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System Reliability Analysis Method Based on Fuzzy Probability

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Abstract Conventional methods for analyzing system reliability involve the use of unit reliability and system composition. However, these methods have limitation on considering the type of failure distribution that exists in unit of system, resulting in inaccurate evaluation of system reliability as the bathtub curve. The process in which unit and system go from a normal working state to a failure state can be described as a fuzzy process. The fuzzy probability of unit reliability is defined in formula, and its curve is drawn based on bathtub curve, reliability function curve and fuzzy probability. For two systems that have the same system reliability, conventional methods also fail to take into account the effect of differences in unit reliability on system reliability. In this study, we introduce a novel method to analyze system reliability that based on fuzzy probability. This method is an assumption that unit reliability is a fuzzy probability-based event, and then these concepts of system failure probability and integrated system reliability are proposed. The system failure probability is a characteristic quantity used to measure the effect of different unit in system reliability. The integrated system reliability is a characteristic quantity that evaluated the comprehensive system reliability. These proposed concepts are applied in the reliability analysis of series system, parallel system and compound system. These results verify the effectiveness of the proposed method by the application and comparison in these examples.

Keywords System reliability · Unit reliability · Failure rate · Bathtub curve · Fuzzy probability

1 Introduction

The system reliability is a basic attribute that ensures normal system operation. Therefore, the study of system reliability is essential. In conventional system reliability analysis theory, probability theory-based calculations have been widely used. The system is made of different unit, whereas the failure probability of a unit is obtained under independent test conditions. At times, it is assumed that the unit has exact probability under failure test and performance level of each unit state is given. But in an actual working environment, it is difficult to obtain sufficient failure data to estimate the precise value of the probability. The multistate system (MSS) model provides a more flexible tool for representing engineering system in actual working environment in [1–4], and recent research has focused on reliability evaluation and optimization of MSS in [5–9]. The multiple failure model typically exhibits performance states going from perfect operation to complete failure in [10–12]. In [13], proposed an efficient logarithmic-encoded binary decision diagram (LBDD)-based method for analyzing MSS. Several papers have been devoted to model common cause failure (CCF) distributions and estimate the effect of CCF on system reliability or availability in [14–16]. A straightforward procedure is suggested for evaluating reliability functions of non-repairable series–parallel multistate system with common cause failure in [17].

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By contrast, using the Monte Carlo method as the basis and combining it with related methods to evaluate the reliability of large complex networks. The randomness of a system and its units can be accurately illustrated in [18, 19]. In [18], a new particle swarm optimization (PSO) based on Monte Carlo simulation (MCS), named MCS-PSO, is proposed to solve complex network reliability optimization problems. In [19], comparisons with the chronological, pseudo-chronological and quasi-sequential MCS are carried out using the IEEE-RTS-96 (reliability test system) and modifications of this system that includes renewable sources.

Some methods are used to analyze hidden failures that cause system units to malfunction and identify weaknesses that hinder system reliability. In [20], it provides the definitions of vulnerability and reliability of protection system to numerically characterize this impact and also proposes an economical system upgrading strategy that can best enhance the protection system reliability under limited budget.

With system has being increasingly complex, predicting the reliability of complex system has become a popular research topic in [21, 22]. In [21], it describes a hierarchical Bayesian model for assessing the early reliability of complex system, for which sparse or no system-level failure data are available, except that which exists for comparable system developed by different categories of manufacturers. In [22], a new estimation method is derived to approximate the vector of failure probabilities associated with a complex, one-shot system.

For many systems, it is often difficult to evaluate the failure probability of unit from past occurrences because the environment of the system changed. It might be necessary to consider possible failure of units even if they have never failed. In [23], a fuzzy-probabilistic method is proposed based on the results of the statistical analyses of the features and employed to assess the condition of circuit breaker through the evaluation of the risk of failure. In [24], based on the failure rate and fuzzy probability, a new method is proposed to evaluate the system reliability. In [25], it proposes a method to incorporate a combined fuzzy and probabilistic load model in power system reliability assessment. In [26], it proposes a new extension of crisp probability theory, which is based on independent inputs and each with a fuzzy probability. In [27], it uses a fuzzy line-segment method to calculate the fuzzy unreliability of a system when only discrete-interval probabilities of stress and strength inside an interference region are available. In many engineering applications, however, it is difficult to evaluate the probabilities and consequences from past experiences, because of dynamic environments of system, and especially because there are situations where past experience does not exist. Some studies combine fuzzy probability, fault tree and event tree to analyze system reliability in [28–36]. In [28], the application of fuzzy event tree is further demonstrated by using a set of event tree and explained for dealing with event tree analysis under uncertainty. In [29], it proposes a method combining fuzzy probability theory with Bayesian network that improves the ability of Bayesian networks to deal with uncertainty and reliability of system.

The process in which unit goes from a normal working state to a failure state can be described as a fuzzy process that is gradual and cumulative. That typically exhibits multiple performance states, going from perfect operation to complete failure, and is named multiple failure models. Thus, the reliability of unit and system is fuzzy at failure time. In this paper, we describe this phenomenon as "the fuzzy probability of reliability." Firstly, the fuzzy probability of unit reliability is described with formula and curve based on bathtub curve, reliability function curve and fuzzy probability. Secondly, the system is consisted of different unit, so the analysis of fuzzy probability can be considered for system reliability. When the system operates, the possibility of failure exists; failure entails the system from a normal working state to a failure state. We describe this probability as the "system failure probability" and denote it as $L_s(t)$, that is a characteristic quantity used to measure the effect of different unit in system reliability. Thirdly, if the fuzzy probability of system reliability and system failure probability are, respectively, assigned different weight such as ω_R and ω_L. We propose another characteristic quantity named "integrated system reliability" and denote it as $Z_s(t)$, which represents a comprehensive evaluation of system reliability. At last, the aforementioned method is applied to analyze the system reliability of series system, parallel system and compound system. By making actual calculation and performing a detailed comparative analysis, the effectiveness of the proposed method is verified.

This paper is organized as follows. In Sect. 2, the method for calculating fuzzy probability of unit reliability is introduced. In Sect. 3, the concept of system failure probability is introduced and applied. In Sect. 4, the concept of integrated system reliability is introduced and applied. In Sect. 5, a conclusion is summarized.

2 Method for Calculating Fuzzy Probability of Unit Reliability

In the theory of reliability analysis, product failure rate refers to the product has been working to the time t. The probability of failure will occur in the unit time after the time t, and it is denoted as $\lambda(t)$, $\lambda(t) \in (0,1)$. Through statistical analysis of a large number of experimental data, the failure rate curve likes a bathtub under the time t, known as the "bathtub curve," as shown in Fig. 1. From the curve, we can see that the product failure rate with the change of time t divided into three periods, which names the early failure period $(0, t_1)$, random failure period (t_1, t_2) and loss failure period (t_2, t_3).

The reliability refers to the product under specified conditions. The probability of failure does not occur in the interval $(0, t)$, and its function is $R(t) = \mathrm{e}^{-\int_0^t \lambda(t)\mathrm{d}t}$ with a curve that is as shown in Fig. 2.

The functions of $\lambda(t)$ and $R(t)$ are obtained through a lot of test data for the unit, that statistics are obtained from the universal law. But specific to the system in the selection of a unit, it also has its own particularity. The key lies in the actual characteristics of the unit itself for the system reliability. It is not appropriate to use Fig. 2 to describe the unit reliability in early failure period in Fig. 1. Similar instances occurred in the actual system, as described in Example 1.

Example 1 An aerospace equipment occurred failure during launch, the reason was that a unit had occurred failure firstly. After tracing, the original installation and commissioning in the first time, the staff found that the unit had occurred failure, but did not pay enough attention. Then the staff selected the spare unit to install and thought it reliable, but the result of the entire launch was failure because that unit had occurred early failure. The reliability of the overall units meets the requirement of the performance in the statistical test, but these selected two units failed in the early failure period, resulting in the failure of the launch.

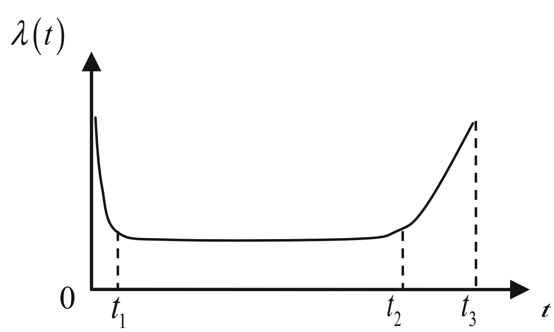

Fig. 1 Bathtub curve

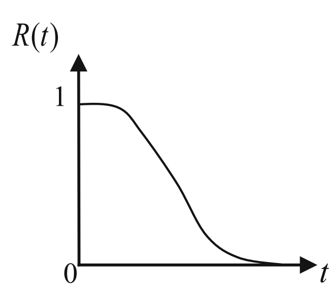

Fig. 2 Reliability function curve

From Example 1, we can find that the expression of unit reliability has limitation with $\lambda(t)$ and $R(t)$, because we cannot find the unit in which failure period. The reliability is a random event, and cannot completely represent the reliability characteristics of a unit, so that description of a unit itself is fuzzy and then is called fuzzy probability of unit reliability.

In this paper, based on the theory of fuzzy probability, we combine the reliability bathtub curve and reliability function curve to make the fuzzy probability of unit reliability as the following assumptions.

1. The domain is $(0, 1)$.
2. The fuzzy set represents "the fuzzy probability of unit reliability."
3. The function of fuzzy probability of unit reliability (denoted as $r(t)$) is as shown in formula (1)

$$r(t) = \begin{cases} 1 - \int_0^{t_1} \lambda(t) & 0 < t < t_1 \\ a & t_1 \le t \le t_2 \\ 1 - \int_0^{t_2} \lambda(t) & t_2 \le t \le t_3 \end{cases} \tag{1}$$

where $\lambda(t) \in (0, 1)$, $a \in (0, 1)$ and a represents the demand unit reliability under system design and is a constant.

1. At the time t, the curve of $r(t)$ is approximate expressed in Fig. 3, based on the bathtub curve in Fig. 1 and the reliability function curve in Fig. 2.

3 Introducing and Applying System Failure Probability

3.1 Introducing System Reliability Based on Fuzzy Probability

The conventional analysis of system reliability involves the use of probability theories. It is assumed that a system is

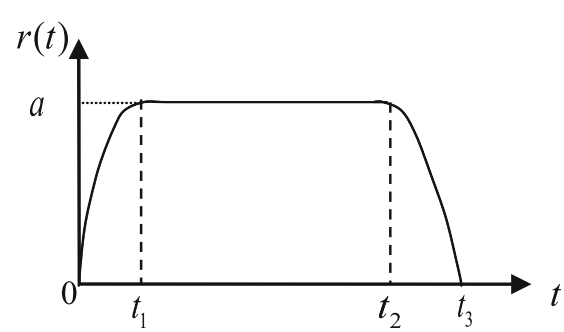

Fig. 3 Curve of the fuzzy probability of unit reliability

Table 1 Formulas for calculating system reliability

	Series system	Parallel system
$R(t)$	$\prod_{i=1}^{n} R_i(t)$	$1 - \prod_{i=1}^{n} F_i(t)$
$F(t)$	$1 - \prod_{i=1}^{n} R_i(t)$	$\prod_{i=1}^{n} F_i(t)$

made up of n mutually independent unit. The symbol $R_i(t)$ represents the probability that the ith unit function normally at time t, whereas the symbol $F_i(t)$ represents the probability that the ith unit failure at time t. The reliability (i.e., $R(t)$) and failure (i.e., $F(t)$) are served as two indicators for the system. Calculation formulas are used to determine the reliability of series and parallel system, as shown in Table 1.

Assuming that the test result shows that the failure rate function of the ith unit is $\lambda(t)$, it is shown in formula (1). The fuzzy probability can be used to find reliability and failure of the ith unit at time t ($r_i(t)$ and $f_i(t)$), respectively, and are shown in formulas (2) and (3).

$$r_i(t) = 1 - \int_0^t \lambda(t)\,\mathrm{d}t \tag{2}$$

$$f_i(t) = \int_0^t \lambda(t)\,\mathrm{d}t \tag{3}$$

Applying calculation method of the fuzzy probability of unit reliability in the system reliability analysis, the fuzzy probability of system reliability denoted as $r(t)$ and fuzzy probability of system failure denoted as $f(t)$. These formulas can be derived as shown in Table 2.

3.2 Introducing System Failure Probability

The analysis combined probability theory and fuzzy theory divides system failures into three sections. In the first section, all system units operate normally and system reliability is represented by the symbol $r_s(t)$. In the second section, all system units fail and system failure is represented by the symbol $f_s(t)$. In the third section, some system units fail but the system continues to operate normally for the parallel system, and some system units have hidden failure but the system continues to operate normally for the series system. These situations lead to the risk of system failure. The system failure probability is represented by the symbol $L_s(t)$. These three sections constitute all possible working state combinations of these units; therefore, they can be used to derive the following mathematical formula.

$$r_s(t) + f_s(t) + L_s(t) = 1 \tag{4}$$

Based on the aforementioned analysis, we define "system failure probability" as the fuzzy probability of system failure during the fuzzy process in which a system goes from a normal working state to a failure state and is denoted as $L_s(t)$. It can be expressed in formula as follows.

$$\begin{cases} r_s(t) = \prod_{i=1}^{n} r_i(t) \\ f_s(t) = \prod_{i=1}^{n} f_i(t) \\ L_s(t) = 1 - r_s(t) - f_s(t) \end{cases} \tag{5}$$

3.3 Applying System Failure Probability

There is a problem that if two systems have the same reliability, how to judge the effect on system reliability for different unit, such as Examples 2, 3 and 4.

Example 2 Assuming that two series systems (i.e., systems E and F) are composed of units A_1 and A_2, as well as A_3 and A_4, respectively (as shown in Figs. 4, 5). The fuzzy probability of unit reliability is calculated as $r_1 = 0.2$, $r_2 = 0.8$, $r_3 = 0.4$ and $r_4 = 0.4$ at time t for these four units. These values are substituted back into the systems to calculate and determine the $r(t), f(t)$ and $L_s(t)$ of systems E and F.

$$\begin{aligned} \text{System}\,E: r_E(t) &= r_1 r_2 = 0.2 \times 0.8 = 0.16 \\ f_E(t) &= 1 - r_E(t) = 0.84 \\ L_{sE}(t) &= 1 - r_1 r_2 - f_1 f_2 \\ &= 1 - r_1 r_2 - (1 - r_1)(1 - r_2) \\ &= 0.68 \end{aligned}$$

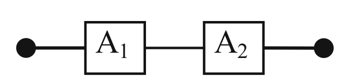

Fig. 4 System E

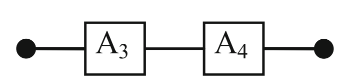

Fig. 5 System F

Table 2 Formulas for calculating the fuzzy probability of system reliability

	Series system	Parallel system
$r(t)$	$\prod_{i=1}^{n} r_i(t)$	$1 - \prod_{i=1}^{n} f_i(t)$
$f(t)$	$1 - \prod_{i=1}^{n} r_i(t)$	$\prod_{i=1}^{n} f_i(t)$

$$\begin{aligned} \text{System } F: r_F(t) &= r_3 r_4 = 0.4 \times 0.4 = 0.16 \\ f_F(t) &= 1 - r_F(t) = 0.84 \\ L_{sF}(t) &= 1 - r_3 r_4 - f_3 f_4 \\ &= 1 - r_3 r_4 - (1 - r_3)(1 - r_4) \\ &= 0.48 \end{aligned}$$

Comparison analysis: $r_E(t) = r_F(t)$ and $f_E(t) = f_F(t)$ indicate that these two systems have the same reliability at time t and there is no difference between these two systems. However, because $L_{sE}(t) > L_{sF}(t)$, when the series system goes from a normal working state to a failure state, the failure probability of the system E is greater than that of system F. Therefore, under the same system reliability, system F is more reliable than system E.

Example 3 Assuming that two parallel systems (i.e., systems G and H) are composed of units A_1 and A_2, as well as A_3 and A_4, respectively (as shown in Figs. 6, 7). The fuzzy probability of unit reliability is calculated as $r_1 = 0.2$, $r_2 = 0.7$, $r_3 = 0.4$ and $r_4 = 0.6$ at time t for these four system units. These values are substituted back into the systems to calculate and determine the $r(t), f(t)$ and $L_s(t)$ of systems G and H.

$$\begin{aligned} \text{System } G: f_G(t) &= f_1 f_2 \\ &= (1 - r_1)(1 - r_2) = 0.24 \\ r_G(t) &= 1 - f_G(t) = 0.76 \\ L_{sG}(t) &= 1 - r_1 r_2 - f_1 f_2 \\ &= 1 - r_1 r_2 - (1 - r_1)(1 - r_2) \\ &= 0.62 \end{aligned}$$

$$\begin{aligned} \text{System } H: f_H(t) &= f_3 f_4 \\ &= (1 - r_3)(1 - r_4) = 0.24 \\ r_H(t) &= 1 - f_H(t) = 0.76 \\ L_{sH}(t) &= 1 - r_3 r_4 - f_3 f_4 \\ &= 1 - r_3 r_4 - (1 - r_3)(1 - r_4) \\ &= 0.52 \end{aligned}$$

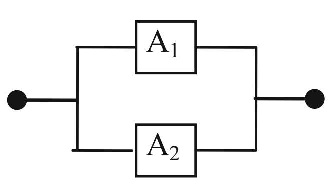

Fig. 6 System G

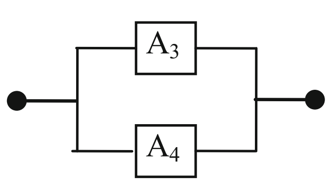

Fig. 7 System H

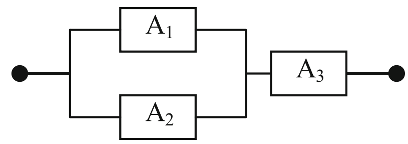

Fig. 8 Compound system

Comparison analysis: $r_G(t) = r_H(t)$ and $f_G(t) = f_H(t)$ indicate that these two systems have the same reliability at time t and there is no difference between these two systems. However, because $L_{sG}(t) > L_{sH}(t)$, when the parallel system goes from a normal working state to a failure state, the failure probability of system G is greater than that of system H. Therefore, under the same system reliability, system H is more reliable than system G.

Example 4 Assuming that a simplified compound system (as shown in Fig. 8) is composed of units A_1, A_2 and A_3. The fuzzy probability of unit reliability of these three system units (i.e., r_1, r_2 and r_3) is calculated using two different choices (i.e., A and B) at time t. The values of r_1, r_2 and r_3 are listed as follows.

$$A = \{r_1 = 0.85, r_2 = 0.89, r_3 = 0.80\}$$

$$B = \{r_1 = 0.80, r_2 = 0.80, r_3 = 0.82\}$$

Next, these two choices are compared and the optimal solution is selected.

$$\begin{aligned} \text{Choice } A: r_{12}(t) &= 1 - f_1 f_2 \\ &= 1 - (1 - r_1)(1 - r_2) = 0.984 \\ r_A(t) &= r_{12}(t) r_3 = 0.787 \\ &= 0.787 \\ L_{sA}(t) &= 1 - r_1 r_2 r_3 - f_1 f_2 f_3 \\ &= 0.210 \end{aligned}$$

$$\begin{aligned} \text{Choice } B: r_{12}(t) &= 1 - f_1 f_2 \\ &= 1 - (1 - r_1)(1 - r_2) = 0.96 \\ r_B(t) &= r_{12}(t) r_3 = 0.787 \\ L_{sB}(t) &= 1 - r_1 r_2 r_3 - f_1 f_2 f_3 \\ &= 0.206 \end{aligned}$$

Comparison analysis: $r_A(t) = r_B(t)$ indicates that these two systems have the same reliability at time t and there is no difference between these two systems. However, because $L_{sA}(t) > L_{sB}(t)$, when the system goes from a normal working state to a failure state, the failure probability of choice A is greater than that of choice B. Therefore, under the same system reliability, the choice B is more suitable.

4 Introducing and Applying Integrated System Reliability

To assess the reliability of system when that is different, this study performs a comprehensive analysis by evaluating the reliability and failure of the system. The concept of integrated system reliability is introduced, and it is represented by the symbol $Z_s(t)$. The calculation procedures are described as follows.

Step 1: Assigning different weights of reliability and failure for system. Allocating the weights based on system operation requirements and their prior working performances. Assuming that reliability and failure of system are given weights of ω_R and ω_L, respectively. The relationship between two weights can be described as follows.

$$\omega_R + \omega_L = 1 \tag{6}$$

Step 2: Multiplying fuzzy probability of system reliability (i.e., $r(t)$) and system failure probability (i.e., $L_s(t)$) by their respective weight (ω_R and ω_L, respectively), and then calculating their sum in formula (7). It is denoted as $Z_s(t)$, and it is named integrated system reliability.

$$Z_s(t) = \omega_R r(t) + \omega_L L_s(t) \tag{7}$$

Step 3: Combining (6) and (7), a comprehensive evaluation method of system reliability is obtained and is shown as follows.

$$\begin{cases} \omega_R + \omega_L = 1 \\ Z_s(t) = \omega_R r(t) + \omega_L L_s(t) \end{cases} \tag{8}$$

Example 5 Assuming that a compound system is composed of units A_1, A_2, A_3 and A_4 at time t (as shown in Fig. 9). The fuzzy probability of unit reliability of these four units (i.e.,r_1, r_2, r_3 and r_4) is calculated at time t. Assuming that two different combinations (i.e., A and B) are available for r_1 and r_2 as $A = \{r_1 = 0.91, r_2 = 0.82\}$, $B = \{r_1 = 0.88, r_2 = 0.85\}$; two different combinations (i.e., C and D) are available for r_3 and r_4 as $C = \{r_3 = 0.86, r_4 = 0.87\}$, $D = \{r_3 = 0.91, r_4 = 0.83\}$.

1. Drawing the block diagram showing the reliability of the simplified system.
2. Selecting suitable combination from A, B, C and D, in which fuzzy probability of system reliability and system failure are assigned weights of 0.8 (i.e., $\omega_R = 0.8$) and 0.2 (i.e., $\omega_L = 0.2$), respectively.

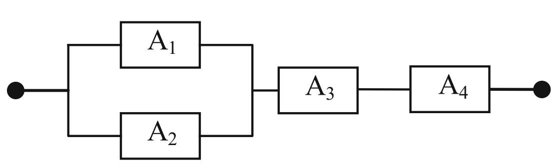

Fig. 9 Block diagram of the compound system

Solutions:

1. Block diagram shows the reliability of a simplified system.

Firstly, the parallel system comprising A_1 and A_2 is simplified to obtain A_{12}. Secondly, the series system comprising A_3 and A_4 is simplified to obtain A_{34}. Finally, the equivalent reliability of the system is obtained, as shown in Fig. 10.

2. Calculating integrated system reliability.

1 The fuzzy probability of system reliability $r(t)$ and system failure $f(t)$ are calculated under the following four sections.

When

$$A = \{r_1 = 0.91, r_2 = 0.82\} \tag{9}$$

$$\begin{aligned} f_A(t) &= f_1 f_2 \\ &= (1 - r_1)(1 - r_2) \\ &= 0.0162 \end{aligned} \tag{9.1}$$

$$\begin{aligned} r_A(t) &= 1 - f_A(t) \\ &= 0.9838 \end{aligned} \tag{9.2}$$

$$\text{When} \quad B = \{r_1 = 0.88, r_2 = 0.85\} \tag{10}$$

$$\begin{aligned} f_B(t) &= f_1 f_2 \\ &= (1 - r_1)(1 - r_2) \\ &= 0.0180 \end{aligned} \tag{10.1}$$

$$\begin{aligned} r_B(t) &= 1 - f_B(t) \\ &= 0.9820 \end{aligned} \tag{10.2}$$

$$\text{When} \quad C = \{r_3 = 0.86, r_4 = 0.87\} \tag{11}$$

$$\begin{aligned} r_C(t) &= r_3 r_4 \\ &= 0.7482 \end{aligned} \tag{11.1}$$

$$\begin{aligned} f_C(t) &= 1 - r_C(t) \\ &= 0.2518 \end{aligned} \tag{11.2}$$

$$\text{When} \quad D = \{r_3 = 0.91, r_4 = 0.83\} \tag{12}$$

$$\begin{aligned} r_D(t) &= r_3 r_4 \\ &= 0.7553 \end{aligned} \tag{12.1}$$

$$\begin{aligned} f_D(t) &= 1 - r_D(t) \\ &= 0.2447 \end{aligned} \tag{12.2}$$

2. Calculating integrated system reliability when different methods are incorporated together.

Calculating combination of A and C:

Fig. 10 Equivalent block diagram of the compound system

$$r_{AC}(t) = r_A(t)r_C(t) = 0.7361 \tag{13}$$

$$f_{AC}(t) = 1 - r_{AC}(t) = 0.2639 \tag{14}$$

$$L_{sAC}(t) = 0.4414 \tag{15}$$

$$Z_{sAC}(t) = \omega_R r_{AC}(t) + \omega_L L_{sAC}(t) = 0.6772 \tag{16}$$

Calculating combination of A and D:

$$r_{AD}(t) = r_A(t)r_D(t) = 0.7431 \tag{17}$$

$$f_{AD}(t) = 1 - r_{AD}(t) = 0.2569 \tag{18}$$

$$L_{sAD}(t) = 0.4361 \tag{19}$$

$$Z_{sAD}(t) = \omega_R r_{AD}(t) + \omega_L L_{sAD}(t) = 0.6817 \tag{20}$$

Calculating combination of B and C:

$$r_{BC}(t) = r_B(t)r_C(t) = 0.7347 \tag{21}$$

$$f_{BC}(t) = 1 - r_{BC}(t) = 0.2653 \tag{22}$$

$$L_{sBC}(t) = 0.5593 \tag{23}$$

$$Z_{sBC}(t) = \omega_R r_{BC}(t) + \omega_L L_{sBC}(t) = 0.6996 \tag{24}$$

Calculating combination of B and D:

$$r_{BD}(t) = r_B(t)r_D(t) = 0.7417 \tag{25}$$

$$f_{BD}(t) = 1 - r_{BD}(t) = 0.2583 \tag{26}$$

$$L_{sBD}(t) = 0.4349 \tag{27}$$

$$Z_{sBD}(t) = \omega_R r_{BD}(t) + \omega_L L_{sBD}(t) = 0.6803 \tag{28}$$

These calculation data are compared in Table 3.

Table 3 Compared data

Result	*AC*	*AD*	*BC*	*BD*
$r(t)$	0.7361	0.7431	0.7347	0.7417
$L_s(t)$	0.4414	0.4361	0.5593	0.4349
$Z_s(t)$	0.6772	0.6817	0.6996	0.6803

Comparing the integrated system reliability results, those are revealed as follows.

$$Z_{sBC}(t) > Z_{sAD}(t) > Z_{sBD}(t) > Z_{sAC}(t) \tag{29}$$

It is shown that combination of B and C is the best integrated system reliability and that is the most favorable combination. The fuzzy probability of unit reliability A_1, A_2, A_3 and A_4, respectively, is $r_1 = 0.88$, $r_2 = 0.85$, $r_3 = 0.86$ and $r_4 = 0.87$. The combination of A and C is the worst.

Example 6 Assuming that, under the requirement of the new technology for Example 5 in the system, it is supposed in the new working environment. The fuzzy probability of system reliability and system failure are assigned weights of 0.3 (i.e.,$\omega_R = 0.3$) and 0.7 (i.e., $\omega_L = 0.7$), respectively. Try to choose the right combination from the combination A, B, C and D.

Solutions:

According to the Example 5 of the known conditions and calculation method, the calculation results are shown as follows.

$$Z_{sAC}(t) = \omega_R r_{AC}(t) + \omega_L L_{sAC}(t) = 0.5298 \tag{30}$$

$$Z_{sAD}(t) = \omega_R r_{AD}(t) + \omega_L L_{sAD}(t) = 0.5282 \tag{31}$$

$$Z_{sBC}(t) = \omega_R r_{BC}(t) + \omega_L L_{sBC}(t) = 0.6119 \tag{32}$$

$$Z_{sBD}(t) = \omega_R r_{BD}(t) + \omega_L L_{sBD}(t) = 0.5269 \tag{33}$$

Comparison of the above calculation result is shown as follows.

$$Z_{sBC}(t) > Z_{sAC}(t) > Z_{sAD}(t) > Z_{sBD}(t) \tag{34}$$

As a result, the combination of B and C is the best combination. The fuzzy probability of unit reliability A_1, A_2, A_3 and A_4 is $r_1 = 0.88$, $r_2 = 0.85$, $r_3 = 0.86$ and $r_4 = 0.87$, respectively. The combination of B and D is the worst.

By comparing those above Examples 5 and 6, we can found that the system performance requirements determine the selection of weights of the fuzzy probability of system reliability and system failure probability.

From these references, the system reliability has not yet found about this aspect. That is, the system is reliable, but there is the risk of failure, how to weigh the pros and cons, for a comprehensive consideration. In the design, must pay attention to the system failure probability and the weight distribution. Need to be assigned according to the experience of the system in the past and the experience of experts.

5 Conclusion

Based on the bathtub curve, reliability function curve and the fuzzy probability, the fuzzy probability of unit reliability is obtained. Next, the system reliability analysis is converted into a fuzzy probability analysis. Then, the concept of system failure probability is introduced, that is, a characteristic quantity for measuring the effect of different unit reliability on system reliability. Subsequently, this study proposes the concept of integrated system reliability to enable a comprehensive evaluation of system reliability, in which the fuzzy probability of system reliability and system failure probability are assigned different weight to evaluate a more complete system reliability analysis.

The aforementioned method is used to analyze system reliability on series system, parallel system and compound system. By making actual calculation and performing a detailed comparative analysis between these results obtained from the analysis and those acquired from the calculation, the present study verified the effectiveness of method. The analysis successfully solves the problem of optimal system evaluation. The method proposed in this study can be used in system reliability-related domains such as system design, evaluation and so on.

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References

1. Yan, M.Q., Yang, B., Wang, Z.: Reliability assessment for multistate system subject to imperfect fault coverage. J. Xi'An Jiao Tong Univ. **45**(10), 109–114 (2011)
2. Fang, Y.R., Kan, S.L., Yang, M., Xie, C., Pei, X.S.: Application of fuzzy multi-state Bayesian networks in redundant hydraulic system's reliability analysis. Comput. Integr. Manuf. Syst. **21**(7), 1856–1864 (2015)
3. Xing, L.D., Dai, Y.S.: A new decision diagram based method for efficient analysis on multi-state systems. IEEE Trans. Dependable Secure Comput. **6**(3), 161–174 (2009)
4. Li, W.J., Pham, H.: Reliability modeling of multi-state degraded systems with multi-competing failures and random shocks. IEEE Trans. Reliab. **54**(2), 297–303 (2005)
5. Yeh, W.C.: The k-out-of-n acyclic multistate-node networks reliability evaluation using the universal generating function method. Reliab. Eng. Syst. Saf. **91**(7), 800–808 (2006)
6. Levitin, G., Xing, L.D.: Reliability and performance of multi-state systems with propagated failures having selective effect. Reliab. Eng. Syst. Saf. **95**(6), 655–661 (2010)
7. Levitin, G., Lisnianski, A., Ben-Haim, H., Elmakis, D.: Redundancy optimization for series–parallel multi-state systems. IEEE Trans. Reliab. **47**(2), 165–172 (1998)
8. Ding, Y., Zuo, M.J., Lisnianski, A., Tian, Z.G.: Fuzzy multi-state system: general definitions and performance assessment. IEEE Trans. Reliab. **57**(4), 589–594 (2008)
9. Li, Y.F., Ding, Y., Zio, E.: Random fuzzy extension of the universal generating function approach for the reliability assessment of multi-state systems under aleatory and epistemic uncertainties. IEEE Trans. Reliab. **63**(1), 13–25 (2014)
10. Huang, J., Zuo, M.J.: Dominant multi-state systems. IEEE Trans. Reliab. **53**(3), 362–368 (2004)
11. Xing, L., Dugan, J.B.: A separable ternary decision diagrams based analysis of generalized phased-mission reliability. IEEE Trans. Reliab. **53**(2), 174–184 (2004)
12. Xing L.: Efficient analysis of systems with multiple states. In: Proceedings of the IEEE 21st International Conference on Advanced Information Networking and Applications, May 2007, pp. 666–672
13. Shrestha, A., Xing, I.D.: A logarithmic binary decision diagram-based method Ion multistate system analysis. IEEE Trans. Reliab. **57**(4), 595–606 (2008)
14. Vaurio, J.K.: The theory and quantification of common-cause shock events for redundant standby systems. Reliab. Eng. Syst. Saf. **43**(3), 289–305 (1994)
15. Chae, K.C., Clark, G.M.: System reliability in the presence of common-cause failures. IEEE Trans. Reliab. **35**(1), 32–35 (1986)
16. Vaurio, J.K.: An implicit method for incorporating common-cause failures in system analysis. IEEE Trans. Reliab. **47**(2), 173–180 (1998)
17. Levitin, G.: Incorporating common-cause failures into non repairable multistate series-parallel system analysis. IEEE Trans. Reliab. **5**(4), 380–388 (2001)
18. Yeh, W.C., Lin, Y.C., Chung, Y.Y., Chih, M.: A particle swarm optimization approach based on Monte Carlo simulation for solving the complex network reliability problem. IEEE Trans. Reliab. **59**(1), 212–221 (2010)
19. González-Fernández, R.A., Leite da Silva, A.M.: Reliability assessment of time-dependent systems via sequential cross-entropy Monte Carlo simulation. IEEE Trans. Power Syst. **26**(4), 2381–2389 (2011)
20. Wang, H., Thorp, J.S.: Optimal locations for protection system enhancement: a simulation of cascading outages. IEEE Trans. Power Deliv. **16**(4), 528–533 (2001)
21. Johnson, V.E.: A hierarchical model for estimating the early reliability of complex systems. IEEE Trans. Reliab. **54**(2), 224–231 (2005)
22. Brian, H., Ali, M.: A reliability growth projection model for one-shot systems. IEEE Trans. Reliab. **57**(1), 174–181 (2008)
23. Ali, A.R.K.: Circuit breaker condition assessment through a fuzzy-probabilistic analysis of actuating coil's current. IET Gener. Transm. Distrib. **10**(1), 48–56 (2016)
24. Sallak, M., Simon, C., Aubry, J.F.: A fuzzy probabilistic approach for determining safety integrity level. IEEE Trans. Fuzzy Syst. **16**(1), 239–248 (2008)
25. Li, W.Y., Zhou, J.Q., Lu, J.P., Yan, W.: Incorporating a combined fuzzy and probabilistic load model in power system reliability assessment. IEEE Trans. Power Syst. **21**(3), 1368–1388 (2007)
26. Dunyak, J., Saad, I.W., Wunsch, D.: A theory of independent fuzzy probability for system reliability. IEEE Trans. Fuzzy Syst. **7**(2), 286–294 (1999)
27. Wang, J.D., Liu, T.S.: Fuzzy reliability using a discrete stress-strength interference model. IEEE Trans. Reliab. **45**(1), 145–149 (1996)
28. Kenarangui, Rasool: Event-tree analysis by fuzzy probability. IEEE Trans. Reliab. **40**(1), 120–124 (1991)
29. Ma, D.Z., Zhou, Z., Yu, X.Y., Fan, S.C., Xing, W.W., Guo, Z.S.: Reliability analysis of multi-state Bayesian networks based on fuzzy probability. Syst. Eng. Electron. **34**(12), 2607–2611 (2012)
30. Tanaka, H., Fan, L.T., Lai, F.S., Toguch, K.I.: Fault-tree analysis by fuzzy probability. IEEE Trans. Reliab. **32**(5), 453–457 (1983)
31. Mentes, A., Heluacioglu, I.H.: An application of fuzzy fault tree analysis for spread mooring systems. Ocean Eng. **38**(2), 285–294 (2011)

32. Song, H., Zhang, H.Y., Chan, C.W.: Fuzzy fault tree analysis based on T-S model with application to INS/GPS navigation system. Soft Comput. **13**(1), 31–40 (2009)
33. Qi, J., Hu, X., Gao, X.: Quantitative risk analysis of subsea pipeline and riser: an experts assessment approach using fuzzy fault tree. Int. J. Reliab. Saf. **8**(1), 33–50 (2014)
34. Sun, L.N., Huang, N., Wu, W.Q., Li, X.K.: Performance reliability of polymorphic systems by fuzzy fault tree based on T-S model. J. Mech. Eng. **52**(10), 191–198 (2016)
35. Liu, P., Cheng, X.Q., Qin, Y., Zhang, Y., Zhang, Z.Y.: Sliding plug door system reliability analysis based on fuzzy fault tree. J. Central South Univ. (Sci. Technol.) **44**(s1), 310–314 (2013)
36. Yao, Y.C., Chen, D.N., Wang, B.: Fuzzy reliability assessment method based on T-S fault tree and Bayesian network. J. Mech. Eng. **50**(2), 193–201 (2014)

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