*_{a1} = 28.2733$, $T^*_{a2} = 35.3643$ and $\hat{\gamma} = 3.6826$. Using the LMI toolbox to solve the LMIs (43)-(46), and by (47), the filter parameters can be obtained as

$$A_{f11} = \begin{bmatrix} -0.9609 & 1.1371 \\ 62.0370 & -235.6692 \end{bmatrix},$$
$$A_{f12} = \begin{bmatrix} -1.1573 & 1.1411 \\ 70.4219 & -236.6697 \end{bmatrix},$$
$$A_{f21} = \begin{bmatrix} -0.9762 & 1.1363 \\ 59.9555 & -235.6900 \end{bmatrix},$$
$$A_{f22} = \begin{bmatrix} -1.2888 & 1.1410 \\ 81.7358 & -236.5881 \end{bmatrix},$$
$$B_{f11} = \begin{bmatrix} 114 \\ -23462 \end{bmatrix},\quad B_{f12} = \begin{bmatrix} 114 \\ -23568 \end{bmatrix},$$
$$B_{f21} = \begin{bmatrix} 114 \\ -23460 \end{bmatrix},\quad B_{f22} = \begin{bmatrix} 115 \\ -23560 \end{bmatrix}.$$

The initial conditions are assumed to be $x(t_0) = [0.4, 404.9]^T$ and $x_f(t_0) = [0.5, 400]^T$. The disturbance input is assumed to be $w(t) = 0.01\exp(-0.007t)\cos(0.5t)$. With the consideration of asynchronous behavior, the output signal $z(t)$ of the switched T–S fuzzy system and the estimated signal $z_f(t)$ of the filter are shown in Figure 1. The filtering error $e(t)$ of the filtering error system with asynchronous behavior is given in Figure 2. As shown in Figure 1 that after every switching, the output signal $z_f(t)$ of the designed filter can estimate the output signal $z(t)$ of the switched T–S fuzzy system quickly. The simulation results demonstrate the effectiveness of our method.

5 Conclusion

In this paper, the H_∞ filtering problem has been investigated for the switched T–S fuzzy systems with asynchronous switching. Every subsystem of the studied switched systems is represented by the T–S fuzzy model. Using the multiple Lyapunov functions approach and mode-dependent average dwell time technique, a more general result is obtained. Based on the obtained results, the desired filters are designed to guarantee the filtering error system to be globally uniformly asymptotically stable with a weighted H_∞ performance index. It is also remarked that the obtained results can be reduced to study

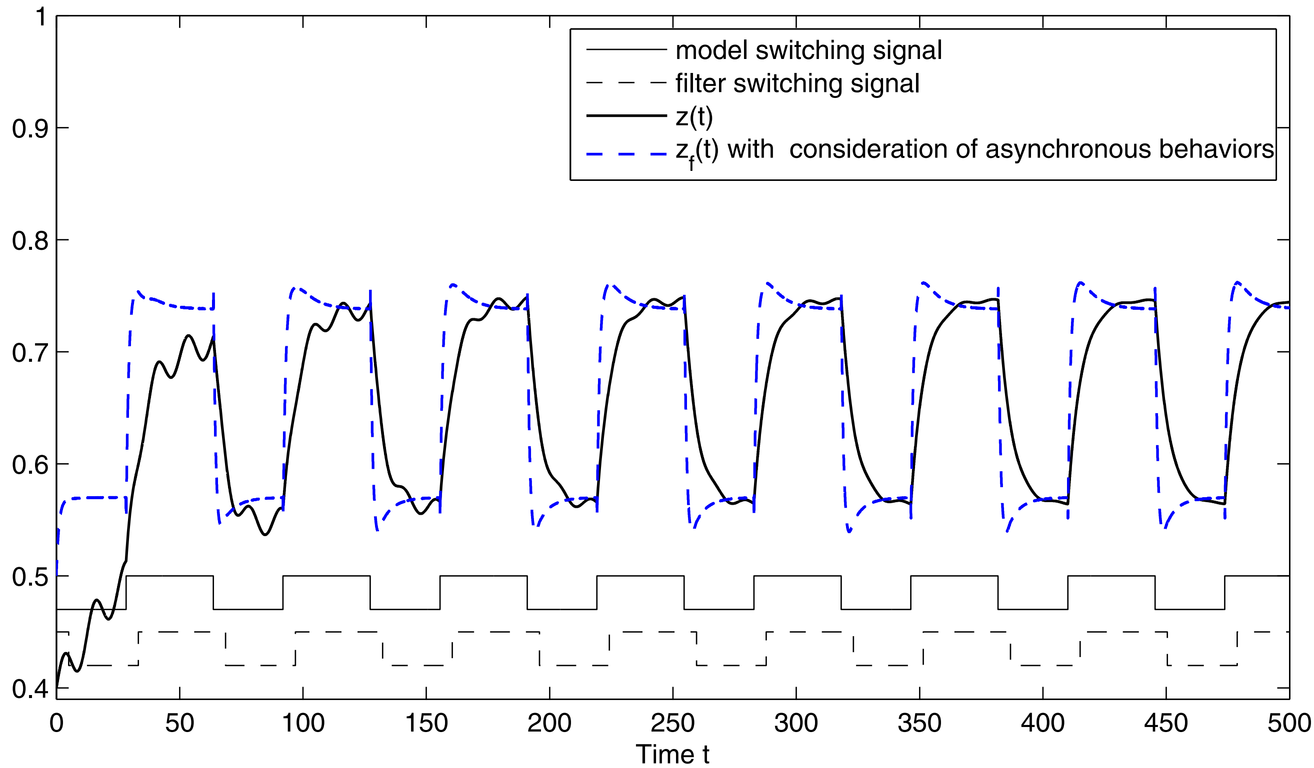

Fig. 1 Output signal $z(t)$ of the switched T–S fuzzy system and the output signal $z_f(t)$ of the designed filter

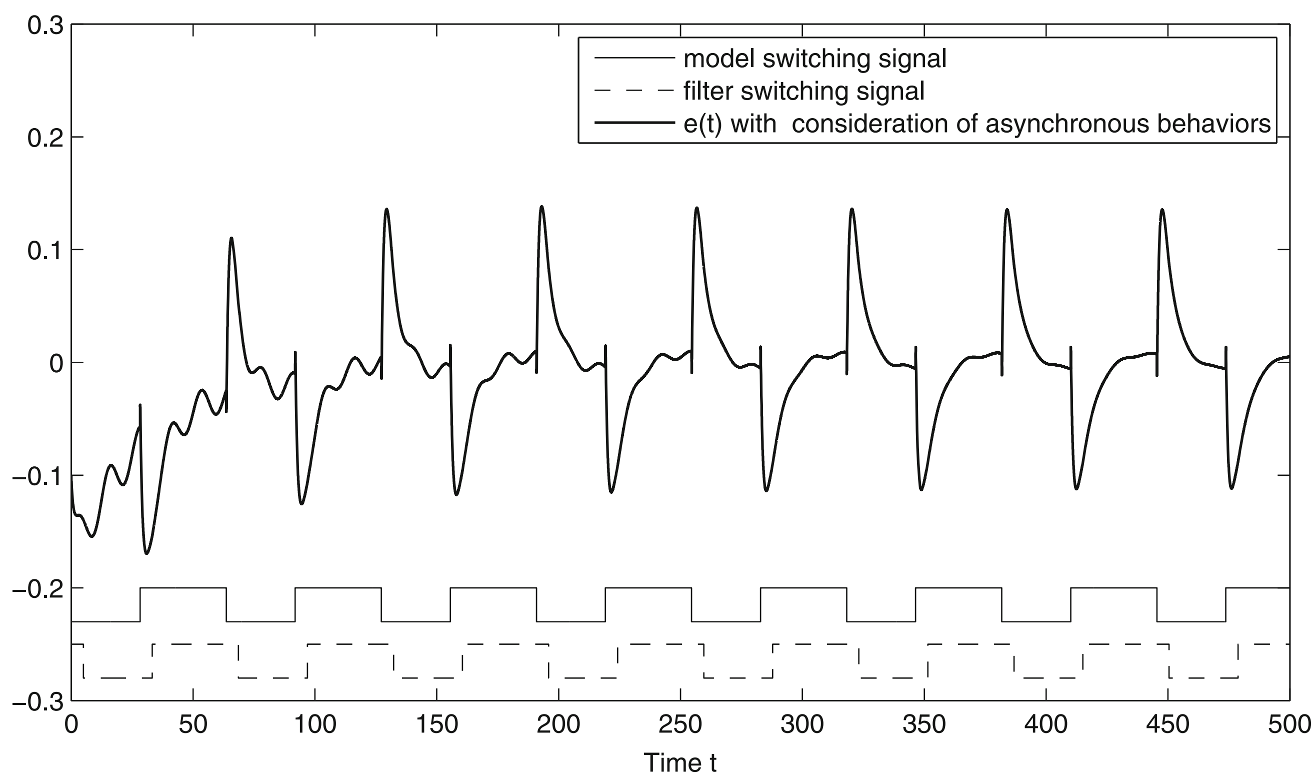

Fig. 2 The filtering error $e(t)$

the fuzzy H_∞ filtering problem for switched systems without asynchronous switching. Finally, a practical example is provided to illustrate the effectiveness of the proposed method.

In our work, all the premise variables are assumed to be measurable. Using the sector nonlinearity approach to derive a T–S formulation from a nonlinear model, the T–S fuzzy systems with unmeasurable premise variables are likely to be considered. This case is much more complex than the condition considered in our work. How the asynchronous filter design for switched T–S fuzzy systems in this case can be implemented has to be left as a future work.

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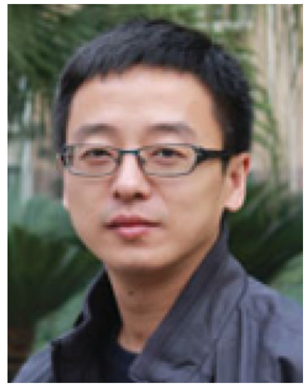

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