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Sensitivity Analysis of the Fuzzy Mean-Entropy Portfolio Model with Transaction Costs Based on Credibility Theory

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Abstract This paper proposes the fuzzy mean-entropy portfolio models with transaction costs based on credibility theory. In the paper, entropy is used as the measurement of risk. Furthermore, sensitivity analysis is discussed for objective function coefficients and constraint coefficients on the right sides in our proposed models. In addition, two numerical examples are given to illustrate the effectiveness of our proposed models and the practicability of sensitivity analysis. More importantly, the obtained results also show that when certain coefficient changes in some value range, we still can obtain the unchanged optimal solutions or unchanged objective function values. Compared with Huang (IEEE Trans Fuzzy Syst 16:1096–1101, 18; Fuzzy Optim Decis Mak 10:71–89, 19), our paper not only proposes the mean-entropy models, but also does research work on sensitivity analysis about objective function coefficients and constraint coefficients in depth in maximizing return model and minimizing risk model. Our results can provide more choices for investors in the practical financial market.

Keywords Sensitivity analysis · Mean-entropy portfolio model · Credibility theory · Transaction costs · Triangular fuzzy number

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1 Introduction

The mean–variance methodology for the portfolio selection problem, proposed originally by Markowitz [1–3], played a critical role in the development of modern finance theory. It combines probability with optimization technique to model the behavior investment. The fundamental principle of the mean–variance model is to use the expected return of the portfolio as the investment return and to use the variance of the return of the portfolio as the investment risk. As we know, the traditional Markowitz's mean–variance models are optimal models by minimizing risk or maximizing return with some constraints. Most of the existing portfolio selection models are based on probability theory. The mean–variance portfolio selection problem has been studied by many researchers including Best et al. [4], Merton [5], Pang [6], Perold [7] and Sharpe [8]. These portfolio models are solved by traditional optimal algorithm (see active set algorithm in [9]) and intelligent algorithm (evolutionary algorithms in [10]).

In the practical financial market, many non-probabilistic factors affect the financial market such that the risky asset's uncertainty is more embodied in the fuzzy uncertainty. Fuzzy uncertainty is more important than the probabilistic uncertainty. Liu [11] presented the credibility theory based on the axiomatic system of fuzzy number. Based on the credibility theory, many researchers investigated fuzzy means and variances into portfolio model. The concept of fuzzy entropy was defined by Liu [11] for measuring the uncertainty of fuzzy variables. Liu [12] presented a deep theoretical study and discussion on credibility theory by some basic concepts and fundamental theorems. Liu [13] also proposed a chance constrained programming model and designed a genetic algorithm to solve the model. Much research work has been obtained when the return rate is regarded as fuzzy number by Amiri [14], Bhattacharyya [15], Dastkhan [16], etc.

At the same time, we also note that risk is also can be measured by entropy. Philippatos [17] pointed out that entropy is more general than variance as an efficient measure of risk because entropy is free from reliance on symmetric probability distributions and can be computed from non-metric data. Huang [18, 19] studied the mean-entropy models for fuzzy portfolio selection and mean-risk models for uncertain portfolio models. Mukesh [20] presented credibilistic mean-entropy models for multi-period portfolio selection with multi-choice aspiration levels. And a real-world empirical application with dataset from an Indian stock market is presented to demonstrate effectiveness of the proposed models. Zhou [21] presented a mean–variance hybrid-entropy model for portfolio selection with fuzzy returns. Finally, the corresponding results show that the proposed models generally perform better than the traditional portfolio selection models.

In the past, many researchers studied fuzzy portfolio selection models with all kinds of complex constraints. Zhang [22] and Sadjadi [23] considered more complex portfolio models with borrowing and lending and transaction costs. Li [24] did some research work on entropy of credibility distributions for fuzzy variables, which was the basis of fuzzy entropy. Liu [25] studied portfolio selection problem with interval-valued returns in which the risk was measured by absolute deviation. Liu [26] also studied a fuzzy portfolio optimization model. Qin [27] considered fuzzy cross-entropy in portfolio selection. Zhou [28] studied a portfolio optimization model based on information entropy with fuzzy time series, both entropy-based models outperform the traditional ones and the fuzzy time series forecasting model also help to further improve the actual performance. Yue [29] studied a new fuzzy multi-objective and higher-order moment portfolio selection model for diversified portfolios, and a new effective multi-objective evolutionary algorithm was designed. Chen [30] did some research work on the hybrid FA-SA algorithm for fuzzy portfolios selection with transaction costs.

As we know, the portfolio model is an optimization model actually. Sensitivity analysis plays an important role in the optimization method. In optimization method, the stability of the optimal solution is studied by using sensitivity analysis to do research on original data inaccuracy and change. Many researchers paid a little attention to the sensitivity analysis about portfolio model. In this paper, the expected return of the portfolio is used as the investment return and the fuzzy entropy of the return of the portfolio is used as the investment risk. We also note that the transaction cost is an important factor in portfolio investment. Thus, the fuzzy mean-entropy model with transaction costs is proposed when the return rates are triangular fuzzy numbers. At the same time, the sensitivity analysis is discussed for the objective function and constraint on coefficients in detail.

The other part of this paper is organized as follows. In Sect. 2, we introduce credibility theory and some basic definitions. In Sect. 3, we propose a mean-entropy model with transaction costs when the returns are triangular fuzzy numbers. In Sect. 4, we use two numerical portfolio examples to demonstrate the effectiveness of our proposed models and the practicability of sensitivity analysis. Furthermore, we investigate sensitivity analysis with right-hand sides of constraints or objective coefficients. In Sect. 5, a brief summary of this paper is given. Finally, scope for future study is presented in Sect. 6.

2 Credibility, Expected Value and Entropy

The concept of fuzzy set was initiated by Zadeh [31] with membership function in 1965. In order to measure a fuzzy set, Zadeh [32] proposed the concept of possibility measure in 1978. Although possibility measure has been widely used, it does not obey the law of truth conservation and is inconsistent with the law of excluded middle and the law of contradiction. The main reason is that possibility measure has no self-duality property. However, a self-dual measure is absolutely needed both in theory and in practice [11].

Liu [11] defined credibility measure with self-dual with strict mathematical basis. The crucial point of credibility theory is self-dual. When the credibility value of a fuzzy event achieves 1, the fuzzy event will surely happen. Therefore, we adopt credibility as the measure of occurrence chance of a fuzzy event in this paper.

Definition 2.1 [11] Let ξ be a fuzzy variable with membership function μ and real number x. Then for any set A of R, the credibility of a fuzzy event $\xi \in A$ is defined as

$$\mathrm{Cr}\{\xi \in \mathrm{A}\} = \frac{1}{2}\left(\sup_{x \in A} \mu(x) + 1 - \sup_{x \in A^C} \mu(x)\right). \quad (1)$$

This above formula is also known as the credibility inversion theorem. Conversely, if ξ is a fuzzy variable, then its membership function is derived from the credibility measure by

$$\mu(x) = (2\mathrm{Cr}\{\xi = x\}) \wedge 1, \quad x \in R. \quad (2)$$

Definition 2.2 [11] Let ξ be a fuzzy variable; then, its expected value is defined by

$$E[\xi] = \int_0^{+\infty} \mathrm{Cr}\{\xi \geq x\}\mathrm{d}x - \int_{-\infty}^0 \mathrm{Cr}\{\xi \leq x\}\mathrm{d}x. \quad (3)$$

Provided that at least one of the above two integrals is finite.

Definition 2.3 [11] *Let* ξ be a continuous fuzzy variable; then, its entropy is defined by

$$H[\xi] = \int_{-\infty}^{+\infty} S(\mathrm{Cr}\{\xi = x\})\mathrm{d}x, \tag{4}$$

where $S(t) = -t\ln t - (1-t)\ln(1-t)$.

Fuzzy entropy is used to measure the uncertainty associated with each fuzzy variable. If ξ has continuous membership function μ, then we have $\mathrm{Cr}\{\xi = x\} = \frac{\mu(x)}{2}$ for any $x \in R$. In this case, it is easy to prove that its entropy is

$$H[\xi] = -\int_{-\infty}^{+\infty}\left(\frac{\mu(x)}{2}\ln\frac{\mu(x)}{2} + (1-\frac{\mu(x)}{2})\ln(1-\frac{\mu(x)}{2})\right)\mathrm{d}x. \tag{5}$$

Example 2.1 A triangular fuzzy variable ξ is fully determined by the triplet (a, b, c) of crisp numbers with $a<b<c$, and its membership function is given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & \text{if } a \le x \le b \\ \dfrac{x-c}{b-c}, & \text{if } b \le x \le c \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

By the formula $\mathrm{Cr}\{\xi \le x\} + \mathrm{Cr}\{\xi \ge x\} = 1$,, we have

$$\mathrm{Cr}\{\xi \le x\} = \begin{cases} 0, & \text{if } x \le a \\ \dfrac{x-a}{2(b-a)}, & \text{if } a \le x \le b \\ \dfrac{x+c-2b}{2(c-b)}, & \text{if } b \le x \le c \\ 1, & \text{if } x \ge c, \end{cases} \tag{7}$$

and

$$\mathrm{Cr}\{\xi \ge x\} = \begin{cases} 1, & \text{if } x \le a \\ \dfrac{2b-a-x}{2(b-a)}, & \text{if } a \le x \le b \\ \dfrac{c-x}{2(c-b)}, & \text{if } b \le x \le c \\ 0, & \text{if } x \ge c. \end{cases} \tag{8}$$

Example 2.2 The triangular fuzzy variable $\xi = (a, b, c)$ has an expected value

$$E[\xi] = \frac{a+2b+c}{4} \tag{9}$$

Example 2.3 Let ξ be a triangular fuzzy variable (a, b, c). Then its entropy is

$$H[\xi] = \frac{c-a}{2}. \tag{10}$$

Theorem 2.1 [11] *Let ξ and η be independent fuzzy variables with finite expected values. Then for any numbers a and b, we have*

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta]. \tag{11}$$

Theorem 2.2 [11] *Let ξ be an uncertain fuzzy variable, let c be a real number. Then*

$$H[\xi + c] = H[\xi]. \tag{12}$$

Theorem 2.3 [11] *Let ξ and η be independent uncertain fuzzy variables. Then for any real numbers a and b, we have*

$$H[a\xi + b\eta] = |a|H[\xi] + |b|H[\eta]. \tag{13}$$

In the portfolio selection problem, since uncertainty causes loss, we use entropy to assess the risk degree of a portfolio. In portfolio model, the smaller the entropy value is, the less uncertainty the portfolio return contains, and thus, the safer the portfolio is. So in this paper, the entropy of portfolio is regarded as the measurement of risk.

3 Fuzzy Mean-Entropy Model with Transaction Costs

Markowitz [1, 2] proposed the classical mean–variance portfolio model by maximizing investment return for a preset level of risk, or by minimizing investment risk for a preset level of investment return. In this paper, we will retain Markowitz's selection principle: using expected value as the measure of return, but use entropy as the measure of risk. The mean-entropy fuzzy portfolio model with transaction costs will be proposed.

For convenience, we introduce the following notations. Let x_i be the investment proportion of the ith asset $(i = 1, 2, \ldots, n)$, and let ξ_i be the ith fuzzy variable representing the return rate of the ith asset. Suppose that an investor invests his/her wealth among n risky assets with the investment proportion vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$. Assume that the transaction cost is a V-shaped function of differences between a new portfolio $\mathbf{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$ and the original portfolio $\mathbf{x}^0 = (x_1^0, x_2^0, \ldots, x_n^0)^{\mathrm{T}}$. In other words, the transaction cost c_i of risky asset can be expressed by $c_i(x_i) = k_i|x_i - x_i^0|$, where k_i is the constant rate of transaction cost for the risky asset. Therefore, the total transaction cost on portfolio $\mathbf{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$ is given by $c(\mathbf{x}) = \sum_{i=1}^n c_i(x_i) = \sum_{i=1}^n k_i|x_i - x_i^0|$. Depending on Markowitz's idea, the corresponding return of the portfolio after paying transaction costs is given by $r = \sum_{i=1}^n x_i\xi_i - \sum_{i=1}^n k_i|x_i - x_i^0|$. Thus, we can propose the maximizing return model with transaction costs for a preset level of risk as follows:

$$\begin{cases} \max E[r] = E\left[\sum_{i=1}^{n} x_i\xi_i - \sum_{i=1}^{n} k_i|x_i - x_i^0|\right] \\ \text{s.t.} H[r] = H\left[\sum_{i=1}^{n} x_i\xi_i - \sum_{i=1}^{n} k_i|x_i - x_i^0|\right] \le \alpha \\ \quad x_1 + x_2 + \cdots + x_n = 1 \\ \quad x_i \ge 0, \quad i = 1, 2, \ldots, n. \end{cases} \tag{14}$$

where α is the maximum entropy level the investors can tolerate, which means that the risk is not more than α and α is given by the investor according to his preference for risk.

The minimizing investment risk model with transaction costs for a preset level of return is:

$$\begin{cases} \min H[r] = \left[\sum_{i=1}^{n} x_i\xi_i - \sum_{i=1}^{n} k_i|x_i - x_i^0|\right] \\ \text{s.t.} E[r] = E\left[\sum_{i=1}^{n} x_i\xi_i - \sum_{i=1}^{n} k_i|x_i - x_i^0|\right] \ge \beta \\ \quad x_1 + x_2 + \cdots + x_n = 1 \\ \quad x_i \ge 0, \quad i = 1, 2, \ldots, n. \end{cases} \tag{15}$$

where β is the lowest return level which is satisfactory to the investor, which means that the return is not less than β and β is given by the investor according to his preference for return.

Now, we consider a portfolio with n kinds of asset, suppose fuzzy asset returns are triangular fuzzy variables $\xi_i = (a_i, b_i, c_i)$ with proportions x_i which are independent for $i = 1, 2, \ldots, n$. Then, by formulae (11) and (13), we have

$$\begin{aligned} E[r] &= E\left[\sum_{i=1}^{n} x_i\xi_i - \sum_{i=1}^{n} k_i|x_i - x_i^0|\right] \\ &= \sum_{i=1}^{n} x_i\left(\frac{a_i + 2b_i + c_i}{4}\right) - \sum_{i=1}^{n} k_i|x_i - x_i^0|. \end{aligned} \tag{16}$$

$$\begin{aligned} H[r] &= H\left[\sum_{i=1}^{n} x_i\xi_i - \sum_{i=1}^{n} k_i|x_i - x_i^0|\right] = \sum_{i=1}^{n} |x_i| H[\xi_i] \\ &= \sum_{i=1}^{n} x_i\left(\frac{c_i - a_i}{2}\right) \quad (x_i \ge 0). \end{aligned} \tag{17}$$

In formula (17), $x_i \ge 0$ means that the portfolio is without short selling. So, when fuzzy variables are triangular, models (14) and (15) can be written as:

$$\begin{cases} \max E[r] = \sum_{i=1}^{n} x_i\left(\frac{a_i + 2b_i + c_i}{4}\right) - \sum_{i=1}^{n} k_i|x_i - x_i^0| \\ \text{s.t.} H[r] = \sum_{i=1}^{n} x_i\left(\frac{c_i - a_i}{2}\right) \le \alpha \\ \quad x_1 + x_2 + \cdots + x_n = 1 \\ \quad x_i \ge 0, \quad i = 1, 2, \ldots, n. \end{cases} \tag{18}$$

and

$$\begin{cases} \min H[r] = \sum_{i=1}^{n} x_i\left(\frac{c_i - a_i}{2}\right) \\ \text{s.t.} E[r] = \sum_{i=1}^{n} x_i\left(\frac{a_i + 2b_i + c_i}{4}\right) - \sum_{i=1}^{n} k_i|x_i - x_i^0| \ge \beta \\ \quad x_1 + x_2 + \cdots + x_n = 1 \\ \quad x_i \ge 0, \quad i = 1, 2, \ldots, n. \end{cases} \tag{19}$$

4 Numerical Examples and Sensitivity Analysis

In order to demonstrate the effectiveness of our proposed expected value and entropy, we consider a real portfolio example. In this example, ten stocks are selected from Shanghai Stock Exchange. Their returns $\xi_i = (a_i, b_i, c_i)$ are regarded as triangular fuzzy numbers, and the transaction rate of risky asset $k_i \equiv 0.005\,(i = 1, 2, \ldots, 10)$. Based on the historical data, the future information, the experts' opinions and the corporations' financial reports, we obtain the following data:

4.1 Sensitivity Analysis of Maximizing Return Portfolio Model

According to the data of Table 1, we can obtain the following maximizing return of linear portfolio model with $\mathbf{x} = (x_1, x_2, \ldots, x_{10})^{\mathrm{T}}$, $k_i = 0.005$ and $\alpha = 1.875$:

$$\begin{cases} \max r = 2.095x_1 + 1.570x_2 + 2.445x_3 + 1.595x_4 + 1.770x_5 + \\ \qquad 2.12x_6 + 1.995x_7 + 2.745x_8 + 1.045x_9 + 1.945x_{10}, \\ \text{s.t.} H = 1.9x_1 + 1.35x_2 + 2.1x_3 + 1.7x_4 + 1.95x_5 + 1.85x_6 + \\ \qquad 1.9x_7 + 2.3x_8 + 1.7x_9 + 2.0x_{10} \le 1.875, \\ \quad x_1 + x_2 + \cdots + x_{10} = 1, \\ \quad x_i \ge 0, i = 1, 2, \cdots, 10. \end{cases} \tag{20}$$

As to model (20), when certain objective function coefficient or constraint on right-hand side changes, then

Table 1 Fuzzy returns of 10 securities and corresponding expected values and entropies (units per stock)

Security i	a_i	b_i	c_i	$E[\xi_i] = \frac{a_i+2b_i+c_i}{4}$	$H[\xi_i] = 0.5(c_i - a_i)$
1	−0.4	2.7	3.4	2.100	1.90
2	−0.1	1.9	2.6	1.575	1.35
3	−0.2	3.0	4.0	2.450	2.10
4	−0.5	2.0	2.9	1.600	1.70
5	−0.6	2.2	3.3	1.775	1.95
6	−0.1	2.5	3.6	2.125	1.85
7	−0.3	2.4	3.5	2.000	1.90
8	−0.1	3.3	4.5	2.750	2.30
9	−0.7	1.1	2.7	1.050	1.70
10	−0.2	2.1	3.8	1.950	2.00

we focus more on basis and optimal solution and the objective value how to change. Next, we will discuss the above questions. Namely, we will do deep sensitivity analysis of model (20).

4.1.1 Sensitivity Analysis of Objective Function Coefficient in Maximizing Return Model

Firstly, solving model (20), we can obtain the following optimal solution:

$$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^{\mathrm{T}},\ r^* = 2.2193,\ H^* = 1.875. \tag{21}$$

Obviously, the basic variables of model (20) are x_2 and x_8, and others are non-basic variables. Furthermore, we do sensitivity analysis to the objective function coefficients by LINGO software. The corresponding sensitivity analysis results show that as to coefficient $c_1 = 2.095$, when the basis is unchangeable, allowable increase amount is 0.1553, allowable decrease amount is infinity, so the coefficient range $c_1 \in [0, 2.2503]$. At this time, the basis is unchangeable, the constraint conditions are unchangeable so optimal solution is also unchangeable. In fact, the objective function coefficient has some change, and it does affect objective function value. In general, there are the following two cases in the objective function coefficient sensitivity analysis: basic variable coefficient and non-basic variable coefficient.

Case1 If the basic variable's coefficient changes in the corresponding range, the optimal basis and optimal solution will be stable. But the value of objective function will change since the values of basic variable are nonzero such as x_2 and x_8

Case2 If the non-basic variable's coefficient changes in the corresponding range, the optimal basis and optimal solution will remain unchanged. Meanwhile, the objective function value is also unchanged since the value of non-basic variables are zeros such as $x_1, x_3, x_4, x_5, x_6, x_7, x_9, x_{10}$ and zero multiplied by any number is still zero

According to the two cases, we can obtain the following data in Table 2:

In order to illustrate the significance of the sensitivity analysis, next we discuss concretely the different values of $c_1 \in [0, 2.2503]$ and $c_2 \in [1.4256, 2.7450]$ in models (22) and (23).

$$\begin{cases} \max r = c_1x_1 + 1.57x_2 + 2.445x_3 + 1.595x_4 + 1.77x_5 + \\ \qquad 2.12x_6 + 1.995x_7 + 2.745x_8 + 1.045x_9 + 1.945x_{10}, \\ \text{s.t.} H = 1.9x_1 + 1.35x_2 + 2.1x_3 + 1.7x_4 + 1.95x_5 + \\ \qquad 1.85x_6 + 1.9x_7 + 2.3x_8 + 1.7x_9 + 2.0x_{10} \le 1.875, \\ \quad x_1 + x_2 + \cdots + x_n = 1, \\ \quad x_i \ge 0, \quad i = 1, 2, \ldots, 10, \end{cases} \tag{22}$$

and

$$\begin{cases} \max r = 2.095x_1 + c_2x_2 + 2.445x_3 + 1.595x_4 + 1.77x_5 + \\ \qquad 2.12x_6 + 1.995x_7 + 2.745x_8 + 1.045x_9 + 1.945x_{10}, \\ s.t. H = 1.9x_1 + 1.35x_2 + 2.1x_3 + 1.7x_4 + 1.95x_5 + \\ \qquad 1.85x_6 + 1.9x_7 + 2.3x_8 + 1.7x_9 + 2.0x_{10} \le 1.875, \\ \quad x_1 + x_2 + \cdots + x_{10} = 1, \\ \quad x_i \ge 0, \quad i = 1, 2, \ldots, 10. \end{cases} \tag{23}$$

As to models (22) and (23), we can obtain the following data when $c_1 \in [0, 2.2503]$ and $c_2 \in [1.4256, 2.7450]$ in Tables 3 and 4.

Table 3 shows that when c_1 changes in the closed interval of $[0, 2.2503]$, optimal basis and optimal solution and objective function value are unchangeable.

Table 4 shows that when c_2 changes in the closed interval of $[1.4256, 2.7450]$, optimal basis and optimal solution are unchangeable, but the objective function value is changeable. Furthermore, we can draw Fig. 1 about the value of c_2 and the value of objective function:

In model (23), since x_2 is a basic variable, when the coefficient c_2 changes in the sensitivity analysis range, the basis and solution are unchangeable, but objective function value will change. From Fig. 1, we can see that when the coefficient c_2 increases, the corresponding objective function value will increase too.

4.1.2 Sensitivity Analysis of Constraint on Right-Hand Sides in Maximizing Return Model

Next, we will do sensitivity analysis of constraint on right-hand sides by LINGO software in model (24):

$$\begin{cases} \max r = 2.095x_1 + 1.57x_2 + 2.445x_3 + 1.595x_4 + 1.77x_5 + \\ \qquad 2.12x_6 + 1.995x_7 + 2.745x_8 + 1.045x_9 + 1.945x_{10} \\ \text{s.t.} H = 1.9x_1 + 1.35x_2 + 2.1x_3 + 1.7x_4 + 1.95x_5 + \\ \qquad 1.85x_6 + 1.9x_7 + 2.3x_8 + 1.7x_9 + 2.0x_{10} \le \alpha, \\ \quad x_1 + x_2 + \cdots + x_{10} = 1, \\ \quad x_i \ge 0, i = 1, 2, \ldots, 10. \end{cases} \tag{24}$$

Table 2 Change descriptions when objective coefficients change in some range compared with the optimal solution (21) in model (20)

Objective variable	Objective coefficient	Objective coefficient ranges	Basis	Optimal solution	Objective value
x_1	2.0950	$[0, 2.2503]$	Unchanged	Unchanged	Unchanged
x_2	1.5700	$[1.4256, 2.7450]$	Unchanged	Unchanged	Changed
x_3	2.4450	$[0, 2.4976]$	Unchanged	Unchanged	Unchanged
x_4	1.5950	$[0, 2.0029]$	Unchanged	Unchanged	Unchanged
x_5	1.7700	$[0, 2.3121]$	Unchanged	Unchanged	Unchanged
x_6	2.1200	$[0, 2.1884]$	Unchanged	Unchanged	Unchanged
x_7	1.9950	$[0, 2.2503]$	Unchanged	Unchanged	Unchanged
x_8	2.7450	$[2.6783, +\infty]$	Unchanged	Unchanged	Changed
x_9	1.0450	$[0, 2.0029]$	Unchanged	Unchanged	Unchanged
x_{10}	1.9450	$[0, 2.3739]$	Unchanged	Unchanged	Unchanged

Table 3 Same solutions and the same objective values as to different values in $c_1 \in [0, 2.2503]$ in model (22)

c_1	$\mathbf{x}^* = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10})^T$	r^*	Basis, solution, objective value
0	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.2193	Unchanged
0.10	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.2193	Unchanged
0.30	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.2193	Unchanged
0.60	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.2193	Unchanged
1.00	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.2193	Unchanged
1.30	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.2193	Unchanged
1.70	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.2193	Unchanged
2.10	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.2193	Unchanged
2.2503	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.2193	Unchanged

Table 4 Same solutions and different objective values as to different values in $c_2 \in [1.4256, 2.7450]$ in model (23)

c_2	$\mathbf{x}^* = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10})^T$	r^*	Basis, solution	Objective value
1.4256	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.1547	Unchanged	Changed
1.6550	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.2574	Unchanged	Changed
1.8550	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.3468	Unchanged	Changed
2.0550	$x^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.4363	Unchanged	Changed
2.2350	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.5168	Unchanged	Changed
2.4550	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.6153	Unchanged	Changed
2.6550	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.7047	Unchanged	Changed
2.7400	$\mathbf{x}^* = (0, 0.4474, 0, 0, 0, 0, 0, 0.5526, 0, 0)^T$	2.7428	Unchanged	Changed

The current right-hand side of model (24) is $\alpha = 1.8750$. By LINGO sensitivity analysis, when the basis is unchangeable, then the allowable increase amount is 0.4250, allowable decrease amount is 0.5250, so the right-hand side range: $\alpha \in [1.8750 - 0.5250, 1.8750 + 0.4250] = [1.3500, 2.300]$. Although the optimal basis is unchangeable, at this time the corresponding constraint conditions change, so the corresponding optimal solution and objective function value will change as well. In the similar way, we do sensitivity analysis of the current right-hand side in the constraint of $x_1 + x_2 + \cdots + x_n = 1$ by LINGO. When the basis is unchanged, the allowable increase amount is 0.3889, allowable decrease amount is 0.1848, so the right-hand side range:$[1 - 0.1848, 1 + 0.3889] = [0.8152, 1.3889]$. As to model (24), we can obtain the following data for $\alpha \in [1.3500, 2.300]$ in Table 5.

Table 5 shows that when α changes in the closed interval of $[1.3500, 2.300]$, optimal basis are unchangeable, but optimal solution and objective function value are changeable. If $\alpha < 1.35$, no feasible solution can be found; if $\alpha < 2.30$, the

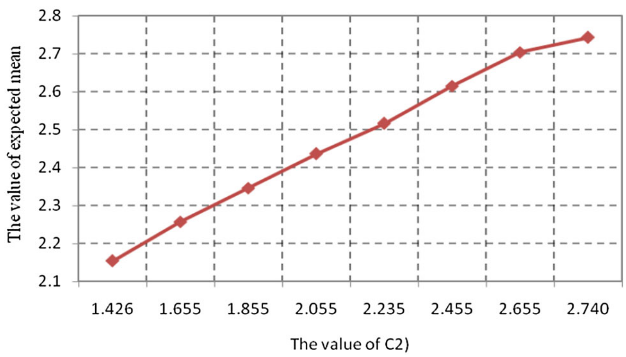

Fig. 1 Relationship of the values of c_2 and the objective function values in model (23)

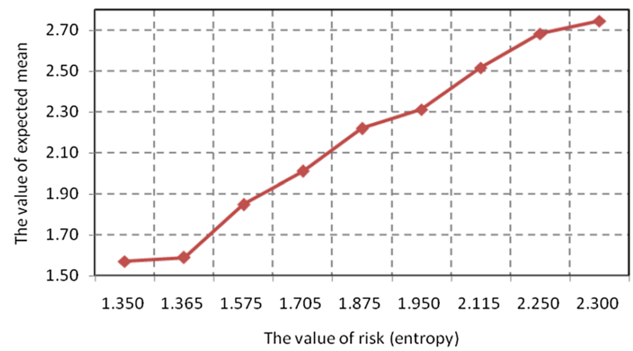

Fig. 2 Effective frontier with different values of α in model (24)

optimal solution is always $\mathbf{x}^* = (0,0,0,0,0,0,0,1,0,0)^{\mathrm{T}}$. According to the data of Table 5, we can draw the effective frontier of model (24):

The effective frontier of Fig. 2 is approximately a straight line, which means that the higher risk (entropy) we bear, the higher return we obtain.

4.2 Sensitivity Analysis of Minimizing Risk Portfolio Model

According to the data of Table 1, on the other hand, by using model (19), we can obtain the following minimizing risk linear portfolio model (25) based on possibility theory with $\mathbf{x} = (x_1, x_2, \ldots, x_{10})^{\mathrm{T}}$ and assume $\beta = 1.9325$:

$$\begin{cases} \min H = 1.9x_1 + 1.35x_2 + 2.1x_3 + 1.7x_4 + 1.95x_5 + \\ \qquad 1.85x_6 + 1.9x_7 + 2.3x_8 + 1.7x_9 + 2.0x_{10}, \\ \text{s.t.} r = 2.095x_1 + 1.57x_2 + 2.445x_3 + 1.595x_4 + 1.77x_5 + \\ \qquad 2.12x_6 + 1.995x_7 + 2.745x_8 + 1.045x_9 + 1.945x_{10} \geq 1.9325, \\ \quad x_1 + x_2 + \cdots + x_{10} = 1, \\ \quad x_i \geq 0, i = 1, 2, \ldots, 10. \end{cases} \tag{25}$$

As to the minimizing risk model (25), we can do a similar discussion work on sensitivity analyses to objective function coefficients. Thus, we still do similar research work as the one in Sect. 4.1: solving the model and working on two kinds of sensitivity analysis.

4.2.1 Sensitivity Analysis of Objective Function Coefficient in Minimizing Risk Model

First, the optimal solution of (25) by taking $\beta = 1.9325$ is:

$$\mathbf{x}^* = (0, 0.6915, 0, 0, 0, 0, 0, 0.3085, 0, 0)^{\mathrm{T}}, \ H^* = 1.6431, \ r^* = 1.9325. \tag{26}$$

Obviously, we find that the basic variables still are x_2 and x_8, and others are non-basic variables. By LINGO software, under the condition that these basis are unchanged, we do sensitivity analysis of x_1 coefficient of objective function: The current coefficient is 1.9000, allowable increase amount is infinity, allowable decrease amount is 0.1255, so the coefficient range is: $[1.900 - 0.1255, 1.900 + \infty] = [1.7745, +\infty]$. In the corresponding range, the basis is unchanged, at the same time the constraint conditions are

Table 5 Different solutions as to different values in $\alpha \in [1.3500, 2.300]$ in model (24)

α	$\mathbf{x}^* = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10})^{\mathrm{T}}$	r^*	H^*	Basis	Solution, objective value
<1.35	No feasible solution found				
1.350	$\mathbf{x}^* = (0,1,0,0,0,0,0,0,0,0)^{\mathrm{T}}$	1.5700	1.350	Unchanged	Changed
1.365	$\mathbf{x}^* = (0,0.9842,0,0,0,0,0,0,0,0.0158)^{\mathrm{T}}$	1.5886	1.365	Unchanged	Changed
1.575	$\mathbf{x}^* = (0,0.7632,0,0,0,0,0,0.2368,0,0)^{\mathrm{T}}$	1.8483	1.575	Unchanged	Changed
1.705	$\mathbf{x}^* = (0,0.6263,0,0,0,0,0,0.3737,0,0)^{\mathrm{T}}$	2.0091	1.705	Unchanged	Changed
1.875	$\mathbf{x}^* = (0,0.4474,0,0,0,0,0,0.5526,0,0)^{\mathrm{T}}$	2.2193	1.875	Unchanged	Changed
1.950	$\mathbf{x}^* = (0,0.3684,0,0,0,0,0,0.6316,0,0)^{\mathrm{T}}$	2.3121	1.950	Unchanged	Changed
2.115	$\mathbf{x}^* = (0,0.1947,0,0,0,0,0,0.8053,0,0)^{\mathrm{T}}$	2.5162	2.115	Unchanged	Changed
2.250	$\mathbf{x}^* = (0,0.0526,0,0,0,0,0,0.9474,0,0)^{\mathrm{T}}$	2.6832	2.250	Unchanged	Changed
2.300	$\mathbf{x}^* = (0,0,0,0,0,0,0,1,0,0)^{\mathrm{T}}$	2.7450	2.300	Unchanged	Changed
>2.30	$\mathbf{x}^* = (0,0,0,0,0,0,0,1,0,0)^{\mathrm{T}}$	2.7450	2.300	Unchanged	Changed

Table 6 Ranges in which the basis is unchanged in model (25)

Objective variable	Objective coefficient	Objective coefficient ranges	Basis, solution	Objective value
x_1	1.9000	$[1.7745, +\infty]$	Unchanged	Unchanged
x_2	1.3500	$[0, 1.4540]$	Unchanged	Changed
x_3	2.1000	$[2.0574, +\infty]$	Unchanged	Unchanged
x_4	1.7000	$[1.3702, +\infty]$	Unchanged	Unchanged
x_5	1.9500	$[1.5117, +\infty]$	Unchanged	Unchanged
x_6	1.8500	$[1.7947, +\infty]$	Unchanged	Unchanged
x_7	1.9000	$[1.6936, +\infty]$	Unchanged	Unchanged
x_8	2.3000	$[1.3500, 2.3571]$	Unchanged	Changed
x_9	1.7000	$[0.9255, +\infty]$	Unchanged	Unchanged
x_{10}	2.0000	$[1.6532, +\infty]$	Unchanged	Unchanged

unchanged (only some objective function coefficient has some change), so unchanged optimal basis means unchanged optimal solution (since some objective function coefficient has changed, the corresponding objective value will change). There are two following cases in the objective function coefficient sensitivity analysis.

Case 1 If the basic variable's coefficient changes in the corresponding range, the optimal basis and optimal solution will be unchangeable. But the value of objective function will change since the values of basic variable are nonzero as to x_2 and x_8

Case 2 If the non-basic variable's coefficient changes in the corresponding range, then the optimal basis and optimal solution will be stable. Furthermore, the objective function value is also unchanged since the values of non-basic variable are zeros such as $x_1, x_3, x_4, x_5, x_6, x_7, x_9, x_{10}$ and zero multiplied by any number is still zero

Depending on the ideas of cases 1 and 2, by LINGO, we can obtain the following value ranges of model (25) in which the basis is unchanged (see Table 6).

4.2.2 Sensitivity Analysis of Constraint on Right-Hand Sides in Minimizing Risk Model

Next, we will discuss the sensitivity analysis by LINGO to the first right-hand side of the following model (27):

$$\begin{cases} \min H = 1.9x_1 + 1.35x_2 + 2.1x_3 + 1.7x_4 + 1.95x_5 + \\ \quad 1.85x_6 + 1.9x_7 + 2.3x_8 + 1.7x_9 + 2.0x_{10}, \\ \text{s.t.} r = 2.095x_1 + 1.57x_2 + 2.445x_3 + 1.595x_4 + 1.77x_5 + \\ \quad 2.12x_6 + 1.995x_7 + 2.745x_8 + 1.045x_9 + 1.945x_{10} \geq \beta, \\ \quad x_1 + x_2 + \cdots + x_{10} = 1, \\ \quad x_i \geq 0, i = 1, 2, \ldots, 10. \end{cases} \tag{27}$$

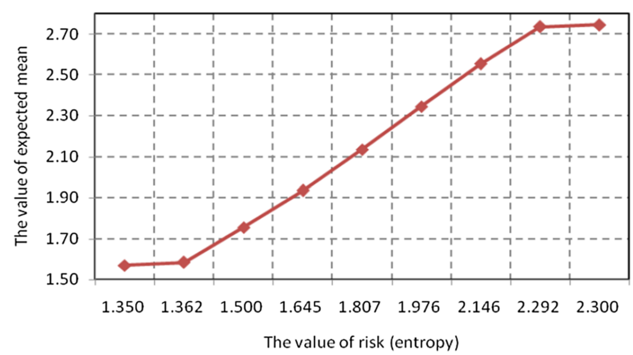

Fig. 3 Effective frontier with different values of β in model (27)

The current right-hand side is $\beta = 1.9325$. We do sensitivity analysis to β by LINGO software, under the condition that the basis is unchanged, the allowable increase amount is 0.8125, allowable decrease amount is 0.3625, then right-hand side range: $[1.9325 - 0.3625, 1.9325 + 0.8125] = [1.5700, 2.7450]$. Although the optimal basis are unchanged, since at this time the corresponding constraint conditions change, then the corresponding optimal solution and optimal objective function value will change in model (27) (see Table 7).According to the data in Table 7, we can draw the effective frontier of model (27) as to different values of β.

From Fig. 3, we can find that the frontier of risk (entropy) and return in the portfolio is almost a straight line, namely, when risk increases, return increases too. In the same way, we can analyze the constraint: $x_1 + x_2 + \cdots + x_n = 1$. The current right-hand side is 1, under the condition the basis is unchanged, the allowable increase amount is 0.2309, allowable decrease amount is 0.2960, then right-hand side range: $[1 - 0.2960, 1 + 0.2309] = [0.7040, 1.2309]$.

Table 7 Different solutions as to different values in $\beta \in [1.5700, 2.7450]$ in model (27)

β	$\mathbf{x}^* = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10})^T$	r^*	H^*	Basis	Solution, objective value
<1.57	$\mathbf{x}^* = (0, 1, 0, 0, 0, 0, 0, 0, 0, 0)^T$	1.5700	1.3500	Unchanged	Changed
1.5700	$\mathbf{x}^* = (0, 1, 0, 0, 0, 0, 0, 0, 0, 0)^T$	1.5700	1.3500	Unchanged	Changed
1.5850	$\mathbf{x}^* = (0, 0.9872, 0, 0, 0, 0, 0, 0.0128, 0, 0)^T$	1.5850	1.3621	Unchanged	Changed
1.7550	$\mathbf{x}^* = (0, 0.8426, 0, 0, 0, 0, 0, 0.1574, 0, 0)^T$	1.7550	1.4996	Unchanged	Changed
1.9350	$\mathbf{x}^* = (0, 0.6894, 0, 0, 0, 0, 0, 0.3106, 0, 0)^T$	1.9350	1.6451	Unchanged	Changed
2.1350	$\mathbf{x}^* = (0, 0.5191, 0, 0, 0, 0, 0, 0.4809, 0, 0)^T$	2.1350	1.8068	Unchanged	Changed
2.3450	$\mathbf{x}^* = (0, 0.3404, 0, 0, 0, 0, 0, 0.6596, 0, 0)^T$	2.3450	1.976	Unchanged	Changed
2.5550	$\mathbf{x}^* = (0, 0.1617, 0, 0, 0, 0, 0, 0.8383, 0, 0)^T$	2.5550	2.1464	Unchanged	Changed
2.7350	$\mathbf{x}^* = (0, 0.0085, 0, 0, 0, 0, 0, 0.9915, 0, 0)^T$	2.7350	2.2919	Unchanged	Changed
2.7450	$\mathbf{x}^* = (0, 0, 0, 0, 0, 0, 0, 1, 0, 0)^T$	2.7450	2.3000	Unchanged	Changed
>2.75	No feasible solution found				

5 Conclusions

The fuzzy mean-entropy models with transaction costs based on credibility theory are considered in our paper. Further, we investigate in depth the sensitivity analysis about the objective coefficients and constraint right coefficients. The data and table of the numerical examples indicate that our proposed model is effective and our sensitivity analysis is useful. It is worth mentioning that the results of sensitivity analysis show that when the corresponding objective coefficients and constraint coefficients change in some value range, the corresponding basis and optimal solution and the objective function values how to change. In particular, the results of two numerical examples show that if return rate of some kind of risky asset changes in some value range and other conditions do not change, then the investor still can obtain the same optimal objective function values, namely the same expected return.

Huang [18] proposed mean-entropy models and presented a hybrid intelligent algorithm for models. Huang [19] introduced a risk curve and developed a mean-risk model. Our paper not only proposes the fuzzy mean-entropy models with transaction costs based on credibility theory, but also does deeper research work of sensitivity analysis about both maximizing return model and minimizing risk model. Compared with Huang [18, 19], our results can provide more information about optimum invest strategies, and our paper can give investors more invest choices. The sensitivity analysis results provide theory guidance with investors according to his/her preference for return and risk in the practical financial market.

6 Scope for Future Study

In the near future, several parameters of objective functions and constraint conditions change simultaneously. It attracts more and more scholars to do research on how the interaction and mutual restriction between parameters affect the optimal solution, the investment return and risk. These results will provide more choices for the investment decision-makers and have guiding significance in the practical portfolio research.

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