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Hesitant Uncertain Linguistic *Z*-Numbers and Their Application in Multi-criteria Group Decision-Making Problems

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Abstract This paper introduces hesitant uncertain linguistic *Z*-numbers (HULZNs) based on *Z*-numbers and linguistic models. HULZNs can serve as a reliable tool to depict complex and uncertain decision-making information and reflect the hesitancy of DMs. This paper focuses on the development of an innovative method to address multi-criteria group decision-making (MCGDM) problems in which the weight information is incompletely known. Handling qualitative information requires the effective support of quantitative tools, after which the linguistic scale function is employed to deal with linguistic information. First, the operations and distance of HULZNs are defined. Then, two power aggregation operators for HULZNs are proposed. Subsequently, a new MCGDM approach is developed by incorporating the power aggregation operators and the VIKOR model. Finally, an illustrative example of ERP system selection is provided for demonstration, and the feasibility and validity of the proposed method are further verified by sensitivity analysis and comparison with an existing method.

Keywords Multi-criteria group decision-making · Hesitant uncertain linguistic *Z*-numbers · Linguistic scale function · Power aggregation operators · VIKOR method

1 Introduction

To deal with fuzzy information, Zadeh [1] proposed fuzzy sets (FSs), which are now considered to be useful tools for decision-making problems [2], pattern recognition [3], and fuzzy inference [4]. However, in some cases, the membership degree alone cannot describe the information precisely. In order to address the uncertainty of non-membership degree, Atanassov [5] introduced intuitionistic fuzzy sets (IFSs). Since their introduction, IFSs have been researched in great detail, and some extensions of IFSs have been developed and applied to multi-criteria decision-making (MCDM) problems [6, 7]. As an extension of traditional fuzzy sets, hesitant fuzzy sets (HFSs) were firstly introduced by Torra and Narukawa [8], which permit the membership degree of an element to be a set of several possible values in [0, 1]. The main purpose of HFSs is to model the uncertainty produced by human doubt when eliciting information [9]. However, there is a limitation that the reliability of decision-making information presented by these classic fuzzy sets is not well taken into account [10]. *Z*-numbers, a new fuzzy-theoretic concept, are proposed by Zadeh [11] to counter this limitation. A *Z*-number is an ordered pair of fuzzy numbers, $Z = (A, B)$, which has a straightforward structure with constraint A and reliability B.

In recent years, researchers from various fields have studied *Z*-numbers and identified a wide range of applications. In general, current researches into *Z*-numbers can be roughly divided into four categories. The first of the four categories is fundamental theory studies. Velammal and Bhanu [12] introduced a new type of intuitionistic *Z*-numbers and their operations. Aliev et al. [13] introduced a direct computation method for *Z*-numbers according to the paradigm of expected utility. Bhanu and Velammal [14] studied how to obtain sum and product operations of *Z*-valuations. Aliev et al. [15] defined some operations of discrete *Z*-numbers according to the general ideas underlying computation with continuous *Z*-numbers, as put forward by Zadeh [11]. Although computation with *Z*-numbers is an important issue, the existing operations for *Z*-numbers are still too

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complicated to apply extensively in practice. The second category of research focuses on the extension of Z-numbers into a tool for computing with words (CW). Patel et al. [16] developed new constructs for Z-numbers in generalized constraint language (GCL) and proposed a new methodology for modelling Z-numbers based on CWShell (a working CW inference engine toolkit). Banerjee and Pal [17] extended Z-numbers into a tool for level-2 CW and designed a Minsky Society for mind-based natural language comprehending machine-mind architecture. The third category addresses some indirect transformation methods through which Z-numbers are converted into classic fuzzy numbers, such as triangular fuzzy numbers, trapezoidal fuzzy numbers, interval-valued fuzzy numbers, and standardized generalized fuzzy numbers [18–20]. The fourth category proposes some decision-making methods based on Z-numbers. Kang et al. [10] developed a new MCDM method to address linguistic decision-making problems based on the arithmetic operations of Z-numbers. Yaakob and Gegov [21] introduced a modified TOPSIS method to deal with stock selection MCGDM problems based on Z-numbers. Kang et al. [18] proposed a new MCDM method for supplier selection by considering the information transformation of Z-numbers. However, the latter two categories of research into Z-numbers inevitably involve the loss and distortion of original information.

Based on the literature discussed above, we know that Z-numbers have more power to describe decision-making information and are considered a more generalized notion than real numbers, interval numbers, random numbers, and fuzzy numbers. However, the nature of general Z-numbers is more complex than that of classic fuzzy numbers, and some theories about Z-numbers are difficult to study and have not yet been figured out. Therefore, in order to effectively invoke the reliability and applicability of Z-numbers to address practical problems, it is necessary to conduct further research into specific types of Z-numbers or a subset of Z-numbers via mature fuzzy decision-making methods.

It is worth noting that the two components contained in Z-numbers are typically described in natural languages [11]. In fact, the best expression of DM's preferences or opinions takes a natural linguistic form in most practical problems because of the complexity of decision-making problems and the inherent vagueness of human preferences. For example, in green product design selection problems, when evaluating the product's recycling potential and re-usability, labels like 'poor', 'fair', and 'good' can be appropriately employed. Furthermore, in most complex and ill-defined decision-making environment, it is preferable for a DM to employ linguistic variables [22] rather than real numbers in his or her assessment. The linguistic variable is a valid tool because the use of linguistic information reinforces the flexibility and reliability of classical decision models [23].

In recent years, linguistic variables whose values are words or sentences from natural or artificial languages have been researched extensively and applied in various fields [24, 25]. Xu [26] defined operations and developed some aggregation operators for linguistic variables represented by a single linguistic term. Subsequently, Xu [27] proposed uncertain linguistic variables, which employ a linguistic interval rather than a single linguistic value in order to depict fuzzy information and suggest the probability that all linguistic values in an interval are equal or obey a specific distribution [28, 29]. With the promotion of preliminary linguistic models, some extended linguistic concepts have been developed. Some linguistic sets consisting of several discrete linguistic terms have been proposed based on linguistic variables and HFSs, such as hesitant fuzzy linguistic term sets (HFLTSs) [30], hesitant fuzzy linguistic sets (HFLSs) [31], linguistic hesitant fuzzy sets (LHFSs) [32], and multi-hesitant fuzzy linguistic term sets (MHFLTSs) [33]. Dealing with decision-making problems that utilize linguistic information implies the need for CW [34]. Many computational models have been developed to deal with linguistic information, and the primary ones can be briefly listed as follows: directly making use of linguistic labels [26, 27, 35, 36], representing linguistic information by fuzzy membership functions [37, 38], resorting to the 2-tuple fuzzy linguistic representation model [39–41], applying the novel cloud model to promote the transformation of linguistic information [42], and employing the linguistic scale function to accommodate different semantic circumstances [32, 33, 43, 44, 49].

Taking into account the widespread applicability of Z-numbers and the effectiveness of the linguistic models discussed above, this paper proposes hesitant uncertain linguistic Z-numbers (HULZNs), which can be treated as a special form or a subclass of Z-numbers. Because the first component of Z-numbers plays the role of fuzzy restriction, an uncertain linguistic variable can be employed to characterize it. Utilizing an interval linguistic value to describe the fuzzy restriction is more appropriate than using a single linguistic value. In reality, the reliability of the first component provided by a DM usually fluctuates among several possible linguistic values; therefore, several consecutive or discrete linguistic terms can be used to depict this hesitance, more effectively presenting incomplete information and providing richer expression than a single linguistic term. Thus, a HULZN can be constructed by integrating an uncertain linguistic variable and several linguistic values. HULZNs can easily represent most decision-making information in real life.

For example, when several DMs are evaluating the flexibility of an ERP system, they each first give a consistent fuzzy restriction using an uncertain linguistic variable, like [*good*, *very good*]; then, each DM provides a linguistic value, or more than one due to hesitance, which denotes the reliability of the given fuzzy restriction, like {*certain*, *somewhat certain*, *very certain*}. Therefore, the evaluation value can be depicted as ([*good*, *very good*], {*certain*, *somewhat certain*, *very certain*}). HULZNs have many advantages; for one, they are more comprehensive and reliable than classic fuzzy sets because they impose the effective representation structure of *Z*-numbers. HULZNs are also more feasible in realistic decision-making processes because they employ linguistic methods, and they can appropriately represent the hesitancy of DMs in expressing their preferences under complex decision-making circumstances by utilizing hesitant values.

The VlseKriterijum-ska Optimizacija I Kompromisno Resenje (VIKOR) method was proposed by Opricovic [45] for multi-attribute optimization of complex problems. The VIKOR method focuses on choosing and ranking options from a set of alternatives with conflicting attributes. This method can identify a compromise solution that consists of one or several feasible alternatives by calculating a maximum group utility for the majority and a minimum individual regret for the opponent, both of which are related to the distances among the evaluation values, PISs, and NISs under each criterion. The VIKOR method has been applied to deal with various linguistic decision-making problems [46, 47]. This method is a reliable ranking tool usually applied in MCDM problems, but it cannot independently solve MCGDM problems. In order to extend the classic VIKOR method to address MCGDM problems with HULZNs, an information fusion tool is needed.

Aggregation operators are important tools for facilitating information fusion in decision-making problems, and they represent a consistently active topic of research. Most aggregation operators suppose that the arguments are mutually independent; however, aggregated values may be correlative in many practical problems. To deal with these kinds of problems, interrelationships among aggregated values must be considered, creating an opportunity to utilize the power average (PA) operator [48]. Proposed by Yager, the PA operator is one of the most important information fusion tools applied in decision-making problems [49, 50]. The PA operator takes into account the interrelationships among aggregated values and allows arguments to support each other in the aggregation process.

The main purpose of this paper is to develop a new MCGDM method for dealing with practical decision-making problems in which the weight information is incompletely known. The evaluation values of alternatives under given criteria are presented in the form of HULZNs; then, a comprehensive approach is developed by integrating the proposed power aggregation operators and the extended VIKOR method. To do this, the rest part of this paper is organized as follows. In Sect. 2, some concepts, such as linguistic term sets, uncertain linguistic variable, linguistic scale functions, and *Z*-numbers, are reviewed briefly. In Sect. 3, the concept of HULZNs is defined, and their operations, comparison method, and distance are proposed. In Sect. 4, two hesitant uncertain linguistic *Z*-numbers aggregation operators are developed. In Sect. 5, the proposed power aggregation operators and extended VIKOR method are combined to develop an innovative MCGDM method. In Sect. 6, an illustrative example is used to verify the validity of the proposed approach, and a sensitivity analysis and comparison analysis are conducted. Finally, the conclusion is drawn in Sect. 7.

2 Preliminaries

This section will introduce concepts including linguistic term sets, uncertain linguistic variable, linguistic scale functions, and *Z*-numbers, which are necessary to the subsequent analysis.

2.1 Linguistic Term Sets

Let $S = \{s_i \mid i = 0, 1, 2, \ldots, 2t\}$ be a finite and completely ordered discrete term set with odd cardinality, where t is a nonnegative integer and s_i represents a possible value for a linguistic variable. It is required that s_i and s_j satisfy the following properties [22]: (1) The set is ordered $s_i \le s_j$, if and only if $i \le j$, and (2) the set obeys negation operation $\text{neg}(s_i) = s_j$, if $i + j = 2t$.

The linguistic term set $S = \{s_i \mid i = 0, 1, 2, \ldots, 2t\}$ is a discrete set, but a continuous set must be employed to solve practical decision-making problems, especially in the process of aggregation operation. Xu [26] extended the discrete linguistic term set to the continuous form, and it can be expressed as $\tilde{S} = \{s_i | i \in k\}$, where k is a large positive real number, and $s_i > s_j$ if $i > j$. If $s_i \in S$, then s_i is the original linguistic term; if $s_i \notin S$, then s_i is the virtual linguistic term. Usually, original linguistic terms are utilized to evaluate alternatives, and virtual linguistic terms only appear in the operations to avoid information loss and enhance the decision-making process.

2.2 Uncertain Linguistic Variable

Definition 1 [27] Let $\tilde{s} = [s_a, s_b]$, $s_a, s_b \in \tilde{S}$, and $0 \le a \le b$; s_a and s_b are the lower and upper limit of $\tilde{s}$, respectively. Then, $\tilde{s}$ is called an uncertain linguistic variable.

2.3 Linguistic Scale Functions

The transformation from linguistic terms to numerical values requires effective support from quantitative tools. The linguistic scale function, which can assign different semantic values to linguistic terms under different situations, has been developed to express semantics more flexibly and impose original information more effectively.

Definition 2 [43] Let $s_i \in S$ be a linguistic term in which $S = \{s_i \mid i = 0, 1, 2, \ldots, 2t\}$. If $\theta_i \in [0, 1]$ is a numerical value, then the linguistic scale function is mapped from s_i to $\theta_i (i = 0, 1, \ldots, 2t)$, and it is defined as follows:

$$H : s_i \rightarrow \theta_i (i = 0, 1, \ldots, 2t), \tag{1}$$

where $0 \leq \theta_0 < \theta_1 < \cdots < \theta_{2t} \leq 1$. It is noted that θ_i reflects the preference of a DM who chooses s_i, so the function H illustrates the semantics of s_i in fact. The linguistic scale function is a strictly monotonously increasing function with respect to the subscript i. Three kinds of linguistic scale functions are shown as follows:

1. The following function is defined based on the subscript function $\mathrm{sub}(s_i) = i$.

   $$\mathrm{LSF1}: \quad H_1(s_i) = \theta_i = \frac{i}{2t}(i = 0, 1, \ldots, 2t).$$

   The evaluation scale of the linguistic information presented above is divided evenly.
2. The following function is defined based on the exponential scale.

   $$\mathrm{LSF2}: \quad H_2(s_i) = \theta_i = \begin{cases} \dfrac{a^t - a^{t-i}}{2a^t - 2}(i = 0, 1, 2, \ldots, t) \\ \dfrac{a^t + a^{i-t} - 2}{2a^t - 2}(i = t+1, t+2, \ldots, 2t) \end{cases}.$$

   Several researches have investigated the parameter a, which generally lies in the interval [1.36, 1.4] [51]. With the extension from the middle of the given linguistic term to both ends, the absolute deviation between adjacent linguistic terms also increases.
3. The following function is defined based on the prospect theory.

   $$\mathrm{LSF3}: \quad H_3(s_i) = \theta_i = \begin{cases} \dfrac{t^\alpha - (t-i)^\alpha}{2t^\alpha}(i = 0, 1, 2, \ldots, t) \\ \dfrac{t^\beta + (i-t)^\beta}{2t^\beta}(i = t+1, t+2, \ldots, 2t) \end{cases}.$$

   Several experiments have been conducted to determine $\alpha = \beta = 0.88$ [52]. If $\alpha = \beta = 1$, then $H_3(s_i)$ is reduced to $H_1(s_i)$. With the extension from the middle of the given linguistic term to both ends, the absolute deviation between adjacent linguistic terms decreases.

To preserve all of the given information and facilitate calculations, the above function can be extended to $H^* : \tilde{S} \rightarrow R^+$, which satisfies $H^*(s_i) = \theta_i$ as a strictly monotonously increasing and continuous function. Therefore, the inverse function of H^* exists, and it can be marked as H^{*-1}.

Based on the preceding discussion, we know that each linguistic scale function possesses exclusive characteristics, which can be graphically shown in Fig. 1 (suppose $t = 4$, $a = 1.4$, and $\alpha = \beta = 0.88$).

2.4 Z-Number

Definition 3 [11] A *Z*-number is an ordered pair of fuzzy numbers denoted as $Z = (A, B)$. It is associated with a real-valued uncertain variable X, where the first component A is a fuzzy restriction on the values that X can take, and the second component B is a measure of reliability of the first component. Typically, A and B are depicted in a natural language, such as (*fair*, *unlikely*), and (*good*, *likely*).

3 Hesitant Uncertain Linguistic *Z*-Numbers and its Relevant Concepts

This section introduces hesitant uncertain linguistic *Z*-numbers (HULZNs) based on linguistic models and *Z*-numbers. Subsequently, some operations, a comparison method, and distance measurement for HULZNs are provided.

3.1 Hesitant Uncertain Linguistic *Z*-Numbers

Definition 4 Let X be a universe of discourse, and $S = \{s_0, s_1, \ldots, s_{2t}\}$, $S' = \{s'_0, s'_1, \ldots, s'_{2r}\}$ be two finite and

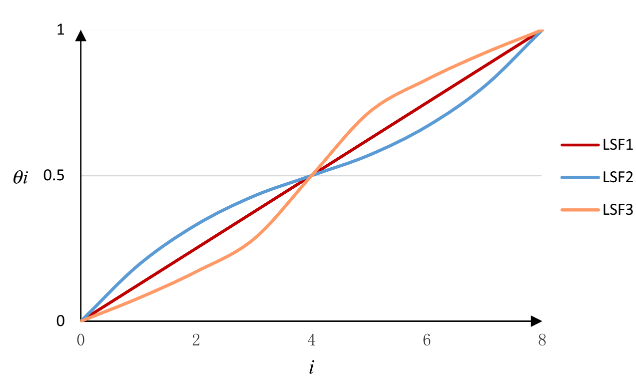

Fig. 1 Illustration of LSF1, LSF2, and LSF3

completely ordered discrete linguistic term sets with odd cardinality, where S and S' represent different semantic situations, and l, $r \in N$. A hesitant uncertain linguistic Z-number (HULZN) on X can be defined in terms of a function that when applied to X returns a subset of Ω as follows:

$$Z = \{(x, A_z(x), B_z(x)) | x \in X\}, \tag{2}$$

where $A_z(x) = [a^l(x), a^u(x)] \subseteq [s_0, s_{2l}]$ is an uncertain linguistic variable, and $B_z(x) = \phi_B(x) = \cup_{b(x)\in\phi_B(x)}\{b(x)\}$ is a set of several linguistic terms from S'. The first component $A_z(x)$ is a fuzzy restriction on the values that X can take, and the second component $B_z(x)$ is a reliability measure of the first component. Usually, the linguistic term sets S and S' are different, and they represent distinct linguistic preference information.

When the two components of a Z-number are described using natural language, the meanings of $A_z(x)$ and $B_z(x)$ are reflected in the corresponding membership functions, which are illustrated by the linguistic information [11]. For a HULZN, the membership function presented by the uncertain linguistic variable $[a^l(x), a^u(x)]$ is a distribution function of random variable with uniform distribution, and the membership function presented by the several linguistic values $\cup_{b(x)\in\phi_B(x)}\{b(x)\}$ is a distribution function of discrete random variables.

In general, the concept of a Z-number can be generalized in various ways [11], and the two components contained in Z-numbers can be described in many forms, including sentences, linguistic values, and fuzzy numbers. Therefore, HULZNs, depicted by two kinds of specific linguistic variables, are merely a special case of Z-numbers.

When X includes only one element, then HULZNs are reduced to a hesitant uncertain linguistic Z-number (HULZN), denoted by $z = (A_z(x), B_z(x))$, and HULZN is an element of HULZNs. Moreover, when $B_z(x) = \{b\}$ has only one linguistic value, the reliability of the fuzzy restriction is b; in this case, HULZN degenerates to an uncertain linguistic Z-number (ULZN). Especially, if $a^l = a^u$ and $B_z(x)$ has only one linguistic value, HULZN is reduced to a linguistic Z-number (LZN). Therefore, HULZN, ULZN, and LZN are all special cases of HULZNs.

3.2 Operations of HULZNs

In the following, some essential algorithms of HULZNs can be established by utilizing linguistic scale functions.

Definition 5 Let $z_i = \left(\left[a_i^l, a_i^u\right], \phi_{B_i}\right)$ and $z_j = \left(\left[a_j^l, a_j^u\right], \phi_{B_j}\right)$ be two arbitrary HULZNs, f^* and g^* be two different linguistic scale functions, where f^{*-1} and g^{*-1} are the inverse functions of f^* and g^*, respectively, and $\lambda > 0$. Then, the operations of HULZNs can be defined as follows:

1. $z_i \oplus z_j = \left(\left[f^{*-1}\left(f^*\left(a_i^l\right) + f^*\left(a_j^l\right)\right), f^{*-1}\left(f^*\left(a_i^u\right) + f^*\left(a_j^u\right)\right)\right], \cup_{b_i\in\phi_{B_i}, b_j\in\phi_{B_j}}\left\{g^{*-1}\left(\frac{f^*(a_i)g^*(b_i)+f^*(a_j)g^*(b_j)}{f^*(a_i)+f^*(a_j)}\right)\right\}\right)$,
   where $f^*(a_i) = f^*\left(a_i^l\right) + f^*\left(a_i^u\right)$, and $f^*(a_j) = f^*\left(a_j^l\right) + f^*\left(a_j^u\right)$;
2. $\lambda z_i = \left(\left[f^{*-1}\left(\lambda f^*\left(a_i^l\right)\right), f^{*-1}\left(\lambda f^*\left(a_i^u\right)\right)\right], \phi_{B_i}\right)$;
3. $z_i \otimes z_j = \left(\left[f^{*-1}\left(f^*\left(a_i^l\right)f^*\left(a_j^l\right)\right), f^{*-1}\left(f^*\left(a_i^u\right)f^*\left(a_j^u\right)\right)\right], \cup_{b_i\in\phi_{B_i}, b_j\in\phi_{B_j}}\left\{g^{*-1}\left(g^*(b_i)g^*(b_j)\right)\right\}\right)$;
4. $z_i^\lambda = \left(\left[f^{*-1}\left(\left(f^*\left(a_i^l\right)\right)^\lambda\right), f^{*-1}\left(\left(f^*\left(a_i^u\right)\right)^\lambda\right)\right], . \cup_{b_i\in\phi_{B_i}}\left\{g^{*-1}\left(\left(g^*(b_i)\right)^\lambda\right)\right\}\right)$;
5. $\text{neg}(z_i) = \left(\left[f^{*-1}\left(f^*(s_{2l}) - f^*\left(a_i^u\right)\right), f^{*-1}\left(f^*(s_{2l}) - f^*\left(a_i^l\right)\right)\right], \cup_{b_i\in\phi_{B_i}}\left\{g^{*-1}\left(g^*\left(s'_{2r}\right) - g^*(b_i)\right)\right\}\right)$.

It is worth noting that, for a HULZN $z = ([a^l, a^u], \cup \{b_i\})$, the linguistic values a^l, a^u and the linguistic values b_i have different semantics; in this case, two different linguistic scale functions, f^* and g^*, respectively, are employed to deal with them in the operations listed above.

Example 1 Assuming l, $r = 4$, then $S = \{s_0 =$ extremely poor, $s_1 =$ very poor, $s_2 =$ poor, $s_3 =$ slightly poor, $s_4 =$ fair, $s_5 =$ slightly good, $s_6 =$ good, $s_7 =$ very good, $s_8 =$ extremely good$\}$ represent a linguistic term set used to present the fuzzy restriction on the evaluation object; furthermore, $S' = \{s'_0 =$ strongly uncertain, $s'_1 =$ very uncertain, $s'_2 =$ uncertain, $s'_3 =$ somewhat uncertain, $s'_4 =$ neutral, $s'_5 =$ somewhat certain, $s'_6 =$ certain, $s'_7 =$ very certain, $s'_8 =$ strongly certain$\}$ represent a linguistic term set used to evaluate the reliability of the given fuzzy restriction. Let $z_1 = \left([s_3, s_4], \left\{s'_4, s'_5\right\}\right)$ and $z_2 = \left([s_4, s_4], \left\{s'_3, s'_4, s'_5\right\}\right)$ be two HULZNs. If $f^*(s_i) = H_1^*(s_i) = \frac{i}{2t}\,(0 \le i \le 2t)$ and $g^*(s_i) = H_2^*(s_i) = \begin{cases} \dfrac{a^t - a^{t-i}}{2a^t - 2}\,(i = 0, 1, 2, \ldots, t) \\ \dfrac{a^t + a^{i-t} - 2}{2a^t - 2}\,(i = t+1, t+2, \ldots, 2t) \end{cases}$ $(a = 1.37)$, then the following results can be calculated.

1. $z_1 \oplus z_2 = \left([s_7, s_8], \left\{s'_{3.43}, s'_{3.92}, s'_4, s'_{4.51}, s'_{4.57}, s'_5\right\}\right)$;
2. $2z_1 = \left([s_6, s_8], \left\{s'_4, s'_5\right\}\right)$;
3. $z_1 \otimes z_2 = \left([s_{1.5}, s_2], \left\{s'_{1.16}, s'_{1.37}, s'_{1.41}, s'_{1.68}, s'_{2.02}\right\}\right)$;
4. $z_1^2 = \left([s_{1.13}, s_2], \left\{s'_{1.41}, s'_{2.02}\right\}\right)$;

5. $\mathrm{neg}(z_1) = \left([s_4, s_5], \{s'_3, s'_4\}\right)$.

It is quite apparent that all the results obtained above are also HULZNs, and there are some desirable properties in terms of the corresponding operations of HULZNs in the following.

Property 1 *Let $z_1 = \left(\left[a_1^l, a_1^u\right], \phi_{B_1}\right)$, $z_2 = \left(\left[a_2^l, a_2^u\right], \phi_{B_2}\right)$, and $z_3 = \left(\left[a_3^l, a_3^u\right], \phi_{B_3}\right)$ be three arbitrary HULZNs, and λ, λ_1, $\lambda_2 > 0$. Then, the following properties can be easily proved:*

1. $z_1 \oplus z_2 = z_2 \oplus z_1$,
2. $z_1 \otimes z_2 = z_2 \otimes z_1$,
3. $(z_1 \oplus z_2) \oplus z_3 = z_1 \oplus (z_2 \oplus z_3)$,
4. $(z_1 \otimes z_2) \otimes z_3 = z_1 \otimes (z_2 \otimes z_3)$,
5. $\lambda(z_1 \oplus z_2) = \lambda z_1 \oplus \lambda z_2$,
6. $(z_1 \otimes z_2)^\lambda = z_1^\lambda \otimes z_2^\lambda$.

3.3 Comparison Method for HULZNs

Definition 6 Let $z_1 = \left(\left[a_1^l, a_1^u\right], \phi_{B_1}\right)$ and $z_2 = \left(\left[a_2^l, a_2^u\right], \phi_{B_2}\right)$ be two arbitrary HULZNs, where all elements in $\phi_{B_j} (j = 1, 2)$ are arranged in ascending order. Let $b_{\phi_{B_j}}^{\sigma(i)} (j = 1, 2)$ be referred to as the *ith* value in ϕ_{B_j}. Then, the method for comparing these HULZNs is as follows:

1. $z_1 \le z_2$, if $a_1^l \le a_2^l$, $a_1^u \le a_2^u$, $b_{\phi_{B_1}}^{\sigma(i)} \le b_{\phi_{B_2}}^{\sigma(i)}$, and $b_{\phi_{B_1}}^{\sigma\left(l_{\phi_{B_1}}\right)} \le b_{\phi_{B_2}}^{\sigma\left(l_{\phi_{B_2}}\right)}$, where $b_{\phi_{B_1}}^{\sigma(i)} \in \phi_{B_1}$, $b_{\phi_{B_2}}^{\sigma(i)} \in \phi_{B_2}$, $i = 1, 2, \ldots, l_{\phi_B}$, and $l_{\phi_B} = \min\left(l_{\phi_{B_1}}, l_{\phi_{B_2}}\right)$ ($l_{\phi_{B_1}}$ and $l_{\phi_{B_2}}$ are the number of elements in ϕ_{B_1} and ϕ_{B_2}, respectively);
2. $z_1 = z_2$, if $z_1 \le z_2$ and $z_1 \ge z_2$.

Although a complete ordering of all HULZNs cannot be determined based on the preceding definition, it is sufficient to demonstrate the property of the proposed generalized distance of HULZNs in the following.

3.4 Distance of HULZNs

Definition 7 Let $z_i = \left(\left[a_i^l, a_i^u\right], \phi_{B_i}\right)$ and $z_j = \left(\left[a_j^l, a_j^u\right], \phi_{B_j}\right)$ be two arbitrary HULZNs, f^* and g^* be two different linguistic scale functions, and $\lambda > 0$. Then, the generalized distance between two HULZNs can be defined as follows:

$$
\begin{aligned}
d_G(z_i, z_j) = {} & \left[\frac{1}{2}\left(\left|f^*\left(a_i^l\right) - f^*\left(a_j^l\right)\right|^\lambda + \left|f^*\left(a_i^u\right) - f^*\left(a_j^u\right)\right|^\lambda\right)\right]^{\frac{1}{\lambda}} \\
& + \left[\frac{1}{4}\left(\frac{1}{l_{\phi_{B_i}}} \sum_{b_i \in \phi_{B_i}} \min_{b_j \in \phi_{B_j}} \left|f^*\left(a_i^l\right) g^*(b_i) - f^*\left(a_j^l\right) g^*(b_j)\right|^\lambda \right.\right. \\
& + \frac{1}{l_{\phi_{B_i}}} \sum_{b_i \in \phi_{B_i}} \min_{b_j \in \phi_{B_j}} \left|f^*\left(a_i^u\right) g^*(b_i) - f^*\left(a_j^u\right) g^*(b_j)\right|^\lambda \\
& + \frac{1}{l_{\phi_{B_j}}} \sum_{b_j \in \phi_{B_j}} \min_{b_i \in \phi_{B_i}} \left|f^*\left(a_j^l\right) g^*(b_j) - f^*\left(a_i^l\right) g^*(b_i)\right|^\lambda \\
& \left.\left. + \frac{1}{l_{\phi_{B_j}}} \sum_{b_j \in \phi_{B_j}} \min_{b_i \in \phi_{B_i}} \left|f^*\left(a_j^u\right) g^*(b_j) - f^*\left(a_i^u\right) g^*(b_i)\right|^\lambda \right)\right]^{\frac{1}{\lambda}}.
\end{aligned} \tag{3}
$$

where $l_{\phi_{B_i}}$ and $l_{\phi_{B_j}}$ are the number of elements in ϕ_{B_i} and ϕ_{B_j}, respectively.

Property 2 *Let $z_1 = \left([a_1^l, a_1^u], \phi_{B_1}\right)$, $z_2 = \left([a_2^l, a_2^u], \phi_{B_2}\right)$, and $z_3 = \left([a_3^l, a_3^u], \phi_{B_3}\right)$ be three arbitrary HULZNs, let all elements in HULZNs be arranged in ascending order, and let $b_{\phi_{B_j}}^{\sigma(i)} (j = 1, 2, 3)$ be referred to as the ith value in ϕ_{B_j}. Then, the generalized distance defined above satisfies the following properties:*

1. $d_G(z_1, z_2) \ge 0$,
2. $d_G(z_1, z_2) = d_G(z_2, z_1)$,
3. If $z_1 \le z_2 \le z_3$, then $d_G(z_1, z_2) \le d_G(z_1, z_3)$, and $d_G(z_2, z_3) \le d_G(z_1, z_3)$.

4 Power Aggregation Operators with HULZNs

This section extends the PA operator to situations in which the input arguments consist of HULZNs. Additionally, two aggregation operators for HULZNs are provided, along with their desirable properties.

Definition 8 [48] Let a_1, a_2, …, a_n be n positive real numbers and Ω be the set of all given values. Then, the PA operator is the mapping PA: $\Omega^n \to \Omega$, defined as

$$
\mathrm{PA}(a_1, a_2, \ldots, a_n) = \frac{\sum_{i=1}^n (1 + T(a_i)) a_i}{\sum_{i=1}^n (1 + T(a_i))}, \tag{4}
$$

where $T(a_i) = \sum_{\substack{j=1 \\ i \ne j}}^n \mathrm{Sup}\left(a_i, a_j\right)$ and $\mathrm{Sup}(a_i, a_j)$ is the support degree of a_i from a_j, which has the following properties:

1. $\mathrm{Sup}(a_i, a_j) \in [0, 1]$,
2. $\mathrm{Sup}(a_i, a_j) = \mathrm{Sup}(a_j, a_i)$,

3. $\text{Sup}(a_i, a_j) > \text{Sup}(x, y)$, if $d(a_i, a_j) < d(x, y)$, where $d(a_i, a_j)$ indicates the distance between a_i and a_j.

4.1 Power Weighted Average Operator with HULZNs

Definition 9 Let $z_i = \left(\left[a_i^l, a_i^u\right], \phi_{B_i}\right)(i = 1, 2, \ldots, n)$ be a collection of HULZNs, Ω be the set of all HULZNs, and $w = (w_1, w_2, \ldots, w_n)$ be the weight vector of z_i $(i = 1, 2, \ldots, n)$, with $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. Then, the hesitant uncertain linguistic Z-numbers power weighted average (HULZPWA) operator is the mapping HULZPWA: $\Omega^n \to \Omega$, and can be defined as follows:

$$\begin{aligned}\text{HULZPWA}(z_1, z_2, \ldots, z_n) &= \frac{w_1(1 + T(z_1))z_1}{\sum_{i=1}^n w_i(1 + T(z_i))} \\ &\oplus \frac{w_2(1 + T(z_2))z_2}{\sum_{i=1}^n w_i(1 + T(z_i))} \\ &\oplus \cdots \oplus \frac{w_n(1 + T(z_n))z_n}{\sum_{i=1}^n w_i(1 + T(z_i))} \\ &= \sum_{i=1}^n \frac{w_i(1 + T(z_i))z_i}{\sum_{i=1}^n w_i(1 + T(z_i))},\end{aligned} \tag{5}$$

where $T(z_i) = \sum_{j=1, j\neq i}^n w_j \text{Sup}\left(z_i, z_j\right)$ is the comprehensive weighted support degree of z_i from $z_j(j = 1, 2, \ldots, n, j \neq i)$ and $\text{Sup}(z_i, z_j)$ is the support for z_i and z_j.

According to the operations of HULZNs provided in Definition 5, the following result can be obtained.

Theorem 1 *Let* $z_i = \left(\left[a_i^l, a_i^u\right], \phi_{B_i}\right)(i = 1, 2, \ldots, n)$ *be a collection of HULZNs, and* $w = (w_1, w_2, \ldots, w_n)$ *be the weight vector of* $z_i(i = 1, 2, \ldots, n)$*, with* $w_i \in [0, 1]$ *and* $\sum_{i=1}^n w_i = 1$*. Then, the aggregated value calculated by the HULZPWA operator is also a HULZN, and*

$$\begin{aligned}&\text{HULZPWA}(z_1, z_2, \ldots, z_n) \\ &= \left(\left[f^{*-1}\left(\sum_{i=1}^n \frac{w_i(1 + T(z_i))f^*\left(a_i^l\right)}{\sum_{i=1}^n w_i(1 + T(z_i))}\right),\right.\right. \\ &\quad \left. f^{*-1}\left(\sum_{i=1}^n \frac{w_i(1 + T(z_i))f^*\left(a_i^u\right)}{\sum_{i=1}^n w_i(1 + T(z_i))}\right)\right], \\ &\quad \cup_{b_1\in\phi_{B_1}, b_2\in\phi_{B_2}, \ldots, b_n\in\phi_{B_n}} \\ &\quad \left.\times\left\{g^{*-1}\left(\frac{\sum_{i=1}^n \frac{w_i(1+T(z_i))f^*(a_i)}{\sum_{i=1}^n w_i(1+T(z_i))}g^*(b_i)}{\sum_{i=1}^n \frac{w_i(1+T(z_i))f^*(a_i)}{\sum_{i=1}^n w_i(1+T(z_i))}}\right)\right\}\right),\end{aligned} \tag{6}$$

where $T(z_i) = \sum_{j=1, j\neq i}^n w_j \text{Sup}\left(z_i, z_j\right)$ *is the comprehensive weighted support degree of* z_i *from* $z_j(j = 1, 2, \ldots, n, j \neq i)$ *and* $f^*(a_i) = f^*\left(a_i^l\right) + f^*\left(a_i^u\right)$*. Theorem 1 can be easily proved by the mathematical induction of n, and the proof is omitted here.*

Theorem 2 (Boundedness) *Let* $z_i = \left(\left[a_i^l, a_i^u\right], \phi_{B_i}\right)(i = 1, 2, \ldots, n)$ *be a collection of HULZNs, and* $p_1 = \left(\left[a_{p_1}^l, a_{p_1}^u\right], b_{p_1}\right) = \left(\left[\min_i\left(a_i^l\right), \min_i\left(a_i^u\right)\right], \min_{b_l\in\phi_{B_1}\cup\phi_{B_2}\cup\cdots\cup\phi_{B_n}}\{b_l\}\right)$, $p_2 = \left(\left[a_{p_2}^l, a_{p_2}^u\right], b_{p_2}\right) = \left(\left[\max_i\left(a_i^l\right), \max_i\left(a_i^u\right)\right], \max_{b_l\in\phi_{B_1}\cup\phi_{B_2}\cup\cdots\cup\phi_{B_n}}\{b_l\}\right)$, *then* $p_1 \leq$ *HULZPWA*$(z_1, z_2, \ldots, z_n) \leq p_2$.

Proof Let $\text{HULZPWA}(z_1, z_2, \ldots, z_n) = z = \left(\left[a^l, a^u\right], \phi_B\right)$, since $a_{p_1}^l \leq a_i^l$, $a_{p_1}^u \leq a_i^u$ and $b_{p_1} \leq b_l$, where $b_l \in \phi_{B_1} \cup \phi_{B_2} \cup \cdots \cup \phi_{B_n}$. Then, there are

$$\begin{aligned}a_{p_1}^l &= f^{*-1}\left(\sum_{i=1}^n \frac{w_i(1 + T(z_i))f^*\left(a_{p_1}^l\right)}{\sum_{i=1}^n w_i(1 + T(z_i))}\right) \\ &\leq f^{*-1}\left(\sum_{i=1}^n \frac{w_i(1 + T(z_i))f^*\left(a_i^l\right)}{\sum_{i=1}^n w_i(1 + T(z_i))}\right) = a^l, \\ a_{p_1}^u &= f^{*-1}\left(\sum_{i=1}^n \frac{w_i(1 + T(z_i))f^*\left(a_{p_1}^u\right)}{\sum_{i=1}^n w_i(1 + T(z_i))}\right) \\ &\leq f^{*-1}\left(\sum_{i=1}^n \frac{w_i(1 + T(z_i))f^*\left(a_i^u\right)}{\sum_{i=1}^n w_i(1 + T(z_i))}\right) = a^u,\end{aligned}$$

and

$$\begin{aligned}b_{p_l} &= g^{*-1}\left(\frac{\sum_{i=1}^n \frac{w_i(1+T(z_i))f^*(a_i)}{\sum_{i=1}^n w_i(1+T(z_i))}g^*\left(b_{p_l}\right)}{\sum_{i=1}^n \frac{w_i(1+T(z_i))f^*(a_i)}{\sum_{i=1}^n w_i(1+T(z_i))}}\right) \\ &\leq b \in \cup_{b_1\in\phi_{B_1}, b_2\in\phi_{B_2}, \ldots, b_n\in\phi_{B_n}} \\ &\quad \times\left\{g^{*-1}\left(\frac{\sum_{i=1}^n \frac{w_i(1+T(z_i))f^*(a_i)}{\sum_{i=1}^n w_i(1+T(z_i))}g^*(b_i)}{\sum_{i=1}^n \frac{w_i(1+T(z_i))f^*(a_i)}{\sum_{i=1}^n w_i(1+T(z_i))}}\right)\right\}.\end{aligned}$$

Therefore, $p_1 \leq \text{HULZPWA}(z_1, z_2, \ldots, z_n)$ can be obtained according to the comparison method of HULZNs presented in Definition 6. In the same way, $\text{HULZPWA}(z_1, z_2, \ldots, z_n) \leq p_2$ can also be acquired. Thus, $p_1 \leq \text{HULZPWA}(z_1, z_2, \ldots, z_n) \leq p_2$.

Theorem 3 (Commutativity) *Let* $z_i = \left(\left[a_i^l, a_i^u\right], \phi_{B_i}\right)(i = 1, 2, \ldots, n)$ *be a collection of HULZNs, and* $(z_1', z_2', \ldots, z_n')$ *be any permutation of* $(z_1, z_2, \cdots, z_n)$*. If the weight of* z_i *is not relevant to the position of arguments, then HULZPWA*$(z_1, z_2, \ldots, z_n) =$ *HULZPWA*$(z_1', z_2', \ldots, z_n')$.

The support degree of z_i is determined by the distance measurement between z_i and $z_j(j = 1, 2, \ldots, n, j \neq i)$, and it will not be affected by its position in the permutation. In this way, Theorem 3 can be easily proven.

It is noted that the HULZPWA operator does not possess idempotency. For example, let $z = \left([s_5, s_6], \left\{s'_4, s'_5\right\}\right)$, and if $z_1 = z_2 = z$, $\text{Sup}(z_1, z_2) = \text{Sup}(z_2, z_1) = 1$, and $w_1 = w_2 = 0.5$, then $\text{HULZPWA}(z_1, z_2) = \left([s_5, s_6], \left\{s'_4, s'_{4.5}, s'_5\right\}\right) \neq z$. Moreover, it is difficult to consider the monotonicity of HULZPWA operator because the support degree must re-calculate and change when the aggregated variables in HULZPWA operator vary.

4.2 Power Weighted Geometric Operator with HULZNs

Definition 10 Let $z_i = \left(\left[a_i^l, a_i^u\right], \phi_{B_i}\right)(i = 1, 2, \ldots, n)$ be a collection of HULZNs, Ω be the set of all HULZNs, and $w = \{w_1, w_2, \ldots, w_n\}$ be the weight vector of z_i $(i = 1, 2, \ldots, n)$, with $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. Then, the hesitant uncertain linguistic Z-numbers power weighted geometric (HULZPWG) operator is the mapping HULZPWG: $\Omega^n \to \Omega$, and can be defined as follows:

$$\begin{aligned}\text{HULZPWG}(z_1, z_2, \ldots, z_n) &= z_1^{\frac{w_1(1+T(z_1))}{\sum_{i=1}^n w_i(1+T(z_i))}} \otimes z_2^{\frac{w_2(1+T(z_2))}{\sum_{i=1}^n w_i(1+T(z_i))}} \\ &\quad \otimes \cdots \otimes z_n^{\frac{w_n(1+T(z_n))}{\sum_{i=1}^n w_i(1+T(z_i))}} \\ &= \prod_{i=1}^n z_i^{\frac{w_i(1+T(z_i))}{\sum_{i=1}^n w_i(1+T(z_i))}},\end{aligned} \tag{7}$$

where $T(z_i) = \sum_{j=1, j\neq i}^n w_j \text{Sup}\left(z_i, z_j\right)$ is the comprehensive weighted support degree of z_i from $z_j (j = 1, 2, \ldots, n$, $j \neq i)$ and $\text{Sup}(z_i, z_j)$ is the support for z_i and z_j.

According to the operations of HULZNs provided in Definition 5, the following result can be obtained.

Theorem 4 *Let $z_i = \left(\left[a_i^l, a_i^u\right], \phi_{B_i}\right)(i = 1, 2, \ldots, n)$ be a collection of HULZNs, and $w = (w_1, w_2, \ldots, w_n)$ be the weight vector of $z_i (i = 1, 2, \ldots, n)$, with $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. Then, the aggregated value calculated by the HULZPWG operator is also a HULZN, and*

$$\begin{aligned}&\text{HULZPWG}(z_1, z_2, \ldots, z_n) \\ &= \left(\left[f^{*-1}\left(\prod_{i=1}^n \left(f^*\left(a_i^l\right)\right)^{\frac{w_i(1+T(z_i))}{\sum_{i=1}^n w_i(1+T(z_i))}}\right),\right.\right. \\ &\quad \left. f^{*-1}\left(\prod_{i=1}^n \left(f^*\left(a_i^u\right)\right)^{\frac{w_i(1+T(z_i))}{\sum_{i=1}^n w_i(1+T(z_i))}}\right)\right], \\ &\quad \left.\cup_{b_1\in\phi_{B_1}, b_2\in\phi_{B_2}, \ldots, b_n\in\phi_{B_n}} \left\{g^{*-1}\left(\prod_{i=1}^n \left(g^*(b_i)\right)^{\frac{w_i(1+T(z_i))}{\sum_{i=1}^n w_i(1+T(z_i))}}\right)\right\}\right).\end{aligned} \tag{8}$$

where $T(z_i) = \sum_{j=1, j\neq i}^n w_j \text{Sup}\left(z_i, z_j\right)$ is the comprehensive weighted support degree of z_i from $z_j (j = 1, 2, \ldots, n, j \neq i)$.

In the same way, Theorem 4 can be proven through mathematical induction of n. Meanwhile, the HULZPWG operator possesses the properties of boundedness and commutativity, but not idempotency and monotonicity.

5 A MCGDM Method using HULZNs

This section establishes the corresponding optimization models for determining the weights of DMs and criteria. Furthermore, a novel MCGDM method is developed by combining the proposed power aggregation operators and the VIKOR method.

MCGDM problems with HULZNs consist of a group of alternatives, denoted by $A = \{a_1, a_2, \ldots, a_n\}$. Suppose $C = \{c_1, c_2, \ldots, c_m\}$ to be the set of criteria, whose weight vector is $w = (w_1, w_2, \ldots, w_m)$, satisfying $w_j \in [0, 1]$ and $\sum_{j=1}^m w_j = 1$; let $D = \{d_1, d_2, \ldots, d_q\}$ be a finite set of DMs, whose weight vector is $v = (v_1, v_2, \ldots, v_q)$, satisfying $v_k \in [0, 1]$ and $\sum_{k=1}^q v_k = 1$. Then for a DM d_k $(k = 1, 2, \ldots, q)$, the evaluation information of $a_i (i = 1, 2, \ldots, n)$ with respect to $c_j (j = 1, 2, \ldots, m)$ is presented in the form of HULZNs, denoted by $z_{ij}^k = \left(\left[a_{ijk}^l, a_{ijk}^u\right], \phi_{B_{ijk}}\right)$, where $\left[a_{ijk}^l, a_{ijk}^u\right]$ indicates the fuzzy restriction of a_i under criterion c_j and $\phi_{B_{ijk}}$ indicates all possible reliability of the fuzzy restriction. Finally, the decision matrix $R^k = \left(z_{ij}^k\right)_{n\times m}$ can be constructed.

5.1 Determining the Weights of DMs and Criteria Based on Optimization Model

The weights of decision-makers (DMs) and criteria are important parameters in MCGDM problems because they directly influence the accuracy of the final results. In practical MCGDM problems, weight information is usually uncertain. Furthermore, because DMs are selected from distinct backgrounds with different degrees of expertise, they cannot be directly endowed with arbitrary weights or equal weights. Moreover, because of the time pressure, problem complexity, and lack of knowledge, criteria weights also should not be determined according to empirical values or subjectively assigned in advance. As a result, for a practical MCGDM problem, weights for DMs and criteria should be regarded as unknown and to be determined.

In general, two primary methods are used to acquire weight information: the subjective weighting method and the objective weighting method. In the subjective weighting method, DMs directly assign weights information in accordance with their preferences and expertise, but this introduces a great deal of subjective randomness. The objective weighting method imposes relevant mathematical models to calculate the weight coefficient based on the distribution of criteria values, but it completely ignores the preferences and subjective judgement of DMs. In order to address the drawbacks of these two weighting methods, many researchers have combined them, unifying subjectivity and objectivity in the process of determining weight. In practical decision-making problems, it is difficult for DMs to provide accurate weight information; however, they can give partial or incomplete weight information, such as the weight of a DM changes in an interval, and one DM is more important than another. Incomplete information about weights can be divided into the following five categories [53]: $\{w_j \le w_i\}$, $\{w_j - w_i \le t_k\}$, $\{w_j \le t_k w_i\}$, $\{t_k \le w_j \le t_k + \varepsilon_k\}$, and $\{w_j - w_i \le w_k - w_l \mid i \ne k \ne j \ne l\}$, where t_k and ε_k are nonnegative constants.

According to the above discussion, two optimization models are established, based on the distance measurement of HULZNs, to calculate the weights of DMs and criteria in the following.

Similarity measurement, a straightforward principle for consensus degree, has been successfully utilized to distinguish DMs [54, 55]. In principle, similarity measurement states that if the overall similarity of evaluation values in decision matrix $R^k = \left(z_{ij}^k\right)_{n\times m}$ provided by the kth DM is considerably greater than other overall similarity of decision matrix $R^l = \left(z_{ij}^l\right)_{n\times m} (l \ne k)$, then the kth DM gives less conflicting and controversial information and plays a relatively important role in decision-making procedure. Therefore, a greater weight should be endowed with this DM. In contrast, low overall similarity indicates that the DM plays a relatively less important role in decision-making procedure, and a smaller weight should be endowed with this DM.

As a result, the following programming model constructs a method for calculating the weights of DMs using similarity between different individual decision matrixes:

$$\max F(v_k) = \sum_{k=1}^{q}\left(\frac{1}{mn(q-1)}\sum_{l=1,l\ne k}^{q}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(1-\frac{1}{2}d_G\left(z_{ij}^k, z_{ij}^l\right)\right)\right)v_k$$
$$\text{s.t.}\begin{cases} v \in V \\ \sum_{k=1}^{q} v_k = 1 \\ v_k \ge 0 \end{cases} \tag{M − 1}$$

where $1-\frac{1}{2}d_G\left(z_{ij}^k, z_{ij}^l\right) \in [0,1]$ indicates the similarity degree between z_{ij}^k and z_{ij}^l, and the partial weight information of DMs is known and denoted by $v \in V$.

Another programming model can obtain weights for criteria based on the maximizing deviation method. If a MCDM problem features marked differences between any two distinct alternatives' evaluation values under the given criterion c_j, then c_j plays a relatively important role in the decision-making process; therefore, a higher weight should be assigned to c_j. In contrast, if a criterion makes the evaluation values of all alternatives appear to be similar, then this criterion plays a less important role in the decision-making process and should be assigned a low weight.

Therefore, taking into account a collective HULZNs decision matrix $R = (z_{ij})_{n\times m}$, a maximizing deviation model can be established to derive the weights of criteria as follows:

$$\max F(w_j) = \sum_{j=1}^{m} w_j \sum_{i=1}^{n}\sum_{l=1,l\ne i}^{n} d_G(z_{ij}, z_{lj})$$
$$\text{s.t.}\begin{cases} w \in W \\ \sum_{w=1}^{m} w_j = 1 \\ w_j \ge 0 \end{cases} \tag{M − 2}$$

where $d_G(z_{ij}, z_{lj})$ signifies the distance between z_{ij} and z_{lj} and the partial weight information of the criteria is known and denoted by $w \in W$.

5.2 A MCGDM Approach Based on Power Aggregation Operators and VIKOR Model with HULZNs

This subsection integrates the proposed power aggregation operators and VIKOR model to develop a MCGDM method. The main procedures can be described as follows:

Step 1 Normalize the evaluation information.

It is necessary to normalize all evaluation values to the same magnitude grade in order to eliminate the influence of different dimensions on the operation process. The normalization of the decision matrix can be expressed as follows:

For benefit criteria, $z_{ij}^{k\prime} = z_{ij}^k = \left(\left[a_{ijk}^l, a_{ijk}^u\right], \phi_{B_{ijk}}\right)$.

For cost criteria, evaluation values can be normalized using the inverse operation of z_{ij}^k, which is $z_{ij}^{k\prime} = \text{neg}\left(z_{ij}^k\right)$.

The normalized decision matrix is expressed as $R^{k\prime} = \left(z_{ij}^{k\prime}\right)_{n\times m}$.

Step 2 Calculate the support degree.

The support degree $\mathrm{Sup}\left(z_{ij}^{k\prime}, z_{ij}^{l\prime}\right)$ can be calculated utilizing the following formula:

$$\mathrm{Sup}\left(z_{ij}^{k\prime}, z_{ij}^{l\prime}\right) = 1 - \frac{1}{2} d_G\left(z_{ij}^{k\prime}, z_{ij}^{l\prime}\right)(i = 1,2,\ldots,n;\ j = 1,2,\ldots,m;\ k,l = 1,2,\ldots,q), \quad (9)$$

where $d_G\left(z_{ij}^{k\prime}, z_{ij}^{l\prime}\right)$ is the distance between $z_{ij}^{k\prime}$ and $z_{ij}^{l\prime}$, as given in Definition 7. The support degree $\mathrm{Sup}\left(z_{ij}^{k\prime}, z_{ij}^{l\prime}\right)$ completely satisfies the three properties listed in Definition 8.

Step 3 Calculate the weight vector $v = (v_1, v_2, \ldots, v_q)$ of the DMs using model (M-1).

Step 4 Calculate the weights with respect to $z_{ij}^{k\prime}(k = 1,2,\ldots,q)$.

$$\gamma_{ij}^{k\prime} = \frac{v_k\left(1 + T\left(z_{ij}^{k\prime}\right)\right) z_{ij}^{k\prime}}{\sum_{k=1}^{q} v_k\left(1 + T\left(z_{ij}^{k\prime}\right)\right)}, \quad (10)$$

where $T\left(z_{ij}^{k\prime}\right) = \sum_{l=1,l\neq k}^{q} v_j \mathrm{Sup}\left(z_{ij}^{k\prime}, z_{ij}^{l\prime}\right)$ is the comprehensive weighted support degree.

Step 5 Obtain the collective evaluation information.

The individual evaluation values $z_{ij}^{\prime}$ can be aggregated by utilizing the HULZPWA or HULZPWG operators, yielding the collective evaluation matrix $R = \left(z_{ij}\right)_{n\times m}$.

Step 6 Compute the weight vector $w = (w_1, w_2, \ldots, w_m)$ of the criteria using model (M-2).

Step 7 Determine the positive ideal solution (PIS) $Z^+ = \left\{z_1^+, z_2^+, \ldots, z_m^+\right\}$ and negative ideal solution (NIS) $Z^- = \left\{z_1^-, z_2^-, \ldots, z_m^-\right\}$.

The PIS z_j^+ can be obtained as follows:

$$z_j^+ = \left(\left[\max_{i=1,2,\ldots,n}\left(a_{ij}^l\right), \max_{i=1,2,\ldots,n}\left(a_{ij}^u\right)\right], \left\{b_j^+\right\}\right), \quad (11)$$

where $b_j^+ = \cup_{b_{1j}\in\phi_{B_{1j}}, b_{2j}\in\phi_{B_{2j}},\ldots,b_{nj}\in\phi_{B_{nj}}} \max\left\{b_{1j}, b_{2j}, \ldots, b_{nj}\right\}$ and $(j = 1, 2, \ldots, m)$.

The NIS Z^- can be obtained as follows:

$$z_j^- = \left(\left[\min_{i=1,2,\ldots,n}\left(a_{ij}^l\right), \min_{i=1,2,\ldots,n}\left(a_{ij}^u\right)\right], \left\{b_j^-\right\}\right), \quad (12)$$

where $b_j^- = \cup_{b_{1j}\in\phi_{B_{1j}}, b_{2j}\in\phi_{B_{2j}},\ldots,b_{nj}\in\phi_{B_{nj}}} \min\left\{b_{1j}, b_{2j}, \ldots, b_{nj}\right\}$ and $(j = 1, 2, \ldots, m)$.

Step 8 Compute the group utility $U(a_i)$ and individual regret $R(a_i)$ for each alternative $a_i (i = 1, 2, \ldots, n)$.

$$U(a_i) = \sum_{j=1}^{m} w_j \frac{d_G\left(z_{ij}, z_j^+\right)}{d_G\left(z_j^-, z_j^+\right)}, \quad (13)$$

$$R(a_i) = \max_{1\le j\le m}\left\{ w_j \frac{d_G\left(z_{ij}, z_j^+\right)}{d_G\left(z_j^-, z_j^+\right)}\right\}. \quad (14)$$

Step 9 Compute the closeness coefficients for each alternative $a_i (i = 1, 2, \ldots, n)$.

$$Q(a_i) = \alpha \cdot \frac{U(a_i) - U^+}{U^- - U^+} + (1-\alpha) \cdot \frac{R(a_i) - R^+}{R^- - R^+}, \quad (15)$$

where $U^+ = \min_{1\le i\le n} U(a_i)$, $U^- = \max_{1\le i\le n} U(a_i)$, $R^+ = \min_{1\le i\le n} R(a_i)$, $R^- = \max_{1\le i\le n} R(a_i)$,, α is the weight of the strategy of maximum group utility, and 1-α is the weight of individual regret, where $\alpha \in [0, 1]$.

Step 10 Rank the order of all alternatives according to $U(a_i)$, $R(a_i)$, and $Q(a_i)$ $(i = 1, 2, \ldots, n)$.

The smaller $U(a_i)$, $R(a_i)$ and $Q(a_i)$ are, the better the alternative a_i will be. The results are three ranking lists.

Step 11 Obtain the best solution or a set of compromise solutions.

Alternative $a_{(1)}$ whose closeness coefficient $Q(a_{(1)})$ is the minimum value is considered to be the best one if the following two conditions are satisfied:

Condition 1 Acceptable advantage: $Q\left(a_{(2)}\right) - Q\left(a_{(1)}\right) \ge \frac{1}{n-1}$, where $a_{(2)}$ is the alternative with the second position in the ranking list acquired by the closeness coefficient, and n is the number of alternatives;

Condition 2 Acceptable stability in decision-making: $a_{(1)}$ must be the best ranked by the group utility $U(a_i)$ and/or individual regret $R(a_i)$.

If one of the above two conditions is not satisfied, then a set of compromise solutions is proposed.

1. If Condition 2 is not satisfied, then $a_{(1)}$ and $a_{(2)}$ are the compromise solutions;
2. If Condition 1 is not satisfied, then all the alternatives $a_{(t)}(t = 1, 2, \ldots, N)$ are the compromise solutions, where the maximum value of N is determined by the inequality $Q\left(a_{(N)}\right) - Q\left(a_{(1)}\right) < \frac{1}{n-1}$.

6 Illustrative Example

This section describes a practical enterprise resource planning (ERP) system selection problem in order to demonstrate the applicability of the proposed method. Its validity is confirmed through a sensitivity analysis, and its strengths are illustrated by a comparative analysis with other existing approaches.

The following background is adapted from Tian et al. [30]. In recent years, unpredictable and ever-changing business environments have driven many enterprises to expand their market shares and promote customer satisfaction. To address these challenges and achieve competitive advantages, a reliable software system known as an ERP system has emerged in the market. The selection of an ERP system is one of a business' most important investment projects because of the adaptability, high cost, and investment risk that the system brings. Numerous companies plan to implement ERP projects to improve operational efficiency and lower costs.

ABC Machinery Manufacturing Co., Ltd. is a medium-sized automotive component manufacturer in China that is mainly involved in the exploitation, manufacture, and sale of automotive components and mechanical products. The company's management team intends to introduce an integrated ERP system to enhance competitiveness. A professional team was formed to assist in decision-making, consisting of an operational management expert, a general manager, and a production manager, denoted by $\{d_1, d_2, d_3\}$. Initially, information on some ERP systems and suppliers was collected. After preliminary filtrating, many unqualified alternatives were weeded out, and four potential ERP systems remained, denoted by $\{a_1, a_2, a_3, a_4\}$. The professional team chose the following four criteria to evaluate these alternatives: (1) c_1 represents the technology and function; (2) c_2 represents the vendor's ability and reputation; (3) c_3 represents the strategic fitness; and (4) c_4 represents the flexibility. After a heated discussion, incomplete information with respect to the weights of DMs and criteria was provided as $V = \{0.2 \le v_1 \le 0.4,\ 0.1 \le v_2 \le 0.35,\ v_3 \le v_2,\ v_2 \le v_1,\ v_1 \le 1.5v_3\}$, $W = \{0.2 \le w_1 \le 0.3,\ \ 0.16 \le w_2 \le 0.35,\ 0.12 \le w_3 \le 0.25,\ \ 0.1 \le w_4 \le 0.25,\ \ 1.5w_3 \le w_1,\ \ 1.5w_4 \le w_2\}$. The two linguistic term sets provided in Example 1 are employed in the evaluation process. The evaluation values are exhibited in the form of HULZNs in Tables 1, 2, and 3.

6.1 An Illustration of the Proposed Method

The main procedure for selecting the best ERP system can be summarized in the following steps. For convenience, let

$f^*(s_i) = H_1^*(s_i) = \frac{i}{2t}(0 \le i \le 2t)$ and $g^*(s_i) = H_2^*(s_i) =$

$$\begin{cases} \dfrac{a^t - a^{t-i}}{2a^t - 2}(i = 0, 1, 2, \ldots, t) \\ \dfrac{a^t + a^{i-t} - 2}{2a^t - 2}(i = t+1,\ t+2, \ldots, 2t) \end{cases} \quad (a = 1.37), \text{ and}$$

$\lambda = 1$ in the following calculation process.

Step 1 Normalize the evaluation information.

There is no need to normalize the evaluation information in this case, because all of the criteria are the benefit type; therefore, $R^{k\prime} = \left(z_{ij}^{k\prime}\right)_{n \times m} = \left(z_{ij}^{k}\right)_{n \times m}$.

Step 2 Calculate the support degree.

Based on Eqs. (3) and (9), the support degree $\text{Sup}\left(z_{ij}^k, z_{ij}^l\right)$ can be obtained as follows:

$$\text{Sup}\left(z_{ij}^1, z_{ij}^2\right) = \text{Sup}\left(z_{ij}^2, z_{ij}^1\right) = \begin{bmatrix} 0.9514 & 0.9655 & 0.9562 & 0.9851 \\ 0.9383 & 0.9492 & 0.9391 & 0.9611 \\ 0.9594 & 0.9345 & 0.7934 & 0.9482 \\ 0.8460 & 1 & 0.9581 & 0.9540 \end{bmatrix},$$

$$\text{Sup}\left(z_{ij}^2, z_{ij}^3\right) = \text{Sup}\left(z_{ij}^3, z_{ij}^2\right) = \begin{bmatrix} 0.8865 & 0.9105 & 0.8762 & 0.9509 \\ 0.9774 & 0.9781 & 0.9382 & 0.9142 \\ 0.9518 & 0.9025 & 0.9017 & 0.8366 \\ 0.7753 & 0.8228 & 0.9246 & 0.9478 \end{bmatrix},$$

$$\text{Sup}\left(z_{ij}^1, z_{ij}^3\right) = \text{Sup}\left(z_{ij}^3, z_{ij}^1\right) = \begin{bmatrix} 0.9529 & 0.8970 & 0.8967 & 0.9603 \\ 0.9529 & 0.9551 & 0.9793 & 0.9515 \\ 0.9048 & 0.9571 & 0.9015 & 0.8968 \\ 0.9196 & 0.8228 & 0.9477 & 0.8944 \end{bmatrix}.$$

Step 3 Calculate the weight vector $v = (v_1, v_2, \ldots, v_q)$ of the DMs using model (M-1).

Because the original evaluation includes inconsistent weight information with respect to the DMs, model (M-1)

Table 1 Decision matrix for d_1

	c_1	c_2	c_3	c_4
a_1	$([s_5, s_6], \{s'_4, s'_5, s'_6\})$	$([s_5, s_5], \{s'_5, s'_7\})$	$([s_5, s_5], \{s'_7\})$	$([s_5, s_6], \{s'_5, s'_6\})$
a_2	$([s_6, s_7], \{s'_6\})$	$([s_6, s_6], \{s'_6, s'_7\})$	$([s_5, s_6], \{s'_5, s'_6\})$	$([s_4, s_6], \{s'_4, s'_5, s'_6\})$
a_3	$([s_3, s_5], \{s'_5, s'_6\})$	$([s_5, s_6], \{s'_6\})$	$([s_4, s_6], \{s'_4, s'_5, s'_6\})$	$([s_6, s_6], \{s'_4, s'_6\})$
a_4	$([s_5, s_7], \{s'_4, s'_5\})$	$([s_4, s_5], \{s'_5, s'_6\})$	$([s_6, s_6], \{s'_6\})$	$([s_5, s_7], \{s'_6\})$

Table 2 Decision matrix for d_2

	c_1	c_2	c_3	c_4
a_1	$([s_5, s_5], \{s'_4, s'_5\})$	$([s_5, s_5], \{s'_6\})$	$([s_5, s_6], \{s'_6, s'_7\})$	$([s_5, s_6], \{s'_4, s'_5\})$
a_2	$([s_6, s_6], \{s'_5, s'_6\})$	$([s_5, s_6], \{s'_5, s'_6, s'_7\})$	$([s_6, s_6], \{s'_6\})$	$([s_4, s_5], \{s'_5, s'_6\})$
a_3	$([s_4, s_5], \{s'_4, s'_5, s'_6\})$	$([s_5, s_7], \{s'_6, s'_7\})$	$([s_7, s_7], \{s'_5, s'_7\})$	$([s_6, s_7], \{s'_5, s'_6\})$
a_4	$([s_6, s_7], \{s'_7\})$	$([s_4, s_5], \{s'_5, s'_6\})$	$([s_6, s_7], \{s'_5, s'_6\})$	$([s_5, s_6], \{s'_6, s'_7\})$

Table 3 Decision matrix for d_3

	c_1	c_2	c_3	c_4
a_1	$([s_5, s_7], \{s'_5, s'_6\})$	$([s_6, s_6], \{s'_5, s'_7\})$	$([s_4, s_5], \{s'_5, s'_6\})$	$([s_6, s_6], \{s'_4, s'_5, s'_6\})$
a_2	$([s_6, s_6], \{s'_6, s'_7\})$	$([s_5, s_6], \{s'_7\})$	$([s_5, s_6], \{s'_5, s'_7\})$	$([s_5, s_6], \{s'_5, s'_6\})$
a_3	$([s_5, s_7], \{s'_5, s'_6\})$	$([s_6, s_6], \{s'_5, s'_6\})$	$([s_6, s_6], \{s'_5, s'_6\})$	$([s_4, s_6], \{s'_4, s'_5\})$
a_4	$([s_5, s_5], \{s'_4, s'_5, s'_6\})$	$([s_5, s_7], \{s'_6, s'_7\})$	$([s_5, s_6], \{s'_6\})$	$([s_6, s_6], \{s'_4, s'_6\})$

can be employed to calculate the optimal weight vector as follows:

$$\max F(v) = 0.9322v_1 + 0.9230v_2 + 0.9152v_3$$
$$\text{s.t.}\begin{cases} 0.2 \le v_1 \le 0.4 \\ 0.1 \le v_2 \le 0.35 \\ v_3 \le v_2 \\ v_2 \le v_1 \\ v_1 \le 1.5v_3 \\ v_1 + v_2 + v_3 = 1 \\ v_1, v_2, v_3 \ge 0 \end{cases}$$

Thus, the optimal weight vector of DMs can be identified as $v = (0.4286, 0.2857, 0.2857)$.

Step 4 Calculate the weights with respect to $z_{ij}^k (k = 1, 2, \ldots, q)$.

Equation (10) can be applied to compute the weights associated with $z_{ij}^k (k = 1, 2, \ldots, q)$ as follows:

$$\gamma_{ij}^1 = \begin{bmatrix} 0.4416 & 0.4399 & 0.4411 & 0.4400 \\ 0.4372 & 0.4376 & 0.4395 & 0.4407 \\ 0.4380 & 0.4405 & 0.4350 & 0.4426 \\ 0.4428 & 0.4433 & 0.4399 & 0.4378 \end{bmatrix},$$
$$\gamma_{ij}^2 = \begin{bmatrix} 0.2911 & 0.2939 & 0.2930 & 0.2928 \\ 0.2927 & 0.2928 & 0.2909 & 0.2919 \\ 0.2943 & 0.2909 & 0.2899 & 0.2920 \\ 0.2877 & 0.2955 & 0.2922 & 0.2945 \end{bmatrix},$$
$$\gamma_{ij}^3 = \begin{bmatrix} 0.2673 & 0.2662 & 0.2659 & 0.2672 \\ 0.2701 & 0.2696 & 0.2696 & 0.2674 \\ 0.2676 & 0.2686 & 0.2750 & 0.2654 \\ 0.2695 & 0.2611 & 0.2678 & 0.2676 \end{bmatrix}.$$

Step 5 Obtain the comprehensive evaluation information. The evaluation information provided by an individual DM can be aggregated by utilizing the HULZPWA operator, and the comprehensive evaluation information is given in Table 4.

Step 6 Compute the weight vector $w = (w_1, w_2, \ldots, w_m)$ of the criteria using model (M-2).

Based on Eq. (3) and the comprehensive evaluation information listed in Table 4, model (M-2) can be constructed as follows:

$$\max F(w) = 2.3687w_1 + 1.4268w_2 + 0.9797w_3 + 1.1894w_4$$
$$\text{s.t.}\begin{cases} 0.2 \le w_1 \le 0.3 \\ 0.16 \le w_2 \le 0.35 \\ 0.12 \le w_3 \le 0.25 \\ 0.1 \le w_4 \le 0.25 \\ 1.5w_3 \le w_1 \\ 1.5w_4 \le w_2 \\ w_1 + w_2 + w_3 + w_4 = 1 \\ w_1, w_2, w_3, w_4 \ge 0 \end{cases}$$

Therefore, the weight vector of the criteria is $w = (0.3, 0.35, 0.12, 0.23)$.

Step 7 Determine the PIS $Z^+ = \{z_1^+, z_2^+, \ldots, z_m^+\}$ and NIS $Z^- = \{z_1^-, z_2^-, \ldots, z_m^-\}$.

According to Eqs. (11) and (12), the PIS Z_j^+ and the NIS Z_j^- involved in the comprehensive evaluation matrix can be identified as follows:

$$z_1^+ = ([s_6, s_{6.4609}], \{s'_{5.6835}\}),\ z_2^+ = ([s_{5.4376}, s_{6.2909}], \{s'_{6.5152}\}),$$
$$z_3^+ = ([s_{5.7322}, s_{6.2922}], \{s'_{6.2468}\}),\ z_4^+ = ([s_{5.4647}, s_{6.4428}], \{s'_{5.6943}\}).$$
$$z_1^- = ([s_{3.8296}, s_5], \{s'_{4.1714}\}),\ z_2^- = ([s_{4.2611}, s_{5.2662}], \{s'_{4.8107}\}),$$
$$z_3^- = ([s_{4.7341}, s_{5.2930}], \{s'_{4.3686}\}),\ z_4^- = ([s_{4.2701}, s_{5.7073}], \{s'_{4.1908}\}).$$

Step 8 Compute the group utility $U(a_i)$ and individual regret $R(a_i)$ for each alternative $a_i (i = 1, 2, \ldots, n)$.

The group utility $U(a_i)$ and individual regret $R(a_i)$ can be computed via Eqs. (13) and (14) as follows:

Table 4 Comprehensive evaluation information

c_1	
a_1	$([s_5, s_{5.9763}], \{s'_{4.1714}, s'_{4.4063}, s'_{4.3269}, s'_{4.4309}, s'_{4.5618}, s'_{4.6659}, s'_{4.5864}, s'_{4.8214}, s'_{4.7867}, s'_{4.9422}, s'_{5.0217}, s'_{5.1772}\})$
a_2	$([s_6, s_{6.4372}], \{s'_{5.1634}, s'_{5.3904}, s'_{5.4565}, s'_{6.6835}\})$
a_3	$([s_{3.8296}, s_5], \{s'_{4.4105}, s'_{4.5864}, s'_{4.6542}, s'_{4.7296}, s'_{4.8276}, s'_{4.8301}, s'_{4.9055}, s'_{4.9733}, s'_{5.0713}, s'_{5.1467}, s'_{5.1492}, s'_{5.3904}\})$
a_4	$([s_{5.2877}, s_{6.4609}], \{s'_{4.8006}, s'_{4.9352}, s'_{5.0658}, s'_{5.1196}, s'_{5.2004}, s'_{5.3848}\})$
c_2	
a_1	$([s_{5.2662}, s_{5.2662}], \{s'_{4.8107}, s'_{5.3958}, s'_{5.6163}, s'_{6.2013}\})$
a_2	$([s_{5.4376}, s_6], \{s'_{5.4556}, s'_{5.6820}, s'_{5.9720}, s'_{5.9988}, s'_{6.1984}, s'_{6.5152}\})$
a_3	$([s_{5.2686}, s_{6.2909}], \{s'_{5.1662}, s'_{5.3904}, s'_{5.5059}, s'_{5.7301}\})$
a_4	$([s_{4.2611}, s_{5.5223}], \{s'_{4.8439}, s'_{5.0625}, s'_{5.1718}, s'_{5.2042}, s'_{5.4228}, s'_{5.3904}, s'_{5.5321}, s'_{5.7507}\})$
c_3	
a_1	$([s_{4.7341}, s_{5.2930}], \{s'_{5.6933}, s'_{5.8852}, s'_{6.0549}, s'_{6.2468}\})$
a_2	$([s_{5.2909}, s_6], \{s'_{4.8350}, s'_{5.1793}, s'_{5.3415}, s'_{5.6858}\})$
a_3	$([s_{5.4202}, s_{6.2899}], \{s'_{4.3686}, s'_{4.5864}, s'_{4.5952}, s'_{4.8130}, s'_{4.8851}, s'_{5.0373}, s'_{5.1117}, s'_{5.2638}, s'_{5.2551}, s'_{5.4817}, s'_{5.5538}, s'_{5.7804},\})$
a_4	$([s_{5.7322}, s_{6.2922}], \{s'_{5.1364}, s'_{5.3904}\})$
c_4	
a_1	$([s_{5.2673}, s_6], \{s'_{4.2528}, s'_{4.4195}, s'_{4.4198}, s'_{4.5864}, s'_{4.5995}, s'_{4.7661}, s'_{4.7664}, s'_{4.9330}\})$
a_2	$([s_{4.2701}, s_{5.7073}], \{s'_{4.3294}, s'_{4.5417}, s'_{4.5688}, s'_{4.5864}, s'_{4.7811}, s'_{4.7987}, s'_{4.9387}, s'_{4.8258}, s'_{5.0381}, s'_{5.1510}, s'_{5.1781}, s'_{5.3904}\})$
a_3	$([s_{5.4647}, s_{6.2943}], \{s'_{4.1908}, s'_{4.3243}, s'_{4.4524}, s'_{4.5859}, s'_{4.8123}, s'_{4.9458}, s'_{5.0740}, s'_{5.2074}\})$
a_4	$([s_{5.2695}, s_{6.4428}], \{s'_{5.0064}, s'_{5.3103}, s'_{5.3904}, s'_{5.6943}\})$

Table 5 Rankings of all alternatives according to $U(a_i)$, $R(a_i)$, and $Q(a_i)$

	Rankings
$U(a_i)$	$a_2 \succ a_4 \succ a_3 \succ a_1$
$R(a_i)$	$a_2 \succ a_1 \succ a_3 \succ a_4$
$Q(a_i)$	$a_2 \succ a_1 \succ a_4 \succ a_3$

$$U(a_1) = 0.4960,\ U(a_2) = 0.2899,\ U(a_3) = 0.4504, \text{ and } U(a_4) = 0.4011.$$

$$R(a_1) = 0.1789,\ R(a_2) = 0.1736,\ R(a_3) = 0.2593, \text{ and } R(a_4) = 0.2755.$$

Step 9 Compute the closeness coefficients for each alternative $a_i (i = 1, 2, \ldots, n)$ utilizing Eq. (15), and $\alpha = 0.5$ here.

$$Q(a_1) = 0.5,\ Q(a_2) = 0.0101,\ Q(a_3) = 0.8075, \text{ and } Q(a_4) = 0.7696.$$

Step 10 Rank the order of all alternatives according to $U(a_i)$, $R(a_i)$, and $Q(a_i)$ $(i = 1, 2, \ldots, n)$. The results are listed in Table 5.

Step 11 Obtain the best solution or a set of compromise solutions.

Because $Q(a_{(2)}) - Q(a_{(1)}) \geq \frac{1}{4-1} = 0.3333$, and a_2 is the best ERP system ranked by $U(a_i)$ and $R(a_i)$, a_2 is the best ERP system.

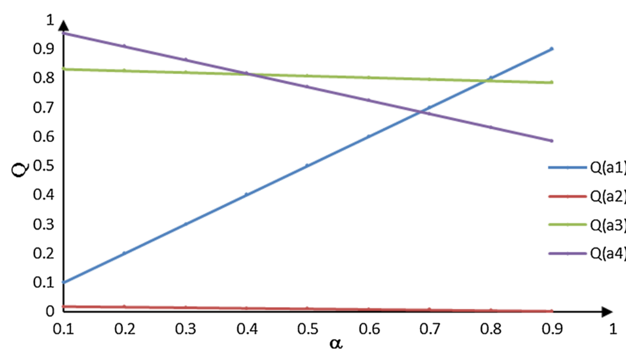

Fig. 2 Variation of alternatives' closeness coefficient with respect to decision mechanism α

Different rankings can be obtained as the parameter α changes, as shown in Fig. 2.

Figure 2 shows that the best ERP system is always a_2. However, the position of a_1, a_3 and a_4 appear to be change; a_1 becomes the worst one, and a_3 and a_4 become better with increases in α. Moreover, there is a linear relationship between each closeness coefficient of alternative $Q(a_i)$ and the decision mechanism α. There is a positive correlation between $Q(a_1)$ and α, and a negative correlation between $Q(a_2)$, $Q(a_3)$, $Q(a_4)$, and α. The DMs can choose different values of α to make decisions according to their own preferences. If $\alpha > 0.5$, then we can say that the DM is risk preferring; if $\alpha < 0.5$, then we can say that the DM is risk averse; otherwise, the DM is risk neutral.

6.2 Sensitivity Analysis

In order to explore the effects of the distance parameters and semantics on the final rankings, different values of λ and different combinations of f^* and g^* are considered. The results are shown in Tables 6 and 7. Because of the limited space, '$a_i \succ a_j \succ a_k \succ a_l$' is substituted with '$a_i$, a_j, a_k, a_l' in the following two tables.

It can be seen that the ranking results obtained via HULZPWA operator change with λ based on different combinations of linguistic scale functions. To be specific, the ranking results change obviously when $1 \leq \lambda \leq 3$, and the ranking results change slightly when $3 \leq \lambda \leq 5$, and the ranking results remain consistent when $\lambda \geq 5$. Thus, we can conclude that the distance measurement and HULZPWA operator are sensitive to the changing of smaller λ, whereas they are dull to the changing of greater λ. Moreover, this result is similar to the decision mechanism α in that a_2 is consistently evaluated as the best alternative, but the position of a_1 changes explicitly and becomes the worst alternative as λ increases. The ranking results also change when λ remains the same under different combinations of linguistic scale functions. The influence of λ and linguistic scale functions on the ranking results are similar when the HULZPWG operator is utilized.

It is worth noting that there is some distinction in the ranking results between the HULZPWA operator and the HULZPWG operator. This occurs because the inherent characteristics of the two operators are different. In general, the HULZPWA operator stresses the compensation between different input arguments and the importance of the comprehensive data, while the HULZPWG operator emphasizes the coordination between different input arguments and the importance of the individual data.

It can be said that λ is correlated with the thinking mode of the DMs: The higher the value of λ, the more optimistic the DMs are; meanwhile, the smaller the value of λ, the more pessimistic the DMs are. This allows the DMs to choose appropriate values for λ and different linguistic scale functions according to their preferences and actual semantic situations to obtain precise results. No matter what semantic combinations and values of λ are used, the best alternative is always a_2, and this fully verifies the stability and accuracy of the proposed method.

6.3 Comparative Analysis and Discussion

In order to further verify the feasibility and validity of the proposed method, we conducted a comparative analysis by applying an existing approach to the illustrative example described above.

Yaakob and Gegov [21] proposed a modified TOPSIS method to deal with MCGDM problems based on Z-numbers, in which the two components of Z-numbers are depicted using different linguistic values. Now, the above illustrative example is addressed utilizing Yaakob and Gegov's method [21].

First, the linguistic values a^l, a^u, and b_i contained in the two components of HULZNs should be, respectively,

Table 6 Ranking results with different $H^*(s_i)$ and λ utilizing HULZPWA operator

	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 4$	$\lambda \geq 5$
$f^*(s_i) = H_1^*(s_i);\ g^*(s_i) = H_2^*(s_i)$	a_2, a_1, a_4, a_3	a_2, a_4, a_3, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1
$f^*(s_i) = H_1^*(s_i);\ g^*(s_i) = H_3^*(s_i)$	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1
$f^*(s_i) = H_2^*(s_i);\ g^*(s_i) = H_1^*(s_i)$	a_2, a_1, a_4, a_3	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1
$f^*(s_i) = H_2^*(s_i);\ g^*(s_i) = H_3^*(s_i)$	a_2, a_1, a_3, a_4	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1
$f^*(s_i) = H_3^*(s_i);\ g^*(s_i) = H_1^*(s_i)$	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_3, a_1, a_4	a_2, a_3, a_1, a_4	a_2, a_3, a_4, a_1
$f^*(s_i) = H_3^*(s_i);\ g^*(s_i) = H_2^*(s_i)$	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3	a_2, a_3, a_1, a_4	a_2, a_3, a_1, a_4	a_2, a_3, a_4, a_1

Table 7 Ranking results with different $H^*(s_i)$ and λ utilizing HULZPWG operator

	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 4$	$\lambda \geq 5$
$f^*(s_i) = H_1^*(s_i);\ g^*(s_i) = H_2^*(s_i)$	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1
$f^*(s_i) = H_1^*(s_i);\ g^*(s_i) = H_3^*(s_i)$	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1
$f^*(s_i) = H_2^*(s_i);\ g^*(s_i) = H_1^*(s_i)$	a_2, a_1, a_3, a_4	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1
$f^*(s_i) = H_2^*(s_i);\ g^*(s_i) = H_3^*(s_i)$	a_2, a_1, a_3, a_4	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1	a_2, a_3, a_4, a_1
$f^*(s_i) = H_3^*(s_i);\ g^*(s_i) = H_1^*(s_i)$	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_3, a_1, a_4	a_2, a_3, a_1, a_4	a_2, a_3, a_4, a_1
$f^*(s_i) = H_3^*(s_i);\ g^*(s_i) = H_2^*(s_i)$	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3	a_2, a_3, a_1, a_4	a_2, a_3, a_1, a_4	a_2, a_3, a_4, a_1

transformed into trapezoidal fuzzy numbers α^l, α^u, and β_i according to the corresponding transformation methods in Ref. [21]. The average $\frac{\alpha^l+\alpha^u}{2}$ and $\frac{\sum \beta_i}{\#\beta_i}$ ($\#\beta_i$ is the number of β_i) should be computed in order to unite α^l, α^u, and β_i into a single trapezoidal fuzzy number. Then, according to other procedures of the modified TOPSIS method in Ref. [21], the closeness coefficient for all alternatives can be obtained as $CC_1 = 0.1863$, $CC_2 = 0.2064$, $CC_3 = 0.1877$, and $CC_4 = 0.1939$. The ranking result obtained from Ref. [21] is $a_2 \succ a_4 \succ a_3 \succ a_1$, which is inconsistent with the results obtained by the proposed method in most cases.

Several possible reasons could explain the difference in rankings. First, the theoretical bases for determining the final ranking differ between the two methods. Yaakob and Gegov's method [21] is based on the modified TOPSIS model, which emphasizes the top-ranking alternative should be as close to the PIS as possible and as far from the NIS as possible; meanwhile, the proposed method in this paper extends the classic VIKOR model to acquire the ranking results, highlighting the maximum group utility for the majority and the minimum individual regret for the opponent. Second, although both the extant method in Ref. [21] and our proposed method employ linguistic values to present *Z*-numbers, the handling methods are fundamentally different. Reference [21] converts linguistic information into trapezoidal fuzzy numbers, which inevitably results in the loss and distortion of original information. However, the proposed method overcomes this drawback by utilizing linguistic scale functions, and this paper applies three linguistic scale functions that are more flexible and applicable under different semantic circumstances. Moreover, Yaakob and Gegov's method [21] employs the simple averaging operation to aggregate the individual evaluation values, while the proposed method uses the PA operators developed in this paper to fuse original information, allowing this method to consider the interrelationships among aggregated values. In addition, the importance of criteria is subjectively rated by DMs in the form of linguistic values in Yaakob and Gegov's method [21], while the proposed method constructs two distance-based models to synthetically obtain the weights of DMs and criteria; this approach unifies subjectivity and objectivity in the process of determining weight and offers greater power in addressing practical problems with incompletely known weight information.

Based on this analysis, the advantages of the proposed method are summarized as follows:

1. The proposed method utilizes linguistic scale functions to achieve the transformation from qualitative linguistic information to quantitative data. In this way, the vagueness of the original information can be fully retained and employed to address practical problems with precision. Moreover, the two components of *Z*-numbers are considered to have different semantics, and different linguistic scale functions are employed to deal with them. This paper offers three linguistic scale functions so that DMs can choose the most appropriate one according to the practical semantic situations. Thus, the proposed approach is more effective and flexible in handling linguistic decision-making problems.
2. The proposed method integrates the power aggregation operators and the VIKOR method, thereby establishing a robust and innovative model to address MCGDM problems. The PA operator takes into account the interrelationships among aggregated values and allows the input arguments to reinforce each other in the aggregation process. After the individual evaluation information is aggregated, the VIKOR method is used to rank all alternatives, reducing the degree of calculation required when only the aggregation operators are employed to handle MCGDM problems involved hesitant values. Moreover, compared with the score function for HULZNs, the ranking results obtained by the proposed method using the distance measurement are more reliable.
3. Many factors involved in complicated MCGDM problems, such as the distance measurement, various semantic situations, incompletely known weight information, and risk preference of DMs, are considered synthetically in the proposed method.

It is undeniable that some tedious calculations are required when the proposed method is employed to address decision-making problems with large numbers of DMs, criteria, and alternatives. Nevertheless, the computing workload can be greatly reduced with the assistance of programming tools such as MATLAB.

7 Conclusion

To deal with situations where DMs employ *Z*-numbers and linguistic values to enhance the reliability and reflect the fuzziness of decision-making information, this paper introduced HULZNs by combining the advantages of *Z*-numbers and linguistic models. First, with the aid of the linguistic scale function, the operations and distance of HULZNs were introduced. Then, an effective MCGDM approach was developed based on the power aggregation operators and extended VIKOR model using HULZNs. Finally, the feasibility and effectiveness of the proposed method was tested through a practical ERP system selection problem, and the comparative analysis demonstrated that the proposed method can provide more precise outcomes than the existing method.

The main contributions of this research can be summarized as follows. First, the robust construct of Z-numbers and the abundant expressions of linguistic models were integrated using HULZNs, which can more reliably depict fuzzy, uncertain, and incomplete information. Second, an innovative method was developed by incorporating the proposed power aggregation operators and extended VIKOR model, which not only reduces the extent of calculation required, but also successfully imposes the effective ranking function of the VIKOR method. Third, the proposed approach proved to be fairly flexible to use, and the results may change with different linguistic scale functions, distance parameters, and DM risk preferences. Finally, the proposed method has explicit construction and favourable logic because it is completely based on the proposed distance measurement of HULZNs.

This paper includes one application example, and we will focus on applying the proposed approach to address more practical decision-making problems in future research. Considering the characteristics of green product design selection problems described in Ref. [49], the proposed approach is completely capable of addressing these problems. Moreover, when other application problems are identified as MCGDM problems, including control system performance evaluation and control system design selection, the proposed method can also be used to successfully address them according to the proposed procedures.

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