

Evidential Supplier Selection Based on Interval Data Fusion

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Abstract Supplier selection is a common and important problem, which is usually modeled in a multi-criterion decision-making framework. In this framework, multiple criteria need to be determined and an expert team needs to be constructed. Generally, different criteria and experts always hold different weights. Due to the complexity of the practical problems, sometimes lots of fuzzy and uncertain information exists inevitably and the weights are tough to determine accurately. Interval number is a simple but effective method to handle the uncertainty. However, most current papers transform an interval number to a crisp number to make decisions, which causes loss of information more or less. To address this issue, a new evidential method based on interval data fusion is proposed in this paper. Dempster–Shafer evidence theory is a widely used method in a supplier selection problem due to its powerful ability in handling uncertainty. In our method, the criteria and decision makers are weighted in the form of interval numbers to generate interval basic probability probabilities, which represent the supporting degree to an event in Dempster–Shafer evidence theory. The obtained interval basic probabilities assignments are fused to make a final decision. Due to the properties of interval data, our method has its own specific advantages, like losing less information, providing different strategies for decision making and so on. The new method is proposed aiming at solving multi-criterion decision-making problems in which the

reliability of decision makers or weights of criteria are described in the form of fuzzy data like linguistic terms or interval data. A numerical example of supplier selection is used to illustrate our method. Also, the results and comparison show the correctness and effectiveness of the new interval data fusing evidential method.

Keywords Dempster–Shafer evidence theory · Interval data · Uncertain information · MCDM · Supplier selection

1 Introduction

The supplier selection is a key component of the supply chain management. Since it is a comprehensive decision-making problem, multiple criteria rather than a preferred single criterion are taking into consideration inevitably in real world. An effective approach to modeling these problems is a multi-criterion decision-making (MCDM) framework, which has been one of the fastest-growing research areas [1]. Compared with those approaches based on experience and intuition, MCDM is apparently more objective and reasonable [2, 3]. Granted that many relevant papers have been proposed [4–6], it is still an open issue to make a decision in a MCDM framework.

In the practical situation, a supplier selection problem is often under a condition occupied with fuzzy and uncertain information. As the complexity of a system grows, the uncertainty of problems and the fuzziness of human's thinking constantly increase accordingly. Hence, it is difficult for people to judge and distribute the reliability or importance of information, which is really a necessary part in a MCDM problem. Although decision making and optimization under uncertainty has been heavily studied [7–9], the uncertain information handling is a still open

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issue [10–12]. Among the current methods, Dempster–Shafer (D–S) evidence theory [13, 14] plays an important role [15–17]. Based on it, D number is also an extended tool to handle uncertainty [18, 19]. Due to the efficient ability of modeling and handling uncertainty, evidence theory is widely used in many fields, like reliability analysis [20], relation evaluation [21, 22], game theory [23], pattern recognition [24] and so on, especially in MCDM problems [25–27]. Also, fuzzy set theory (FST) proposed by Zadeh [28] is an effective approach to handling uncertainty [29–32], which has wide applications in MCDM problems [33–35], like supplier selection [36], group decision-making [37, 38], quality assessing [39], fault diagnosis [40], etc. Besides, interval number is another simple but effective tool to deal with fuzzy and uncertain data [41–43], which has been widely applied in decision-making problems [44, 45]. Among them, interval-valued intuitionistic fuzzy set is a well-developed approach [46–48]. However, there is still not an authority definition for the distance of interval-valued intuitionistic fuzzy numbers. Also, the uncertainty of the reliability of information sources is less taken into consideration. In a MCDM problem under uncertainty, the weights of criteria and decision makers (DMs) are often given ahead as fuzzy numbers or interval numbers, or derived by a specific decision matrix [38, 49]. Sometimes, the interval weights can be adopted directly to make a decision by measuring distance between intervals [50, 51]. However under some circumstances where the interval weights are intersected or cannot be ranked, some methods to handle the interval weights are necessary. Most of current methods convert the interval numbers to crisp numbers to help do the final decision making [52, 53]. For example, Deng and Chan [54] converted an interval number to a crisp weight based on a technique for order preference by similarity to ideal solution (TOPSIS) method, and proposed a method to combine D–S evidence theory and fuzzy set theory to address MCDM problems, whose shortcomings will be discussed in detail later in Sect. 3. Generally, interval data represent the uncertain information with less information lost. The information losing is inevitable when converting an interval number to a crisp one, which does disbenefit to a decision making sometimes.

To address the information losing, in this paper, we propose an evidential method based on interval data fusion which keeps relatively full information in the decision-making process to solve a supplier selection problem. In our method, a MCDM problem is modeled in an evidential framework, and a basic probabilities assignment (BPA) is used to represent the supporting degree to suppliers. The weights of DMs and criteria are represented in interval data (or fuzzy data which can be transformed to a interval form). Differing from the methods of converting the

interval data to crisp ones, in our method, interval data are reserved during fusion process to generate interval BPAs. The final decision can be made according to the derived interval BPAs, which has some appreciable properties as follows: (a) interval data reflect concrete and detailed information of objects in a great extent as less information is lost. (b) It provides applications of different decision-making criteria, which can be adopted in some specific problems. Sometimes decision can be made based on risk aversion or risk preference strategy. For example, when the system requires extremely high degree of accuracy, risk aversion strategy will be adopted. We can only use the lower limiting value of interval data and abandon the rest information to make a decision. (c) The reliability of information sources or the weights of criteria are allowed to be modeled as both fuzzy linguistic variables and interval numbers. The fuzzy linguistic description can be converted to interval data. This property is quite useful in that not only quantitative data but also qualitative representation is widely used in practical decision-making problems. (d) The fusion of interval BPAs is a generalization of the classical one. Because crisp number is a special form of interval number (like 0.5 can be seen as [0.5,0.5]), which conforms with the universal cognition.

The rest of this paper is organized as follows. The preliminaries of the basic theory employed are briefly presented in Sect. 2. Then, our new fusion method based on interval data is proposed in Sect. 3. Section 4 takes a numerical example of supplier selection to show the efficiency of the method. Finally, the paper is concluded in Sect. 5.

2 Preliminaries

In this section, some preliminaries such as interval number, fuzzy set theory, Dempster–Shafer theory and pignistic probability transformation (PPT) are briefly introduced.

2.1 Interval Number

Definition 1 (*Interval number*) An interval number \tilde{a} is defined as $\tilde{a} = [a^L, a^U] = \{x | a^L \leq x \leq a^U\}$ where a^L is the lower limiting value and a^U is the upper limiting value, while $x \in [0, 1]$.

Let \tilde{a} and \tilde{b} be two arbitrary positive closed interval numbers. The basic algorithm of interval number is given as follows [55]:

$$\tilde{a} + \tilde{b} = [a^L + b^L, a^U + b^U], \quad (1)$$

$$\tilde{a} \times \tilde{b} = [a^L b^L, a^U b^U], \quad (2)$$

$$\tilde{a} \div \tilde{b} = \left[\frac{a^L}{b^L}, \frac{a^U}{b^U} \right], \tag{3}$$

$$k\tilde{a} = [ka^L, ka^U], \tag{4}$$

$$\frac{1}{\tilde{a}} = \left[\frac{1}{a^U}, \frac{1}{a^L} \right]. \tag{5}$$

For \tilde{a} and \tilde{b} , let norm $\|\tilde{a} - \tilde{b}\| = |a^L - b^L| + |a^U - b^U|$ be the distance between interval numbers \tilde{a} and \tilde{b} . Apparently, the larger $\|\tilde{a} - \tilde{b}\|$ is, the more \tilde{a} and \tilde{b} differ. Especially, interval number \tilde{a} equals \tilde{b} completely when $\|\tilde{a} - \tilde{b}\| = 0$.

2.2 Fuzzy Set Theory

Fuzzy set introduced by Zadeh [28] is an extension of classic set, which is an efficient tool to model linguistic variables.

2.2.1 Fuzzy Number

A fuzzy set allows its members to have different grades of membership in the interval [0,1]. It consists of two components: a set and a membership function corresponding to it.

Definition 2 (Fuzzy set) Let X be the collection of objects denoted by x . \tilde{A} is a fuzzy subset of X , which is a set of ordered pairs [56]:

$$\tilde{A} = \{ (x, \mu_{\tilde{A}}(x) | x \in X) \}, \tag{6}$$

where $\mu_{\tilde{A}}(x)$ is called as the membership function (generalized characteristic function) which maps X to a membership space M . Its range is the subset of nonnegative real members whose supreme is finite.

Definition 3 (Triangular fuzzy number) A fuzzy number is a fuzzy subset of X . Then, a triangular fuzzy number \tilde{A} can be defined by a triplet (a, b, c) shown in Fig. 1. Its membership function is defined as [57]:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x > c \end{cases} \tag{7}$$

2.2.2 Linguistic Variable

Linguistic variable is a variable with linguistic words in a natural language [58]. It is widely used in practical life as one of the most classical fuzzy information [59, 60]. When dealing with conditions which are too complex or ill-

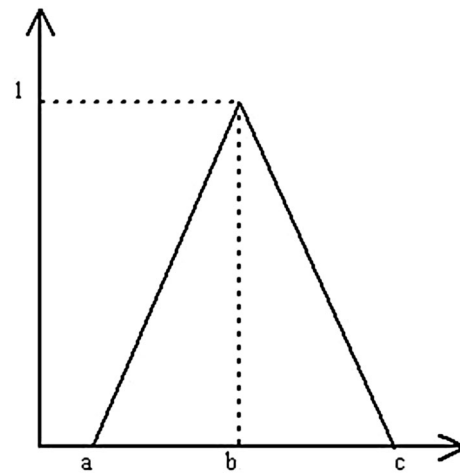


Fig. 1 A triangular fuzzy number

defined to be accurately described in conventional quantitative expressions, it is convenient and reasonable to do a qualitative description. Generally, each linguistic variable corresponds to a fuzzy set. For example, these linguistic variables can be expressed in positive triangular fuzzy numbers [61] as Table 1. Virtually, the concrete models used to represent linguistic items are flexible and changeable. That which kind of representative method will be used depends on the realistic application systems and opinions of experts. Compared with FST, our method adopts the way of converting linguistic variables into interval data.

2.3 Dempster–Shafer Evidence Theory

Dempster–Shafer theory is a mathematical theory of evidence which is a powerful tool to combine separate pieces of evidences. In an evidential framework, the set of possible hypotheses are collectively called the frame of discernment Θ , which is defined as follows [14]:

$$\Theta = \{H_1, H_2, H_3, \dots, H_n\},$$

where n is the number of exclusive and exhaustive elements in Θ . Let $P(\theta)$ denote the power set composed with 2^n subsets of Θ :

Table 1 Linguistic variables for importance in triangular fuzzy number [61]

Terms	Triangular fuzzy number
Very low (VL)	(0, 0.1, 0.3)
Low (L)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
High (H)	(0.5, 0.7, 0.9)
Very high (VH)	(0.7, 0.9, 1.0)

$$P(\theta) = \{\emptyset, \{H_1\}, \{H_2\}, \dots, \{H_n\}, \{H_1 \cup H_2\}, \{H_1 \cup H_3\}, \dots, \theta\},$$

where \emptyset denotes the empty set. Let A be a subset of $P(\theta)$, then a mass function m is defined as $m(A) \in [0, 1]$ to distribute the belief across the frame meeting the following conditions:

$$\sum_{A \subseteq P(\theta)} m(A) = 1,$$

$$m(\emptyset) = 0.$$

Under these circumstances, the mass functions can only be assigned to non-empty subsets and must sum to 1. To combine evidences from multiple sources, for example, to fuse the evidences m_1 and m_2 , Dempster’s combination rule (denoted as $m_{12} = m_1 \oplus m_2$) is defined as:

$$m_{12}(A) = \frac{\sum_{\forall X, Y: X \cap Y = A} m_1(X) \cdot m_2(Y)}{1 - K} \tag{8}$$

with

$$K = \sum_{\forall X, Y: X \cap Y = \emptyset} m_1(X) \cdot m_2(Y), \tag{9}$$

where A is an element of $P(\theta)$, and K is called the conflict coefficient which reflects the conflict degree of evidences in some degrees.

2.4 Pignistic Probability Transformation

The term ‘‘pignistic’’ proposed by Smets is originated from the word pignus, meaning ‘bet’ in Latin. Pignistic probability transformation (PPT) is used to assign the basic probability of multiple-element set to singleton sets. In other word, a belief interval is distributed into crisp ones determined as [62]:

$$\text{Bet}(A_i) = \sum_{A_j \subseteq A_k} \frac{m(A_k)}{|A_k|}, \tag{10}$$

where $|A_k|$ (called as cardinality) denotes the number of elements in set A_k and Eq. (10) is also called as PPT.

3 Proposed Method

In this section, our new method based on interval data fusion is proposed. In general, a basic MCDM problem can be modeled as follows: for a certain problem, there is a committee of k DMs $\{DM_1, DM_2, DM_3, \dots, DM_k\}$ to evaluate it. Each DM holds m alternatives $\{A_1, A_2, A_3, \dots, A_m\}$. For each alternative, n criteria $\{C_1, C_2, C_3, \dots, C_n\}$ are considered to make decisions. Usually, the same

criteria are shared for all the DMs. The following is a succinct model proposed by Hwang and Yoon [63] to express MCDM framework in a matrix.

$$DM_k = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ A_1 & \left[\begin{matrix} r_{11} & r_{12} & \dots & r_{1n} \end{matrix} \right] \\ A_2 & \left[\begin{matrix} r_{21} & r_{22} & \dots & r_{2n} \end{matrix} \right] \\ \vdots & \left[\begin{matrix} \vdots & \vdots & \dots & \vdots \end{matrix} \right] \\ A_m & \left[\begin{matrix} r_{m1} & r_{m2} & \dots & r_{mn} \end{matrix} \right] \end{matrix},$$

where r_{mn} is the rating of alternative A_m with respect to criteria C_n . In our method, r_{mn} is allowed to be both crisp and interval. For now, the faced problem is how to acquire r_{mn} .

In practice, we often consider ranking the alternatives and making the best selection as the final aim. Hence, the final scores of every alternative are not cared too much. Considering that, Hwang and Yoon proposed a technique for order preference by similarity to ideal solution (TOPSIS) method to solve MCDM problems [64]. The principle is that the chosen alternative should have the shortest distance to the positive ideal solution and the farthest distance to the positive ideal solution. TOPSIS method is also widely applied in complex network [65], multi-objective optimization [66], best-worst method [67, 68], etc. Based on TOPSIS, Deng and Chan [54] proposed a method using FST combining with D–S evidence theory. After determining the ideal solution and negative ideal solution, the distances of an alternative between them can be derived. Then, the classical BPAs are used to describe the distance from the alternative to both ideal solution and negative ideal solution.

In the classical TOPSIS method, the weights are in the form of crisp numbers. Hence, Deng and Chan [54] converted the fuzzy MCDM problem into a crisp one via using a distance function. However, this method only averages the lower limitation and the upper limitation of an interval, and new crisp weights are generated according to the average in the essence. It means that one criterion holds the weight of [0.1, 0.9] measures the same as another one holds [0.4, 0.6], which is apparently not reasonable and convincing enough. For one criterion whose weight is in an interval number \tilde{a} , a smaller result of $a^U - a^L$ represents that the information about this criterion is clearer when the sum of a^L and a^U is constant. Considering that, it is rational for us to distribute a larger crisp weight to a criterion which weighs [0.4, 0.6] than another one weighting [0.1, 0.9]. To improve it, the interval data are preserved during the fusion procedure and interval BPAs are generated in our method. Based on TOPSIS method, the propositions of our interval BPA are $\{IS(\text{ideal solution})\}$, $\{NS(\text{negative solution})\}$ and $\{IS, NS\}$. The frame of discernment is $\{IS, NS\}$. The

following is an example of the interval BPA of an alternative:

$$\begin{aligned}
 m(\{IS\}) &= [a^L, a^U], \\
 m(\{NS\}) &= [b^L, b^U], \\
 m(\{IS, NS\}) &= [c^L, c^U],
 \end{aligned}$$

which means that: (1) The hypothesis “the alternative is an ideal solution” is upheld with belief degree from a^L to a^U . (2) The hypothesis “the alternative is a negative ideal solution” is upheld with belief degree from b^L to b^U . (3) The hypothesis “the alternative is perceived as a discernment, namely it is likely to be an ideal solution or a negative solution” is upheld with belief degree from c^L to c^U .

It is worth mentioning that $m(\{IS, NS\}) = 1 - m(\{IS\}) - m(\{NS\})$. Hence, it is easy to know that $c^L = 1 - a^U - b^U$ and $c^U = 1 - a^L - b^L$. For an interval BPA ($\{IS\}, \{NS\}, \{IS, NS\}$), there is another way to express it as $\{[a^L, b^L, c^L], [a^U, b^U, c^U]\}$. Then, the left part and right part can do fusion independently. When it comes to making a comprehensive decision, we can fuse the interval BPAs into classical ones. In the ultimate, PPT is used to help rank the order of alternatives. According to what mentioned above, our new method can be stated step by step as follows:

Step 1 Generate classical BPAs based on TOPSIS.

Here, we follow the TOPSIS method used in [54]. Since it is not the key progress of our method, we only briefly introduce the process as follows: (1) Determine the ideal solution and negative ideal solution. (2) Calculate the decision’s distance from IS and NS, respectively. (3) Generate classical BPAs of each alternative based on the calculated distance.

Step 2 Interval BPAs fusion of different criteria.

Convert the criteria’s weights into interval numbers, which includes three situations: (1) The weights are given as interval numbers ahead. (2) The weights are given as crisp numbers. Then extend the crisp number as an interval whose values of left and right parts are both the crisp number itself. (3) The weights are described as fuzzy linguistic variables. Same as the fuzzy linguistic variables, in this case, the transform rules between terms and interval data differ in different situations. With the basis of Table 1, here, we adopt the rules illustrated in Table 2. The derived weight of criterion C_i are denoted as $W_{C_i} = [w_{C_i}^L, w_{C_i}^U], i = 1, 2, \dots, n$.

Normalize the weights of criteria as follows:

$$W_{C_i} = \left[\frac{w_{C_i}^L}{w_{C_{\max}}}, \frac{w_{C_i}^U}{w_{C_{\max}}} \right] \tag{11}$$

where $w_{C_{\max}}$ is the largest number among all limit values of criterion intervals.

Table 2 Convert linguistic variables into interval data

Terms	Interval data
Very low (VL)	[0, 0.3]
Low (L)	[0.1, 0.5]
Medium (M)	[0.3, 0.7]
High (H)	[0.5, 0.9]
Very high (VH)	[0.7, 1.0]

Then, discount classical BPAs with interval data to generate the interval BPA of each criterion. Denote the new weight of criterion C_i as $W_{C_i} = [W_{C_{i,\min}}, W_{C_{i,\max}}]$, then we can determine the corresponding interval BPA as follows:

$$m_{C_i}(\{IS\}) = \left[a_{C_i}^L W_{C_{i,\min}}, a_{C_i}^U W_{C_{i,\max}} \right], \tag{12}$$

$$m_{C_i}(\{NS\}) = \left[b_{C_i}^L W_{C_{i,\min}}, b_{C_i}^U W_{C_{i,\max}} \right], \tag{13}$$

$$\begin{aligned}
 m_{C_i}(\{IS, NS\}) &= \left[1 - a_{C_i}^L W_{C_{i,\min}} - b_{C_i}^L W_{C_{i,\min}}, \right. \\
 &\quad \left. 1 - a_{C_i}^U W_{C_{i,\max}} - b_{C_i}^U W_{C_{i,\max}} \right]. \tag{14}
 \end{aligned}$$

The newly obtained interval BPAs are consisted of two classical BPAs served as the left part and right part, respectively. Therefore, the integrated BPA of criterion C_i is expressed as:

$$\begin{aligned}
 m_{C_i}(\{IS\}, \{NS\}, \{IS, NS\}) &= \left(\begin{aligned} &\left[a_{C_i}^L W_{C_{i,\min}}, a_{C_i}^U W_{C_{i,\max}} \right], \left[b_{C_i}^L W_{C_{i,\min}}, b_{C_i}^U W_{C_{i,\max}} \right], \\ &\left[1 - a_{C_i}^L W_{C_{i,\min}} - b_{C_i}^L W_{C_{i,\min}}, \right. \\ &\quad \left. 1 - a_{C_i}^U W_{C_{i,\max}} - b_{C_i}^U W_{C_{i,\max}} \right] \end{aligned} \right). \tag{15}
 \end{aligned}$$

which can be also denoted as:

$$\begin{aligned}
 m_{C_i}(\{IS\}, \{NS\}, \{IS, NS\}) &= (m_{C_i}(\text{left}), m_{C_i}(\text{right})) \tag{16}
 \end{aligned}$$

with

$$\begin{aligned}
 m_{C_i}(\text{left}) &= \left[a_{C_i}^L W_{C_{i,\min}}, b_{C_i}^L W_{C_{i,\min}}, \right. \\
 &\quad \left. 1 - a_{C_i}^L W_{C_{i,\min}} - b_{C_i}^L W_{C_{i,\min}} \right], \\
 m_{C_i}(\text{right}) &= \left[a_{C_i}^U W_{C_{i,\min}}, b_{C_i}^U W_{C_{i,\min}}, \right. \\
 &\quad \left. 1 - a_{C_i}^U W_{C_{i,\min}} - b_{C_i}^U W_{C_{i,\min}} \right]
 \end{aligned}$$

Fuse the left and right parts of the interval BPAs of all n criteria to obtain the comprehensive evaluation of an alternative separately from one DM as follows:

$$m_{C_i}(\text{left}) = \left[a_{C_i}^L W_{C_{i\min}}, b_{C_i}^L W_{C_{i\min}}, 1 - a_{C_i}^L W_{C_{i\min}} - b_{C_i}^L W_{C_{i\min}} \right], \tag{17}$$

$$m_{C_i}(\text{right}) = \left[a_{C_i}^L W_{C_{i\max}}, b_{C_i}^L W_{C_{i\max}}, 1 - a_{C_i}^L W_{C_{i\max}} - b_{C_i}^L W_{C_{i\max}} \right]. \tag{18}$$

Repeat the above process to obtain the interval BPAs of all k DMs.

Step 3 Interval BPAs fusion of different DMs.

Generally, this step follows the same process as step 2, except for the fusion object is the interval BPAs of different DMs. Convert the decision makers' weights into interval numbers with the same method in step 2. Normalize the weights of DMs as follows:

$$W_{DM_i} = \left[w_{DM_i}^L, w_{DM_i}^U \right] / w_{DM_{\max}}, \tag{19}$$

where $w_{DM_{\max}}$ is the largest number among all limit values of DM weight intervals. Similar to Eqs. (12–16), discount

classical BPAs with interval data to generate the interval BPA of each DM, which is denoted as:

$$m^{DM_i} = (m^{DM_i}(\text{left}), m^{DM_i}(\text{right})).$$

Then fuse the left and right parts of the interval BPAs of all k DMs to obtain the final evaluation of an alternative separately:

$$m(\text{left}) = m^{DM_1}(\text{left}) \oplus \dots \oplus m^{DM_i}(\text{left}) \oplus \dots \oplus m^{DM_n}(\text{left}), \tag{20}$$

$$m(\text{right}) = m^{DM_1}(\text{right}) \oplus \dots \oplus m^{DM_i}(\text{right}) \oplus \dots \oplus m^{DM_n}(\text{right}). \tag{21}$$

Repeat the above process to obtain the interval BPAs of all m alternatives.

Step 4 Interval BPAs fusion of left and right parts.

The final evaluation for n criteria from k DMs has been obtained as two parts. For now, we have been able to make a decision based on different strategies. If the risk aversion strategy is adopted, a decision can be made according to the left part of the interval BPAs $m(\text{left})$ for all alternatives.

Table 3 Data of the supplier selection problem in [54]

Alternatives	C_1 ({IS}, {NS}, {IS, NS})	C_2 ({IS}, {NS}, {IS, NS})	C_3 ({IS}, {NS}, {IS, NS})	C_4 ({IS}, {NS}, {IS, NS})
DM₁ [0.20, 0.45]				
Weights	[0.20, 0.35]	[0.30, 0.55]	[0.05, 0.30]	[0.25, 0.50]
Supplier1	(0.60, 0.20, 0.20)	(0.6429, 0.0714, 0.2857)	(0.60, 0.20, 0.20)	(0.60, 0.20, 0.20)
Supplier2	(0.60, 0.20, 0.20)	(0.6429, 0.0714, 0.2857)	(0.50, 0.50, 0)	(0.50, 0.50, 0)
Supplier3	(0.50, 0.50, 0)	(0.50, 0.50, 0)	(0.60, 0.20, 0.20)	(0.6667, 0, 0.3333)
Supplier4	(0.66667, 0, 0.3333)	(0.6667, 0, 0.3333)	(0.50, 0.50, 0)	(0.50, 0.50, 0)
Supplier5	(0, 0.6667, 0.3333)	(0, 0.6667, 0.3333)	(0, 0.6667, 0.3333)	(0, 0.6667, 0.3333)
Supplier6	(0.20, 0.60, 0.20)	(0.0714, 0.6429, 0.2857)	(0.6667, 0, 0.3333)	(0, 0.6667, 0.3333)
DM₂ [0.35, 0.55]				
Weights	[0.25, 0.45]	[0.20, 0.55]	[0.05, 0.3]	[0.20, 0.60]
Supplier1	(0.60, 0.20, 0.20)	(0.6429, 0.0714, 0.2857)	(0.50, 0.50, 0)	(0.60, 0.20, 0.20)
Supplier2	(0.60, 0.20, 0.20)	(0.6429, 0.0714, 0.2857)	(0, 0.6667, 0.3333)	(0.50, 0.50, 0)
Supplier3	(0.50, 0.50, 0)	(0.50, 0.50, 0)	(0.6667, 0, 0.3333)	(0.6667, 0, 0.3333)
Supplier4	(0.66667, 0, 0.3333)	(0.6667, 0, 0.3333)	(0.50, 0.50, 0)	(0.50, 0.50, 0)
Supplier5	(0, 0.6667, 0.3333)	(0, 0.6667, 0.3333)	(0, 0.6667, 0.3333)	(0.20, 0.60, 0.20)
Supplier6	(0.20, 0.60, 0.20)	(0.0714, 0.6429, 0.2857)	(0.6667, 0, 0.3333)	(0, 0.6667, 0.3333)
DM₃ [0.70, 0.95]				
Weights	[0.20, 0.55]	[0.20, 0.70]	[0.10, 0.40]	[0.20, 0.60]
Supplier1	(0.60, 0.20, 0.20)	(0.6429, 0.0714, 0.2857)	(0.5714, 0.2857, 0.1429)	(0.6667, 0, 0.3333)
Supplier2	(0.60, 0.20, 0.20)	(0.6429, 0.0714, 0.2857)	(0.6667, 0, 0.3333)	(0.2857, 0.5714, 0.1429)
Supplier3	(0.50, 0.50, 0)	(0.50, 0.50, 0)	(0.6667, 0, 0.3333)	(0.6667, 0, 0.3333)
Supplier4	(0.66667, 0, 0.3333)	(0.6667, 0, 0.3333)	(0.5714, 0.2857, 0.1429)	(0.6667, 0, 0.3333)
Supplier5	(0, 0.6667, 0.3333)	(0, 0.6667, 0.3333)	(0, 0.6667, 0.3333)	(0.2857, 0.5714, 0.1429)
Supplier6	(0.20, 0.60, 0.20)	(0.0714, 0.6429, 0.2857)	(0.6667, 0, 0.3333)	(0, 0.6667, 0.3333)

On the contrary, if the risk preference strategy is adopted, a decision can be made according to the right part of the interval BPAs $m(\text{right})$ for all alternatives. However, if we want to make a decision based on the comprehensive evaluation, we can continue to fuse the left part and right part of interval BPAs to obtain the final evaluation of each alternative as follows:

$$m = m(\text{left}) \oplus m(\text{right}). \tag{22}$$

One advantage of using interval data is that it can preserve original information as much as possible during the fusion process. The fusion result is relatively more reliable as less information is lost during the previous steps.

Step 5 Rank the order.

Sometimes, the proposition $\{IS, NS\}$ may cause difficulty to make a decision. To address it, one of the most classical methods is using PPT [Eq. (10)] to transform BPA to a probability distribution. Here, we follow this method, and rank the order of alternatives by comparing $Bet(\{IS\})$ of m alternatives.

4 Numerical Example

Supplier selection is a typical MCDM problem where lots of fuzzy and uncertain information exists. In reality, although managers claim that quality is the most important attribute for a supplier, they actually choose suppliers based largely on cost and delivery performance. Actually, in the modern society, the decision making has become increasingly complex [69]. Various risk factors can influence the supplier selection greatly. To decrease risk and obtain a comprehensive evaluation, a company usually requires a group of experts in the decision-making process. To compare our new interval data fusion method with the existing ones, the numerical example used in [54, 70] is adopted in this section. The initial conditions, such as classical BPAs of each alternative, the weights of criteria and DMs are shown in Table 3. In this problems, three DMs (DM₁ to DM₃) provide their evaluation results toward six suppliers (supplier1 to supplier6). There are four criteria (C_1 to C_4) taken into consideration, including product late delivery, cost, risk factor and suppliers' service performance detailed as following:

C_1 : Product late delivery. The delivery process can reflect the service ability of a supplier. It is considered to investigate whether the supplier can supply stable and constant appreciation serve for the enterprise.

C_2 : Cost. A good price measures quite a lot in reducing cost and increasing competitive force.

C_3 : Risk factor. If we want to make long-term cooperation with a supplier, then we must take its risk factor

(political factor, economic factor, the reputation, etc.) into account.

C_4 : Supplier's service performance. Service performance means the sustaining promotion of the product and service (like product quality acceptance level, technological support, information process), which is deemed as the core factor.

Before applying our method, a flow chart is illustrated in Fig. 2 to summarize the whole procedure of applying our method in a supplier selection problem. Based on it, the detailed processes is illustrated step by step in the following.

Step 1 Generate classical BPAs based on TOPSIS.

Since the classical BPAs of alternatives are already given in Table 3, we will implement the following steps directly.

Step 2 Interval BPAs fusion of different criteria.

Since all the original weights of criteria are given in the form of interval data, what we need to do is to normalize interval numbers based on Eq. (11). Let take the weights of

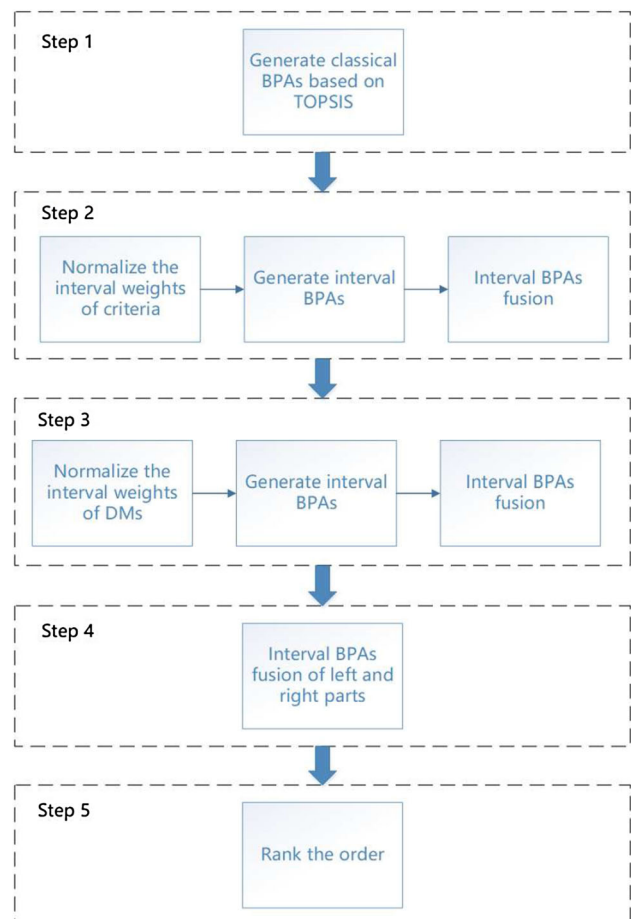


Fig. 2 Supplier selection based on interval data fusion

Table 4 Generate the interval BPAs of four criteria respectively

Alternatives	C_1 ({IS}, {NS}, {IS, NS})	C_2 ({IS}, {NS}, {IS, NS})	C_3 ({IS}, {NS}, {IS, NS})	C_4 ({IS}, {NS}, {IS, NS})
DM₁				
Weights	[0.2857, 0.5]	[0.4286, 0.7857]	[0.0714, 0.4286]	[0.3571, 0.7143]
Supplier1	([0.1714, 0.3], [0.0571, 0.1], [0.7715, 0.6])	([0.2755, 0.5051], [0.0306, 0.0561], [0.6939, 0.4388])	([0.0428, 0.2572], [0.0143, 0.0857], [0.9429, 0.6571])	([0.2143, 0.4286], [0.0714, 0.1429], [0.7143, 0.4285])
Supplier2	([0.1714, 0.3], [0.0571, 0.1], [0.7715, 0.6])	([0.2755, 0.5051], [0.0306, 0.0561], [0.6939, 0.4388])	([0.0357, 0.2143], [0.0357, 0.2143], [0.9286, 0.5714])	([0.1785, 0.3572], [0.1785, 0.3572], [0.6430, 0.2856])
Supplier3	([0.1429, 0.25], [0.1429, 0.25], [0.7142, 0.5])	([0.1429, 0.3928], [0.1429, 0.3928], [0.7242, 0.2144])	([0.0428, 0.2572], [0.0143, 0.0857], [0.9429, 0.6571])	([0.2381, 0.4762], [0, 0], [0.7619, 0.5238])
Supplier4	([0.1905, 0.3333], [0, 0], [0.8095, 0.6667])	([0.2857, 0.5238], [0, 0], [0.7143, 0.4762])	([0.0357, 0.2143], [0.0357, 0.2143], [0.9286, 0.5714])	([0.1785, 0.3572], [0.1785, 0.3572], [0, 0])
Supplier5	([0, 0], [0.1905, 0.3333], [0.8095, 0.6667])	([0, 0], [0.2857, 0.5238], [0.7143, 0.4762])	([0, 0], [0.0476, 0.2857], [0.9524, 0.7143])	([0, 0], [0.2381, 0.4762], [0.7619, 0.5238])
Supplier6	([0.0571, 0.1], [0.0714, 0.3], [0.7715, 0.6])	([0.0306, 0.0561], [0.2755, 0.5051], [0.6939, 0.4388])	([0.0476, 0.2857], [0, 0], [0.9524, 0.7143])	([0, 0], [0.2381, 0.4762], [0.7619, 0.5238])
DM₂				
Weights	[0.3571, 0.6428]	[0.2857, 0.7857]	[0.0714, 0.4286]	[0.2857, 0.8571]
Supplier1	([0.2143, 0.3857], [0.0714, 0.1286], [0.7143, 0.4857])	([0.1837, 0.5051], [0.0204, 0.0561], [0.7957, 0.4388])	([0.0357, 0.2143], [0.0357, 0.2143], [0.9286, 0.5714])	([0.1714, 0.5143], [0.0571, 0.1714], [0.7715, 0.3143])
Supplier2	([0.2143, 0.3857], [0.0714, 0.1286], [0.7143, 0.4857])	([0.1837, 0.5051], [0.0204, 0.0561], [0.7957, 0.4388])	([0, 0], [0.0476, 0.2857], [0.9524, 0.7143])	([0.1429, 0.4285], [0.1429, 0.4285], [0.7142, 0.1430])
Supplier3	([0.1785, 0.3214], [0.1785, 0.3214], [0.6430, 0.3572])	([0.1429, 0.3928], [0.1429, 0.3928], [0.7142, 0.2144])	([0.0476, 0.2857], [0, 0], [0.9524, 0.7143])	([0.1905, 0.5714], [0, 0], [0.8095, 0.4286])
Supplier4	([0.2381, 0.4286], [0, 0], [0.7619, 0.5714])	([0.1905, 0.5238], [0, 0], [0.8095, 0.4762])	([0.0357, 0.2143], [0.0357, 0.2143], [0.9286, 0.5714])	([0.1429, 0.4285], [0.1429, 0.4285], [0.7142, 0.1430])
Supplier5	([0, 0], [0.2381, 0.4286], [0.7619, 0.5714])	([0, 0], [0.1905, 0.5238], [0.8095, 0.4762])	([0, 0], [0.0476, 0.2857], [0.9524, 0.7143])	([0.0571, 0.1714], [0.1714, 0.5143], [0.7715, 0.3143])
Supplier6	([0.0714, 0.1286], [0.2143, 0.3857], [0.7143, 0.4857])	([0.0204, 0.0561], [0.1837, 0.5031], [0.7957, 0.4388])	([0.0476, 0.2857], [0, 0], [0.9524, 0.7143])	([0, 0], [0.1905, 0.5714], [0.8095, 0.4286])
DM₃				
Weights	[0.2857, 0.7857]	[0.2857, 1]	[0.1429, 0.5714]	[0.2857, 0.8571]
Supplier1	([0.1714, 0.4714], [0.0571, 0.1571], [0.7715, 0.3715])	([0.1837, 0.6429], [0.0204, 0.0714], [0.7957, 0.2857])	([0.0817, 0.3265], [0.0408, 0.1632], [0.8775, 0.5103])	([0.1905, 0.5714], [0, 0], [0.8095, 0.4286])
Supplier2	([0.1714, 0.4717], [0.0571, 0.1571], [0.7715, 0.3715])	([0.1837, 0.6429], [0.0204, 0.0714], [0.7957, 0.2857])	([0.0952, 0.3810], [0, 0], [0.9048, 0.6190])	([0.0816, 0.2449], [0.1632, 0.4897], [0.7552, 0.2654])
Supplier3	([0.1429, 0.3928], [0.1429, 0.3928], [0.7142, 0.2144])	([0.1429, 0.5], [0.1429, 0.5], [0.7142, 0])	([0.0952, 0.3810], [0, 0], [0.9048, 0.6190])	([0.1905, 0.5714], [0, 0], [0.8095, 0.4286])
Supplier4	([0.1905, 0.5238], [0, 0], [0.8095, 0.4762])	([0.1905, 0.6667], [0, 0], [0.8095, 0.3333])	([0.0817, 0.3265], [0.0408, 0.1632], [0.8775, 0.5103])	([0.1905, 0.5714], [0, 0], [0.8095, 0.4286])
Supplier5	([0, 0], [0.1905, 0.5238], [0.8095, 0.4762])	([0, 0], [0.1905, 0.6667], [0.8095, 0.3333])	([0, 0], [0.0952, 0.3810], [0.9048, 0.6190])	([0.0816, 0.2449], [0.1632, 0.4897], [0.7552, 0.2654])
Supplier6	([0.0571, 0.1571], [0.1714, 0.4714], [0.7715, 0.3715])	([0.0204, 0.0714], [0.1837, 0.1429], [0.7959, 0.2857])	([0.0952, 0.3810], [0, 0], [0.9048, 0.6190])	([0, 0], [0.1905, 0.5714], [0.8095, 0.4286])

DM₁'s four criteria as an example, the normalized weights are obtained as following, respectively:

$$W_{C_1} = [0.20, 0.35]/0.70 = [0.2857, 0.5],$$

$$W_{C_2} = [0.30, 0.55]/0.70 = [0.4286, 0.7857],$$

$$W_{C_3} = [0.20, 0.35]/0.70 = [0.0714, 0.4286],$$

$$W_{C_4} = [0.20, 0.35]/0.70 = [0.3571, 0.7142].$$

Then, let us take DM₁'s evaluation to C_1 of supplier1 as an example to normalize the weights:

$$m_{C_1}(\{IS\}) = [0.2875 \times 0.6, 0.5 \times 0.6] = [0.1714, 0.3],$$

$$m_{C_1}(\{NS\}) = [0.2875 \times 0.2, 0.5 \times 0.2] = [0.0571, 0.1],$$

$$m_{C_1}(\{IS, NS\}) = [0.7715, 0.6].$$

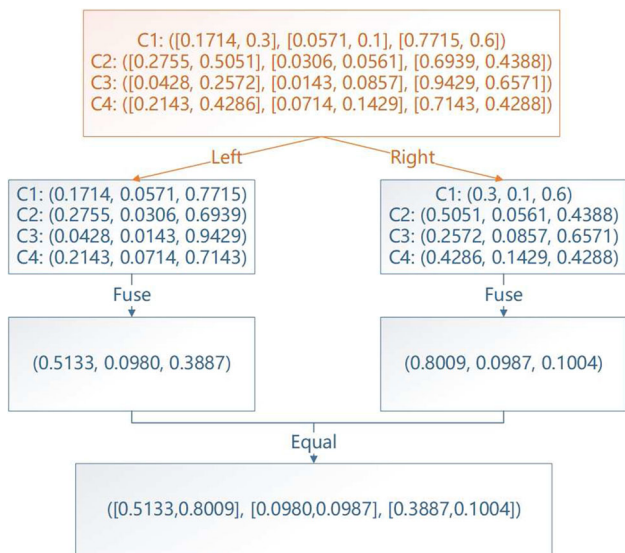


Fig. 3 Fuse DM₁'s evaluation to supplier1

By using Eq. (15), the rest interval BPAs of each alternative are generated (listed in Table 4).

Following, we do the interval BPAs fusion of all criteria. Fig. 3 illustrates the process of fusing DM₁'s evaluation to supplier1. Then, by using Eqs. (17) and (18), the four left and right part BPAs are fused together separately. In the same way, we can obtain all the interval BPAs which

represent the evaluation (considering all criteria) of each supplier from each DM (shown as Table 5).

Step 3 Interval BPAs fusion of different DMs.

This step is similar to step 2. It aims to fuse the evaluation from all DMs for one supplier. First of all, using Eq. (19) to normalize the weights of DMs:

$$W_{DM_1} = [0.20, 0.45] / 0.95 = [0.2105, 0.4739],$$

$$W_{DM_2} = [0.35, 0.55] / 0.95 = [0.3684, 0.5789],$$

$$W_{DM_3} = [0.70, 0.95] / 0.95 = [0.7368, 1].$$

In the following, discount the interval BPAs and do the fusion like in step 2. Still take supplier1 as an example, the results are obtained in Table 6. After the fusion, we can obtain the final interval BPA which represents the overall information about supplier1. Repeat the above process, all the suppliers' final interval BPAs are obtained (shown in Table 7). The rest steps are to compare these interval BPAs and rank order to make a decision.

Step 4 Interval BPAs fusion of left and right parts.

For the moment, we consider making a decision based on the comprehensive evaluation first. Hence, use Eq. (22) to fuse the two parts of an interval BPA.

Step 5 Rank the order.

According to Eq. (10), $m(\{IS, NS\})$ can be handled as follows:

Table 5 Fuse interval data to get comprehensive information containing all criteria

Alternatives	The left part of interval BPAs ({IS}, {NS}, {IS, NS})	The right part of interval BPAs ({IS}, {NS}, {IS, NS})
DM₁		
Supplier1	(0.5133, 0.0980, 0.3887)	(0.8009, 0.0987, 0.1004)
Supplier2	(0.4596, 0.1772, 0.3632)	(0.6881, 0.2353, 0.0766)
Supplier3	(0.4003, 0.1895, 0.4102)	(0.6817, 0.2520, 0.0663)
Supplier4	(0.4938, 0.1246, 0.3815)	(0.7447, 0.1782, 0.0771)
Supplier5	(0, 0.5804, 0.4196)	(0, 0.8812, 0.1188)
Supplier6	(0.0734, 0.5131, 0.4135)	(0.1203, 0.7369, 0.1428)
DM₂		
Supplier1	(0.4502, 0.1128, 0.4370)	(0.8108, 0.1268, 0.0624)
Supplier2	(0.3929, 0.1800, 0.4271)	(0.6438, 0.3116, 0.0446)
Supplier3	(0.3920, 0.2114, 0.3966)	(0.7549, 0.1995, 0.0456)
Supplier4	(0.4502, 0.1086, 0.4412)	(0.7774, 0.1821, 0.0405)
Supplier5	(0.0343, 0.5015, 0.4642)	(0.0387, 0.8905, 0.0708)
Supplier6	(0.0854, 0.4518, 0.4628)	(0.0996, 0.7475, 0.1529)
DM₃		
Supplier1	(0.4722, 0.0699, 0.4579)	(0.9206, 0.0456, 0.0338)
Supplier2	(0.4015, 0.1829, 0.4155)	(0.8124, 0.1506, 0.0370)
Supplier3	(0.4015, 0.1829, 0.4155)	(0.7903, 0.2097, 0)
Supplier4	(0.5034, 0.0221, 0.4746)	(0.9460, 0.0131, 0.0409)
Supplier5	(0.0500, 0.4868, 0.4631)	(0.0309, 0.9357, 0.0334)
Supplier6	(0.1051, 0.4620, 0.4833)	(0.0982, 0.8494, 0.0524)

Table 6 Fuse the evaluation of supplier1 from three DMs

	The left part of interval BPA ({IS}, {NS}, {IS, NS})	The right part of interval BPA ({IS}, {NS}, {IS, NS})
DM ₁	(0.1080, 0.0206, 0.8714)	(0.3795, 0.0468, 0.5737)
DM ₂	(0.1659, 0.0416, 0.7925)	(0.4694, 0.0734, 0.4572)
DM ₃	(0.3479, 0.0515, 0.6006)	(0.9206, 0.0456, 0.0338)
Fusion result	(0.4950, 0.0733, 0.4317)	(0.8849, 0.1017, 0.0135)

Table 7 Fuse the evaluation of all suppliers from three DMs

Alternatives	The left part of interval BPAs ({IS}, {NS}, {IS, NS})	The right part of interval BPAs ({IS}, {NS}, {IS, NS})
Supplier1	(0.4950, 0.0733, 0.4317)	(0.9696, 0.0201, 0.0103)
Supplier2	(0.4106, 0.1516, 0.4378)	(0.8849, 0.1017, 0.0135)
Supplier3	(0.4019, 0.1878, 0.4104)	(0.8851, 0.1149, 0)
Supplier4	(0.5135, 0.0485, 0.4380)	(0.9765, 0.0110, 0.0125)
Supplier5	(0.0336, 0.5337, 0.4328)	(0.0096, 0.9811, 0.0094)
Supplier6	(0.0845, 0.4895, 0.4260)	(0.0473, 0.9339, 0.0189)

Table 8 Convert interval BPAs back to classical BPAs

Alternatives	Combining results	Bet({IS})	Bet({IS}) in [54]	Final ranking order
Supplier1	(0.9833, 0.0119, 0.0048)	0.9857	0.9775	2
Supplier2	(0.9177, 0.0752, 0.0072)	0.9213	0.9008	3
Supplier3	(0.9129, 0.0873, 0)	0.9129	0.8888	4
Supplier4	(0.9879, 0.0063, 0.0058)	0.9908	0.9864	1
Supplier5	(0.0050, 0.9910, 0.0042)	0.0071	0.0130	6
Supplier6	(0.0287, 0.9625, 0.0090)	0.0332	0.0445	5

$$\text{Bet}(\{IS\}) = m(\{IS\}) + m(\{IS, NS\})/2 \tag{23}$$

The final evaluations of each supplier are shown in Table 8. According to the data, the order is easily ranked as supplier4 > supplier1 > supplier2 > supplier3 > supplier6 > supplier5. Apparently, supplier4 is the best selection, which coincides with the results presented in paper [54]. In addition, the Bet({IS}) of our method is higher than the classical one, which proves the feasibility and validity of our new method. However, in some particular situations, the decision can be made based on a risk aversion strategy or risk preference strategy. In this case, a decision can be made with the basis of the data in Table 7. The results of order ranking based on different strategies are shown in Fig. 4, where the labels above cylinders represent the ranking order of suppliers. If the decision maker adopts a risk aversion strategy, the ranking order will be supplier4 > supplier1 > supplier3 > supplier2 > supplier6 > supplier5, which is slightly different from Table 8. In a similar way, the ranking order based on a risk preference strategy can be obtained as supplier4 > supplier1 > supplier3 > supplier2 > supplier6 > supplier5. As we can see, when taking a comprehensive consideration of all information, the results

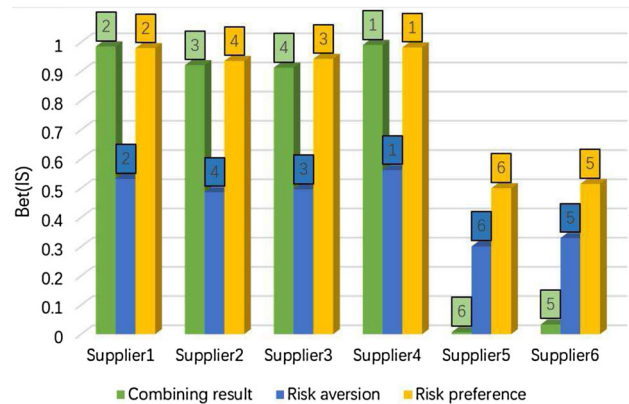


Fig. 4 Order ranking based on different strategies

of our method is same as the original ones. However, when taking different strategies to make a decision, the results are slightly different, which is rational as only a part of information is used in the decision-making process. In sum, our method has following advantages:

1. Fuzzy and uncertain information is well handled in the form of interval data.

2. The decision made in comprehensive consideration is more rational as less information is lost.
3. Different decision-making strategies, like risk aversion strategy and risk preference strategy, can be adopted in our method, which is very useful in some specific applications.
4. In some degrees, our method is the generalization of the classical one in [54] as an interval number will degenerate into a crisp number when no uncertainty exists.

5 Conclusion

Supplier selection is common and significant problem which is always modeled in a MCDM framework. In reality, a mass of fuzzy information exists in MCDM problems inevitably. Sometimes, uncertainty exists in the decision opinions from different DMs and the weights of both different criteria and DMs. To handle the uncertainty, an evidential method based on interval data fusion is proposed in this paper. We follow the classical D–S evidence theory to handle the uncertainty of decision opinions. As for the uncertainty of different weights, interval number is adopted to handle it since interval number is a simple but quiet effective tool to handle fuzzy and uncertain information. When uncertainty exists in the weights, our method degenerates to a classical one in [54]. The interval weights are used to discount and generate interval BPAs. Then, we do the fusion of different criteria, DMs in sequence to make a final decision. Compared with the most current paper, the interval data are conserved in the fusion process. It means that relatively full information is used to make a comprehensive decision in our method as the information will be lost inevitably when convert an interval number to a crisp one. Also, different strategies can be adopted in our method, which could bring some difference to the final decision making. It is very useful in some specific applications, like it is better to adopt risk aversion strategy in a military application. An example of supplier selection is used to illustrate the detailed procedures of our method. The ranking result agrees with the original methods when making a decision with the usage of full information; however, some slight differences occur when adopting other decision-making strategies. In future research, we will try to apply the interval fusion method to other models rather than fixed in an evidential framework.

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References

1. Jato-Espino, D., Castillo-Lopez, E., Rodriguez-Hernandez, J., Canteras-Jordana, J.C.: A review of application of multi-criteria decision making methods in construction. *Autom. Constr.* **45**(Supplement C), 151–162 (2014)
2. Mateos, A., Jiménez-Martín, A., Aguayo, E., Sabio, P.: Dominance intensity measuring methods in MCDM with ordinal relations regarding weights. *Knowl. Based Syst.* **70**, 26–32 (2014)
3. Hung, Y.H., Huang, T.L., Hsieh, J.C., Tsuei, H.J., Cheng, C.C., Tzeng, G.H.: Online reputation management for improving marketing by using a hybrid MCDM model. *Knowl. Based Syst.* **35**, 87–93 (2012)
4. Gong, Y.B.: Fuzzy multi-attribute group decision making method based on interval type-2 fuzzy sets and applications to global supplier selection. *Int. J. Fuzzy Syst.* **15**(4), 392–400 (2013)
5. Wu, X.H., Wang, J.Q., Peng, J.J., Chen, X.H.: Cross-entropy and prioritized aggregation operator with simplified neutrosophic sets and their application in multi-criteria decision-making problems. *Int. J. Fuzzy Syst.* **18**(6), 1–13 (2016)
6. Liu, T., Deng, Y., Chan, F.: Evidential supplier selection based on DEMATEL and game theory. *Int. J. Fuzzy Syst.* (2017). <https://doi.org/10.1007/s40815-017-0400-4>
7. Deng, X., Jiang, W.: An evidential axiomatic design approach for decision making using the evaluation of belief structure satisfaction to uncertain target values. *Int. J. Intell. Syst.* (2017, in press). <https://doi.org/10.1002/int.21.929>
8. Lin, M., Xu, Z., Zhai, Y., Yao, Z.: Multi-attribute group decision-making under probabilistic uncertain linguistic environment. *J. Oper. Res. Soc.* 1–15 (2017). <https://doi.org/10.1057/s41274-017-0182-y>
9. Rezaei, J., Hemmes, A., Tavasszy, L.: Multi-criteria decision-making for complex bundling configurations in surface transportation of air freight. *J. Air Transp. Manag.* **61**, 95–105 (2017)
10. Xu, S., Jiang, W., Deng, X., Shou, Y.: A modified physarum-inspired model for the user equilibrium traffic assignment problem. *Appl. Math. Model.* (2017, in press). <https://doi.org/10.1016/j.apm.2017.07.032>
11. Jiang, W., Wei, B., Tang, Y., Zhou, D.: Ordered visibility graph average aggregation operator: an application in produced water management. *Chaos Interdiscip. J. Nonlinear Sci.* **27**(2), Article ID 023117 (2017)
12. Huang, Y., Li, T., Luo, C., Fujita, H., jinn Horng, S.: Dynamic variable precision rough set approach for probabilistic set-valued information systems. *Knowl. Based Syst.* **122**, 131–147 (2017)
13. Dempster, A.P.: Upper and lower probabilities induced by a multivalued mapping. *Ann. Math. Stat.* **38**(2), 325–339 (1967)
14. Shafer, G.: A mathematical theory of evidence. *Technometrics* **20**(1), 242 (1978)
15. Deng, X., Xiao, F., Deng, Y.: An improved distance-based total uncertainty measure in belief function theory. *Appl. Intell.* **46**(4), 898–915 (2017)
16. Jiang, W., Wang, S.: An uncertainty measure for interval-valued evidences. *Int. J. Comput. Commun. Control* **12**(5), 631–644 (2017)
17. Fu, C., Yang, J.B., Yang, S.L.: A group evidential reasoning approach based on expert reliability. *Eur. J. Oper. Res.* **246**(3), 886–893 (2015)
18. Mo, H., Deng, Y.: A new aggregating operator in linguistic decision making based on D numbers. *Int. J. Uncertain. Fuzziness Knowl. Based Syst.* **24**(6), 831–846 (2016)

19. Zhou, X., Deng, X., Deng, Y., Mahadevan, S.: Dependence assessment in human reliability analysis based on D numbers and AHP. *Nucl. Eng. Des.* **313**, 243–252 (2017)
20. Zheng, X., Deng, Y.: Dependence assessment in human reliability analysis based on evidence credibility decay model and IOWA operator. *Ann. Nucl. Energy* **112**, 673–684 (2018)
21. Zheng, H., Deng, Y.: Evaluation method based on fuzzy relations between Dempster–Shafer belief structure. *Int. J. Intell. Syst.* (2017). <https://doi.org/10.1002/int.21956>
22. Zheng, H., Deng, Y., Hu, Y.: Fuzzy evidential influence diagram and its evaluation algorithm. *Knowl. Based Syst.* **131**, 28–45 (2017)
23. Deng, X., Jiang, W., Zhang, J.: Zero-sum matrix game with payoffs of Dempster–Shafer belief structures and its applications on sensors. *Sensors* **17**(4), Article ID 922 (2017)
24. Jiang, W., Zhan, J.: A modified combination rule in generalized evidence theory. *Appl. Intell.* **46**(3), 630–640 (2017). <https://doi.org/10.1007/s10489-016-0851-6>
25. Wang, X., Zhu, J., Song, Y., Lei, L.: Combination of unreliable evidence sources in intuitionistic fuzzy MCDM framework. *Knowl. Based Syst.* **97**, 24–39 (2016)
26. Dezert, J., Han, D., Tacnet, J.M., Carladous, S., Yang, Y.: *Decision-Making with Belief Interval Distance*. Springer, Cham (2016)
27. Li, X., Song, Y., Quan, W.: Evaluating evidence reliability based on intuitionistic fuzzy MCDM model. *J. Intell. Fuzzy Syst.* **31**(3), 1167–1182 (2016)
28. Zadeh, L.A.: Fuzzy sets. *Inf. Control* **8**(65), 338–353 (1965)
29. Shih, Y.Y., Su, S.F., Rudas, I.J.: Fuzzy based compensation for image stabilization in a camera hand-shake emulation system. *Int. J. Fuzzy Syst.* **16**(3), 350–357 (2014)
30. Jiang, W., Wei, B., Liu, X., Li, X., Zheng, H.: Intuitionistic fuzzy power aggregation operator based on entropy and its application in decision making. *Int. J. Intell. Syst.* (2017) Published online. <https://doi.org/10.1002/int.21939>
31. Chou, C.C.: A generalized similarity measure for fuzzy numbers. *J. Intell. Fuzzy Syst.* **30**(2), 1147–1155 (2016)
32. Zhang, R., Ashuri, B., Deng, Y.: A novel method for forecasting time series based on fuzzy logic and visibility graph. *Adv. Data Anal. Classif.* (2017). <https://doi.org/10.1007/s11634-017-0300-3>
33. Luo, A.C., Chen, S.W., Fang, C.Y.: Gaussian successive fuzzy integral for sequential multi-decision making. *Int. J. Fuzzy Syst.* **17**(2), 1–16 (2015)
34. Zhang, R., Ran, X., Wang, C., Deng, Y.: Fuzzy evaluation of network vulnerability. *Qual. Reliab. Eng. Int.* **32**(5), 1715–1730 (2016)
35. Fei, L., Wang, H., Chen, L., Deng, Y.: A new vector valued similarity measure for intuitionistic fuzzy sets based on OWA operators. *Iran. J. Fuzzy Syst.* (2017) (accepted)
36. Afzali, A., Rafsanjani, M.K., Saeid, A.B.: A fuzzy multi-objective linear programming model based on interval-valued intuitionistic fuzzy sets for supplier selection. *Int. J. Fuzzy Syst.* **18**(5), 864–874 (2016)
37. Mahmoudi, A., Sadi-Nezhad, S., Makui, A.: A hybrid fuzzy-intelligent system for group multi-attribute decision making. *Int. J. Fuzzy Syst.* **18**(6), 1–14 (2016)
38. Liu, W., Liao, H.: A bibliometric analysis of fuzzy decision research during 1970–2015. *Int. J. Fuzzy Syst.* **19**(1), 1–14 (2017)
39. Hu, Y.C.: Fuzzy multiple-criteria decision making in the determination of critical criteria for assessing service quality of travel websites. *Expert Syst. Appl.* **36**(3), 6439–6445 (2009)
40. Jiang, W., Xie, C., Zhuang, M., Tang, Y.: Failure mode and effects analysis based on a novel fuzzy evidential method. *Appl. Soft Comput.* **57**, 672–683 (2017)
41. Su, S.F., Chuang, C.C., Tao, C.W., Jeng, J.T., Hsiao, C.C.: Radial basis function networks with linear interval regression weights for symbolic interval data. *IEEE Trans. Syst. Man Cybern. Part B* **42**(1), 69–80 (2012)
42. Lei, Q., Xu, Z., Bustince, H., Fernandez, J.: Intuitionistic fuzzy integrals based on archimedean t-conorms and t-norms. *Inf. Sci.* **327**, 57–70 (2016)
43. Wang, N., Liu, X., Wei, D.: A modified D numbers' integration for multiple attributes decision making. *Int. J. Fuzzy Syst.* 1–12 (2017). <https://doi.org/10.1007/s40815-017-0323-0>
44. Zhang, Z.: Several new interval-valued intuitionistic fuzzy hamacher hybrid operators and their application to multi-criteria group decision making. *Int. J. Fuzzy Syst.* **18**(5), 829–848 (2015)
45. Liu, S., Yu, F., Xu, W., Zhang, W.: New approach to MCDM under interval-valued intuitionistic fuzzy environment. *Int. J. Mach. Learn. Cybern.* **4**(6), 671–678 (2013)
46. Chen, S.M., Huang, Z.C.: Multiattribute decision making based on interval-valued intuitionistic fuzzy values and linear programming methodology. *Inf. Sci.* **381**, 341–351 (2017)
47. Wang, C.Y., Chen, S.M.: Multiple attribute decision making based on interval-valued intuitionistic fuzzy sets, linear programming methodology, and the extended TOPSIS method. *Inf. Sci.* **397–398**, 155–167 (2017)
48. Tang, H.: Decision making based on interval-valued intuitionistic fuzzy soft sets and its algorithm. *J. Comput. Anal. Appl.* **23**(1), 119–131 (2017)
49. Kahraman, C., Onar, S.C., Oztaysi, B.: Fuzzy multicriteria decision-making: a literature review. *Int. J. Comput. Intell. Syst.* **8**(4), 637–666 (2015)
50. Tao, Z., Liu, X., Chen, H., Zhou, L.: Ranking interval-valued fuzzy numbers with intuitionistic fuzzy possibility degree and its application to fuzzy multi-attribute? decision making. *Int. J. Fuzzy Syst.* **19**(3), 646–658 (2017)
51. Joshi, D., Kumar, S.: Interval-valued intuitionistic hesitant fuzzy choquet integral based topsis method for multi-criteria group decision making. *Eur. J. Oper. Res.* **248**(1), 183–191 (2016)
52. Jahanshahloo, G.R., Lotfi, F.H., Izadikhah, M.: An algorithmic method to extend topsis for decision-making problems with interval data. *Appl. Math. Comput.* **175**(2), 1375–1384 (2006)
53. Yue, Z.: A method for group decision-making based on determining weights of decision makers using TOPSIS. *Appl. Math. Modell.* **35**(4), 1926–1936 (2011)
54. Deng, Y., Chan, F.T.S.: A new fuzzy Dempster MCDM method and its application in supplier selection. *Expert Syst. Appl.* **38**(8), 9854–9861 (2011)
55. Bosc, P., Prade, H.: *An Introduction to the Fuzzy Set and Possibility Theory-Based Treatment of Flexible Queries and Uncertain or Imprecise Databases*. Springer, New York (1997)
56. Gupta, M.M.: *Fuzzy set theory and its applications*. *Fuzzy Sets Syst.* **47**(1), 101–109 (1992)
57. Zimmermann, H.J.: *Fuzzy Set Theory—and its Applications*. Springer Science & Business Media (2011)
58. Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning—II. *Inf. Sci.* **8**(75), 301–357 (1975)
59. Zhang, X., Mahadevan, S., Deng, X.: Reliability analysis with linguistic data: an evidential network approach. *Reliab. Eng. Syst. Saf.* **162**, 111–121 (2017)
60. Zhang, X., Deng, Y., Chan, F.T.S., Adamatzky, A., Mahadevan, S.: Supplier selection based on evidence theory and analytic network process. *Proc. Inst. Mech. Eng. Part B J. Eng. Manuf.* **230**(3), 562–573 (2016)
61. Kaufmann, A., Gupta, M.M.: *Introduction to Fuzzy Arithmetic: Theory and Applications*. Van Nostrand Reinhold Co., New York (1985)

62. Smets, P., Kennes, R.: The transferable belief model. *Artif. Intell.* **66**(94), 191–234 (1994)
63. Hwang, C.L., Yoon, K.: *Multiple Attribute Decision Making*. Springer, Berlin (1981b)
64. Hwang, C.L., Yoon, K.: *Methods for Multiple Attribute Decision Making*. Springer, Berlin (1981a)
65. Zhang, Q., Li, M., Deng, Y.: Measure the structure similarity of nodes in complex networks based on relative entropy. *Phys. A Stat. Mech. Appl.* (2017). <https://doi.org/10.1016/j.physa.2017.09.042>
66. Hamdan, S., Cheaitou, A.: Supplier selection and order allocation with green criteria: an MCDM and multi-objective optimization approach. *Comput. Oper. Res.* **81**, 282–304 (2017)
67. Guo, S., Zhao, H.: Fuzzy best-worst multi-criteria decision-making method and its applications. *Knowl. Based Syst.* **121**, 23–31 (2017)
68. Ren, J., Liang, H., Chan, F.T.: Urban sewage sludge, sustainability, and transition for eco-city: multi-criteria sustainability assessment of technologies based on best-worst method. *Technol. Forecast. Soc. Change* **116**, 29–39 (2017)
69. Brown, J.R., Bushuev, M.A., Kretinin, A.A., Guiffrida, A.L.: Recent developments in green supply chain management: sourcing and logistics. In: Khan, M., Hussain, M., Ajmal, M.M (eds.) *Green Supply Chain Management for Sustainable Business Practice*, 197–217 (2016)
70. Wu, D.: Supplier selection in a fuzzy group setting: a method using gray related analysis and Dempster–Shafer theory. *Expert Syst. Appl.* **36**, 8892–8899 (2009)



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