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Relaxed Stabilization Conditions for the T–S Fuzzy System with Input Constraints

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Abstract The stabilization conditions of the discrete Takagi–Sugeno (T–S) fuzzy system are reduced by considering possible switching subregions. In addition, the stabilization conditions for the T–S fuzzy system are relaxed by representing the interactions among the fuzzy subsystems in a single matrix. However, these two concepts have not been applied together to the discrete T–S fuzzy system with constraints on the control input. The aim of this paper is to relax the stabilization conditions for the discrete T–S fuzzy system with constraints on the control input. The possible switching subregions fired by two successive states of the system are analyzed and utilized to reduce the stabilization conditions. The interactions of fuzzy subsystems within two subregions are integrated into a single matrix to relax the stabilization conditions. The relaxation and effectiveness of the proposed stabilization conditions are demonstrated by a numerical example and a mass-spring-damper system.

Keywords Discrete Takagi–Sugeno (T–S) fuzzy system · Input constraint · Relaxed stabilization conditions · Switching subregions

1 Introduction

In the past few decades, Takagi–Sugeno (T–S) fuzzy control systems have been extensively investigated, since they can successfully control many nonlinear systems (see [1–10] and references therein). The parallel distributed compensation (PDC) [1, 11] control law that shares the same fuzzy sets with the T–S fuzzy model in the premise parts is primarily used to stabilize the T–S fuzzy system. Owing to the nonlinear weighted summation in the T–S fuzzy control system, the stabilization conditions of the PDC control law are usually derived based on the Lyapunov direct method. During the early development of T–S fuzzy control systems, the common quadratic Lyapunov function (CQLF) was employed to prove the stability of the control designs [1, 11, 12]. Based on the CQLF, the PDC stabilization criterion is to obtain a common positive definite matrix such that all the Lyapunov inequalities with respect to the subsystems of the T–S fuzzy system are satisfied. The stabilization conditions are often represented as linear matrix inequalities (LMI) and then solved using existing LMI tools [13]. However, the structural information between the subsystems and the membership functions is ignored in the CQLF-based stabilization conditions. In contrast, deriving the stabilization conditions for the T–S fuzzy control system by using the piecewise quadratic Lyapunov function (PQLF) involves the structural information of the T–S fuzzy system [2, 14–22]. Further, the PQLF has more Lyapunov function candidates than the CQLF. If a piece of the PQLF strictly decreases in the particular subregion(s) of the state space, the local stability of the system in the subregion(s) can be guaranteed. If sufficient pieces of the Lyapunov candidates guarantee the local stability of all subregions, and the boundary conditions are met, then the global stability of the system can be guaranteed by the summation of all piecewise Lyapunov candidates [14–20]. In [14, 20], the boundary condition is that the PQLF has to remain continuous across subregion boundaries of the state space, whereas in [15–19, 21, 22], the PQLF has to strictly decrease not only in each

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subregion but also across subregion boundaries. Further analysis of the possible switching subregions has been considered such that the stabilization conditions of the discrete T–S fuzzy system are reduced [16, 17, 19].

In practical control systems, the control signal always falls in a definite range owing to the limitations induced by the mechanical structure, driver circuit devices, and so on; this is called the input constraint [1, 2, 18, 23–31]. The input constraint limits the performance of the control system and may affect its stability. The T–S fuzzy control system design with input constraints has been applied to many practical systems [23–31]. In [23], the use of extra LMIs is proposed to ensure that the input constraints are met, and the control design with input constraints is applied to a hovercraft system. The fuzzy control laws with input constraints have been applied to the overhead crane [24], inverted pendulum on a cart [25], inverted pendulum [26], self-sustaining bicycle [27], mobile robot [28], aircraft [29], ship positioning systems [30], and electric power steering system [31]. However, there has been little discussion about the relaxation of stabilization conditions for the discrete T–S fuzzy system with input constraints.

This study proposes relaxed stabilizations for the discrete T–S fuzzy systems with input constraints by considering possible switching subregions. In this study, the PQLF is employed to prove that the discrete T–S fuzzy system is stable. The T–S fuzzy system is represented as an equivalent switching T–S fuzzy system, in which the state space is divided into several subregions according to the firing principle of fuzzy rules. The possible switching subregions, respectively, fired by two successive states of the system are estimated based on the system and input constraints. Since, the stabilization conditions are considered only with respect to the possible switching subregions, fewer stabilization conditions are derived as compared to the traditional stabilization criteria. The stabilization conditions are also relaxed by integrating the interactions of fuzzy subsystems within two subregions, into a single matrix [12, 32]. The stabilization conditions and input constraints are both represented in the LMI form and solved by using LMI tools. Consequently, we concluded that the stabilizations for the discrete T–S fuzzy systems with input constraints can be relaxed.

2 Preliminary

The fuzzy rule of a discrete T–S fuzzy system is presented as follows:

$$\begin{aligned}\text{Rule } i: &\text{ If } x_1(k) \text{ is } M_{i1},\ x_2(k) \text{ is } M_{i2}, \ldots, x_n(k) \text{ is } M_{in}, \\ &\text{then } \boldsymbol{x}(k+1) = \boldsymbol{A}_i\boldsymbol{x}(k) + \boldsymbol{B}_i\boldsymbol{u}(k),\end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, r$; r and n denote the numbers of fuzzy rules and state variables, respectively. M_{ij} is the membership function of the state variable x_j in the i-th fuzzy rule. $\boldsymbol{A}_i \in \Re^n \times n$ and $\boldsymbol{B}_i \in \Re^{n\times m}$ are the system and input matrices, respectively. $\boldsymbol{x}(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T$ is the state vector. $\boldsymbol{u}(k) = [u_1(k), u_2(k), \ldots, u_m(k)]^T$ is the input vector. By the weighting average defuzzification, the output of the system is inferred as

$$\boldsymbol{x}(k+1) = \sum_{i=1}^{r} w_i(k)[\boldsymbol{A}_i\boldsymbol{x}(k) + \boldsymbol{B}_i\boldsymbol{u}(k)], \tag{2}$$

where

$$\begin{aligned}& w_i(k) = \frac{\prod_{j=1}^{n} M_{ij}(x_j(k))}{\sum_{i=1}^{r}\prod_{j=1}^{n} M_{ij}(x_j(k))}; \quad 0 \le w_i(k) \le 1; \\ & \sum_{i=1}^{r} w_i(k) = 1,\end{aligned} \tag{3}$$

where $M_{ij}(x_j(k))$ denotes the grade of $x_j(k)$ in the membership function M_{ij}. The PDC law [1, 11] is applied to the system (2) as follows:

$$\begin{aligned}\text{Rule } i: &\text{ If } x_1(k) \text{ is } M_{i1},\ x_2(k) \text{ is } M_{i2}, \ldots, x_n(k) \text{ is } M_{in}, \text{ then} \\ &\boldsymbol{u}(k) = -\boldsymbol{K}_i\boldsymbol{x}(k),\end{aligned} \tag{4}$$

where $i = 1, 2, \ldots, r$. According to the PDC law, the fuzzy control rules (4) use the same fuzzy sets as the system rules (1). Then, using the same inference as (2), the defuzzified controller is

$$\boldsymbol{u}(k) = -\sum_{i=1}^{r} w_i(k)\boldsymbol{K}_i\boldsymbol{x}(k). \tag{5}$$

For practical systems, the control input is always limited in a definite range. This is called an input constraint [1]. Herein, the input constraint on the discrete T–S fuzzy system is represented as

$$\|\boldsymbol{u}(k)\|_2 < \bar{u}. \tag{6}$$

By applying the PDC (5) to (2), the following discrete T–S fuzzy control system is obtained.

$$\boldsymbol{x}(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)w_j(k)[\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_j]\boldsymbol{x}(k). \tag{7}$$

Based on the CQLF $V(k) = \boldsymbol{x}^T(k)\boldsymbol{P}\boldsymbol{x}(k)$, the Lyapunov stabilization criteria for the discrete T–S fuzzy system with the input constraint (6) are presented below.

Lemma 1 [1]: *Consider the discrete T–S fuzzy control system (7) with the input constraint (6) and assume* $\|\boldsymbol{x}(0)\|_2 < \bar{x}_0$. *The system (7) is asymptotically stable if there are matrices* $\boldsymbol{M}_i = \boldsymbol{K}_i\boldsymbol{X}$ *and* $\boldsymbol{X} = \boldsymbol{P}^{-1}$ *such that*

$$X > \bar{x}_0^2 I \quad (8)$$

$$\begin{bmatrix} X & * \\ M_i & \bar{u}^2 I \end{bmatrix} > 0, \quad (9)$$

$$\begin{bmatrix} X & * \\ A_i X - B_i M_i & X \end{bmatrix} > 0, \quad (10)$$

$$\begin{bmatrix} X & * \\ (A_i X - B_i M_j + A_j X - B_j M_i)/2 & X \end{bmatrix} > 0, \quad (11)$$

where $i<j$; $i, j = 1, 2, \cdots, r$. The asterisk denotes the transposed matrices for symmetric positions.

Lemma 1 is a typical stabilization criterion for the discrete T–S fuzzy system with an input constraint. If the LMIs (8) and (9) hold, the constraint (6) is guaranteed for $k \geq 0$. For details, see [1]. LMIs (10) and (11) are typical stabilization conditions for a discrete T–S fuzzy system. The relaxation of LMIs (10) and (11) can be achieved by integrating the interactions between all fuzzy subsystems into a single matrix as follows.

Lemma 2 [12]: *The discrete T–S fuzzy control system (7) is asymptotically stable if there are matrices $M_i = K_i X$, $Q_{ij} = Q_{ij}^T$, for $i, j = 1, 2, \cdots, r$, $i \leq j$, and $X = P^{-1} > 0$ such that*

$$\begin{bmatrix} X - Q_{ii} & * \\ A_i X - B_i M_i & X \end{bmatrix} > 0 \quad (12)$$

$$\begin{bmatrix} X - Q_{ij} & * \\ (A_i X - B_i M_j + A_j X - B_j M_i)/2 & X \end{bmatrix} > 0, \quad (13)$$

$$Q \equiv \begin{pmatrix} Q_{11} & * & \cdots & * \\ Q_{12} & Q_{22} & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ Q_{1r} & Q_{2r} & \cdots & Q_{rr} \end{pmatrix} > 0. \quad (14)$$

By integrating Lemmas 1 and 2, the relaxed Lyapunov stabilization criterion for the discrete T–S fuzzy system with the input constraint (6) is derived as follows.

Theorem 1 *Consider the discrete T–S fuzzy control system (7) with the input constraint (6), and assume that $\|x(0)\|_2 < \bar{x}_0$. The system (7) is asymptotically stable if there are matrices $M_i = K_i P^{-1}$, $Q_{ij} = Q_{ij}^T$, for $i, j = 1, 2, \cdots, r$, $i \leq j$, and $X = P^{-1} > 0$, such that the inequalities (8)–(9) and (12)–(14) are satisfied.*

The above stabilization conditions are derived based on the CQLF. The further relaxed stability conditions for the discrete T–S fuzzy system are expected to derive by adopting the PQLF. For convenience, the discrete T–S fuzzy system is represented as an equivalent discrete switching T–S fuzzy system.

3 Main Results

The rules of a discrete switching T–S fuzzy system include two levels: region rules and local fuzzy rules. The region rules are constructed based on the firing property of fuzzy rules. The overall state space Ψ is divided into several nonoverlapping subregions, wherein the same local fuzzy rules are always fired simultaneously. Therefore, the fuzzy rule (1) is represented as follows [17]:

$$\begin{aligned} &\textbf{Region Rule}\, j: \text{If } x(k) \in S_j, \text{ then} \\ &\quad \textbf{Local Fuzzy Rule } LR_{js}: \text{If } x_1(k) \text{ is } M_{js1},\ x_2(k) \\ &\qquad \text{is } M_{js2}, \ldots, x_n(k) \text{ is } M_{jsn}, \text{then} \\ &\qquad\quad x(k+1) = A_{js} x(k) + B_{js} u(k), \end{aligned} \quad (15)$$

where S_j denotes the j-th subregion for $j = 1, 2, \ldots, \sigma$; LR_{js} denotes the s-th local fuzzy rule in S_j for $s = 1, 2, \cdots, \beta(j)$. Moreover, $\cup_{j=1}^{\sigma} S_j = \Psi$, and $S_i \cap S_j = \emptyset$ for $i \neq j$. The final output of (15) is inferred as

$$x(k+1) = \sum_{j=1}^{\sigma} \sum_{s=1}^{\beta(j)} g_j(k) w_{js}(k) \left[A_{js} x(k) + B_{js} u(k) \right], \quad (16)$$

where

$$g_j(k) = \begin{cases} 1, & x(k) \in S_j \\ 0, & \text{otherwise} \end{cases}, \quad (17)$$

$$w_{js}(k) = \frac{\prod_{d=1}^{n} M_{jsd}(x_d(k))}{\sum_{s=1}^{\beta(j)} \prod_{d=1}^{n} M_{jsd}(x_d(k))}, \quad 0 \leq w_{js}(k) \leq 1, \quad \sum_{s=1}^{\beta(j)} w_{js}(k) = 1. \quad (18)$$

Similarly, the PDC control law is rewritten in the switching T–S fuzzy system form.

$$\begin{aligned} &\textbf{Region Rule}\, j: \text{If } x(k) \in S_j, \text{ then} \\ &\quad \textbf{Local Fuzzy Rule } LR_{js}: \\ &\qquad \text{If } x_1(k) \text{ is } M_{js1},\ x_2(k) \text{ is } M_{js2}, \cdots, x_n(k) \text{ is } M_{jsn}, \\ &\qquad\quad \text{then } u(k) = -K_{js} x(k), \end{aligned} \quad (19)$$

where $j = 1, 2, \ldots, \sigma$, and $s = 1, 2, \ldots, \beta(j)$. The switching fuzzy controller is inferred as

$$u(k) = -\sum_{j=1}^{\sigma} \sum_{s=1}^{\beta(j)} g_j(k) w_{js}(k) K_{js} x(k). \quad (20)$$

By applying (20) to (16), the following closed-loop switching T–S fuzzy control system is obtained.

$$\begin{aligned}\boldsymbol{x}(k+1) &= \sum_{j=1}^{\sigma}\sum_{s=1}^{\beta(j)}\sum_{l=1}^{\beta(j)} g_j(k)w_{js}(k)w_{jl}(k)[\boldsymbol{A}_{js}-\boldsymbol{B}_{js}\boldsymbol{K}_{jl}]\boldsymbol{x}(k)\\ &= \sum_{j=1}^{\sigma}\sum_{s=1}^{\beta(j)} g_j(k)w_{js}^2(k)\boldsymbol{G}_{jss}\boldsymbol{x}(k)\\ &\quad + 2\sum_{j=1}^{\sigma}\sum_{s=1}^{\beta(j)}\sum_{l>s}^{\beta(j)} g_j(k)w_{js}(k)w_{jl}(k)\boldsymbol{\Lambda}_{jsl}\boldsymbol{x}(k),\end{aligned} \tag{21}$$

where $\boldsymbol{G}_{jsl}=\boldsymbol{A}_{js}-\boldsymbol{B}_{js}\boldsymbol{K}_{jl}$, and $\boldsymbol{\Lambda}_{jsl}=(\boldsymbol{G}_{jsl}+\boldsymbol{G}_{jls})/2$. In the subregion $\boldsymbol{S}_j$, a state $\boldsymbol{x}(k)$ can be represented by the interpolation of vertices $\boldsymbol{x}_{jc}^v$

$$\boldsymbol{x}(k)=\sum_{c=1}^{2^n} w_{jc}^v(k)\boldsymbol{x}_{jc}^v, \quad \boldsymbol{x}(k)\in\boldsymbol{S}_j \tag{22}$$

where

$$0\le w_{jc}^v(k)\le 1, \quad \sum_{c=1}^{2^n} w_{jc}^v(k)=1. \tag{23}$$

According to (16), (22)–(23) and the constraint on the control input (6), $||\Delta\boldsymbol{x}(k)||_\infty$ for $\boldsymbol{x}(k)\in\boldsymbol{S}_j$ is estimated as follows:

$$\begin{aligned}\|\Delta\boldsymbol{x}(k)\|_\infty &= \|\boldsymbol{x}(k+1)-\boldsymbol{x}(k)\|_\infty\\ &= \left\|\sum_{s=1}^{\beta(j)} w_{js}(k)[\boldsymbol{A}_{js}\boldsymbol{x}(k)+\boldsymbol{B}_{js}\boldsymbol{u}(k)]-\boldsymbol{x}(k)\right\|_\infty\\ &= \left\|\sum_{s=1}^{\beta(j)} w_{js}(k)[\boldsymbol{A}_{js}-\boldsymbol{I}]\boldsymbol{x}(k)+\sum_{s=1}^{\beta(j)} w_{js}(k)\boldsymbol{B}_{js}\boldsymbol{u}(k)\right\|_\infty.\\ &\le \left\|\sum_{s=1}^{\beta(j)}\sum_{c=1}^{2^n} w_{js}(k)w_{jc}^v(k)[\boldsymbol{A}_{js}-\boldsymbol{I}]\boldsymbol{x}_{jc}^v\right\|_\infty\\ &\quad + \left\|\sum_{s=1}^{\beta(j)} w_{js}(k)\boldsymbol{B}_{js}\right\|_\infty \cdot\bar{u}, \quad \text{for} \quad \boldsymbol{x}(k)\in\boldsymbol{S}_j.\end{aligned} \tag{24}$$

According to (18) and (23),

$$\|\Delta\boldsymbol{x}(k)\|_\infty \le \max_{s,\,c}\left\|(\boldsymbol{A}_{js}-\boldsymbol{I})\boldsymbol{x}_{jc}^v\right\|_\infty + \max_s\|\boldsymbol{B}_{js}\|_\infty\cdot\bar{u}, \tag{25}$$

for $\boldsymbol{x}(k)\in\boldsymbol{S}_j$.

If $\boldsymbol{x}(k)\in\boldsymbol{S}_j$, the next state $\boldsymbol{x}(k+1)$ will only be located at following subregions

$$\boldsymbol{x}(k+1)\in\boldsymbol{S}_i, \text{ for } i=j\pm(1,\ 2,\ldots,\ \delta_j), \quad 0<i\le\sigma, \tag{26}$$

where

$$\delta_j = ceil\left(\|\Delta\boldsymbol{x}(k)\|_\infty/L_j\right), \tag{27}$$

where L_j is the minimal width of subregion $\boldsymbol{S}_j$; $ceil(z)$ rounds z to the nearest integer greater than or equal to z. Let Ω be the set of the subregion switching from $\boldsymbol{S}_j$ to $\boldsymbol{S}_i$, i.e., $\Omega := \{\langle j,\ i\rangle | \boldsymbol{x}(k)\in\boldsymbol{S}_j,\ \boldsymbol{x}(k+1)\in\boldsymbol{S}_i\}$. Then,

$$\Omega = \{\langle j,\ i\rangle | j=1,\ 2,\ldots,\sigma;\ i=j\pm(1,\ 2,\ldots,\ \delta_j),\ 0<i\le\sigma\}. \tag{28}$$

Based on the PQLF, $V(k)=\sum_{j=1}^{\sigma} g_j(k)\boldsymbol{x}^T(k)\boldsymbol{P}_j\boldsymbol{x}(k)$ and the set Ω, the relaxed stabilization criterion for the discrete T–S fuzzy system with the input constraint (6) is derived.

Theorem 2 *Consider the discrete T–S fuzzy control system (21) with the input constraint (6), and assume that* $\|\boldsymbol{x}(0)\|_2<\bar{x}_0$. *The system (21) is asymptotically stable if there are matrices* $\boldsymbol{M}_{js}=\boldsymbol{K}_{js}\boldsymbol{X}_j$, $\boldsymbol{Q}_{ji}^{sl}=(\boldsymbol{Q}_{ji}^{sl})^T$, $\boldsymbol{Q}_{ji}^{s}=(\boldsymbol{Q}_{ji}^{s})^T$, *and* $\boldsymbol{X}_j=\boldsymbol{P}_j^{-1}$, *for* $j=1,\ 2,\cdots,\ \sigma$, *and* $s,\ l=1,\ 2,\cdots,\ \beta(j)$, $s<l$, *such that*

$$\boldsymbol{X}_j > \bar{x}_0^2\boldsymbol{I} \tag{29}$$

$$\begin{bmatrix}\boldsymbol{X}_j & *\\ \boldsymbol{M}_{js} & \bar{u}^2\boldsymbol{I}\end{bmatrix} > 0, \tag{30}$$

$$\begin{bmatrix}\boldsymbol{X}_j-\boldsymbol{Q}_{ji}^s & *\\ \boldsymbol{A}_{js}\boldsymbol{X}_j-\boldsymbol{B}_{js}\boldsymbol{M}_{js} & \boldsymbol{X}_i\end{bmatrix} > 0, \tag{31}$$

$$\begin{bmatrix}\boldsymbol{X}_j-\left(\boldsymbol{Q}_{ji}^{sl}+\boldsymbol{Q}_{ji}^{ls}\right)/2 & *\\ (\boldsymbol{A}_{js}\boldsymbol{X}_j-\boldsymbol{B}_{js}\boldsymbol{M}_{jl}+\boldsymbol{A}_{jl}\boldsymbol{X}_j-\boldsymbol{B}_{jl}\boldsymbol{M}_{js})/2 & \boldsymbol{X}_i\end{bmatrix} > 0, \tag{32}$$

$$\hat{\boldsymbol{Q}}_{ji} \equiv \begin{bmatrix}\boldsymbol{Q}_{ji}^1 & \boldsymbol{Q}_{ji}^{12} & \cdots & \boldsymbol{Q}_{ji}^{1\beta(j)}\\ \boldsymbol{Q}_{ji}^{21} & \boldsymbol{Q}_{ji}^{2} & \cdots & \boldsymbol{Q}_{ji}^{2\beta(j)}\\ \vdots & \vdots & \ddots & \vdots\\ \boldsymbol{Q}_{ji}^{\beta(j)1} & \boldsymbol{Q}_{ji}^{\beta(j)2} & \cdots & \boldsymbol{Q}_{ji}^{\beta(j)}\end{bmatrix} > 0, \tag{33}$$

where $s,\ l=1,\ 2,\ldots,\ \beta(j)$, $s<l$, and $\langle j,\ i\rangle\in\Omega$. The asterisk denotes the transposed matrices for symmetric positions.

Proof For the PQLF, $V(k)=\sum_{j=1}^{\sigma} g_j(k)\boldsymbol{x}^T(k)\boldsymbol{P}_j\boldsymbol{x}(k)$, since (17) and $\langle j,\ i\rangle\in\Omega$, the difference between $V(k+1)$ and $V(k)$ is represented as

$$\begin{aligned}\Delta V(k) &= V(k+1)-V(k) = \boldsymbol{x}^T(k+1)\boldsymbol{P}_i\boldsymbol{x}(k+1)\\ &\quad -\boldsymbol{x}^T(k)\boldsymbol{P}_j\boldsymbol{x}(k), \text{ for } \boldsymbol{x}(k)\in\boldsymbol{S}_j, \quad \langle j,\ i\rangle\in\Omega.\end{aligned} \tag{34}$$

From (21),

$$\begin{aligned}
\Delta V(k) &= \boldsymbol{x}^T(k)\left[\left(\sum_{s=1}^{\beta(j)}\sum_{l=1}^{\beta(j)} w_{js}(k)w_{jl}(k)\boldsymbol{G}_{jsl}\right)^T \boldsymbol{P}_i\left(\sum_{s=1}^{\beta(j)}\sum_{l=1}^{\beta(j)} w_{js}(k)w_{jl}(k)\boldsymbol{G}_{jsl}\right) - \boldsymbol{P}_j\right]\boldsymbol{x}(k) \\
&\le \sum_{s=1}^{\beta(j)} w_{js}^2(k)\boldsymbol{x}^T(k)\left[\boldsymbol{G}_{jss}^T\boldsymbol{P}_i\boldsymbol{G}_{jss} - \boldsymbol{P}_j\right]\boldsymbol{x}(k) \\
&\quad + \sum_{s=1}^{\beta(j)}\sum_{s>l}^{\beta(j)} w_{js}(k)w_{jl}(k)\boldsymbol{x}^T(k)\left[\boldsymbol{\Lambda}_{jsl}^T\boldsymbol{P}_i\boldsymbol{\Lambda}_{jsl} - \boldsymbol{P}_j + \boldsymbol{\Lambda}_{jls}^T\boldsymbol{P}_i\boldsymbol{\Lambda}_{jls} - \boldsymbol{P}_j\right]\boldsymbol{x}(k).
\end{aligned} \tag{35}$$

If

$$\boldsymbol{G}_{jss}^T\boldsymbol{P}_i\boldsymbol{G}_{jss} - \boldsymbol{P}_j < -\boldsymbol{Y}_{ji}^s, \quad \langle j,\ i\rangle \in \Omega, \quad s = 1,\ 2,\ldots,\ \beta(j), \tag{36}$$

$$\begin{aligned}
&\boldsymbol{\Lambda}_{jsl}^T\boldsymbol{P}_i\boldsymbol{\Lambda}_{jsl} - \boldsymbol{P}_j + \boldsymbol{\Lambda}_{jls}^T\boldsymbol{P}_i\boldsymbol{\Lambda}_{jls} - \boldsymbol{P}_j < -\boldsymbol{Y}_{ji}^{sl} - \boldsymbol{Y}_{ji}^{ls}, \\
&\langle j,\ i\rangle \in \Omega, \quad 1 \le s < l \le \beta(j),
\end{aligned} \tag{37}$$

then

$$\begin{aligned}
\Delta V(k) &\le -\sum_{s=1}^{\beta(j)} w_{js}^2(k)\boldsymbol{x}^T(k)\boldsymbol{Y}_{ji}^s\boldsymbol{x}(k) \\
&\quad -\sum_{s=1}^{\beta(j)}\sum_{l>s}^{\beta(j)} w_{js}(k)w_{jl}(k)\boldsymbol{x}^T(k)[\boldsymbol{Y}_{ji}^{sl} + \boldsymbol{Y}_{ji}^{ls}]\boldsymbol{x}(k) \\
&= -\begin{bmatrix} w_{j1}\boldsymbol{x}(k) & & & \\ & w_{j2}\boldsymbol{x}(k) & & \\ \vdots & & & \\ w_{j\beta(j)}\boldsymbol{x}(k) & & & \end{bmatrix}^T \begin{bmatrix} \boldsymbol{Y}_{ji}^1 & \boldsymbol{Y}_{ji}^{12} & \cdots & \boldsymbol{Y}_{ji}^{1\beta(j)} \\ \boldsymbol{Y}_{ji}^{21} & \boldsymbol{Y}_{ji}^2 & \cdots & \boldsymbol{Y}_{ji}^{2\beta(j)} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{Y}_{ji}^{\beta(j)1} & \boldsymbol{Y}_{ji}^{\beta(j)2} & \cdots & \boldsymbol{Y}_{ji}^{\beta(j)} \end{bmatrix} \begin{bmatrix} w_{j1}\boldsymbol{x}(k) \\ w_{j2}\boldsymbol{x}(k) \\ \vdots \\ w_{j\beta(j)}\boldsymbol{x}(k) \end{bmatrix}.
\end{aligned} \tag{38}$$

$\Delta V(k) < 0$ holds, if

$$\hat{\boldsymbol{Y}}_{ji} \equiv \begin{bmatrix} \boldsymbol{Y}_{ji}^1 & \boldsymbol{Y}_{ji}^{12} & \cdots & \boldsymbol{Y}_{ji}^{1\beta(j)} \\ \boldsymbol{Y}_{ji}^{21} & \boldsymbol{Y}_{ji}^2 & \cdots & \boldsymbol{Y}_{ji}^{2\beta(j)} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{Y}_{ji}^{\beta(j)1} & \boldsymbol{Y}_{ji}^{\beta(j)2} & \cdots & \boldsymbol{Y}_{ji}^{\beta(j)} \end{bmatrix} > 0. \tag{39}$$

Multiplying the inequality (39) on the right and left by $diag(\overbrace{\boldsymbol{P}_j^{-1},\ \boldsymbol{P}_j^{-1},\ \cdots,\ \boldsymbol{P}_j^{-1}}^{\beta(j)})$, and defining $\boldsymbol{X}_j = \boldsymbol{P}_j^{-1}$, $\boldsymbol{X}_j\boldsymbol{Y}_{ji}^s\boldsymbol{X}_j = \boldsymbol{Q}_{ji}^s$, and $\boldsymbol{X}_j\boldsymbol{Y}_{ji}^{sl}\boldsymbol{X}_j = \boldsymbol{Q}_{ji}^{sl}$, then (33) is obtained. Moreover, by Schur complements, (36) and (37) are equivalent to

$$\begin{bmatrix} \boldsymbol{P}_j - \boldsymbol{Y}_{ji}^s & * \\ \boldsymbol{A}_{js} - \boldsymbol{B}_{js}\boldsymbol{K}_{js} & \boldsymbol{P}_i^{-1} \end{bmatrix} > 0, \tag{40}$$

$$\begin{bmatrix} \boldsymbol{P}_j - \left(\boldsymbol{Y}_{ji}^{sl} + \boldsymbol{Y}_{ji}^{ls}\right)/2 & * \\ (\boldsymbol{A}_{js} - \boldsymbol{B}_{js}\boldsymbol{K}_{jl} + \boldsymbol{A}_{jl} - \boldsymbol{B}_{jl}\boldsymbol{K}_{js})/2 & \boldsymbol{P}_i^{-1} \end{bmatrix} > 0. \tag{41}$$

Multiplying the inequalities (40) and (41) on the right and left by $diag(\boldsymbol{P}_j^{-1},\ \boldsymbol{I})$, and defining $\boldsymbol{M}_{js} = \boldsymbol{K}_{js}\boldsymbol{X}_j$, then (31) and (32) are obtained. The proof is completed. □

Remark 1 Based on the PQLF, the matrices $\boldsymbol{X}_j = \boldsymbol{P}_j^{-1}$ and $\boldsymbol{X}_i = \boldsymbol{P}_i^{-1}$ are with respect to the piece of the PQLF candidates for $\boldsymbol{x}(k) \in \boldsymbol{S}_j$ and $\boldsymbol{x}(k+1) \in \boldsymbol{S}_i$, respectively. In a discrete system, the successive two states $\boldsymbol{x}(k)$ and $\boldsymbol{x}(k+1)$ may be in any subregions, and consequently, $i, j = 1,\ 2, \cdots,\ \sigma$ should be considered in traditional stabilization conditions. However, by estimating the relative positions of two consecutive states $\boldsymbol{x}(k)$ and $\boldsymbol{x}(k+1)$, only $\langle j,\ i\rangle \in \Omega$ pairs should be satisfied in Theorem 2. Obviously, under the consideration of $\langle j,\ i\rangle \in \Omega$, the stabilization conditions are reduced.

Remark 2 The interactions of fuzzy subsystems within two subregions, for $\boldsymbol{x}(k) \in \boldsymbol{S}_j$ and $\boldsymbol{x}(k+1) \in \boldsymbol{S}_i$, are integrated into a single matrix $\hat{\boldsymbol{Q}}_{ji}$. Also, only $\langle j,\ i\rangle \in \Omega$ pairs should be considered for $\hat{\boldsymbol{Q}}_{ji}$. Following Lemma 2, the integrating matrices $\hat{\boldsymbol{Q}}_{ji}$ can relax the stabilization conditions for the discrete T–S fuzzy system with an input constraint.

4 Illustrative Examples

Two examples are shown in this section. Example 1 is a numerical example and demonstrates the relaxation of the stabilization criteria. In Example 2, the control design for a nonlinear mass-spring-damper system illustrates the relaxation and effectiveness of the proposed theorem.

Example 1 Consider the following switching T–S fuzzy system with the input constraint $\|\boldsymbol{u}(k)\|_2 < \bar{u} = 1$.

$$\begin{aligned}
&\textbf{Region Rule}\, j: \text{If } \boldsymbol{x}(k) \in \boldsymbol{S}_j, \text{ then} \\
&\quad \textbf{Local Fuzzy Rule } \boldsymbol{LR}_{js}: \\
&\quad\quad \text{If } x_1(k) \text{ is } M_{js1},\ x_2(k) \text{ is } M_{js2}, \text{then } \boldsymbol{x}(k+1) \\
&\quad\quad = \boldsymbol{A}_{js}\boldsymbol{x}(k) + \boldsymbol{B}_{js}\boldsymbol{u}(k),
\end{aligned} \tag{42}$$

where

$$\boldsymbol{A}_{11} = \begin{bmatrix} a & 0.4 \\ 0.5 & 0.8 \end{bmatrix}, \quad \boldsymbol{A}_{12} = \boldsymbol{A}_{21} = \begin{bmatrix} 0.9 & 0.3 \\ 0.5 & 0.2 \end{bmatrix},$$
$$\boldsymbol{A}_{22} = \boldsymbol{A}_{31} = \begin{bmatrix} 0.9 & 0.3 \\ 0.1 & 0.7 \end{bmatrix}, \quad \boldsymbol{A}_{32} = \begin{bmatrix} 0.7 & 0.2 \\ 0.4 & 0.9 \end{bmatrix},$$

$$\boldsymbol{A}_{13} = \begin{bmatrix} 0.9 & 0.5 \\ 0.5 & 0.1 \end{bmatrix}, \quad \boldsymbol{A}_{14} = \boldsymbol{A}_{23} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.8 \end{bmatrix},$$
$$\boldsymbol{A}_{24} = \boldsymbol{A}_{33} = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.9 \end{bmatrix}, \quad \boldsymbol{A}_{34} = \begin{bmatrix} 0.8 & 0.1 \\ 0.4 & 0.8 \end{bmatrix},$$

$\boldsymbol{B}_{11} = [b \quad 0]^T$, and $\boldsymbol{B}_{js} = [0.5 \quad 0]^T$ for $j = 1, 2, 3$ and $s = 1, 2, 3, 4$ except $\boldsymbol{B}_{11}$,

where a and b are variables used to demonstrate the relaxation of the stabilization conditions brought by the proposed theorems. Figure 1 shows the membership functions and rule table of this system.

Figure 1 also displays the vertices $\boldsymbol{x}_{jc}^{v}$ of subregion $\boldsymbol{S}_j$, for $j = 1, 2, 3$ and $c = 1, 2, 3, 4$. Therefore, the δ_j can be obtained by (24)–(27) with $\|\boldsymbol{u}(k)\|_2 < \bar{u} = 1$. Herein, $\delta_2 = \delta_3 = 1$ and $\delta_1 = 2$ regardless of the value of a and b. Consequently, $\Omega = \{\langle 1, 1\rangle, \langle 1, 2\rangle, \langle 1, 3\rangle, \langle 2, 1\rangle, \langle 2, 2\rangle, \langle 2, 3\rangle, \langle 3, 2\rangle, \langle 3, 3\rangle\}$. Assume that the initial condition $\boldsymbol{x}(0)$ is inside the subregion $\boldsymbol{S}_j$, for $j = 1, 2, 3$, therefore, $\|\boldsymbol{x}(0)\|_2 < \bar{x}_0 = \sqrt{10}$. Then, the above stabilization conditions are solved using Matlab LMI toolbox [13]. The adopted instruction of LMI solver is FEASP with the options [0 100 1e9 10 0]. The comparisons between the Lemma 1 [1], Theorems 1 and 2, Theorem 3.1 of [19] and the modified Theorem 3 of [17] are considered. Figure 2 shows the feasible areas for the stabilization conditions under $-2 \le a \le 2$ and $-4 \le b \le 5$. The Theorem 3 of [17] is modified to suit the T–S fuzzy system with input constraints. Clearly, the modified Theorem 3 of [17] and proposed Theorem 2 have the stabilization conditions significantly more relaxed than in Lemma 1, Theorem 1, and Theorem 3.1 of [19]. The relaxation can be explained by integrating interactions among fuzzy subsystems of subregion $\boldsymbol{S}_j$ into a single matrix $\hat{\boldsymbol{Q}}_{ji}$ or $\hat{\boldsymbol{Q}}_j$(in Theorem 3 of [17]). Furthermore, the proposed Theorem 2 relaxes the stabilization conditions more than in modified Theorem 3 of [17]. The explanation for this is that the nondiagonal matrices (i.e., $\boldsymbol{Q}_{ji}^{st}$) of the integrating matrix $\hat{\boldsymbol{Q}}_{ji}$ herein are asymmetric [32] and $\hat{\boldsymbol{Q}}_{ji}$ has additional degrees of freedom compared to $\hat{\boldsymbol{Q}}_j$ in Theorem 3 of [17].

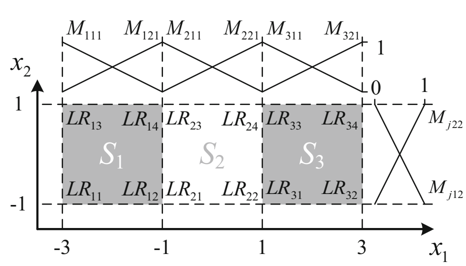

Fig. 1 Membership functions and rule table of Example 1

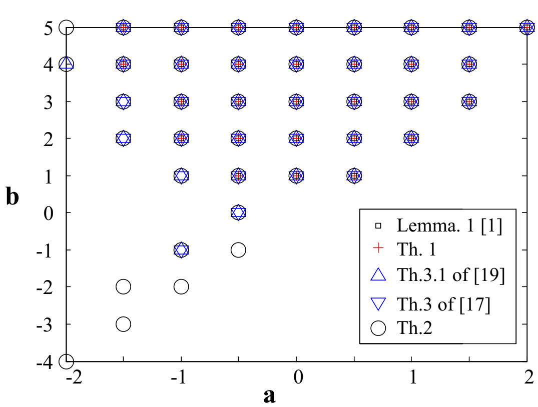

Fig. 2 Feasible areas for the compared stabilization criteria

Example 2 Consider a nonlinear mass-spring-damper system [33–36] as shown in Fig. 3.

The dynamic equation of the nonlinear mass-spring-damper system is expressed as follows:

$$M\ddot{y}(t) + \gamma(y(t), \dot{y}(t)) + \kappa(y(t)) = \phi(\dot{y}(t))u(t), \tag{43}$$

where M and $u(t)$ are the mass and input force, respectively. $\gamma(y(t), \dot{y}(t))$ is the reaction force of the damper. $\kappa(y(t))$ is the reaction force of the spring. From [33–36], $\gamma(y(t), \dot{y}(t)) = c_1 y(t) + c_2\dot{y}(t)$, $\kappa(y(t)) = c_3 y(t) + c_4 y(t)^3$, and $\phi(\dot{y}(t)) = 1 + c_5\dot{y}(t)^3$ for $y(t) \in [-1.5 \quad 1.5]$ and $\dot{y}(t) \in [-1.5 \quad 1.5]$. Assume $M = 1$, $c_1 = 0.01$, $c_2 = 0.1$, $c_3 = 0.01$, $c_4 = 0.67$, and $c_5 = 0$. Using the sector nonlinearity method [1], the nonlinear terms can be represented by the fuzzy combinations. To demonstrate the relaxation of the stabilization conditions brought by Theorem 2, multiple sectors are considered in this example. The considered six sectors include $-1.5 \le y(t) \le -1$, $-1 \le y(t) \le -0.5$, $-0.5 \le y(t) \le 0$, $0 \le y(t) \le 0.5$, $0.5 \le y(t) \le 1$, and $1 \le y(t) \le 1.5$. Let $\boldsymbol{x}(t)^T = [x_1(t) \;\; x_2(t)] = [y(t) \;\; \dot{y}(t)]$, then the mass-spring-damper system (43) is represented by the following continuous-time switching T–S fuzzy model:

$$\begin{aligned}&\textbf{Region Rule}\, j: \text{If } \boldsymbol{x}(t) \in \boldsymbol{S}_j, \text{ then}\\ &\quad\textbf{Local Fuzzy Rule } \boldsymbol{LR}_{js}:\\ &\quad\quad \text{If } x_1(t) \text{ is } M_{js1},\ x_2(t) \text{ is } M_{js2}, \text{ then } \dot{\boldsymbol{x}}(t) = \bar{\boldsymbol{A}}_{js}\boldsymbol{x}(t) + \bar{\boldsymbol{B}}_{js}\boldsymbol{u}(t)\end{aligned}, \tag{44}$$

where $j = 1, 2, 3, \cdots, 6$, $s = 1, 2, 3, 4$,

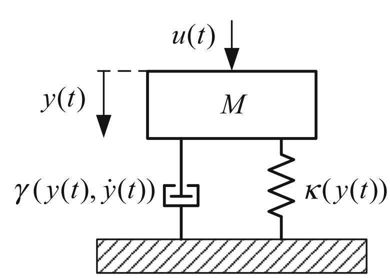

Fig. 3 Mass-spring-damper system

$$\bar{A}_{11} = \bar{A}_{62} = \begin{bmatrix} 0 & 1 \\ -1.5275 & 0 \end{bmatrix}, \quad \bar{A}_{12} = \bar{A}_{21} = \bar{A}_{52} = \bar{A}_{61} = \begin{bmatrix} 0 & 1 \\ -0.69 & 0 \end{bmatrix},$$

$$\bar{A}_{22} = \bar{A}_{31} = \bar{A}_{42} = \bar{A}_{51} = \begin{bmatrix} 0 & 1 \\ -0.1875 & 0 \end{bmatrix}, \quad \bar{A}_{32} = \bar{A}_{41} = \begin{bmatrix} 0 & 1 \\ -0.02 & 0 \end{bmatrix},$$

$$\bar{A}_{13} = \bar{A}_{64} = \begin{bmatrix} 0 & 1 \\ -1.5275 & -0.225 \end{bmatrix}, \quad \bar{A}_{14} = \bar{A}_{23} = \bar{A}_{54} = \bar{A}_{63} = \begin{bmatrix} 0 & 1 \\ -0.69 & -0.225 \end{bmatrix},$$

$$\bar{A}_{24} = \bar{A}_{33} = \bar{A}_{44} = \bar{A}_{53} = \begin{bmatrix} 0 & 1 \\ -0.1875 & -0.225 \end{bmatrix}, \quad \bar{A}_{34} = \bar{A}_{43} = \begin{bmatrix} 0 & 1 \\ -0.02 & -0.225 \end{bmatrix},$$

$$\bar{B}_{11} = \bar{B}_{12} = \cdots = \bar{B}_{64} = [0 \quad 1]^T.$$

The continuous-time T–S fuzzy system can be converted into the discrete T–S fuzzy system by using zero-order hold over the sampling period $T_s = 0.2$. The discrete T–S fuzzy model is as follows:

$$\begin{aligned} &\textbf{Region Rule}\, j: \text{If } \boldsymbol{x}(k) \in \boldsymbol{S}_j, \text{ then} \\ &\quad \textbf{Local Fuzzy Rule}\, \boldsymbol{LR}_{js}: \\ &\quad\quad \text{If } x_1(k) \text{ is } M_{js1},\ x_2(k) \text{ is } M_{js2}, \text{ then } \boldsymbol{x}(k+1) \\ &\quad\quad = \boldsymbol{A}_{js}\boldsymbol{x}(k) + \boldsymbol{B}_{js}\boldsymbol{u}(k) \end{aligned}, \tag{45}$$

where $j = 1, 2, 3, \ldots, 6$, $s = 1, 2, 3, 4$,

$$A_{11} = A_{62} = \begin{bmatrix} 0.9696 & 0.1980 \\ -0.3024 & 0.9696 \end{bmatrix}, \quad A_{12} = A_{21} = A_{52} = A_{61} = \begin{bmatrix} 0.9862 & 0.1991 \\ -0.1374 & 0.9862 \end{bmatrix},$$

$$A_{22} = A_{31} = A_{42} = A_{51} = \begin{bmatrix} 0.9963 & 0.1998 \\ -0.0375 & 0.9963 \end{bmatrix}, \quad A_{32} = A_{41} = \begin{bmatrix} 0.9996 & 0.2000 \\ -0.004 & 0.9996 \end{bmatrix},$$

$$A_{13} = A_{64} = \begin{bmatrix} 0.9701 & 0.1936 \\ -0.2957 & 0.9265 \end{bmatrix}, \quad A_{14} = A_{23} = A_{54} = A_{63} = \begin{bmatrix} 0.9864 & 0.1947 \\ -0.1343 & 0.9426 \end{bmatrix},$$

$$A_{24} = A_{33} = A_{44} = A_{53} = \begin{bmatrix} 0.9963 & 0.1953 \\ -0.0366 & 0.9524 \end{bmatrix}, \quad A_{34} = A_{43} = \begin{bmatrix} 0.9996 & 0.1955 \\ -0.0039 & 0.9556 \end{bmatrix},$$

$$B_{11} = B_{62} = \begin{bmatrix} 0.0199 \\ 0.1980 \end{bmatrix}, \quad B_{12} = B_{21} = B_{52} = B_{61} = \begin{bmatrix} 0.02 \\ 0.1991 \end{bmatrix}, \quad B_{32} = B_{41} = \begin{bmatrix} 0.02 \\ 0.2 \end{bmatrix},$$

$$B_{22} = B_{31} = B_{42} = B_{51} = \begin{bmatrix} 0.02 \\ 0.1998 \end{bmatrix}, \quad B_{13} = B_{64} = \begin{bmatrix} 0.0196 \\ 0.1936 \end{bmatrix}, \quad B_{34} = B_{43} = \begin{bmatrix} 0.0197 \\ 0.1955 \end{bmatrix},$$

$$B_{14} = B_{23} = B_{54} = B_{63} = \begin{bmatrix} 0.0197 \\ 0.1947 \end{bmatrix}, \quad B_{24} = B_{33} = B_{44} = B_{53} = \begin{bmatrix} 0.0197 \\ 0.1953 \end{bmatrix}.$$

The input constraint limits the performance of the control system and may affect its stability. For a control system, smaller input constraints might lead to more conservative stabilization conditions. First, by decreasing the input constraints, this example demonstrates the relaxation of the proposed theorem and the input constraint may affect the stability of the control system. Lemma 1 and Theorem 1 both can give feasible solutions for this example under $\bar{u} \geq 0.9409$. However, Theorem 2 can give feasible solutions for this example under $\bar{u} \geq 0.8560$. If the input constraint of this system is too small, such as $\bar{u} = 0.9$, neither Lemma 1 nor Theorem 1 can be used to stabilize this system. Obviously, Theorem 2 is more relaxed than Theorem 1 and Lemma 1 due to the consideration of the possible switching subregions. Next, the control design under $\|\boldsymbol{u}(k)\|_2 < \bar{u} = 0.9$ shows the effectiveness of the proposed Theorem 2.

Consider the initial condition $\boldsymbol{x}(0)$ inside the considered state space $x_1(k) \in [-1.5 \quad 1.5]$ and $x_2(k) \in [-1.5 \quad 1.5]$, that is, $\|\boldsymbol{x}(0)\|_2 < \bar{x}_0 = \sqrt{4.5}$. Then, the $\delta_1 = \delta_2 = 1$ and $\delta_3 = 2$ are obtained by (24)–(27) with $\|\boldsymbol{u}(k)\|_2 < \bar{u} = 0.9$. Hence, $\Omega = \{\langle 1, 1\rangle, \langle 1, 2\rangle, \langle 2, 1\rangle, \langle 2, 2\rangle, \langle 2, 3\rangle, \langle 3, 1\rangle, \langle 3, 2\rangle, \langle 3, 3\rangle\}$. From Theorem 2, the control design is as follows:

$$K_{11} = K_{61} = [0.1756 \quad 0.5135], \quad K_{12} = K_{62} = [-0.2869 \quad 0.3673],$$

$$K_{13} = K_{63} = [0.1732 \quad 0.4981], \quad K_{14} = K_{64} = [-0.1804 \quad 0.4271],$$

$$K_{21} = K_{51} = [0.3366 \quad 0.4761], \quad K_{22} = K_{52} = [0.0570 \quad 0.5146],$$

$$K_{23} = K_{53} = [0.3531 \quad 0.4444], \quad K_{24} = K_{54} = [0.0868 \quad 0.5038],$$

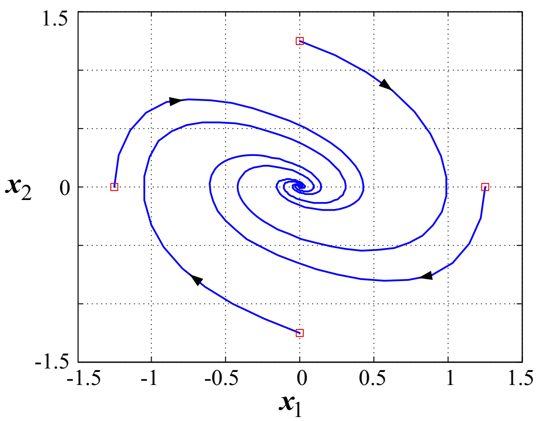

Fig. 4 Compensated state responses of the system (Example 2)

$$K_{31} = K_{41} = [0.3846 \quad 0.4536], \quad K_{32} = K_{42} = [0.3331 \quad 0.4746],$$

$$K_{33} = K_{43} = [0.4108 \quad 0.3974], \quad K_{34} = K_{44} = [0.3529 \quad 0.4408].$$

In addition,

$$P_1 = P_6 = \begin{bmatrix} 0.3192 & 0.0679 \\ 0.0679 & 0.3503 \end{bmatrix}, \quad P_2 = P_5 = \begin{bmatrix} 0.3076 & 0.0887 \\ 0.0887 & 0.3666 \end{bmatrix}, \quad P_3 = P_4 = \begin{bmatrix} 0.2958 & 0.1027 \\ 0.1027 & 0.3713 \end{bmatrix}.$$

Figure 4 shows the state responses of the control design. The square markers denote the different initial conditions $x(0)$ as follows:

$$[1.25 \quad 0]^T, \quad [-1.25 \quad 0]^T, \quad [0 \quad 1.25]^T, \quad [0 \quad -1.25]^T.$$

The simulation demonstrates the effectiveness of Theorem 2.

5 Conclusion

In this paper, the relaxation of stabilization conditions for the discrete T–S fuzzy system with input constraints has been investigated. The piecewise Lyapunov function is adopted to derive relaxed stabilization conditions. The stabilization conditions are reduced under the consideration of possible switching subregions. The stabilization conditions are further relaxed by integrating interactions of the fuzzy subsystems within two subregions into a single matrix. However, the reduction of the stabilization conditions may be little if the successive switching subregions have too many possibilities. The proposed estimation for the possible switching subregions can be applied to the discrete T–S fuzzy systems with constraints on the control input.

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