Int. J. Fuzzy Syst. (2018) 20(3):1015–1026
https://doi.org/10.1007/s40815-017-0341-y

An Interval Programming Approach for Multi-period and Multi-product Aggregate Production Planning by Considering the Decision Maker's Preference

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Received: 9 February 2017 / Revised: 27 April 2017 / Accepted: 12 June 2017 / Published online: 23 June 2017

Abstract This paper presents an interval programming approach for solving a typical multi-period and multi-product aggregate production planning (MPMP-APP) problem. Firstly, a MPMP-APP model based on interval programming is developed, in which the decision maker's risk preference is taken into consideration. Next, to solve the MPMP-APP model based on interval numbers, the original uncertain objective function is replaced by two crisp objective functions which are equivalent to minimizing the interval value and the deviation of the uncertain objective function, respectively, and uncertain constraints are transformed into their corresponding crisp equivalents by using the possibility degree based on six possible relations between two intervals. And then, the linear weighted sum method is adopted to transform the above two-objective model into a single one which can be solved by LINGO software. Finally, an industrial example is used to illustrate the validity and flexibility of the proposed method. It is expected that this study can provide a useful reference for decision makers to make a rational production plan in uncertain environment.

Keywords Production planning · Uncertainty modeling · Interval programming · Risk preference

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1 Introduction

MPMP-APP is a medium range capacity planning that typically encompasses multi-period from 3 to 18 months and involves multiple products. MPMP-APP is about determining optimal production, workforce and inventory levels, backorder level, volume of hiring, volume of firing, overtime work for each period to meet the demand for all products over a finite planning horizon with limitations of capacities or resources, so that the total cost of production can be kept to the minimum [10, 14, 19, 28]. As a typical optimization problem in the fields of production planning management, MPMP-APP has attracted increasing attentions from both researchers and practitioners. There is a great deal of approaches proposed to solve the MPMP-APP [2, 6, 7, 12, 22, 32, 35, 37]. But, the earlier studies about MPMP-APP are mainly carried out in deterministic environment, in which parameters are generally regarded as deterministic values. However, in the real world, some input data, such as market demand, the production cost, the subcontracting cost and the inventory holding cost and so on, are usually imprecise or fuzzy because some information is incomplete, or collecting precise data is very hard. Compared to those deterministic models, the consideration of uncertainty in MPMP-APP model would generate a more practical production planning results [30]. Therefore, much more attention has been paid to the research about uncertain MPMP-APP problem in recent years [23, 24, 39].

According to the differences of uncertainty definition, the methods about resolving uncertain MPMP-APP problem can be classified into the following categories. (1) Stochastic optimization approach, in which some uncertain parameters are described as random numbers with the associated probability distribution. Bitran and Yanasse [4] had considered a single-item lot-sizing production planning problem with random demands, in which shortage probability was incorporated, and also presented an approach to transform stochastic programming model to its corresponding deterministic one. Mirzapour Al-e-hashem et al. [28] proposed a stochastic programming method to solve an uncertain MPMP-APP problem for a medium-term plans, in which the market demands were supposed to be uncertainty and obeyed a certain distribution. Ramezanian and Saidi-Mehrabad [34] developed a stochastic mathematical model for solving lot-sizing and scheduling problem in a MPMP production system. To transform the stochastic problem into the deterministic one, the probability distributions and CCP theory were applied to deal with the uncertainty. Kumar and Goswami [21] analyzed some factors such as shortages of materials, machine failure, workforce level and so on that affect the production process, and presented an economic production quantity model based on stochastic programming, in which the demand rate was regarded as a fuzzy random variable. (2) Soft constrained optimization approach, in which the imprecise input data or parameters were generally defined as fuzzy set. Wang and Liang [43] presented a linear programming (LP) approach based on possibility for dealing with MPMP-APP problem in which imprecise parameters were formulated as triangular possibility distribution. Torabia et al. [40] incorporated the fuzzy set idea into the hierarchical production planning system in which two decision making levels had been considered. At the first level, the fuzzy linear programming was applied to solve APP problem at the product family level, and at the second level, a disaggregated production plan was obtained by another fuzzy linear programming. Wang and Liang [42] firstly constructed a multi-objective linear programming (MOLP) model for solving MPMP-APP problem and then integrated fuzzy set theory into MOLP methods. Jia et al. [15] proposed a fuzzy linear programming approach for a multi-objectives APP problem with fuzzy price, fuzzy cost, fuzzy production capacity and fuzzy market demands by describing these uncertain data as trapezoidal fuzzy numbers. Tang et al. [38] employed a kind of fuzzy approach in modeling for MPMP-APP problem within the constraints of fuzzy requirements and fuzzy capacities. Baykasoglu and Gocken [3] studied a fuzzy multi-objective MPMP-APP problem, in which the uncertain parameters were described with triangular fuzzy numbers. For obtaining the solution, the authors used different ranking methods of fuzzy numbers and TS algorithm. Figueroa-García et al. [9] defined a mixed production problem with fuzzy demands by both type-1 fuzzy sets and interval type-2 fuzzy sets. And the fuzzy optimization algorithm was applied to solve the problem. Gholamian et al. [12] built a fuzzy multi-objective optimization model for a supply chain to address a multi-site, multi-period and multi-product APP problem, in which four conflicting objectives are considered. Kothyari et al. [20] considered carbon emission problem in the purchasing process. Thus, a fuzzy mixed integer linear programming model was developed to minimize carbon cost and other costs such as purchasing, ordering and so on. (3) Robust optimization approach, in which uncertainty data were represented through setting up various scenarios. Leung et al. [24] developed a robust optimization model to address a multi-site MPMP-APP problem motivated by a multi-national company. The model was to minimize the total costs, such as production cost, workforce cost, inventory cost and so on, under consideration of different economic growth scenarios. Mirzapour Al-e-hashem et al. [27] proposed a novel nonlinear robust MIP model to address a multi-objective MPMP-APP problem by considering multi-suppliers, multi-manufacturers and multi-customers in a supply chain. Rahmani et al. [33] developed a robust optimization model to solve a two-stage capacitated production problem, in which uncertain parameters such as production costs, demand and so on, were considered by introducing possible scenarios. Modarres and Izadpanahi [29] developed a linear aggregate planning model with three objective functions, in which energy saving and carbon emission were considered. To deal with uncertain parameters, the robust optimization approach was applied in this paper.

But in the above studies, the imprecise parameters are generally described as random variables or fuzzy set in an uncertain MPMP-APP model, and the related probability distributions or membership function are obtained through mathematical statistics method based on a large amount of statistics data. However, it is difficult to collect sufficient available statistic data in actual production planning. Therefore, there are unreasonable points in the use of the above conventional uncertain optimization methods [18]. As a result, the interval analysis method has drawn more and more attention due to its practicality and flexibility [8, 13, 17, 18, 26, 36, 44]. In interval mathematics, an interval is a closed bounded set of real numbers with the property that any number that lies between two numbers in the set is also included in the set. The left and right bounds of the imprecise parameters are only needed by applying interval method, unnecessarily acquiring their precise probability distributions [16]. Qiu et al. [31] studied ranking method of interval numbers by using probability reliability distribution. And the order relation of intervals was defined to establish the ranking rule. Xiao et al. [46] studied two kinds of definitions of possibility for two interval numbers and concluded that the ranking method proposed in reference [48] was more suitable for an accurate comparison of interval numbers. Wolfe [45] introduced some applications of interval mathematics to the solution of systems of linear and nonlinear algebraic equations and to the solution of unconstrained and constrained nonlinear optimization problems. Bouloukat and Foroud [5] presented some superiority by applying the interval linear programming for modeling inherent stochastic nature of the renewable energy resources. Afzali et al. [1] proposed a fuzzy multi-objective linear programming model for supplier selection, in which linguistic variables were defined as intervals. Lin et al. [25] constructed a multi-objective optimization model with interval-valued objective functions to optimize the integrated production planning for the steelmaking continuous casting-hot rolling process in the steel industry. And they proposed a modified interval multi-objective optimization evolutionary algorithm to solve the model.

The main contributions of this paper are to (1) develop a MPMP-APP model based on interval programming method, in which the decision maker's risk preference is taken into consideration, (2) describe uncertain parameters in the objective function and constraints as interval numbers. By this way, only the lower and upper bounds of imprecise parameters are required, while the probability distributions or membership functions of the uncertain parameters are not required, (3) enable decision makers easy to conduct an efficient production planning in an uncertain environment without collecting a large amount of statistics data.

The rest of this paper is organized as follows. In Sect. 2, a typical MPMP-APP problem is formalized. In Sect. 3, the general uncertain optimization model with interval number is demonstrated, and the treatment of both the uncertain objective function and constraints are presented in detail. In Sect. 4, the crisp equivalent model based on the interval programming method is provided, and the strategy for dealing with the uncertain objective function and constraints is presented. An industrial case to verify the feasibility of applying the proposed approach to MPMP-APP decision problem is presented in Sect. 5. Finally, some concluding remarks are given in Sect. 6.

2 Problem Formulation

MPMP-APP is a traditional production planning problem that companies have to cope with. The problem is to make decisions about appropriate regular time/overtime production quantities, inventory, subcontracting/backordering quantities, and workface level to satisfy market demands over a given planning period. Some characteristics of the problem are summarized as follows.

- Suppose that a company will manufacture N types of products to satisfy fluctuating demand over a given planning period T. The uncertain demand in each planning period will be described as interval number in the following sections.
- The feasible means that can be chosen by the decision makers include adjusting production output, overtime, inventory levels, subcontracting, backordering and workforce changing and so on.
- Some parameters which are not known exactly both in the objective function and in constraints will be represented by interval numbers in the following sections.
- Maximum machine and inventory capacities in each planning period may be also imprecise and can be estimated by interval numbers.

2.1 Notations

2.1.1 Decision Variables

Q_{it}	The number of product i manufactured in the regular production time during period t (units)
O_{it}	The number of product i manufactured in the overtime production time during period t (units)
S_{it}	The number of product i subcontracting in period t (units)
I_{it}	The inventory of product i in period t (units)
B_{it}	The backorder number for product i in period t (units)
H_t	The number of workers hired in period t (labors/period t)
L_t	The number of workers laid off in period t (labors/period t)
W_t	The number of workers required in period t (labors/period t)

2.1.2 Parameters

N	The number of product category
T	Planning period
$\tilde{D}_{it}$	The forecast demand for product i in period t (units)
$\tilde{q}_{it}$	The production cost in regular working time to produce one unit of ith product in period t ($/unit)
$\tilde{o}_{it}$	The production cost in working overtime to produce one unit of ith product in period t ($/unit)
$\tilde{s}_{it}$	The subcontracting cost for per unit of ith product in period t ($/unit)
$\tilde{h}_{it}$	The inventory holding cost for per unit of ith product in period t ($/unit)
$\tilde{b}_{it}$	The backorder cost for ith product in period t ($/unit)
$\tilde{hr}_t$	The cost to hire one worker in period t ($/man-day)

$\tilde{lo}_t$ The cost to layoff one worker in period t (\$/man-day)

$\tilde{w}_t$ The labor cost in period t (\$/man-day)

$\tilde{M}_{t\mathrm{Max}}$ The maximum available machine capacity in period t (machine-hours)

$V_{t\max}$ The maximum inventory capacity available in period t (units)

$\tilde{e}_i$ The man hours required to produce ith product (man-hours/unit)

ρ The working hours of a labor in each period t (hours/period t)

α_t The fraction of regular production time available for overtime in period t

$\tilde{r}_i$ The machining time for producing one unit ith product (machine-hours/unit)

$\tilde{v}_i$ The inventory holding space occupied by ith product

2.2 Objective Function

The objective function of the uncertain MPMP-APP problem can be represented as follows:

$$\begin{aligned}\mathrm{Min}\ \tilde{f} = &\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\tilde{q}_{it}Q_{it} + \tilde{o}_{it}O_{it} + \tilde{s}_{it}S_{it}\right)\\ &+ \sum_{t=1}^{T}\sum_{i=1}^{N}\tilde{h}_{it}I_{it} + \sum_{t=1}^{T}\sum_{i=1}^{N}\tilde{b}_{it}B_{it}\\ &+ \sum_{t=1}^{T}\left(\tilde{hr}_tH_t + \tilde{lo}_tL_t + \tilde{w}_tW_t\right)\end{aligned} \quad (1)$$

where the first term $\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\tilde{q}_{it}Q_{it} + \tilde{o}_{it}O_{it} + \tilde{s}_{it}S_{it}\right)$ in Eq. (1) is the total production cost including the regular time production, overtime production and subcontracting cost, $\sum_{t=1}^{T}\sum_{i=1}^{N}\tilde{h}_{it}I_{it}$ is the total inventory costs, $\sum_{t=1}^{T}\sum_{i=1}^{N}\tilde{b}_{it}B_{it}$ is the total backorder costs, and the last term $\sum_{t=1}^{T}\left(\tilde{hr}_tH_t + l\tilde{o}_tL_t + \tilde{w}_tW_t\right)$ is the labor-related costs over all planning period. $\tilde{q}_{it}, \tilde{o}_{it}, \tilde{s}_{it}, \tilde{h}_{it}, \tilde{b}_{it}, \tilde{hr}_t, \tilde{lo}_t, \tilde{w}_t$ in Eq. (1) are parameters, some of which may be imprecise.

2.3 Constraints

(1) Constraint on quantity balance

$$Q_{it} + O_{it} + S_{it} + I_{i(t-1)} + B_{it} - I_{it} - B_{i(t-1)} = \tilde{D}_{it} \quad (2)$$

Equation (2) is relevant to satisfy market demands, where $\tilde{D}_{it}$ represents the imprecise market demand of ith product in period t.

(2) Constraints on labor levels

$$W_t = W_{t-1} + H_t - L_t \quad t = 1, 2, \ldots, T, \quad (3)$$

Equation (3) is relevant to current workforce level in period t.

$$\sum_{i=1}^{N}\tilde{e}_iQ_{it} \le \rho W_t \quad t = 1, 2, \ldots, T, \quad (4)$$

$$\sum_{i=1}^{N}\tilde{e}_iO_{it} \le \alpha_t\rho W_t \quad t = 1, 2, \ldots, T, \quad (5)$$

Inequality (4) and (5) denote workforce capacity constraints, which limit regular time and overtime production to available workers in each planning period t, respectively.

(3) Constraints on machine capacity:

$$\sum_{i=1}^{N}\tilde{r}_i(Q_{it} + O_{it}) \le \tilde{M}_{t\max} \ t = 1, 2, \ldots, T \quad (6)$$

Inequality (6) denotes the limits of available machine capacity in each period t.

(4) Constraint on warehouse space:

$$\sum_{i=1}^{N}\tilde{v}_iI_{it} \le V_{t\max} \quad t = 1, 2, \ldots, T \quad (7)$$

Inequality (7) is associated with the limit of available inventory capacity in each period t.

(5) Constraint on subcontracting volume

$$S_{it} \le S_{it}^{\max} \quad t = 1, 2, \ldots, T \quad (8)$$

Inequality (8) denotes that the subcontracting level in each period t should be less than or equal to the available subcontracting capacity.

(6) Non-negativity constraints on decision variables:

$$Q_{it}, O_{it}, S_{it}, I_{it}, B_{it}, H_t, L_t, W_t \ge 0 \quad \forall i, \forall t\,. \quad (9)$$

3 Interval Programming Method

3.1 The General Optimization Model with Interval Number

The general interval linear programming (ILP) model can be represented as follows:

$$\begin{aligned}&\min_x f(\mathbf{x}, \tilde{\mathbf{c}}) = \sum_{i}^{n}\tilde{c}_ix_i\\ &\text{s.t.} \quad g_j(\mathbf{x}, \tilde{\mathbf{a}}) = \sum_{i=1}^{n}\tilde{a}_{ij}x_i \le (=, \ge)\tilde{b}_j, \quad j = 1, 2, \ldots, l\\ &\tilde{c}_i \in \left[c_i^{\mathrm{L}}, c_i^{\mathrm{R}}\right], \quad \tilde{a}_{ij} \in \left[a_{ij}^{\mathrm{L}}, a_{ij}^{\mathrm{R}}\right], \quad \tilde{b}_j \in \left[b_j^{\mathrm{L}}, b_j^{\mathrm{R}}\right]\\ &x_i \ge 0\end{aligned} \quad (10)$$

where $f(\mathbf{x}, \tilde{\mathbf{c}})$ denotes the objective function and $g_j(\mathbf{x}, \tilde{\mathbf{a}})$ represents the jth constraint, x is an n-dimensional optimal vector and x_i represents its ith decisional variable. $\tilde{\mathbf{c}}$ is a n-dimensional uncertain vector, $\tilde{c}_i$ is its ith component. $\tilde{\mathbf{a}}$ is a $n \times l$ dimensional uncertain coefficient matrix, and $\tilde{a}_{ij}$ is its component. l denotes the number of the constraints in ILP model. $\tilde{b}_j$ represents the allowable interval of the jth constraint. The superscripts L and R represent the lower and upper bounds of an interval number, respectively. For example, $[b_j^{\mathrm{L}}, b_j^{\mathrm{R}}]$ is a bounded set of the interval number $\tilde{b}_j$, and b_j^{L} is its lower bound, b_j^{R} is its upper bound. According to the above definitions, the value of both the objective function and constraints will also be an interval number rather than a real number due to the existence of uncertain parameters. Therefore, the above ILP model cannot be solved directly by conventional optimization methods and needs to transform the uncertain objective function and constraints into the corresponding equivalent deterministic ones, respectively. In the following sections, some treatment methods will be given to deal with the above problems.

3.2 Deterministic Transformation of the Uncertain Objective Function

Since the imprecise parameters in the objective function are represented by interval numbers, the value of the objective function will also be an interval number rather than a precise real number. The method of deterministic transformation of the uncertain objective function is based on the interval order relation which implies that an interval number is larger or less than another. How to determine the order relation between two interval numbers depends on the decision makers' preferences. Ma [26] summarizes the common five kinds of order relations between two interval numbers $\tilde{A}$ and $\tilde{B}$ and presented the following definition for the maximization problem:

$$[1]\ \tilde{A} \le_{\mathrm{LR}} \tilde{B},\ \text{ if } A^{\mathrm{L}} \le B^{\mathrm{L}} \text{ and } A^{\mathrm{R}} \le B^{\mathrm{R}} \tag{11}$$

where the symbol $\le_{\mathrm{LR}}$ represents the preference of the decision maker to both a higher lower bound and a higher upper bound for an interval.

$$[2]\ \tilde{A} \le_{\mathrm{CW}} \tilde{B},\ \text{ if } A^{\mathrm{C}} \le B^{\mathrm{C}} \text{ and } A^{\mathrm{W}} \ge B^{\mathrm{W}} \tag{12}$$

$$A^{\mathrm{C}} = \frac{A^{\mathrm{L}} + A^{\mathrm{R}}}{2},\quad B^{\mathrm{C}} = \frac{B^{\mathrm{L}} + B^{\mathrm{R}}}{2},\quad A^{\mathrm{W}} = \frac{A^{\mathrm{R}} - A^{\mathrm{L}}}{2},\quad B^{\mathrm{W}} = \frac{B^{\mathrm{R}} - B^{\mathrm{L}}}{2}$$

where the symbol $\le_{\mathrm{CW}}$ represents the preference of the decision maker to both a higher expectation value and a lower uncertainty for an interval.

$$[3]\ \tilde{A} \le_{\mathrm{LC}} \tilde{B},\ \text{ if } A^{\mathrm{L}} \le B^{\mathrm{L}} \text{ and } A^{\mathrm{C}} \le B^{\mathrm{C}} \tag{13}$$

where the symbol $\le_{\mathrm{LC}}$ represents the preference of the decision maker to both a higher lower bound and a higher expectation value for an interval.

$$[4]\ \tilde{A} \le_{\mathrm{L}} \tilde{B},\ \text{ if } A^{\mathrm{L}} \le B^{\mathrm{L}} \tag{14}$$

where the symbol $\le_{\mathrm{L}}$ represents the preference of the decision maker to a higher lower bound for an interval, and which shows that the decision makers are more conservative.

$$[5]\ \tilde{A} \le_{\mathrm{R}} \tilde{B},\ \text{ if } A^{\mathrm{R}} \le B^{\mathrm{R}} \tag{15}$$

where the symbol $\le_{\mathrm{R}}$ represents the preference of the decision maker to a higher upper bound for an interval, and which shows that the decision makers are more optimistic.

Synthesizing the above definition for different order relations, in this paper we hope that the imprecise objective function can not only reflect the decision maker's risk preference, but also reduce the uncertainty caused by imprecise parameters. Thus, the original uncertain objective function can be replaced by the following two crisp objective functions:

$$\min_x Z_1(\mathbf{x}, \tilde{c}) = f^{\mathrm{L}}(\mathbf{x}, \tilde{\mathbf{c}}) + \zeta(f^{\mathrm{R}}(\mathbf{x}, \tilde{\mathbf{c}}) - f^{\mathrm{L}}(\mathbf{x}, \tilde{\mathbf{c}})) \tag{16}$$

$$\min_{\mathbf{x}} Z_2(\mathbf{x}, \tilde{\mathbf{c}}) = f^{\mathrm{R}}(\mathbf{x}, \mathbf{e}) - f^{\mathrm{L}}(\mathbf{x}, \mathbf{e}) \tag{17}$$

$$\begin{gathered} f^{\mathrm{L}}(\mathbf{x}, \tilde{\mathbf{c}}) = \min_{\mathbf{c} \in \Gamma} f(\mathbf{x}, \tilde{\mathbf{c}}),\quad f^{\mathrm{R}}(\mathbf{x}, \tilde{\mathbf{c}}) = \max_{c \in \Gamma} f(\mathbf{x}, \tilde{\mathbf{c}}), \\ \Gamma = \{\tilde{\mathbf{c}} | \mathbf{c}^{\mathrm{L}} \le \tilde{\mathbf{c}} \le \mathbf{c}^{\mathrm{R}}\},\quad \zeta \in [0, 1] \end{gathered} \tag{18}$$

where ζ in the first objective function (16) represents the decision maker's risk preference level to an interval, when $\zeta = 0$ represents the preference to the lower bound of an interval, when $\zeta = 0.5$ represents the preference to the expected value (or mean value) of an interval, and when $\zeta = 1.0$ represents the preference to the upper bound of an interval. And the second objective function (17) is analogous to minimize the variance of the uncertain objective function, i.e., the interval range of the objective function will be decreased.

3.3 Deterministic Transformation of the Uncertain Constraints

The possibility degree describes quantitatively the degree to which an interval number is larger or smaller than another. There are some scholars who have proposed the possibility degree formula, such as references Wan and Dong [41], Zhang et al. [48], Xu and Da [47] and so on. Jiang [16] summarized six possible relations between interval numbers $\tilde{A}$ and $\tilde{B}$ based on three relations proposed in the literature [48] and proposed a modified definition for the possibility degree $P_{\tilde{A}\le\tilde{B}}$, see Eq. (19). For more details about the calculation method of possibility degree, see references Gao [11] and Xiao et al. [46].

$$P_{\tilde{A}\le\tilde{B}}=\begin{cases} 0, & A^{\mathrm{L}}\ge B^{\mathrm{R}} \\ 0.5\dfrac{B^{\mathrm{R}}-A^{\mathrm{L}}}{A^{\mathrm{R}}-A^{\mathrm{L}}}\cdot\dfrac{B^{\mathrm{R}}-A^{\mathrm{L}}}{B^{\mathrm{R}}-B^{\mathrm{L}}}, & B^{\mathrm{L}}\le A^{\mathrm{L}}<B^{\mathrm{R}}\le A^{\mathrm{R}} \\ \dfrac{B^{\mathrm{L}}-A^{\mathrm{L}}}{A^{\mathrm{R}}-A^{\mathrm{L}}}+0.5\cdot\dfrac{B^{\mathrm{R}}-B^{\mathrm{L}}}{A^{\mathrm{R}}-A^{\mathrm{L}}}, & A^{\mathrm{L}}<B^{\mathrm{L}}<B^{\mathrm{R}}\le A^{\mathrm{R}} \\ \dfrac{B^{\mathrm{L}}-A^{\mathrm{L}}}{A^{\mathrm{R}}-A^{\mathrm{L}}}+\dfrac{A^{\mathrm{R}}-B^{\mathrm{L}}}{A^{\mathrm{R}}-A^{\mathrm{L}}}\cdot\dfrac{B^{\mathrm{R}}-A^{\mathrm{R}}}{B^{\mathrm{R}}-B^{\mathrm{L}}}+0.5\cdot\dfrac{A^{\mathrm{R}}-B^{\mathrm{L}}}{A^{\mathrm{R}}-A^{\mathrm{L}}}\cdot\dfrac{A^{\mathrm{R}}-B^{\mathrm{L}}}{B^{\mathrm{R}}-B^{\mathrm{L}}}, & A^{\mathrm{L}}<B^{\mathrm{L}}\le A^{\mathrm{R}}<B^{\mathrm{R}} \\ \dfrac{B^{\mathrm{R}}-A^{\mathrm{R}}}{B^{\mathrm{R}}-B^{\mathrm{L}}}+0.5\cdot\dfrac{A^{\mathrm{R}}-A^{\mathrm{L}}}{B^{\mathrm{R}}-B^{\mathrm{L}}}, & B^{\mathrm{L}}\le A^{\mathrm{L}}<A^{\mathrm{R}}<B^{\mathrm{R}} \\ 1, & A^{\mathrm{R}}<B^{\mathrm{L}} \end{cases} \tag{19}$$

where $P_{\tilde{A}\le\tilde{B}}$ represents the possibility degree that $\tilde{A}$ is less than or equal to $\tilde{B}$.

When $\tilde{B}$ is degenerated into a real number b, the degenerated possibility degree $P_{\tilde{A}\le b}$ can be rewritten as follows:

$$P_{\tilde{A}\le b}=\begin{cases} 0, & b\le A^{\mathrm{L}} \\ \dfrac{b-A^{\mathrm{L}}}{A^{\mathrm{R}}-A^{\mathrm{L}}}, & A^{\mathrm{L}}<b\le A^{\mathrm{R}} \\ 1, & b>A^{\mathrm{R}} \end{cases} \tag{20}$$

Similarly, the possibility degree $P_{a\le\tilde{B}}$ can be also obtained when $\tilde{A}$ is degenerated into a real number a:

$$P_{a\le\tilde{B}}=\begin{cases} 1, & a\le B^{\mathrm{L}} \\ \dfrac{B^{\mathrm{R}}-a}{B^{\mathrm{R}}-B^{\mathrm{L}}}, & B^{\mathrm{L}}<a\le B^{\mathrm{R}} \\ 0, & a>B^{\mathrm{R}} \end{cases} \tag{21}$$

Based on the above definition of the possibility degree, the uncertain constraint $g_j(\mathbf{x},\tilde{\mathbf{a}})\le[b_j^{\mathrm{L}},b_j^{\mathrm{R}}]$ in the constraint (10) can be converted into the following form by introducing the possibility degree:

$$P_{\tilde{A}_j\le\tilde{B}_j}\ge\lambda_j \tag{22}$$

where $P_{\tilde{A}_j\le\tilde{B}_j}$ represents the possibility degree to which $\tilde{\boldsymbol{A}}_j$ is less than or equal to $\tilde{\boldsymbol{B}}_j$. λ_j $(0\le\lambda_j\le1)$ is a predetermined confidence level for the possibility degree and can be adjusted by decision makers according to their risk preference. The higher the value of λ_j is, the smaller the feasible range of the decision variable $\mathbf{x}$ will be. $\tilde{\boldsymbol{A}}_j=\left[g_j^{\mathrm{L}}(\mathbf{x},\widetilde{\mathbf{a}}),g_j^{\mathrm{R}}(\mathbf{x},\widetilde{\mathbf{a}})\right]$, $\tilde{\boldsymbol{B}}_j=\left[b_j^{\mathrm{L}},b_j^{\mathrm{R}}\right]$, where $g_j^{\mathrm{L}}(\mathbf{x},\widetilde{\mathbf{a}})$ and $g_j^{\mathrm{R}}(\mathbf{x},\widetilde{\mathbf{a}})$ are the lower and upper bound of the uncertain constraint $g_j(\mathbf{x},\widetilde{\mathbf{a}})$, respectively.

where

$$g_j^{\mathrm{L}}(\mathbf{x},\widetilde{\mathbf{a}})=\min_{\mathbf{a}\in\boldsymbol{\Gamma}}g_j(\mathbf{x},\tilde{\mathbf{a}}),\quad g_j^{\mathrm{R}}(\mathbf{x},\widetilde{\mathbf{a}})=\max_{\mathbf{a}\in\boldsymbol{\Gamma}}g_j(\mathbf{x},\tilde{\mathbf{a}}), \tag{23}$$

Similarly, for the inequality constraint $g_j(\mathbf{x},\tilde{\mathbf{a}})\ge\left[b_j^{\mathrm{L}},b_j^{\mathrm{R}}\right]$ in the constraint (10), it can be changed into $[b_j^{\mathrm{L}},b_j^{\mathrm{R}}]\le g_j(\mathbf{x},\tilde{\mathbf{a}})$, and it can also be treated with Eq. (19).

Finally, for the uncertain equality constraint $g_j(\mathbf{x},\tilde{\mathbf{a}})=\left[b_j^{\mathrm{L}},b_j^{\mathrm{R}}\right]$, it could be replaced by the following two inequality constraints:

$$\begin{cases} g_j(\mathbf{x},\tilde{\mathbf{a}})\le b_j^{\mathrm{R}} \\ b_j^{\mathrm{L}}\le g_j(\mathbf{x},\tilde{\mathbf{a}}) \end{cases} \tag{24}$$

Inequality (24) denotes that the original interval number $[b_j^{\mathrm{L}},b_j^{\mathrm{R}}]$ is degenerated into two real numbers, i.e., b_j^{L} and b_j^{R}. Therefore, inequality (24) can be dealt with Eqs. (20) and (21), respectively.

4 Crisp Equivalent Model Based on the Interval Programming

4.1 Strategy of Deterministic Equivalent Transformation

According to the method of treating uncertain objective function, the original uncertain objective function (1) can be changed into the two crisp objective functions which are equivalent to minimizing the interval value according to the decision maker's preference and the deviation of the uncertain objective function, respectively.

Recalling the above uncertain MPMP-APP model, constraints (2), (4), (5), (6) and (7) are soft constraints due to imprecise parameters incorporated in them. And others are deterministic constraints which need no more transformation. Those soft constraints have the same form like the constraint (10); therefore, they can be transformed into the corresponding form by introducing the possibility degree. For example, constraint (6) is a soft constraint because the imprecise coefficients exist in both sides of it, so its possibility degree can be defined by Eq. (19), and then through introducing confidence level λ to finish corresponding deterministic transformation.

4.2 Deterministic Equivalent Model of the Uncertain MPMP-APP

Through the above treatment, the original uncertain MPMP-APP problem can be replaced by the following deterministic equivalent model, which is a two-objective linear programming (TOLP) model.

Table 1 Forecast demands for each product (units)

Product (i)	Period (t)					
	1	2	3	4	5	6
$\tilde{D}_{1t}$	[900, 1080]	[1300, 1500]	[1400, 1680]	[1850, 2100]	[1500, 1650]	[1200, 1300]
$\tilde{D}_{2t}$	[900, 1080]	[1500, 1650]	[2750, 3200]	[2300, 2650]	[1850, 2100]	[1850, 2100]

Table 2 Operation cost data

Product (i)	$\tilde{q}_{it}$ (\$/unit)	$\tilde{o}_{it}$ (\$/unit)	$\tilde{s}_{it}$ (\$/unit)	$\tilde{h}_{it}$ (\$/unit)	$\tilde{b}_{it}$ (\$/unit)
1	[17, 22]	[26, 33]	[60, 70]	[0.27, 0.32]	[80, 98]
2	[8, 11]	[12, 17]	[30, 35]	[0.13, 0.16]	[50, 63]

$$\min_{x} Z_1(\mathbf{x},\tilde{\mathbf{c}}) = f^{\mathrm{L}}(\mathbf{x},\tilde{\mathbf{c}}) + \zeta(f^{\mathrm{R}}(\mathbf{x},\tilde{\mathbf{c}}) - f^{\mathrm{L}}(\mathbf{x},\tilde{\mathbf{c}})) \tag{25}$$

$$\min_{\mathbf{x}} Z_2(\mathbf{x},\tilde{\mathbf{c}}) = f^{\mathrm{R}}(\mathbf{x},\tilde{\mathbf{c}}) - f^{\mathrm{L}}(\mathbf{x},\tilde{\mathbf{c}}) \tag{26}$$

s.t.

$$Q_{it} + O_{it} + S_{it} + I_{i(t-1)} + B_{it} - I_{it} - B_{i(t-1)} \geq D_{it}^{\mathrm{L}} \tag{27}$$

$$Q_{it} + O_{it} + S_{it} + I_{i(t-1)} + B_{it} - I_{it} - B_{i(t-1)} \leq D_{it}^{\mathrm{R}} \tag{28}$$

$$P_{1t}\left(\sum_{i=1}^{N} \tilde{e}_i Q_{it} \leq \rho W_t\right) \geq \lambda_{1t} \tag{29}$$

$$P_{2t}\left(\sum_{i=1}^{N} \tilde{e}_i O_{it} \leq \alpha_t \rho W_t\right) \geq \lambda_{2t} \tag{30}$$

$$P_{3t}\left(\sum_{i=1}^{N} \tilde{r}_i (Q_{it} + O_{it}) \leq \tilde{M}_{t\max}\right) \geq \lambda_{3t} \tag{31}$$

$$P_{4t}\left(\sum_{i=1}^{N} \tilde{v}_i I_{it} \leq V_{t\max}\right) \geq \lambda_{4t} \tag{32}$$

$$\text{Eqs. (3), (8) and (9)}, \quad t = 1, 2, \ldots, T \tag{33}$$

In the above TOLP model, both of the constraints (27) and (28) are rigid constraints, which are equivalent to the constraint (2). But inequalities (29)–(32) are soft constraints due to imprecise parameters embedded in them, and the symbols λ_{1t}, λ_{2t}, λ_{3t} and λ_{4t} are possibility confidence level that the decision maker predetermine for soft constraints (4), (5), (6) and (7), respectively, and $0 \leq \lambda_{1t}, \lambda_{2t}, \lambda_{3t}, \lambda_{4t} \leq 1$. There are many effective approaches to solve multi-objective linear programming model, such as the main objective method, the linear weighted sum method, the min–max method, the ideal point method and so on. Since the objective function (26) has the same dimension with the objective function (25), the linear weighted sum method is an appropriate method that can be used to transform the above two-objective model into a single one. Accordingly, the objective function (25) and (26) can be formalized as the following single-objective function by adopting the linear weighted sum method.

$$\min_{\mathbf{x}} Z = \sqrt{\left(Z_1 - Z_1^*\right)^2 + \left(Z_2 - Z_2^*\right)^2} \tag{34}$$

where Z_1* and Z_2* represent the ideal point for the objective function $\min Z_1(\mathbf{x},\tilde{\mathbf{c}})$ and $\min Z_2(\mathbf{x},\tilde{\mathbf{c}})$, respectively. In this manner, the final single-objective linear programming model has been built, which can be solved by some mature professional software efficiently, such as LINGO, MATLAB and so on.

5 Numerical Example

5.1 Case Description

In this section, the data provided by a manufacturing enterprise in Xi'an, China, are used to illustrate the applicability and effectiveness of the proposed method. The MPMP-APP problem for this company focuses on finding out an efficient method to minimize the total cost within the limited resources and capacities constraints. In this case, the planning horizon is 6 months long ($T = 6$), from May to October, and includes two types of products ($N = 2$), namely products 1 and 2.

Table 3 Maximum labor, machine and warehouse capacity data

Period (t)	$W_{t\max}$ (labors)	$\tilde{M}_{t\max}$ (machine-hours)	$V_{t\max}$ (ft^2)
1	35	[4500, 5500]	10,000
2	35	[5500, 6500]	10,000
3	35	[5500, 6500]	10,000
4	35	[5500, 6500]	10,000
5	35	[5500, 6500]	10,000
6	35	[2500, 3500]	10,000

Table 4 Labor cost and workforce changing costs

Period (t)	Labor cost (\$/man-day)	Hiring cost (\$/man-day)	Layoff cost (\$/man-day)
1	115	70	58
2	115	70	58
3	115	70	58
4	115	70	58
5	115	70	58
6	115	70	58

Table 5 Labor and machine relevant time data

Product (i)	Labor time $\tilde{e}_i$ (h/unit)	Machine time $\tilde{r}_i$ (h/unit)
1	1.6	[1.2, 1.5]
2	1.8	[1.5, 1.7]

5.2 Basic Data for Numerical Example

In order to analyze the impact of uncertainty on the objective function easily, it is assumed that only the constraint (31) is considered as the soft constraint in this case, i.e., parameters $\tilde{r}$ and $\tilde{M}_{t\max}$ are uncertain and described as interval numbers, and other parameters in constraints (29), (30) and (32) are assumed to be deterministic. The symbol P_{3t} in constraint (31) denote the possibility degree that the actual machine capacity requirements in period t is less than or equal to the maximum available machine capacity in corresponding planning period, λ_{3t} $(0 \le \lambda_{3t} \le 1)$ is a predetermined confidence level. And the concrete data for this case are presented in Tables 1, 2, 3, 4, 5, 6, 7 and 8.

Table 1 corresponds to the imprecise market demands for two products in different period t. Table 2 shows relevant operation cost data. Table 3 lists the maximum available capacity for workforce, machines and inventory, respectively, in which the maximum capacity of machine is considered as the imprecise parameter and described as interval number. Labor costs and hiring and layoff costs are listed in Table 4. Labor and machine relevant times are presented in Table 5, in which hours of machine usage per unit for six planning periods are defined as interval numbers.

Other relevant data are explained below:

1. Assume that the initial inventory in period 1 is 400 units for product 1 and 200 units for product 2. And the end inventory in period 6 is 300 units for product 1 and 200 units for product 2.
2. Inventory spaces occupied by per unit of products 1 and 2 are 2 and 3 ft^2, respectively.

Table 6 Optimization results with different values of preference coefficients ζ and λ

	$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
	Min Z_1	Min Z_2	Min Z_1	Min Z_2	Min Z_1	Min Z_2
$\zeta = 0.0$	261,854.7	79,320.23	262,362.4	79,585.17	262,827.1	79,856.55
$\zeta = 0.5$	301,514.9	79,320.27	302,156.0	79,585.36	302,755.7	79,856.60
$\zeta = 1.0$	341,174.6	79,320.18	341,947.6	79,585.17	342,683.5	79,856.53

Table 7 Optimization results under different possibility degree when $\zeta = 0.5$

Confidence level λ	Two products during six planning horizons					
	Sum of regular time (units)	Sum of overtime (units)	Sum of subcontracting (units)	Sum of workforce (labors)	Min Z_1 (\$)	Min Z_2 (\$)
1.0	15,911	2095	1194	194	325,527.4	80,498.06
0.8	16,257	2943	0	198	302,755.7	79,856.60
0.6	16,385	2815	0	200	302,155.6	79,585.26
0.4	16,510	2690	0	201	301,516.6	79,320.47
0.2	16,642	2558	0	203	300,891.9	79,040.34
0.0	17,227	1973	0	210	297,911.2	77,776.23

Table 8 Multi-period production plan with different confidence level λ when $\zeta = 0.5$

Period t	1			2			3			4			5			6		
Confidence level λ	0.2	0.6	1.0	0.2	0.6	1.0	0.2	0.6	1.0	0.2	0.6	1.0	0.2	0.6	1.0	0.2	0.6	1.0
Product 1																		
Regular time	502	502	920	1303	1303	880	1395	1395	1400	1852	1854	1850	1501	1501	1508	1497	1495	1492
Overtime	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Subcontracting	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	2	2	420	5	5	0	0	0	0	2	4	0	3	5	8	300	300	300
Backorder	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Product 2																		
Regular time	2276	2276	1835	1564	1564	1940	1482	1482	1477	1076	1074	1077	1388	1388	1381	806	551	151
Overtime	0	0	0	110	367	519	816	816	523	816	816	526	816	816	524	0	0	3
Subcontracting	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1194
Inventory	1576	1576	1135	1750	2007	2094	1298	1555	1344	890	1145	647	1244	1499	702	200	200	200
Backorder	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hiring	5	5	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Layoff	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	10	16
Workforce	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	28	25	19
P_{3t}	0.988	0.988	1.0	1.0	1.0	1.0	0.858	0.858	1.0	0.869	0.869	1.0	0.861	0.861	1.0	0.200	0.600	1.0

3. For convenience of comparison, we assume that the confidence level λ_{3t} in constraint (31) in different planning period t are equal, that is, $\lambda_{31}=\lambda_{32}=\cdots=\lambda_{36}=\lambda$.
4. Other relevant data are $\alpha_t = 0.3$, $\rho = 140$. Initial workforce level is 30.

5.3 Results Analysis

Through the interval programming method proposed in this paper, the above uncertain MPMP-APP problem is finally transformed to a deterministic single-objective LP model. Therefore, LINGO computer software can be applied to solve this model and the computational results are shown in Tables 6, 7 and 8.

Table 6 shows the different optimization results when $\lambda = 0.4$, 0.6 and 0.8, and $\zeta = 0.0$, 0.5 and 1.0, respectively. The result indicates that the smaller both the values of λ and ζ are, the lower the total cost is.

Table 7 lists the sum of some decision variables during six planning periods under different confidence level λ based on statistic data in Table 8. It shows that the objective function values are different with different λ. With the decrease in λ from 1.0 to 0.0, the values of both of the objective functions Z_1 and Z_2 become smaller and the sum of both of the regular time and overtime production quantities becomes larger. This is because that a larger λ may result in a more strict constraint, and it will lead to a smaller production capacity and a higher cost. When $\lambda = 1.0$, it means that the original soft constraint becomes a deterministic rigid constraint, and the sum of subcontracting of two products during six planning periods is no longer equal to zero due to insufficient machine capacity. Therefore, it has to rely on subcontracting to satisfy some of the demands. As the confidence level λ falls, the available machine capacity is improved properly, the sum of both of the regular and the overtime production quantities becomes larger, and subcontracting is no longer needed. This is because that both of the regular and the overtime production can fully satisfy the market demands. And when λ falls to zero, namely $\lambda = 0$, it means that the current constraint has no longer effect on the objective function. And furthermore, the results also imply that the larger the confidence level λ, the lower the expected value of the objective function.

Table 8 shows the final optimization results of the decision variables when $\zeta = 0.5$ and $\lambda = 0.0$, 0.2, 0.4, 0.6, 0.8 and 1.0, respectively.

6 Conclusions

In this paper, an interval programming method has been proposed for resolving uncertain MPMP-APP problem. The proposed approach attempts to minimize total cost with reference to regular and overtime production, subcontracting and backordering levels, labor levels, and machine and warehouse capacity. The interval programming model developed in this paper extends the conventional uncertain MPMP-APP problem which deals with the uncertainty mainly based on probability distributions or membership function. The interval analysis method is applied in this paper to reformulate the MPMP-APP problems with imprecise parameters. In comparison with the conventional methods such as stochastic programming and fuzzy optimization method and so on, the interval programming method only requires the lower and upper bounds of the uncertain parameters without necessarily knowing the probability distributions or membership functions. In order to solve this model, the original uncertain interval model would be converted into a deterministic two-objective LP model through introducing the possibility degree and the order relation between two interval numbers.

An industrial case demonstrates that the proposed method is feasible for handling imprecise parameters in the uncertain MPMP-APP decision problems. Moreover, some limitations of the proposed model should be taken into consideration. Firstly, both the objective function and constraints in the interval programming model are required to be linear, the further research can consider nonlinear fuzzy APP model, in which either the objective function or constraints might be nonlinear. Secondly, the proposed model is based on the interval analysis method, in which imprecise parameters are described as interval numbers, and further research may explore mixed type description for uncertain parameters. For example, market demands during different planning period would be described as random variables that obey some kinds of probability distribution, and other uncertain parameters are described as intervals. Finally, the value of this research may be increased if the multi-objective and multi-stage model for MPMP-APP problems would be considered in the further research.

Acknowledgements The authors gratefully acknowledge the financial support of the National Natural Science Foundation of China (NSFC). Grant No. 51605041; and the Innovation team funds of the Central Universities, China (Grant No. 310825153403).

References

1. Afzali, A., Rafsanjani, M.K., Saeid, A.B.: A fuzzy multi-objective linear programming model based on interval-valued intuitionistic fuzzy sets for supplier selection. Int. J. Fuzzy Syst. **18**(5), 864–874 (2016)
2. Baykasoglu, A.: Moapps 1.0: aggregate production planning using the multiple-objective tabu search. Int. J. Prod. Res. **39**(16), 3685–3702 (2001)
3. Baykasoglu, A., Gocken, T.: Multi-objective aggregate production planning with fuzzy parameters. Adv. Eng. Softw. **41**(9), 1124–1131 (2010)
4. Bitran, G.R., Yanasse, H.H.: Deterministic approximations to stochastic production problems. Manag. Sci. **32**(5), 999–1018 (1984)
5. Boloukat, M.H.S., Foroud, A.A.: Stochastic-based resource expansion planning for a grid-connected microgrid using interval linear programming. Energy **113**, 776–787 (2016)
6. Corominas, A., Lusa, A., Olivella, J.: A detailed workforce planning model including non-linear dependence of capacity on the size of the staff and cash management. Eur. J. Oper. Res. **216**(2), 445–458 (2012)
7. Entezaminia, A., Heydari, M., Rahmani, D.: A multi-objective model for multi-product multi-site aggregate production planning in a green supply chain: considering collection and recycling centers. J. Manuf. Syst. **40**(1), 63–75 (2016)
8. Feng, J.W., Wu, D., Gao, W., et al.: Uncertainty analysis for structures with hybrid random and interval parameters using mathematical programming approach. Appl. Math. Model. **48**, 208–232 (2017)
9. Figueroa-García, J.C., Kalenatic, D., Lopez-Bello, C.A.: Multi-period mixed production planning with uncertain demands: fuzzy and interval fuzzy sets approach. Fuzzy Sets Syst. **206**, 21–38 (2012)
10. Gansterer, M.: Aggregate planning and forecasting in make-to-order production systems. Int. J. Prod. Econ. **170**(12), 521–528 (2015)
11. Gao, F.J.: Possibility degree and comprehensive priority of interval numbers. Syst. Eng. Theory Pract. **33**(8), 2033–2040 (2013)
12. Gholamian, N., Mahdavi, I., Reza, T.M., et al.: Comprehensive fuzzy multi-objective multi-product multi-site aggregate production planning decisions in a supply chain under uncertainty. Appl. Soft Comput. **37**, 585–607 (2015)
13. Ishibuchi, H., Tanaka, H.: Multiobjective programming in optimization of the interval objective. Eur. J. Oper. Res. **48**(2), 219–225 (1990)
14. Jamalnia, A., Soukhakian, M.A.: A hybrid fuzzy goal programming approach with different goal priorities to aggregate production planning. Comput. Ind. Eng. **56**, 1474–1486 (2009)
15. Jia, W.F., Liao, X.X., Wang, M.Z., Shen, Y.: Aggregate production planning with multi-objectives in uncertain environment. Syst. Eng. **20**, 50–54 (2002)
16. Jiang, C.: Uncertainty Optimization Theory and Algorithm Based on Interval Number. Hunan University for the degree of Doctor of Philosophy (2008)
17. Jiang, C., Han, X., Guan, F.J., Li, Y.H.: An uncertain structural optimization method based on nonlinear interval number programming and interval analysis method. Eng. Struct. **29**, 3168–3177 (2007)
18. Jiang, C., Han, X., Liu, G.R., Liu, G.P.: A nonlinear interval number programming method for uncertain optimization problems. Eur. J. Oper. Res. **188**(1), 1–13 (2008)
19. Khalili-Damghan, K., Shahrokh, A.: Solving a new multi-period multi-objective multi-product aggregate production planning problem using fuzzy goal programming. Ind. Eng. Manag. Syst. **13**(4), 369–382 (2014)
20. Kothyari, A., Singh, S.P., Kaur, H.: Fuzzy modeling for low-carbon dynamic procurement problem. Int. J. Fuzzy Syst. (2016). doi:10.1007/s40815-016-0238-1
21. Kumar, R.S., Goswami, A.: A continuous review production–inventory system in fuzzy random environment: minmax distribution free procedure. Comput. Ind. Eng. **79**, 65–75 (2015)
22. Lan, Y.F., Liu, Y.K., Sun, G.J.: Modeling fuzzy multi-period production planning and sourcing problem with credibility service levels. J. Comput. Appl. Math. **231**, 208–221 (2009)
23. Lan, Y.F., Liu, Y.K., Sun, G.J.: An approximation-based approach for fuzzy multi-period production planning problem with credibility objective. Appl. Math. Model. **34**, 3202–3215 (2010)
24. Leung, S.C.H., Tsang, S.O.S., Ng, W.L., Wu, Y.: A robust optimization model for multi-site production planning problem in an uncertain environment. Eur. J. Oper. Res. **181**, 224–238 (2007)
25. Lin, J.H., Liu, M., Hao, J.H., et al.: A multi-objective optimization approach for integrated production planning under interval uncertainties in the steel industry. Comput. Oper. Res. **72**, 189–203 (2016)
26. Ma, L.H.: Research on Method and Application of Robust Optimization for Uncertainty System. Zhejiang University for the degree of Doctor of Philosophy (2002)
27. Mirzapour Al-e-hashem, S.M.J., Malekly, H., Aryanezhad, M.B.: A multi-objective robust optimization model for multi-product multi-site aggregate production planning in a supply chain under uncertainty. Int. J. Prod. Econ. **134**(1), 28–42 (2011)
28. Mirzapour Al-e-hashem, S.M.J., Baboli, A., Sazvar, Z.: A stochastic aggregate production planning model in a green supply chain: considering flexible lead times, nonlinear purchase and shortage cost functions. Eur. J. Oper. Res. **230**(1), 26–41 (2013)
29. Modarres, M., Izadpanahi, E.: Aggregate production planning by focusing on energy saving: a robust optimization approach. J. Clean. Prod. **133**, 1074–1085 (2016)
30. Mula, J., Poler, R., García-Sabater, J.P., Lario, F.C.: Models for production planning under uncertainty: a review. Int. J. Prod. Econ. **103**, 271–285 (2006)
31. Qiu, D.S., He, C., Zhu, X.M.: Ranking method research of interval numbers based on probability reliability distribution. Control Decis. **27**(12), 1894–1898 (2012)
32. Qu, T., Chen, X.D., Zhang, Y.F., Yang, H.D., Huang, G.Q.: Analytical target cascading enabled optimal configuration platform for production service systems. Int. J. Comput. Integr. Manuf. **24**(5), 457–470 (2012)
33. Rahmani, D., Ramezanian, R., Heydari, P.F.M.: A robust optimization model for multi-product two-stage capacitated production planning under uncertainty. Appl. Math. Model. **37**(20–21), 8957–8971 (2013)
34. Ramezanian, R., Saidi-Mehrabad, M.: Hybrid simulated annealing and MIP-based heuristics for stochastic lot-sizing and scheduling problem in capacitated multi-stage production system. Appl. Math. Model. **37**, 5134–5147 (2013)
35. Rodrigueza, M.A., Montagnab, J.M., Vecchiettib, A.: Generalized disjunctive programming model for the multi-period production planning optimization: an application in a polyurethane foam manufacturing plant. Comput. Chem. Eng. **103**(4), 69–80 (2017)
36. Sengupta, A., Pal, T.K., Chakraborty, D.: Interpretation of inequality constraints involving interval coefficients and a solution to interval linear programming. Fuzzy Sets Syst. **119**, 129–138 (2001)
37. Su, T.S., Lin, Y.F.: Fuzzy multi-objective procurement/production planning decision problems for recoverable manufacturing systems. J. Manuf. Syst. **37**, 396–408 (2015)

38. Tang, J.F., Wang, D.W., Xu, B.D.: Fuzzy modeling approach to aggregate production planning with multi-product. J. Manag. Sci. China **6**(1), 44–50 (2003)
39. Tavakkoli-Moghaddam, R., Rabbani, M., Gharehgozli, A.H., Zaerpour, N.: A fuzzy aggregate production planning model for make-to-stock environments. In: 2007 IEEE International Conference on Industrial Engineering and Engineering Management, pp. 1609–1613 (2007)
40. Torabia, S.A., Ebadianb, M., Tanha, R.: Fuzzy hierarchical production planning (with a case study). Fuzzy Sets Syst. **161**, 1511–1529 (2010)
41. Wan, S.P., Dong, J.Y.: A possibility degree method for interval-valued intuitionistic fuzzy multi-attribute group decision making. J. Comput. Syst. Sci. **80**(1), 237–256 (2014)
42. Wang, R.C., Liang, T.F.: Application of fuzzy multi-objective linear programming to aggregate production planning. Comput. Ind. Eng. **46**(1), 17–41 (2004)
43. Wang, R.C., Liang, T.F.: Applying possibilistic linear programming to aggregate production planning. Int. J. Prod. Econ. **98**(3), 328–341 (2005)
44. Wei, G.W.: Approaches to interval intuitionistic trapezoidal fuzzy multiple attribute decision making with incomplete weight information. Int. J. Fuzzy Syst. **17**(3), 484–489 (2015)
45. Wolfe, M.A.: Interval mathematics, algebraic equations and optimization. J. Comput. Appl. Math. **124**, 263–280 (2000)
46. Xiao, J., Zhang, Y., Fu, C.: Comparison between methods of interval number ranking based on possibility. J. Tianjin Univ. **44**(8), 705–711 (2011)
47. Xu, Z.S., Da, Q.L.: Possibility degree method for ranking interval numbers and its application. J. Syst. Eng. **18**(1), 67–70 (2003)
48. Zhang, Q., Fan, Z., Pan, D.: A ranking approach for interval numbers in uncertain multiple attribute decision making problems. Syst. Eng. Theory Pract. **19**(5), 129–133 (1999)

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