

# Adaptive Fuzzy Control for Nonlinear Pure-feedback Systems with External Disturbance and Unknown Dead Zone Output

Zhikang Lin<sup>1</sup> · Xikui Liu<sup>1</sup> · Yan Li<sup>2</sup>

Received: 6 May 2016/Revised: 30 September 2016/Accepted: 3 November 2016/Published online: 22 November 2016  
© Taiwan Fuzzy Systems Association and Springer-Verlag Berlin Heidelberg 2016

**Abstract** By using the adaptive backstepping technique, a novel adaptive fuzzy backstepping control scheme is proposed for the nonlinear pure-feedback systems with external disturbance and unknown dead zone output in this paper. The proposed control scheme not only guarantees that all the signals in the closed-loop system are semi-globally bounded, but also makes the tracking error converge to a small neighborhood of the origin by suitable choice of design parameters. Fuzzy logic systems are utilized to approximate the unknown nonlinear functions. The primary characteristic of this thesis is that the unknown dead zone output nonlinearity and external disturbance of pure-feedback systems is introduced. Finally, an instance is used to prove the superiority of scheme.

**Keywords** Dead zone output · Pure-feedback · Young's inequality · Adaptive fuzzy backstepping control

## 1 Introduction

By applying the universal approximation property [1–4], amounts of adaptive fuzzy control schemes have been investigated and significant advances have been obtained in nonlinear systems [5–12]. In the past few decades, the combination of backstepping technique and fuzzy logic system for adaptive control of strict-feedback nonlinear systems got rapid development and research [8–13]. There is no doubt the controllers ensure all the signals which are bounded in [1–14]. However, all the aforementioned works do neglect the existence of nonsmooth nonlinearities, for instance, dead zone and backlash [14] which generally exist in a lot of practical systems.

As is well known, dead zone in many components of control systems is the crucial non-smooth nonlinearities in lots of industrial projects which have gravely deteriorated the system performance, on account of the characteristics of dead zone nonlinearity in actuators are poorly known. A typical dead zone example is dry friction in electromechanical systems [15]. Some scholars directly adopt the most straightforward approach to deal with dead zone nonlinearities by utilizing their inverses based on [16, 17]. Although the above method reduces difficult coefficient of the tracking deviation significantly, the model of dead zone is a simplification for physical properties.

To cope with discrete-time plants with unknown dead zone output, some scholars introduce a novel structure of controller. By [18] the new adaptive control schemes keep the closed-loop signal bounded even if the slopes of the dead zone are unequal. Recently, an adaptive fuzzy backstepping controller which does not construct the dead zone inverse is applied to solve the problem of the nonlinear systems with dead zone and dynamics, see [18, 19]. A novel smooth inverse model was proposed which

---

This work is supported by NSF of China (61170054, 61402265).

---

✉ Xikui Liu  
liuxikuiqd@163.com  
Zhikang Lin  
qdlinzhikang@163.com  
Yan Li  
liyanhd@163.com

<sup>1</sup> College of Mathematics and System Science, Shandong University of Science and Technology, Qingdao 266590, Shandong, People's Republic of China

<sup>2</sup> College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, Shandong, People's Republic of China

compensates the impact of dead zone in controller design instead of applying traditional nonsmooth input models to describe output dead zone in [20, 21]. The large-scale nonlinear system and unified stochastic nonlinear interconnected systems have been discussed in depth by some researchers in [22, 23]. Nonlinear systems with dead zone input have been attracting a lot of attention in the adaptive fuzzy control community since few decades ago, numerous professors have dedicated endeavor to enhance the property of nonlinear systems with dead zone at the input in [24–28]. However, there are few research on the output nonlinearity. The recursive least squares (RLS) algorithm can be used to avoid constructing dead zone model in sensors, see [29].

By applying a Nussbaum function and an input-driven filter, the nonlinearity in the output mechanism is resolved in [30]. However, the schemes are not suitable for more universal systems. In addition, a novel smooth model is introduced to deal with dead zone in [31]. A number of gratifying results have been obtained to the above systems by applying the backstepping technique and the fuzzy logic systems (FLS); however, a few schemes are usable for working out pure-feedback control of nonlinear systems which represents a class of more ordinary systems [32]. It is shown that the mentioned pure-feedback control of nonlinear systems has no affine appearance of variables [33]. Other related research literatures on adaptive fuzzy output-feedback control, see [34–37].

In this paper, the tracking control issue of pure-feedback nonlinear systems with external disturbance and unknown dead zone output is researched. We will think about the adaptive control for nonlinear pure-feedback systems with uncertain nonlinearity and unknown dead zone output, which is a challenging and significant work. In this research, we have the following assumption. The virtual control signal and the actual control must be independent of the variables  $x_i$  in order to guarantee the controllers gainable. The primary contributions of this paper are that:

(1) The tracking control problem of pure-feedback nonlinear systems with external disturbance and unknown dead zone output is investigated, firstly. On account of the mean value theorem, the non-affine problem is solved.

(2) It is worth noticing that the pure-feedback systems synchronously take the external disturbance and unknown dead zone output into account, comparing with the now available results on pure-feedback issue. In addition, the adaptive fuzzy controller ensures all the signals are bounded in the closed-loop system and the system output converges to a small neighborhood of the initial signal.

The preparatory work is introduced, firstly. A new adaptive fuzzy control design is proposed in Sect. 3. In Sect. 4, the simulation examples are provided to prove the superiority of the scheme.

## 2 Problem Formulation and Preliminaries

The nonlinear pure-feedback system is given as:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i, x_{i+1}) + \psi_i(t), & i = 1, 2, \dots, n - 1 \\ \dot{x}_n = f_n(\bar{x}_n, u) + \psi_n(t), \\ y = \chi(x_1) \end{cases} \quad (2.1)$$

where  $\bar{x}_i = [x_1, x_2, \dots, x_i] \in R^i$  is called the state variable, and  $f_i(\cdot)$  are unknown smooth functions,  $\psi_i(t)$  is the unknown external disturbance,  $u \in R$  is called the input of the system,  $y \in R$  is called the output of the system which is the directly measurable variable. It is assumed that  $x_1(t)$  are immeasurable, but  $x_2(t), \dots, x_n(t)$  are measurable variables.

The unknown dead zone output  $\chi(x_1)$  is considered as the following form:

$$y = \chi(x_1) = \begin{cases} \pi_r(x_1), & x_1 \geq h_r \\ 0, & h_r \leq x_1 \leq h_l \\ \pi_l(x_1), & x_1 \leq h_l \end{cases} \quad (2.2)$$

where the unknown parameters  $h_r < 0$  and  $h_l > 0$  and  $\pi_r(x_1)$  and  $\pi_l(x_1)$  are unknown nonlinear terms.

*Remark 1* Literatures [29, 31–35] discussed the issue of adaptive control of nonlinear non-affine pure-feedback systems, which is more general than the previous systems.

Applying the mean value theorem (see [35]), the non-affine functions in (2.1) is rewritten as

$$f_i(\bar{x}_i, x_{i+1}) - f_i(\bar{x}_i, x_{i+1}^0) = v_i(\bar{x}_i, \varsigma_i)(x_{i+1} - x_{i+1}^0), \quad i = 1, 2, \dots, n \quad (2.3)$$

where  $(x_1^0, x_2^0, \dots, x_n^0, u^0)^T$  is an operating point of interest,  $x_{n+1} = u$ ,  $x_{n+1}^0 = u^0$  and  $\varsigma_i$  is some point between  $x_{i+1}^0$  and  $x_{i+1}$ . Substituting (2.3) into (2.1), the system dynamics have the following form:

$$\begin{cases} \dot{x}_i = v_i(\bar{x}_i, \varsigma_i)(x_{i+1} - x_{i+1}^0) + f_i(\bar{x}_i, x_{i+1}^0) + \psi_i(t), & i = 1, 2, \dots, n - 1 \\ \dot{x}_n = v_n(\bar{x}_n, \varsigma_n)(u - u^0) + f_n(\bar{x}_n, u^0) + \psi_n(t), \\ y = \chi(x_1) \end{cases} \quad (2.4)$$

A fuzzy logic system is applied to approximate a continuous function  $f(x)$  on some compact set  $\Omega$  (see[20]). Adopt the singleton fuzzifier, the product inference, and the center-average defuzzifier to conclude the following fuzzy rules:

$R^i$ : If  $x_1$  is  $F_1^i$  and ... and  $x_n$  is  $F_n^i$ . Then  $y$  is  $G^i$ ,  $i = 1, 2, \dots, N$ .

$\bar{x}_n = [x_1, x_2, \dots, x_n] \in R^n$  is the input of the fuzzy system and  $y \in R$  is the fuzzy system output,  $F_j^i$  and  $G^i$  are the fuzzy sets in  $R$ .  $N$  is the amount of the rules. By the above

discussion, it is not hard to obtain the output of the fuzzy system:

$$y(x) = \frac{\sum_{i=1}^N \Phi_i \prod_{j=1}^n \mu F_j^i(x_j)}{\sum_{i=1}^N \left[ \prod_{j=1}^n \mu F_j^i(x_j) \right]}$$

where

$$\Phi_i = \max_{y \in R} \mu G^i(y), \quad \Phi = (\Phi_1, \Phi_2, \dots, \Phi_N)^T.$$

Let

$$G_i(x) = \frac{\prod_{j=1}^n \mu F_j^i(x_j)}{\sum_{i=1}^N \left[ \prod_{j=1}^n \mu F_j^i(x_j) \right]},$$

$$G(x) = (G_1(x), G_2(x), \dots, G_N(x))^T,$$

then

$$y(x) = \Phi^T G(x).$$

For the sake of proofing the main results, some lemmas and assumptions will be introduced.

**Lemma 1** (see[5]): By applying FLS theory, we introduce a continuous function  $f(x)$  on a compact set  $\Omega$ .

$$\sup_{x \in \Omega} |f(x) - \Phi^T P(x)| \leq \varepsilon, \quad \forall \varepsilon > 0.$$

**Lemma 2** (see[29]): We can find a smooth function  $\Gamma(\cdot) > 0$  and an unknown term  $\Gamma_0 \in R$  such that

$$|f_1(x_1, x_2^0)| \leq \Gamma(|y|) + \Gamma_0.$$

*Remark 2* Due to  $x_1$  is not accessible for measurement, how to build the relationship between the unknown function  $f_1(x_1, x_2^0)$  and the output  $y$  is a crux to construct a backstepping design method for (2.1). Because of Lemma 2, the difficulty of  $x_1$  unavailable is overcome. As a result, the following intermediate signal  $\alpha_i$  and the actual control are independent of the variable  $x_1$ .

**Assumption 1** (see[8]): The reference signal  $y_0$  has  $n$ th-order time derivatives which are continuous and bounded.

**Assumption 2** (see[34]): The sign of  $v_i(\bar{x}_i, x_{i+1})$  does not change.  $c_m$  and  $c_M$  are unknown constants such that

$$0 < c_m < |v_i(\bar{x}_i, x_{i+1})| < c_M < \infty,$$

$$\forall (\bar{x}_i, x_{i+1}) \in R^i \times R, \quad i = 1, \dots, n,$$

according to the needs of this thesis, it is further assumed that the signs  $v_i(\bar{x}_i, x_{i+1}) \geq c_m$ . In addition, the constants  $c_m$  and  $c_M$  are unknown.

**Assumption 3** (see[5]): It is assumed that there are known parameters  $\bar{p}_i (i = 1, 2, \dots, n)$ , satisfying

$$|\psi_i(t)| \leq \bar{p}_i.$$

**Assumption 4** (see[32]): It is assumed that there are unknown positive constants  $b_M$  such that

$$|\chi'(x_1)| = \left| \frac{d\chi(z)}{dz} \right|_{z=x_1} \leq b_M, \quad \forall x_1 \in R.$$

For simplicity of our discussion, we give the following definitions.

**Definition 1** (see[30]):  $H(\varphi)$  is a Nussbaum-type function such that:

$$\lim_{a \rightarrow +\infty} \sup \frac{1}{a} \int_0^a H(\varphi) d\varphi = +\infty,$$

$$\lim_{a \rightarrow -\infty} \inf \frac{1}{a} \int_0^a H(\varphi) d\varphi = -\infty.$$

We can list the functions which meet the definition, such as  $\varphi^2 \cos(\varphi)$ ,  $\varphi^2 \sin(\varphi)$ ,  $\exp(\varphi^2) \cos(\varphi)$ .

**Lemma 3** (see[31]):  $V(\cdot)$  and  $\varphi(\cdot)$  are the smooth functions,  $H(\cdot)$  is a smooth Nussbaum-type function. Then we have:

$$0 \leq V(t) \leq b_0 + \int_0^t (q(x(\tau))H(\varphi) + 1) \dot{\varphi} e^{-b_1(t-\tau)} d\tau, \quad \forall t \in [0, t_f],$$

(2.5)

where  $b_0$  and  $b_1 \geq 0$  are suitable constant,  $q(x(\tau))$  is a time-varying parameter that takes values in  $I = [l_-, l_+]$  with  $0 \notin I$ ,  $V(t)$ ,  $\varphi(t)$ ,  $\int_0^t (q(x(\tau))H(\varphi) + 1) \dot{\varphi} d\tau$  are bounded on  $[0, t_f]$ .

### 3 The Design of Adaptive Fuzzy Control and Its Stability Analysis

In order to develop the  $n$  step backstepping approach, we make the following state transformation:

$$\omega_1 = y - y_0,$$

$$\omega_i = x_i - \alpha_{i-1}, \quad i = 2, \dots, n,$$

(3.1)

where  $\alpha_i$  is an intermediate control that is determined up to the  $i$ th step.

We define a constant before developing a backstepping-based design procedure,

$$\eta = \max\{\|\theta_i\|^2, \quad i = 0, 1, 2, \dots, n\},$$

$\|\theta_i\|$  is unknown, so  $\eta$  is an unknown positive constant.  $\hat{\eta}$  is the estimate of  $\eta$ , and  $\tilde{\eta} = \eta - \hat{\eta}$ . According to the work in [8], it is straightforward to obtain that if  $\hat{\eta}(0) \geq 0$ , then  $\hat{\eta}(t) \geq 0, \forall t \geq 0$ .

Step 1: Think about the Lyapunov function candidate

$$V_1 = \frac{\omega_1^2}{2} + \frac{\tilde{\eta}^2}{2r},$$

(3.2)

where  $r$  is a design parameter.

The time derivative of  $V_1$  is

$$\dot{V}_1 = \omega_1 \dot{\omega}_1 - \frac{\hat{\eta}}{r} \dot{\eta}, \tag{3.3}$$

$$\begin{aligned} \omega_1 \dot{\omega}_1 &= \omega_1 [\chi'(x_1)(v_1(\bar{x}_1, \varsigma_1)(x_2 - x_2^0) \\ &\quad + f_1(\bar{x}_1, x_2^0) + \psi_1(t)) - \dot{y}_o] \\ &= \omega_1 [\rho_1 \omega_2 v_1(x_1, \varsigma_1) + \rho_1 \alpha_1 v_1(x_1, \varsigma_1) \\ &\quad - \rho_1 x_2^0 v_1(x_1, \varsigma_1) + \rho_1 f_1(x_1, x_2^0) + \rho_1 \psi_1(t) - \dot{y}_o], \end{aligned} \tag{3.4}$$

where  $\rho_1 = \chi'(x_1), |\rho_1| \leq b_M$ .

To deal with the issue that  $\rho_1$  is varying, the first virtual control law is constructed by introducing an even Nussbaum-type function  $H(\varphi)$ .

$$\alpha_1 = -H(\varphi)\bar{\alpha}_1, \tag{3.5}$$

where  $\bar{\alpha}_1$  is the auxiliary virtual controller and  $H(\varphi)$  is a Nussbaum-type function such that

$$H(\varphi) = \varphi^2 \cos(\varphi), \quad \dot{\varphi} = -\gamma \bar{\alpha}_1 \omega_1 g_1,$$

where  $\gamma > 0$  is a parameter.

Through the forementioned discussion, we have

$$\begin{aligned} \omega_1 \rho_1 \alpha_1 v_1(x_1, \varsigma_1) &= -\omega_1 \rho_1 H(\varphi) \bar{\alpha}_1 v_1(x_1, \varsigma_1) \\ &\quad - \omega_1 \bar{\alpha}_1 v_1(x_1, \varsigma_1) + \omega_1 \bar{\alpha}_1 v_1(x_1, \varsigma_1) \\ &= -\omega_1 v_1(x_1, \varsigma_1) (\rho_1 H(\varphi) + 1) \bar{\alpha}_1 \\ &\quad + \omega_1 v_1(x_1, \varsigma_1) \bar{\alpha}_1. \end{aligned} \tag{3.6}$$

By using Young's inequality, Assumptions 3, 4 and Lemma 2, we can obtain the following inequalities:

$$\omega_1 \rho_1 \psi_1(t) \leq \frac{b_M^2 \bar{p}_1^2 \omega_1^2}{2a_1^2} + \frac{a_1^2}{2}, \tag{3.7}$$

$$\begin{aligned} \omega_1 \rho_1 f_1(x_1, x_2^0) &\leq |\omega_1| (\Gamma(|y|) + \Gamma_0) \\ &\leq \frac{b_M^2 \Gamma^2(|y|) \omega_1^2}{2b_1^2} + \frac{b_M^2 \Gamma_0^2 \omega_1^2}{2b_1^2} + b_1^2, \end{aligned} \tag{3.8}$$

where  $a_1$  and  $b_1$  are arbitrary positive constants.

Based on (3.6, 3.7, 3.8 and 3.4), one has

$$\begin{aligned} \omega_1 \dot{\omega}_1 &\leq \omega_1 (\rho_1 \omega_2 v_1(x_1, \varsigma_1) + v_1(x_1, \varsigma_1) (\bar{\alpha}_1 - x_2^0) + \bar{l}_1) \\ &\quad + \frac{1}{\gamma} (\rho_1 H(\varphi) + 1) \dot{\varphi} + \frac{a_1^2}{2} + b_1^2, \end{aligned} \tag{3.9}$$

where  $\bar{l}_1 = \frac{b_M^2 \bar{p}_1^2 \omega_1^2}{2a_1^2} + \frac{b_M^2 \Gamma^2(|y|) \omega_1^2}{2b_1^2} + \frac{b_M^2 \Gamma_0^2 \omega_1^2}{2b_1^2} - \dot{y}_o$ .

Because of Lemma 1, there is a fuzzy logic system  $\theta_1^T G_1(X_1)$  such that

$$\bar{l}_1 = \theta_1^T G_1(X_1) + \delta_1(X_1), \quad |\delta_1(X_1)| \leq \varepsilon_1, \quad \forall \varepsilon_1 > 0,$$

where  $X_1 = (y, y_o, \dot{y}_o)$ .

It can be shown that:

$$\omega_1 \bar{l}_1 \leq \frac{\eta}{2c_1^2} \omega_1^2 G_1^T(X_1) G_1(X_1) + \frac{c_1^2}{2} + \frac{\omega_1^2}{2} + \frac{\varepsilon_1^2}{2}, \tag{3.10}$$

where  $c_1$  is a parameter.

Through the above discussion, we select the auxiliary virtual controller as follows:

$$\bar{\alpha}_1 = -c_M^{-1} \left[ \left( h_1 + \frac{1}{2} \right) \omega_1 - \frac{\hat{\eta}}{2c_1^2} \omega_1 G_1^T(X_1) G_1(X_1) \right] + x_2^0, \tag{3.11}$$

where  $h_1 > 0$  is a design constant.

Substituting (3.10 and 3.11) into (3.9), we will obtain

$$\begin{aligned} \omega_1 \dot{\omega}_1 &\leq -h_1 \omega_1^2 + \frac{\hat{\eta}}{2c_1^2} \omega_1^2 P_1^T(X_1) P_1(X_1) + \omega_1 \rho_1 \omega_2 c_M \\ &\quad + \frac{1}{\gamma} (\rho_1 H(\varphi) + 1) \dot{\varphi} + \frac{a_1^2}{2} + b_1^2 + \frac{c_1^2}{2} + \frac{\varepsilon_1^2}{2}. \end{aligned} \tag{3.12}$$

Combing (3.3) with (3.12), the time derivative of  $V_1$  has the following form:

$$\begin{aligned} \dot{V}_1 &\leq \rho_1 \omega_1 \omega_2 c_M - h_1 \omega_1^2 + \frac{1}{\gamma} (\rho_1 H(\varphi) + 1) \dot{\varphi} + \pi_1 \\ &\quad - \frac{\hat{\eta}}{r} \left( \dot{\eta} - \frac{r}{2c_1^2} \omega_1^2 G_1^T(X_1) G_1(X_1) \right), \end{aligned} \tag{3.13}$$

where  $\pi_1 = \frac{a_1^2}{2} + b_1^2 + \frac{c_1^2}{2} + \frac{\varepsilon_1^2}{2}$ .

Step 2: Select the Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2} \omega_2^2, \tag{3.14}$$

Similarly to step 1, it is also easy to see that the time derivative of  $V_2$ ,

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \omega_2 (v_2(\bar{x}_2, \varsigma_2) \omega_3 + v_2(\bar{x}_2, \varsigma_2) (\alpha_2 - x_3^0) \\ &\quad + f_2(\bar{x}_2, x_3^0) + \psi_2(t) - \dot{\alpha}_1), \end{aligned} \tag{3.15}$$

where

$$\begin{aligned} \dot{\alpha}_1 &= \frac{\partial \alpha_1}{\partial y} \rho_1 (v_1(\bar{x}_1, \varsigma_1) (x_2 - x_2^0) + f_1(\bar{x}_1, x_2^0) + \psi_1(t)) \\ &\quad + \frac{\partial \alpha_1}{\partial \dot{y}_o} \dot{y}_o - \frac{\partial \alpha_1}{\partial \varphi} \gamma \omega_1 \bar{\alpha}_1 v_1(x_1, \varsigma_1) + \frac{\partial \alpha_1}{\partial \hat{\eta}} \dot{\eta}. \end{aligned} \tag{3.16}$$

By applying Assumptions 2, 3 and 4, Lemma 2 and Young's inequality, the following inequalities are viewed as :

$$-\omega_2 \frac{\partial \alpha_1}{\partial y} \rho_1 \psi_1(t) \leq \frac{b_M^2 \bar{p}_1^2 \omega_2^2}{2a_2^2} \left( \frac{\partial \alpha_1}{\partial y} \right)^2 + \frac{a_2^2}{2}, \tag{3.17}$$

$$\begin{aligned} -\omega_2 \frac{\partial \alpha_1}{\partial y} \rho_1 f_1(x_1, x_2^0) &\leq \frac{b_M^2 \Gamma^2(|y|) \omega_2^2}{2b_1^2} \left( \frac{\partial \alpha_1}{\partial y} \right)^2 \\ &\quad + \frac{b_M^2 \Gamma_0^2 \omega_2^2}{2b_2^2} \left( \frac{\partial \alpha_1}{\partial y} \right)^2 + b_2^2, \end{aligned} \tag{3.18}$$

$$\rho_1 \omega_1 \omega_2 v_1(x_1, \varsigma_1) \leq \frac{c_M^2 b_M^2 \omega_1^2 \omega_2^2}{2k_0^2} + \frac{k_0^2}{2}, \tag{3.19}$$

$$-\omega_2 \frac{\partial \alpha_1}{\partial y} \rho_1 x_2 v_1(x_1, \varsigma_1) \leq \frac{b_M^2 c_M^2 \omega_2^2 x_2^2}{2e_2^2} \left( \frac{\partial \alpha_1}{\partial y} \right)^2 + \frac{e_2^2}{2}, \tag{3.20}$$

$$\omega_2 \frac{\partial \alpha_1}{\partial y} \rho_1 x_2^0 v_1(x_1, \varsigma_1) \leq \frac{b_M^2 c_M^2 \omega_2^2 x_2^{02}}{2m_2^2} \left( \frac{\partial \alpha_1}{\partial y} \right)^2 + \frac{m_2^2}{2}, \tag{3.21}$$

where  $a_2, b_2, k_0, e_2$  and  $m_2$  are arbitrary positive constants.

Substituting (3.17, 3.18, 3.19, 3.20, 3.21) into (3.15) yields

$$\begin{aligned} \dot{V}_2 \leq & -h_1 \omega_1^2 + \frac{1}{\gamma} (\rho_1 H(\varphi) + 1) \dot{\varphi} + \pi_1 \\ & - \frac{\bar{\eta}}{r} \left( \dot{\eta} - \frac{r}{2c_1^2} \omega_1^2 G_1^T(X_1) G_1(X_1) \right) + \frac{a_2^2}{2} + b_2^2 + \frac{k_0^2}{2} + \frac{e_2^2}{2} \\ & + \frac{m_2^2}{2} + \omega_2 \left( \omega_3 v_2(\bar{x}_2, \varsigma_2) + \alpha_2 v_2(\bar{x}_2, \varsigma_2) + \bar{l}_2 + \varphi_2(X_2) - \frac{\partial \alpha_1}{\partial \eta} \dot{\eta} \right), \end{aligned} \tag{3.22}$$

where

$$\begin{aligned} \bar{l}_2 = & -\varphi_2(X_2) + \frac{b_M^2 \bar{p}_1^2 \omega_2}{2a_2^2} \left( \frac{\partial \alpha_1}{\partial y} \right) + \frac{b_M^2 \Gamma^2(|y|) \omega_2}{2b_1^2} \left( \frac{\partial \alpha_1}{\partial y} \right)^2 \\ & + \frac{b_M^2 \Gamma_0^2 \omega_2}{2b_2^2} \left( \frac{\partial \alpha_1}{\partial y} \right)^2 + \frac{c_M^2 b_M^2 \omega_1^2 \omega_2}{2k_0^2} - \frac{\partial \alpha_1}{\partial y_o} \dot{y}_o \\ & + \frac{b_M^2 c_M^2 \omega_2^2 x_2^{02}}{2m_2^2} \left( \frac{\partial \alpha_1}{\partial y} \right)^2 + \frac{b_M^2 c_M^2 \omega_2 x_2^2}{2e_2^2} \left( \frac{\partial \alpha_1}{\partial y} \right)^2 - \frac{\partial \alpha_1}{\partial y_o} \ddot{y}_o \\ & + \frac{\partial \alpha_1}{\partial \varphi} \gamma \omega_1 \bar{\alpha}_1 v_1(x_1, \varsigma_1) + f_2(\bar{x}_2, x_3^0) + \psi_2(t), \end{aligned}$$

with

$$\begin{aligned} \varphi_2(X_2) = & -h_0 \bar{\eta} \frac{\partial \alpha_1}{\partial \eta} - \omega_2 \frac{r}{2c_2} \left| \omega_2 \frac{\partial \alpha_1}{\partial \eta} \right| \\ & + \frac{r}{2c_2} \frac{\partial \alpha_1}{\partial \eta} \omega_1^2 G_1^T(X_1) G_1(X_1), \end{aligned} \tag{3.23}$$

where  $a_0, c_2$  are positive design parameters.

Similarly to the discussion of (3.10), one has

$$\bar{l}_2 = \theta_2^T G_2(X_2) + \delta_2(X_2), \quad |\delta_2(X_2)| \leq \varepsilon_2, \tag{3.24}$$

where  $X_2 = (y, y_o, \dot{y}_o, \ddot{y}_o, x_2, \eta, \rho)$ .

It is means that the following inequalities

$$\omega_2 \bar{l}_2 \leq \frac{\eta}{2c_2^2} \omega_2^2 G_2^T(X_2) G_2(X_2) + \frac{c_2^2}{2} + \frac{\omega_2^2}{2} + \frac{\varepsilon_2^2}{2}, \tag{3.25}$$

can be obtained, where  $c_2$  is a parameter.

Similarly to (3.11), we have

$$\begin{aligned} \alpha_2(X_2) & = -c_M^{-1} \left[ \left( h_2 + \frac{1}{2} \right) \omega_2 - \frac{\bar{\eta}}{2c_2^2} \omega_2 G_2^T(X_2) G_2(X_2) \right] + x_3^0, \end{aligned} \tag{3.26}$$

where  $h_2$  is a positive design constant.

Combing (3.15) and (3.23, 3.24, 3.25, 3.26) with (3.22), the time derivative of  $V_2$  has the following form:

$$\begin{aligned} \dot{V}_2 \leq & -\sum_{i=1}^2 h_i \omega_i^2 + \omega_2 \omega_3 c_M + \frac{1}{\gamma} (\rho_1 H(\varphi) + 1) \dot{\varphi} \\ & + \omega_2 \left( \varphi_2(X_2) - \frac{\partial \alpha_1}{\partial \eta} \dot{\eta} \right) \\ & - \frac{\bar{\eta}}{r} \left( \dot{\eta} - \sum_{i=1}^2 \frac{r}{2c_i^2} \omega_i^2 G_i^T(X_i) G_i(X_i) \right) + \pi_2, \end{aligned} \tag{3.27}$$

where  $\pi_2 = \pi_1 + \frac{a_2^2}{2} + b_2^2 + \frac{c_2^2}{2} + \frac{k_0^2}{2} + \frac{m_2^2}{2} + \frac{e_2^2}{2} + \frac{\varepsilon_2^2}{2}$ .

Step  $i$  : Select the Lyapunov function candidate as

$$V_i = V_{i-1} + \frac{1}{2} \omega_i^2, \quad (3 \leq i < n) \tag{3.28}$$

similar to the discussion in Step 2, the time derivative of  $V_i$  is shown as the following form:

$$\begin{aligned} \dot{V}_i = & \dot{V}_{i-1} + \omega_i (v_i(\bar{x}_i, \varsigma_i) \omega_{i+1} + v_i(\bar{x}_i, \varsigma_i) (\alpha_i - x_{i+1}^0) \\ & + f_i(\bar{x}_i, x_{i+1}^0) + \psi_i(t) - \dot{\alpha}_{i-1}) \end{aligned} \tag{3.29}$$

where

$$\begin{aligned} \dot{\alpha}_{i-1} = & -\frac{\partial \alpha_{i-1}}{\partial \varphi} \gamma \omega_1 \bar{\alpha}_1 v_1(x, \varsigma_1) + \frac{\partial \alpha_{i-1}}{\partial \eta} \dot{\eta} \\ & + \frac{\partial \alpha_{i-1}}{\partial y} \rho_1 (v_1(x_1, \varsigma_1) (x_2 - x_2^0) + f_1(x_1, x_2^0) + \psi_1(t)) \\ & + \sum_{j=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (v_j(\bar{x}_j, \varsigma_j) (x_{j+1} - x_{j+1}^0) \\ & + f_j(\bar{x}_j, x_{j+1}^0) + \psi_j(t)) + \sum_{j=1}^i \frac{\partial \alpha_{i-1}}{\partial y_o^{j-1}} y_o^j. \end{aligned} \tag{3.30}$$

Similar to the analysis of (3.17, 3.18, 3.20) and (3.21), we can obtain the following inequalities:

$$-\omega_i \frac{\partial \alpha_{i-1}}{\partial y} \rho_1 \psi_1(t) \leq \frac{b_M^2 \bar{p}_1^2 \omega_i^2}{2a_i^2} \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 + \frac{a_i^2}{2}, \tag{3.31}$$

$$\begin{aligned} -\omega_i \frac{\partial \alpha_{i-1}}{\partial y} \rho_1 f_1(x_1, x_2^0) \leq & \frac{b_M^2 \Gamma^2(|y|) \omega_i^2}{2b_i^2} \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 \\ & + \frac{b_M^2 \Gamma_0^2 \omega_i^2}{2b_i^2} \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 + b_i^2, \end{aligned} \tag{3.32}$$

$$-\omega_i \frac{\partial \alpha_{i-1}}{\partial y} \rho_1 x_2 v_1(x_1, \varsigma_1) \leq \frac{b_M^2 c_M^2 \omega_i^2 x_2^2}{2e_i^2} \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 + \frac{e_i^2}{2}, \tag{3.33}$$

$$\omega_i \frac{\partial \alpha_{i-1}}{\partial y} \rho_1 x_2^0 v_1(x_1, \varsigma_1) \leq \frac{b_M^2 c_M^2 \omega_i^2 x_2^2}{2m_i^2} \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 + \frac{m_i^2}{2}, \tag{3.34}$$

where  $a_i, b_i, e_i$  and  $m_i$  are arbitrary positive constants.

We take the same way in Step 2, then

$$\begin{aligned} \dot{V}_i \leq & - \sum_{j=1}^{i-1} h_j \omega_j^2 + \frac{1}{\gamma} (\rho_1 H(\varphi) + 1) \dot{\varphi} + \pi_{i-1} \\ & - \frac{\tilde{\eta}}{r} \left( \dot{\hat{\eta}} - \sum_{j=1}^{i-1} \frac{r}{2c_j^2} \omega_j^2 G_j^T(X_j) G_j(X_1) \right) \\ & + \omega_i \left( \omega_{i+1} v_i(\bar{x}_i, \varsigma_i) + v_i(\bar{x}_i, \varsigma_i) (\alpha_i - x_{i+1}^0) \right. \\ & \left. + \bar{l}_i + \varphi_i(X_i) - \frac{\partial \alpha_{i-1}}{\partial \hat{\eta}} \dot{\hat{\eta}} \right) + \frac{a_i^2}{2} + b_i^2 + \frac{m_i^2}{2} + \frac{e_i^2}{2}, \end{aligned} \tag{3.35}$$

where

$$\begin{aligned} \bar{l}_i = & -\varphi_i(X_i) + \frac{b_M^2 \bar{p}_1^2 \omega_i}{2a_i^2} \left( \frac{\partial \alpha_{i-1}}{\partial y} \right) + \frac{b_M^2 \Gamma^2(|y|) \omega_i}{2b_i^2} \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 \\ & + \frac{b_M^2 \Gamma_0^2 \omega_i}{2b_i^2} \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 + \frac{b_M^2 c_M^2 \omega_i x_i^2}{2e_i^2} \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 \\ & + \frac{b_M^2 c_M^2 \omega_i^2 x_2^2}{2m_i^2} \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 - \sum_{j=1}^i \frac{\partial \alpha_{i-1}}{\partial y_o^{(j-1)}} y_o^{(j)} \\ & + \frac{\partial \alpha_{i-1}}{\partial \varphi} \gamma \omega_1 \bar{\alpha}_1 v_1(x_1, \varsigma_1) \\ & + \sum_{j=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left( v_j(\bar{x}_j, \varsigma_j) (x_{j+1} - x_{j+1}^0) + f_j(\bar{x}_j, x_{j+1}^0) + \psi_j(t) \right) \\ & + \psi_i(t) + f_i(\bar{x}_i, x_{i+1}^0) + \omega_i \omega_{i+1} v_i(\bar{x}_i, \varsigma_i), \end{aligned}$$

with

$$\begin{aligned} \varphi_i(X_i) = & -h_0 \hat{\eta} \frac{\partial \alpha_{i-1}}{\partial \hat{\eta}} - \sum_{j=2}^i \omega_j \frac{r}{2c_i} \left| \omega_j \frac{\partial \alpha_{i-1}}{\partial \hat{\eta}} \right| \\ & + \sum_{j=1}^{i-1} \frac{r}{2c_i} \frac{\partial \alpha_{i-1}}{\partial \hat{\eta}} \omega_j^2 G_j^T(X_j) G_j(X_j), \end{aligned} \tag{3.36}$$

where  $c_i$  are positive design parameters.

From (3.24), there is a fuzzy logic system  $\theta_i^T G_i(X_i)$ ; it satisfies

$$\bar{l}_i = \theta_i^T G_i(X_i) + \delta_i(X_i), \quad |\delta_i(X_i)| \leq \varepsilon_i, \tag{3.37}$$

where  $X_i = (y, \bar{y}_o^i, x_2, \dots, x_i, \hat{\eta}, \rho)$ .

We can prove

$$\omega_i \bar{l}_i \leq \frac{\eta}{2c_i^2} \omega_i^2 G_i^T(X_i) G_i(X_i) + \frac{c_i^2}{2} + \frac{\omega_i^2}{2} + \frac{\varepsilon_i^2}{2}, \tag{3.38}$$

where  $c_i$  is a parameter.

Similarly to (3.11), we have

$$\alpha_i(X_i) = -c_M^{-1} \left\{ \left( h_i + \frac{1}{2} \right) \omega_i - \frac{\hat{\eta}}{2c_i^2} \omega_i G_i^T(X_i) G_i(X_i) \right\} + x_{i+1}^0, \tag{3.39}$$

where  $h_i > 0$  is a design constant.

Combing (3.36, 3.37, 3.38, 3.39) with (3.35):

$$\begin{aligned} \dot{V}_i \leq & - \sum_{j=1}^i h_j \omega_j^2 + \frac{1}{\gamma} (\rho_1 H(\varphi) + 1) \dot{\varphi} + \sum_{j=2}^i \omega_j \left( \varphi_j(X_j) - \frac{\partial \alpha_{j-1}}{\partial \hat{\eta}} \dot{\hat{\eta}} \right) \\ & - \frac{\tilde{\eta}}{r} \left( \dot{\hat{\eta}} - \sum_{j=1}^i \frac{r}{2c_j^2} \omega_j^2 G_j^T(X_j) G_j(X_j) \right) + \pi_i + \omega_i \omega_{i+1} c_M, \end{aligned} \tag{3.40}$$

where  $\pi_i = \pi_{i-1} + \frac{a_i^2}{2} + b_i^2 + \frac{c_i^2}{2} + \frac{e_i^2}{2} + \frac{m_i^2}{2} + \frac{\varepsilon_i^2}{2}$ .

Step  $n$ : Select the Lyapunov function candidate as

$$V_n = V_{n-1} + \frac{1}{2} \omega_n^2. \tag{3.41}$$

Similarly,

$$\dot{V}_n = \dot{V}_{n-1} + \omega_n (v_n(\bar{x}_n, \varsigma_n) (u - u^0) + f_i(\bar{x}_n, u^0) + \psi_n(t) - \dot{\alpha}_{n-1}) \tag{3.42}$$

where

$$\begin{aligned} \dot{\alpha}_{n-1} = & - \frac{\partial \alpha_{n-1}}{\partial \varphi} \gamma \omega_1 \bar{\alpha}_1 v_1(x, \varsigma_1) + \frac{\partial \alpha_{n-1}}{\partial \hat{\eta}} \dot{\hat{\eta}} \\ & + \frac{\partial \alpha_{n-1}}{\partial y} \rho_1 (v_1(x_1, \varsigma_1) (x_2 - x_2^0) + f_1(x_1, x_2^0) + \psi_1(t)) \\ & + \sum_{i=2}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i} (v_i(\bar{x}_i, \varsigma_i) (x_{i+1} - x_{i+1}^0) \\ & + f_i(\bar{x}_i, x_{i+1}^0) + \psi_i(t)) + \sum_{i=1}^n \frac{\partial \alpha_{n-1}}{\partial y_o^{i-1}} y_o^i. \end{aligned} \tag{3.43}$$

Based on the above discussion, taking (3.31, 3.32, 3.33, 3.34) with  $i = n$  into account that

$$-\omega_n \frac{\partial \alpha_{n-1}}{\partial y} \rho_1 \psi_1(t) \leq \frac{b_M^2 \bar{p}_1^2 \omega_n^2}{2a_n^2} \left( \frac{\partial \alpha_{n-1}}{\partial y} \right)^2 + \frac{a_n^2}{2}, \tag{3.44}$$

$$\begin{aligned} -\omega_n \frac{\partial \alpha_{n-1}}{\partial y} \rho_1 f_1(x_1, x_2^0) \leq & \frac{b_M^2 \Gamma^2(|y|) \omega_n^2}{2b_n^2} \left( \frac{\partial \alpha_{n-1}}{\partial y} \right)^2 \\ & + \frac{b_M^2 \Gamma_0^2 \omega_n^2}{2b_n^2} \left( \frac{\partial \alpha_{n-1}}{\partial y} \right)^2 + b_n^2, \end{aligned} \tag{3.45}$$

$$-\omega_n \frac{\partial \alpha_{n-1}}{\partial y} \rho_1 x_2 v_1(x_1, \varsigma_1) \leq \frac{b_M^2 c_M^2 \omega_n^2 x_2^2}{2e_n^2} \left( \frac{\partial \alpha_{n-1}}{\partial y} \right)^2 + \frac{e_n^2}{2}, \tag{3.46}$$

$$\omega_n \frac{\partial \alpha_{n-1}}{\partial y} \rho_1 x_2^0 v_1(x_1, \varsigma_1) \leq \frac{b_M^2 c_M^2 \omega_n^2 x_2^2}{2m_n^2} \left( \frac{\partial \alpha_{n-1}}{\partial y} \right)^2 + \frac{m_n^2}{2}, \tag{3.47}$$

where  $a_n, b_n, e_n$  and  $m_n$  are arbitrary positive constants.

Substituting (3.44, 3.35, 3.36, 3.47) into (3.42), it yields

$$\begin{aligned} \dot{V}_n \leq & - \sum_{i=1}^{n-1} h_i \omega_i^2 + \frac{1}{\gamma} (\rho_1 H(\varphi) + 1) \dot{\varphi} + \pi_{n-1} \\ & - \frac{\tilde{\eta}}{r} \left( \dot{\eta} - \sum_{i=1}^{n-1} \frac{r}{2c_i^2} \omega_i^2 G_i^T(X_i) G_i(X_i) \right) + \frac{a_n^2}{2} + b_n^2 + \frac{e_n^2}{2} \\ & + \frac{m_n^2}{2} + \omega_n \left( v_n(\bar{x}_n, \varsigma_n) (u - u^0) + \bar{l}_n + \varphi_n(X_n) - \frac{\partial \alpha_{i-1}}{\partial \hat{\eta}} \dot{\eta} \right), \end{aligned} \tag{3.48}$$

where

$$\begin{aligned} \bar{l}_n = & -\varphi_n(X_n) + \frac{b_M^2 \bar{p}_1^2 \omega_n}{2a_i^2} \left( \frac{\partial \alpha_{n-1}}{\partial y} \right) + \frac{b_M^2 \Gamma^2(|y|) \omega_n}{2b_i^2} \\ & \times \left( \frac{\partial \alpha_{n-1}}{\partial y} \right)^2 + \frac{b_M^2 \Gamma_0^2 \omega_n}{2b_i^2} \left( \frac{\partial \alpha_{n-1}}{\partial y} \right)^2 \\ & + \frac{b_M^2 c_M^2 \omega_n x_i^2}{2e_n^2} \left( \frac{\partial \alpha_{n-1}}{\partial y} \right)^2 + \frac{b_M^2 c_M^2 \omega_n^2 x_2^2}{2m_n^2} \left( \frac{\partial \alpha_{n-1}}{\partial y} \right)^2 \\ & + \sum_{i=2}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i} (x_{i+1} v_i(\bar{x}_i) + f_i(\bar{x}_i) + \psi_i(t)) \\ & - \sum_{i=1}^n \frac{\partial \alpha_{n-1}}{\partial y_o^{(i-1)}} y_o^{(i)} + \frac{\partial \alpha_{n-1}}{\partial \varphi} \gamma \omega_1 \bar{\alpha}_1 v_1(x_1, \varsigma_1) \\ & + f_n(\bar{x}_n, u_0) + \psi_n(t), \end{aligned}$$

with

$$\begin{aligned} \varphi_n(X_n) = & -h_0 \hat{\eta} \frac{\partial \alpha_{n-1}}{\partial \hat{\eta}} - \sum_{i=2}^n \omega_n \frac{r}{2c_n} \left| \omega_i \frac{\partial \alpha_{n-1}}{\partial \hat{\eta}} \right| \\ & + \sum_{i=1}^{n-1} \frac{r}{2c_n} \frac{\partial \alpha_{n-1}}{\partial \hat{\eta}} \omega_i^2 G_i^T(X_i) G_i(X_i), \end{aligned} \tag{3.49}$$

where  $c_n$  are positive design parameters.

Similarly, one has

$$\bar{l}_n = \theta_n^T G_n(X_n) + \delta_n(X_n), \quad |\delta_n(X_n)| \leq \varepsilon_n, \quad \varepsilon_n > 0, \tag{3.50}$$

where  $X_n = (y, \bar{y}_o^{(n)}, x_2, \dots, x_n, \hat{\eta}, \rho)$ .

It is easy to show that

$$\omega_n \bar{l}_n \leq \frac{\eta}{2c_n^2} \omega_n^2 G_n^T(X_n) G_n(X_n) + \frac{c_n^2}{2} + \frac{\omega_n^2}{2} + \frac{\varepsilon_n^2}{2}, \tag{3.51}$$

where  $c_n$  is a parameter.

The control law is selected as

$$u = -c_M^{-1} \left[ \left( h_n + \frac{1}{2} \right) \omega_n - \frac{\hat{\eta}}{2c_n^2} \omega_n G_n^T(X_n) G_n(X_n) \right] + u_0, \tag{3.52}$$

where  $h_n$  is a positive design constant.

Together with (3.49, 3.50, 3.51, 3.52) and (3.48), the time derivative of  $V_n$  is shown as:

$$\begin{aligned} \dot{V}_n \leq & - \sum_{i=1}^n h_i \omega_i^2 + \frac{1}{\gamma} (\rho_1 H(\varphi) + 1) \dot{\varphi} + \sum_{i=2}^n \omega_i (\varphi_i(X_i) - \frac{\partial \alpha_{i-1}}{\partial \hat{\eta}} \dot{\eta}) \\ & - \frac{\tilde{\eta}}{r} \left( \dot{\eta} - \sum_{i=1}^n \frac{r}{2c_i^2} \omega_i^2 G_i^T(X_i) G_i(X_i) \right) + \pi_n, \end{aligned} \tag{3.53}$$

where  $\pi_n = \pi_{n-1} + \frac{a_n^2}{2} + b_n^2 + \frac{c_n^2}{2} + \frac{e_n^2}{2} + \frac{m_n^2}{2} + \frac{\varepsilon_n^2}{2}$ .

We employ the adaptive law as

$$\dot{\hat{\eta}} = \sum_{i=1}^n \frac{r}{2c_i^2} \omega_i^2 G_i^T(X_i) G_i(X_i) - h_0 \hat{\eta}, \quad \hat{\eta}(0) \geq 0. \tag{3.54}$$

By applying the work in [29], it can be shown that

$$\sum_{i=2}^n \omega_i \left( \varphi_i(X_i) - \frac{\partial \alpha_{i-1}}{\partial \hat{\eta}} \dot{\eta} \right) \leq 0. \tag{3.55}$$

It is noted that

$$\tilde{\eta} \dot{\hat{\eta}} = \tilde{\eta} (\eta - \hat{\eta}) \leq -\frac{\tilde{\eta}^2}{2} + \frac{\eta^2}{2}. \tag{3.56}$$

Combing (3.53, 3.54, 3.55, 3.56), the time derivative of  $V_n$  satisfies

$$\dot{V}_n \leq - \sum_{i=1}^n h_i \omega_i^2 + \frac{1}{\gamma} (\rho_1 H(\varphi) + 1) \dot{\varphi} + \pi_n - \frac{h_0}{2r} \tilde{\eta}^2 + \frac{h_0}{2r} \eta^2. \tag{3.57}$$

Define

$$\begin{aligned} D = & \min\{2h_i, h_0\} > 0, \quad (1 \leq i \leq n), \\ \pi = & \pi_n + \frac{h_0}{2r} \eta^2 = \sum_{i=1}^n \frac{a_i^2}{2} + \sum_{i=1}^n b_i^2 + \sum_{i=1}^n \frac{c_i^2}{2} + \sum_{i=1}^n \frac{e_i^2}{2} \\ & + \sum_{i=2}^n \frac{m_i^2}{2} + \sum_{i=1}^n \frac{\varepsilon_i^2}{2} + \frac{h_0}{2r} \eta^2, \end{aligned} \tag{3.58}$$

then we get

$$\dot{V}_n \leq -DV_n + \pi + \frac{1}{\gamma} (\rho_1 H(\varphi) + 1) \dot{\varphi}. \tag{3.59}$$

With above discussions, then we present our contribution in this thesis.

**Theorem 1** By applying the fuzzy logic systems and Assumptions 1, 2, 3 and 4, the unknown functions  $\bar{l}_i$  are approximated to some bounded term for the nonlinear system

(2.4). By choosing the control law (3.52) and the intermediate virtual control (3.5, 3.11, 3.26, 3.39) and the adaptive law (3.50), it makes all the signals which are mentioned in this system be bounded. In addition, the tracking deviation  $\omega_1 = y - y_0$  meets the following condition

$$\lim_{t \rightarrow \infty} S_1^2 \leq \frac{2\pi}{D} + \frac{2\sigma}{\gamma}, \quad 1 \leq i \leq n, \tag{3.60}$$

where  $D$  and  $\pi$  are defined in (3.58), and  $\sigma$  defined in (3.62).

*Proof* Integrating (3.59) over  $[0,t]$ , we can obtain the following inequality:

$$V_n(t) \leq e^{-Dt} (V_n(0) - \frac{\pi}{D}) + \frac{\pi}{D} + \frac{e^{-Dt}}{\gamma} \int_0^t (\rho_1 H(\varphi) + 1) \dot{\varphi} e^{D\tau} d\tau. \tag{3.61}$$

$V_n(t)$ ,  $\varphi(t)$  and  $\int_0^t (\rho_1 H(\varphi) + 1) \dot{\varphi} d\tau$  are bounded in  $[0, t_f]$  by Lemma 3. In addition,  $\int_0^t (\rho_1 H(\varphi) + 1) \dot{\varphi} e^{-D(t-\tau)} d\tau \leq \int_0^t (\rho_1 H(\varphi) + 1) \dot{\varphi} d\tau$ , it is shown that  $\int_0^t (\rho_1 H(\varphi) + 1) \dot{\varphi} e^{-D(t-\tau)} d\tau$  are bounded on  $[0, t_f]$ .

Define

$$\sigma = \max_{t \in [0, t_f]} \left| \int_0^t (\rho_1 H(\varphi) + 1) \dot{\varphi} e^{-D(t-\tau)} d\tau \right|. \tag{3.62}$$

By (3.60) and (3.61), we have  $\frac{\omega_1^2}{2} \leq V_n(t) \leq e^{-Dt} (V_n(0) - \frac{\pi}{D}) + \frac{\pi}{D} + \frac{\sigma}{\gamma}$ .

Because of  $D > 0$ , it is not hard to elicit that all the signals mentioned in this system are bounded. Therefore, when  $t \rightarrow +\infty$ , (3.60) holds.

*Remark 3* It is evident that the tracking deviation depends on  $a_i, b_i, c_i, e_i, h_i, r, \gamma$  and unknown term  $\eta$  by the above analysis. It is guaranteed that the tracking deviation is in a small neighborhood of the origin. According to (3.57), increasing  $h_i, r$  and  $\gamma$ , meanwhile reducing  $a_i, b_i, c_i$  and  $e_i$ , leads to a small tracking error.

### 4 Simulation Examples

The second-order nonlinear system is introduced to examine the availability of the proposed scheme.

$$\begin{cases} \dot{x}_1 = x_2 \ln(10 + x_1^2) + x_1^2 \sin x_1 + 0.01e^{-t} \cos t, \\ \dot{x}_2 = u(1 + e^{-x_1^2 - x_2^2}) + \frac{x_1 x_2}{1 + x_1^2 + x_2^2} + 0.5 \cos(2t), \\ y = \chi(x_1). \end{cases} \tag{4.1}$$

The dead zone  $\chi(x_1)$  is seen as:

$$y = \chi(x_1) = \begin{cases} 1.5(x_1 - 0.25)^{1.34}, & x_1 \geq 0.25 \\ 0, & 0.2 < x_1 < 0.25 \\ 0.5(x_1 + 0.2)^{1.5}, & x_1 \leq 0.2. \end{cases} \tag{4.2}$$

The reference signal is hypothesized as  $y_r = 0.5 \sin t + \sin(0.5t)$ . Because (4.1) not only contains the nonlinear term  $f_1(\bar{x}_1), f_2(\bar{x}_1), g_1(\bar{x}_1), g_2(\bar{x}_2)$  and disturbance terms  $\psi_1(t), \psi_2(t)$ , but also has the dead zone output  $y = \chi(x_1)$ , the control schemes proposed earlier is very inappropriate to this system.

For the fuzzy control, we select the following membership functions:

$$\begin{aligned} \mu_{F_i^1}(x) &= \exp[-(x + 2.5)^2], & \mu_{F_i^2}(x) &= \exp[-(x + 1.5)^2], \\ \mu_{F_i^3}(x) &= \exp[-(x + 0.5)^2], & \mu_{F_i^4}(x) &= \exp[-(x - 0.5)^2], \\ \mu_{F_i^5}(x) &= \exp[-(x - 1.5)^2], & \mu_{F_i^6}(x) &= \exp[-(x - 4)^2]. \end{aligned} \tag{4.3}$$

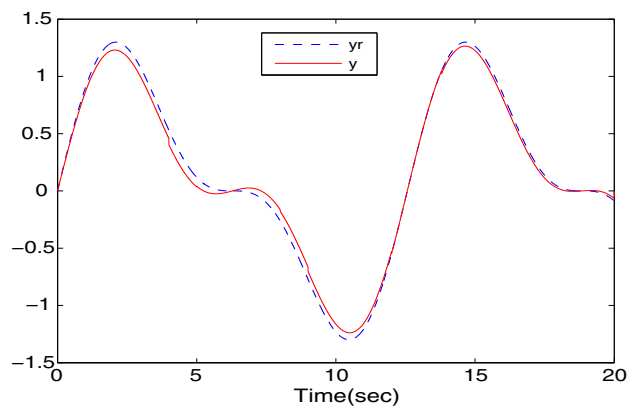


Fig. 1 Trajectories of  $y$ (solid line) and  $y_r$ (dashed line) for Case 1

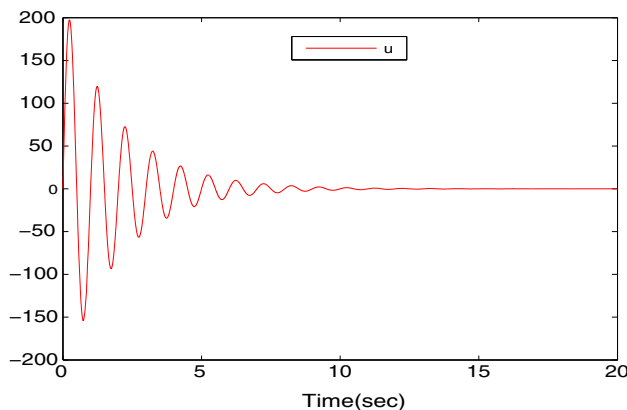
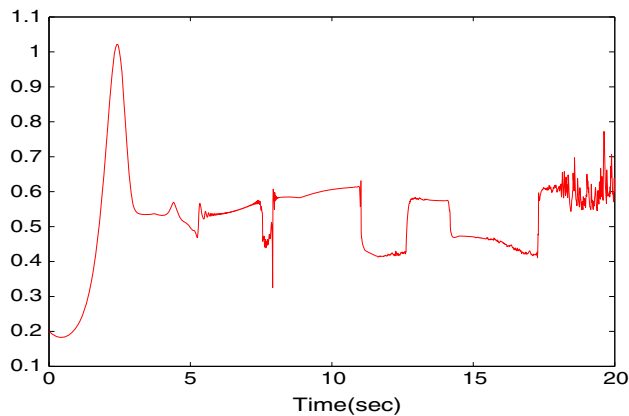
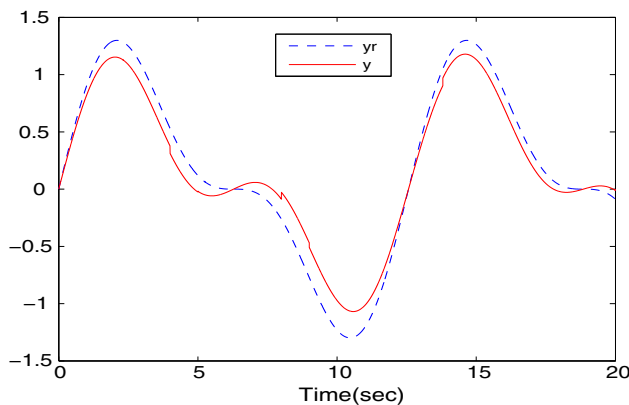


Fig. 2 Trajectories of control  $u$  for Case 1





**Fig. 3** Trajectories of  $\hat{\eta}$  for Case 1



**Fig. 4** Trajectories of  $y$  for Case 2

Choose the intermediate virtual control functions (3.9), the actual control (3.47) and the parameter adaptive law (3.49) with  $c_1 = 3.4$ ,  $c_2 = 4.2$ ,  $h_1 = 30$ ,  $h_2 = 20$ ,  $k_0 = 5$ ,  $r = 400$  and  $\gamma = 40$  for Case 1 and  $c_1 = 1.2$ ,  $c_2 = 2.5$ ,  $h_1 = 40$ ,  $h_2 = 35$ ,  $k_0 = 5$ ,  $r = 400$  and  $\gamma = 40$  for Case 2. The initial conditions are chosen as  $[x_1(0), x_2(0), \hat{\eta}(0)] = [0.1, 0.2, 0]$ .

The simulation consequences are revealed in Figs. 1, 2, 3 and 4, where Fig. 1 reveals the output  $y$  and the reference signal  $y_r$  for the Case 1, Fig. 2 displays the trajectories of the control  $u$ , Fig. 3 shows the trajectories of the parameter  $\hat{\eta}$ , and Fig. 4 presents the trajectories of  $y$  and the reference signal  $y_r$  for the Case 2.

**Acknowledgements** The authors are grateful to editor and anonymous reviewers for their helpful comments and suggestions on the paper. This work is supported by NSF of China (Grant Nos. 61170054, and 61402265), Supported by SDUST Excellent Teaching Team Construction Plan (Grant No. JXTD20160507).

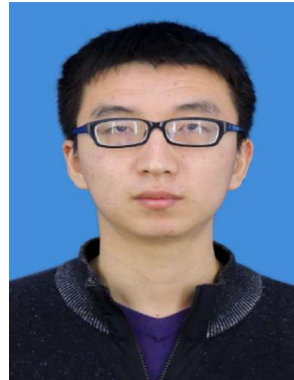
## Compliance with Ethical Standards

**Conflict of interest** The authors declare that there is no conflict of interests regarding the publication of this article.

## References

- Kosko, B.: Fuzzy systems as universal approximators. *IEEE Trans. Comput.* **43**, 1329–1333 (1994)
- Wang, L.X.: Stable adaptive fuzzy control of nonlinear systems. *IEEE Trans. Fuzzy Syst.* **1**, 146–155 (1993)
- Chen, B., Liu, X.P., Lin, C., Ge, S.S.: Adaptive fuzzy control of a class of nonlinear systems by fuzzy approximation approach. *IEEE Trans. Fuzzy Syst.* **20**, 1012–1020 (2012)
- Zhou, Q., Shi, P., Tian, Y., Wang, M.Y.: Approximation-based adaptive tracking control for MIMO nonlinear systems with input saturation. *IEEE Trans. Cybern.* **45**, 2119–2128 (2015)
- Wang, H.Q., Chen, B., Lin, C.: Direct adaptive neural control for strict feedback stochastic nonlinear systems. *Nonlinear Dyn.* **67**, 2703–2718 (2012)
- Wu, L.B., Yang, G.H.: Adaptive fuzzy tracking control for a class of uncertain nonaffine nonlinear systems with dead-zone inputs. *Fuzzy Sets Syst.* **290**, 1–21 (2016)
- Tong, S.C., Li, C.Y., Li, Y.M.: Fuzzy adaptive observer backstepping control for MIMO nonlinear systems. *Fuzzy Sets Syst.* **160**, 2755–2775 (2009)
- Zhou, W.D., Liao, C.Y., Zheng, L., Liu, M.M.: Adaptive fuzzy output feedback control for a class of nonaffine nonlinear systems with unknown dead-zone input. *Nonlinear Dyn.* **79**, 2609–2621 (2015)
- Shi, W.X.: Adaptive fuzzy control for multi-input multi-output nonlinear systems with unknown dead-zone inputs. *Appl. Soft Comput. J.* **30**, 36–47 (2015)
- Tong, S., Li, Y.: Adaptive fuzzy output feedback tracking backstepping control of strict-feedback nonlinear systems with unknown dead zones. *IEEE Trans. Fuzzy Syst.* **20**, 168–180 (2012)
- Wang, H.Q., Liu, X.Q., Liu, P.X.Q., Li, S.: Robust adaptive fuzzy fault-tolerant control for a class of non-lower-triangular nonlinear systems with actuator failures. *Inf. Sci.* **336**, 60–74 (2016)
- Chen, W.S., Zhang, Z.Q.: Globally stable adaptive backstepping fuzzy control for output-feedback systems with unknown high-frequency gain sign. *Fuzzy Sets Syst.* **161**, 821–836 (2010)
- Wang, F., Liu, Z., Zhang, Y., Philip, C.L.: Adaptive quantized controller design via backstepping and stochastic small-gain approach. *IEEE Trans. Fuzzy Syst.* **24**, 330–343 (2016)
- Lai, G.Y., Liu, Z., Zhang, Y., Chen, P.: Asymmetric actuator backlash compensation in quantized adaptive control of uncertain networked nonlinear systems. *IEEE Trans. Neural Netw. Learn. Syst.* **99**, 1–14 (2015)
- Ku, R.T., Athans, M.: Further results on the uncertainty threshold principle. *IEEE Trans. Autom. Control* **22**, 866–868 (1977)
- Tao, G., Kokotovic, P.V.: Adaptive control of plants with unknown dead-zones. *IEEE Trans. Autom. Control* **39**, 59–68 (1994)
- Zhou, J., Wen, C., Zhang, Y.: Adaptive output control of nonlinear systems with uncertain dead zone nonlinearity. *IEEE Trans. Autom. Control* **51**, 504–511 (2006)
- Alonso-Quesada, S., DelaSen, M., Bilbao-Guillerna, A., Ibeas, A.: A semiempirical reduced-order identification modeling tool for partially unknown discrete-time plants by using a multi-estimation scheme. *Instrum Sci. Technol.* **35**, 419–436 (2007)
- Wang, F., Liu, Z., Zhang, Y., Chen, X., Chen, C.L.P.: Adaptive fuzzy dynamic surface control for a class of nonlinear systems

- with fuzzy dead zone and dynamic uncertainties. *Nonlinear Dyn.* **79**, 1693C1709 (2015)
20. Wang, X.J., Wang, S.P.: Adaptive fuzzy robust control of PMSM with smooth inverse based dead zone compensation. *Int. J. Control Autom. Syst.* **14**, 378–388 (2016)
  21. Liu, Z., Wang, F., Zhang, Y., Chen, X., Chen, C.L.P.: Adaptive tracking control for a class of nonlinear systems with a fuzzy dead-zone input. *IEEE Trans. Fuzzy Syst.* **23**, 193–203 (2015)
  22. Yoo, S.J.: Decentralised fault compensation of time-delayed interactions and dead-zone actuators for a class of large-scale non-linear systems. *IET Control Theory Appl.* **9**, 1461–1471 (2015)
  23. Wang, H.Q., Liu, X.Q., Liu, K.F.: Robust adaptive neural tracking control for a class of stochastic nonlinear interconnected systems. *IEEE Trans. Neural Netw. Learn. Syst.* **27**, 510–523 (2016)
  24. Li, D.J.: Adaptive neural network control for unified chaotic systems with dead-zone input. *J. Vib. Control* **21**, 2446–2451 (2015)
  25. Zhang, Z.Q., Xu, S.Y., Zhang, B.Y.: Exact tracking control of nonlinear systems with time delays and dead-zone input. *Automatica* **52**, 272–276 (2015)
  26. Zhang, B., Mao, Z.Z.: Adaptive control of stochastic Hammerstein systems with dead-zone input nonlinearity. *Trans. Inst. Meas. Control* **37**, 746–759 (2015)
  27. Wigren, Torbjorn, Nordsjo, Anders E.: Compensation of the RLS algorithm for output nonlinearities. *IEEE Trans. Autom. Control* **44**, 1913–1918 (1999)
  28. Liu, Z., Lai, G.Y., Zhang, Y., Chen, P.C.L.: Adaptive fuzzy tracking control of nonlinear time-delay systems with dead zone output mechanism based on a novel smooth model. *IEEE Trans. Fuzzy Syst.* **10**, 1–13 (2015)
  29. Wang, F., Liu, Z., Lai, G.Y.: Fuzzy adaptive control of nonlinear uncertain plants with unknown dead zone output. *Fuzzy Sets Syst.* **263**, 27–48 (2015)
  30. Nussbaum, Roger D.: Some remarks on adaptive control a conjecture in parameter. *Syst. Control Lett.* **3**, 243–246 (1983)
  31. Ge, S.C., Hong, F., Lee, T.H.: Adaptive neural control of nonlinear time-delay systems with unknown virtual control coefficients. *IEEE Trans. Syst. Man Cybern. Part B: Cybern.* **34**, 499–517 (2004)
  32. Zhang, T.P., Wen, H., Zhu, Q.: Adaptive fuzzy control of nonlinear systems in pure feedback form based on input-to-state stability. *IEEE Trans. Fuzzy Syst.* **18**, 80–93 (2010)
  33. Li, Y.M., Tong, S.C., Li, T.S.: Adaptive fuzzy output feedback dynamic surface control of interconnected nonlinear pure-feedback systems. *IEEE Trans. Cybern.* **45**, 138–149 (2015)
  34. Zou, A.M., Hou, Z.G., Tan, M.: Adaptive control of a class of nonlinear pure-feedback systems using fuzzy backstepping approach. *IEEE Trans. Fuzzy Syst.* **16**, 886–897 (2008)
  35. Wang, F., Liu, Z., Zhang, Y., Chen, P.C.L.: Adaptive fuzzy control for a class of stochastic pure-feedback nonlinear systems with unknown hysteresis. *IEEE Trans. Fuzzy Syst.* **10**, 1–12 (2015)
  36. Li, Y.M., Tong, S.C.: Adaptive fuzzy output-feedback control of pure-feedback uncertain nonlinear systems with unknown dead-zone. *IEEE Trans. Fuzzy Syst.* **22**, 1341–1347 (2014)
  37. Li, Y.M., Tong, S.C., Li, T.S.: Observer-based adaptive fuzzy tracking control of MIMO stochastic nonlinear systems with unknown control direction and unknown dead-zones. *IEEE Trans. Fuzzy Syst.* **23**, 1228–1241 (2015)



**Zhikang Lin** received the B.S. degree from Heze University, China, in 2014. He is working toward the M.S. degree in Shandong University of Science and Technology. His current research interests include fuzzy control, adaptive control and stochastic control.



**Xikui Liu** received the M.S. degree from Shandong University of Science and Technology, and the Ph.D. degree from Huazhong University of Science and Technology, China, in 2000 and 2004, respectively. He is an Associate Professor of Shandong University of Science and Technology. His interests include graph theory and DNA computing.



**Yan LI** received M.S. and Ph.D. degrees from Shandong University of Science and Technology, China, in 2006 and 2015, respectively. She is a Lecture of Shandong University of Science and Technology. Her research interests include linear and nonlinear stochastic control.