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A Novel Approach to Group Decision-Making with Interval-Valued Intuitionistic Fuzzy Preference Relations via Shapley Value

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Abstract This paper proposes a novel weighting approach to group decision-making (GDM) with interval-valued intuitionistic fuzzy preference relations (IVIFPRs) based on a fuzzy cooperative game method and the continuous interval-valued intuitionistic fuzzy ordered weighted averaging (CIVIFOWA) operator. First of all, a continuous IVIFPR (CIVIFPR) is defined based on the CIVIFOWA operator, and then considering the contribution of each decision-maker (DM), an iterative algorithm is designed to redistribute weights of DMs by using cooperative method. Moreover, a logarithm least optimal model is developed to deriving interval priority weights of IFPR and a two-stage resolution process is proposed for the GDM with IVIFPRs. Finally, a practical example with cooperation and competition is provided to verify the feasibility and efficiency of the proposed method. The characteristics of the proposed method are as follows: (1) the iterative algorithm is devoted to deriving DMs' weights in GDM by using fuzzy cooperative game based on the CIVIFOWA operator in which the contribution of each DM's opinion to the group indicates the rationality and importance in GDM of their opinions; (2) the weighting algorithm can be adjusted by modifying the attitude parameter based on the CIVIFOWA operator, which makes the proposed method more flexible.

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1 Introduction

Group decision-making (GDM) with preference relations gained extensive attentions in recent researches. Problems of GDM with preference relations can be solved by using the general GDM with suitable aggregation techniques. In the previous literature, GDM problems with multiplicative preference relations [1], fuzzy preference relations [2], linguistic preference relations [3, 4], and intuitionistic preference relations [5, 6] are finely discussed and demonstrated. Nevertheless, constrained by the high complexity of socioeconomic environments and the insufficient level of knowledge in real-life decision-making problems, it is reasonable for DMs utilizing interval variables [7], interval linguistic variables [8], or interval-valued intuitionistic fuzzy variables [9] to express their preferences over alternatives. Thus, preference relations are extended to fuzzy environment, such as the uncertain preference relations [7, 10–14], the uncertain linguistic preference relations [8, 15], and the interval-valued intuitionistic fuzzy preference relations [16, 17].

In GDM problems with preference relations, the absence of consistency or consensus may lead to misleading conclusions, therefore fruitful results are investigated to measure the consistency and consensus of preference relations. Saaty and Vargas [18] proposed a consistency degree by measuring the divergence between any two multiplicative preference relations. To estimate the consistency of linguistic preference relations, Dong and Herrera-Viedma [19] converted linguistic preference relations into interval preference relations. As for intuitionistic fuzzy preference relations (IFPRs), a multiplicative consistency definition was proposed by Xu et al. [20]. To perfect IFPRs consistency, Liao and Xu [16] introduced an advanced consistency index. Aiming at overcoming the deficiencies of IFPRs consistency, Liao et al. [21] put forward an iterative algorithm and a consensus model is built to improve the consensus among experts. Afterward, Liao et al. [22] enhanced the consensus model constructed in [21] and set up a reaching model. Moreover, Xu et al. [23] developed some mathematical programming models to reach consistency and consensus GDM with intuitionistic fuzzy preference relations, and Liao et al. [24] defined the multiplicative consistency of IVIFPR. Wan et al. [25] investigated a GDM method based on additive consistent IVIFPRs and likelihood comparison algorithm.

To aggregate all information about alternatives' preferences precisely, Yager [26] initially introduced the ordered weighted averaging (OWA) operator, which provides a family of aggregation operators including the maximum, the minimum, and the average criteria. Then, Refs. [13, 14, 26–31] extended the OWA operator to different forms. Among all extensions, the continuous ordered weighted averaging (COWA) operator [31] is an excellent one with certain practicability, where the argument takes the form of interval number. Owing to flexibility of the COWA operator, it has been further applied in [13, 14, 32–35]. Zhou and Chen [34] proposed the CIVIFOWA operator on basis of the COWA operator considering the risk attitude of DMs.

Particularly, when aggregating all evaluations of various DMs, how much value should every expert share is of great importance [36–38]. In order to deriving the optimal DMs' weights regarding GDM problems with AIFPRs, Ureña et al. [39] introduced a new concept of experts' confidence, based on the confidence degree and experts' consistency, a new aggregation operator is defined. In view of DMs' risk attitudes toward uncertainty, Wan et al. [40] constructed an intuitionistic fuzzy program and proposed three different solving methods. In the available literatures regarding GDM problems with IVIFPRs, different importance values were investigated to assign them to each DM [24, 41, 42] objectively.

Particularly, game theory is an efficient tool in decision-making. Chen and Larbani [43] used a two-person zero-sum game with an uncertain payoff matrix for deriving the weights of attributes. By utilizing the cooperative games, AL-Mutairi [44] proposed a fuzzy preference decision-making with two DMs. Madani and Lund [45] regarded the multi-criteria DM problems as strategic game which can be handled by noncooperative game theory. By the Shapley value [46], an objective way [47, 48] is provided to deriving each player' importance value in the cooperative game.

The core of the GDM is the DMs who come from various research areas with different knowledge structure, analytical ability, and evaluation levels. They will give the individual preference under their own practical experience or unique perspective. Hence, it is common to see that the decision-makers give different degrees of IVIFPRs for the same GDM problem. So it is meaningful to give each decision-maker a weight which reflects their corresponding importance in the group. Ref. [37] proposed a consensus model which not only deriving the weights to the DMs, but also give the personal decision suggestion to change their preferences for a better consistency in the GDM problems. How to determine the weights of DMs objectively is playing a dominant role in GDM process. Recently, many researches have been done for assigning the DMs' weights. For example, Xu [49] determined the weights of the DMs by some formulas which come from the deviation measures between additive linguistic preference relations. Yue [50] used the degree of similarity between each individual preference with an optimistic coefficient parameter to determine the weights of DMs, but pay no attention to each DM's contribution to the group preference. Wan et al. [51] also pointed out that DMs' weights play an important role whether in homogenous or heterogeneous decision environments. For the GDM under IVIFPR environment, there is no investigation on the determination of DMs' weights objectively. Some existing methods [24, 41, 42] assumed that each DM has equal importance or assigned DMs' weights in advance, which may lead to the subjective randomness.

However, it is noted that all the aforementioned methods to determine the DM's weights always come from the perspective of the coherence between each individual preference, but pay no attention to the contribution of each individual to the group. That is to say, in the process of GDM, the different contribution of the individual will bring different group decision effects. And it is more valuable to obtain the best decision of the group rather than the individual. Therefore, one of the main goals of this paper is to develop a novel approach to deriving the DMs' weights under IVIFPRs environment from the perspective of the contribution of each of decision-makers to the group decision.

This paper is to develop a new approach to GDM with IVAIFPRs based on a fuzzy cooperative game method and the CIVIFOWA operator. We first define a CIVIFPR by using the CIVIFOWA operator and then put forward an algorithm to deriving DMs by using the Shapley value based on the contribution of each DM. We further present a logarithm least optimal model to determine interval priority weights of IFPR and develop a two-stage resolution process for the GDM with IVIFPRs. And finally, an example is provided to verify the feasibility and efficiency of the proposed method.

The rest of paper is organized as follows. In Sect. 2, some basic concepts are briefly reviewed. Section 3 proposes a Shapley value method, a weighting algorithm based on the CIVIFOWA operator and a logarithm least optimal model are put forward. In Sect. 4, a practical example of the new approach is presented and a comparison example is illustrated to verify the feasibility and validity of the proposed method. At last, in Sect. 5, conclusions of this paper are demonstrated.

2 Preliminaries

In this section, fundamental concepts are briefly reviewed, including IFPRs, IVIFPRs, CIVIFOWA operator, and method of Shapley value in cooperative game.

2.1 The IFPRs and IVIFPRs

Definition 1 [52] Let X be a non-empty set of universe, then $\tilde{F} = \{x, \tilde{\mu}_{\tilde{F}}(x), \tilde{v}_{\tilde{F}}(x) | x \in X\}$ is called an intuitionistic fuzzy set (IFS), where $\mu_{\tilde{F}} : X \to [0,1]$ is the degree of membership, $v_{\tilde{F}} : X \to [0,1]$ is the degree of non-membership of $x \in X$ to $\tilde{F}$, $\tilde{\pi}_{\tilde{F}} = 1 - \tilde{\mu}_{\tilde{F}} - \tilde{v}_{\tilde{F}}$ is the third parameter called hesitation degree.

Specially, $\tilde{a} = (\tilde{\mu}_{\tilde{a}}, \tilde{v}_{\tilde{a}})$ is called an intuitionistic fuzzy number (IFN) [53], where $\tilde{\mu}_{\tilde{a}} \in [0,1]$, $\tilde{v}_{\tilde{a}} \in [0,1]$, $\tilde{\mu}_{\tilde{a}} + \tilde{v}_{\tilde{a}} \in [0,1]$, $\tilde{\mu}_{\tilde{a}} + \tilde{v}_{\tilde{a}} + \tilde{\pi}_{\tilde{a}} = 1$.

For any two IFNs $\tilde{a}_1 = (\tilde{\mu}_{\tilde{a}_1}, \tilde{v}_{\tilde{a}_1})$, $\tilde{a}_2 = (\tilde{\mu}_{\tilde{a}_2}, \tilde{v}_{\tilde{a}_2})$ and $k \geq 0$, then the operational laws on IFNs are listed as follows [53]:

(1) Addition operation: $\tilde{a}_1 \oplus \tilde{a}_2 = (\tilde{\mu}_1 + \tilde{\mu}_2 - \tilde{\mu}_1 \cdot \tilde{\mu}_2, \tilde{\mu}_1 \cdot \tilde{\mu}_2)$;
(2) Multiplication operation: $\tilde{a}_1 \otimes \tilde{a}_2 = (\tilde{\mu}_1 \cdot \tilde{\mu}_2, \tilde{\mu}_1 + \tilde{\mu}_2 - \tilde{\mu}_1 \cdot \tilde{\mu}_2)$;
(3) Scalar multiplication operation: $k\tilde{a}_1 = (1 - (1 - \tilde{\mu}_1)^k, \tilde{\mu}_2^k)$.

Definition 2 [24] $\tilde{R} = (\tilde{r}_{ij})_{n \times n}$ is called an intuitionistic fuzzy preference relation, where $\tilde{r}_{ij} = ((x_i, x_j), \tilde{\mu}(x_i, x_j), \tilde{v}(x_i, x_j))$, x_i denotes the ith criterion or alternative, $1 \leq i, j \leq n$.

As a matter of convenience, set $\tilde{r}_{ij} = (\tilde{\mu}_{ij}, \tilde{v}_{ij})_{n \times n}$, where $\tilde{\mu}_{ij}$ and $\tilde{v}_{ij}$ measure the degree to which x_i is preferred and non-preferred to x_j. Furthermore, $\tilde{\mu}_{ij}$ and $\tilde{v}_{ij}$ satisfy the following condition:

$$\tilde{\mu}_{ij} = \tilde{v}_{ji}, \tilde{v}_{ij} = \tilde{\mu}_{ji}, \tilde{\mu}_{ii} = \tilde{v}_{ii} = 0.5, 0 \leq \tilde{\mu}_{ij} + \tilde{v}_{ij} \leq 1, 1 \leq i, \ j \leq n.$$

Definition 3 [54] $\tilde{R} = (\tilde{r}_{ij})_{n \times n}$ is called an IVIFPR, where $\tilde{r}_{ij} = ((x_i, x_j), \ \tilde{\mu}(x_i, x_j), \ \tilde{v}(x_i, x_j)), i, j = 1, 2, \ldots, n$, and generally, $\tilde{r}_{ij} = (\tilde{\mu}_{ij}, \tilde{v}_{ij})$ satisfies the following conditions:

(1) $\tilde{\mu}_{ij} = [\tilde{\mu}_{ij}^L, \tilde{\mu}_{ij}^U] \subset [0,1]$, $\tilde{v}_{ij} = [\tilde{v}_{ij}^L, \tilde{v}_{ij}^U] \subset [0,1]$;
(2) $\tilde{\mu}_{ij} = \tilde{v}_{ji}$, $\tilde{v}_{ij} = \tilde{\mu}_{ji}$, $\tilde{\mu}_{ii} = \tilde{v}_{ii} = [0.5, 0.5]$;
(3) $\tilde{\mu}_{ij}^L + \tilde{v}_{ij}^L \leq 1$, $\tilde{\mu}_{ij}^U + \tilde{v}_{ij}^U \leq 1$, $i, j = 1, 2, \ldots, n$.

Xu and Chen [54] called $(\tilde{\mu}, \tilde{v}) = ([\tilde{\mu}^L, \tilde{v}^U], [\tilde{v}^L, \tilde{v}^U])$ an interval-valued intuitionistic fuzzy number (IVIFN), where $[\tilde{\mu}^L, \tilde{\mu}^U] \in [0,1]$, $[\tilde{v}^L, \ \tilde{v}^U] \in [0, 1]$, $\tilde{v}^L + \tilde{\mu}^U \leq 1$.

2.2 CIVIFOWA Operator

The continuous ordered weighted average (COWA) operator is developed by Yager in 2004 [31], which is an extension of the OWA (ordered weighted averaging) operator and can be defined as follows.

Definition 4 [31] A mapping $f : M \to R^+$ is called the COWA operator, if

$$f_Q(\tilde{a}) = f_Q([\tilde{a}^L, \tilde{a}^U]) = \int_0^1 \frac{\mathrm{d}Q(y)}{\mathrm{d}y} (\tilde{a}^U - y(\tilde{a}^U - \tilde{a}^L))\mathrm{d}y, \quad (1)$$

where $\tilde{a} = [\tilde{a}^L, \tilde{a}^U] \in M, M$ refers to the set of all non-negative interval numbers.

If $\lambda = \int_0^1 Q(y)\mathrm{d}y$ is the attitudinal character of Q, then the formula of $f_Q(\tilde{a})$ can be generally expressed as the following:

$$f_Q(\tilde{a}) = f_Q([\tilde{a}^L, \tilde{a}^U]) = \lambda \tilde{a}^U + (1 - \lambda)\tilde{a}^L, \quad (2)$$

where $\lambda = 1$ indicates that the DM's decisional attitude is absolutely positive, while $\lambda = 0$ shows that DM's attitude is absolutely negative. Obviously, the COWA operator guided by the function Q is equivalent to an aggregation where the arguments are valued in $[\tilde{a}^L, \tilde{a}^U]$. Therefore, the aggregation $f_Q([\tilde{a}^L, \tilde{a}^U])$ can be used to replace the interval $[\tilde{a}^L, \tilde{a}^U]$. For convenience, denote the COWA operator f_Q by f_λ in the following content.

Definition 5 [35] A mapping $g : \sum \to \Omega$ is a continuous interval-valued intuitionistic fuzzy ordered weighted averaging (CIVIFOWA) operator, if it is associated with the monotonic function Q, which is defined on a unit interval; and

$$g_Q(\tilde{a}) = (\tilde{\mu}_{g_Q(\tilde{a})}, \tilde{v}_{g_Q(\tilde{a})}) = (f_\lambda([\tilde{\mu}_{\tilde{a}}^L, \tilde{\mu}_{\tilde{a}}^U], f_\lambda([\tilde{v}_{\tilde{a}}^L, \tilde{v}_{\tilde{a}}^U])), \quad (3)$$

where $\tilde{a} = (\tilde{\mu}_{\tilde{a}}, \tilde{v}_{\tilde{a}}) = ([\tilde{\mu}_{\tilde{a}}^L, \tilde{\mu}_{\tilde{a}}^U], [\tilde{v}_{\tilde{a}}^L, \tilde{v}_{\tilde{a}}^U]) \in \sum$, the COWA operator is determined by Eq. (4) and $Q : [0,1] \to [0,1]$ is a monotonic function defined by a basic unit interval with $Q(0) = 0, Q(1) = 1$.

Theorem 1 [35] *Let λ be the attitudinal character of Q, then*

$$g_Q(\tilde{a}) = (\lambda\tilde{\mu}_{\tilde{a}}^L + (1-\lambda)\tilde{\mu}_{\tilde{a}}^U, \lambda\tilde{v}_{\tilde{a}}^L + (1-\lambda)\tilde{v}_{\tilde{a}}^U).$$

Obviously, it can be concluded that $g_Q(\tilde{a})$ is an IFN. Particularly, if $\tilde{\mu}_{\tilde{a}}^U = \tilde{\mu}_{\tilde{a}}^L$ and $\tilde{v}_{\tilde{a}}^U = \tilde{v}_{\tilde{a}}^L$, then $g_Q(\tilde{a})$ degenerates to IFN.

In addition, it also can be concluded that each element of IVIFPR is a double uncertain value composed by interval membership degree as well as interval non-membership degree. This defect may potentially decrease efficiency in data processing. However, the CIVIFOWA operator can ameliorate this problem by transforming the IVIFV into IFV by a controlling parameter λ. This would effectively enhance efficiency and decrease complexity of data processing. Additionally, by utilizing the CIVIFOWA operator to transform interval-valued preferred and non-preferred memberships into parametric real numbers, interval values, and parametric real numbers differ in format, but in essence they do not change. Therefore, the using of parametric variables can prevent the loss of the information. For convenience, we denote CIVIFOWA operator g_Q by g_λ in following paragraphs.

Definition 6 Let $\tilde{A} = (\tilde{a}_{ij})_{n\times n} = ((\tilde{\mu}_{ij}, \tilde{v}_{ij}))_{n\times n} = (([\tilde{\mu}_{ij}^L, \tilde{\mu}_{ij}^U], [\tilde{v}_{ij}^L, \tilde{v}_{ij}^U]))_{n\times n}$ be an IVIFPR, and λ be the attitudinal character of Q. Then, $\tilde{A}_\lambda = (\tilde{a}_{\lambda,ij})_{n\times n}$ is called the continuous interval-valued intuitionistic fuzzy matrix, where $\tilde{a}_{\lambda,ij}$ satisfies following conditions:

(1) $\tilde{a}_{\lambda,ij} = (\tilde{\mu}_{\lambda,ij}, \tilde{v}_{\lambda,ij})$, $\tilde{\mu}_{\lambda,ij} = \lambda\tilde{\mu}_{ij}^U + (1-\lambda)\tilde{\mu}_{ij}^L$, $\tilde{v}_{\lambda,ij} = \lambda\tilde{v}_{ij}^U + (1-\lambda)\tilde{v}_{ij}^L$, $i<j$;
(2) $\tilde{a}_{\lambda,ij} = (0.5, 0.5)$, $i = j$
(3) $\tilde{a}_{\lambda,ij} = (\tilde{v}_{\lambda,ji}, \tilde{\mu}_{\lambda,ji})$, $i > j$.

As we can see, the continuous interval-valued intuitionistic fuzzy matrix $\tilde{A}_\lambda$ is an IFPR. Thus, the continuous interval-valued intuitionistic fuzzy matrix is also called the continuous interval-valued intuitionistic fuzzy preference relation (CIVIFPR).

In order to measure the distance of two IVIFNs, a continuous interval-valued intuitionistic fuzzy distance (CIVIFD) measure is defined as follows.

Definition 7 [55] Let $\tilde{B}_1 = ([\tilde{\mu}_1^L, \tilde{\mu}_1^U], [\tilde{v}_1^L, \tilde{v}_1^U])$ and $\tilde{B}_2 = ([\tilde{\mu}_2^L, \tilde{\mu}_2^U], [\tilde{v}_2^L, \tilde{v}_2^U])$ be two IVIFNs, then

$$d_\lambda(\tilde{B}_1, \tilde{B}_2) = \frac{1}{2}(|\lambda(\tilde{\mu}_1^U - \tilde{\mu}_2^U) + (1-\lambda)(\tilde{\mu}_1^L - \tilde{\mu}_2^L)| + |\lambda(\tilde{v}_1^U - \tilde{v}_2^U) + (1-\lambda)(\tilde{v}_1^L - \tilde{v}_2^L)|), \tag{4}$$

is called the CIVIFD measure.

3 A Novel GDM with IVIFPRs

In this section, a new approach to GDM with IVIFPRs is developed, in which a method to obtain weights of DMs is put forward based on Shapley value [56–58] and a logarithm least optimal model is presented to deriving the priority weights.

3.1 To Deriving DMs' Weights by Using Fuzzy Cooperative Game

Assume that a set of DMs give their opinions on a finite set $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$ of possible alternatives indicated as $\hat{R} = \{\tilde{R}^{(1)}, \tilde{R}^{(2)}, \ldots, \tilde{R}^{(K)}\}$ $(K \geq 2)$. where $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{n\times n}$ represents the kth DM's opinion (judgment) of pairwise comparison on x_i over x_j using the IVIFPR, and $\tilde{r}_{ij}^{(k)} = (\tilde{\mu}_{ijk}, \tilde{v}_{ijk})$. Therefore, GDM with IVIFPRs is capable of possible best options selection based on $\{\tilde{R}_\lambda^{(k)}, k \in \{1, 2, \ldots, K\}\}$. Note that GDM with the IVIFPRs is presented as $<X, \hat{R}>$ below.

With $<X, \hat{R}>$, it is possible for individuals to expect their personal opinions to be in line with decision of the group. Despite deviation, the new method contributes to develop a modified solution which balances up all forms of deviation.

Definition 8 [54] Let $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{n\times n} = (([u_{ij,k}^-, u_{ij,k}^+], [v_{ij,k}^-, v_{ij,k}^+]))_{n\times n}$ be the IVIFPRs provided by kth DM, and $w = (w_1, w_2, \ldots, w_K)^{\mathrm{T}}$ be a weighting vector satisfying $w_k \geq 0$ and $\sum_{k=1}^K w_k = 1$. If $\tilde{R} = (\tilde{r}_{ij}^{(k)})_{n\times n} = (([u_{ij}^-, u_{ij}^+], [v_{ij}^-, v_{ij}^+]))_{n\times n}$ satisfies:

$$u_{ij}^- = \sum_{k=1}^K w_k u_{ij,k}^-, u_{ij}^+ = \sum_{k=1}^K w_k u_{ij,k}^+, \quad v_{ij}^- = \sum_{k=1}^K w_k v_{ij,k}^-, v_{ij}^+ = \sum_{k=1}^K w_k v_{ij,k}^+, \tag{5}$$

then $\tilde{R}$ is called the group interval-valued intuitionistic fuzzy matrix. Clearly, $\tilde{R}$ is an IVIFPR. For each $x_i \in X$, the following deviation of each individual and the group IVIFPR is defined to measure the distance between each individual and the group.

Definition 9 Let $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{n\times n}$ be the IVIFPR provided by kth DM, and $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$ be the group IVIFPR, then

$$deq_{\lambda,ik} = \sum_{j=1}^n d_\lambda(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}), \tag{6}$$

is called the continuous interval-valued intuitionistic fuzzy deviation measure of kth DM and the group on ith alternative with λ, where d_λ is the continuous interval-valued intuitionistic fuzzy distance measure.

Proposition 1 $deq_{\lambda,ik} = deq_{\lambda,ki}$.

Definition 10 Assume that $deq_{\lambda,ik}$ is as before. Then, $Cdeq_\lambda = (deq_{\lambda,ik})_{n\times K}$ is called a continuous complete deviation matrix with λ.

Definition 11 Let $Cdeq_\lambda$ be the continuous complete deviation matrix with λ, then for a certain λ, $E_\lambda = Cdeq_\lambda^{\mathrm{T}} \cdot Cdeq_\lambda = (e_{\lambda,ij})_{K\times K}$ is called a continuous group decision error information matrix with λ.

Proposition 2 *The continuous group decision error information matrix E_λ is symmetric.*

The proof is straightforward from Definition 11, because the element on the primary diagonal of E_λ is the expression of the decision error of each corresponding individual.

3.2 Algorithm to Determine DMs' Weights

This subsection proposes a novel algorithm in which cooperative game is applied to deriving weighting vector of DMs by using the reciprocal of mean error. The goal of each stage and computing methods within are explained in detail as follows.

Algorithm 1

Step 1. To define an initial weighting vector of DMs

The initial weighting vector of DMs is defined as $w_\lambda = (w_{\lambda,1}, w_{\lambda,2}, \ldots, w_{\lambda,K})^{\mathrm{T}}$ by using the following formula:

$$w_{\lambda,k} = \left(\sum_{k=1}^{K} e_{\lambda,kk}^{-1}\right)^{-1} \cdot e_{\lambda,kk}^{-1}, k = 1, 2, \ldots, K, \tag{7}$$

where $e_{\lambda,kk}$ is the kth diagonal element of E_λ.

Step 2. Characteristic functions of coalitions

For the set of DMs, there are 2^K coalitions of DMs. Using the initial weighting vector, a characteristic function of coalition $s \in 2^K$ can be obtained as below:

$$q_\lambda(s) = -\Lambda_s^{\mathrm{T}} E_{\lambda,s} \Lambda_s. \tag{8}$$

where $\Lambda_s = \{\lambda_1, \lambda_2, \ldots, \lambda_{|s|}\}$, λ_i is the weights of ith DM in coalition s, and $E_{\lambda,s} = (e_{\lambda,tk})_{|s|\times|s|}$, |s| is the cardinality of s.

Step 3. Determining individual's contribution in GDM

The cooperative result of the coalition s can be measured as the characteristic value $q_\lambda(s)$. For the ith DM, his/her contribution to the coalition is calculated according to Eq. (5).

Step 4. Determining the new weighting vector of DMs

The Shapley [46] value is defined as

$$\phi_i(N; v) = v(N \cap [1, i]) - v(N \cap [1, i-1]). \tag{9}$$

There are a finite number of N game players (the grand coalition s) in the cooperative games, with a feature function $v: 2^N \to R$ coming from all available payers in relation to another payment set meeting at $v(\phi) = 0$.

Therefore, each player's contribution in GDM can be represented by the average of the contribution above, denoted as $\phi_i(q)$, $i = \{i, \ldots, K\}$, which is calculated as

$$\phi_i(q) = \sum_{i\in s} \frac{(k-|s|)! \cdot (|s|-1)!}{K!}[q(s) - q(s - \{i\})], \tag{10}$$

Owen [59] has proved that:

$$\sum_{k=1}^{K} \phi_i(q) = q(s), \tag{11}$$

where $s = \{1, 2, \ldots, K\}$ the largest coalition. Equation (5) indicates that the sum of the individual's contributions is equal to the result of the group's cooperation.

Thus, the weight of a DM should be in proportion to his/her "contribution," because the "contribution" is measured by using the decision error, so a new weight can be computed by using the following formula:

$$w_{\lambda,t} = \frac{1}{\phi_{\lambda,k}(q)} \Bigg/ \sum_{k=1}^{K} \frac{1}{\phi_{\lambda,k}(q)}, k = 1, 2, \ldots, K. \tag{12}$$

Assume that δ is the stopping condition, if

$$w_{\lambda,t}^{\mathrm{T}} E_{\lambda,t} w_{\lambda,t} - w_{\lambda,t+1}^{\mathrm{T}} E_{\lambda,t+1} w_{\lambda,t+1} \le \delta, \tag{13}$$

then the iteration process of the algorithm is finished, otherwise, return to stage 2.

Based on stages as above, the new algorithm for determining the DMs' weights in GDM with IFPRs is depicted in Fig. 1.

Theorem 2 *Let $E_{\lambda,t}$ be the decision error information matrix and $w_{\lambda,t}$ be the weighting vector of DMs in t-th iteration of Algorithm* 1*, respectively. Then it follows that*

$$w_{\lambda,t}^{\mathrm{T}} E_{\lambda,t} w_{\lambda,t} \ge w_{\lambda,t+1}^{\mathrm{T}} E_{\lambda,t+1} w_{\lambda,t+1}. \tag{14}$$

Proof Owen [59] deduced that Eq. (13) is a valid payment plan for $v_\lambda(M)$, which satisfies the following conditions:

$$v_\lambda(\emptyset) = 0 \text{ and } v_\lambda(M) \ge \sum_{k=1}^{K} v_\lambda(k).$$

□

Similarly, for the payment plan of the new coalition $\tilde{v}_\lambda(M)$, we have $\tilde{v}_\lambda(\emptyset) = 0$ and $\tilde{v}_\lambda(M) \ge \sum_{k=1}^{K} \tilde{v}_\lambda(k)$, where

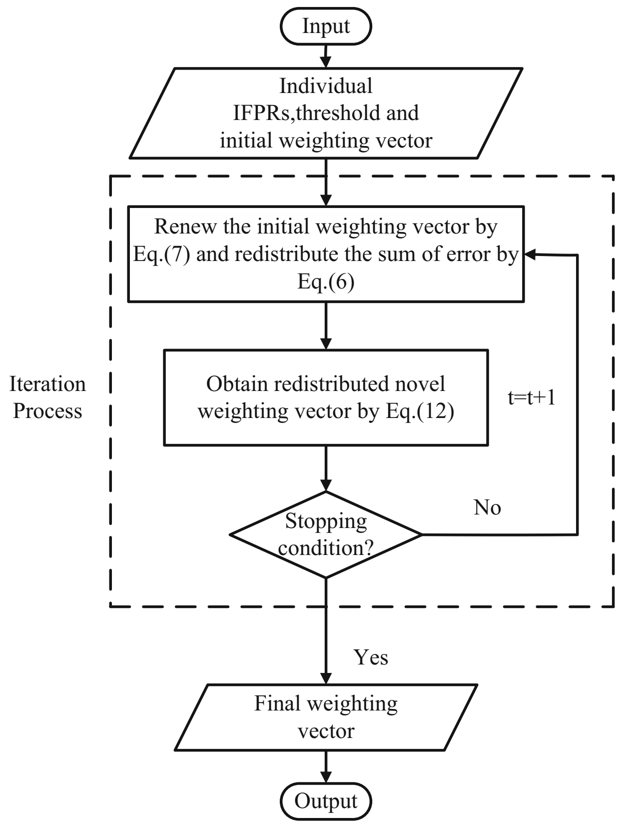

Fig. 1 Algorithm for determining weighting vector of DMs via cooperative method

$\tilde{v}_\lambda(M) = \sum_{k=1}^{K} \tilde{\tau}_{\lambda,k}(v)$, $\tilde{\tau}_{\lambda,k}(v)$ is the payment of kth DM in the new coalition. Assume that $\tilde{T}_\lambda(v) = (\tilde{\tau}_{\lambda,1}(v), \tilde{\tau}_{\lambda,2}(v), \ldots, \tilde{\tau}_{\lambda,K}(v))$ represents the Shapley value of the cooperative game corresponding to the weighting vector w.

When redistributing the earning of coalition by utilizing the weighting vector, it can be derived that $\tilde{\tau}_{\lambda,k}(v) \geq \tau_{\lambda,k}(v)$, $k = 1, 2, \ldots, K$, otherwise, such redistribution would be discontinued. Therefore, the result can be obtained directly as follows:

$$\tilde{v}_\lambda(M) = \sum_{k=1}^{K} \tilde{\tau}_{\lambda,k}(v) \geq \sum_{k=1}^{K} \tau_{\lambda,k}(v) = v_\lambda(M). \quad (15)$$

Since $v_\lambda(M) = -\Lambda^{\mathrm{T}} E_\lambda \Lambda$ and $\tilde{v}_\lambda(M) = -w_\lambda^{\mathrm{T}} E_\lambda w_\lambda$, then by Eq. (14),

$$-w_\lambda^{\mathrm{T}} E_\lambda w_\lambda \geq -\Lambda^{\mathrm{T}} E_\lambda \Lambda. \quad (16)$$

Therefore, with the inequality (15), $w_{\lambda,t}^{\mathrm{T}} E_{\lambda,t} w_{\lambda,t} \geq w_{\lambda,t+1}^{\mathrm{T}} E_{\lambda,t+1} w_{\lambda,t+1}$ can be proved.

Repeating Algorithm 1, series of weighting vectors can be obtained as $w_{\lambda,1}, w_{\lambda,2}, w_{\lambda,3}, \ldots$, the decision error information matrices are derived as $E_{\lambda,1}, E_{\lambda,2}, E_{\lambda,3}, \ldots$. From Theorem 3, we can get that

$$w_{\lambda,0}^{\mathrm{T}} E_{\lambda,0} w_{\lambda,0} \geq w_{\lambda,1}^{\mathrm{T}} E_{\lambda,1} w_{\lambda,1} \geq w_{\lambda,2}^{\mathrm{T}} E_{\lambda,2} w_{\lambda,2} \geq \cdots. \quad (17)$$

The following theorem can be obtained.

Theorem 3 *Let $S_\lambda(t) = w_{\lambda,t}^{\mathrm{T}} E_{\lambda,t} w_{\lambda,t}$ be the sum of decision errors in t-th iteration of Algorithm* 1. *Then, the sequence $\{S_\lambda(t)\}$ is convergent as $t \to +\infty$.*

Proof For the sequence of $\{S_\lambda(t)\}$, it is monotonic in reduction as it can be directly obtained from Eq. (16). The weights and the decision error matrix constructed by the distance measure are positive, so $S_\lambda(t) \geq 0$, which means that sequence $\{S_\lambda(t)\}$ has lower bound. This makes the sequence $\{S_\lambda(t)\}$ convergent which completes the proof of Theorem 4. □

Theorem 3 suggest that a weighting vector of DMs for the GDM derived from the Algorithm 1 in accordance with Shapley Value is reasonable and objective. As a result, Shapley function can be appropriately used in obtaining weights, in line with the contribution of each individual DM.

The main advantages of Algorithm 1 are as follows:

- The redistributing process of the algorithm aims to get an optimal DM's combination, in which the contribution of each DM's opinions (judgments) to the group indicates their opinions (judgments)'s rationality and importance in GDM.
- On the basis of the decision accuracy and certain computation efficiency that the DMs prefer, the stopping condition of the iteration can be adjusted flexibly.
- DMs can revise the algorithm by adjusting the parameter λ based on their decision-making attitude, and after a constant iterating process, the initial information still can be preserved to the maximum.

3.3 The Logarithm Least Optimal Model to Deriving Priority Weights

In the following section, based on the Gong's optimization model developed in [60], a new optimal model is introduced for getting the priority vector of group IFPR in which the priority weights are represented as interval values. Suppose that group IFPR $\tilde{R}_\lambda = (\tilde{r}_{\lambda,ij})_{n\times n} = (\tilde{\mu}_{\lambda,ij}, \tilde{v}_{\lambda,ij})_{n\times n}$ and $\xi_i = [\xi_{il}, \xi_{iu}]$ is the ith weight, $i = 1, 2, \ldots, n$. According to the consistency condition and Definition 4, we have

$$\xi_{il} - \tilde{\mu}'_{\lambda,ij}\xi_{iu} = 0 \xi_{iu} - \tilde{v}'_{\lambda,ij}\xi_{il} = 0, i,j \in N, \quad (18)$$

where $\tilde{\mu'}_{\lambda,ij} = 3^{5(\tilde{\mu}_{\lambda,ij}-0.5)}$, $\tilde{v'}_{\lambda,ij} = 3^{5(0.5-\tilde{v}_{\lambda,ij})}$. However, in practical environment, the equality given by Eq. (18) does not always hold, i.e., the deviation cannot avoid. Hence, the following logarithm least optimal model is proposed to obtain the priority vector for GDM.

$$\min J = \sum_{i=1}^{n}\sum_{j=1}^{n} \log\left|\xi_{il} - \tilde{\mu}'_{\lambda,ij}\xi_{ju}\right| + \log\left|\xi_{iu} - \tilde{v}'_{\lambda,ij}\xi_{jl}\right|$$

$$s.t.\begin{cases} \xi_{il} + \sum\limits_{j=1, j\neq i}^{n} \xi_{ju} \geq 1, & i \in N; \\ \xi_{iu} + \sum\limits_{j=1, j\neq i}^{n} \xi_{jl} \geq 1, & i \in N; \\ \xi_{iu} - \xi_{il} \geq 0, & i \in N; \\ \xi_{iu} \geq 0, \xi_{il} \geq 0, & i \in N. \end{cases} \tag{19}$$

In this model, the goal is to minimize the consistency of group IFPR. The smaller the logarithm values, the better the consistency of group IFPR. By considering the optimal priority vector of group IFPR, it can be seen that the greater the value of $\xi_i = [\xi_{il}, \xi_{iu}]$ is, the more important of the ith alternative is.

Note that professional software, such as MATLAB and Lingo, can be applied into the process of solving the optimal model.

3.4 Resolution Process of the GDM Problem

In this subsection, a two-stage resolution process is proposed for the GDM with IVIFPRs.

Stage 1 Aggregation

In this stage, after transforming all individual IVIFPRs into individual IFPRs and by aggregating all individual IFPRs $\tilde{R}_{\lambda}^{(k)} = (\tilde{r}_{\lambda,ij}^{(k)})_{n\times n} = (u_{\lambda,ij}^{(k)},\ v_{\lambda,ij}^{(k)})_{n\times n},\ k = 1, 2, \ldots, K,$ through the final weighting vector $w = \{w_1, w_2, \ldots, w_K\}^{\mathrm{T}}$, the group IFPR [61] $\tilde{R}_{\lambda} = (\tilde{r}_{\lambda,ij})_{n\times n} = ((u_{\lambda,ij}, v_{\lambda,ij}))_{n\times n}$ is calculated according to Eq. (1), where

$$u_{\lambda,ij} = \sum_{k=1}^{K} w_k u_{\lambda,ij}^{(k)}, v_{\lambda,ij} = \sum_{k=1}^{K} w_k v_{\lambda,ij}^{(k)}. \tag{20}$$

Note that before aggregating individual's IFPRs, the consistency should be tested and readjusted through the method developed by Wan et al. [51].

Stage 2 Selection

In this stage, the rank of alternatives is determined. By utilizing the logarithm least optimal model, the interval priority vector $\xi_{\lambda,i} = [\xi_{\lambda,il}, \xi_{\lambda,iu}]$ of group IFPR can be obtained. Construct the priority vector as $\xi'_{\lambda,i} = (\xi_{\lambda,il}, 1 - \xi_{\lambda,iu})$, and the score of each alternative is defined as $\Delta x_{\lambda,i} = \xi_{\lambda,il} + \xi_{\lambda,iu} - 1$, which can be used to rank and select the best alternative(s). The bigger the score is, the better the alternative is. And the whole process for the aggregation and selection stage is shown in Fig. 2.

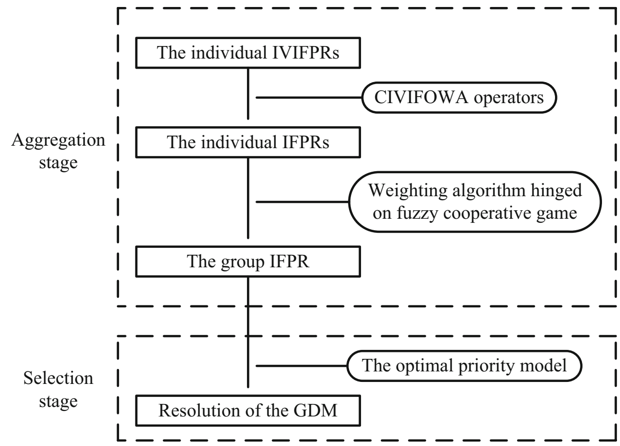

Fig. 2 The process of GDM

4 Illustrative Example

In this section, a practical example is used to demonstrate the application of proposed method. Meanwhile, the ideal superiorities of proposed method are revealed by comparison analysis.

In this constantly changeable market environment, it is increasingly challenging for an enterprise to meet demand of market by itself alone. Under this circumstance, virtual enterprise is established based on multiple members, who share kinds of resources such as fund and techniques within the "group."

For an enterprise that is developing a new product, it is possible that the product cannot be produced by the enterprise alone. Therefore, enterprise leaders may turn to other competent enterprises (member) for help. Once the production begins to run, the key and member enterprises give all their best for whole process. Obviously, efficient member enterprises will contribute to production. Therefore, based on the cooperative manner, selecting appropriate partners is crucial for the key enterprise.

Take AHEAD Information Technology Co., LTD as an example, which is a reputable software company in China [50]. AHEAD has been concentrating on medical information integrating and service since 2003 when its establishment was started. The target of AHEAD is to up a brand-new rural cooperative medical care management information system, which comprises a hardware device (with chips integrated) and software (some essential software systems). As an information company, AHEAD is deeply troubled by the hardware device. Therefore, it is necessary for AHEAD to select a suitable and responsible partner to take charge of producing hardware device.

First of all, AHEAD chose four candidates and asked three DMs to test the four candidates in order to make the best choice. For this purpose, the proposed cooperative method is applied to obtain the best choice out of four candidates.

By conducting pairwise comparisons on four partners, DMs furnish their IVIFPRs as

$$\tilde{R}^{(1)} = \begin{pmatrix} ([0.5000,0.5000],[0.5000,0.5000]) & ([0.4500,0.5000],[0.1000,0.2000]) & ([0.5000,0.6000],[0.0271,0.0707]) & ([0.8000,0.8500],[0.0028,0.0167]) \\ ([0.1000,0.2000],[0.4500,0.5000]) & ([0.5000,0.5000],[0.5000,0.5000]) & ([0.7500,0.8000],[0.1500,0.2000]) & ([0.6500,0.7000],[0.0094,0.0254]) \\ ([0.0271,0.0707],[0.5000,0.6000]) & ([0.1500,0.2000],[0.7500,0.8000]) & ([0.5000,0.5000],[0.5000,0.5000]) & ([0.8500,0.8500],[0.0500,0.1000]) \\ ([0.0028,0.0167],[0.8000,0.8500]) & ([0.0094,0.0254],[0.6500,0.7000]) & ([0.0500,0.1000],[0.8000,0.8000]) & ([0.5000,0.5000],[0.5000,0.5000]) \end{pmatrix}$$

$$\tilde{R}^{(2)} = \begin{pmatrix} ([0.5000,0.5000],[0.5000,0.5000]) & ([0.6500,0.6700],[0.0500,0.3300]) & ([0.6200,0.6500],[0.0360,0.3171]) & ([0.7800,0.8200],[0.0060,0.1246]) \\ ([0.0500,0.3300],[0.6500,0.7000]) & ([0.5000,0.5000],[0.5000,0.5000]) & ([0.5000,0.5500],[0.3000,0.4000]) & ([0.7500,0.7500],[0.1000,0.2000]) \\ ([0.0360,0.3171],[0.6200,0.6500]) & ([0.3000,0.4000],[0.5000,0.5500]) & ([0.5000,0.5000],[0.5000,0.5000]) & ([0.7000,0.7500],[0.1000,0.2000]) \\ ([0.0060,0.1246],[0.7800,0.8200]) & ([0.0800,0.1800],[0.7500,0.8000]) & ([0.1000,0.2000],[0.7000,0.7500]) & ([0.5000,0.5000],[0.5000,0.5000]) \end{pmatrix}$$

$$\tilde{R}^{(3)} = \begin{pmatrix} ([0.5000,0.5000],[0.5000,0.5000]) & ([0.4500,0.5000],[0.1000,0.2000]) & ([0.5000,0.6000],[0.271,0.0707]) & ([0.8000,0.8500],[0.0028,0.0167]) \\ ([0.1000,0.2000],[0.4500,0.5000]) & ([0.5000,0.5000],[0.5000,0.5000]) & ([0.7500,0.8000],[0.1500,0.2000]) & ([0.6500,0.7000],[0.0094,0.0254]) \\ ([0.0271,0.0707],[0.5000,0.6000]) & ([0.1500,0.2000],[0.7500,0.8000]) & ([0.5000,0.5000],[0.5000,0.5000]) & ([0.8500,0.8500],[0.0500,0.1000]) \\ ([0.0028,0.0167],[0.8000,0.8500]) & ([0.0094,0.0254],[0.6500,0.7000]) & ([0.0500,0.1000],[0.8000,0.8000]) & ([0.5000,0.5000],[0.5000,0.5000]) \end{pmatrix}$$

Step 1 Transform the IVIFPRs into the IFPRs by utilizing the CIVIFOWA operator, without loss of generality, we take $Q(y) = y^{2/3}$ and then $\lambda = \int_0^1 Q(y)\mathrm{d}y = 0.6$, the IFPRs $\tilde{R}_\lambda^{(k)} = (\tilde{r}_{\lambda,ij}^{(k)})_{n\times n}$, $k = 1,2,3$, are as follows:

$$\tilde{R}_\lambda^{(1)} = \begin{pmatrix} [0.5000,0.5000] & [0.1400,0.6500] & [0.6200,0.0860] & [0.8300,0.0230] \\ [0.6500,0.1400] & [0.5000,0.5000] & [0.7300,0.0400] & [0.9000,0.0094] \\ [0.0860,0.6200] & [0.0400,0.7300] & [0.5000,0.5000] & [0.5500,0.1000] \\ [0.0234,0.8300] & [0.0094,0.9000] & [0.1000,0.5500] & [0.5000,0.5000] \end{pmatrix},$$

$$\tilde{R}_\lambda^{(2)} = \begin{pmatrix} [0.5000,0.5000] & [0.6500,0.0500] & [0.6200,0.0360] & [0.7800,0.0060] \\ [0.0500,0.6500] & [0.5000,0.5000] & [0.5000,0.3000] & [0.7500,0.0800] \\ [0.0360,0.6200] & [0.3000,0.5000] & [0.5000,0.5000] & [0.7000,0.1000] \\ [0.0060,0.7800] & [0.0800,0.7500] & [0.1000,0.7000] & [0.5000,0.5000] \end{pmatrix},$$

$$\tilde{R}_\lambda^{(3)} = \begin{pmatrix} [0.5000,0.5000] & [0.4500,0.1000] & [0.5000,0.0271] & [0.8000,0.0028] \\ [0.1000,0.4500] & [0.5000,0.5000] & [0.7500,0.1500] & [0.6500,0.0094] \\ [0.0271,0.5000] & [0.1500,0.7500] & [0.5000,0.5000] & [0.8000,0.0500] \\ [0.0028,0.8000] & [0.0094,0.6500] & [0.0500,0.8000] & [0.5000,0.5000] \end{pmatrix}.$$

Step 2 By giving initial weighting vector $w_0 = (0.4232, 0.1819, 0.3949)$ and Eqs. (6)–(8), we obtain the deviation matrix and decision error matrix:

$$deq = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{ccc} \mathrm{IDL}_1 & \mathrm{IDL}_2 & \mathrm{IDL}_3 \end{array} \\ \begin{pmatrix} 0.0234 & 0.0143 & 0.0090 \\ 0.0560 & 0.0364 & 0.0213 \\ 0.0700 & 0.0500 & 0.0323 \\ 0.0833 & 0.0547 & 0.0416 \end{pmatrix} \end{array}$$

$$E_{0.6} = \begin{array}{c} \\ \mathrm{IDL}_1 \\ \mathrm{IDL}_2 \\ \mathrm{IDL}_3 \end{array} \begin{array}{c} \begin{array}{ccc} \mathrm{IDL}_1 & \mathrm{IDL}_2 & \mathrm{IDL}_3 \end{array} \\ \begin{pmatrix} 0.0155 & 0.0104 & 0.0071 \\ 0.0104 & 0.0070 & 0.0048 \\ 0.0071 & 0.0048 & 0.0033 \end{pmatrix} \end{array}$$

Then, set the threshold $\delta = 10^{-6}$ and the iterative process of the Algorithm 1 is finished. The results are listed in Table 1, and the final weighting vector of three DMs is (0.2349, 0.5115, 0.2536).

Step 3 Solve the logarithm least optimal model Eq. (19) for deriving the IFPR priority weighting vector of alternatives based on the group IFPR, and we have the optimal solution is

$$\xi_{0.6} = ([0.0831, 0.2042], [0.1764, 0.4150], [0.1374, 0.2815], [0.2690, 0.5363]).$$

Step 4 Employ the score function to determine the values of alternatives:

$$\Delta x_1 = -0.7795, \Delta x_2 = -0.5143, \Delta x_3 = -0.5546, \Delta x_4 = -0.0488.$$

which indicates that $x_4 \succ x_2 \succ x_3 \succ x_1$.

In order to measure how the parameter λ plays the role on DMs' weights, priority weights of group IFPR, and the ranking of alternatives, we consider $\lambda = 0, 0.1, 0.2, \ldots, 0.9, 1$, and the results of DMs' weights, priority weights of group IFPR, and the ranking of alternatives with different λ are listed in Table 2, and the results of DMs' weights and score function values of alternatives are shown in Figs. 3 and 4.

From Fig. 3, we can see that three weights are fluctuating between 0.2 and 0.55, with different λ, each DM gets the chance to be dominant, means that giving the most contribution to the group IFPRs. However, $w_1 > w_3 > w_2$ as $\lambda < 0.1425$, which means that if DMs are very pessimistic, then the opinion of d_1 is the most important decision. Similarly, $w_1 > w_2 > w_3$ as $\lambda > 0.6519$, which indicates that if DMs are optimistic, then the opinion of d_1 is also the most important.

From Fig. 4, when $\lambda < 0.1326$ and $\lambda > 0.6575$, the best alternative is x_2, while $0.1326 \le \lambda \le 0.6575$, x_4 is superior to other alternatives.

Furthermore, by using different attitudal characters for different DMs, we can obtain the weights of DMs and the ranking results, which are shown in Table 3. From Table 3, we can see when the attitudes of the DMs as well as their judgments are different, the weight of each DM changes while the ranking order reserves. The results illustrate that the attitudinal character will not produce a loss of information and the ranking result is stable under any given conditions.

Compared to [51], we observe:

(1) Reference [51] mainly focuses on the consistency test of individual IVIFPR and consistency adjusting for finishing the GDM process. The proposed method is devoted to deriving DMs' weights in GDM by using fuzzy cooperative game based on the CIVIFOWA operator, which is different from [51].

(2) Reference [51] built an optimization model to get DMs weights by minimizing the deviations between each individual IVIFPR and the collective one, while the proposed method developed an iterative algorithm to get optimal DMs' weights in which the contribution of each DM to the group is calculated

based on Shapley value method and the CIVIFOWA operator.

(3) The weighting algorithm proposed in this paper can be adjusted by modifying the attitude parameter based on the CIVIFOWA operator, which makes the proposed method more flexible.

In the following, the proposed method is applied to the Xu's example [62]. Results of Xu's method and proposed method are compared, and the advantages of proposed method are demonstrated.

Step 1 Take the IVIFPRs from Xu's example [62] are as follows,

$$\tilde{C}^{(1)}=\begin{pmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.2,0.3]) & ([0.3,0.5],[0.2,0.4]) \\ ([0.1,0.2],[0.6,0.7]) & ([0.5,0.5],[0.5,0.5]) & ([0.4,0.6],[0.1,0.2]) & ([0.6,0.7],[0.1,0.3]) \\ ([0.2,0.3],[0.5,0.6]) & ([0.1,0.2],[0.4,0.6]) & ([0.5,0.5],[0.5,0.5]) & ([0.3,0.4],[0.5,0.6]) \\ ([0.2,0.4],[0.3,0.5) & ([0.1,0.3],[0.6,0.7]) & ([0.5,0.6],[0.3,0.4]) & ([0.5,0.5],[0.5,0.5]) \end{pmatrix}$$

$$\tilde{C}^{(2)}=\begin{pmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.2,0.3],[0.5,0.6]) & ([0.5,0.7],[0.1,0.2]) & ([0.2,0.4],[0.1,0.3]) \\ ([0.5,0.6],[0.2,0.3]) & ([0.5,0.5],[0.5,0.5]) & ([0.5,0.8],[0.1,0.2]) & ([0.3,0.6],[0.2,0.3]) \\ ([0.1,0.2],[0.5,0.7]) & ([0.1,0.2],[0.5,0.8]) & ([0.5,0.5],[0.5,0.5]) & ([0.4,0.6],[0.1,0.4]) \\ ([0.1,0.3],[0.2,0.4]) & ([0.2,0.3],[0.3,0.6]) & ([0.1,0.4],[0.4,0.6]) & ([0.5,0.5],[0.5,0.5]) \end{pmatrix}$$

$$\tilde{C}^{(3)}=\begin{pmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.4,0.5],[0.2,0.3]) & ([0.6,0.7],[0.1,0.2]) & ([0.5,0.7],[0.2,0.3]) \\ ([0.2,0.3],[0.4,0.5]) & ([0.5,0.5],[0.5,0.5]) & ([0.5,0.6],[0.2,0.4]) & ([0.7,0.8],[0.1,0.2]) \\ ([0.1,0.2],[0.6,0.7]) & ([0.2,0.4],[0.5,0.6]) & ([0.5,0.5],[0.5,0.5]) & ([0.6,0.7],[0.1,0.3]) \\ ([0.2,0.3],[0.5,0.7]) & ([0.1,0.2],[0.7,0.8]) & ([0.1,0.3],[0.6,0.7]) & ([0.5,0.5],[0.5,0.5]) \end{pmatrix}$$

Step 2 Transform the IVIFPRs into the IFPRs by using the CIVIFOWA operator and predefine the parameter $\lambda = 0.6$.

$$\tilde{C}_{\lambda}^{(1)}=\begin{pmatrix} [0.50,0.50] & [0.64,0.14] & [0.54,0.24] & [0.38,0.28] \\ [0.14,0.64] & [0.50,0.50] & [0.48,0.14] & [0.64,0.18] \\ [0.24,0.54] & [0.14,0.48] & [0.50,0.50] & [0.34,0.54] \\ [0.28,0.38] & [0.18,0.64] & [0.54,0.34] & [0.50,0.50] \end{pmatrix}$$

$$\tilde{C}_{\lambda}^{(2)}=\begin{pmatrix} [0.50,0.50] & [0.24,0.54] & [0.58,0.14] & [0.28,0.18] \\ [0.54,0.24] & [0.50,0.50] & [0.62,0.14] & [0.42,0.24] \\ [0.14,0.58] & [0.14,0.62] & [0.50,0.50] & [0.48,0.22] \\ [0.18,0.28] & [0.24,0.42] & [0.22,0.48] & [0.50,0.50] \end{pmatrix}$$

$$\tilde{C}_{\lambda}^{(3)}=\begin{pmatrix} [0.50,0.50] & [0.44,0.24] & [0.64,0.14] & [0.58,0.24] \\ [0.24,0.44] & [0.50,0.50] & [0.54,0.28] & [0.74,0.14] \\ [0.14,0.64] & [0.28,0.54] & [0.50,0.50] & [0.64,0.18] \\ [0.24,0.58] & [0.14,0.74] & [0.18,0.64] & [0.50,0.50] \end{pmatrix}$$

Step 3 Utilize Algorithm 1 for computing the weighting vector according to Eqs. (6)–(8), and the deviation matrix and decision error matrix can be obtained:

$$deq=\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}\begin{array}{c} \begin{array}{ccc} IDL_1 & IDL_2 & IDL_3 \end{array} \\ \begin{pmatrix} 0.0147 & 0.0232 & 0.0105 \\ 0.0280 & 0.0508 & 0.0212 \\ 0.0442 & 0.0605 & 0.0381 \\ 0.0574 & 0.0788 & 0.0591 \end{pmatrix} \end{array}, \quad E_{0.6}=\begin{array}{c} \\ IDL_1 \\ IDL_2 \\ IDL_3 \end{array}\begin{array}{c} \begin{array}{ccc} IDL_1 & IDL_2 & IDL_3 \end{array} \\ \begin{pmatrix} 0.0062 & 0.0090 & 0.0058 \\ 0.0090 & 0.0130 & 0.0083 \\ 0.0058 & 0.0083 & 0.0055 \end{pmatrix} \end{array}.$$

And the final weights of DMs are $w = (0.4284,\ 0.2589,\ 0.3127)$. The results of iterative process are listed in Table 3. The final weighting vector of DMs is (0.4284, 0.2589, 0.3127).

Step 4 Solve the logarithm least optimal model Eq.(19) for deriving the IFPR priority vector of alternatives based on the group IFPRs. The optimal solution is

Table 1 Iterative results of weighting vector and iterative times

The number of iterations	$w_{0.6,1}$	$w_{0.6,2}$	$w_{0.6,3}$
0	0.4232	0.1819	0.3949
1	0.5311	0.2398	0.2291
2	0.2675	0.2408	0.4917
3	0.2453	0.5233	0.2314
4	0.2618	0.3217	0.4165
5	0.2349	0.5115	0.2536

$$\xi = ([0.1772, 0.3310], [0.1531, 0.2505], [0.1895, 0.2967], [0.2665, 0.4344]).$$

Then, employing the score function, the alternatives values are

$$\Delta x_1 = -0.4918, \Delta x_2 = -0.5965, \Delta x_3 = -0.5138, \Delta x_4 = -0.2991.$$

It indicates that the ranking order is $x_4 \succ x_1 \succ x_3 \succ x_2$ and the best alternative is x_4. Priority vectors and ranking of alternatives with different λ are listed in Table 4 and shown in Figs. 5 and 6.

As we can see from Fig. 5, w_1 is monotonically increasing as λ increases, while w_2 and w_3 decrease monotonically as λ increases. However, when $\lambda < 0.267$, $w_3 > w_1 > w_2$, and when $\lambda > 0.267$, $w_1 > w_3 > w_2$, while $\lambda = 0.267$, $w_1 = w_3 > w_2$. The results indicate that when $\lambda < 0.267$, d_3 is the most important DM, while $\lambda > 0.267$, w_1 takes up the dominant degree in GDM, and the importance degree of d_2 is stable.

From Fig. 6, it is obvious that alternative x_4 is at optimal status throughout this changing process of λ. Although each sequencing value has a change corresponding to λ, the superiority and inferiority of the alternatives are apparent and not changed (Table 5).

Moreover, compared to [62], we find:

(1) The focuses of the approach developed in Ref. [62] and the proposed method are different. Method [62] investigated the aggregating the IVIFPRs into a collective one by using interval-valued intuitionistic fuzzy aggregation operators, while the proposed method focuses on deriving DMs' weights in GDM by using fuzzy cooperative game based on the CIVIFOWA operator.

(2) Ref. [62] proposed a new weighting method to obtain DMs weights based on the normal distribution, while the proposed method developed a weighting iterative algorithm to get optimal DMs' weights based on Shapley value method and the

Table 2 DM's weights, priority weights, and the ranking with different λ

λ	w_λ	ξ_λ	Ranking
0	(0.4602, 0.2501, 0.2897)	([0.1593, 0.3224][0.1649, 0.2433] [0.2024, 0.3773][0.2308, 0.2824])	$x_2 \succ x_4 \succ x_3 \succ x_1$
0.1	(0.4602, 0.2501, 0.2897)	([0.1543, 0.3211][0.1546, 0.2129] [0.2026, 0.3839][0.2239, 0.2590])	$x_2 \succ x_4 \succ x_3 \succ x_1$
0.2	(0.2252, 0.5448, 0.2300)	([0.0956, 0.2032][0.1757, 0.4183] [0.1286, 0.2451][0.2348, 0.4991])	$x_4 \succ x_2 \succ x_3 \succ x_1$
0.3	(0.2588, 0.5246, 0.2316)	([0.0090, 0.1279][0.1997, 0.2489] [0.1728, 0.2387][0.4120, 0.5013])	$x_4 \succ x_2 \succ x_3 \succ x_1$
0.4	(0.2569, 0.3070, 0.4361)	([0.0092, 0.1499][0.1983, 0.2718] [0.1871, 0.2684][0.4119, 0.5230])	$x_4 \succ x_2 \succ x_3 \succ x_1$
0.5	(0.2519, 0.3131, 0.4278)	([0.0838, 0.1423][0.1865, 0.2621] [0.1736, 0.2565][0.3815, 0.4926])	$x_4 \succ x_2 \succ x_3 \succ x_1$
0.6	(0.2349, 0.5115, 0.2536)	([0.0831, 0.2042][0.1764, 0.4150] [0.1374, 0.2815][0.2690, 0.5363])	$x_4 \succ x_2 \succ x_3 \succ x_1$
0.7	(0.5106, 0.2543, 0.2351)	([0.1359, 0.3312][0.1434, 0.1904] [0.2236, 0.4565][0.2450, 0.2526])	$x_4 \succ x_2 \succ x_3 \succ x_1$
0.8	(0.4897, 0.2689, 0.2414)	([0.1274, 0.2199][0.3189, 0.4530] [0.1461, 0.2545][0.2023, 0.2737])	$x_4 \succ x_2 \succ x_3 \succ x_1$
0.9	(0.4691, 0.2834, 0.2475)	([0.1217, 0.3136][0.1519, 0.2192] [0.2206, 0.4592][0.2699, 0.3026])	$x_4 \succ x_2 \succ x_3 \succ x_1$
1.0	(0.4816, 0.2746, 0.2438)	([0.1213, 0.2284][0.3203, 0.4804] [0.1501, 0.2747][0.2108, 0.2970])	$x_4 \succ x_2 \succ x_3 \succ x_1$

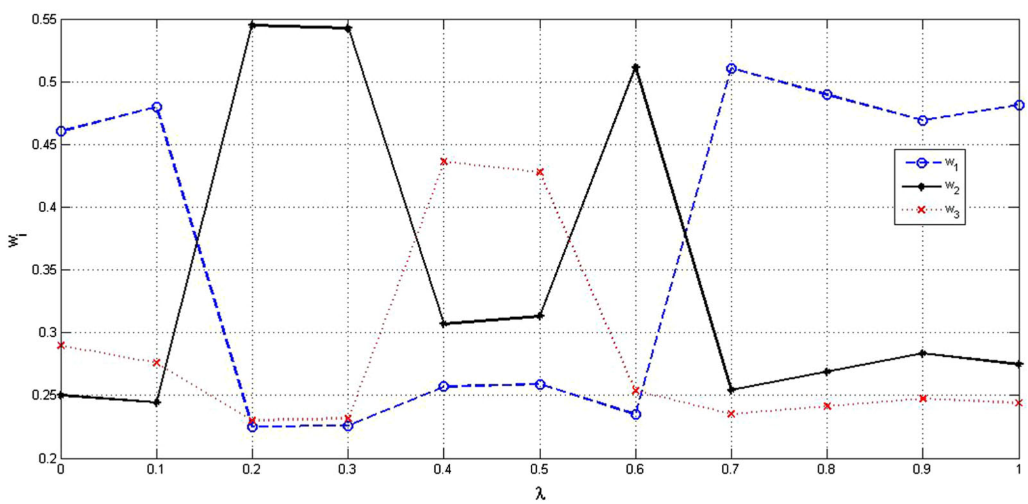

Fig. 3 DM's weights change with λ

CIVIFOWA operator which takes the contribution of each DM to the group into account.

In the following, we will compare the proposed approach with other previous methods. In [17], Xu developed a method based on distance measure for group decision-making with IVIFPRs. We firstly denote $\tilde{R}_i^{(k)} = (\tilde{R}_{i1}^{(k)}, \tilde{R}_{i2}^{(k)} \tilde{R}_{i3}^{(k)} \quad \tilde{R}_{i4}^{(k)})$ as the final preference vector corresponding to the alternative x_i and the kth DM. Then, we use the proposed measure to calculate the distance between the preference vector $\tilde{R}_i^{(k)}$ and the uncertain intuitionistic fuzzy ideal solution $\tilde{\alpha}^* = (\tilde{\alpha_1}^*, \tilde{\alpha_2}^*, \tilde{\alpha_3}^*, \tilde{\alpha_4}^*)$, we obtain:

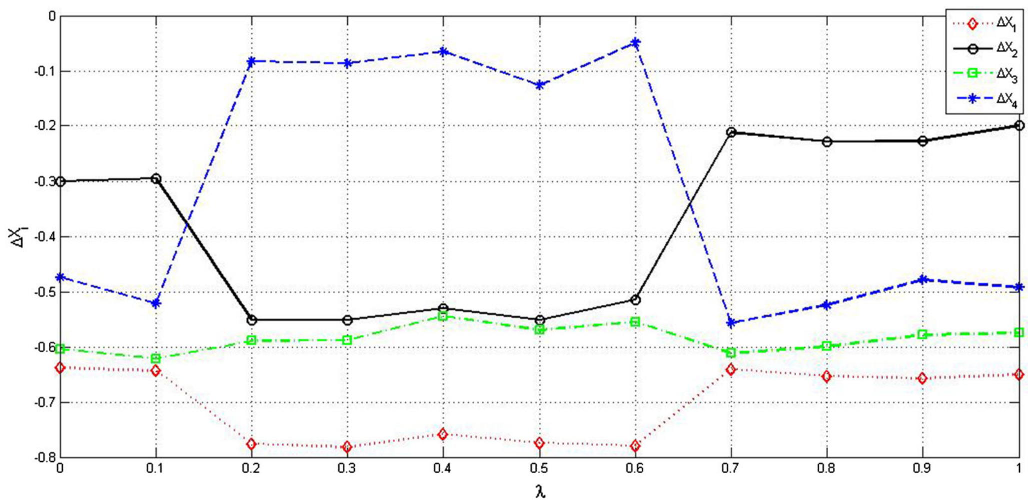

Fig. 4 Alternative values change with λ

Table 3 DM's weights and ranking results with different attitudes of the DMs

λ_1	λ_2	λ_3	ω	$(\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4)$	Ranking
0.2	0.2	0.2	(0.2252, 0.5448, 0.2300)	(− 0.7758, − 0.5518, − 0.5895, − 0.0825)	$x_4 \succ x_2 \succ x_3 \succ x_1$
		0.5	(0.2298, 0.2414, 0.5287)	(− 0.7822, − 0.6010, − 0.5701, − 0.0591)	$x_4 \succ x_2 \succ x_3 \succ x_1$
		0.8	(0.2457, 0.5227, 0.2316)	(− 0.7659, − 0.5305, − 0.5936, − 0.1358)	$x_4 \succ x_2 \succ x_3 \succ x_1$
	0.5	0.2	(0.5145, 0.2515, 0.2340)	(− 0.6435, − 0.2476, − 0.6326, − 0.5750)	$x_2 \succ x_4 \succ x_3 \succ x_1$
		0.5	(0.5005, 0.2613, 0.2381)	(− 0.6508, − 0.2627, − 0.6299, − 0.5594)	$x_2 \succ x_4 \succ x_3 \succ x_1$
		0.8	(0.4842, 0.2728, 0.2430)	(− 0.6596, − 0.2812, − 0.6269, − 0.5402)	$x_2 \succ x_4 \succ x_3 \succ x_1$
	0.8	0.2	(0.4569, 0.2920, 0.2511)	(− 0.6765, − 0.2901, − 0.5990, − 0.4762)	$x_2 \succ x_4 \succ x_3 \succ x_1$
		0.5	(0.4235, 0.3163, 0.2602)	(− 0.6952, − 0.3290, − 0.5920, − 0.4275)	$x_2 \succ x_4 \succ x_3 \succ x_1$
		0.8	(0.3923, 0.3412, 0.2665)	(− 0.6830, − 0.2995, − 0.5446, − 0.3169)	$x_2 \succ x_4 \succ x_3 \succ x_1$
0.4	0.2	0.2	(0.4741, 0.2460, 0.2799)	(− 0.6148, − 0.2228, − 0.5745, − 0.4622)	$x_2 \succ x_4 \succ x_3 \succ x_1$
		0.5	(0.4695, 0.2474, 0.2831)	(− 0.6251, − 0.2440, − 0.5839, − 0.4701)	$x_2 \succ x_4 \succ x_3 \succ x_1$
		0.8	(0.4726, 0.2465, 0.2809)	(− 0.6575, − 0.3079, − 0.6249, − 0.5269)	$x_2 \succ x_4 \succ x_3 \succ x_1$
	0.5	0.2	(0.2372, 0.5036, 0.2592)	(− 0.7868, − 0.5437, − 0.5803, − 0.1012)	$x_4 \succ x_2 \succ x_3 \succ x_1$
		0.5	(0.2591, 0.3131, 0.4278)	(− 0.7739, − 0.5513, − 0.5699, − 0.1259)	$x_4 \succ x_2 \succ x_3 \succ x_1$
		0.8	(0.2418, 0.2698, 0.4884)	(− 0.7659, − 0.5469, − 0.5389, − 0.0301)	$x_4 \succ x_3 \succ x_2 \succ x_1$
	0.8	0.2	(0.4753, 0.2790, 0.2457)	(− 0.6376, − 0.2005, − 0.5666, − 0.4610)	$x_2 \succ x_4 \succ x_3 \succ x_1$
		0.5	(0.4571, 0.2919, 0.2510)	(− 0.6460, − 0.2178, − 0.5598, − 0.4312)	$x_2 \succ x_4 \succ x_3 \succ x_1$
		0.8	(0.4373, 0.3061, 0.2566)	(− 0.6559, − 02381, − 0.5535, − 0.3985)	$x_2 \succ x_4 \succ x_3 \succ x_1$
0.6	0.2	0.2	(0.4642, 0.2490,0.2869)	(− 0.6500, − 0.2918, − 0.6008, − 0.4879)	$x_2 \succ x_4 \succ x_3 \succ x_1$
		0.5	(0.4585, 0.2506, 0.2909)	(− 0.6208, − 0.2330, − 0.5643, − 0.4355)	$x_2 \succ x_4 \succ x_3 \succ x_1$
		0.8	(0.4506, 0.2529, 0.2965)	(− 0.6361, − 0.2649, − 0.5770, − 0.4435)	$x_2 \succ x_4 \succ x_3 \succ x_1$
	0.5	0.2	(0.2517, 0.4548, 0.2935)	(− 0.7795, − 0.5342, − 0.5759, − 0.1243)	$x_4 \succ x_2 \succ x_3 \succ x_1$
		0.5	(0.2448, 0.4782, 0.2770)	(− 0.7822, − 0.5357, − 0.5781, − 0.1173)	$x_4 \succ x_2 \succ x_3 \succ x_1$
		0.8	(0.2272, 0.5378, 0.2350)	(− 0.7905, − 0.5444, − 0.5823, − 0.0921)	$x_4 \succ x_2 \succ x_3 \succ x_1$
	0.8	0.2	(0.2522, 0.4532, 0.2947)	(− 0.7953, − 0.5391, − 0.5766, − 0.1361)	$x_4 \succ x_2 \succ x_3 \succ x_1$
		0.5	(0.5030, 0.2596, 0.2374)	(− 0.6479, − 0.2181, − 0.6046, − 0.5429)	$x_2 \succ x_4 \succ x_3 \succ x_1$
		0.8	(0.4897, 0.2689, 0.2414)	(− 0.6527, − 0.2281, − 0.5994, − 0.5241)	$x_2 \succ x_4 \succ x_3 \succ x_1$

Table 4 Iterative results of weighting vector and iterations

Iterations	w_1	w_2	w_3
0	0.4232	0.1819	0.3949
1	0.4284	0.2589	0.3127

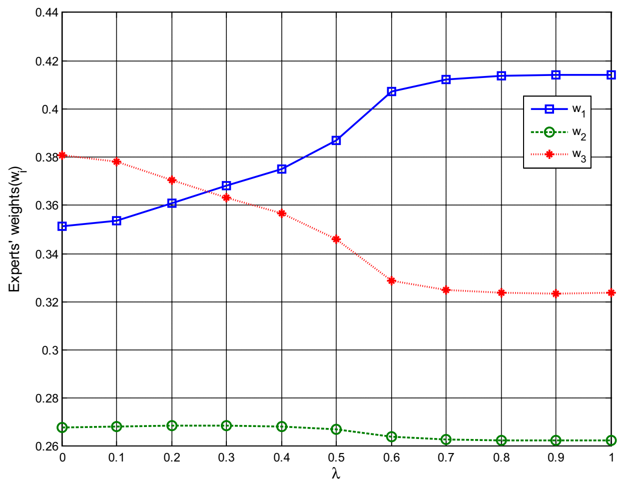

Fig. 5 Experts' weights change with λ

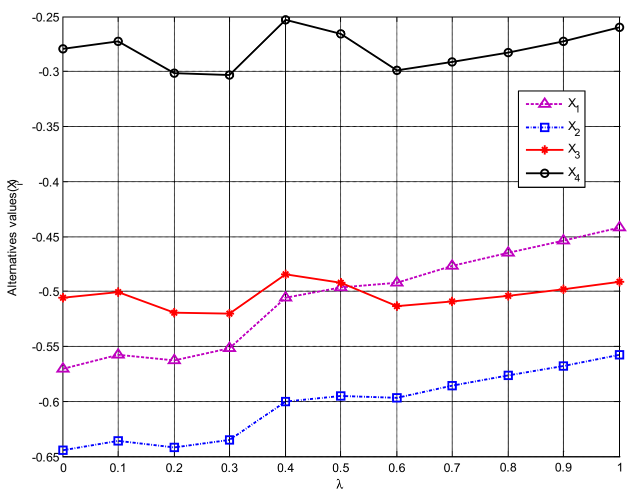

Fig. 6 Alternative values change with λ

$$d(\tilde{R}_1^{(1)}, \tilde{\alpha}^*) = 1.6154, d(\tilde{R}_2^{(1)}, \tilde{\alpha}^*) = 1.0180,$$
$$d(\tilde{R}_3^{(1)}, \tilde{\alpha}^*) = 2.4510, d(\tilde{R}_4^{(1)}, \tilde{\alpha}^*) = 3.0851,$$
$$d(\tilde{R}_1^{(2)}, \tilde{\alpha}^*) = 1.2516, d(\tilde{R}_2^{(2)}, \tilde{\alpha}^*) = 1.8525,$$
$$d(\tilde{R}_3^{(2)}, \tilde{\alpha}^*) = 2.0592, d(\tilde{R}_4^{(2)}, \tilde{\alpha}^*) = 2.9869,$$
$$d(\tilde{R}_1^{(3)}, \tilde{\alpha}^*) = 1.3358, d(\tilde{R}_2^{(3)}, \tilde{\alpha}^*) = 1.6236,$$
$$d(\tilde{R}_3^{(3)}, \tilde{\alpha}^*) = 2.2156, d(\tilde{R}_4^{(3)}, \tilde{\alpha}^*) = 3.1315,$$

where $\tilde{\alpha}_j = ([1,1],[0,0]), j = 1,2,3,4$. Based on the weighting vector $\lambda = (0.4, 0.3, 0.3)$ and the weighted averaging operator, all the distances can be aggregated as

$$d(\tilde{R}_1, \tilde{\alpha}^*) = 1.4224, d(\tilde{R}_2, \tilde{\alpha}^*) = 1.4500,$$
$$d(\tilde{R}_3, \tilde{\alpha}^*) = 2.2628, d(\tilde{R}_4, \tilde{\alpha}^*) = 3.0696,$$

Thus, we rank all the alternative x_i $(i = 1, 2, \ldots, n)$ as follows:

$$x_4 \succ x_3 \succ x_2 \succ x_1.$$

These rankings of the alternatives are slightly different. For instance, the alternative x_4 and x_1 are ranked first and last, respectively, and the rankings of the other two alternatives are reversed. The algorithm is simple and efficient, but the weights of the DMs are predefined subjectively.

Moreover, Xu and Chen [54] proposed some aggregation operators to deal with the interval-valued intuitionistic fuzzy preferences. Now, we follow Xu and Chen's approach to solve the problem we mentioned above. First of all, we use the IIFA operator which was introduced in paper [54] to aggregate all the elements $\tilde{R}_{ij}^{(k)}$, $i = 1,2,3,4$, and we obtain:

$$\tilde{R}_1^{(1)} = ([0.5918, 0.6673], [0.1599, 0.2692]),$$
$$\tilde{R}_2^{(1)} = ([0.7378, 0.7985], [0.0716, 0.1133]),$$
$$\tilde{R}_3^{(1)} = ([0.3334, 0.3661], [0.3879, 0.4745]),$$
$$\tilde{R}_4^{(1)} = ([0.1877, 0.2297], [0.6732, 0.7117]),$$
$$\tilde{R}_1^{(2)} = ([0.6522, 0.6807], [0.0482, 0.2842]),$$
$$\tilde{R}_2^{(2)} = ([0.5064, 0.5833], [0.2972, 0.3941]),$$
$$\tilde{R}_3^{(2)} = ([0.4360, 0.5243], [0.3528, 0.4348]),$$
$$\tilde{R}_4^{(2)} = ([0.1991, 0.2680], [0.6727, 0.7043]),$$
$$\tilde{R}_1^{(3)} = ([0.5928, 0.6500], [0.0441, 0.1042]),$$
$$\tilde{R}_2^{(3)} = ([0.5545, 0.6064], [0.1335, 0.1888]),$$
$$\tilde{R}_3^{(3)} = ([0.4637, 0.5141], [0.3112, 0.3936]),$$
$$\tilde{R}_4^{(3)} = ([0.1724, 0.1896], [0.6753, 0.7091]),$$

Then, we use the interval-valued intuitionistic hybrid averaging operator [54] with the predefined DMs' weights vector $w = (0.4, 0.3, 0.3)$ to aggregate the preferences:

$$\tilde{R}_1 = ([0.6149, 0.6689], [0.0579, 0.1601]),$$
$$\tilde{R}_2 = ([0.6614, 0.7260], [0.1117, 0.1662]),$$
$$\tilde{R}_3 = ([0.4347, 0.4917], [0.3307, 0.4142]),$$
$$\tilde{R}_4 = ([0.1935, 0.2450], [0.6681, 0.7023]),$$

In order to compare the IVIFNs, we calculate the scores [54] of $\tilde{R}_i$:

Table 5 Priority vectors and ranking of alternatives with different λ

	w	ξ	Ranking order
0	(0.3513, 0.2679, 0.3808)	([0.1803, 0.2490][0.1627, 0.1930] [0.2368, 0.2576][0.3515, 0.3691])	$x_4 \succ x_3 \succ x_1 \succ x_2$
0.1	(0.3536, 0.2681, 0.3782)	([0.2324, 0.2673][0.3430, 0.3848] ([0.1795, 0.2625][0.1621, 0.2023])	$x_4 \succ x_3 \succ x_1 \succ x_2$
0.2	(0.3609, 0.2685, 0.3705)	([0.1724, 0.2648][0.1545, 0.2034] [0.2162, 0.2464][0.3515, 0.3824])	$x_4 \succ x_3 \succ x_1 \succ x_2$
0.3	(0.3681, 0.2686, 0.3633)	([0.1716, 0.2768][0.1527, 0.2121] [0.2986, 0.2761][0.3025, 0.3941])	$x_4 \succ x_1 \succ x_3 \succ x_2$
0.4	(0.3751, 0.2683, 0.3566)	([0.1834, 0.3106][0.1622, 0.2377] [0.2163, 0.2993][0.3109, 0.4365])	$x_4 \succ x_1 \succ x_3 \succ x_2$
0.5	(0.3869, 0.2672, 0.3459)	([0.1813, 0.3225][0.1588, 0.2458] [0.2053, 0.3023][0.2922, 0.4422])	$x_4 \succ x_1 \succ x_3 \succ x_2$
0.6	(0.4073, 0.2638, 0.3288)	([0.1772, 0.3310][0.1530, 0.2505] [0.1895, 0.2967][0.2665, 0.4344])	$x_4 \succ x_1 \succ x_3 \succ x_2$
0.7	(0.4123, 0.2628, 0.3249)	([0.1767, 0.3467][0.1520, 0.2626] [0.1845, 0.3067][0.2571, 0.4515])	$x_4 \succ x_1 \succ x_3 \succ x_2$
0.8	(0.4138, 0.2625, 0.3238)	([0.1746, 0.3602][0.1500, 0.2734] [0.1795, 0.3165][0.2481, 0.4692])	$x_4 \succ x_1 \succ x_3 \succ x_2$
0.9	(0.4141, 0.2624, 0.3236)	([0.1724, 0.3738][0.1479, 0.2844] [0.1747, 0.3270][0.2396, 0.4882])	$x_4 \succ x_1 \succ x_3 \succ x_2$
1.0	(0.4140, 0.2624, 0.3236)	([0.1702, 0.3878][0.1459, 0.2960] [0.1703, 0.3382][0.2317, 0.5086])	$x_4 \succ x_1 \succ x_3 \succ x_2$

$$s(\tilde{R}_1) = 0.5329, s(\tilde{R}_1) = 0.5548, s(\tilde{R}_3) = 0.0907,$$
$$s(\tilde{R}_4) = -0.4660.$$

By using the same method for the comparison in [54], we have $\tilde{R}_2 > \tilde{R}_1 > \tilde{R}_3 > \tilde{R}_4$, and thus the ranking of the alternatives is $x_2 \succ x_1 \succ x_3 \succ x_4$.

In the above-mentioned example, the rankings of the alternatives derived by the two approaches are different. The reason is that Xu and Chen's approach for the weights of DMs is predefined rather than using the individual's opinions. And the proposed method emphasizes the contribution of each DMs in the GDM, which makes it more reasonable and distinctive.

5 Conclusions

In this paper, based on cooperative games method, we introduced a new weighting algorithm for GDM with IVIFPRs by using the CIVIFOWA operator. After using the CIVIFOWA operator to transform the IVIFPRs into CIVIFPRs, an iterative weighting algorithm was designed on the basis of Shapley value method. Meanwhile, by reallocating the sum of the continuous group decision error, we can deriving a new weighting vector in each iteration, which is measured by the continuous interval-valued intuitionistic fuzzy distances between decision information of each individual and the weighted group decision information. Then, a novel optimal model is raised, which will be used to rank the alternatives. At last, we illustrate some practical examples, including a numerical example and some comparative examples, to demonstrate its efficiency and application area. What can be generalized is that the proposed method can be applied to GDM with other fuzzy relations [63–65] with great efficiency.

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