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An Exact Expected Value-Based Method to Prioritize Engineering Characteristics in Fuzzy Quality Function Deployment

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Abstract Quality function deployment (QFD) is a comprehensive and systematic method that contributes to transforming customer requirements (CRs) into appropriate engineering characteristics (ECs). In QFD, one fundamental and crucial step is to derive the importance of ECs. Since the QFD process involves huge amounts of subjective perceptions or evaluations made by both customers and decision-makers, the importance of ECs naturally becomes fuzzy. This paper focuses on how the importance of ECs can be correctly measured and rated in fuzzy environments, and proposes an approach for deriving the exact expected values and the rankings of the importance of ECs without any simulations or approximations. A design case is also illustrated to show the performance of the proposed method and compare with the traditional h-cut method. The results show that through our method, not only can more reasonable and reliable rankings of ECs be obtained, but also the burden of the complex arithmetic processes which the approximation or simulation algorithms generally involve can be eliminated. Finally, some extensive applications of the exact expected values of the fuzzy importance of ECs are demonstrated including the determination of the overall customer satisfaction and the cost–benefit analysis of ECs.

1 Introduction

Manufacturing enterprises, which are confronted with an increasing competitive environment in today's global market, realize that the efficient products design and manufacture based on customer needs are crucial for both of their survival and long-term development. In essence, if manufacturers were able to develop new products which satisfy customer preferences, this would give them a competitive advantage [5]. A widespread acceptance in the industry to ensure and improve quality during the product development is the application of quality function deployment (QFD), which is a proven customer-oriented planning approach first introduced in Japan in the late 1960s [1]. It is a comprehensive and systematic method that devotes to transforming customer requirements (CRs) to appropriate engineering characteristics (ECs) of the product in order to increase customer satisfaction.

The most significant tool of QFD, the house of quality (HoQ), is a kind of conceptual map that provides means for interfunctional planning and communications [18]. As is shown in Fig. 1, the left side of the HoQ is the relative weights vector of CRs, while the body of the HoQ is the relationship matrix between CRs and ECs. In addition, the correlation matrix among ECs is showed on the roof of the HoQ, and the importance vector of ECs is given on the bottom of the HoQ. The process of establishing a HoQ is a quantitative and qualitative analysis procedure, after which

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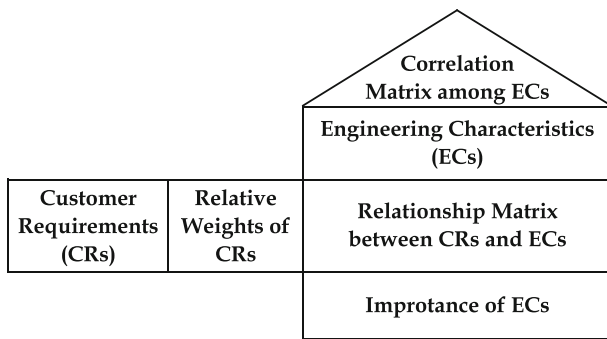2 School of Management, Shenzhen Polytechnic, Shenzhen 518055, China

Fig. 1 The house of quality

we can achieve the conversion from customer feedback to engineering information.

It is generally considered that in the design process, planners should pay particular attention to engineering characteristics of a new or existing product or service from the viewpoints of customer desires. However, due to time and budget limitations, it is hard to show great concern for all ECs in the product development. The design team needs to be able to make trade-offs, while selecting the ECs based on the order of their relative importance ranking to achieve more customer satisfaction without violating the business constraints [22]. Therefore, determining the importance ranking of ECs is such a core issue towards successful QFD realization that enterprise resources can then be properly assigned to ECs. In the meantime, the importance of ECs and its prioritization are also key results of QFD as they help the design team select design features without violating the business constraints including time, budget or feasible technology and so on [15].

However, the successful implementation of QFD often requires a significant number of subjective judgements from both customers in a targeted market and a QFD design team in a firm [38]. Therefore, in the QFD process, most of the inputs consist of linguistic data, i.e. the human perception and evaluation involved in the relative weights of CRs and the relationships between CRs and ECs, which are usually subjective and ambiguous. Under the background, the fuzzy set theory originated by [40] and various ranking methods for fuzzy numbers [2, 12, 33] provide a sound theoretical basis for ranking ECs in fuzzy QFD. As an important branch of QFD study, more and more studies and researches have been conducted on how to derive the priority ratings of ECs in fuzzy environments.

On the one hand, many scholars have focused on deriving the priority rankings of ECs by calculating the importance of ECs. For example, Khoo and Ho [20] developed a framework of fuzzy QFD system to handle the ambiguity involved in the QFD process, and the simple arithmetic mean was applied to defuzzify the fuzzy numbers. Kao and Liu [19] proposed the h-cut representation to

calculate the importance of ECs, and for each membership value h, a pair of fractional programming problems was formulated to find the h-cut of the importance of ECs. These two are original ways to obtain the importance of ECs. Based on these, Chen et al. [10] calculated the technical importance of ECs using a fuzzy weighted average method, and the fuzzy expected value operator proposed by Liu and Liu [29] was utilized to prioritize ECs. Geng et al. [16] applied the fuzzy ANP and the modified fuzzy logarithmic least squares method to determine the technical importance ratings of ECs. Besides, Chen et al. [7] commended a fuzzy-based quantitative approach to determine the importance priorities or achievement degrees of ECs. A procedure for constructing the HoQ in terms of the proposed approach was also presented. Actually, apart from these, numerous approximation or simulation algorithms are also frequently utilized in fuzzy QFD [21, 23, 25, 30, 36, 41]. Notably, these approaches suggested various fuzzy approximations or simulations to determine the importance of ECs. However, no matter which approach is applied, the error between the real and approximate value always exists. Sometimes, it may be extremely large that contributes to erroneous result. In addition, the computing processes of these methods are often very complicated.

On the other hand, some studies considered to obtain the priority ratings of ECs directly without calculating the importance of ECs. Kwong et al. [24] suggested a fuzzy group decision-making method which integrated the fuzzy weighted average method with a consensus ordinal ranking technique, and then the customers' preferences on the ranking of ECs were synthesized via 0–1 integer programming. Yan et al. [39] employed an alternative approach to prioritize ECs in QFD based on the order-based semantics of linguistic information and fuzzy preference relations of linguistic profiles, under uncertain interpretations of customers, design team and CRs. Moreover, Fiorenzo et al. [13] proposed an alternative approach deriving from the ordinal prioritization method which initially was proposed by Yager [37] for the prioritization of a set of alternatives basing on a set of ordered criteria. This method addressed the problem of aggregating preference orderings of multiple, ordered decision-makers with respect to a set of possible alternatives. However, although the limitation of information loss caused by the approximation can be avoided and the burden of quantifying qualitative concepts can be eliminated in these methods, and the importance of ECs is not determined, which would be an inconvenience for following QFD processes, such as obtaining the overall customer satisfaction, and determining the target values of the ECs. Therefore, most approaches for ranking ECs employed in fuzzy QFD do not work well.

In order to overcome the aforementioned deficiency, based on the method of Chen et al. [10], this paper aims to

devise a method to obtain both the exact expected values and rankings of the importance of ECs in fuzzy QFD. As the theoretical basis, we first present a method to provide a general form on the exact expected value of the product of two non-negative triangular fuzzy numbers that frequently occur in applying QFD. Then, based on the general form, we calculate the exact expected values of the importance of ECs and prioritize ECs in fuzzy QFD, in which the two sets of input data are expressed as triangular fuzzy numbers (TFNs), namely, the relative weights of CRs and relationships between CRs and ECs. Compared with the traditional h-cut method in QFD, our method can improve the accuracy and reduce the computational complexity. Additionally, the exact expected values obtained by our method are much better in reflecting the average values of the fuzzy importance of ECs, and the final rankings of ECs are also more reliable. Finally, we show the extensive applications of the exact expected values of the importance of ECs, that is, determining the overall customer satisfaction and cost–benefit analysis of ECs.

The rest of the article is organized as follows. In the next section, some important concepts of TFN that is used in fuzzy QFD are introduced. Section 3 presents a method to compute the exact expected value of the product of two triangular fuzzy numbers. Afterwards, in Sect. 4, our method is applied to determine the exact expected value of the importance of ECs and prioritize ECs in fuzzy QFD. Section 5 illustrates a numerical example about the design of a flexible manufacturing system, and the ranking results are compared with those obtained by the frequently used h-cut method in QFD. More than that, some other applications of the exact expected value of the fuzzy importance of ECs are also presented.

The following list contains all acronyms and abbreviations which are used in this paper.

- QFD: quality function deployment;
- HoQ: house of quality;
- CR: customer requirement;
- EC: engineering characteristic;
- TFN: triangular fuzzy number;
- FMS: flexible manufacturing system.

2 Distribution and Expected Value of a TFN

Fuzzy set theory has been well developed and applied in a wide variety of real problems. As the main instrument used in the fuzzy set theory, a fuzzy number is a mathematical model of a vaguely perceived quantitative piece of information, defined as a function from a possibility space to the set of real numbers. For practical purposes, TFN as the most commonly used form of fuzzy numbers have been applied widely. In this section, we briefly review the concept of TFN which will be adopted to interpret the

fuzziness of CRs and ECs in this paper. Meanwhile, the expressions of its membership function, credibility distribution, inverse credibility distribution and expected value are also presented, which will be used in our problem.

2.1 Definition of a TFN

Let Θ be a nonempty set, $\mathcal{P}(\Theta)$ the power set of Θ and Pos a possibility measure. Then, the triplet $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$ is called a possibility space. Based on this, we have the following mathematical definition of fuzzy number.

Definition 1 A fuzzy number is defined as a function from a possibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$ to the set of real numbers.

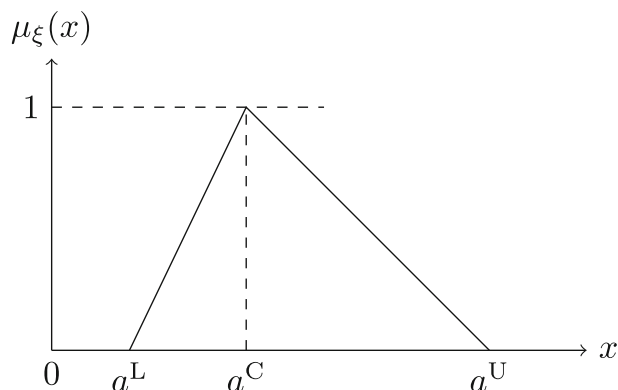Therewith, the membership function of a fuzzy number can be defined as follows.

Definition 2 Let ξ be a fuzzy number defined on the possibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$. Then its membership function is derived from the possibility measure by

$$\mu(x) = \mathrm{Pos}\{\theta \in \Theta | \xi(\theta) = x\}, \quad x \in \Re. \quad (1)$$

Let ξ be a TFN fully determined by a triplet of crisp numbers as $\xi = (a^{\mathrm{L}}, a^{\mathrm{C}}, a^{\mathrm{U}})$, where a^{C} is the central value with membership 1 describing the most possible value of ξ, and a^{L} and a^{U} indicate the lower and upper limit values of ξ, respectively. Thus, the membership function of ξ can be given by

$$\mu(x) = \begin{cases} \dfrac{x - a^{\mathrm{L}}}{a^{\mathrm{C}} - a^{\mathrm{L}}}, & \text{if } a^{\mathrm{L}} \le x < a^{\mathrm{C}} \\ \dfrac{a^{\mathrm{U}} - x}{a^{\mathrm{U}} - a^{\mathrm{C}}}, & \text{if } a^{\mathrm{C}} \le x \le a^{\mathrm{U}} \\ 0, & \text{otherwise}, \end{cases} \quad (2)$$

which is depicted in Fig. 2. Particularly, we call $\xi = (a^{\mathrm{L}}, a^{\mathrm{C}}, a^{\mathrm{U}})$ a symmetric TFN if it satisfies $a^{\mathrm{C}} - a^{\mathrm{L}} = a^{\mathrm{U}} - a^{\mathrm{C}}$.

Fig. 2 The membership function of $\xi = (a^{\mathrm{L}}, a^{\mathrm{C}}, a^{\mathrm{U}})$

2.2 Credibility Distribution of a TFN

In order to obtain the credibility distribution of a TFN, let us introduce some related concepts first. Suppose that ξ is a fuzzy number with the membership function μ. Then, the possibility of the fuzzy event $\{\xi \geq x\}$ is defined by

$$\text{Pos}\{\xi \geq x\} = \sup_{y \geq x} \mu(y), \tag{3}$$

whereas the necessity of $\{\xi \geq x\}$ is defined as the impossibility of $\{\xi < x\}$, i.e.

$$\text{Nec}\{\xi \geq x\} = 1 - \text{Pos}\{\xi < x\} = 1 - \sup_{y < x} \mu(y). \tag{4}$$

Furthermore, Liu and Liu [29] initially suggested that the credibility of a fuzzy event is defined as the average of its possibility and necessity. That is, the credibility of $\{\xi \geq x\}$ is

$$\text{Cr}\{\xi \geq x\} = \frac{1}{2}(\text{Pos}\{\xi \geq x\} + \text{Nec}\{\xi \geq x\}). \tag{5}$$

Subsequently, in order to describe fuzzy numbers, the concept of the credibility distribution is adopted.

Definition 3 ([26]) The credibility distribution $\Phi : \Re \to [0, 1]$ of a fuzzy number ξ is defined by

$$\Phi(x) = \text{Cr}\{\theta \in \Theta | \xi(\theta) \leq x\}. \tag{6}$$

That is, $\Phi(x)$ is the credibility that the fuzzy number ξ takes a value less than or equal to x. Liu [27] proved that the credibility distribution Φ is a nondecreasing function on $\Re$ with $\Phi(-\infty) = 0$ and $\Phi(+\infty) = 1$, which can be calculated via the membership function of ξ as follows.

Theorem 1 ([28]) Let ξ be a fuzzy number with the membership function μ. Then, its credibility distribution is

$$\Phi(x) = \frac{1}{2}\left(\sup_{y \leq x} \mu(y) + 1 - \sup_{y > x} \mu(y)\right), \quad \forall x \in \Re. \tag{7}$$

As for a TFN $\xi = (a^L, a^C, a^U)$, to obtain its credibility distribution $\Phi(x)$, we need to calculate the credibility via the possibility and necessity by virtue of Eqs. (2) $\sim$ (5) as follows:

$$\text{Pos}\{\xi \geq x\} = \begin{cases} 1, & \text{if } x < a^C \\ \dfrac{-x + a^U}{a^U - a^C}, & \text{if } a^C \leq x < a^U \\ 0, & \text{if } x \geq a^U, \end{cases} \tag{8}$$

$$\text{Pos}\{\xi \leq x\} = \begin{cases} 0, & \text{if } x < a^L \\ \dfrac{x - a^L}{a^C - a^L}, & \text{if } a^L \leq x < a^C \\ 1, & \text{if } x \geq a^C, \end{cases} \tag{9}$$

$$\text{Nec}\{\xi \geq x\} = \begin{cases} 1, & \text{if } x < a^L \\ \dfrac{-x + a^C}{a^C - a^L}, & \text{if } a^L \leq x < a^C \\ 0, & \text{if } x \geq a^C, \end{cases} \tag{10}$$

$$\text{Nec}\{\xi \leq x\} = \begin{cases} 0, & \text{if } x < a^C \\ \dfrac{x - a^C}{a^U - a^C}, & \text{if } a^C \leq x < a^U \\ 1, & \text{if } x \geq a^U, \end{cases} \tag{11}$$

$$\text{Cr}\{\xi \geq x\} = \begin{cases} 1, & \text{if } x < a^L \\ \dfrac{-x + 2a^C - a^L}{2(a^C - a^L)}, & \text{if } a^L \leq x < a^C \\ \dfrac{-x + a^U}{2(a^U - a^C)}, & \text{if } a^C \leq x < a^U \\ 0, & \text{if } x \geq a^U, \end{cases} \tag{12}$$

$$\text{Cr}\{\xi \leq x\} = \begin{cases} 0, & \text{if } x < a^L \\ \dfrac{x - a^L}{2(a^C - a^L)}, & \text{if } a^L \leq x < a^C \\ \dfrac{x + a^U - 2a^C}{2(a^U - a^C)}, & \text{if } a^C \leq x < a^U \\ 1, & \text{if } x \geq a^U. \end{cases} \tag{13}$$

Then according to Eq. (6), the credibility distribution of ξ is obtained as

$$\Phi(x) = \text{Cr}\{\xi \leq x\} = \begin{cases} 0, & \text{if } x < a^L \\ \dfrac{x - a^L}{2(a^C - a^L)}, & \text{if } a^L \leq x < a^C \\ \dfrac{x + a^U - 2a^C}{2(a^U - a^C)}, & \text{if } a^C \leq x < a^U \\ 1, & \text{if } x \geq a^U, \end{cases} \tag{14}$$

which is depicted in Fig. 3. It can be seen from Fig. 3 that the credibility distribution of $\xi = (a^L, a^C, a^U)$ is strictly increasing in the closed interval $[a^L, a^U]$. If ξ is an asymmetric TFN, i.e. $a^C - a^L \neq a^U - a^C$, there will be a turning point $(a^C, 0.5)$.

2.3 Inverse Credibility Distribution of a TFN

Generally speaking, credibility distributions of any fuzzy numbers are nondecreasing. Moreover, regarding the fuzzy number with a strictly increasing credibility distribution, the definition of inverse credibility distribution was proposed by Zhou et al. [42] as follows.

Definition 4 ([42]) Let ξ be a fuzzy number with a continuous and strictly increasing credibility distribution Φ. Then, the inverse function Φ^{-1} is called the inverse credibility distribution of ξ.

Note that the inverse credibility distribution Φ^{-1} is well defined on the open interval (0, 1). If required, we may extend the domain via
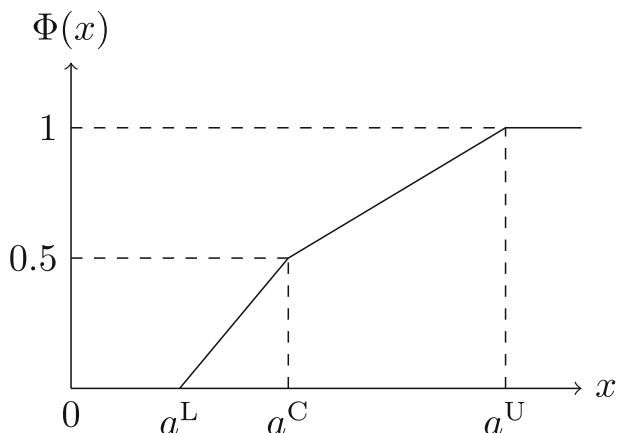
Fig. 3 The credibility distribution of $\xi = (a^L, a^C, a^U)$

$$\Phi^{-1}(0) = \lim_{\alpha \downarrow 0} \Phi^{-1}(\alpha), \quad \Phi^{-1}(1) = \lim_{\alpha \uparrow 1} \Phi^{-1}(\alpha). \tag{15}$$

From Fig. 3, we can see that the credibility distribution of the TFN $\xi = (a^L, a^C, a^U)$ is continuous and strictly increasing at each point x with $0 < \Phi(x) < 1$, which follows immediately that the inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0, 1)$. Thus, the inverse credibility distribution of ξ can be deduced through inverse operation directly and figured out as follows (see Fig. 4):

$$\Phi^{-1}(\alpha) = \begin{cases} 2(a^C - a^L)\alpha + a^L, & \text{if } 0 \leq \alpha < 0.5 \\ 2(a^U - a^C)\alpha + 2a^C - a^U, & \text{if } 0.5 \leq \alpha \leq 1. \end{cases} \tag{16}$$

Furthermore, as a direct result, the function $\Phi^{-1}(1 - \alpha)$ can be conducted from Eq. (16) as

$$\Phi^{-1}(1 - \alpha) = \begin{cases} 2(a^U - a^C)(1 - \alpha) + 2a^C - a^U, & \text{if } 0 \leq \alpha < 0.5 \\ 2(a^C - a^L)(1 - \alpha) + a^L, & \text{if } 0.5 \leq \alpha \leq 1 \end{cases}$$
$$= \begin{cases} a^U - 2(a^U - a^C)\alpha, & \text{if } 0 \leq \alpha < 0.5 \\ 2a^C - a^L - 2(a^C - a^L)\alpha, & \text{if } 0.5 \leq \alpha \leq 1. \end{cases} \tag{17}$$
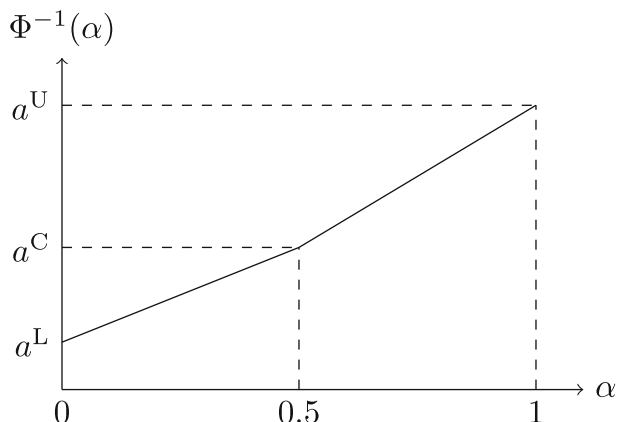
which will be used in the following context.

Fig. 4 The inverse credibility distribution of $\xi = (a^L, a^C, a^U)$

2.4 Expected Value of a TFN

The expected value of a fuzzy number is the average value in the sense of fuzzy measure. For fuzzy numbers, there are many ways to define an expected value operator. In this paper, we use the definition of the expected value operator for fuzzy numbers given by Liu and Liu [29]. This definition is applicable to not only continuous fuzzy numbers but also discrete ones.

Definition 5 ([29]) Let ξ be a fuzzy number. Then, the expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} \text{Cr}\{\xi \geq x\}\mathrm{d}x - \int_{-\infty}^0 \text{Cr}\{\xi \leq x\}\mathrm{d}x \tag{18}$$

provided that at least one of the two integrals is finite.

Note that Eq. (18) can also be expressed as another form. Before introducing that, the concepts of the optimistic and pessimistic functions of a fuzzy number are given.

Definition 6 ([32]) Let ξ be a fuzzy number on the possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$. Then, the optimistic function, $\xi_{\sup}(h)$, of ξ is defined as

$$\xi_{\sup}(h) = \sup\{r | \text{Pos}\{\theta \in \Theta | \xi(\theta) \geq r\} \geq h\}, \quad h \in [0, 1], \tag{19}$$

while the pessimistic function, $\xi_{\inf}(h)$, of ξ is defined as

$$\xi_{\inf}(h) = \inf\{r | \text{Pos}\{\theta \in \Theta | \xi(\theta) \leq r\} \geq h\}, \quad h \in [0, 1]. \tag{20}$$

By Definition 6, it has been proved by Liu and Liu [32] that the expected value of ξ can be calculated as

$$E[\xi] = \frac{1}{2} \int_0^1 \left(\xi_{\sup}(h) + \xi_{\inf}(h)\right)\mathrm{d}h, \tag{21}$$

which is an equivalent form of Eq. (18). Meanwhile, Eq. (21) is also a special case of f-weighted possibilistic mean value of a fuzzy number presented by Fuller and Majlender [14], that is, **1**-weighted possibilistic mean value. Fore more details, readers can refer to Fuller and Majlender [14].

As for the TFN $\xi = (a^L, a^C, a^U)$, in accordance with Eqs. (12) and (13), its expected value $E[\xi]$ can be obtained as

$$E[\xi] = \int_0^{+\infty} \text{Cr}\{\xi \geq x\}\mathrm{d}x - \int_{-\infty}^0 \text{Cr}\{\xi \leq x\}\mathrm{d}x$$
$$= \frac{a^L + 2a^C + a^U}{4}. \tag{22}$$

With Eq. (21), the same result can be achieved.

Regarding the independence of fuzzy numbers, it has been discussed by many researchers from different angles. In this paper, we focus on the definition together with an equivalent condition given by Liu and Gao [31] as follows.

Definition 7 ([31]) The fuzzy numbers $\xi_1, \xi_2, \ldots, \xi_n$ are said to be independent if

$$\text{Cr}\left\{\bigcap_{i=1}^{n}\{\xi_i \in B_i\}\right\} = \min_{1 \leq i \leq n} \text{Cr}\{\xi_i \in B_i\} \qquad (23)$$

for any sets $B_1, B_2, \ldots, B_n$ of $\Re$.

Theorem 2 ([31]) The fuzzy numbers $\xi_1, \xi_2, \ldots, \xi_n$ are independent if and only if

$$\text{Cr}\left\{\bigcup_{i=1}^{n}\{\xi_i \in B_i\}\right\} = \max_{1 \leq i \leq n} \text{Cr}\{\xi_i \in B_i\} \qquad (24)$$

for any sets $B_1, B_2, \ldots, B_n$ of $\Re$.

Subsequently, considering the linearity of expected value operator, a related theorem has been put forward on the basis of independence of fuzzy numbers.

Theorem 3 ([32]) Let ξ and η be independent fuzzy numbers with finite expected values. Then, for any real numbers λ and δ, we have

$$E[\lambda\xi + \delta\eta] = \lambda E[\xi] + \delta E[\eta]. \qquad (25)$$

Suppose that $\xi = (a^L, a^C, a^U)$ and $\eta = (b^L, b^C, b^U)$ are two independent TFNs. By applying Eqs. (22) and (25), it is easy to acquire

$$\begin{aligned} E[\lambda\xi + \delta\eta] &= \lambda E[\xi] + \delta E[\eta] \\ &= \frac{\lambda}{4}(a^L + 2a^C + a^U) + \frac{\delta}{4}(b^L + 2b^C + b^U). \end{aligned} \qquad (26)$$

3 Expected Value of Product of Two TFNs

It is acknowledged that a fuzzy number created by the product of two TFNs is no longer a TFN, and its membership function is nonlinear and complicated. Consequently, it is not easy to get its exact expected value in most cases. Since our problem refers to the product of two TFNs, this section focuses on how to obtain the exact expected value of the product of two TFNs.

3.1 Derivation of the Exact Expected Value of the Product of Two TFNs

In order to describe our method, a theorem proved by Zhou et al. [42] is introduced first.

Theorem 4 ([42]) Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent fuzzy numbers with continuous and strictly increasing credibility distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If the function $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then the expected value of the fuzzy number $\xi = f(\xi_1, \ldots, \xi_m, \xi_{m+1}, \ldots, \xi_n)$ is

$$\begin{aligned} E[\xi] &= \int_0^1 f(\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1} \\ &\quad (1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha))\text{d}\alpha. \end{aligned} \qquad (27)$$

Then let us discuss the product of two TFNs. Assume that $\xi_1 = (a^L, a^C, a^U)$ and $\xi_2 = (b^L, b^C, b^U)$ are two TFNs with credibility distributions Φ_1 and Φ_2, respectively. Then, according to Definition 5, the expected value of ξ is defined by

$$\begin{aligned} E[\xi] &= E[\xi_1\xi_2] \\ &= \int_0^{+\infty} \text{Cr}\{\xi_1\xi_2 \geq x\}\text{d}x - \int_{-\infty}^0 \text{Cr}\{\xi_1\xi_2 \leq x\}\text{d}x. \end{aligned} \qquad (28)$$

However, if both ξ_1 and ξ_2 are nonnegative TFNs (i.e. $a^L \geq 0$ and $b^L \geq 0$), we can know that $\xi = \xi_1\xi_2$ is strictly increasing with respect to ξ_1 and ξ_2. Since the inputs ξ_1 and ξ_2 are independent TFNs with inverse credibility distributions Φ_1^{-1} and Φ_2^{-1}, based on Theorem 4, the expected value of $\xi = \xi_1\xi_2$ can be calculated by

$$E[\xi] = \int_0^1 \Phi_1^{-1}(\alpha)\Phi_2^{-1}(\alpha)\text{d}\alpha. \qquad (29)$$

Afterwards, by taking inverse credibility distributions $\Phi_1^{-1}(\alpha)$ and $\Phi_2^{-1}(\alpha)$ in Eq. (16) into the above equation, we obtain

$$\begin{aligned} E[\xi] &= \int_0^1 \Phi_1^{-1}(\alpha)\Phi_2^{-1}(\alpha)\text{d}\alpha \\ &= \int_0^{0.5} \left(2(a^C - a^L)\alpha + a^L\right)\left(2(b^C - b^L)\alpha + b^L\right)\text{d}\alpha \\ &\quad + \int_{0.5}^1 \left(2(a^U - a^C)\alpha + 2a^C - a^U\right) \\ &\quad \times \left(2(b^U - b^C)\alpha + 2b^C - b^U\right)\text{d}\alpha \\ &= \frac{1}{6}\left(a^Lb^L + 2a^Cb^C + a^Ub^U\right) \\ &\quad + \frac{1}{12}\left(a^Lb^C + a^Cb^L + a^Cb^U + a^Ub^C\right). \end{aligned} \qquad (30)$$

In particular, if both ξ_1 and ξ_2 are symmetric TFNs, that is, $a^C - a^L = a^U - a^C$ and $b^C - b^L = b^U - b^C$, Eq. (30) can be further expressed as

$$E[\xi] = \frac{1}{3}\left(a^L b^L + a^U b^U\right) + \frac{1}{6}\left(a^L b^U + a^U b^L\right). \qquad (31)$$

Remark 1 It should be noted that the exact expected value of $\xi = \xi_1 \xi_2$ can also be easily derived from Eq. (21) applying the well-known Nguyen's theorem [34] for nonnegative triangular fuzzy numbers. Obviously, the same result can be obtained via the two different methods.

Remark 2 If ξ_1 is a nonnegative TFN (i.e. $a^L \geq 0$), and ξ_2 is nonpositive (i.e. $b^U \leq 0$), we can know that $-\xi_2 = (-b^U, -b^C, -b^L)$ is nonnegative. Then, $E[\xi] = E[\xi_1 \xi_2] = -E[\xi_1 \cdot (-\xi_2)]$, which can be computed as the product of two nonnegative TFNs via Eq. (30). Similarly, If both ξ_1 and ξ_2 are nonpositive TFNs, then $E[\xi] = E[\xi_1 \xi_2] = E[(-\xi_1) \cdot (-\xi_2)]$, which can also be calculated as the product of two nonnegative TFNs via Eq. (30) immediately.

Example 1 Let $\xi_1 = (0, 2, 7)$ and $\xi_2 = (1, 2, 10)$. Then based on Eq. (30), the expected value of $\xi = \xi_1 \xi_2$ can be calculated as

$$\begin{aligned} E[\xi] &= \frac{1}{6}(0 \times 1 + 2 \times 2 + 7 \times 10) + \frac{1}{12}(0 \times 2 + 2 \\ &\quad \times 1 + 2 \times 10 + 7 \times 2) \\ &= 16. \end{aligned} \qquad (32)$$

Example 2 Let $\xi_1 = (2, 2, 4)$, $\xi_2 = (2, 3, 6)$, $\xi_3 = (3, 4, 5)$, $\xi_4 = (-6, -5, 0)$, $\xi_5 = (-3, -1, -1)$, $\xi_6 = (-5, -2, -1)$ and $\xi = 3\xi_1 \xi_2 + \xi_3 \xi_4 + 2\xi_5 \xi_6$. Then, according to Theorem 3, we obtain

$$\begin{aligned} E[\xi] &= E[3\xi_1 \xi_2 + \xi_3 \xi_4 + 2\xi_5 \xi_6] \\ &= 3E[\xi_1 \xi_2] + E[\xi_3 \xi_4] + 2E[\xi_5 \xi_6]. \end{aligned} \qquad (33)$$

Afterwards, based on Eq. (30), Eq. (33) can be calculated as

$$\begin{aligned} E[\xi] &= 3E[\xi_1 \xi_2] + E[\xi_3 \xi_4] + 2E[\xi_5 \xi_6] \\ &= 3\left[\frac{1}{6}(2 \times 2 + 2 \times 2 \times 3 + 4 \times 6)\right. \\ &\quad \left. + \frac{1}{12}(2 \times 3 + 2 \times 2 + 2 \times 6 + 4 \times 3)\right] \\ &\quad - \left[\frac{1}{6}(3 \times 0 + 2 \times 4 \times 5 + 5 \times 6)\right. \\ &\quad \left. + \frac{1}{12}(3 \times 5 + 4 \times 0 + 4 \times 6 + 5 \times 5)\right] \\ &\quad + 2\left[\frac{1}{6}(1 \times 1 + 2 \times 1 \times 2 + 3 \times 5)\right. \\ &\quad \left. + \frac{1}{12}(1 \times 2 + 1 \times 1 + 1 \times 5 + 3 \times 2)\right] = 20.5. \end{aligned} \qquad (34)$$

On the whole, our proposed algorithm consists of the following two steps. The first step is deriving the inverse credibility distribution $\Phi_i^{-1}(\alpha)$ of the TFN ξ_i, $i = 1, 2$, via Eq. (16). The second step is deriving the expected value of $\xi = \xi_1 \xi_2$ based on Theorem 4. Through the proposed algorithm, Eq. (30) is obtained, which can be considered as a general form on the expected value of the product of two nonnegative TFNs.

3.2 Comparisons with the *h*-Cut Method

In order to obtain the expected value of a fuzzy number which has a nonlinear and complex membership function, many previous studies gave advice to provide an approximation or simulation, typical is the use of the *h*-cut representation of fuzzy sets and interval analysis [10, 19]. For a comparison purpose, we briefly recall the *h*-cut method. As mentioned in Sect. 2.4, the expected value of ξ can be calculated via Eq. (21) based on Definition 6. In general, when the membership function of ξ is not exactly known, the fuzzy expected value of ξ can be obtained by enumerating some discrete *h* values, that is,

$$E[\xi] \simeq \frac{1}{2L} \sum_{l=1}^{L} \left(\xi_{\sup}(h_l) + \xi_{\inf}(h_l)\right), \qquad (35)$$

where $\xi_{\sup}(h_l)$ and $\xi_{\inf}(h_l)$ are the optimistic and pessimistic functions of ξ in the h_l-cut, $l = 1, 2, \ldots, L$. The choice of the total number of *h* values, i.e. *L*, determines the closeness of the computational result to the real expected value. Theoretically, by setting infinite *h* values, this approach can obtain the exact expected value of the product of two TFNs. However, for simplicity, a finite set of *h* values which usually contains 11 *h* values from 0 to 1, i.e. $\{0, 0.1, \ldots, 0.9, 1\}$, is selected in many previous methods.

In order to provide the comparisons of our method with the *h*-cut method, Examples 1 and 2 are recalculated. Tables 1 and 2 list the results of the expected value of $\xi = \xi_1 \xi_2$ in Example 1 and $\xi = 3\xi_1 \xi_2 + \xi_3 \xi_4 + 2\xi_5 \xi_6$ in Example 2 by applying the *h*-cut method, respectively.

It can be seen that the results obtained by the two methods are not the same. In Example 1, the result derived by our method is 16, whereas the result by the *h*-cut method is 16.35. In Example 2, the result derived by our method is 20.5, whereas that by the *h*-cut method is 22.8. Obviously, with the increase of the number of TFNs ξ_i, $i = 1, 2, \ldots, n$ in the function $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$, the difference between the two results obtained by our method and the *h*-cut method gradually becomes large. In fact, it is quite in accord with intuition.

Table 1 The expected value of ξ in Example 1 using the h-cut method

$h =$		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	$E[\xi]$
ξ	Sup	70	59.8	50.4	41.8	34	27	20.8	15.4	10.8	7	4	16.35
	Inf	0	0.22	0.48	0.78	1.12	1.5	1.92	2.38	2.88	3.42	4	

Table 2 The expected value of ξ in Example 2 using the h-cut method

$h =$		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	$E[\xi]$
ξ	Sup	102	88.85	76.4	64.65	53.6	43.25	33.6	24.65	16.4	8.85	2	22.8
	Inf	−4	−3.49	−2.96	−2.41	−1.84	−1.25	−0.64	−0.01	0.64	1.31	2	

From the above comparison results, it is clear that (1) our method can get the exact expected value and thus improve the accuracy; (2) our method can reduce the computational complexity compared with the h-cut method; (3) for the h-cut method, the number of TFNs $\xi_i, \quad i = 1, 2, \ldots, n$ in the function $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ has a negative influence on the accuracy, but it has no influence on the results obtained by our method.

In some cases, our method can be used in many practical problems involving the related arithmetic operations on TFNs such as in QFD. Then in the next section, we will calculate the exact expected value of the importance of ECs and prioritize ECs in fuzzy QFD by applying our method.

4 Ranking the Importance of ECs in Fuzzy QFD

In this section, based on the method of Chen et al. [10], we manage to derive both the exact expected values and the rankings of the importance of ECs. Instead of using any fuzzy simulation or approximation algorithms, our method can obtain the exact expected values of the importance of ECs with a relatively simple calculation.

The proposed method is composed of three stages. The first stage is to collect the personal evaluations from target customers on the importance of CRs and individual judgements from the product planning team on the relationships between CRs and ECs. Then, the fuzzy importance of ECs is derived according to the synthesized importance of CRs and synthesized relationships between CRs and ECs. Finally, the final rankings of ECs are obtained based on the corresponding exact expected values of their fuzzy importance.

4.1 Collections of Individual Evaluations and Judgements

Different from traditional QFD, the input data in fuzzy QFD are expressed in fuzzy numbers rather than crisp numbers. In order to calculate the importance of ECs, the two sets of input data should be expressed in fuzzy

numbers, that is, the relative weights of CRs and the relationship measures between CRs and ECs [8].

The relative weight of each CR is one of the key inputs in QFD. Generally, the more important a CR is, the higher weight it should get. Formally, a set of CRs is represented as

$$CR = \{CR_1, CR_2, \ldots, CR_M\}. \tag{36}$$

In general, CRs are gathered by analysing questionnaires and surveys with regard to the product. The preferences of different customers on a specific product differ with personal tastes and individual needs. In order to obtain the relative weights of CRs, K customers $C_1, C_2, \ldots, C_K$ are surveyed in a target market. The kth customer's individual preference on CR_m is denoted by W_m^k as shown in Table 3. In QFD, it is natural and reasonable to suppose that $W_m^k, m = 1, 2, \ldots, M, \quad k = 1, 2, \ldots, K$, are independent and nonnegative TFNs, denoted by $(a_{mk}^L, a_{mk}^C, a_{mk}^U)$, respectively.

Furthermore, the company's product planning team should develop a set of ECs to capture the CRs in measurable technical terms. Formally, a set of ECs is collected from the expert team involved in the design of a particular product denoted as

$$EC = \{EC_1, EC_2, \ldots, EC_N\}. \tag{37}$$

The translating process from CRs to ECs in a QFD problem is carried out by a product planning team composed with S experts $D_1, D_2, \ldots, D_S$. Then, the relationship measure between CR_m and EC_n with respect to D_S is denoted by U_{mn}^s as shown in Table 3. In QFD, we are similarly allowed to assume that $U_{mn}^s, \quad m = 1, 2, \ldots, M, \quad n = 1, 2, \ldots, N, \quad s = 1, 2, \ldots, S$, are independent and nonnegative TFNs, denoted by $(b_{mns}^L, b_{mns}^C, b_{mns}^U)$, respectively.

4.2 Formulation of the Fuzzy Importance of ECs

To formulate the problem of consensus ranking of the importance of CRs, by synthesizing the fuzzy weights W_m^k of K customers, the final importance weight of CR_m can be obtained as

Table 3 Fuzzy linguistic data of a HoQ

CRs	Importance of CRs	ECs			
		EC_1	EC_2	$\cdots$	EC_N
CR_1	$C_1 : W_1^1$	$D_1 : U_{11}^1$	$D_1 : U_{12}^1$	$\cdots$	$D_1 : U_{1N}^1$
	$C_2 : W_1^2$	$D_2 : U_{11}^2$	$D_2 : U_{12}^2$	$\cdots$	$D_2 : U_{1N}^2$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	$C_K : W_1^K$	$D_S : U_{11}^S$	$D_S : U_{12}^S$	$\cdots$	$D_S : U_{1N}^S$
CR_2	$C_1 : W_2^1$	$D_1 : U_{21}^1$	$D_1 : U_{22}^1$	$\cdots$	$D_1 : U_{2N}^1$
	$C_2 : W_2^2$	$D_2 : U_{21}^2$	$D_2 : U_{22}^2$	$\cdots$	$D_2 : U_{2N}^2$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	$C_K : W_2^K$	$D_S : U_{21}^S$	$D_S : U_{22}^S$	$\cdots$	$D_S : U_{2N}^S$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
CR_M	$C_1 : W_M^1$	$D_1 : U_{M1}^1$	$D_1 : U_{M2}^1$	$\cdots$	$D_1 : U_{MN}^1$
	$C_2 : W_M^2$	$D_2 : U_{M1}^2$	$D_2 : U_{M2}^2$	$\cdots$	$D_2 : U_{MN}^2$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	$C_K : W_M^K$	$D_S : U_{M1}^S$	$D_S : U_{M2}^S$	$\cdots$	$D_S : U_{MN}^S$

$$W_m = \frac{1}{K} \sum_{k=1}^{K} W_m^k, \quad m = 1, 2, \ldots, M \qquad (38)$$

representing a trade-off among the preferences of customers surveyed. Since W_m^k, $m = 1, 2, \ldots, M$, $k = 1, 2, \ldots, K$, are independent and nonnegative TFNs, the importance of CRs W_m, $m = 1, 2, \ldots, M$, is also independent and nonnegative TFNs, denoted by (a_m^L, a_m^C, a_m^U).

Similar to the relative weights of CRs, by aggregating the evaluation of S experts, the final relationship measure between the mth CR and the nth EC can be derived from the following equation:

$$U_{mn} = \frac{1}{S} \sum_{s=1}^{S} U_{mn}^s, \quad m = 1, 2, \ldots, M, \qquad n = 1, 2, \ldots, N,$$
$$(39)$$

representing a balance of the relationship measures between CRs and ECs judged by all the consulted experts. Since U_{mn}^s, $m = 1, 2, \ldots, M$, $n = 1, 2, \ldots, N$, $s = 1, 2, \ldots, S$, are independent and nonnegative TFNs, the relationship measures between CRs and ECs, i.e. U_{mn}, $m = 1, 2, \ldots, M$, $n = 1, 2, \ldots, N$, are also independent and nonnegative TFNs, denoted by $(b_{mn}^L, b_{mn}^C, b_{mn}^U)$.

Now, let us consider the importance of ECs. In this paper, we assume that there are no correlations among ECs. Owing to the uncertainties in the product design process, the relative importance vector of CRs W and the

relationship matrix between CRs and ECs U are fuzzy. Accordingly, the importance of ECs Y is also fuzzy.

Generally, there are two main forms to be adopted for the calculation of the importance of ECs. One is that the fuzzy importance of ECs is computed by multiplying the relative importance of CRs and the relationships between CRs and ECs, which is defined by the following equation:

$$Y_n = \sum_{m=1}^{M} W_m U_{mn}. \qquad (40)$$

The other is that the importance of ECs is calculated by the fuzzy weighted average, which can be expressed as follows:

$$Y_n = \frac{\sum_{m=1}^{M} W_m U_{mn}}{\sum_{m=1}^{M} W_m}. \qquad (41)$$

To the second form, i.e. Eq.(41), although it shows the idea of normalization, the relations among W_m, U_{mn} and Y_n are not be reflected intuitively. Moreover, disadvantages of this form are apparent in the increased complexity for calculating the fuzzy importance of ECs. Finally, a critical drawback is the information loss, since approximation processes are necessary [39]. For all these reasons we think that it is more reasonable to apply the first form, that is, the relative importance of CRs and the relationships between CRs and ECs are aggregated to determine the fuzzy importance of ECs. Thus, based on Eq. (40), we have

$$Y = (W^T U)^T. \qquad (42)$$

Therefore, the fuzzy importance of EC_n, denoted by Y_n, can be calculated by

$$Y_n = \sum_{m=1}^{M} W_m U_{mn}, \quad n = 1, 2, \ldots, N. \qquad (43)$$

Since W_m and U_{mn} are fuzzy numbers, Y_n is also a fuzzy number.

4.3 Prioritization of the Fuzzy Importance of ECs

Now the problem at hand is how to prioritize these ECs. Quite a lot of researches have been conducted such as Fiorenzo et al. [13], Ko and Chen [21], Yan et al. [39] and Yan and Ma [38] in recent years. However, these methods either just got the rankings of ECs without calculating the importance of ECs, or suggested fuzzy approximation to measure the importance of ECs. It is thus necessary to develop a robust prioritization approach for deriving both the importance of ECs and their rankings effectively. Since it is natural that the fuzzy expected value operator can best reflect the amount of information conveyed by the underlying fuzzy set, a method based on the exact expected value

of the importance of ECs is proposed in this subsection to rank ECs in fuzzy QFD.

As is shown in Eq. (43), the expected value of the fuzzy importance of EC_n, i.e. Y_n, can be given by

$$E[Y_n] = E\left[\sum_{m=1}^{M} W_m U_{mn}\right], \quad n = 1, 2, \ldots, N. \quad (44)$$

After that, according to the interpretation above, in the QFD process, $W_1 U_{1n}, W_2 U_{2n}, \ldots, W_M U_{Mn}$ are independent of each other. Then based on Theorem 3, the expected value of the fuzzy importance of EC_n in Eq. (44) can be calculated by

$$E[Y_n] = \sum_{m=1}^{M} E[W_m U_{mn}], \quad n = 1, 2, \ldots, N. \quad (45)$$

We have already known that $W_1, W_2, \ldots, W_M, U_{1n}, U_{2n}, \ldots, U_{Mn}$ are all nonnegative TFNs, then in line with Eq. (30), we can obtain

$$\begin{aligned} E[Y_n] &= \sum_{m=1}^{M} E[W_m U_{mn}] \\ &= \sum_{m=1}^{M} \left[\frac{1}{6}(a_m^L b_{mn}^L + 2a_m^C b_{mn}^C + a_m^U b_{mn}^U) \right. \\ &\quad \left. + \frac{1}{12}(a_m^L b_{mn}^C + a_m^C b_{mn}^L + a_m^C b_{mn}^U + a_m^U b_{mn}^C) \right]. \end{aligned} \quad (46)$$

Obviously, the fuzzy importance of ECs can be ranked according to their corresponding exact expected value, i.e. $E[Y_n]$. For example, if $E[Y_a] > E[Y_b]$, we say EC_a is more important than EC_b, and then EC_a can get a higher ranking than EC_b.

4.4 Summary of the Proposed Approach

In summary, our proposed approach for determining the importance of ECs and their rankings is described as follows.

Step 1: Aggregations of individual evaluations and judgements.

- Gather CRs and identify customers to be surveyed in a target market.
- Gather ECs and identify experts involved in the product planning development.
- Collect the personal linguistic evaluations from customers on the importance of each CR and the individual linguistic judgements from experts on the relationships between CRs and ECs.

Step 2: Formulation of the fuzzy importance of ECs.

- Formulate the synthesized importance of CRs via Eq. (38).

- Formulate the synthesized relationships between CRs and ECs via Eq. (39).
- Derive the fuzzy importance of ECs via Eq. (43).

Step 3: Prioritization of the fuzzy importance of ECs.

- Calculate the exact expected value of the fuzzy importance of ECs via Eq. (46).
- Prioritize ECs according to their exact expected values of the fuzzy importance of ECs.

So far, the detailed implementation procedure of ranking the importance of ECs based on the proposed method has been presented. Since the computing process is easier and the results are more precise, our method provides a practical convenience in the whole QFD process.

It should be noted that the differences between our method and Chen et al.'s [10] method mainly consist of the following two parts. The one is that our method uses Eq. (40) which is considered to be more reasonable to determine the importance of ECs, while Chen et al.'s [10] method uses Eq. (41) to determine them. The other is that our method can obtain the exact expected values of the importance of ECs through Eq. (46), whereas Chen et al.'s [10] method suggests the use of h-cut to derive their approximate expected values.

In this section, the relative importance of CRs and the relationships between CRs and ECs are assumed to be nonnegative TFNs. Actually, there also exists nonpositive relations in the QFD matrix. As an example, Cheng and Chiu [11] considered the negative relationships between CRs and ECs, since some technical attributes were not always positively met with all customer needs. In addition, both positive and negative relations were often taken into account to estimate the correlations among ECs in many previous studies. When such situations occur, we can also calculate the exact expected value of the importance of ECs in fuzzy QFD according to Eq. (30) (see Remark 2).

5 Numerical Example

The design case of a flexible manufacturing system (FMS) [10, 20, 24] is applied in this paper to show the potential applications of the proposed approach for determining the importance of ECs and their rankings. In general, CRs are gathered by analysing questionnaire surveys and interviews with regard to the product, which is conducted by the marketing department. Since customers words are too general or detailed to be directly used as customer requirements, they are usually organized as a tree-like hierarchical structure to form various levels of CRs and, according to the situation, those at a specific level are

chosen as the final CRs [4, 17]. In this example, eight major CRs are identified to represent the paramount needs of the customers, namely, "high production volume" (CR_1), "short setup time" (CR_2), "load carrying capacity" (CR_3), "user friendliness" (CR_4), "associated functions" (CR_5), "modularity" (CR_6), "wide tool variety" (CR_7) and "wide product variety" (CR_8).

The determination of ECs is usually led by the engineering department. The design team should utilize their experience and knowledge to collect one or more ECs to meet each CR. In QFD, ECs may not be specific design details or solutions, but there must be some characteristics that can be measured and set target values. In accordance with the design team's experience and knowledge, 10 major ECs are identified to meet eight major CRs, namely, "automatic gauging" (EC_1), "the tool change system" (EC_2), "tool monitoring system" (EC_3), "coordinate measuring machine" (EC_4), "automated guided vehicle" (EC_5), "conveyor" (EC_6), "programmable logic controller" (EC_7), "storage and retrieval system" (EC_8), "modular fixturing" (EC_9) and "robots" (EC_{10}). The design team needs to translate eight CRs into technical specifications from 10 ECs. Consequently, the design team needs to prioritize these ECs in developing a new FMS.

The relative weights of CRs are classified into seven levels to describe the difference of importance, that is, very unimportant (VN), quite unimportant (QN), unimportant (N), some important (SI), moderately important (MI), important (I) and very important (VI). The TFNs (0, 0, 0.2), (0, 0.2, 0.4), (0.2, 0.35, 0.5), (0.3, 0.5, 0.7), (0.5, 0.65, 0.8), (0.6, 0.8, 1) and (0.8, 1, 1) are used to quantity these seven linguistic terms. Figure 5 shows the membership functions of these TFNs.

Suppose that 10 customers are surveyed in the target market, represented by C_k, $k = 1, 2, \ldots, 10$. Their personal evaluations on each CR are summarized in Table 4.

After synthesizing individual weights of the 10 customers on the eight CRs by Eq. (38), the final relative weights of eight CRs W can be obtained as $W_1 = (0.71, 0.91, 0.98)$, $W_2 = (0.40, 0.58, 0.75)$, $W_3 = (0.61, 0.80, 0.94)$, $W_4 = (0.57, 0.76, 0.88)$, $W_5 = (0.30, 0.47, 0.64)$, $W_6 = (0.47, 0.65, 0.83)$, $W_7 = (0.52, 0.70,$
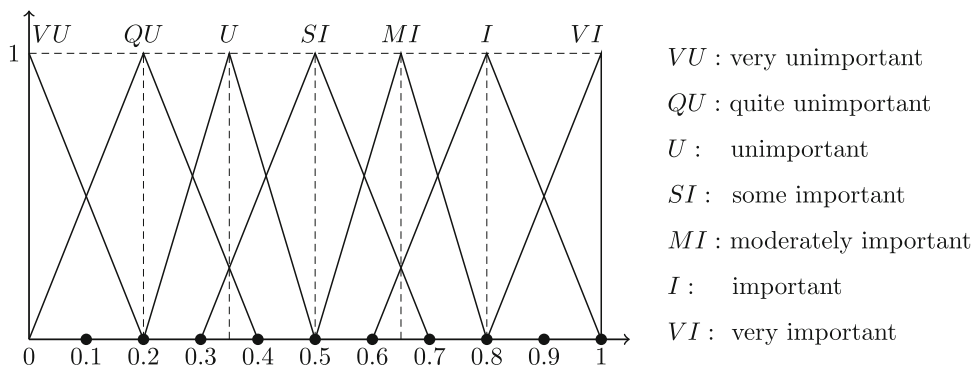

Fig. 5 The triangular fuzzy weights of CRs

Table 4 The relative fuzzy weights of eight CRs assessed by 10 customers

	CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7	CR_8
C_1	I	MI	VI	VI	MI	MI	SI	VI
C_2	VI	SI	I	I	MI	MI	I	VI
C_3	VI	MI	I	MI	MI	I	SI	VI
C_4	VI	SI	VI	MI	MI	I	SI	I
C_5	VI	MI	I	I	MI	I	SI	I
C_6	VI	SI	MI	MI	MI	I	SI	I
C_7	VI	SI	I	I	SI	MI	SI	I
C_8	I	MI	MI	MI	SI	I	I	MI
C_9	I	MI	I	I	SI	MI	I	MI
C_{10}	MI	SI	MI	MI	SI	I	I	MI

Table 5 The relationship matrix between CRs and ECs and relative importance of CRs

Customer requirements	Fuzzy weights of customer requirements	Engineering characteristics									
		Fuzzy relationship matrix between customer requirements and engineering characteristics									
		EC₁	EC₂	EC₃	EC₄	EC₅	EC₆	EC₇	EC₈	EC₉	EC₁₀
CR₁	W₁ (0.71, 0.91, 0.98)	(0.31, 0.54, 0.76)	(0.16, 0.36, 0.60)	(0.27, 0.40, 0.73)	(0.37, 0.61, 0.80)	(0.13, 0.29, 0.53)	(0.17, 0.39, 0.61)	(0.36, 0.57, 0.79)	(0.17, 0.39, 0.61)	(0.13, 0.29, 0.53)	(0.17, 0.32, 0.56)
CR₂	W₂ (0.40, 0.58, 0.75)	(0.09, 0.25, 0.50)	(0.47, 0.71, 0.91)	(0.55, 0.82, 0.96)	(0.13, 0.32, 0.56)	(0.13, 0.29, 0.53)	(0.24, 0.46, 0.69)	(0.34, 0.57, 0.80)	(0.27, 0.50, 0.73)	(0.47, 0.71, 0.87)	(0.04, 0.18, 0.44)
CR₃	W₃ (0.61, 0.80, 0.94)	(0.56, 0.82, 0.96)	(0.27, 0.50, 0.73)	(0.61, 0.89, 1.00)	(0.13, 0.36, 0.63)	(0.17, 0.36, 0.59)	(0.13, 0.36, 0.59)	(0.53, 0.79, 0.91)	(0.17, 0.39, 0.61)	(0.09, 0.25, 0.50)	(0.09, 0.29, 0.53)
CR₄	W₄ (0.57, 0.76, 0.88)	(0.39, 0.64, 0.90)	(0.27, 0.50, 0.73)	(0.23, 0.46, 0.70)	(0.24, 0.46, 0.69)	(0.13, 0.32, 0.56)	(0.17, 0.36, 0.59)	(0.24, 0.46, 0.70)	(0.20, 0.43, 0.66)	(0.23, 0.46, 0.70)	(0.09, 0.25, 0.50)
CR₅	W₅ (0.30, 0.47, 0.64)	(0.56, 0.82, 0.96)	(0.59, 0.86, 1.00)	(0.39, 0.61, 0.83)	(0.59, 0.86, 0.96)	(0.09, 0.25, 0.50)	(0.13, 0.36, 0.59)	(0.27, 0.50, 0.73)	(0.31, 0.54, 0.76)	(0.34, 0.57, 0.76)	(0.41, 0.64, 0.83)
CR₆	W₆ (0.47, 0.65, 0.83)	(0.13, 0.36, 0.59)	(0.13, 0.32, 0.56)	(0.29, 0.50, 0.71)	(0.39, 0.61, 0.83)	(0.09, 0.32, 0.56)	(0.17, 0.39, 0.61)	(0.09, 0.29, 0.53)	(0.09, 0.32, 0.56)	(0.13, 0.36, 0.59)	(0.13, 0.32, 0.56)
CR₇	W₇ (0.52, 0.70, 0.86)	(0.09, 0.32, 0.56)	(0.33, 0.54, 0.74)	(0.36, 0.57, 0.79)	(0.04, 0.25, 0.50)	(0.29, 0.50, 0.71)	(0.31, 0.61, 0.83)	(0.56, 0.82, 1.00)	(0.36, 0.57, 0.79)	(0.09, 0.32, 0.56)	(0.04, 0.29, 0.53)
CR₈	W₈ (0.63, 0.82, 0.93)	(0.21, 0.43, 0.64)	(0.09, 0.29, 0.53)	(0.39, 0.61, 0.79)	(0.31, 0.54, 0.76)	(0.04, 0.25, 0.50)	(0.17, 0.39, 0.61)	(0.50, 0.75, 0.96)	(0.36, 0.57, 0.79)	(0.09, 0.32, 0.56)	(0.16, 0.39, 0.63)

0.86) and $W_8 = (0.63, 0.82, 0.93)$, which are shown on the left wall of the HoQ in Table 5.

Likewise, the relationships between CRs and ECs are linguistically judged as none (N), weak (W), moderate (M), strong (S), or very strong (VS), which can be expressed by the TFNs (0, 0, 0.3), (0, 0.3, 0.5), (0.3, 0.5, 0.7), (0.5, 0.75, 1) and (0.7, 1, 1), respectively, and the membership functions of these TFNs are shown in Fig. 6.

Meanwhile, it is assumed that seven experts, denoted by D_s, $s = 1, 2, \ldots, 7$, are involved in evaluating the relationships between CRs and ECs. After aggregating all the assessments of each expert using Eq. (39), the relationship matrix between eight CRs and 10 ECs U can be obtained as shown in the room of the HoQ in Table 5.

5.1 The Proposed Exact Expected Value-Based Method

Since the relative weights of CRs and the relationship measures between CRs and ECs are TFNs, the proposed method is employed to calculate the exact expected value of the importance of ECs. Firstly, on the basis of Eq. (43), the fuzzy importance of EC_1, denoted by Y_1, can be expressed as

$$Y_1 = \sum_{m=1}^{8} W_m U_{m1} = W_1 U_{11} + W_2 U_{21} + \cdots + W_8 U_{81}. \tag{47}$$

Since $W_1, W_2, \ldots, W_8, U_{11}, U_{21}, \ldots, U_{81}$ are all independent and nonnegative TFNs, the parameters $W_m = (a_m^L, a_m^C, a_m^U)$ and $U_{m1} = (b_{m1}^L, b_{m1}^C, b_{m1}^U)$, $m = 1, 2, \ldots, 8$ as shown in Table 5 can be brought into Eq. (46). Thus, we can obtain the exact expected value of the fuzzy importance of EC_1, i.e. $E[Y_1]$ as

$$\begin{aligned} E[Y_1] = \sum_{m=1}^{8} \Bigg[&\frac{1}{6} (a_m^L b_{m1}^L + 2a_m^C b_{m1}^C + a_m^U b_{m1}^U) \\ &+ \frac{1}{12} (a_m^L b_{m1}^C + a_m^C b_{m1}^L + a_m^C b_{m1}^U + a_m^U b_{m1}^C) \Bigg] \\ = &\ 3.1184. \end{aligned} \tag{48}$$

Following the similar calculation of the exact expected value of the importance of EC_1, the exact expected values of the fuzzy importance of $EC_2, EC_3, \ldots, EC_{10}$ can also be obtained. The results are summarized in the first line of Table 6.

All ECs are sorted in accordance with the values of $E[Y_n]$. The larger the value of $E[Y_n]$ is, the higher priority EC_n can get. Therefore, it can be seen that the ranking of ECs based on the proposed approach and the stated criterion can be obtained as follows:
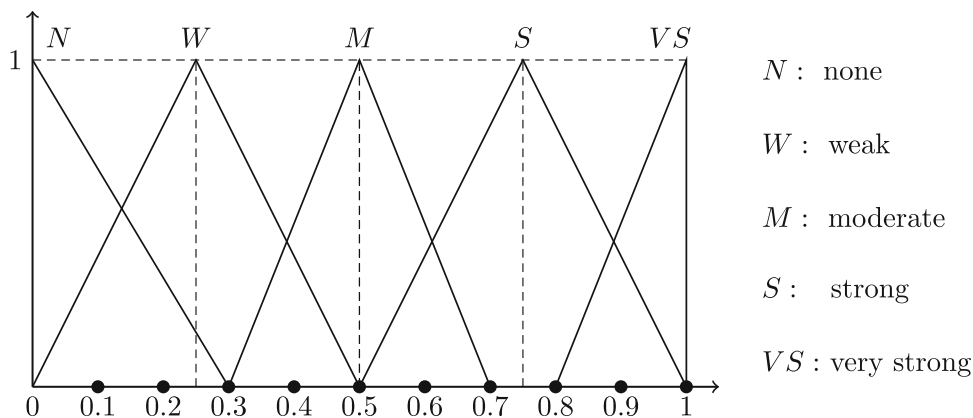
Fig. 6 The triangular fuzzy relationship measures between CRs and ECs

Table 6 The fuzzy expected values and the ordinal ranking results of the importance of 10 ECs

	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7	EC_8	EC_9	EC_{10}
$E[Y_j]$	3.1184	2.9859	3.5507	2.9883	2.0345	2.4867	3.5453	2.7633	2.3992	2.0839
Ranking	3	5	1	4	10	7	2	6	8	9

$EC_3 \succ EC_7 \succ EC_1 \succ EC_4 \succ EC_2 \succ EC_8 \succ EC_6 \succ EC_9$
$\succ EC_{10} \succ EC_5,$

where $\succ$ means "is more preferred than". The second line of Table 6 shows the above rankings of 10 ECs.

5.2 Comparisons with the *h*-Cut Method in QFD

In this subsection, since the *h*-cut is applied by many previous methods in QFD, particularly by Chen et al.'s [10] method which is the basis of our method, we compare our proposed method with the *h*-cut method in QFD. Note that in the method of Chen et al. [10], Eq. (2) is used to determine the fuzzy importance of ECs, which is considered to be unreasonable as mentioned earlier. Thus, a direct comparison between our method and Chen et al.'s [10] method makes no sense. In order to obtain more meaningful comparison results, we choose Eq. (40) to determine the importance of ECs in this subsection. Then, the *h*-cut method is adopted to calculate the expected values of the importance of ECs, in which 11 *h* values $\{0, 0.1, \ldots, 0.9, 1\}$ are selected and the corresponding expected value of the importance of an EC can be obtained by Eq. (35). Table 7 shows the rankings of the fuzzy importance of 10 ECs obtained by the *h*-cut method for the design of the FMS.

It can be seen that 10 ECs are prioritized by the *h*-cut method as

$EC_7 \succ EC_3 \succ EC_1 \succ EC_2 \succ EC_4 \succ EC_8 \succ EC_6 \succ EC_9$
$\succ EC_{10} \succ EC_5,$

which differs from the rankings obtained by our proposed method in that the ranking between EC_3 and EC_7 is opposite, as well as between EC_2 and EC_4. Our proposed method prioritizes these four ECs as $EC_3 \succ EC_7$ and $EC_4 \succ EC_2$, whereas the *h*-cut method ranks these four ECs as $EC_7 \succ EC_3$ and $EC_2 \succ EC_4$. The results show that (1) compared with the approximate expected values obtained by the *h*-cut method, the exact expected values obtained by our method are much better in reflecting the average values of the fuzzy importance of ECs; (2) our proposed approach prioritizes ECs according to their exact expected values of the fuzzy importance, whereas the *h*-cut method prioritizes ECs based on their approximate expected values. Thus, the final ranking results derived by our method are more reliable; (3) compared with the *h*-cut method, our method has a relatively easier computing process, and the final results are more accurate, which is more suitable for the QFD team to apply to the real work.

5.3 Other Applications of the Exact Expected Value of the Importance of ECs

In QFD, the expected value of the importance of an EC plays a significant role. It not only can be used to rank ECs, but also allows decision-makers to get more useful and reliable information such as overall customer satisfaction, marginal benefit of ECs and so on. Under these circumstances, combined with the above example of the design of the FMS, two concrete instances are presented to show its extensive applications.

Table 7 The ranking results of 10 ECs using the *h*-cut method

		h = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	$E[Y_j]$	Ranking
EC_1	Sup	4.9951	4.9596	4.7309	4.5072	4.2887	4.0752	3.8668	3.6636	3.4654	3.2722	2.9748	3.1008	3
	Inf	1.2282	1.4550	1.6095	1.7707	1.9384	2.1123	2.2939	2.4815	2.6758	2.8767	2.9748		
EC_2	Sup	4.8332	4.8037	4.5811	4.3630	4.1495	3.9406	3.7362	3.5363	3.3411	3.1503	2.8542	2.9677	4
	Inf	1.0918	1.3225	1.4775	1.6394	1.8081	1.9837	2.1661	2.3553	2.5514	2.7544	2.8542		
EC_3	Sup	5.5064	5.4326	5.2232	4.9921	4.7657	4.5441	4.3271	4.1150	3.9076	3.7049	3.4121	3.5334	2
	Inf	1.6011	1.8287	1.9898	2.1573	2.3310	2.5112	2.6976	2.8904	3.0896	3.2951	3.4121		
EC_4	Sup	4.8435	4.8243	4.5939	4.3688	4.1488	3.9340	3.7243	3.5199	3.3206	3.1265	2.7968	2.9657	5
	Inf	1.1072	1.3562	1.5059	1.6620	1.8247	1.9939	2.1696	2.3518	2.5406	2.7358	2.7968		
EC_5	Sup	3.8247	3.7259	3.4991	3.2780	3.0626	2.8529	2.6488	2.4504	2.2577	2.0707	1.8438	2.0281	10
	Inf	0.5674	0.6921	0.8034	0.9201	1.0423	1.1699	1.3029	1.4414	1.5853	1.7346	1.8438		
EC_6	Sup	4.3541	4.2725	4.0445	3.8218	3.6046	3.3927	3.1863	2.9853	2.7896	2.5994	2.3528	2.4777	7
	Inf	0.7801	0.9417	1.0791	1.2230	1.3735	1.5306	1.6942	1.8644	2.0413	2.2247	2.3528		
EC_7	Sup	5.5695	5.5017	5.2584	5.0204	4.7879	4.5607	4.3388	4.1223	3.9112	3.7055	3.4434	3.5347	1
	Inf	1.5812	1.7799	1.9448	2.1164	2.2947	2.4797	2.6714	2.8698	3.0749	3.2866	3.4434		
EC_8	Sup	4.6648	4.5962	4.3624	4.1340	3.9110	3.6935	3.4813	3.2746	3.0733	2.8774	2.6119	2.7941	6
	Inf	1.9300	1.1768	1.3190	1.4675	1.6225	1.7839	1.9517	2.1260	2.3065	2.4935	2.6119		
EC_9	Sup	4.2364	4.1713	3.9419	3.7179	3.4991	3.2857	3.0776	2.8747	2.6772	2.4850	2.2136	2.3862	8
	Inf	0.7329	0.9091	1.0389	1.1749	1.3169	1.4651	1.6195	1.7799	1.9465	2.1192	2.2136		
EC_{10}	Sup	3.8547	3.7924	3.5643	3.3418	3.1250	2.9139	2.7084	2.5087	2.3147	2.1264	1.8492	2.0694	9
	Inf	0.5486	0.7179	0.8318	0.9513	1.0764	1.2070	1.3432	1.4850	1.6323	1.7852	1.8492		

5.3.1 Determining the Overall Customer Satisfaction

Facing with fierce competition in marketplaces, companies try to determine the settings such that the best customer satisfaction of products could be obtained [6]. The overall customer satisfaction, S, which indicates the degree of satisfaction of CR_m in comparison with those of the competitors, can be considered as a mathematical aggregation of Y_n, $n = 1, 2, \ldots, N$ [9]. In our problem, assume that four main competitors, i.e. Comp1 (our corporation), Comp2, Comp3 and Comp4 are considered in the above example of the design of the FMS, and let $X = (x_1, x_2, \ldots, x_{10})^T$ be the decision vector of the level of attainment of ECs, in which x_n is the level of attainment of EC_n, $0 \leq x_n \leq 1$, $n = 1, 2, \ldots, 10$. Then, technical measure data have been collected from the company and its main competitors by testing, and the current target values of ECs of all competitors can be normalized as follows:

$$(X_1, X_2, X_3, X_4)^T$$
$$= \begin{pmatrix} 0.42 & 0.50 & 0 & 0.80 & 0.54 & 0.85 & 0.70 & 0.62 & 0 & 0.30 \\ 0.86 & 0.33 & 0.52 & 0.90 & 0.86 & 0.78 & 0 & 0.62 & 0.70 & 0.45 \\ 0.86 & 0.64 & 0.90 & 0.80 & 0.50 & 0.67 & 0.35 & 0.50 & 0.42 & 0.88 \\ 0.54 & 0 & 0.67 & 0 & 0 & 0.37 & 0.90 & 0.84 & 0.50 & 0.70 \end{pmatrix}.$$
$$(49)$$

Then the fuzzy expected value of the overall customer satisfaction, $E[S]$, can be calculated by

$$E[S] = E[Y^T X] = E\left[\sum_{m=1}^{10} Y_n x_n\right] = \sum_{m=1}^{10} E[Y_n] x_n, \quad (50)$$

where $E[Y_n]$ is the expected value of fuzzy importance of EC_n. Since we have obtained $E[Y_n]$ as listed in Table 6, for convenience of calculation, we normalize $E[Y_n]$, $n = 1, 2, \ldots, 10$. Then, the relative importance of EC_n, i.e. the normalized expected value of Y_n, denoted by $E[Y'_n]$, can be expressed as

$$E[Y'_n] = \frac{E[Y_n]}{\sum_{n=1}^{N} E[Y_n]}, \quad n = 1, 2, \ldots, N, \quad (51)$$

where $0 < E[Y'_n] < 1$. For example, the normalized expected value of Y_1, i.e. $E[Y'_1]$, can be calculated by

$$E[Y'_1] = \frac{E[Y_1]}{E[Y_1] + E[Y_2] + \cdots + E[Y_{10}]}$$
$$= \frac{3.1184}{3.1184 + 2.9859 + \cdots + 2.0839} \quad (52)$$
$$= 0.1115.$$

Likewise, similar calculations for $Y'_2, Y'_3, \ldots, Y'_{10}$ can be done and the results are listed in the second line of Table 8.

Then by using $E[Y'_n]$, the expected value of the overall customer satisfaction can be scaled from 0 to 1 as

Table 8 The normalized fuzzy expected values of the importance of 10 ECs

	EC$_1$	EC$_2$	EC$_3$	EC$_4$	EC$_5$	EC$_6$	EC$_7$	EC$_8$	EC$_9$	EC$_{10}$
$E[Y_j]$	3.1184	2.9859	3.5507	2.9883	2.0345	2.4867	3.5453	2.7633	2.3992	2.0839
$E[Y_j']$	0.1115	0.1068	0.1270	0.1069	0.0728	0.0889	0.1268	0.0988	0.0858	0.0745

$$E[S'] = \sum_{n=1}^{10} E[Y_n']x_n, \tag{53}$$

where $E[S']$ is the normalized expected value of the overall customer satisfaction. This facilitates quickly and visually identifying the highest overall customer satisfaction of products among the competitors. Table 9 shows the relative fuzzy expected values and the ranking results of the overall customer satisfaction for products of the four competitors.

As can be seen from the ranking results in Table 9, on the one hand, the existing design of our corporation (Comp$_1$) currently has a low score of $E[S']$ (0.4731) and is ranked third, which means that it is much less competitive. On the other hand, Comp$_3$ achieves the highest customer satisfaction (0.6555) among four competitors, so it is necessary for our company to improve the existing design in order to enhance its competitiveness and elevate client satisfaction.

5.3.2 Cost–Benefit Analysis of ECs

In order to complete the assessments and rankings of ECs more practical, some studies considered the costs of improvement [3, 35]. Denote that T_n, $n = 1, 2, \ldots, 10$ is the total cost for improving EC$_n$ in the design of the FMS,

Table 9 The relative fuzzy expected values of the overall customer satisfaction

	Comp$_1$	Comp$_2$	Comp$_3$	Comp$_4$
$E[S']$	0.4731	0.5803	0.6555	0.4705
Ranking	3	2	1	4

which are the crisp numbers and listed in the second line of Table 10, respectively. Then, the marginal benefit U_n of ECs can be calculated through the ratio between benefit and cost, which is expressed by

$$U_n = \frac{Y_n}{T_n}, \quad n = 1, 2, \ldots, 10. \tag{54}$$

Since the importance of EC$_n$, i.e. Y_n is a fuzzy number, in order to get the cost–benefit analysis results, defuzzified values should be calculated. Due to its information richness, the expected value of Y_n, $E[Y_n]$, is usually utilized to defuzzify the fuzzy number. Then, the marginal benefit U_n can be obtained by

$$U_n = \frac{E[Y_n]}{T_n}, \quad n = 1, 2, \ldots, 10. \tag{55}$$

In accordance with Eq. (55), the final marginal benefits of ECs and their rankings are shown in the last two rows of Table 10.

In particular, the greater the U_n value, the higher the improvement priority of the corresponding EC. Table 9 indicates that EC$_7$ which gets the highest score is the one which has the highest impact on CRs, and should be a top priority by the corporation to enhance both customer satisfaction and comprehensive benefits. Moreover, contrast that with the previous ranking results of the fuzzy importance of ECs in Table 5. When the total cost for improving ECs is not an issue, EC$_3$ gains the highest priority. Yet because improving the load carrying capacity (EC$_3$) of the FMS carries a very high capital cost, it has a lower marginal benefit and ranks only the 7th in Table 9. Hence, in economic terms, the firm should not put more energy and efforts on EC$_3$ in line with the cost–benefit analysis of ECs.

Table 10 Costs and marginal benefits of ECs

	EC$_1$	EC$_2$	EC$_3$	EC$_4$	EC$_5$	EC$_6$	EC$_7$	EC$_8$	EC$_9$	EC$_{10}$
$E[Y_n]$	3.1184	2.9859	3.5507	2.9883	2.0345	2.4867	3.5453	2.7633	2.3992	2.0839
T_n	0.55	0.725	0.9	0.75	0.4	0.85	0.6	0.55	0.725	0.9
U_n	5.6699	4.1185	3.9452	3.9844	5.0863	2.9254	5.9089	5.0242	3.3093	2.3154
Ranking	2	5	7	6	3	9	1	4	8	10

6 Conclusions

One of the core problems of QFD is the determination of the importance of ECs, which can provide essential information to a design team to carry out resource allocation and product planning. In fuzzy QFD, the relative weights of CRs and the relationship measures between CRs and ECs are often expressed as fuzzy numbers, while TFN as one of the most common types of fuzzy numbers has been applied by many researchers, and was also adopted to measure the fuzziness of CRs and ECs in this paper.

To sum up, our contributions to the related research area mainly lie in the following three aspects. First of all, we proposed a method to calculate the exact expected value of the product of two TFNs, which has higher accuracy and easier computing process. In addition, we applied this method to derive the exact expected values of the fuzzy importance of ECs and the rankings of ECs. The performance of this method was well certified by a practical product development example. Compared with the *h*-cut method, our method obviously gave a more reliable ranking result of ECs. Finally, we demonstrated the widespread use of the exact expected values of the importance of ECs through some concrete instances, namely, determining the overall customer satisfaction and cost–benefit analysis of ECs in fuzzy QFD.

In the future research, deep research is needed to take into account the correlations among ECs and the more benchmarking information compared to competitors to enrich our study in the area of determination of the importance of ECs and its applications. In addition, it should be noted that our consideration was only confined to TFNs. The proposed approach can be extended to the trapezoidal fuzzy number or other types of fuzzy numbers.

References

1. Akao, Y.: Quality Function Deployment: Integrating Customer Requirements into Product Design (Mazur, G. Trans.). Productivity Press, Cambridge (1990)
2. Ban, A.I., Coroianu, L.: Simplifying the search for effective ranking of fuzzy numbers. IEEE Trans. Fuzzy Syst. **23**(2), 327–339 (2015)
3. Bottani, E., Rizzi, A.: Strategic management of logistics service: a fuzzy QFD approach. Int. J. Prod. Econ. **103**(2), 585–599 (2006)
4. Chan, L.K., Kao, H.P., Wu, M.L.: Rating the importance of customer needs in quality function deployment by fuzzy and entropy methods. Int. J. Prod. Res. **37**(11), 2499–2518 (1999)
5. Chan, K.Y., Kwong, C.K., Law, M.C.: Modelling customer satisfaction for product developent using genetic programming. J. Eng. Des. **22**(1), 55–68 (2011)
6. Chan, K.Y., Kwong, C.K., Wong, T.C.: A fuzzy ordinary regression method for modeling customer preference in tea maker design. Neurocomputing **142**, 147–154 (2014)
7. Chen, L.H., Ko, W.C., Tseng, C.Y.: Fuzzy approaches for constructing house of quality in QFD and its applications: a group decision-making method. IEEE Trans. Eng. Manag. **60**(1), 77–87 (2013)
8. Chen, Y., Tang, J., Fung, R.Y.K., Ren, Z.: Fuzzy regression-based mathematical programming model for quality function deployment. Int. J. Prod. Res. **42**(5), 1009–1027 (2004)
9. Chen, Y., Fung, R.Y.K., Tang, J.: Fuzzy expected value modelling approach for determining target values of engineering characteristics in QFD. Int. J. Prod. Res. **43**(17), 3583–3604 (2005)
10. Chen, Y., Fung, R.Y.K., Tang, J.: Rating technical attributes in fuzzy QFD by integrating fuzzy weighted average method and fuzzy expected value operator. Eur. J. Oper. Res. **174**(3), 1553–1566 (2006)
11. Cheng, B.W., Chiu, W.H.: Two-dimensional quality function deployment: an application for deciding quality strategy using fuzzy logic. Total Qual. Manag. Bus. Excell. **18**(4), 451–470 (2007)
12. Ezzatia, R., Allahviranloob, T., Khezerlooa, S., Khezerloob, M.: An approach for ranking of fuzzy numbers. Expert Syst. Appl. **39**(1), 690–695 (2012)
13. Fiorenzo, F., Maurizio, G., Domenico, M., Luca, M.: Prioritisation of engineering characteristics in QFD in the case of customer requirements orderings. Int. J. Prod. Res. **53**(13), 3975–3988 (2015)
14. Fuller, R., Majlender, P.: On weighted possibilistic mean and variance of fuzzy numbers. Fuzzy Sets Syst. **136**(3), 363–374 (2003)
15. Fung, R.Y.K., Tang, J., Tu, P.Y., Chen, Y.: Modelling of quality function deployment planning with resource allocation. Res. Eng. Des. **14**(4), 247–255 (2003)
16. Geng, X., Chu, X., Xue, D., Zhang, Z.: An integrated approach for rating engineering characteristics final importance in product-service system development. Comput. Ind. Eng. **59**(4), 585–594 (2010)
17. Griffin, A., Hauser, J.: The voice of the customer. Mark. Sci. **12**(1), 1–27 (1993)
18. Hauser, J.R., Clausing, D.: The house of quality. Harv. Bus. Rev. **66**(3), 63–73 (1988)
19. Kao, C., Liu, S.T.: Fractional programming approach to fuzzy weighted average. Fuzzy Sets Syst. **120**(3), 435–444 (2001)
20. Khoo, L.P., Ho, N.C.: Framework of a fuzzy quality function deployment system. Int. J. Prod. Res. **34**(2), 299–311 (1996)
21. Ko, W.C., Chen, L.H.: An approach of new product planning using quality function deployment and fuzzy linear programming model. Int. J. Prod. Res. **52**(6), 1728–1743 (2014)
22. Kwong, C.K., Chen, Y., Bai, H., Chan, D.S.K.: A methodology of determining aggregated importance of engineering characteristics in QFD. Comput. Ind. Eng. **53**(4), 667–679 (2007)
23. Kwong, C.K., Chen, Y., Chan, K.Y., Luo, X.: A generalised fuzzy least-squares regression approach to modelling relationships in QFD. J. Eng. Des. **21**(5), 601–613 (2010)
24. Kwong, C.K., Ye, Y., Chen, Y., Choy, K.L.: A novel fuzzy group decision-making approach to prioritizing engineering characteristics in QFD under uncertainties. Int. J. Prod. Res. **49**(19), 5801–5820 (2011)
25. Lee, A.H.I., Lin, C.Y.: An integrated fuzzy QFD framework for new product development. Flex. Serv. Manuf. J. **23**(1), 26–47 (2011)
26. Liu, B.: Theory and Practice of Uncertain Programming. Physica-Verlag, Heidelberg (2002)
27. Liu, B.: Uncertainty Theory. Springer, Berlin (2004)
28. Liu, B.: Uncertainty Theory, 2nd edn. Springer, Berlin (2007)

29. Liu, B., Liu, Y.-K.: Expected value of fuzzy variable and fuzzy expected value models. IEEE Trans. Fuzzy Syst. **10**(4), 445–450 (2002)

30. Liu, Y., Chen, Y., Zhou, J., Zhong, S.: Fuzzy linear regression models for QFD using optimized h values. Eng. Appl. Artif. Intell. **39**, 45–54 (2015)

31. Liu, Y.-K., Gao, J.: The independence of fuzzy variables in credibility theory and its applications. Int. J. Uncertain. Fuzziness Knowl. Based Syst. **15**(Suppl.2), 1–20 (2007)

32. Liu, Y.-K., Liu, B.: Expected value operator of random fuzzy variable and random fuzzy expected value models. Int. J. Uncertain. Fuzziness Knowl. Based Syst. **11**(2), 195–215 (2003)

33. Nehi, H.M.: A new ranking method for intuitionistic fuzzy numbers. Int. J. Fuzzy Syst. **12**(1), 80–86 (2010)

34. Nguyen, H.T.: A note on the extension principle for fuzzy sets. J. Math. Anal. Appl. **64**(2), 369–380 (1978)

35. Trappey, C.V., Trappey, A.J.C., Hwang, S.J.: A computerized quality function deployment approach for retail services. Comput. Ind. Eng. **30**(4), 611–622 (1996)

36. Wang, Y.: A fuzzy-normalisation-based group decision-making approach for prioritising engineering design requirements in QFD under uncertainty. Int. J. Prod. Res. **50**(23), 6963–6977 (2012)

37. Yager, R.R.: Fusion of multi-agent preference orderings. Fuzzy Sets Syst. **117**(1), 1–12 (2001)

38. Yan, H.B., Ma, T.: A group decision-making approach to uncertain quality function deployment based on fuzzy preference relation and fuzzy majority. Eur. J. Oper. Res. **241**(3), 815–829 (2015)

39. Yan, H.B., Ma, T., Li, Y.: A novel fuzzy linguistic model for prioritising engineering design requirements in quality function deployment under uncertainties. Int. J. Prod. Res. **51**(21), 6336–6355 (2013)

40. Zadeh, L.A.: Fuzzy sets. Inf. Control **8**(3), 338–353 (1965)

41. Zhong, S., Zhou, J., Chen, Y.: Determination of target values of engineering characteristics in QFD using a fuzzy chance-constrained modelling approach. Neurocomputing **142**, 125–135 (2014)

42. Zhou J., Yang F., Wang K.: Fuzzy arithmetic on LR fuzzy numbers with applications to fuzzy programming. J. Intell. Fuzzy Syst. (2015). doi:10.3233/IFS-151712.

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