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A Fuzzy-Based Risk Assessment Methodology for Construction Projects Under Epistemic Uncertainty

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Abstract In this paper, a methodology for construction projects risk assessment under epistemic uncertainty (i.e., uncertainty arising from lack of data/knowledge) has been proposed. In practice, as the sufficient data from historical sources for probabilistic analysis are quite difficult to obtain, qualitative risk assessment methodologies based on expert's judgments are commonly used in construction industry. However, these insufficient probabilistic data combined with experts' judgments can be used in the risks evaluation process to reduce uncertainties and biasness. All the methodologies developed so far have assumed that the degrees of uncertainties (i.e., levels of uncertainties) involved in individual risk event are equal. However, in practice, the degree of uncertainties that involved in each risk event may vary due to the variation in the availability or quality of data obtained from multiple sources (e.g., from experts' opinions and past data from similar projects). Therefore, evaluation of risks considering the degree of uncertainty involved in individual risk events may assist project manager in setting-up response strategies to mitigate threat to the project objectives. This paper proposes a risk assessment methodology using triangular fuzzy numbering system to compute risk value by combining expert's opinion and insufficient historical data. A modified form of general ramp-type fuzzy membership function for quantification of uncertainty range of each risk event and an extended VIKOR method for risk ranking with these uncertainty ranges have been used. The most notable difference with other fuzzy risk assessment methods is the use of algorithm to handle the uncertainties involved in individual risk event. The proposed risk assessment methodology is illustrated for two practical example problems: (1) a steel-frame structured building and (2) a rehabilitation project of a building.

Keywords Risk assessment · Epistemic uncertainty · Degree of uncertainty · Fuzzy logic · VIKOR

1 Introduction

The construction industry is plagued by various risks which are often responsible for poor performance with increasing cost and time delay, even project failure [37]. Risk is inherent in all projects and can never be eliminated completely. However, it can be managed to reduce its effects to an acceptable level. Therefore, a systematic and proactive risk management framework is needed to enhance the chance of success and improve performance. All potential risks and uncertain factors should be identified at the initial phase of the project life cycle and managed effectively for avoiding potential loss. The successful management of risk requires the identification of risks, assessment of risk magnitude and implementation of response strategies to reduce threats to the project objectives [10].

A risk is an uncertain future event that has negative impact on the project objectives, such as scope, schedule, cost or quality [24]. Other definitions of risk are also available in the literature, for example, "risk is the potential barrier for project completion and achieving

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goal" [17], "the possibility of financial losses, physical damages or injuries, delays and detrimental events occurring to the project" [3, 5], "negative deviation from desired level" [12]. Although various researchers define risk in various ways, some common characteristics are found in all definitions. A risk is an unexpected future event with the involvement of substantial uncertainties that have detrimental effects to the project objectives.

Risk is raised when there are uncertainties and these uncertainties are integral part in any risk assessment process [4, 11, 21, 26]. Therefore, without considering the uncertainty that is associated with risks, the risk assessment process will remain inefficient. The uncertainty that involved in risk assessment process can be divided into two types: aleatory uncertainty and epistemic uncertainty [9]. In real-life problems, both types of uncertainties should be accounted for in risk analysis process [1, 9]. Aleatory uncertainty is irreducible and also known as random uncertainty. It refers to the inherent randomness that comes from natural variability. On the other hand, epistemic uncertainty is reducible and often arises from limited or imprecise data, measurement limitations and approximations in mathematical model. By gathering more information and precise data, these types of uncertainty can be reduced. In the literature, epistemic uncertainty previously has been expressed by probability distributions [35], subjective probabilities [25], fuzzy sets [14], etc.

Construction projects are associated with greater inherent risks due to the involvement of many stakeholders [29]. There are many risk sources and factors involved in construction projects that should be identified and assessed for effective risk managements. In risk analysis process, there exist both qualitative and quantitative data. However, in many circumstances, for construction project, it is very hard to obtain sufficient amount of risk data from historical sources due to its non-routine and unique characteristics. Due to the scarcity of sufficient data for probabilistic analysis, construction project risks are being managed based on experts' judgments and experiences [37]. Therefore, the data type for risk studies is mostly qualitative rather than quantitative [18]. Note that this qualitative data may induce imprecision and biasness in the decision-making process [27]. Moreover, these qualitative data are often found as linguistic variables. These linguistic variables express imprecise and vague information instead of sharp numerical values. In these situations, the risk assessment cannot be exact but approximate. Fuzzy set theory [34] provides an effective tool to quantify or capture the vagueness in the linguistic variables. However, depending on the data types, availability and sources, both probabilistic and subjective judgments can be used in risk analysis process simultaneously. Due to its suitability for handling both quantitative and qualitative data, fuzzy logic has been used in risk assessment process for a long time [3, 8, 24, 27, 37]. Concepts other than fuzzy logic have also been used in risk management over the last two decades, such as multicriteria decision-making (MCDM) approach [13, 32, 36, 38], fault tree analysis/event tree analysis [15], Dempster-Shafer theory [33], influence diagram [10], brain storming, merging fuzzy with MCDM tools [16, 22]. In real-life problem, although it is very hard to obtain sufficient statistical data, it may be possible to evaluate risk using these insufficient data merging with subjective judgments. A very few researchers have attempted to develop their risk assessment models using data from both sources: historical data from similar projects and subjective judgments from experts [7] (Choi et al., 2008). This paper evaluates risks considering data from both historical source (i.e., insufficient statistical data) and subjective judgments from many experts under fuzzy environments.

Several studies on the construction project risk assessment are reported in the literature to deal with uncertainty. A model for risk assessment considering associated uncertainty was developed by Zeng et al. [37], based on fuzzy reasoning and Analytic Hierarchy Process (AHP). A Factor Index was introduced in this risk analysis process to evaluate all possible uncertainty associated with two parameters: risk likelihood and risk severity. However, the biasness in subjective judgments is ignored in this model; therefore, evaluation of uncertainty is still not comprehensive. Asan et al. [2] proposed a fuzzy prioritizing approach to project risk management considering the uncertainty raised from subjective judgments. This model gives satisfactory results in respect of handling biasness in subjectivities but still incapable of handling modeling uncertainty. Islam and Nepal [18] proposed a fuzzy-Bayesian model for making realistic budget and avoiding cost overrun by identifying the critical risk in the preliminary stage of the project life cycle. They used expert's judgments for developing the model with Bayesian belief networks, which overcome the drawback of biasness in subjective judgments. However, the uncertainty that involved in probabilistic parameter estimation was not explicitly considered in this risk analysis model. All these existing models are developed under aleatory uncertainty alone, which leads poor performance in practice as real-life problem includes both aleatory and epistemic uncertainty. These models are also found incapable of handling epistemic uncertainty and complex relationships among the risk factors properly. Therefore, as the epistemic uncertainty is reducible, this must be incorporated into the risk assessment framework and managed separately for better project performance.

In the uncertain environment, interval-valued numbers are the simplest way of representing uncertainty in the decision-making problems. Since the assessment of risk is basically the measure of uncertainty, it is difficult or even impossible to express the risk with exact point value. Therefore, in this situation, it is more appropriate to express them as intervals. Basically, risks are assessed for prioritizing them in order to set up risk response strategies against only higher-order risks because of the limitations of time and money. Numerous methods for ranking with interval numbers are available in the literature such as a two-grade approach [31], interval-valued intuitionistic fuzzy sets theory [6, 23, 30], Monte Carlo method [19], extended TOPSIS [20], extended VIKOR [28].

Although there is now an extensive volume of methods available for ranking with interval numbers, all these methods have only been studied with respect to the decision-making problems. However, this concept may also be employed in construction project risk assessment process. Since the construction project is associated with substantial epistemic uncertainties, interval number can be the way of representing the degree of uncertainties involved in each risk event. All the risk assessment methodologies developed so far have assumed that the degrees of uncertainties involved in individual risk event are equal. However, in practice, the degree of uncertainties involved in each risk event may vary due to the variations in availability or quality of data. None of the existing risk assessment methods take into account the degrees of uncertainties that involved among different risk events as a variable factor and interval numbers to express risk values. Therefore, there is a need for an efficient risk assessment methodology that evaluates construction project risks with interval numbers considering the degree of uncertainty involved in each risk event. This paper proposes a methodology for risk assessment of construction project under epistemic uncertainty using fuzzy concept. The proposed method evaluates construction project risk in terms of uncertainty interval that represents the degree of uncertainties involved in individual risk. It also provides a ranking of risks based on these uncertainty intervals.

The rest of the paper is organized as follows: Sect. 2 describes the proposed risk assessment model using fuzzy logic under epistemic uncertainty. In Sect. 3, two case problems: (1) a steel-frame structured building and (2) a rehabilitation project of a building are used to illustrate the proposed methodology. Results and discussion are given in Sect. 4. Section 5 provides conclusions and suggestions for future work.

2 Proposed Risk Assessment Model

A typical risk management process consists of four steps: risk identification, risk assessment, risk response, and risk monitoring and controlling. It should cover all aspects of risks in construction project and demonstrate risks with potential causes, effects and their corrective actions. All the previously proposed fuzzy-based risk assessment methodologies have three common steps as follows:

Step 1: *Definition and fuzzification*—all the fundamental parameters are defined basically with vague data or linguistic terms and then these parameters are converted into suitable fuzzy numbers or membership functions (MFs).

Step 2: *Fuzzy inference system*—the relationships between inputs and output parameters are defined by the appropriated fuzzy mathematical operations or *if–then* rules.

Step 3: *Defuzzification*—the output result in the form of fuzzy number is converted into appropriate numerical value that can adequately represent it.

This paper proposes a risk assessment model under epistemic uncertainty based on fuzzy concept as shown in Fig. 1. In this risk assessment framework, the algorithm of risk model consists of four phases: preliminary phase, data collection phase, risk measurement phase and uncertainty measurement phase. In brief, the risk assessment team must go through these four phases to implement the proposed construction project risk assessment model. The following four phases are basically concerned with the following tasks:

Phase 1: The review of risks data, definition of fuzzy linguistic variables and selection of their corresponding fuzzy membership function. Here, in this paper, triangular fuzzy number (TFN) is used to map the membership values to take advantage of its simplicity and familiarity.

Phase 2: Identification of risks sources and gathering risk-related information such as risk likelihood (RL) and risk severity (RS) from diversified sources to reduce biasness.

Phase 3: Application of the appropriate fuzzy operations for aggregation of data obtained from multiple sources and computation of risk value (RV) through fuzzy inference system (FIS).

Phase 4: Determination of uncertainty range of each risk event by selecting appropriate fuzzy membership curve and prioritization of risk events based on their uncertainty ranges.

The details of the risk assessment methodology are described in the following subsections.

2.1 Preliminary Phase

2.1.1 Establish a Risk Assessment Team

Risk assessment process is basically a team work, and its success mainly depends on how well the risk assessment

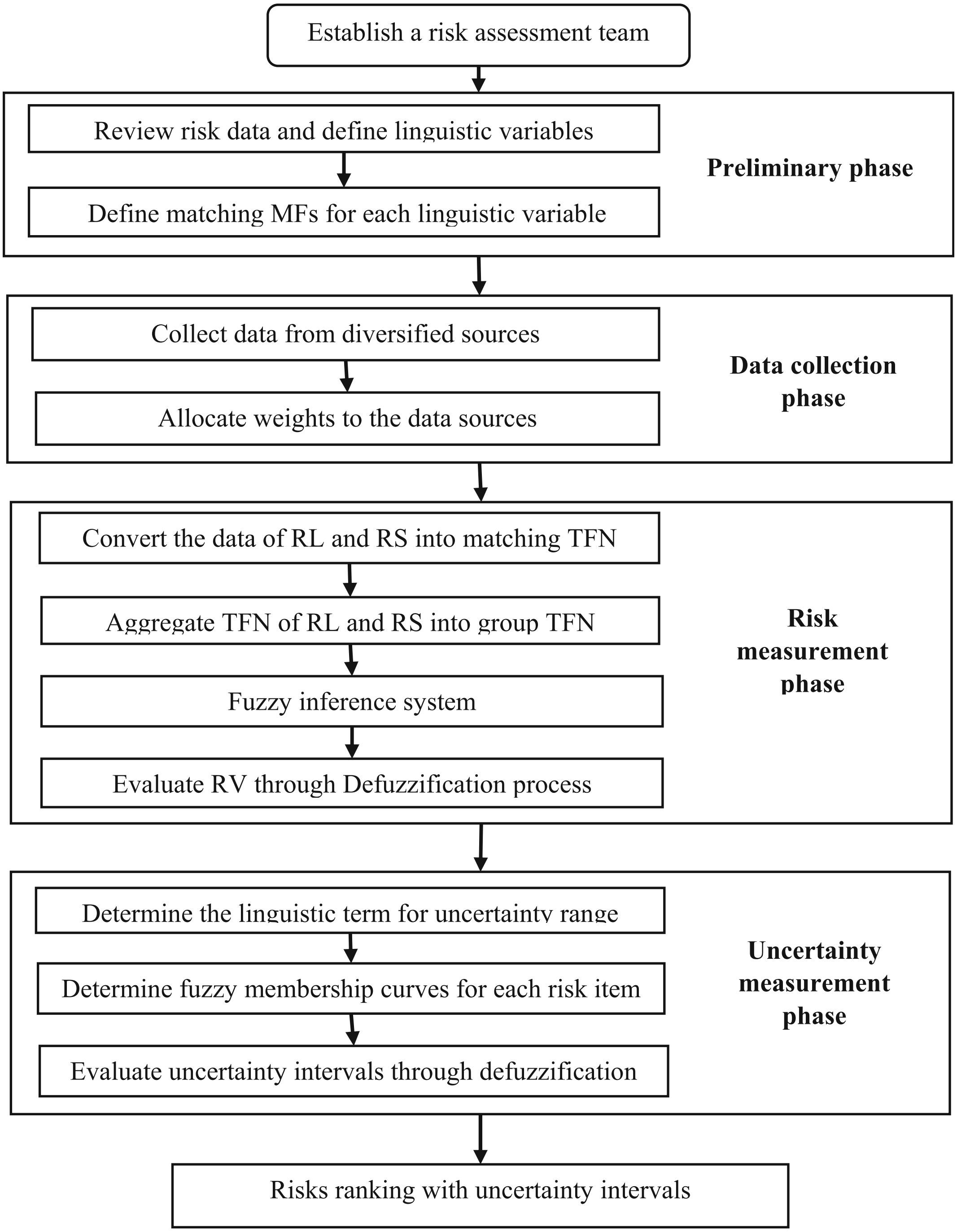

Fig. 1 A fuzzy-based risk assessment model

team is formed. Therefore, selection of members in the risk assessment team is very crucial and needs great attention of senior management. The team should be formed with the experts from different backgrounds and disciplines and have a high degree of knowledge and previous experience of working in similar construction projects. This team may include the following experts: project managers, site engineers, construction managers, project team members, subject specialists, etc. The size of the team is also important; too big can create many opinions which often lead to lack of coordination, and too small may lead to biasness with incomplete viewpoints. The authors suggest that the perfect team size may vary from 3 to 7 members depending on the project's type, size and length. Once the risk assessment team is formed, they will undertake the responsibility of carrying out the whole risk assessment process from the beginning to the very end.

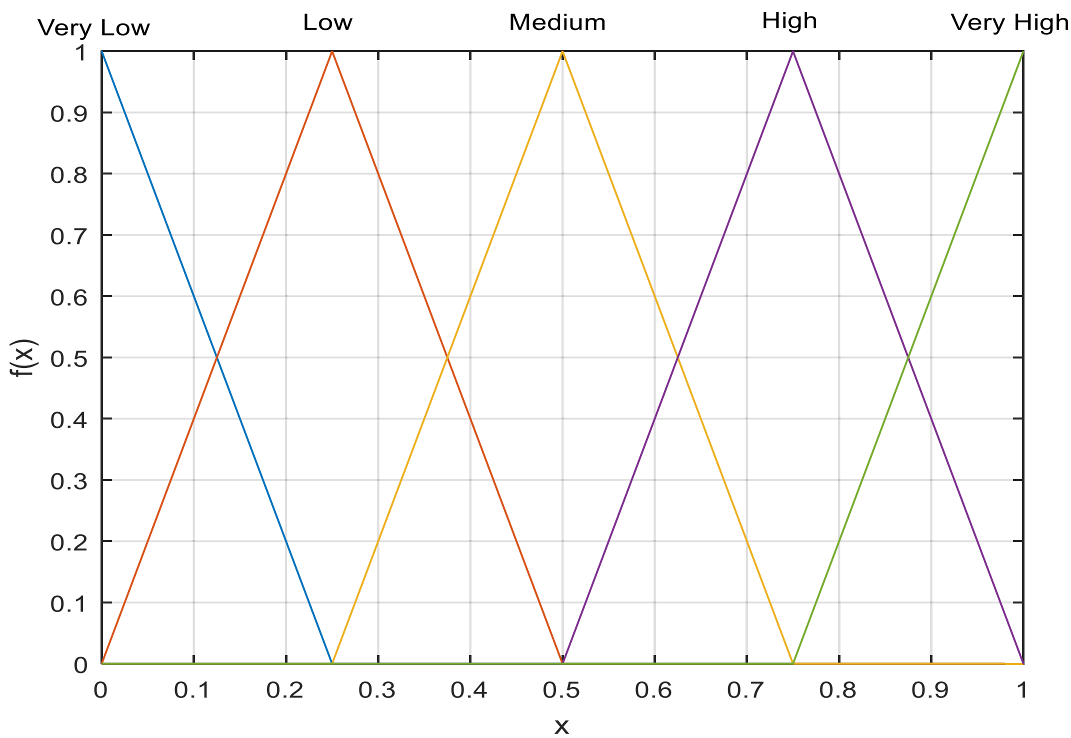

Fig. 2 TFN for linguistic variables

2.1.2 *Review Risk Data and Define Linguistic Variables*

All the members of risk assessment team are required to review the risk-related information and should be clarified by themselves if they have any doubts about the risk assessment procedures. All the risk parameters such as RL, RS and associated linguistic variables should be defined by the risk assessment team at the very beginning of the risk assessment process. It is extremely difficult to quantify the construction project risks with an exact numerical value due to the involvement of huge uncertainties. If the risk assessment group has imprecise, imperfect or lack of information about risks associated with a project, then the assessment of risk cannot be exact but approximate. In these situations, the judgments of the risk assessment group members are expressed by means of linguistic terms instead of numerical values or real numbers. The variable which can take words in natural languages as its value is called linguistic variable. For example, the occurrence probability of a risk event can be expressed with simple linguistic terms such as "high," "low," "very low," and "very high," instead of exact numerical values such as 2/10, 4/100. For evaluating the risk parameters with this risk assessment model, RL and RS are defined by five linguistics terms: "very low," "low," "medium," "high," and "very high."

2.1.3 *Define Matching MFs for Each Linguistic Variable*

The linguistics terms must be converted into a matching fuzzy number by using appropriate conversion scale for numerical quantification of risks. The linguistic variables are characterized by fuzzy membership functions defined in the universe of discourse in which the variable is defined. Various types of fuzzy membership functions are available, such as triangular, trapezoidal, Gaussian and S-shaped MFs. However, triangular and trapezoidal MFs are the most frequently used MFs in construction project risk analysis in practice because of their simplicity. Figure 2 shows the TFN for the associated linguistic variables of RS and RL. It is seen that the TFN for linguistic term "very low" is (0, 0, 0.25), for "low," it is (0, 0.25, 0.5) and so on.

2.2 Data Collection Phase

2.2.1 *Collect Data from Diversified Sources*

In construction project risk analysis, sufficient amount of historical or statistical data is often hard to obtain. Therefore, most of the existing models use only data from expert's judgments. However, these insufficient statistical data along with expert's judgments can be used in risk evaluation process for better performance. Although the projects are characterized as unique, one-time endeavor, there are some common risk events that exist for all types of projects and some are specific to a particular project. Therefore, risk data are available for these common risk events in the historical sources. This paper evaluates risks considering both data from historical source (i.e., insufficient statistical data) and subjective judgments from many experts. If data are collected from m number of experts, then total number of data source will be $n = m + 1$, because data from statistical source should be considered as one source. It is important to note that data should be collected from as diversified and multiple sources as possible to reduce biasness, because different experts will see the problems from their own viewpoints. Here, the term diversified is used to mean the experts from different backgrounds and sectors.

2.2.2 Allocate Weights to the Data Sources

As different sources of data have different impacts on the final decision, weights are introduced into the project risk analysis model. Weights (Ws) will be allocated to experts on the basis of experience, knowledge and expertise and to the statistical source on the basis of data quality, quantity and credibility. If data are collected from n number of sources, then the kth data source S_k is assigned a weight factor W_k, where $W_k \in [0, 1]$, and $W_1 + W_2 + \cdots + W_n = 1$.

2.3 Risk Measurement Phase

2.3.1 Convert the Data of RL and RS into Matching TFN

In this step, all the risk data related to RL and RS obtained from experts' opinions and historical source should be converted into appropriate fuzzy number. In this risk assessment model, TFN is used for its simplicity and popularity. Experts' judgments in the form of linguistic variables are needed to convert into matching TFN as defined earlier by the risk assessment team. For example, an expert might say that the occurrence probability for the kth risk event is "high"; then, according to the definition, the matching TFN is (0.5, 0.75, 1.0). Experts are also allowed to give any intermediate values of TFN about RL and RS directly without any help of linguistic variables. Suppose, it is possible to put (0.3 0.4 0.5) directly as TFN for both RL and RS. In case of statistical data source, single numerical values are obtained about RL and RS from probabilistic analysis such as frequency analysis, Monte Carlo simulation, Bayesian approach. Data obtained from probabilistic analysis also need to be converted into TFN to take advantages of merging with TFN obtained from other sources in the aggregation process. If "a" be the measured value of RS or RL by probabilistic analysis, then TFN is converted as (a, a, a). For example, if the occurrence probability of a risk event is found as 0.3, then TFN will be (0.3, 0.3, 0.3).

2.3.2 Aggregate Individual TFN of RL and RS into Group TFN

The aim of this step is to apply appropriate operator to aggregate the individual TFN of RL and RS obtained from various sources into group TFN. The aggregation of TFN scores is performed by applying the fuzzy weighted triangular averaging operator, which is defined by

$$A_k^* = \left(A_{i1}^* \otimes W_1 \oplus A_{i2}^* \otimes W_2 \oplus \cdots \oplus A_{in}^* \otimes W_k\right) \quad (1)$$

where A_k^* is the fuzzy aggregated TFN score and $A_{ik}^*(\text{for } k = 1, 2, \ldots, m+1)$ are the measured TFN of m numbers of experts from diversified field and one from statistical source. Here, $\otimes$ and $\oplus$ denote the fuzzy multiplication and fuzzy addition operators, respectively. W_1, W_2,…, W_{m+1} are the weights allocated to experts, E_1, E_2,…, E_m and $W_1 + W_2 + \cdots + W_{m+1} = 1$.

2.3.3 Fuzzy Inference system

Fuzzy inference system (FIS) is the process of transferring from a given input mapping to an output mapping using fuzzy logic. In the fuzzy inference phase, the aggregated TFNs of RL and RS are converted into matching fuzzy sets of RV. Therefore, this fuzzy inference system has two inputs RL and RS and one output variable RV. Here, the value of the output RV depends on both values of RL and RS. Therefore, according to fuzzy set theory, the logical operation between RL and RS is "fuzzy intersection" or "AND." In other words, according to *truth table* of standard Boolean logic, RV is "truth" when both RL and RS are "truth." The classical fuzzy operator for this function is: *min,* but the fuzzy *T-norm* operator (i.e., triangular norm) enables us to customize the AND operator. The intersection of two fuzzy sets A and B is defined in general by a binary mapping T, which aggregates two membership functions as follows:

$$\mu_{A \cap B}(x) = T(\mu_A(x), \mu_B(x)) \quad (2)$$

where binary operator T represents the *product* of $\mu_A(x)$ and $\mu_B(x)$.

The classical method of fuzzy intersection (i.e., "min") considers only the minimum value of the two input variables. This implies that the value of RV is equal to the minimum value between them that may come from either RL or RS value ignoring the maximum value. However, there is great impact of both inputs RL and RS to the output RV. In this respect, *prod* operator considers the effects of both inputs to the output RV, which is desirable.

2.3.4 Evaluate RV through Defuzzification

Defuzzification of fuzzy numbers is the process of producing non-fuzzy number that is needed for decision making in a fuzzy environment. There are many defuzzification methods available, any one of which can be selected according to the requirements for reflecting the real situation and viewpoints of the decision maker. Centroid, bisector, middle of maximum, largest of maximum, smallest of maximum and α-cut are the very popular defuzzification methods. In this phase, the centroid method is selected as it is relatively easy to apply, which can be mathematically defined as:

$$RV = \frac{\int_0^1 xf(x)dx}{\int_0^1 f(x)dx} \tag{3}$$

where f(x) denotes the membership function of RV.

2.4 Uncertainty Measurement Phase

2.4.1 Determine the Linguistic Variables for Uncertainty Range

This risk assessment model uses both expert's judgments and insufficient historical data in risk analysis. Therefore, uncertainties are involved in both processes: probabilistic analysis and subjective judgments. The uncertainties involved in probabilistic estimations of RV and RS are basically due to (1) unreliable/insufficient data or (2) approximation in statistical analysis methods. On the other hand, the factors influencing the uncertainties in subjective judgments are: (3) the complexity of work/conditions and (4) the level of education and experience of the experts. Based on these four factors, a linguistic variable of "close to ∼" type is determined to consider the degree of uncertainties involved in each risk event as shown in Table 1. It is seen that five linguistic variables such as "very very close to," "very close to," "close to," "fairly close to" and "fairly fairly close to" are used to evaluate a proper uncertainty range. Here, this "close to ∼" type linguistic variables are basically used to mean how close the determined risk value to the actual value (i.e., the RV with zero degree of uncertainty). In general, four possible grades of uncertainties such as "very small," "small," "normal" and "large" are assumed in these four uncertainty factors. Table 1 shows the classification of linguistics variables that represent the degree of uncertainties according to different combinations of the four possible uncertainty grades.

2.4.2 Determine Fuzzy Membership Curve for Each Risk Item

In this step, after determination of appropriate linguistic variables for the degree of uncertainties involved in each risk event, a fuzzy membership curve is drawn based on the determined linguistic variables. The fuzzy membership functions of "close to ∼"type have been developed earlier by Choi et al. (2008) to represent the uncertainty range involved in probability of occurrence. Here, in this paper, these membership functions are used to compute the

Table 1 Factors for determining uncertainty range of RV (proposed by Choi et al. 2008)

Subjective judgments		Probabilistic parameter estimations		Determined uncertainty range
Complexity of work	Level of education, experience and confidence	Unreliable/ insufficient data	Approximation in statistical analysis	
Very small	Very small	Very small	Very small	Very very close
Very small	Small	Very small	Small	
Small	Very small	Small	Very small	Very close
Very small	Normal	Very small	Normal	
Normal	Very small	Normal	Very small	
Small	Small	Small	Small	Close
Small	Normal	Small	Normal	
Normal	Small	Normal	Small	
Very small	Large	Very small	Large	
Large	Very small	Large	Very small	Fairly close
Normal	Normal	Normal	Normal	
Small	Large	Small	Large	
Large	Small	Large	Small	Fairly fairly close
Normal	Large	Normal	Large	
Large	Large	Large	Large	

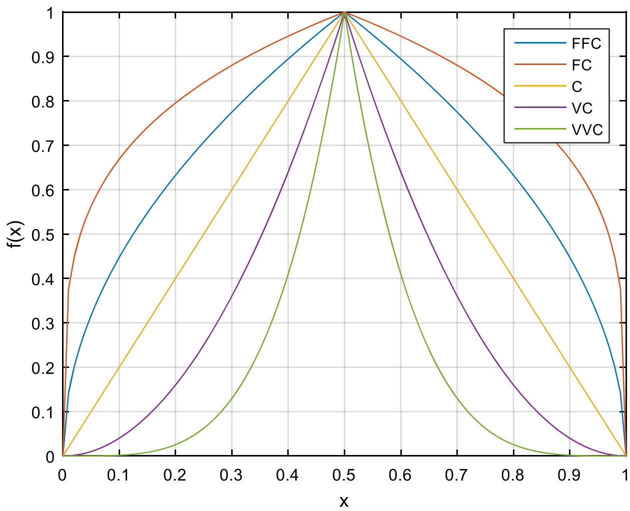

Fig. 3 Membership curves with different degrees of uncertainty

uncertainty interval involved in individual risk event. Figure 3 shows the sample membership curves for a risk event with RV of 0.5 which are drawn for all the five defined linguistic variables as described in the previous subsection. If x be the RV of a risk event, then the fuzzy membership curve is defined for "close to x" type as follows:

$$f(x\prime) = \begin{cases} \left\{\left[\left(2x\prime^{1/y}\right)^{y}\right]^{p}\right\}, & \text{for} \quad 0.0^{y} \le x\prime \le 0.5^{y} \\ \left\{\left[\left(2 - 2x\prime^{1/y}\right)^{y}\right]^{p}\right\}, & \text{for} \quad 0.5^{y} \le x\prime \le 1.0^{y} \end{cases} \tag{4}$$

where x' is the transformed axis such that estimated or determined value of each risk event is located at the mid-point (0.5) of x-axis. Thus, $x^{y}= x'$; where y is calculated by using the value of the fuzzy number at the midpoint, so that $0.5^{y} = x'$. Here, y is the midpoint transfer function and p is the coefficient of power according to linguistic variables.

For example, assume that the risk value (RV) of a risk event A is determined as 0.204 through fuzzy inference system, then the corresponding value of midpoint transfer function y will be 2.293 as shown below:

$0.5^{y} = 0.204$ or $y = 2.293$

Table 2 shows the equations which are associated with the five linguistic variables.

2.4.3 Evaluate Uncertainty Intervals Through Defuzzification

The uncertainty ranges of these linguistic expressions of each risk event are evaluated quantitatively by using α-cut defuzzification method. Although there exist numerous defuzzification methods, especially for this defuzzification step, α-cut method is recommended due to its capability to produce interval data from membership functions. Here, α represents the degree of membership functions or belief functions. The optimistic decision makers will have higher values of α than the pessimistic decision makers.

Table 2 Membership functions to capture uncertainty ranges [7]

Linguistic variable	Values ($f(x')$)	Limit
Very very close (VVC)	$\left[\left(2x'^{1/y}\right)^{y}\right]^{4}$	$0.0^{y} \le x' \le 0.5^{y}$
	$\left[\left(2 - 2x'^{1/y}\right)^{y}\right]^{4}$	$0.5^{y} \le x' \le 1.0^{y}$
Very close (VC)	$\left[\left(2x'^{1/y}\right)^{y}\right]^{2}$	$0.0^{y} \le x' \le 0.5^{y}$
	$\left[\left(2 - 2x'^{1/y}\right)^{y}\right]^{2}$	$0.5^{y} \le x' \le 1.0^{y}$
Close (C)	$\left(2x'^{1/y}\right)^{y}$	$0.0^{y} \le x' \le 0.5^{y}$
	$\left(2 - 2x'^{1/y}\right)^{y}$	$0.5^{y} \le x' \le 1.0^{y}$
Fairly close (FC)	$\left[\left(2x'^{1/y}\right)^{y}\right]^{1/2}$	$0.0^{y} \le x' \le 0.5^{y}$
	$\left[\left(2 - 2x'^{1/y}\right)^{y}\right]^{1/2}$	$0.5^{y} \le x' \le 1.0^{y}$
Fairly fairly close (FFC)	$\left[\left(2x'^{1/y}\right)^{y}\right]^{1/4}$	$0.0^{y} \le x' \le 0.5^{y}$
	$\left[\left(2 - 2x'^{1/y}\right)^{y}\right]^{1/4}$	$0.5^{y} \le x' \le 1.0^{y}$

2.5 Risk Ranking with Uncertainty Intervals

In this section, risks are ranked based on uncertainty range by applying extended VIKOR method which is developed by Sayadi et al. [28]. They proposed the model for the purpose of solving general MCDM problems. However, in this risk analysis model, the problem is more specific and ranking or prioritizing of risks is made based on only uncertainty interval of each risk event. Therefore, a minor modification is made in order to simplify the model for ranking risks with the help of uncertainty intervals. The simplified extended VIKOR method consists of the following steps:

Step 1: Determine the positive ideal solution (PIS) and negative ideal solution (NIS).

$$A^{*} = \left\{x_{1}^{*}, x_{2}^{*} \ldots x_{n}^{*}\right\} = \left\{\left(\min_{i} x_{ij}^{L} | j \in J\right)\right\}, \quad j = 1, 2, \ldots n \tag{5a}$$

$$A^{-} = \left\{x_{1}^{*}, x_{2}^{*} \ldots x_{n}^{*}\right\} = \left\{\left(\max_{i} x_{ij}^{U} | j \in J\right)\right\}, \quad j = 1, 2, \ldots n \tag{5b}$$

where J denotes cost criteria. A^{*} and A^{-} are PIS and NIS, respectively.

Step 2: In this step, the $\left[S_i^L, S_i^U\right]$ and $\left[R_i^L, R_i^U\right]$ intervals are calculated as follows:

$$S_i^L = \sum_{j \in J} W_j \left(\frac{x_{ij}^L - x_j^*}{x_j^* - x_j^-} \right), \quad \text{where,} \quad i = 1, 2\ldots, m \tag{6a}$$

$$S_i^U = \sum_{j \in J} W_j \left(\frac{x_{ij}^U - x_j^*}{x_j^* - x_j^-} \right), \quad \text{where,} \quad i = 1, 2\ldots, m \tag{6b}$$

$$R_i^L = \max \left\{ \left(W_j \left(\frac{x_{ij}^L - x_j^*}{x_j^- - x_j^*} \right) | j \in J \right) \right\}, \quad \text{where,} \quad i = 1, 2, \ldots, m \tag{7a}$$

$$R_i^U = \max \left\{ \left(W_j \left(\frac{x_{ij}^U - x_j^*}{x_j^- - x_j^*} \right) | j \in J \right) \right\}, \quad \text{where,} \quad i = 1, 2, \ldots, m \tag{7b}$$

Step 3: Compute the interval Q_i = [Q_i^L,$\mathrm{Q_i^U}$], i = 1, 2 … m, by the following equations:

$$Q_i^L = v \frac{(S_i^L - S^*)}{(S^- - S^*)} + (1 - v) \frac{\left(R_i^L - R^*\right)}{(R^- - R^*)} \tag{8a}$$

$$Q_i^U = v \frac{(S_i^U - S^*)}{(S^- - S^*)} + (1 - v) \frac{\left(R_i^U - R^*\right)}{(R^- - R^*)} \tag{8b}$$

where

$$S^* = \min_i S_i^L, \quad \text{and} \quad S^- = \max_i S_i^U \tag{9a}$$

$$R^* = \min_i R_i^L, \quad \text{and} \quad R^- = \max_i R_i^U \tag{9b}$$

where v represents weight of the strategy of "the dominant part of criteria."

Note that in this risk assessment problem, the value of v will always be equal to one, because "uncertainty interval" is the only criterion for ranking the alternatives. For the same reason, the values of the Q_i and R_i interval will be the same as the S_i interval.

Step 3: According to the VIKOR method, the alternative which has minimum Q_i is the best alternative and it is chosen as compromise solution. However, here in risk analysis model, the risks with higher values of Q_i will get higher priority in the ranking order, which is opposite to the VIKOR method because in this risk assessment model, the worst alternative will get higher priority in the ranking. To rank all construction risks with Q_i interval numbers, pairwise comparisons among all risks are made. The next step shows the method for comparison of two interval numbers.

Step 4: Suppose that $[a^L, a^U]$ and $[b^L, b^U]$ are two interval numbers and the maximum interval number has to be chosen from them. Therefore, these two interval numbers may have four possible states:

(a) If there is no intersection between these two interval numbers, the maximum interval is that one which has higher values. In different words: If $a^U \le b^L$, then interval $[b^L, b^U]$ is the maximum one.

(b) If two interval numbers are the same, then two have similar priority.

(c) In circumstances that $a^L \le b^L < b^U \le a^U$, the maximum interval number is computed as follows: If $\beta(b^L - a^L) \ge (1 - \beta)(a^U - b^U)$, then $[b^L, b^U]$ is the maximum interval number, else $[a^L, a^U]$ is maximum interval number.

(d) In circumstances that $a^L < b^L < a^U < b^U$, and if $\beta(b^L - a^L) \ge (1 - \beta)(b^U - a^U)$, then $[b^L, b^U]$ is the maximum interval number, else $[a^L, a^U]$ is maximum interval number.

Here, β is introduced as optimism level of the decision maker $(0 < \beta \le 1)$. The optimistic decision maker will use higher value of β than the pessimistic decision maker. In this situation, the final ranking is obtained by the proposed modified VIKOR method with pairwise comparisons of interval numbers.

Therefore, once the ranking of the construction project risk is obtained by the method described above, risk response strategies are taken against only for the higher-order risks due to the limitation of time and money. In the following section, two practical example problems have been illustrated to demonstrate the applicability of the proposed risk assessment methodology.

3 Numerical Examples

In this section, in order to illustrate the effectiveness and applicability of the proposed risk assessment model, it was applied to two types of building construction projects: (1) a steel-frame structured building in Bangladesh and (2) a rehabilitation project of a building in University of Cartagena, Spain. Although the model is applied to building construction project, the method and procedure can be applied to various types of other construction projects, namely bridges, highways, other types of buildings, with minor modification according to the characteristics of the projects.

3.1 Example-1: A Case Study on Building Construction Project

The studied steel-frame structured commercial building project was located at Ashulia Industrial Area, Savar, Project Risks

- Technical Risks:
  - Design mistakes
  - Design Changes
- Operational Risks:
  - Delays
  - Injuries/Accidents
  - Construction mistakes
- Management Risks:
  - Inexperience team members
  - Lack of adequate process
  - Lack of resource
- Financial Risks:
  - Cost overrun
- Logistics:
  - Raw materials supply delay
- Political:
  - Political instability
- Natural:
  - Flood
  - Heavy rainfall, etc.

Fig. 4 Risks in building construction project

Dhaka. As the construction project is associated with a large numbers of risk events, nevertheless a very few common risk events are considered in this building construction project's risk assessment case study as shown in Fig. 4. After a critical review of these risks data, all the possible risks events that involved in a construction project can be categorized into some major groups such as technical risks, operational risks, managerial risks, political risks. For example, the risk events, *design mistakes* and *design changes* can be grouped into the technical risk, whereas *delays*, *injuries/accidents*, *construction mistakes* are the operational type risk. To demonstrate the proposed model and to simplify the calculations, only the technical and operational risks are assessed in this case study. Rest of the risk groups can also be assessed in the similar way.

The step-by-step risk assessment procedures of the proposed model are discussed in the following subsections.

3.1.1 Preliminary Phase

3.1.1.1 Establish a Risk Assessment Team A risk assessment team of three members were formed for undertaking risk assessment of the building construction project by using the proposed methodology. The team was made by a project manager with 12 years of experience, a construction manager with 10 years of experience and a chief engineer with 15 years of experience. A team leader was selected from the members based on their knowledge, experiences and qualifications. This team has carried out the whole risk assessment process from data collection to the final risk ranking. All the members of the risk assessment team were allowed to give their own judgments about risks as well data obtained from other sources.

3.1.1.2 Review Risk Data and Define the Linguistic Variables First, the risk data of similar previous projects were critically reviewed by the risk assessment team and the potential risks with their sources were identified. Before data collections, all the linguistic variables related to RL and RS were defined by the risk assessment team. These linguistic variables helped the team in collecting data from experts in the form of defined linguistic terms. In this case study, five simple linguistic variables such as "very high," "high," "medium," "low" and "very low" are defined for both RL and RS with different meanings. For clarification, in case of RL, linguistic variable "low" means "unlikely to occur," while for the case of RS, it means "involved small impact." Table 3 describes the parameters that have been used in this case study. The linguistic variables under each parameter with their corresponding interpretations are shown in Table 4.

3.1.1.3 Define Matching MFs for each Linguistic Variable In this step, a matching TFN was defined for each linguistic term to evaluate the risks of building construction project. The matching TFNs for each linguistic variable of both factors RL and RS are shown in the last column of Table 4. It is seen that the matching TFNs are defined for the linguistic terms "very high" as (0.75, 1, 1), whereas for the linguistic term "high," it is (0.5, 0.75, 1) and so on for the rest. Experts were allowed to give their judgments about RL and RS with these defined linguistic terms as well as with TFN directly. For instance, it is possible to put any intermediate value of TFN for both RL and RS, if any expert wishes to do that. Therefore, (0.20, 0.3, 0.45) is also possible to take as TFN for RL or RS. In Table 5, it is seen that expert E_2 gave his judgments about RL and RS for risk

Table 3 Descriptions of WDS, RL, RS and RV

Parameters	General interpretation
Weights to the data sources (WDS)	Different data sources have different impact on risks related decision making. Therefore, weights (W_i) are given to individual data source based on their quality, preciseness and quantity of data. Note that summation of all WDS must be equal to one, i.e., $\sum W_i=1.0$
Risk likelihood (RL)	This parameter denotes how likely a risk event to occur, i.e., probability of occurrence
Risk severity (RS)	It expresses that if a risk occurs, then how much it can affect the project objectives, i.e., consequences of a risk
Risk value (RV)	Here, the parameter RV has been used to present the output of fuzzy inference system, where RL and RS are two input parameters

Table 4 Descriptions of the parameters under WDS, RL, RS and RV

Weights of the data source (WDS)	Descriptions	Weight (W_i)
Expert 1 ($E1$)	Project manager (team leader)	$W_1 = 0.23$
Expert 2 ($E2$)	Construction manager	$W_2 = 0.20$
Expert 3 ($E3$)	Chief engineer	$W_3 = 0.30$
Statistical data (SD)	Data from previous project	$W_4 = 0.27$
Total		$\sum W_i$=1.0
Risk likelihood (RL)	**Descriptions**	**Fuzzy number**
Very low	Very rarely to occur	(0.0, 0.0, 0.25)
Low	Unlikely to occur	(0.0, 0.25, 0.5)
Medium	Occurrence is usual	(0.25, 0.5, 0.75)
High	Very likely to occur	(0.5, 0.75, 1.0)
Very high	Occurrence is almost inevitable	(0.75, 1.0, 1.0)
Risk severity (RS)	**Descriptions**	**Fuzzy number**
Very low	Impact is quite negligible	(0.0, 0.0, 0.25)
Low	Involved small impact	(0.0, 0.25, 0.5)
Medium	Moderate impact is involved	(0.25, 0.5, 0.75)
High	Involved highly impact	(0.5, 0.75, 1.0)
Very high	Very high impact is involved	(0.75, 1.0, 1.0)

Table 5 Aggregated (Ag.) and individual source's TFNs of RL and RS parameters

Risks	Data sources	Measure of RL	Measure of RS
Design mistakes	E_1	(0.25, 0.50, 0.75)	(0.50, 0.75, 1)
	E_2	(0.25, 0.35, 0.50)	(0.50, 0.70, 0.90)
	E_3	(0.20, 0.30, 0.40)	(0.25, 0.40, 0.60)
	SD	(0.30, 0.30, 0.30)	(0.35, 0.35, 0.35)
	Ag.	(0.258, 0.356, 0.474)	(0.384, 0.527, 0.686)
Changes in design	E_1	(0.30, 0.50, 0.75)	(0.40, 0.70, 0.90)
	E_2	(0.30, 0.45, 0.65)	(0.50, 0.70, 0.90)
	E_3	(0.20, 0.35, 0.50)	(0.25, 0.40, 0.60)
	SD	(0.32, 0.32, 0.32)	(0.29, 0.29, 0.29)
	Ag.	(0.275, 0.396, 0.539)	(0.345, 0.498, 0.645)
Delays	E_1	(0.50, 0.75, 1.0)	(0.25, 0.50, 0.75)
	E_2	(0.50, 0.75, 1.0)	(0.50, 0.75, 1.0)
	E_3	(0.50, 0.75, 1.0)	(0.25, 0.50, 0.75)
	SD	(0.63, 0.63, 0.63)	(0.30, 0.30, 0.30)
	Ag.	(0.585, 0.768, 0.9)	(0.314, 0.496, 0.678)
Injuries/accidents	E_1	(0.0, 0.25, 0.50)	(0.0, 0.25, 0.50)
	E_2	(0.25, 0.50, 0.75)	(0.25, 0.50, 0.75)
	E_3	(0.0, 0.25, 0.50)	(0.25, 0.50, 0.75)
	SD	(0.26, 0.26, 0.26)	(0.28, 0.28, 0.28)
	Ag.	(0.12, 0.303, 0.485)	(0.201, 0.383, 0.565)
Construction mistakes	E_1	(0.0, 0.25, 0.50)	(0.0, 0.25, 0.50)
	E_2	(0.25, 0.50, 0.75)	(0.25, 0.50, 0.75)
	E_3	(0.20, 0.40, 0.60)	(0.15, 0.25, 0.45)
	SD	(0.10, 0.10, 0.10)	(0.25, 0.25, 0.25)
	Ag.	(0.137, 0.305, 0.472)	(0.163, 0.3, 0.467)

event "design mistakes" with his own defined TFNs such as (0.25, 0.35, 0.50) for RL and (0.50, 0.70, 0.90) for RS, respectively.

3.1.2 Data Collection Phase

3.1.2.1 Collect Data from Diversified Sources For the purpose of risk assessment of the illustrated building construction project, data were collected from three experts working in diversified working areas to reduce biasness. The first expert is a project manager with 12 years of experience, second expert is a construction manager with 10 years of experience, and third expert is a chief engineer with 15 years of experience. All the collected data are shown in Table 5, in the form of TFN system. Here, E_1, E_2 and E_3 represent the first, second and third experts, respectively. The data from historical source of previously completed similar projects are also taken into account in this risk analysis method. The simple frequency analysis method was employed to analyze the statistical data (SD). A crisp or single numerical value about RL and RS for each risk event was obtained from statistical source by probabilistic analysis. However, this value was also converted into TFN to make ease in calculation with the data from subjective judgment. For example, in Table 5, RL value for design mistakes is found as 0.20 by statistical analysis and then converted to the corresponding TFN value as (0.20, 0.20, 0.20).

3.1.2.2 Allocate Weights to the Data Sources Weights (W_s) were allocated to three experts on the basis of their experience, knowledge and expertise and to the statistical data on the basis of quality, quantity and credibility. These weights are allocated by risk assessment team. Here, for four data sources such as E_1, E_2, E_3 and SD, the weights are determined as W_1, W_2, W_3 and W_4, respectively. Table 4 shows the weights for four data sources as $W_1 = 0.23$, $W_2 = 0.20$, $W_3 = 0.30$ and $W_4 = 0.27$, respectively.

3.1.3 Risk Measurement Phase

3.1.3.1 Convert the Data of RL and RS into Matching TFN The risk data of RL and RS are found in different forms from different sources. For example, data from experts—some were found in the linguistic forms, and some were found as TFNs directly—and data from statistical source were found as point data by probabilistic analysis. Therefore, in this step, all the collected data from different sources in different forms were converted into matching TFN as defined in Sect. 3.1.1.3. Table 5 shows the summary of data from four sources about RL and RS in the converted TFN forms.

3.1.3.2 Aggregate TFN of RL and RS into Group TFN In this step, the collected risk data of RL and RS from four individual sources were aggregated into group TFN. The aggregation of TFN scores was performed by applying fuzzy weighted triangular averaging operator which is defined in Eq. (1) as illustrated in Sect. 2.3.2. This aggregated TFN scores of RL and RS were used as inputs in the next fuzzy inferences phase to evaluate output RV. The aggregated TFNs of RV are shown in Table 5.

3.1.3.3 Fuzzy Inference Phase In the fuzzy inference process, there are two input variables RL and RS and one output variable RV. The aggregated TFNs of RL and RS were converted into TFN of RV through the fuzzy inference system where fuzzy intersection operator was employed. Using Eq. (2), described in Sect. 2.3.3, the input TFNs of RL and RS were converted into output TFN of RV. The RV values in TFN forms are shown in the fourth column of Table 6.

3.1.3.4 Evaluate RV Through Defuzzification Since the output RV of fuzzy inference system is a fuzzy number, an appropriate defuzzification method was employed to convert it into matching numerical value. Center of area or centroid method (Eq. (3)) is applied as defuzzification method to convert the triangular fuzzy number into matching numerical value of RV. The defuzzied RVs are shown in the last column of Table 6.

3.1.4 Uncertainty Measurement Phase

3.1.4.1 Determine the Linguistic Variable for each Risk Event The linguistic variables that represent the degree of uncertainties involved in each risk event were selected based on four factors as described in Sect. 2.4.1. Table 7

Table 6 Evaluation of RV through defuzzification

Risks	RL	RS	TFN of RV	RV
Design mistakes	(0.258, 0.356, 0.474)	(0.384, 0.527, 0.686)	(0.096, 0.188, 0.324)	0.204
Changes in design	(0.275, 0.396, 0.539)	(0.345, 0.498, 0.645)	(0.095, 0.198, 0.348)	0.214
Delays	(0.585, 0.768, 0.9)	(0.314, 0.496, 0.678)	(0.183, 0.381, 0.611)	0.392
Injuries/accidents	(0.12, 0.303, 0.485)	(0.201, 0.383, 0.565)	(0.024, 0.116, 0.274)	0.138
Construction mistakes	(0.137, 0.305, 0.472)	(0.163, 0.3, 0.467)	(0.022, 0.091, 0.221)	0.111

Table 7 Degree of uncertainties involved in each risk event

Risks	Subjective judgments		Probabilistic parameter estimations		Determined linguistic variable
	Complexity of work	Level of education and experience	Unreliable/ insufficient data	Approximation in statistical analysis	
Design mistakes	Small	Large	Small	Large	Fairly close
Changes in design	Very small	Normal	Very small	Normal	Very close
Delays	Very small	Large	Very small	Large	Close
Injuries/accidents	Very small	Small	Very small	Small	Very very close
Construction mistakes	Normal	Large	Normal	Large	Fairly fairly close

shows the determined linguistic variables for five risk events that were selected subjectively considering the four uncertainty factors as described Sect. 2.4.1. For instance, the linguistic variable "fairly close" was selected for the risk event "design mistakes." It means that the calculated RV is fairly close to the actual RV indicating that a high level of uncertainty is involved.

3.1.4.2 Determine the Fuzzy Membership Curve for each Risk Event Fuzzy membership function for each risk event was selected based on linguistic variables as described in Sect. 2.4.2. The fuzzy membership curves for the representation of the degrees of uncertainty for the risk events are shown in Figs. 5, 6, 7, 8 and 9.

3.1.4.3 Evaluate Uncertainty Intervals Through Defuzzification The uncertainty ranges for each risk event were evaluated quantitatively through the application of appropriate defuzzification process. In this step, α-cut defuzzification method was employed because of its pertinence. Here, α represents the degree of belief function that is represented by y axis of the fuzzy membership curve. At $\alpha = 0.8$, uncertainty range for the risk event "delays" was obtained as 0.31–0.48, from the membership curve shown

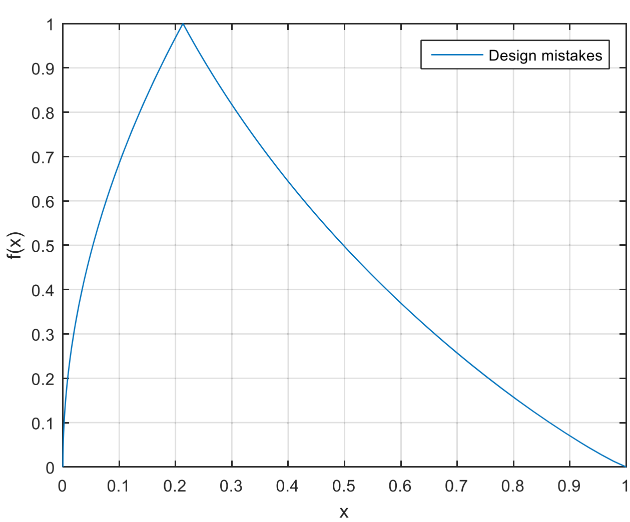

Fig. 5 Membership curve for "design mistakes"

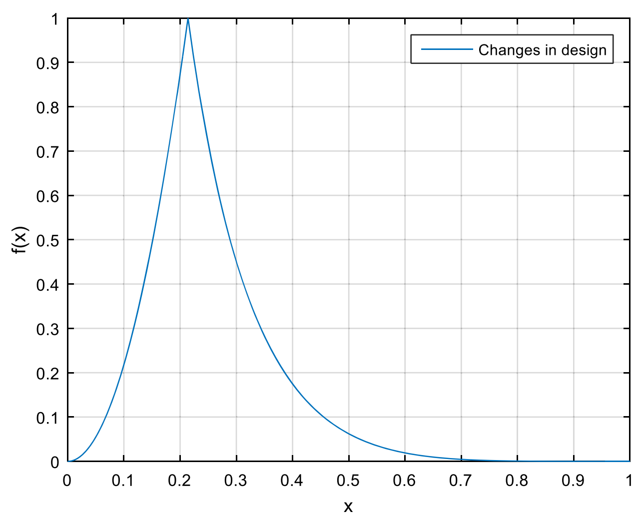

Fig. 6 Membership curve for "changes in design"

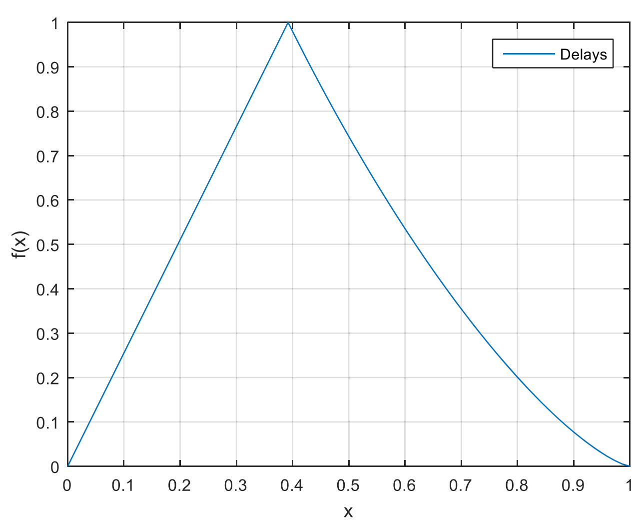

Fig. 7 Membership curve for "delays"

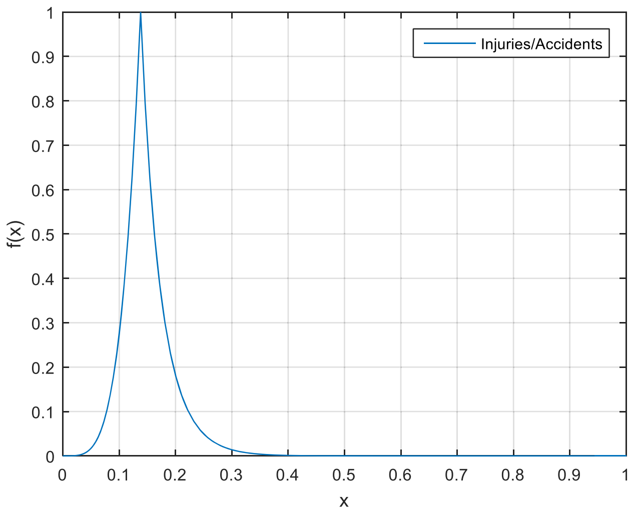

Fig. 8 Membership curve for "injuries/accidents"

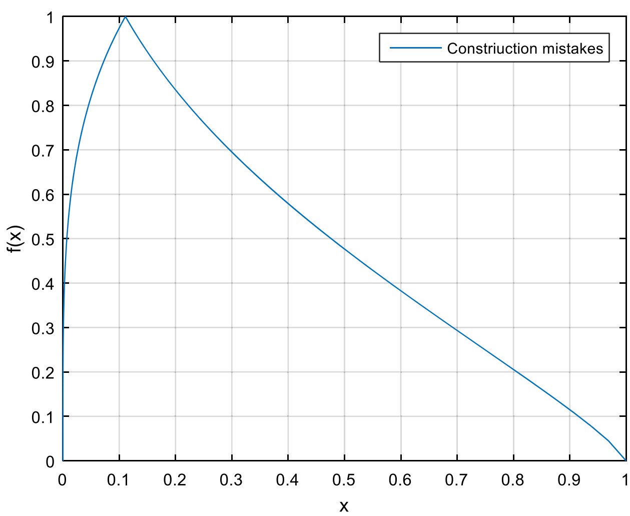

Fig. 9 Membership curve for "construction mistakes"

in Fig. 7. The ranges of uncertainties of all technical and operational risks at $\alpha = 0.8$ are shown in Table 8.

3.1.5 Risk Ranking with Uncertainty Intervals

In this subsection, the proposed modified extended VIKOR method was used to rank the risks with their uncertainty intervals. In order to solve the example problem by modified extended VIKOR method, risk assessment team went through the following steps:

(1) PIS and NIS were computed by Eqs. (5a) and (5b), respectively, as shown in Table 9.

(2) The Q_i intervals were computed by using Eqs. (8a) and (8b). The results are presented in Table 10.

(3) Using step 4, described in Sect. 2.5 and taking optimism level $\beta = 0.8$, final ranking of technical and operational risks was obtained by pairwise comparison as follows:

$$\text{Pairwise comparisons}\left\{\begin{array}{c}\text{Design mistakes} > \text{Changes in design}\\ \text{Design mistakes} < \text{Delays}\\ \text{Design mistakes} > \text{Injuries/Accidents}\\ \text{Design mistakes} > \text{Construction mistakes}\\ \text{Changes in design} < \text{Delays}\\ \text{Changes in design} > \text{Injuries/Accidents}\\ \text{Changes in design} > \text{Construction mistakes}\\ \text{Delays} > \text{Injuries/Accidents}\\ \text{Delays} > \text{Construction mistakes}\\ \text{Injuries/Accidents} > \text{Construction mistakes}\end{array}\right.$$

The final ranking was obtained as follows:

$$\rightarrow \text{Delays} > \text{Design mistakes} > \text{Changes in design}$$
$$> \text{Construction mistakes} > \text{Injuries/Accidents}$$

3.2 Example-2: A Rehabilitation Project of a Building

In order to compare the effectiveness of the proposed risk assessment methodology with the existing similar methods, another case problem on rehabilitation project of a building in University of Cartagena, Spain, was adopted from Nieto-Morote and Ruz-Vila [24] and solved by the proposed methodology.

The project completion time was identified as the critical objective as per project requirements. Therefore, the risk sources which can lead to the time overrun were needed to be assessed for securing the success of the project. There were many possible risk sources, such as lack of resources, design changes, lack of supply quality and so on. This kind of risks was difficult to assess due to the lack of information and uncertainties involved. Thus, a risk assessment group of four experts, formed on the basis of

Table 8 Calculated uncertainty interval for each risk event

Risks	RV	Degree of uncertainty	Uncertainty range
Design mistakes	0.204	Fairly close	0.13–0.298
Changes in design	0.214	Very close	0.185–0.243
Delays	0.392	Close	0.31–0.48
Injuries/accidents	0.138	Very very close	0.13–0.146
Construction mistakes	0.111	Fairly fairly close	0.046–0.22

Table 9 Interval decision matrix and PIS and NIS

Risks	Uncertainty range	PIS and NIS
Design mistakes	0.13–0.298	$A^* = x^* = 0.046$
Changes in design	0.185–0.243	$A^- = x^- = 0.48$
Delays	0.31–0.48	
Injuries/accidents	0.13–0.146	
Construction mistakes	0.046–0.22	

Table 10 Q_i interval numbers

Risks	$[Q_i^L, Q_i^U]$
Design mistakes	[0.194, 0.581]
Changes in design	[0.320, 0.454]
Delays	[0.608, 1.000]
Injuries/accidents	[0.194, 0.230]
Construction mistakes	[0.000, 0.401]

their experience and qualification, identified the risks and constructed a risk hierarchy as shown in Fig. 10. The team was made by a civil engineer, an architect, an archeologist and a project manager; all of them are experts in rehabilitation of buildings. However, the defined risk hierarchy consists of four groups, namely "engineering risks," "execution risks," "suppliers' risks" and "project management risks." For instance, under project management group, there are four major risks which might affect the completion time of the rehabilitation project. These are "lack of adequate process," "lack of resources," "inexperienced team members" and "lack of motivating attitudes." In the following subsection, all the risks are assessed by using the proposed methodology.

3.2.1 Risks Measurement Phase

From the beginning to this phase, the proposed risk assessment methodology is almost similar to the method carried out by Nieto-Morote and Ruz-Vila [24]. However, they used different symbols and techniques for defining the same parameters and operations, respectively. For instance, instead of using risk likelihood (RL), risk severity (RS) and risk value (RV), they used Risk Probability (RP), Risk Impact (RI) and Overall Risk Factor (ORF), respectively. For the data aggregation process, they used simple fuzzy arithmetic average, whereas weighted fuzzy triangular averaging technique has been used in the proposed model. Another difference is that they used trapezoidal fuzzy number (TPFN), while triangular fuzzy number (TFN) has been used in the proposed methodology. However, in both models, centroid method is employed as a defuzzification process for converting fuzzy numbers into sharp numerical value. It is clear that the basic concepts for evaluating RV are quite analogous in both models. Nevertheless, the consideration of degree of uncertainties that involved in individual risk and determining the ranking based on the interval numbers have made the proposed model different from the existing models.

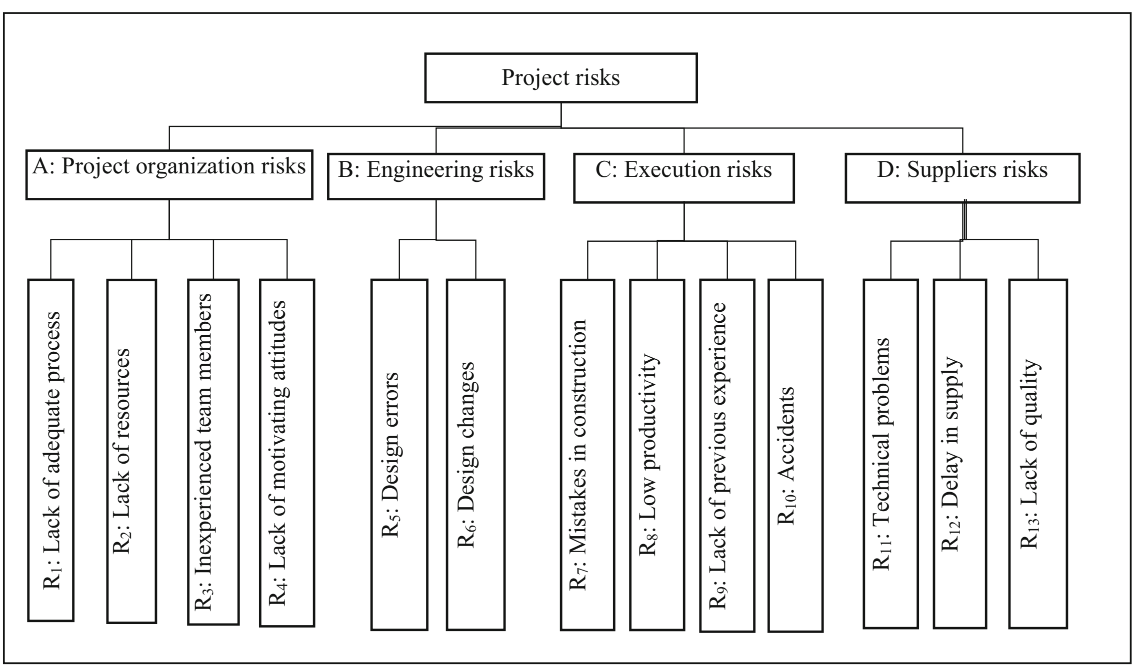

Fig. 10 Hierarchical structure of risks

Table 11 Outputs of fuzzy inference system

Risks	Trapezoidal fuzzy number of RV	RV	Ranking
R_1: Lack of adequate process	(0.0065, 0.0544, 0.1222, 1.2320)	0.6369	1
R_2: Lack of resources	(0.0031, 0.0201, 0.0348, 0.2303)	0.1934	7
R_3: Inexperienced team members	(0.0128, 0.1016, 0.1883, 1.8119)	0.5977	2
R_4: Lack of motivating attitudes	(0.0220, 0.1044, 0.1818, 0.8612)	0.0508	10
R_5: Design errors	(0.0363, 0.2888, 0.4385, 7.2472)	0.3183	3
R_6: Design changes	(0.0239, 0.1050, 0.1584, 1.0540)	0.2460	5
R_7: Mistakes construction	(0.0406, 0.2266, 0.4531, 3.9401)	0.1106	9
R_8: Low productivity	(0.0032, 0.0219, 0.0369, 0.2725)	0.0426	12
R_9: Lack of previous experience	(0.0258, 0.1326, 0.2524, 1.7567)	0.0406	13
R_{10}: Accidents	(0.0086, 0.0549, 0.1783, 1.8078)	0.1806	8
R_{11}: Technical problems	(0.0074, 0.0482, 0.0842, 0.6112)	0.0450	11
R_{12}: Delay in supply	(0.0068, 0.0254, 0.0385, 0.1514)	0.2742	4
R_{13}: Lack of quality	(0.0029, 0.0192, 0.0314, 0.1871)	0.1978	6

Table 12 Measurement of uncertainty intervals and calculations of Q_i interval

Risks	Degree of uncertainty	Uncertainty intervals	PIS and NIS	$[Q_i^L, Q_i^U]$
R_1	Very very close	0.603–0.669	$\begin{cases} A^* = 0.016 \\ A^- = 0.784 \end{cases}$	0.764–0.850
R_2	Close	0.155–0.236		0.181–0.286
R_3	Fairly close	0.378–0.784		0.471–1.000
R_4	Very close	0.043–0.583		0.035–0.738
R_5	Fairly fairly close	0.129–0.568		0.147–0.718
R_6	Very close	0.227–0.287		0.275–0.353
R_7	Fairly close	0.072–0.152		0.073–0.177
R_8	Close	0.032–0.053		0.021–0.048
R_9	Fairly fairly close	0.016–0.087		0.000–0.092
R_{10}	Very close	0.161–0.201		0.188–0.241
R_{11}	Very close	0.040–0.051		0.031–0.045
R_{12}	Close	0.220–0.329		0.266–0.407
R_{13}	Fairly close	0.125–0.290		0.142–0.357

Table 11 shows the results of the rehabilitation project that obtained by Nieto-Morote and Ruz-Vila [24]. The details of the calculation can be found in their work. To apply the proposed model to this project, the values of RV have been taken directly from the original work as they used quite similar method. The values of RV in the third column in Table 11 have been calculated by employing defuzzification method from the trapezoidal fuzzy numbers of RV as given in the second column. These trapezoidal fuzzy numbers were obtained as outputs in fuzzy inference system. While they concluded their work by making the ranking based on RV, the proposed method has calculated uncertainty intervals considering degree of uncertainties involved in each risk for determining the final ranking. The details of calculation of uncertainty intervals are given in the following subsection.

3.2.2 Uncertainty Measurement Phase

Basically, this phase of the proposed risk assessment model makes it different from the existing models. In this phase, degree of uncertainties involved in individual risk is measured in terms of interval numbers. It is quite natural that degree of uncertainties involved in individual risk will vary from risk to risk, owing to the variation in quality and quantity of available data. Therefore, in order to measure the degree of uncertainties involved in the rehabilitation project's risks, linguistic variables for drawing fuzzy membership curves are arbitrarily assumed. The determined linguistic variables are shown in the second column in Table 12. Based on these linguistic variables, the fuzzy membership curve for individual risk is drawn; for the sake of illustration, membership curves for the risks under "project management risks" group are shown in Figs. 11, 12, 13 and 14. Thereafter, uncertainty intervals (shown in

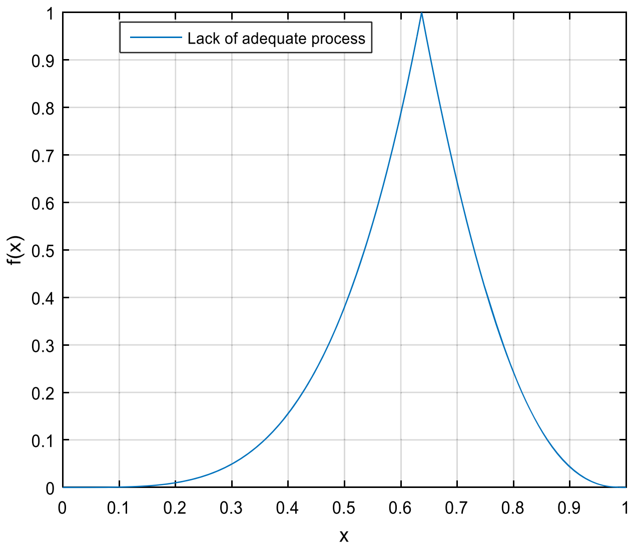

Fig. 11 Membership curve for "lack of adequate process"

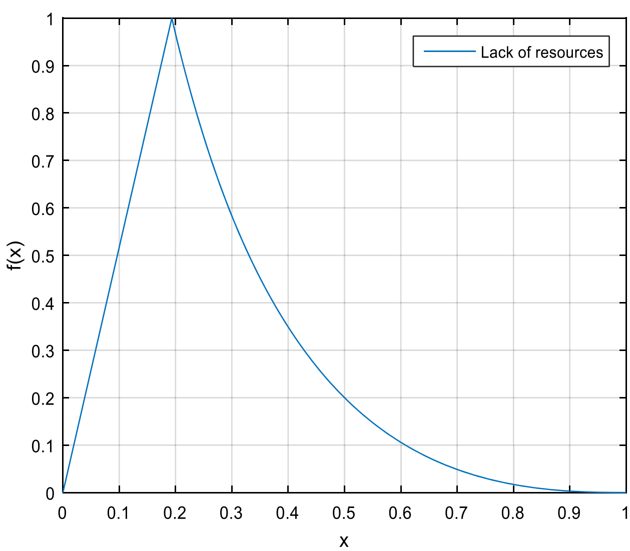

Fig. 12 Membership curve for "lack of resources"

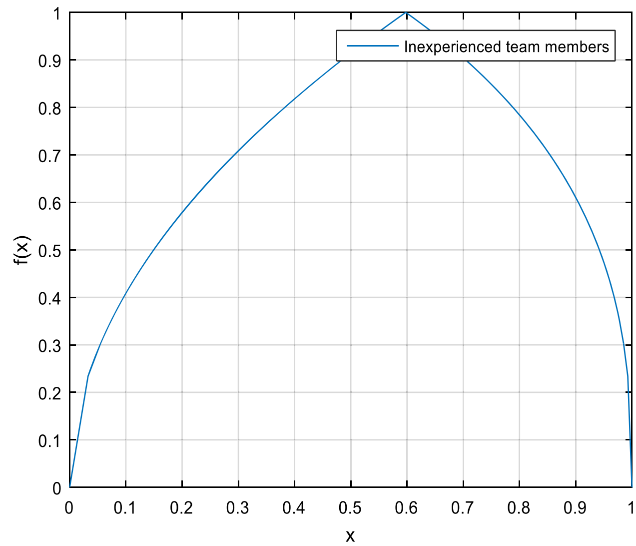

Fig. 13 Membership curve for "inexperienced team members"

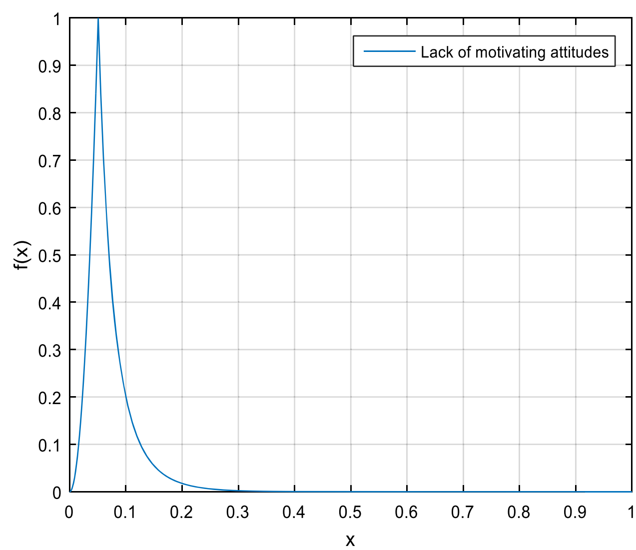

Fig. 14 Membership curve for "lack of motivating attitudes"

the third column in Table 12) are calculated by applying α-cut defuzzification method as described in Sect. 2.4.3. Here, the value of α is taken as equal to 0.8. For making the final ranking based on uncertainty intervals, extended VIKOR method is employed as illustrated in Sect. 2.5. Using Eqs. (5a)–(5b), PIS and NIS are calculated as shown in the fourth column in Table 12. The Q_i intervals given in the last column in Table 12 are calculated using Eqs. (8a)–(8b). Then, the final ranking is made by pairwise comparison as shown later in the last column of Table 14. To compare between two intervals, β value is taken as 0.8.

4 Results and Discussion

Table 13 shows the ranking of operational and technical risks of the studied steel-frame structured building construction project based on both RV and uncertainty intervals. It is seen that ranking based on uncertainty intervals is slightly different from the ranking based on RV. In both cases, the risk "delays" comes first in the ranking, but the ranking of the rest of the risks has been changed. For example, the risk "design mistakes" comes up with position 2 in the ranking based on uncertainty interval, whereas it was at position 3 in the ranking based on RV. The consideration of the degree of uncertainty involved in the individual risk event has brought this change into the results. This is due to the fact that the degree of uncertainty

Table 13 Risk ranking of steel-frame structured building

Risks	Ranking based on RV	Ranking based on uncertainty interval
Design mistakes	3	2
Changes in design	2	3
Delays	1	1
Injuries/accidents	4	5
Construction mistakes	5	4

Table 14 Risk ranking of rehabilitation project of a building

Risks	Ranking based on RV	Ranking based on uncertainty intervals
R_1: Lack of adequate process	1	2
R_2: Lack of resources	7	6
R_3: Inexperienced team members	2	1
R_4: Lack of motivating attitudes	10	12
R_5: Design errors	3	5
R_6: Design changes	5	3
R_7: Mistakes construction	9	9
R_8: Low productivity	12	10
R_9: Lack of previous experience	13	13
R_{10}: Accidents	8	7
R_{11}: Technical problems	11	11
R_{12}: Delay in supply	4	4
R_{13}: Lack of quality	6	8

involved in risk event “design mistakes” is higher than that of the risk “changes in design.”

The same picture in the ranking orders is also observed for the rehabilitation project of a building in University of Cartagena as shown in Table 14. The ranking based on uncertainty intervals, which is obtained by the proposed method, is quite different from the ranking based on RV obtained by the existing similar model. Therefore, both results indicate that there is a great impact of the degree of uncertainty involved in individual risk in case of risk ranking or prioritization. Since the preventive actions are taken against only higher-order risks, a logical question arises that which ranking should be followed for better performances.

The application of the proposed risk assessment methodology to both types of the building construction projects leads to the following conclusions. In real-life problems, the involvement of uncertainty level varies from one risk event to another due to the existence of variations in the data availability, data quality and complexity level. Therefore, its quantification is quite logical and effective for risk ranking. Not only does the proposed risk assessment model consider the mean values of risks, but it also considers their associated uncertainty ranges in the risk ranking process. Thus, the result obtained by the proposed model is expected to be more robust in comparison with the ones obtained by the existing models. In the proposed model, degree of uncertainties involved in individual risk has been expressed with interval number, which basically represents the possible range of deviation from the calculated mean value (i.e., uncertainty range). The higher degree of uncertainties is involved in a risk, and the larger interval number is obtained to express it. Since risk arises as a consequence of uncertainty, risk ranking based on the uncertainty interval using the proposed method is quite reasonable and effective. As the preventive actions are taken against only higher-order risks, this result will provide valuable information to the risk management team in making effective risk response strategies. Note, however, that change in α value may lead to changes in risk ranking.

5 Conclusions

The construction projects are becoming more complex and dynamic in nature day by day. Additionally, the sources of uncertainties are also increasing with the involvement of too many stakeholders. Therefore, project risk management is an essential and crucial task for the project team to avoid project losses. This paper proposes a fuzzy-based risk assessment methodology for construction project incorporating epistemic uncertainties into conventional risk assessment framework. Because of the fact that determining the sharp or exact value of the risk is difficult or even impossible, it is more appropriate to consider them as interval numbers. This paper presents the risks values as interval numbers and ranks them by using modified extended VIKOR method with their associated interval numbers. Basically, risks are assessed at the earlier stage for taking preventive measures against only the identified top-order risks that have tremendous impact on project failure or loss. It is not always possible and not even be a wise decision to take actions against too many risk events because of the limitations in time and budget. Also, impacts of all risks to the project objectives are not severe and considerable.

Based on the results from the case studies, it may be stated that the proposed risk assessment and uncertainty representation methodology is capable of solving any construction risk assessment problem effectively and efficiently. In conclusions, the proposed methodology can be very much useful for risk assessment problem, especially where epistemic uncertainty exists. The proposed methodology is quite general and can be successfully applied to any kind of project risk assessment with only minor modifications.

There are many factors that are responsible for the uncertainty involvement in a project. These factors have not always been dealt with adequately, often resulting in poor performance with increasing costs and delays. In this paper, four major factors are considered in uncertainty evaluation process. Therefore, this research can be expanded with the consideration of more uncertainty factors. In additions, other types of fuzzy membership functions like Gaussian, trapezoidal and *S*-shaped membership functions can also be applied to estimate the uncertainties in risk assessment process. Many methods for ranking with interval numbers are available such as extended TOPSIS, fuzzy intuitionistic approach, which could also be applied in this situation.

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