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Nussbaum-Based Adaptive Fuzzy Tracking Control of Unmanned Surface Vehicles with Fully Unknown Dynamics and Complex Input Nonlinearities

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Abstract In this paper, subject to both fully unknown dynamics and complex input nonlinearities including unknown control directions and dead zones, a Nussbaum-based adaptive fuzzy trajectory tracking control scheme of an unmanned surface vehicle is addressed by combining adaptive fuzzy backstepping technique with Nussbaum approach. The dead-zone input nonlinearity is firstly divided into input-dependent functions and time-varying input coefficients which can be treated as system uncertainties. Together with disturbances, unknown dynamics and uncertainties, the lumped nonlinearity is online approximated by employing an adaptive fuzzy approximator. Within the backstepping framework, a Nussbaum gain function is further designed to tackle unknown control directions, and thereby devising an adaptive fuzzy trajectory tracking control scheme which is constructed recursively to deal with complex input nonlinearities and fully unknown dynamics. Theoretical analysis reveals that all signals of the closed-loop tracking system are bounded and tracking errors can converge to an arbitrarily small neighborhood of zero. Simulation studies demonstrate the effectiveness and superiority of the proposed approach.

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1 Introduction

Tracking control of an unmanned surface vehicle (USV) is a critical and challenging issue which has attracted great attention from both marine and control fields [1–10]. Due to harsh environments, system uncertainties, unknown dynamics and complex nonlinearities, etc, it becomes extremely involved to synthesize an effective model-based tracking controller for an USV.

Various results have been proposed for nonlinear complex USV systems by using advanced control techniques including feedback linearization [11], robust control [12–14] and sliding-mode control [15–18]. Note that the foregoing approaches require explicit parametric dynamics and bounded uncertainties and/or disturbances. However, an USV would inevitably suffer from complex uncertainties, hydrodynamics and unknown disturbances, thereby resulting in great difficulties in tracking controller design and synthesis for such a complex USV system.

In recent years, approximation-based control methods via fuzzy logic systems and/or neural networks [19–23] have been developed to deal with model uncertainties and unknown disturbances associated with nonlinear systems. In [24], a novel adaptive fuzzy control method for tracking an USV system was proposed, whereby an online constructive fuzzy approximator is created to deal with unmodeled dynamics and external disturbances. Combining the backstepping technique with adaptive approximation [25], tracking a fully actuated marine surface vessel was addressed by an adaptive neural network controller which can tackle multiple output constraints. An finite-time disturbance observer-based accurate tracking control scheme for an USV with unknown disturbances was proposed in [26], whereby unknown uncertainties and disturbances can be exactly rejected. Leonessa et al. [27] introduced neural networks to handle unmodeled dynamics of the USV, and thereby enhancing the tracking performance and the robustness. In [28], a novel self-constructing fuzzy neural network was developed to approximate system uncertainties and unknown disturbances. Unlike predefined-structure approximation approaches, the self-constructing fuzzy neural network is able to online self-construct dynamic-structure fuzzy neural approximation by generating and pruning fuzzy rules and achieve accurate approximation and trajectory tracking, simultaneously. It should be pointed out that the aforementioned results in [24–27] are only available for the USV with exactly known inertia dynamics. In practice, inertia masses and hydrodynamics can hardly be identified accurately. To address the foregoing challenges, a direct adaptive fuzzy tracking control scheme in [29] is proposed for an USV with fully unknown inertia dynamics, whereby the backstepping technique and fuzzy approximation are incorporated such that tracking errors can converge to an arbitrarily small neighborhood of zero. It should be noted that previous results did not consider involved issues on control input nonlinearites and/or constraints.

Recently, input saturations pertaining to an USV have been extensively investigated by employing cascaded kinematic and dynamic linearizations [30], adaptive auxiliary compensation [31], and nested saturation [32–35], respectively. In this context, designed control input signals can ensure predefined boundedness. In addition to aforementioned input saturation issue, much more involved input nonlinearities including unknown control directions and actuator dead zones represent typical nonsmooth constraints on control input signals which widely appear in an USV. In practice, actuator dead zones would critically degrade control system performance, and give rise to undesirable inaccuracy, and even would destroy system stability. However, to our best knowledge, in comparison with flight vehicles [36, 37], little attention in the literature has been paid to foregoing issues on complex input nonlinearities composed by unknown control directions and dead zones pertaining to an USV.

Motivated by above observations, for an USV system with complex input nonlinearities including unknown control directions and dead zones in addition to fully unknown dynamics, a Nussbaum-based adaptive fuzzy trajectory tracking control approach is investigated in this paper. To be specific, the dead-zone input nonlinearity is firstly divided into input-dependent functions and time-varying input coefficients which can be treated as system uncertainties. Together with fully unknown dynamics and disturbances, complex input nonlinearities are encapsulated into lumped unknown dynamics which can be further identified online by an adaptive fuzzy approximator. Furthermore, a Nussbaum gain function is employed to solve the unknown control direction problem, and thereby contributing to the entire control scheme which can deal with complex input nonlinearities and fully unknown dynamics, simultaneously. Eventually, trajectory tracking errors can be rendered to arbitrarily small neighborhood of zero.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are given in Sect. 2. An adaptive fuzzy trajectory tracking control scheme is addressed in Sect. 3, and the corresponding stability analysis is presented in Sect. 4. Simulation studies are conducted in Sect. 5. Section 6 concludes this work.

2 Problem Formulation

2.1 USV Model

Consider an USV dynamic system with unknown disturbances and complex input nonlinearities, as shown in Fig. 1, as follows:

$$\begin{aligned} \dot{\eta} &= \mathbf{R}(\varphi)\omega \\ \dot{\omega} &= -\mathbf{M}^{-1}[\mathbf{C}(\omega)\omega + \mathbf{D}(\omega)\omega \\ &\quad + \boldsymbol{g}(\eta,\omega) + \tau_d - \rho\tau(v)] \end{aligned} \tag{1}$$

where $\eta = [x, y, \varphi]^{\mathrm{T}}$ are the position (x, y) and heading angle (φ) of the USV in the earth-fixed frame, $\omega = [u, \omega, r]^{\mathrm{T}}$ denote the corresponding linear velocities (u, ω) and angular rate (r) in the body-fixed frame, $\tau = [\tau_u, \tau_\omega, \tau_r]^{\mathrm{T}}$ and $\tau_d = [\tau_{du}, \tau_{d\omega}, \tau_{dr}]^{\mathrm{T}}$ are control input nonlinearities and the unknown disturbances, respectively, $\boldsymbol{g} = [g_u, g_\omega, g_r]^{\mathrm{T}}$ is the vector of gravitational/buoyancy forces

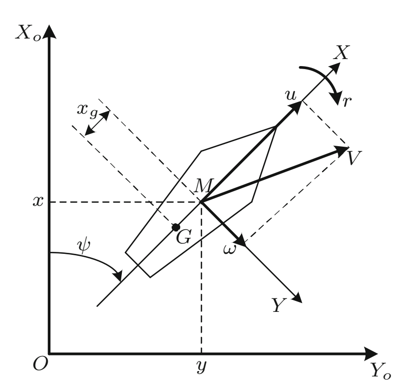

Fig. 1 Earth-fixed OX_oY_o and body-fixed AXY coordinate frames

and moments, $0 < \rho < 1$ is the bound unknown parameter and is referred to as the control coefficient.

To be specific, as shown in Fig. 2, the control input $\tau(\boldsymbol{v}) := [\tau_u(v_u), \tau_\omega(v_\omega), \tau_r(v_r)]^{\mathrm{T}}$ in (1) consists of nonsymmetric dead zones and is defined as follows:

$$\tau_i(v_i) = \begin{cases} \beta(v_i - b_{ri}), & \text{if } v_i \geq b_{ri} \\ 0, & \text{if } -b_{li} < v_i < b_{ri} \\ \beta(v_i + b_{li}), & \text{if } v_i \leq -b_{li} \end{cases} \tag{2}$$

where $\boldsymbol{v} := [v_u, v_\omega, v_r]^{\mathrm{T}}$ is the input of dead-zone, β stands for the slopes of the dead-zone characteristic with $0 < \beta_{\min} < \beta < \beta_{\max}$, $\boldsymbol{b}_r = [b_{ru}, b_{r\omega}, b_{rr}]^{\mathrm{T}}$ and $\boldsymbol{b}_l = [b_{lu}, b_{l\omega}, b_{lr}]^{\mathrm{T}}$ represent the breakpoints of the input nonlinearity.

In addition, $\mathbf{R}(\varphi)$ is a rotation matrix given by

$$\mathbf{R}(\varphi) = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

with the following properties:

$$\begin{aligned} &\mathbf{R}^{\mathrm{T}}(\varphi)\mathbf{R}(\varphi) = \mathbf{I}, \text{ and} \\ &\quad \|\mathbf{R}(\varphi)\| = 1, \ \forall \ \varphi \in [0, 2\pi] \end{aligned} \tag{4}$$

The inertia matrix $\mathbf{M}(t) = \mathbf{M}^{\mathrm{T}}(t) > 0, \forall t$, the skew-symmetric matrix $\mathbf{C}(\omega) = -\mathbf{C}^{\mathrm{T}}(\omega)$ of Coriolis and centripetal and the damping matrix $\mathbf{D}(\omega)$ are provided by

$$\mathbf{M} = \begin{bmatrix} m_{11}(t) & 0 & 0 \\ 0 & m_{22}(t) & m_{23}(t) \\ 0 & m_{32}(t) & m_{33}(t) \end{bmatrix} \tag{5}$$

$$\mathbf{C}(\omega) = \begin{bmatrix} 0 & 0 & c_{13}(\omega) \\ 0 & 0 & c_{23}(\omega) \\ -c_{13}(\omega) & -c_{23}(\omega) & 0 \end{bmatrix} \tag{6}$$

$$\mathbf{D}(\omega) = \begin{bmatrix} d_{11}(\omega) & 0 & 0 \\ 0 & d_{22}(\omega) & d_{23}(\omega) \\ 0 & d_{32}(\omega) & d_{33}(\omega) \end{bmatrix} \tag{7}$$

where detailed definitions can be found in [38]. Note that the parameters $c_{13}, c_{23}, d_{11}, d_{22}, d_{23}, d_{32}$ and d_{33} are all regarded as unknown nonlinearities due to the complex hydrodynamics, thereby resulting in unknown dynamics which can hardly be obtained accurately in practice.

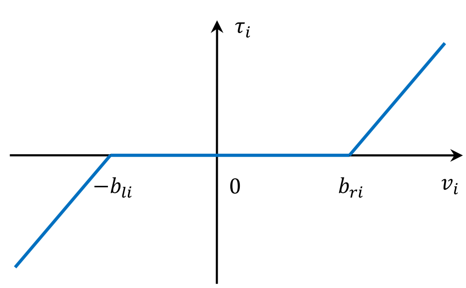

Fig. 2 Control input nonlinearity with dead zones

In order to facilitate our control objective, generic assumptions are required as follows:

Assumption 1 [29] The inertia matrix $\mathbf{M}(t)$ satisfies

$$0 < M_1 \leq \lambda(\dot{\mathbf{M}}(t)) \leq M_2, \forall t \tag{8}$$

where M_1 and M_2 are unknown constants, $\lambda(\cdot)$ denotes the eigenvalue of a square matrix.

Assumption 2 [39–41] The slope β in the control input nonlinearity (2) is nonzero, i.e., $\beta \neq 0$.

In this context, the dead-zone nonlinearity (2) can be reformulated as a slowly time-varying input-dependent function in the following form:

$$\tau(\boldsymbol{v}) = \beta\boldsymbol{v} + \boldsymbol{h} \tag{9}$$

with $\boldsymbol{h} := [h_u, h_\omega, h_r]^{\mathrm{T}}$ given by

$$h_i = \begin{cases} -\beta b_{ri}, & \text{if } v_i \geq b_{ri} \\ -\beta v_i, & \text{if } -b_{li} < v_i < b_{ri} \\ \beta b_{li}, & \text{if } v_i \leq -b_{li} \end{cases} \tag{10}$$

Note that the nonlinearity $\boldsymbol{h}(t)$ is bounded, i.e., $|h_i| \leq \bar{h}_i := \beta \max\{b_{ri}, b_{li}\}, i \in \{u, \omega, r\}$.

Together with (1) and (10), the USV system with control input nonlinearities can be rewritten as follows:

$$\begin{aligned} \dot{\eta} &= \mathbf{R}(\varphi)\omega \\ \dot{\omega} &= -\mathbf{M}^{-1}(t)[\mathbf{C}(\omega)\omega + \mathbf{D}(\omega)\omega \\ &\quad + \boldsymbol{g}(\eta, \omega) + \tau_d - \boldsymbol{h}(t)] \\ &\quad + \bar{\rho}\mathbf{M}^{-1}(t)\boldsymbol{v}(t) \end{aligned} \tag{11}$$

where $\bar{\rho} = \rho\beta$.

2.2 Nussbaum Function Properties

In order to deal with unknown control directions, the Nussbaum gain technique is employed in the sequel.

Definition 1 [42–44] A function $N(\xi)$ is called a Nussbaum-type function if it has the following properties:

$$\lim_{s\to\infty} \sup \frac{1}{s}\int_0^s N(\xi)\mathrm{d}_\xi = \infty \tag{12}$$

$$\lim_{s\to\infty} \inf \frac{1}{s}\int_0^s N(\xi)\mathrm{d}_\xi = -\infty \tag{13}$$

From Definition 1, one can find that Nussbaum functions should have infinite gains and infinite switching frequencies. There are many functions satisfying the foregoing conditions, e.g., $\exp(\xi^2)\cos((\pi/2)\xi)$, $\xi^2\cos(\xi)$, and $\xi^2\sin(\xi)$.

In this paper, an even Nussbaum function is chosen as $\xi^2\cos(\xi)$. A key result on the property of Nussbaum function gain is frequently used in controller design and recalled here.

Lemma 1 [40, 45] *Consider a special Nussbaum function* $N(\xi) = \xi^2\cos(\xi)$*, and let* $V(t)$ *and* $\xi(t)$ *be smooth functions defined on* $[0, t_f)$ *with* $V(t) \geq 0\ \forall t \in [0, t_f)$*. If the following inequality satisfies*

$$\dot{V}(t) \leq -aV + (\bar{\rho}N'(\xi) - 1)d\dot{\xi} + b \tag{14}$$

where $N'(\xi) = \partial N(\xi)/\partial\xi, a > 0$ and $b > 0$ *are constants,* $\bar{\rho}$ *is a nonzero constant, and* d *is some suitable constant, then* $V(\cdot), \xi(\cdot)$*, and* $(\bar{\rho}N'(\xi) - 1)d\dot{\xi}$ *must be bounded on* $[0, t_f)$.

2.3 Control Objective and Implementation

In this context, our control objective is to design a Nussbaum-based adaptive fuzzy control scheme for trajectory tracking the complex USV system in (1) with the ability to tackle both complex input nonlinearities and fully unknown dynamics, simultaneously, such that the actual trajectory η can track the desired trajectory η_d as precise as possible.

To be specific, the proposed Nussbaum-based adaptive fuzzy control scheme will be implemented by incorporating adaptive fuzzy approximation and Nussbaum gain function into the backstepping framework. In this context, adaptive fuzzy approximation and Nussbaum gain function techniques are expected to deal with complex unknowns and input nonlinearities, respectively.

3 Adaptive Fuzzy Tracking Control Scheme

In this section, a trajectory tracking controller is designed by combining the backstepping technique with fuzzy approximation.

Define tracking errors as follows:

$$z_1 = \eta - \eta_d \tag{15}$$

$$z_2 = \omega - \alpha \tag{16}$$

where η_d is the desired trajectory, and α is a virtual control signal designed as follows:

$$\alpha = \mathbf{R}^{\mathrm{T}}(\varphi)(-\mathbf{K}_1 z_1 + \dot{\eta}_d) \tag{17}$$

where $\mathbf{K}_1 = \mathbf{K}_1^{\mathrm{T}} > 0$ is the design parameter.

Taking time derivatives of tracking errors z_1 and z_2 along (11) yields

$$\dot{z}_1 = -\mathbf{K}_1 z_1 + \mathbf{R}(\varphi) z_2 \tag{18}$$

$$\dot{z}_2 = \mathbf{M}^{-1}(t)[\boldsymbol{f}(\eta, \omega, \dot{\alpha}) + \bar{\rho}\boldsymbol{v}(t)] \tag{19}$$

where

$$\begin{aligned} \boldsymbol{f}(\eta, \omega, \dot{\alpha}) = & -\mathbf{C}(\omega)\omega - \mathbf{D}(\omega)\omega - \boldsymbol{g}(\eta, \omega) \\ & - \tau_d + \boldsymbol{h}(t) - \mathbf{M}\dot{\alpha} \end{aligned} \tag{20}$$

with

$$\begin{aligned} \dot{\alpha} = & -\dot{\mathbf{R}}^{\mathrm{T}}\mathbf{K}_1\eta - \mathbf{R}^{\mathrm{T}}\mathbf{K}_1\mathbf{R}\omega + \dot{\mathbf{R}}^{\mathrm{T}}\mathbf{K}_1\eta_d \\ & + (\dot{\mathbf{R}} + \mathbf{R}^{\mathrm{T}}\mathbf{K}_1)\dot{\eta}_d + \mathbf{R}^{\mathrm{T}}\ddot{\eta}_d \end{aligned} \tag{21}$$

Note that the term $\boldsymbol{f}(\eta, \omega, \dot{\alpha})$ is a lumped unknown nonlinearity encapsulated by unknown dynamics, input nonlinearities and disturbances. The universal approximation ability of a fuzzy approximator is used in [46–48], the nonlinearity $\boldsymbol{f}$ in (20) can be optimally approximated as follows:

$$\boldsymbol{f}(\boldsymbol{x}) = \theta^{*\mathrm{T}}\varphi(\boldsymbol{x}) + \varepsilon \tag{22}$$

where $\boldsymbol{x} = [\eta^{\mathrm{T}}, \boldsymbol{v}^{\mathrm{T}}, \dot{\alpha}^{\mathrm{T}}]^{\mathrm{T}}, \varepsilon$ is the optimal approximation error and is bounded, i.e., $\|\varepsilon\| \leq \varepsilon^*$, and θ^* is optimal parameters given by

$$\theta^* = \arg\min_{\theta}\left\{\sup_{x \in U_x}\left\|\theta^{\mathrm{T}}\varphi(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{x})\right\|\right\} \tag{23}$$

However, the optimal parameter θ^* cannot be known in advance and requires adaptive mechanism.

In this context, a fuzzy approximator $\hat{\boldsymbol{f}}(\cdot)$ is devised to adaptive estimate the lumped unknown dynamics $\boldsymbol{f}(\cdot)$ online, and is designed as follows:

$$\hat{\boldsymbol{f}}(\boldsymbol{x}) := [\hat{f}_1, \hat{f}_2, \ldots, \hat{f}_n]^{\mathrm{T}} = \hat{\theta}^{\mathrm{T}}\varphi(\boldsymbol{x}) \tag{24}$$

where $\boldsymbol{x} = [\eta^{\mathrm{T}}, \boldsymbol{v}^{\mathrm{T}}, \dot{\alpha}^{\mathrm{T}}]^{\mathrm{T}}$, and the output weight estimate matrix $\hat{\theta}$ and regressor vector $\varphi(\boldsymbol{x})$ are defined as follows:

$$\hat{\theta} = [\hat{\theta}_1, \ldots, \hat{\theta}_n] \in \mathbb{R}^{N \times n}, \quad \hat{\theta}_j = [\hat{\theta}_j^1, \ldots, \hat{\theta}_j^N]^{\mathrm{T}} \tag{25}$$

$$\varphi(\boldsymbol{x}) = [\varphi_1(\boldsymbol{x}), \varphi_2(\boldsymbol{x}), \ldots, \varphi_N(\boldsymbol{x})]^{\mathrm{T}} \tag{26}$$

where the regressor vector $\varphi(\boldsymbol{x})$ can be composed by Gaussian functions and can refer to [29] for details.

Eventually, design the Nussbaum-based adaptive fuzzy control law $\boldsymbol{v}$ as follows:

$$\boldsymbol{v} = N'(\xi)\left[-\mathbf{K}_2 z_2 - \mathbf{R}^{\mathrm{T}}(\varphi) z_1 - \hat{\theta}^{\mathrm{T}}\varphi(\boldsymbol{x})\right] \tag{27}$$

where

$$\dot{\xi} = \frac{z_2^{\mathrm{T}}}{d}\left[-\mathbf{K}_2 z_2 - \mathbf{R}^{\mathrm{T}}(\varphi) z_1 - \hat{\theta}^{\mathrm{T}}\varphi(\boldsymbol{x})\right] \tag{28}$$

$$\dot{\hat{\theta}}_i = \gamma_i z_{2i}\varphi(\boldsymbol{x}) + \sigma_i\hat{\theta}_i, \quad i = 1, 2, 3 \tag{29}$$

where d is an appropriate constant, $\gamma_i > 0$ and $\sigma_i > 0$.

Remark 1 Unlike previous works focusing on input saturations [30–35], complex input nonlinearities including unknown control directions and actuator dead zones have been intensively addressed by virtue of a Nussbaum-based adaptive fuzzy control scheme governed by (27)–(29), which can tackle both complex input constraints and fully unknown dynamics, simultaneously.

Remark 2 From (27), one can see that in addition to adaptive fuzzy control effort, a dynamic Nussbaum-dependent term $N'(\xi)$ is deployed to regulate the control input, thereby achieving composite adaptation to unknown control directions, actuator dead zones and fully unknown dynamics, simultaneously. Clearly, adaptive approximation-based approaches [7, 19, 23, 24, 27–29] which cannot tackle input nonlinearities become special cases (i.e., $N'(\xi) = 1$) of the proposed Nussbaum-based framework in (27).

4 Stability Analysis

It is essential to prove that the proposed tracking control scheme can guarantee the stability of a closed-loop USV tracking system, and the tracking error can converge to an arbitrarily small neighborhood of zero.

Theorem 1 *Under Assumptions* 1 *and* 2*, consider the USV dynamic system* (1) *using the controller* (27) *with adaptive laws* (28)–(29) *and the virtual control signal* (17)*, all signals of the closed-loop system are bounded, and the tracking error can converge to an arbitrarily small neighborhood of zero.*

Proof Applying (22) to (18)–(19) yields

$$\dot{z}_1 = -\mathbf{K}_1 z_1 + \mathbf{R}(\varphi) z_2 \tag{30}$$

$$\dot{z}_2 = \mathbf{M}^{-1}\left[\hat{\theta}^{\mathrm{T}}\varphi(\boldsymbol{x}) + \bar{\rho}\boldsymbol{v}(t) - \tilde{\theta}^{\mathrm{T}}\varphi(\boldsymbol{x}) + \varepsilon\right] \tag{31}$$

where $\tilde{\theta} = \hat{\theta} - \theta^* = [\tilde{\theta}_1, \tilde{\theta}_2, \tilde{\theta}_3]$.

Consider the following Lyapunov function:

$$V = \frac{1}{2}\left(z_1^{\mathrm{T}} z_1 + z_2^{\mathrm{T}}\mathbf{M} z_2 + \mathrm{tr}\left\{\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}\right\}\right) \tag{32}$$

Differentiating V along (30)–(31) yields

$$\begin{aligned}\dot{V} = &-z_1^{\mathrm{T}}\mathbf{K}_1 z_1 + z_1^{\mathrm{T}}\mathbf{R}(\varphi) z_2 \\ &+ z_2^{\mathrm{T}}\left[\hat{\theta}^{\mathrm{T}}\varphi(\boldsymbol{x}) - \varepsilon + \bar{\rho}\boldsymbol{v}(t) - \tilde{\theta}^{\mathrm{T}}\varphi(\boldsymbol{x})\right] \\ &+ \frac{1}{2} z_2^{\mathrm{T}}\dot{\mathbf{M}} z_2 + \sum_{i=1}^{3}\gamma_i^{-1}\tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i\end{aligned} \tag{33}$$

Using Assumption 1 and the Young's inequalities, we have

$$z_2^{\mathrm{T}}\dot{\mathbf{M}} z_2 \le M_2\|z_2\|^2 \tag{34}$$

$$-z_2^{\mathrm{T}}\varepsilon \le \frac{1}{2}\|z_2\|^2 + \frac{1}{2}\varepsilon^{*2} \tag{35}$$

Substituting control law (27) and inequalities (34)–(35) into (33) gives

$$\begin{aligned}\dot{V} \le &-z_1^{\mathrm{T}}\mathbf{K}_1 z_1 - z_2^{\mathrm{T}}\mathbf{K}_2 z_2 + (\bar{\rho}N'(\xi) - 1)d\dot{\xi} \\ &+ \sum_{i=1}^{3}\gamma_i^{-1}\tilde{\theta}_i^{\mathrm{T}}\left(\dot{\hat{\theta}}_i - \gamma_i z_{2i}\varphi(\boldsymbol{x})\right) \\ &+ \frac{1}{2}(M_2 + 1)\|z_2\|^2 + \frac{1}{2}\varepsilon^{*2}\end{aligned} \tag{36}$$

Using adaptive law (29) and the following inequality:

$$-\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i \le -\frac{1}{2}\|\tilde{\theta}_i\|^2 + \frac{1}{2}\|\theta_i^*\|^2 \tag{37}$$

we further have

$$\begin{aligned}\dot{V} \le &-z_1^{\mathrm{T}}\mathbf{K}_1 z_1 - z_2^{\mathrm{T}}\mathbf{K}_2 z_2 \\ &+ \frac{1}{2}(M_2 + 1)\|z_2\|^2 + \frac{1}{2}\varepsilon^{*2} \\ &+ (\bar{\rho}N'(\xi) - 1)d\dot{\xi} - \frac{1}{2}\sum_{i=1}^{3}\frac{\sigma_i}{\gamma_i}\|\tilde{\theta}_i\|^2 \\ &+ \frac{1}{2}\sum_{i=1}^{3}\frac{\sigma_i}{\gamma_i}\|\theta_i^*\|^2 \\ \le &-\lambda_{\min}(\mathbf{K}_1)\|z_1\|^2 \\ &- (\lambda_{\min}(\mathbf{K}_2) - M_2 - \frac{1}{2})\|z_2\|^2 \\ &+ (\bar{\rho}N'(\xi) - 1)d\dot{\xi} \\ &- \frac{\lambda_{\min}(\sigma)}{2}\mathrm{tr}\left\{\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}\right\} \\ &+ \frac{\lambda_{\max}(\sigma)}{2}\mathrm{tr}\left\{\theta^{*\mathrm{T}}\Gamma^{-1}\theta^*\right\} \\ &+ \frac{1}{2}\varepsilon^{*2} \\ \le &-aV + (\bar{\rho}N'(\xi) - 1)d\dot{\xi} + b\end{aligned} \tag{38}$$

where

$$a = \min\left\{2\lambda_{\min}(\mathbf{K}_1), \frac{2\lambda_{\min}(\mathbf{K}_2) - 2M_2 - 1}{\lambda_{\max}(\mathbf{M})}, \lambda_{\min}(\sigma)\right\}$$

$$b = \frac{\lambda_{\max}(\sigma)}{2}\mathrm{tr}\left\{\theta^{*\mathrm{T}}\Gamma^{-1}\theta^*\right\} + \frac{1}{2}\varepsilon^{*2}$$

with parameters $\mathbf{K}_1, \mathbf{K}_2$ and σ satisfying $\lambda_{\min}(\mathbf{K}_1) > 0, \lambda_{\min}(\mathbf{K}_2) - M_2 - 1/2 > 0$ and $\lambda_{\min}(\sigma) > 0$.

From (38), we have

$$0 \le V(t) \le V(0) + e^{-at}\int_0^t d(\bar{\rho}N'(\xi) - 1)\dot{\xi}e^{a\tau}\mathrm{d}\tau + b \tag{39}$$

Using Lemma 1, we immediately obtain $(\bar{\rho}N'(\xi) - 1)d\dot{\xi}$ is bounded on $[0, t_f)$. Accordingly, define $b_{\max} = \max_{t\in[0,t_f)}(\bar{\rho}N'(\xi) - 1)\ d\dot{\xi}$. Together with (39), we further have

$$\dot{V} \le -aV + \bar{b} \tag{40}$$

where $\bar{b} = b + b_{\max}$.

It follows that

$$V(t) \le e^{-at}V(0) + \bar{b}/a \tag{41}$$

In this context, choosing appropriate parameters would make all signals $\eta, \omega, \alpha, \dot{\alpha}, \hat{\theta}(t)$ and $\boldsymbol{v}(t)$ are bounded. From (41), we have $\|\eta - \eta_d\| \le \sqrt{2V(0)}e^{-at} + \sqrt{2\bar{b}/a}$. Clearly, the term $(2\bar{b}/a)^{\frac{1}{2}}$ can be made as small as possible by choosing parameters $\mathbf{K}_1, \mathbf{K}_2$ and σ appropriately. Denotes $(2\bar{b}/a)^{\frac{1}{2}} \le \mu/2$. There exists a finite time T such that $\|\eta(t) - \eta_d(t)\| \le \mu,\ \forall t \ge T$. This concludes the proof. □

5 Simulation Studies

To demonstrate the effectiveness and superiority of the proposed Nussbaum-based adaptive fuzzy control scheme, simulation studies on a well-known surface vehicle CyberShip II [49] with unknown dynamics are conducted.

The desired trajectory $\eta_d = [\sin(t), \cos(t), \sin(t)]^T$ which is expected to be tracked by the proposed scheme with high accuracy. The initial conditions of the USV are as follows: $\eta(0) = [-0.5, -0.5, 0.5]^{\mathrm{T}}$ and $\omega(0) = [0.2, 0, 0]^{\mathrm{T}}$. Define the fuzzy sets $U_x = [-2, 2]^6 \subset \mathbf{R}^6$ with the widths uniformly set as 2.

For the sake of simulation studies, external disturbances are assumed as follows:

$$\tau_d = \begin{bmatrix} 5 + 0.1u^3 \\ 2 + 0.1u^2 \\ -0.1r^3 + \sin(\omega) \end{bmatrix},$$

and unknown dynamics are assumed as follows:

$$\begin{aligned} c_{13}(\omega) &= -24.6612\omega - 1.0948r, \\ c_{23}(\omega) &= 25.8u, \\ d_{11}(\omega) &= 0.7225 + 1.3274|u| + 5.8664u^2, \\ d_{22}(\omega) &= 0.8612 + 36.2823|\omega| + 8.05|r|, \\ d_{23}(\omega) &= -0.1079 + 0.845|\omega| + 3.45|r|, \\ d_{32}(\omega) &= -0.1052 - 5.0437|\omega| - 0.13|r|, \quad \text{and} \\ d_{33}(\omega) &= 1.9 - 0.08|\omega| + 0.75|r| \text{and}. \end{aligned}$$

In addition, unknown inertia parameters are assumed as follows:

$$\begin{aligned} m_{11}(t) &= 20 + 50\sin^2(0.1\pi t), \\ m_{22}(t) &= 20 + 50\sin^2(0.2\pi t), \\ m_{23}(t) &= 1 + \cos^2(0.1\pi t), \\ m_{32}(t) &= 1 + \cos^2(0.1\pi t), \text{ and} \\ m_{33}(t) &= 10 + 20\sin^2(0.5\pi t). \end{aligned}$$

Input nonlinearity parameters are as follows: $\beta = 1, \boldsymbol{b}_r = [30, 30, 30]^{\mathrm{T}}$, and $\boldsymbol{b}_l = [30, 30, 30]^{\mathrm{T}}$.

The user-defined parameters of the Nussbaum-based adaptive fuzzy controller are selected as follows: $\mathbf{K}_1 = \mathrm{diag}(0.7, 0.7, 0.7), \mathbf{K}_2 = \mathrm{diag}(16, 16, 16), \Gamma_1 = 0.1,\ \Gamma_2 = 0.1, \Gamma_3 = 0.1, \sigma_1 = 0.05,\ \sigma_2 = 0.05, \sigma_3 = 0.05, \rho = 0.8$, and $d = 30$.

Furthermore, in order to demonstrate the superiority of the proposed Nussbaum-based adaptive fuzzy control scheme, we conduct comprehensive comparisons of our proposed approach with Nussbaum-based adaptive fuzzy control without considering control coefficients (i.e., W/O Coef., $\rho = 1$) and without considering dead zones (i.e., W/O DZ, $\tau(\boldsymbol{v}) = \boldsymbol{v}$), respectively.

Simulation results and comparisons are shown in Figs. 3, 4, 5 and 6, which clearly demonstrate that the proposed Nussbaum-based adaptive fuzzy control scheme can track the USV with complex input nonlinearities and fully unknown dynamics to the desired trajectory with high accuracy while the W/O Coef. and W/O DZ approaches can only track roughly the desired trajectory. To make matters worse, if control coefficients and/or actuator dead zones cannot be addressed, i.e., the W/O Coef. and W/O DZ approaches, both position and velocity tracking errors of surge and sway dynamics become

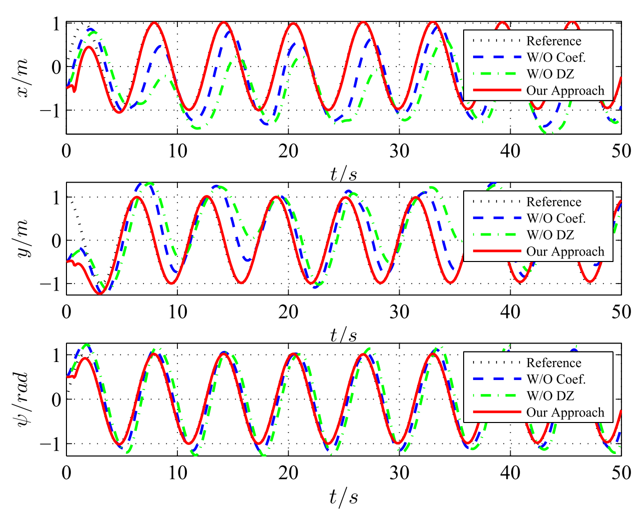

Fig. 3 Desired and actual states x, y and φ

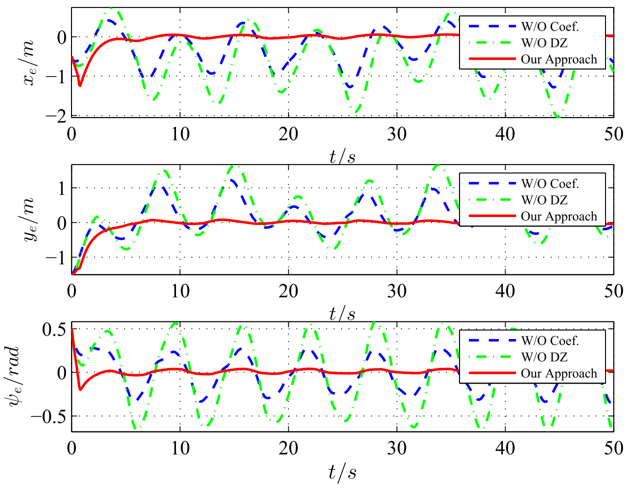

Fig. 4 Tracking errors of x, y and φ

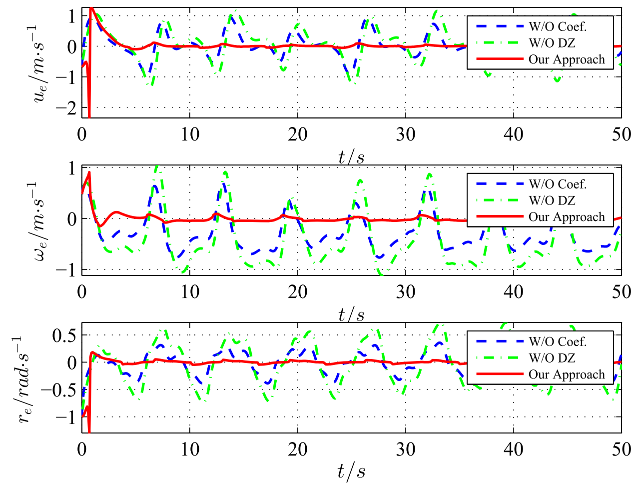

Fig. 6 Tracking errors of u, ω, r

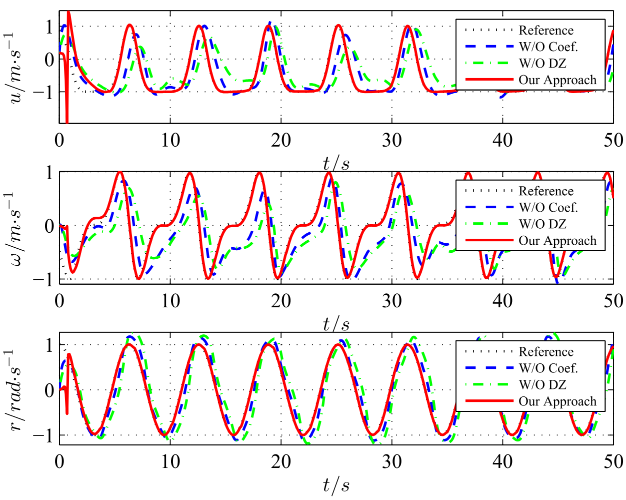

Fig. 5 Desired and actual states u, ω, r

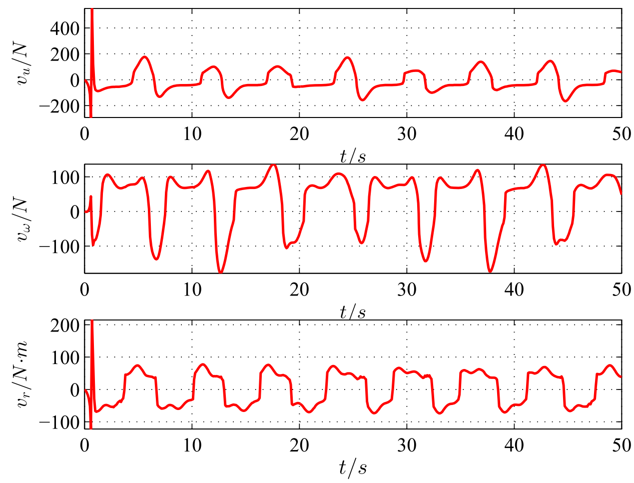

Fig. 7 Control inputs v_u, v_ω, v_r

significantly large. In fact, the proposed Nussbaum-based adaptive fuzzy control scheme can render actual trajectories adapt to complex input constraints in addition to unknown dynamics, while the W/O Coef. and W/O DZ approaches cannot compensate unknown control coefficients and/or dead zones, and thereby leading to apparent tracking delays which can be clearly observed from Figs. 3, 4, 5 and 6. In this context, the superiority and effectiveness can be sufficiently validated. In essence, dead-zone-hold control inputs $\boldsymbol{v} = [v_u, v_\omega, v_r]^T$ shown in Fig. 7 contribute to the remarkable tracking performance. To be specific, actuators stay idle whenever the required control efforts fall into dead zones, and thereby not only avoiding frequent chattering within dead zones but also enhancing adaptation to input nonlinearities. As a bypass advantage, the proposed Nussbaum-based adaptive fuzzy control scheme can prevent unwanted wear and tear of actuators.

6 Conclusion

In this paper, a novel Nussbaum-based adaptive fuzzy control scheme for trajectory tracking of an USV in the presence of complex unknown nonlinearities and fully unknown dynamics has been proposed. By virtue of adaptive fuzzy approximation, the lumped unknown nonlinearity involves input nonlinearities, unknown dynamics and disturbances can be approximated online. In combination with the backstepping technique and the Nussbaum gain property, a Nussbaum-based adaptive fuzzy tracking control scheme has been developed to handle complex input nonlinearities and fully unknown dynamics, simultaneously, which have not been addressed in the literature.

Moreover, stability analysis guarantees high tracking accuracy since tracking errors can converge to an arbitrarily small neighborhood of zero. The effectiveness and superiority of the proposed control scheme have also been demonstrated by simulation studies. In future work, the proposed Nussbaum-based adaptive fuzzy control scheme is expected to apply to experimental prototypes.

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