Int. J. Fuzzy Syst. (2017) 19(1):257–271
DOI 10.1007/s40815-015-0086-4

Exponential Stability and Asynchronous Stabilization of Nonlinear Impulsive Switched Systems via Switching Fuzzy Lyapunov Function Approach

Qunxian Zheng[1] · Hongbin Zhang[1]

Received: 12 April 2015 / Revised: 13 July 2015 / Accepted: 10 September 2015 / Published online: 17 October 2015

Abstract In this paper, the Takagi–Sugeno (T-S) fuzzy model is first used to deal with the exponential stability and asynchronous stabilization problem of a class of continuous-time nonlinear impulsive switched systems with asynchronous behaviors. In order to reduce the conservativeness resulting from the quadratic Lyapunov functions (QLFs) and nonlinearity, the switching fuzzy Lyapunov functions (FLFs) are proposed using the switching information and structural information of membership function in the rule base. Using the switching FLFs approach and the mode-dependent average dwell time (MDADT) technique, we obtain stability conditions for the open-loop nonlinear impulsive switched systems and stabilization conditions for the closed-loop nonlinear impulsive switched systems. Moreover, the stability and stabilization results are formulated in the form of LMIs. Finally, a numerical example and a chemical process example are given to demonstrate the advantage and applicability of the proposed method.

Keywords Nonlinear impulsive switched systems · Asynchronous switching · Takagi–Sugeno (T-S) fuzzy model · Switching fuzzy Lyapunov functions (FLFs) · Mode-dependent average dwell time (MDADT)

✉ Hongbin Zhang
zhanghb@uestc.edu.cn

[1] School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu 611731, Sichuan, People's Republic of China

1 Introduction

Switched systems are an important class of hybrid systems encountered in numerous practical circumstances. The stability problem is a main concern in the field of switched systems [1–6]. Up to now, two stability issues have been addressed in the literature, i.e., the stability under arbitrary switching and the stability under constrained switching. As for the arbitrary switching issue, the study is mainly based on a common Lyapunov function for all subsystems [1, 6]. As for the constrained switching issue, the multiple Lyapunov functions play an important role in the stability analysis [7–9]. As one typical example of the constrained switching, the average dwell time (ADT) logic is proposed in [3]. The ADT is widely used to investigate the problems of stability and stabilization of switched systems [10–12]. However, most works assumed that the ADT is independent of the system models. Thus, the acquired results have more or less conservativeness compared with the case if the ADT can be extended to the mode-dependent average dwell time (MDADT) [13]. Recently several, though not many, works have studied the control problem for switched systems with MDADT [14, 15].

It is well known that many practical systems exhibit impulsive dynamical behaviors because of sudden changes at certain instants during the dynamical process. The switched systems with impulsive effect can be modeled as impulsive switched systems [16]. Due to the existence of the impulsive effect, the traditional pure continuous or pure discrete models cannot well describe these systems, so it is important and necessary for us to study the impulsive switched systems. As for such systems, some useful results on stability and stabilization have been achieved [16–18].

In this paper, we are interested in investigating the asynchronous stabilization problem of nonlinear impulsive switched systems. As is well known, due to the existence of nonlinearity, it is difficult to analyze the nonlinear systems directly. The T-S fuzzy model is proven to be an effective tool in approximating most complex nonlinear systems [19], which utilizes local linear system description for each rule. The issue of stability and controller synthesis of T-S fuzzy systems has been studied extensively [20–28]. Similar to [27, 28], we use the T-S fuzzy model to represent each nonlinear subsystem of nonlinear impulsive switched systems in this paper.

It is well known that the results based on a common quadratic Lyapunov function might be conservative. By taking into consideration the information of membership functions, the authors in [29] presented the fuzzy Lyapunov functions (FLFs) which are defined by fuzzily blending multiple quadratic Lyapunov functions (QLFs). Stability analysis and controller synthesis results based on FLFs can be seen in [29–31]. The FLFs method only needs to search for a local common positive matrix in fuzzy model. In this paper, we investigate the nonlinear impulsive switched systems by employing the T-S fuzzy model. The switching FLFs are proposed using the switching information and structural information of membership function in the rule base. The candidate Lyapunov function is switching according to the system switching among several FLFs, which is based on premise membership functions to reduce conservativeness introduced by nonlinearity. Due to the above advantages, we consider nonlinear impulsive switched systems based on switching FLFs method.

In practice, when the systems are switching among the subsystems, the switching of the matched controller of each subsystem has a lag to the switching of the corresponding subsystem, which results in asynchronous switching in the switched systems. The asynchronous behaviors usually bring unsatisfactory performance or even make the switched systems out of control. Recently, several works have explored the effect of asynchronous behaviors on the switched systems [11, 12, 15, 32].

To the best of our knowledge, there is very little work on the use of the T-S fuzzy model to study the nonlinear impulsive switched systems, not to mention the nonlinear impulsive switched systems with asynchronous behaviors. In this paper, we fully considered the effects of various factors on the systems and used the T-S fuzzy model to study the nonlinear impulsive switched systems with asynchronous behaviors. Furthermore, the results obtained in this paper can also apply to the nonlinear switched systems without impulsive behaviors or asynchronous switching.

The main contributions of this paper are as follows: (i) The previous work studied the asynchronous switching problem mainly focused on the switched linear systems. In our work, the T-S fuzzy model is first used to study the nonlinear impulsive switched systems with asynchronous switching. (ii) In order to further reduce the conservativeness resulting from the nonlinearity and the quadratic Lyapunov functions approach, the switching fuzzy Lyapunov functions approach is proposed, and this approach can also be applied to study other nonlinear switched systems. (iii) The case $\beta_p < 0$ is also considered in this paper, which means that the Lyapunov functions can still decrease in the asynchronous state, so the results obtained in this paper can be much looser compared with the results in [11].

The remainder of the paper is organized as follows: System descriptions and preliminaries are presented in Sect. 2. The main results are given in Sect. 3. In Sect. 4, a numerical example and a chemical process example are presented. Finally, some conclusions are obtained in Sect. 5.

Notations: The notations used in this paper are fairly standard. N and N^+ denote the set of the natural numbers and the set of positive integers, respectively. I represents the identity matrix. The symbol "*" in a matrix stands for the transposed elements in the symmetric positions. The superscript "T" is the matrix transposition. R^n denotes the n-dimensional Euclidean space. The notation $\|\cdot\|$ refers to the Euclidean vector norm. $\mathcal{C}^1$ denotes the space of continuously differentiable functions. $L_2[0,\infty)$ is the space of square-integrable, and for $v(t) \in L_2[0,\infty)$ its norm is given by $\|v(t)\|_2 = \sqrt{\int_0^\infty v(t)^T v(t)dt}$. We use $P > 0\,(\geq, <, \leq)$ to denote a positive definite (semi-positive definite, negative definite, semi-negative definite) matrix P. We use $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$, respectively, to denote the maximum and minimum eigenvalues of P. If not explicitly stated, matrices are assumed to have compatible dimensions. Throughout this paper, $(p,q) \in S \times S, p \neq q$, and $(m,n,u,v) \in \{1,2,\ldots,k\}$.

2 System Descriptions and Preliminaries

In this paper, let us consider the following class of nonlinear impulsive switched systems:

$$\begin{cases} \dot{x}(t) = f_{\sigma(t_i)}(x(t), u(t)), & t \neq t_i \\ \Delta x(t) = D_{\sigma(t_i)}x(t) + g(t, x(t)), & t = t_i\,, \\ x(t_0^+) = x_0 \end{cases} \tag{1}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ denote the state vector and input vector, respectively. $D_{\sigma(t_i)}$ is a known matrix. $f_{\sigma(t)}$ and $g(t,x(t))$ are nonlinear functions, and $g(t,0) \equiv 0$ for all $t \in [t_0,\infty)$. $\sigma(t)$ is defined as a switching signal, which is a piecewise constant function of time and takes its values in the finite set $S = \{1,2,\ldots,M\}$, where M is the number of subsystems. For a switching sequence $0 < t_0 < t_1 < \cdots < t_i < t_{i+1} < \cdots$, $\sigma(t)$ is continuous from right everywhere. When $t \in [t_i, t_{i+1})$, we say that the $\sigma(t_i)$ subsystem is activated, and $\sigma(t_i) = p,\ p \in S$. $\Delta x(t_i) = x(t_i^+) - x(t_i^-) = x(t_i^+) - x(t_i)$, with $x(t_i^+) = \lim_{h\to 0^+} x(t_i + h)$, $x(t_i^-) = \lim_{h\to 0^-}$

$x(t_i + h) = x(t_i)$, meaning that the solution of the nonlinear impulsive switched systems (1) is left continuous. This implies that the impulses will affect the state $x(t)$ at the switching instant.

The T-S fuzzy model which is described by fuzzy IF-THEN rules [19] is employed here to represent each subsystem of systems (1). By introducing the T-S fuzzy model, the subsystem p of the nonlinear impulse switched systems (1) is described in the following form:

Rule m for the subsystem p: IF $z_{p1}(t)$ is M_{p1m} and $\cdots$ and $z_{pg}(t)$ is M_{pgm}, THEN

$$\begin{cases} \dot{x}(t) = A_{pm}x(t) + B_{pm}u(t), & t \neq t_i \\ \Delta x(t) = D_p x(t) + g(t, x(t)), & t = t_i \,, \\ x(t_0^+) = x_0 \end{cases} \tag{2}$$

where $z_{pl}(t)$ are some measurable premise variables and M_{plm} are fuzzy sets $(l = 1, 2, \ldots, g)$. A_{pm} and B_{pm} are constant real matrices of the mth local model of the pth subsystem.

Using "fuzzy blending", the final output of the pth subsystem is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum\limits_{m=1}^{k} h_{pm}(t)[A_{pm}x(t) + B_{pm}u(t)], & t \neq t_i \\ \Delta x(t) = D_p x(t) + g(t, x(t)), & t = t_i \\ x(t_0^+) = x_0 \end{cases}, \tag{3}$$

where $h_{pm}(t) = w_{pm}(t) / \sum\limits_{m=1}^{k} w_{pm}(t), w_{pm}(t) = \prod\limits_{l=1}^{g} M_{plm}$ $(z_{pl}(t))$, k is the number of IF-THEN rules, and $M_{plm}(z_{pl}(t))$ is the grade of the membership function of z_{pl} in M_{plm}. It is assumed that $w_{pm}(t) \geq 0$ for all t, $m = 1, 2, \ldots, k$. Therefore, the normalized membership function $h_{pm}(t)$ satisfies

$$h_{pm}(t) \geq 0, \quad \sum_{m=1}^{k} h_{pm}(t) = 1. \tag{4}$$

In this paper, we design a fuzzy controller for the system (3) via parallel distributed compensation (PDC) [20]. In the PDC design, the designed fuzzy controller shares the same premise variables with the fuzzy model (3). In view of the asynchronous behaviors, the controller $u(t)$ is divided into two parts $\bar{u}(t)$ and $\hat{u}(t)$, where $\bar{u}(t)$ denotes the unmatched controller, and $\hat{u}(t)$ represents the matched controller. For the fuzzy model (3), we can construct the following fuzzy controller via the PDC:

$$\begin{cases} \bar{u}(t) = \sum\limits_{n=1}^{k} h_{qn}(t)K_{qn}x(t), & t \in (t_i, \bar{t}_i] \\ \hat{u}(t) = \sum\limits_{n=1}^{k} h_{pn}(t)K_{pn}x(t), & t \in (\bar{t}_i, t_{i+1}] \end{cases}, \tag{5}$$

where notation $\bar{t}_i$ $(t_i \leq \bar{t}_i < t_{i+1})$ denotes the starting-operating instant of the matched controller, and K_{qn} and K_{pn} are constant matrices. Substituting (5) into (3), we can obtain the following closed-loop nonlinear impulsive switched system:

$$\begin{cases} \dot{x}(t) = \bar{A}_p(t)x(t), & t \in (t_i, \bar{t}_i] \\ \dot{x}(t) = \hat{A}_p(t)x(t), & t \in (\bar{t}_i, t_{i+1}] \\ \Delta x(t) = D_p x(t) + g(t, x(t)), & t = t_i \\ x(t_0^+) = x_0 \end{cases}, \tag{6}$$

where the $\bar{A}_p(t)$ and $\hat{A}_p(t)$ are defined as follows:

$$\begin{aligned} \bar{A}_p(t) &= A_p(t) + B_p(t)K_q(t) \\ &= \sum_{m=1}^{k}\sum_{n=1}^{k} h_{pm}(t)h_{qn}(t)(A_{pm} + B_{pm}K_{qn}), \\ \hat{A}_p(t) &= A_p(t) + B_p(t)K_p(t) \\ &= \sum_{m=1}^{k}\sum_{n=1}^{k} h_{pm}(t)h_{pn}(t)(A_{pm} + B_{pm}K_{pn}). \end{aligned}$$

We assume that there is no impulsive and asynchronous effects at the initial instant.

Now, we introduce the following assumptions, definitions and lemmas, which are useful in the following derivation.

Assumption 1 Let the nonlinear function $g(t, x(t))$ satisfy the following inequality:

$$\|g(t, x(t))\| \leq \eta \|x(t)\|$$

for all $t \in [t_0, \infty)$, where η is a positive constant.

Definition 1 Suppose that a switching signal $\sigma(t)$ is given. The nonlinear impulsive switched systems (1) with $u(t) \equiv 0$ are exponentially stable under the switching signal $\sigma(t)$ if for any initial conditions $x(t_0)$

$$\|x(t)\| \leq \Gamma \|x(t_0)\| e^{-\gamma(t - t_0)}, \quad \forall t \geq t_0,$$

where $\Gamma > 0$ and $\gamma > 0$ are constants.

Definition 2 [13] For switching signal $\sigma(t)$ and each $T \geq t \geq 0$. Let $N_{\sigma_p}(T, t)$ be the switching numbers such that the pth subsystem is activated over the interval $[t, T]$, and $T_p(T, t)$ denotes the total running time of the pth subsystem over the interval $[t, T]$, $\forall p \in S$. We say that $\sigma(t)$ has a mode-dependent average dwell time (MDADT) T_{ap} if there exist positive numbers N_{0p} (N_{0p} denotes mode-dependent chatter bounds here) and T_{ap} such that

$$N_{\sigma_p}(T, t) \leq N_{0p} + T_p(T, t)/T_{ap}, \quad \forall T \geq t \geq 0.$$

Definition 3 [29] Equation (7) is said to be a fuzzy Lyapunov function for the pth subsystem of the T-S fuzzy system (3) if there exists a positive definite matrix P_{pu} and the time derivative of $V_p(x(t))$ is always negative at $x(t) \neq 0$.

$$V_p(x(t)) = x(t)^T P_p(t) x(t), \tag{7}$$

where $P_p(t) = \sum_{u=1}^{k} h_{pu}(t)P_{pu}$ and $\dot{P}_p(t) = \sum_{u=1}^{k} \dot{h}_{pu}(t)P_{pu}$.

Lemma 1 [33] Let $P \in R^{n\times n}$ be a given symmetric positive definite matrix and let $Q \in R^{n\times n}$ be a given symmetric matrix. Then

$$\lambda_{\min}(P^{-1}Q)\Omega(t) \le x(t)^T Qx(t) \le \lambda_{\max}(P^{-1}Q)\Omega(t)$$

for all $x(t) \in R^n$, where $\Omega(t) = x(t)^T Px(t)$, $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote, respectively, the largest and the smallest eigenvalues of the matrix inside the brackets.

Lemma 2 [34] Given matrices M, E, and F with compatible dimensions and F satisfying $F^TF \le I$, the following inequality holds for any $\varepsilon > 0$:

$$MFE + E^TF^TM^T \le \varepsilon MM^T + \varepsilon^{-1}E^TE.$$

3 Main Results

In this paper, to deal with the asynchronous switching of switched systems, the Lyapunov function is allowed to increase with a bounded rate. Here, the parameter α_p represents the decaying rate of the Lyapunov function, which corresponds to the convergence rate of the system in synchronous state. And the parameter β_p denotes the increasing rate of the Lyapunov function, which corresponds to the divergence rate of the system in asynchronous state. In a sense, the purpose of controller design is to design the appropriate α_p and β_p parameters to make the systems reach the desired control performance.

For concise notation, let $T(t_{i+1}, t_i) = t_{i+1} - t_i$ represent the length of the running time interval of each subsystem. By (6), we can see that $T(t_{i+1}, t_i)$ is divided into two parts, $T_\uparrow(t_i, t_{i+1})$ and $T_\downarrow(t_i, t_{i+1})$, where $T_\uparrow(t_{i+1}, t_i) = \bar{t}_i - t_i$ and $T_\downarrow(t_{i+1}, t_i) = t_{i+1} - \bar{t}_i$. During $T_\uparrow(t_i, t_{i+1})$ the Lyapunov function may increase or decrease, which represents the running time of the unmatched controllers in $(t_i, t_{i+1}]$, while during $T_\downarrow(t_i, t_{i+1})$ the Lyapunov function is strictly decreasing with the matched controllers.

For brevity, we introduce the following notations: $\sigma_i = \sigma(t_i)$ and

$$Q_p = (I+D_p)^T\left(\sum_{m=1}^{k} P_{pm}\right)(I+D_p) + \left(\frac{\eta}{\varepsilon} + \eta\lambda_{\max}\left(\sum_{m=1}^{k} P_{pm}\right)\right)I \\ +\varepsilon(I+D_p)^T\left(\sum_{m=1}^{k} P_{pm}\right)\left(\sum_{m=1}^{k} P_{pm}\right)(I+D_p),$$

where the parameters η and ε are given in assumption 1 and lemma 2. The parameter μ_p used in the following lemmas and theorems is $\mu_p = \max_q\{\mu_{qp}, 1\}$, $\forall q \in S$, $q \ne p$, with $\mu_{qp} = \lambda_{\max}\left(\left(\sum_{n=1}^{k} P_{qn}\right)^{-1} Q_p\right)$.

3.1 Stability Analysis

In this section, we consider the stability analysis problem of the nonlinear impulsive switched systems. Without control input, the open-loop system for (6) is listed as follows:

$$\begin{cases} \dot{x}(t) = \sum_{m=1}^{k} h_{pm}(t)A_{pm}x(t), & \forall t \in (t_i, t_{i+1}] \\ \Delta x(t) = D_px(t) + g(t, x(t)), & t = t_i \\ x(t_0^+) = x_0 \end{cases}. \quad (8)$$

Lemma 3 Consider the open-loop nonlinear impulsive switched system (8), and let $\eta > 0$, $\varepsilon > 0$, and $\alpha_p < 0$ be given constants. If there exists positive definite $\mathcal{C}^1$ function $V_{\sigma(t_i)} : R^n \to R, \sigma(t_i) \in S$ with $V_{\sigma(t_0)}(x(t_0)) \equiv 0$ satisfying

$$\dot{V}_p(t) \le \alpha_p V_p(t), \quad t \in (t_i, t_{i+1}], \quad (9)$$

then the system (8) is exponentially stable for any switching signal satisfying

$$\tau_{op} \ge \tau_{op}^* = \frac{\ln \mu_p}{-\alpha_p}. \quad (10)$$

Proof From definition 3, at the switching instant t_i, we can obtain

$$\begin{aligned} V_p(x(t_i^+)) &= \left[(I+D_p)x(t_i) + g(t_i, x(t_i))\right]^T \\ &\quad P_p(t_i)\left[(I+D_p)x(t_i) + g(t_i, x(t_i))\right] \\ &= x(t_i)^T(I+D_p)^TP_p(t_i)(I+D_p)x(t_i) \\ &\quad + 2x(t_i)^T(I+D_p)^TP_p(t_i)g(t_i) \\ &\quad + g(t_i, x(t_i))^TP_p(t_i)g(t_i, x(t_i)). \end{aligned}$$

Then, by Assumption 1, Lemmas 1 and 2, we have

$$\begin{aligned} V_p(x(t_i^+)) &\le x(t_i)^T(I+D_p)^TP_p(t_i)(I+D_p)x(t_i) \\ &\quad + \varepsilon x(t_i)^T(I+D_p)^TP_p(t_i)P_p(t_i)(I+D_p)x(t_i) \\ &\quad + \frac{1}{\varepsilon}g(t_i, x(t_i))^Tg(t_i, x(t_i)) \\ &\quad + \lambda_{\max}(P_p(t_i))g(t_i, x(t_i))^Tg(t_i, x(t_i)) \\ &\le x(t_i)^T\left[(I+D_p)^T\left(\sum_{m=1}^{k} P_{pm}\right)(I+D_p)\right. \\ &\quad +\varepsilon(I+D_p)^T\left(\sum_{m=1}^{k} P_{pm}\right)\left(\sum_{m=1}^{k} P_{pm}\right)(I+D_p) \\ &\quad \left.+\left(\frac{\eta}{\varepsilon} + \eta\lambda_{\max}\left(\sum_{m=1}^{k} P_{pm}\right)\right)\right]x(t_i) \\ &= x(t_i)^TQ_px(t_i). \end{aligned} \quad (11)$$

By Lemma 1 and (11), we have

$$V_p(x(t_i^+)) \le x(t_i)^T Q_p x(t_i) \le \lambda_{\max}\left(P_q^{-1}(t_i)Q_p\right)V_q(x(t_i)). \tag{12}$$

Due to $P_q^{-1}(t_i) = \left(\sum_{n=1}^k h_{qn}(t_i)P_{qn}\right)^{-1}$, we can conclude that the inequality (12) holds if the following inequality (13) holds

$$\begin{aligned} V_p(x(t_i^+)) &\le x(t_i)^T Q_p x(t_i) \\ &\le \lambda_{\max}\left(\left(\sum_{n=1}^k P_{qn}\right)^{-1} Q_p\right)V_q(x(t_i)). \end{aligned} \tag{13}$$

Let $\mu_p = \max_q\{\mu_{qp}, 1\}$, $\forall q \in S$, $q \ne p$ with $\mu_{qp} = \lambda_{\max}\left(\left(\sum_{n=1}^k P_{qn}\right)^{-1} Q_p\right)$, then we get

$$V_p(x(t_i^+)) \le \mu_p V_q(x(t_i)). \tag{14}$$

By integrating (9) we have

$$V_p(t) \le e^{\alpha_p(t-t_i)}V_p(t_i). \tag{15}$$

Combining (14) and (15), $t \in (t_i, t_{i+1}]$, we have

$$\begin{aligned} V_{\sigma_i}(x(t)) &\le \mu_{\sigma_i} e^{\alpha_{\sigma_i}(t-t_i)} V_{\sigma_{i-1}}(x(t_i)) \\ &\le \mu_{\sigma_i} e^{\alpha_{\sigma_i}(t-t_i)+\alpha_{\sigma_{i-1}}(t_i-t_{i-1})} V_{\sigma_{i-1}}(x(t_{i-1}^+)) \\ &\le \cdots \\ &\le \mu_{\sigma_i}\mu_{\sigma_{i-1}}\cdots\mu_{\sigma_1} e^{\alpha_{\sigma_i}(t-t_i)+\cdots+\alpha_{\sigma_0}(t_1-t_0)} V_{\sigma_0}(x(t_0)). \end{aligned}$$

From definition 2, let N_{σ_i} denote $N_{\sigma_i}(t, t_0)$ for simplicity. The following inequality holds

$$V(t) \le \exp\left\{\sum_{p=1}^M N_{\sigma_i}(\alpha_p \tau_{op} + \ln \mu_p)\right\}V(t_0).$$

If supposing

$$\alpha_p \tau_{op} + \ln \mu_p \le 0, \tag{16}$$

we obtain a sufficient condition that guarantees the exponential stability of the system (8). The inequality (16) is equivalent to

$$\tau_{op} \ge \tau_{op}^* = \frac{\ln \mu_p}{-\alpha_p}.$$ □

The stability conditions for the system (8) can be summarized in the following theorem.

Theorem 1 Assume that

$$\left|\dot{h}_{ps}(t)\right| \le \varepsilon_{ps}, \quad s = 1, 2, \ldots, k, \tag{17}$$

where $\varepsilon_{ps} \ge 0$. Let $\eta > 0$, $\varepsilon > 0$, and $\alpha_p < 0$ be given constants. The system (8) is exponentially stable for any switching signal satisfying (10), if there exist matrices $P_{pu} > 0$ satisfying

$$P_{pl} \ge P_{pk}, \quad l = 1, 2, \ldots, k-1, \tag{18}$$

and

$$\Theta_{pmu} + \Theta_{pum} < 0, \quad m \le u, \tag{19}$$

where

$\Theta_{pmu} = A_{pm}^T P_{pu} + P_{pu}A_{pm} + \sum_{s=1}^{k-1} \varepsilon_{ps}(P_{ps} - P_{pk}) - \alpha_p P_{pu}$.

Proof Differentiating (4) implies $\dot{h}_{pk}(t) = -\sum_{m=1}^{k-1}\dot{h}_{pm}(t)$, so we have

$$\dot{P}_p(t) = \sum_{u=1}^{k-1} \dot{h}_{pu}(t)(P_{pu} - P_{pk}). \tag{20}$$

Combining (17), (18) with (20) implies

$$\dot{P}_p(t) \le \sum_{u=1}^{k-1} \varepsilon_{pu}(P_{pu} - P_{pk}). \tag{21}$$

Along with the solution of the system (8), we have

$$\begin{aligned} \dot{V}_p(t) - \alpha_p V_p(t) = x(t)^T&\left[A_p(t)^T P_p(t) + P_p(t)A_p(t)\right. \\ &\left. + \dot{P}_p(t) - \alpha_p P_p(t)\right]x(t). \end{aligned}$$

Since $A_p(t) = \sum_{m=1}^k h_{pm}(t)A_{pm}$, $P_p(t) = \sum_{u=1}^k h_{pu}(t)P_{pu}$ and $\dot{P}_p(t) = \sum_{s=1}^k \dot{h}_{ps}(t)P_{ps}$, then

$$\begin{aligned} \dot{V}_p(t) - \alpha_p V_p(t) &\le \sum_{m=1}^k h_{pm}^2(t)x(t)^T \Theta_{pmm} x(t) \\ &+ \sum_{m<u}^k h_{pm}(t)h_{pu}(t)x(t)^T\left[\Theta_{pmu} + \Theta_{pum}\right]x(t). \end{aligned}$$

From (19) we can conclude that

$$\dot{V}_p(t) - \alpha_p V_p(t) \le 0.$$

So the system (8) is exponentially stable for any switching signal satisfying (10). This completes the proof. □

Remark 1 By setting $P_{pu} = P_p$, we get Corollary 1 where the QLFs are used. Compared with the QLFs method, the computational complexity of the results based on the FLFs method will be greater. But the results based on the FLFs method have less conservativeness than the results based on the QLFs method.

Corollary 1 Let $\eta > 0$, $\varepsilon > 0$, and $\alpha_p < 0$ be given constants. The system (8) is exponentially stable for any switching signal satisfying (10), if there exist matrices $P_p > 0$ satisfying

$$A_{pm}^T P_p + P_p A_{pm} - \alpha_p P_p \le 0. \tag{22}$$

Proof The proof is similar to that of Theorem 1, with the function $V_p(t)$ given by

$$V_p(t) = x(t)^T P_p x(t).$$

It is omitted here. □

3.2 Controller design

In this section, our objective is to design a set of model-dependent controllers and find a set of admissible switching signal such that the closed-loop nonlinear impulsive switched system (6) is exponentially stable with asynchronous switching.

Lemma 4 Consider the closed-loop nonlinear impulsive switched system (6), and let $\eta > 0$, $\varepsilon > 0$, $\alpha_p < 0$ and β_p be given constants. If there exists positive definite $\mathcal{C}^1$ function $V_{\sigma(t_i)} : R^n \to R, \sigma(t_i) \in S$ with $V_{\sigma(t_0)}(x(t_0)) \equiv 0$ satisfying

$$\dot{V}_p(t) \le \begin{cases} \beta_p V_p(t), & t \in T_\uparrow(t_i, t_{i+1}) \\ \alpha_p V_p(t), & t \in T_\downarrow(t_i, t_{i+1}) \end{cases}, \tag{23}$$

then the system is exponentially stable for any switching signal satisfying

$$\begin{cases} \tau_p \ge \tau_p^* = \dfrac{\ln \mu_p}{-\alpha_p}, & \beta_p < \alpha_p \\ \tau_p \ge \tau_p^* = \dfrac{T_{pM}(\beta_p - \alpha_p) + \ln \mu_p}{-\alpha_p}, & \beta_p \ge \alpha_p \end{cases}, \tag{24}$$

where $T_{pM} \overset{\Delta}{=} \max T_{p\uparrow}(t_{i+1}, t_i), \forall i \in N^+$.

Proof Let $S_1 = \{0, 1, \ldots, r\}$, $S_2 = \{r+1, \ldots, M\}$, $r \ge 0$. Now based on the value of β_p, all the subsystems are divided into two parts. If the unmatched controller can stabilize the current subsystem, i.e., $\beta_p < 0$, then the subsystem is contained in set S_1, otherwise, it is contained in set S_2. For any $T > 0$, let $t_0 = 0$ and denote the switching times on the interval [0, T] as $t_1, t_2, \ldots t_i, t_{i+1}, \ldots t_{N_\sigma(T,0)}$, then

$$N_\sigma(T,0) = \sum\nolimits_{p=1}^{r} N_p(T,0) + \sum\nolimits_{p=r+1}^{M} N_p(T,0).$$

Let $T_{p\downarrow}(T,0)$ denote the total running time of the pth subsystem controlled by the matched controller and $T_{p\uparrow}(T,0)$ denote the total running time of the pth subsystem controlled by the unmatched controller. We get $T_p(T,0) = T_{p\downarrow}(T,0) + T_{p\uparrow}(T,0)$.

Since $T_{pM} \overset{\Delta}{=} \max T_{p\uparrow}(t_{i+1}, t_i)$, by Definition 2, we have

$$T_{p\uparrow}(T,0) \le T_{pM} N_p(T,0) \le T_{pM}\left(N_{0p} + \frac{T_p(T,0)}{\tau_p}\right). \tag{25}$$

By integrating (23) and together with (14) for $t \in (t_i, t_{i+1}]$, we have

$$\begin{aligned}
V_{\sigma_i}(x(t)) &\le e^{\alpha_{\sigma_i} T_\downarrow(t,t_i) + \beta_{\sigma_i} T_\uparrow(t,t_i)} \mu_{\sigma_i} V_{\sigma_{i-1}}(x(t_i)) \\
&\le \exp\{\alpha_{\sigma_i} T_\downarrow(t,t_i) + \beta_{\sigma_i} T_\uparrow(t,t_i) + \alpha_{\sigma_{i-1}} T_\downarrow(t_i,t_{i-1}) \\
&\quad + \beta_{\sigma_{i-1}} T_\uparrow(t_i,t_{i-1})\} \mu_{\sigma_i} V_{\sigma_{i-1}}\left(x(t_{i-1}^+)\right) \le \cdots \\
&\le \prod_{n=1}^{i} \mu_{\sigma_n} \exp\left\{\alpha_{\sigma_i} T_\downarrow(t,t_i) + \beta_{\sigma_i} T_\uparrow(t,t_i) + \sum_{n=1}^{i} \alpha_{\sigma_{n-1}} T_\downarrow(t_n,t_{n-1}) \right. \\
&\quad \left. + \sum_{n=2}^{i} \beta_{\sigma_{n-1}} T_\uparrow(t_n,t_{n-1})\right\} V_{\sigma_0}(x(t_0)) \\
&= \prod_{p=1}^{M} \mu_p^{N_p(T,0)} \exp\left\{\sum_{p=1}^{M} \left[\alpha_p T_{p\downarrow}(T,0) + \beta_p T_{p\uparrow}(T,0)\right]\right\} V_{\sigma_0}(x(t_0)) \\
&= \Omega_1 \Omega_2 V_{\sigma_0}(x(t_0)),
\end{aligned} \tag{26}$$

where

$$\Omega_1 = \prod_{p=1}^{r} \mu_p^{N_p(T,0)} \exp\left\{\sum_{p=1}^{r} \left[\alpha_p T_{p\downarrow}(T,0) + \beta_p T_{p\uparrow}(T,0)\right]\right\},$$

$$\Omega_2 = \prod_{p=r+1}^{M} \mu_p^{N_p(T,0)} \exp\left\{\sum_{p=r+1}^{M} \left[\alpha_p T_{p\downarrow}(T,0) + \beta_p T_{p\uparrow}(T,0)\right]\right\}.$$

As for Ω_1, we introduce the following notations:

$$\Omega_{11} = \prod_{p=1}^{r_1} \mu_p^{N_p(T,0)} \exp\left\{\sum_{p=1}^{r_1} \left[\alpha_p T_{p\downarrow}(T,0) + \beta_p T_{p\uparrow}(T,0)\right]\right\},$$

where $\beta_p < \alpha_p$, $0 \le r_1 \le r$;

$$\Omega_{12} = \prod_{p=r_1+1}^{r} \mu_p^{N_p(T,0)} \exp\left\{\sum_{p=r_1+1}^{r} \left[\alpha_p T_{p\downarrow}(T,0) + \beta_p T_{p\uparrow}(T,0)\right]\right\},$$

where $\alpha_p \le \beta_p \le 0$, $0 \le r_1 \le r$.

As for Ω_{11}, we have

$$\begin{aligned}
\Omega_{11} &\le \prod_{p=1}^{r_1} \mu_p^{N_p(T,0)} \exp\left\{\sum_{p=1}^{r_1} \left[\alpha_p T_{p\downarrow}(T,0) + \alpha_p T_{p\uparrow}(T,0)\right]\right\} \\
&= \exp\left\{\sum_{p=1}^{r_1} \left[N_p(T,0) \ln \mu_p + \alpha_p T_p(T,0)\right]\right\} \\
&\le \exp\left\{\sum_{p=1}^{r_1} \left[N_{0p} \ln \mu_p + T_p(T,0)(\alpha_p + \frac{\ln \mu_p}{\tau_p})\right]\right\} \\
&= \hat{\Omega}_{11}
\end{aligned} \tag{27}$$

As for Ω_{12}, we get

$$\Omega_{12} = \prod_{p=r_1+1}^{r} \mu_p^{N_p(T,0)} \exp\left\{ \sum_{p=r_1+1}^{r} \left[\alpha_p T_p(T,0) + (-\alpha_p + \beta_p) T_{p\uparrow}(T,0)\right] \right\}$$
$$= \exp\left\{ \sum_{p=r_1+1}^{r} \left[N_p(T,0)\ln\mu_p + \alpha_p T_p(T,0) + (-\alpha_p + \beta_p) T_{p\uparrow}(T,0)\right] \right\}$$
$$\leq \exp\left\{ \sum_{p=r_1+1}^{r} \left[N_{0p}\ln\mu_p + \alpha_p T_p(T,0) + (-\alpha_p + \beta_p) T_{p\uparrow}(T,0) + \frac{T_p(T,0)\ln\mu_p}{\tau_p}\right] \right\}. \quad (28)$$

Substituting (25) into (28), we have

$$\Omega_{12} \leq \exp\left\{ \sum_{p=r_1+1}^{r} \left[N_{0p}\left(\ln\mu_p + T_{pM}(-\alpha_p + \beta_p)\right) + T_p(T,0)\left(\alpha_p + \frac{\ln\mu_p}{\tau_p}\right) + (-\alpha_p + \beta_p) T_{pM}\frac{T_p(T,0)}{\tau_p}\right] \right\} = \hat{\Omega}_{12}. \quad (29)$$

Combining (27) and (29), we can obtain

$$\Omega_1 = \Omega_{11}\Omega_{12} \leq \hat{\Omega}_{11}\hat{\Omega}_{12}. \quad (30)$$

As for Ω_2, similar to the derivations of (28) and (29), and letting

$$\hat{\Omega}_2 = \exp\left\{ \sum_{p=r+1}^{M} \left[N_{0p}\left(\ln\mu_p + T_{pM}(-\alpha_p + \beta_p)\right) + T_p(T,0) \times (\alpha_p + \frac{\ln\mu_p}{\tau_p}) + (-\alpha_p + \beta_p) T_{pM}\frac{T_p(T,0)}{\tau_p}\right] \right\},$$

we have

$$\Omega_2 \leq \hat{\Omega}_2. \quad (31)$$

By (26), (30), and (31), we have

$$V_{\sigma_i}(x(t)) \leq \hat{\Omega}_{11}\hat{\Omega}_{12}\hat{\Omega}_2 V_{\sigma_0}(x(t_0))$$
$$= \exp\left\{ \sum_{p=1}^{r_1} N_{0p}\ln\mu_p + \sum_{p=r_1+1}^{M}\left[N_{0p}\left(\ln\mu_p + T_{pM}(-\alpha_p+\beta_p)\right)\right] \right\}$$
$$\times \exp\left\{ \sum_{p=1}^{r_1}\left[T_p(T,0)\left(\alpha_p + \frac{\ln\mu_p}{\tau_p}\right)\right] + \sum_{p=r_1+1}^{M}\left[T_p(T,0)\left(\alpha_p + \frac{\ln\mu_p}{\tau_p}\right) + (-\alpha_p+\beta_p)T_{pM}\frac{T_p(T,0)}{\tau_p}\right] \right\} V_{\sigma_0}(x(t_0)). \quad (32)$$

If (24) holds, we conclude that $V_{\sigma_i}(x(t))$ converges to zero as $T \to \infty$. Let

$$\Gamma_1 = \exp\left\{ \sum_{p=1}^{r_1} N_{0p}\ln\mu_p + \sum_{p=r_1+1}^{M}\left[N_{0p}\left(\ln\mu_p + T_{pM}(-\alpha_p+\beta_p)\right)\right] \right\} \quad (33)$$

and

$$-\gamma_1 = \max\left\{ \max_{p\in\{1,\ldots,r_1\}}\left\{\alpha_p + \frac{\ln\mu_p}{\tau_p}\right\}, \max_{p\in\{r_1+1,\ldots,M\}}\left\{\alpha_p + \frac{\ln\mu_p}{\tau_p} + \frac{(-\alpha_p+\beta_p)T_{pM}}{\tau_p}\right\}\right\}. \quad (34)$$

By (32)–(34), we have

$$V_{\sigma_i}(x(t)) \leq \Gamma_1 e^{-\gamma_1 t} V_{\sigma_0}(x(t_0)). \quad (35)$$

Letting $\gamma = \gamma_1/2$ and $\Gamma = \Gamma_1^{1/2}\left[\min_{p\in S}\left\{\lambda_{\min}\left(\sum_{m=1}^{k} P_{pm}\right)\right\}\right]^{-\frac{1}{2}}\left[\max_{p\in S}\left\{\lambda_{\max}\left(\sum_{m=1}^{k} P_{pm}\right)\right\}\right]^{\frac{1}{2}}$, by Lemma 1 and together with (35), we get

$$\|x(t)\| \leq \Gamma e^{-\gamma t}\|x(t_0)\|.$$

By Definition 1, we can conclude that the system is exponentially stable for any switching signal satisfying (24). This completes the proof. □

Remark 2 Although the main method in the proof of Lemma 4 is literally similar to that of [32], there is an essential difference between these two methods. The Lyapunov function $V_p(x(t))$ in [32] is $V_p(x(t)) = x(t)^T P_p x(t)$, which is a quadratic form. However, the Lyapunov function used in our work is a fuzzy Lyapunov function to treat the nonlinearity, which is given by $V_p(x(t)) = x^T(t) P_p(t)x(t)$, where $P_p(t) = \sum_{m=1}^{k} h_{pm}(t)P_{pm}$. Moreover, we will show that the quadratic Lyapunov function is a special case of the fuzzy Lyapunov function.

Lemma 5 Let $\eta > 0$, $\varepsilon > 0$, $\alpha_p < 0$ and β_p be given constants. The closed-loop nonlinear impulsive switched system (6) is exponentially stable for any switching signal satisfying (24), if there exist matrices $P_p(t) > 0$ satisfying

$$P_p(t)\bar{A}_p(t) + \bar{A}_p^T(t)P_p(t) + \dot{P}_p(t) - \beta_p P_p(t) < 0, \quad (36)$$

$$P_p(t)\hat{A}_p(t) + \hat{A}_p^T(t)P_p(t) + \dot{P}_p(t) - \alpha_p P_p(t) < 0. \quad (37)$$

Proof Suppose pth subsystem is activated and the former one is qth subsystem. Considering $t \in T_\uparrow(t_i, t_{i+1})$, from the system (6) and definition 3, we have

$$\dot{V}_p(t) - \beta_p V_p(t) = x(t)^T\left[P_p(t)\bar{A}_p(t) + \bar{A}_p^T(t)P_p(t) + \dot{P}_p(t) - \beta_p P_p(t)\right]x(t). \quad (38)$$

Similarly, for $t \in T_\downarrow(t_i, t_{i+1})$, we obtain

$$\dot{V}_p(t) - \alpha_p V_p(t) = x(t)^T \Big[P_p(t)\hat{A}_p(t) + \hat{A}_p{}^T(t)P_p(t) + \dot{P}_p(t) - \alpha_p P_p(t)\Big]x(t). \quad (39)$$

Inequalities (36) and (37) imply (38) <0 and (39) <0. So we obtain the following inequality:

$$\dot{V}_p(t) \le \begin{cases} \beta_p V_p(t), & t \in T_{\uparrow}(t_i, t_{i+1}) \\ \alpha_p V_p(t), & t \in T_{\downarrow}(t_i, t_{i+1}) \end{cases}.$$

According to Lemma 4, the system (6) is exponentially stable for any switching signal satisfying (24). This completes the proof.

Lemma 5 provides a sufficient condition for the controller design. However, the matrix variables $P_p(t)$ are coupled with system parameter matrices in (36) and (37), and thus it is difficult to design the controller directly. To overcome this difficulty, a decoupling technique is needed. In such a way, the following lemma is introduced. □

Lemma 6 Let $\eta > 0$, $\varepsilon > 0$, $\alpha_p < 0$ and β_p be given constants. If there exist matrices $\bar{P}_p(t) > 0$, $L_p(t)$ and X satisfying

$$\begin{bmatrix} \Pi_{p1} & \Pi_{p2} \\ * & -X - X^T \end{bmatrix} < 0, \quad (40)$$

$$\begin{bmatrix} \Pi_{p3} & \Pi_{p4} \\ * & -X - X^T \end{bmatrix} < 0, \quad (41)$$

where

$$\Pi_{p1} = A_p(t)X + X^T A_p(t)^T + B_p(t)L_q(t) + L_q(t)^T B_p(t)^T - \beta_p \bar{P}_p(t) + \dot{\bar{P}}_p(t),$$
$$\Pi_{p2} = \bar{P}_p(t) - X + X^T A_p(t)^T + L_q(t)^T B_p(t)^T,$$
$$\Pi_{p3} = A_p(t)X + X^T A_p(t)^T + B_p(t)L_p(t) + L_p(t)^T B_p(t)^T - \alpha_p \bar{P}_p(t) + \dot{\bar{P}}_p(t),$$
$$\Pi_{p4} = \bar{P}_p(t) - X + X^T A_p(t)^T + L_p(t)^T B_p(t)^T,$$

we can conclude that the inequalities (36) and (37) hold.

Proof In order to decouple the matrix variables $P_p(t)$ and system parameter matrices, we introduce a slack matrix H. Moreover, introducing slack matrices can also reduce the design conservativeness [35]. By introducing a slack matrix H, we introduce the following inequalities:

$$\begin{bmatrix} \bar{A}_p(t)^T H + H^T \bar{A}_p(t) - \beta_p P_p(t) + \dot{P}_p(t) & P_p(t) - H^T + \bar{A}_p(t)^T H \\ * & -H - H^T \end{bmatrix} < 0, \quad (42)$$

$$\begin{bmatrix} \hat{A}_p(t)^T H + H^T \hat{A}_p(t) - \alpha_p P_p(t) + \dot{P}_p(t) & P_p(t) - H^T + \hat{A}_p(t)^T H \\ * & -H - H^T \end{bmatrix} < 0. \quad (43)$$

Multiplying (42) from the left and right, respectively, by $\hat{\Lambda}_p = \begin{bmatrix} I & \bar{A}_p(t)^T \end{bmatrix}$ and its transpose, and multiplying (43) from the left and right, respectively, by $\bar{\Lambda}_p = \begin{bmatrix} I & \hat{A}_p(t)^T \end{bmatrix}$, we can conclude that (36) and (37) hold. Notice that if the conditions in (42) and (43) hold, the matrix H is nonsingular. Now, we define the following matrices:

$$X = H^{-1}, \quad L_p(t) = K_p(t)X, \quad \bar{P}_p(t) = X^T P_p(t) X. \quad (44)$$

Multiplying (42) from the left and right, respectively, by diagonal matrix diag(H^{-T},H^{-T}) and its transpose, we can obtain

$$\begin{bmatrix} \Pi_{p5} & \Pi_{p6} \\ * & -H^{-1} - H^{-T} \end{bmatrix} < 0, \quad (45)$$

where

$$\Pi_{p5} = H^{-T}\bar{A}_p(t)^T + \bar{A}_p(t)H^{-1} - \beta_p H^{-T} P_p(t) H^{-1} + H^{-T}\dot{P}_p(t)H^{-},$$
$$\Pi_{p6} = H^{-T}P_p(t)H^{-1} - H^{-1} + H^{-T}\bar{A}_p(t)^T$$

Similarly, for (43), we have

$$\begin{bmatrix} \Pi_{p7} & \Pi_{p8} \\ * & -H^{-1} - H^{-T} \end{bmatrix} < 0 \quad (46)$$

where

$$\Pi_{p7} = H^{-T}\hat{A}_p(t)^T + \hat{A}_p(t)H^{-1} - \alpha_p H^{-T} P_p(t) H^{-1} + H^{-T}\dot{P}_p(t)H^{-}, \Pi_{p8} = H^{-T}P_p(t)H^{-1} - H^{-1} + H^{-T}\hat{A}_p(t)^T.$$

LMIs (40) and (41) imply (45) and (46), respectively. Thus, if (40) and (41) hold, we can conclude that (36) and (37) hold. This completes the proof. □

Remark 3 With the introduction of some new additional matrices $L_p(t)$ and X, we obtain the linear matrix inequalities in which the Lyapunov matrix $P_p(t)$ is not involved in any product with the state matrices $A_p(t)$. This feature enables us to derive conditions with less conservativeness due to the extra degrees of freedom.

Based on the above lemmas, the asynchronous stabilization problem can be addressed by the following theorem.

Theorem 2 Assume that

$$\left|\dot{h}_{ps}(t)\right| \le \varepsilon_{ps}, \quad s = 1, 2, \ldots, k, \quad (47)$$

where $\varepsilon_{ps} \ge 0$. Let $\eta > 0$, $\varepsilon > 0$, $\alpha_p < 0$ and β_p be given constants. If there exist matrices $\bar{P}_{pu} > 0$, L_{pn} and X satisfying

$$\bar{P}_{pl} \ge \bar{P}_{pk}, \quad l = 1, 2, \ldots, k-1 \quad (48)$$

$$\bar{\Theta}_{pmnu} + \bar{\Theta}_{punm} < 0, m \le u \quad (49)$$

$$\begin{cases} \hat{S}_{pmmm} = \hat{\Theta}_{pmmm} < 0 \\ \hat{S}_{pmmn} = \frac{1}{3}\left[\hat{\Theta}_{pmmn} + \hat{\Theta}_{pmnm} + \hat{\Theta}_{pnmm}\right] < 0, m \neq n \\ \hat{S}_{pmnu} = \frac{1}{6}\left[\hat{\Theta}_{pmnu} + \hat{\Theta}_{pmun} + \hat{\Theta}_{pnmu} + \hat{\Theta}_{pnum} \right. \\ \qquad \left. + \hat{\Theta}_{punm} + \hat{\Theta}_{pumn}\right] < 0, m < n < u \end{cases}$$

for all $m, n, u \in \{1, 2, \ldots, k\}$, where

$$\bar{\Theta}_{pmnu} = \begin{bmatrix} \Phi_{p1} & \Phi_{p2} \\ * & -X - X^T \end{bmatrix}, \hat{\Theta}_{pmnu} = \begin{bmatrix} \Psi_{p1} & \Psi_{p2} \\ * & -X - X^T \end{bmatrix}$$

and

$$\begin{aligned} \Phi_{p1} &= A_{pm}X + X^T A_{pm}^T + B_{pm}L_{qn} + L_{qn}^T B_{pm}^T - \beta_p \bar{P}_{pu} \\ &\quad + \sum_{s=1}^{k-1} \varepsilon_{ps}(\bar{P}_{ps} - \bar{P}_{pk}), \\ \Phi_{p2} &= \bar{P}_{pu} - X + X^T A_{pm}^T + L_{qn}^T B_{pm}^T, \\ \Psi_{p1} &= A_{pm}X + X^T A_{pm}^T + B_{pm}L_{pn} + L_{pn}^T B_{pm}^T - \alpha_p \bar{P}_{pu} \\ &\quad + \sum_{s=1}^{k-1} \varepsilon_{ps}(\bar{P}_{ps} - \bar{P}_{pk}), \\ \Psi_{p2} &= \bar{P}_{pu} - X + X^T A_{pm}^T + L_{pn}^T B_{pm}^T, \end{aligned}$$

The system (6) is exponentially stable for any switching signal satisfying (24). Moreover, if a feasible solution exists, the admissible controller gains can be given by

$$\begin{cases} K_{qn} = L_{qn}X^{-1}, & t \in (t_i, \bar{t}_i] \\ K_{pn} = L_{pn}X^{-1}, & t \in (\bar{t}_i, t_{i+1}]. \end{cases} \tag{51}$$

Proof Denote the left side of (40) and (41) as $\bar{\Theta}_p(t)$ and $\hat{\Theta}_p(t)$, respectively. If the conditions in Theorem 2 are satisfied, we can obtain

$$\begin{aligned} \bar{\Theta}_i(t) &\leq \sum_{m=1}^{k}\sum_{n=1}^{k}\sum_{u=1}^{k} h_{pm}(t)h_{qn}(t)h_{pu}(t)\bar{\Theta}_{pmnu} \\ &= \sum_{n=1}^{k} h_{qn}(t)\left[\sum_{m=1}^{k} h_{pm}^2(t)\bar{\Theta}_{pmnm} \right. \\ &\quad \left. + \sum_{m=1}^{k}\sum_{m<u}^{k} h_{pm}(t)h_{pu}(t)(\bar{\Theta}_{pmnu} + \bar{\Theta}_{punm})\right] < 0 \end{aligned}$$

and

$$\begin{aligned} \hat{\Theta}_p(t) &\leq \sum_{m=1}^{k}\sum_{n=1}^{k}\sum_{u=1}^{k} h_{pm}(t)h_{pn}(t)h_{pu}(t)\hat{\Theta}_{pmnu} \\ &= \sum_{m=1}^{k} h_{pm}^3(t)\hat{S}_{pmmm} + 3\sum_{m=1}^{k}\sum_{\substack{n=1 \\ n \neq m}}^{k} h_{pm}(t)h_{pn}(t)\hat{S}_{pmmn} \\ &\quad + 6\sum_{m=1}^{k}\sum_{n>m}\sum_{u>n} h_{pm}(t)h_{pn}(t)h_{pu}(t)\hat{S}_{pmnu} < 0. \end{aligned}$$

From Lemmas 4, 5, and 6, we know that the controller design problem is solved. By (44), we can conclude that the gains in (5) can be constructed by (51). This completes the proof. □

Remark 4 Theorem 2 provides a sufficient condition to guarantee the exponential stability of the system (6) with asynchronous switching. This theorem can also be used to study other nonlinear switched systems. For example, if $T_{pM} = 0$, the results obtained above can be used to study the nonlinear impulsive switched systems without asynchronous switching. And if $\Delta x(t) = 0$, the above theorem can be applicable to the asynchronous control of nonlinear switched systems without impulsive behaviors.

Remark 5 By setting $\bar{P}_{pu} = \bar{P}_p$, we get Corollary 2 where the QLFs are used.

Corollary 2 Let $\eta > 0$, $\varepsilon > 0$, $\alpha_p < 0$, and β_p be given constants. If there exist matrices $\bar{P}_p > 0$, L_{pn} and X satisfying

$$\begin{cases} \bar{\Lambda}_{pmn} + \bar{\Lambda}_{pnm} < 0 \\ \hat{\Lambda}_{pmn} + \hat{\Lambda}_{pnm} < 0 \end{cases}, \quad m \leq n, \tag{52}$$

for all $m, n \in \{1, 2, \ldots, k\}$, where

$$\bar{\Lambda}_{pmn} = \begin{bmatrix} \phi_{p1} & \Phi_{p2} \\ * & -X - X^T \end{bmatrix}, \quad \hat{\Lambda}_{pmn} = \begin{bmatrix} \psi_{p1} & \psi_{p2} \\ * & -X - X^T \end{bmatrix}$$

and

$$\begin{aligned} \phi_{p1} &= A_{pm}X + X^T A_{pm}^T + B_{pm}L_{qn} + L_{qn}^T B_{pm}^T - \beta_p \bar{P}_p, \\ \phi_{p2} &= \bar{P}_p - X + X^T A_{pm}^T + L_{qn}^T B_{pm}^T, \\ \psi_{p1} &= A_{pm}X + X^T A_{pm}^T + B_{pm}L_{pn} + L_{pn}^T B_{pm}^T - \alpha_p \bar{P}_p, \\ \psi_{p2} &= \bar{P}_p - X + X^T A_{pm}^T + L_{pn}^T B_{pm}^T, \end{aligned}$$

then the system (6) is exponentially stable for any switching signal satisfying (24). Moreover, if a feasible solution exists, the admissible controller gains can be given by

$$\begin{cases} K_{qn} = L_{qn}X^{-1}, & t \in (t_i, \bar{t}_i] \\ K_{pn} = L_{pn}X^{-1}, & t \in (\bar{t}_i, t_{i+1}] \end{cases}.$$

Proof The proof is similar to that of Theorem 2 with the function $V_p(t)$ given by

$$V_p(t) = x(t)^T P_p x(t).$$

It is omitted here. □

4 Example

Example 1 Here, a numerical example is given to illustrate the effectiveness and advantage of our results obtained above. Consider the following continuous-time nonlinear impulsive switched system consisting of two subsystems:

Subsystem 1:

$$\begin{cases} \dot{x}_1(t) = (-0.2 + 0.03z_{11}(t))x_1(t) + (0.01 + 0.04z_{11}(t))x_2(t) + \\ \qquad (-0.21 + 0.09z_{11}(t))u_1(t) + (0.01 - 0.02z_{11}(t))u_2(t) \\ \dot{x}_2(t) = (0.16 - 0.06z_{11}(t))x_2(t) + (0.22 + 0.03z_{11}(t))u_2(t) \end{cases},$$

Subsystem 2:

$$\begin{cases} \dot{x}_1(t) = (0.04 + 0.01z_{21}(t))x_1(t) - (0.02 - 0.01z_{21}(t))x_2(t) + \\ \qquad (0.3 - 0.1z_{21}(t))u_1(t) - 0.01u_2(t) \\ \dot{x}_2(t) = 0.02x_1(t) + (0.05z_{21}(t) - 0.35)x_2(t) \\ \qquad -(0.3 + 0.1z_{21}(t))u_2(t) \end{cases},$$

where

$$\begin{cases} z_{11}(t) = \dfrac{\sin^2(x_1(t) + 0.5)e^{\sin^2(x_1(t)+0.5)}}{e} \\ z_{21}(t) = \cos^2(x_2(t) + 0.5) \end{cases}.$$

Using the local sector nonlinearity method [20], we obtain the fuzzy model as follows:

Subsystem 1:

Rule 1: IF $z_{11}(t)$ is 0, THEN $\dot{x}(t) = A_{11}x(t) + B_{11}u(t)$,
Rule 2: IF $z_{11}(t)$ is 1, THEN $\dot{x}(t) = A_{12}x(t) + B_{12}u(t)$,

Subsystem 2:

Rule 1: IF $z_{21}(t)$ is 0, THEN $\dot{x}(t) = A_{21}x(t) + B_{21}u(t)$,
Rule 2: IF $z_{21}(t)$ is 1, THEN $\dot{x}(t) = A_{22}x(t) + B_{22}u(t)$,
where

$$A_{11} = \begin{bmatrix} -0.2 & 0.01 \\ 0 & 0.16 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -0.17 & 0.05 \\ 0 & 0.1 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} -0.21 & 0.01 \\ 0 & 0.22 \end{bmatrix},$$
$$B_{12} = \begin{bmatrix} -0.12 & -0.01 \\ 0 & 0.25 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0.04 & -0.02 \\ 0.02 & -0.35 \end{bmatrix},$$
$$A_{22} = \begin{bmatrix} 0.05 & -0.01 \\ 0.02 & -0.3 \end{bmatrix},$$
$$B_{21} = \begin{bmatrix} 0.3 & -0.01 \\ 0 & -0.3 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0.2 & -0.01 \\ 0 & -0.4 \end{bmatrix}.$$

The normalized membership functions are calculated as follows:

$$\begin{cases} h_{11}(x_1(t)) = 1 - \dfrac{\sin^2(x_1(t) + 0.5)e^{\sin^2(x_1(t)+0.5)}}{e}, \\ h_{12}(x_1(t)) = 1 - h_{11}(x_1(t)) \\ h_{21}(x_2(t)) = 1 - \cos^2(x_2(t) + 0.5), \\ h_{22}(x_2(t)) = 1 - h_{21}(x_2(t)) \end{cases}.$$

The nonlinear functions $g_p(t, x(t))$ and the parameters D_p are given as

$$g_1 = g_2 = \begin{bmatrix} 0.01\sin(x_1(t)) \\ 0.01\sin(x_2(t)) \end{bmatrix}, \quad D_1 = D_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

By the method of [29], we can calculate the parameters ε_{ps} as $\varepsilon_{11} = \varepsilon_{12} = 1$ and $\varepsilon_{21} = \varepsilon_{22} = 0.5$. The initial conditions are assumed to be $x(t_0) = [1, 2]^T$. The state response of subsystem 1 and subsystem 2 is shown in Fig. 1. As we see from Fig. 1 that both of these two subsystems are unstable.

We will use our results to design a set of model-dependent controllers such that the system (6) is exponentially stable. In practice, the asynchronous switching between the system models and the controllers generally exists. First, we study the asynchronous switching between the system models and the controllers, while the controllers are designed by only considering the synchronous conditions. Given $\eta = 0.01$, $\varepsilon = 0.01$, $\alpha_1 = -0.12$, $\alpha_2 = -0.2$, $\beta_1 = 0.08$, $\beta_2 = 0.045$ and $T_M = 8$, with these parameters, we can obtain $\mu_1 = 2.1892$, $\mu_2 = 2.2699$, $\tau_1^* = 19.8626$ and $\tau_2^* = 13.8987$. Using the LMI toolbox in Matlab to solve (48) and (50), the controller gains can be obtained as

$$K_{11} = \begin{bmatrix} 7.9722 & -0.4578 \\ -0.1675 & -9.1560 \end{bmatrix}, \quad K_{12} = \begin{bmatrix} 8.3959 & 0.5866 \\ 0.0438 & -6.5639 \end{bmatrix},$$
$$K_{21} = \begin{bmatrix} -7.3636 & 0.2791 \\ 0.1263 & 5.0258 \end{bmatrix}, \quad K_{22} = \begin{bmatrix} -6.3776 & 0.2231 \\ 0.0955 & 2.7234 \end{bmatrix}.$$

The state response for this case is shown in Fig. 2. As shown in Fig. 2, the system becomes unstable if we ignore the effects of asynchronous behaviors on the controllers design.

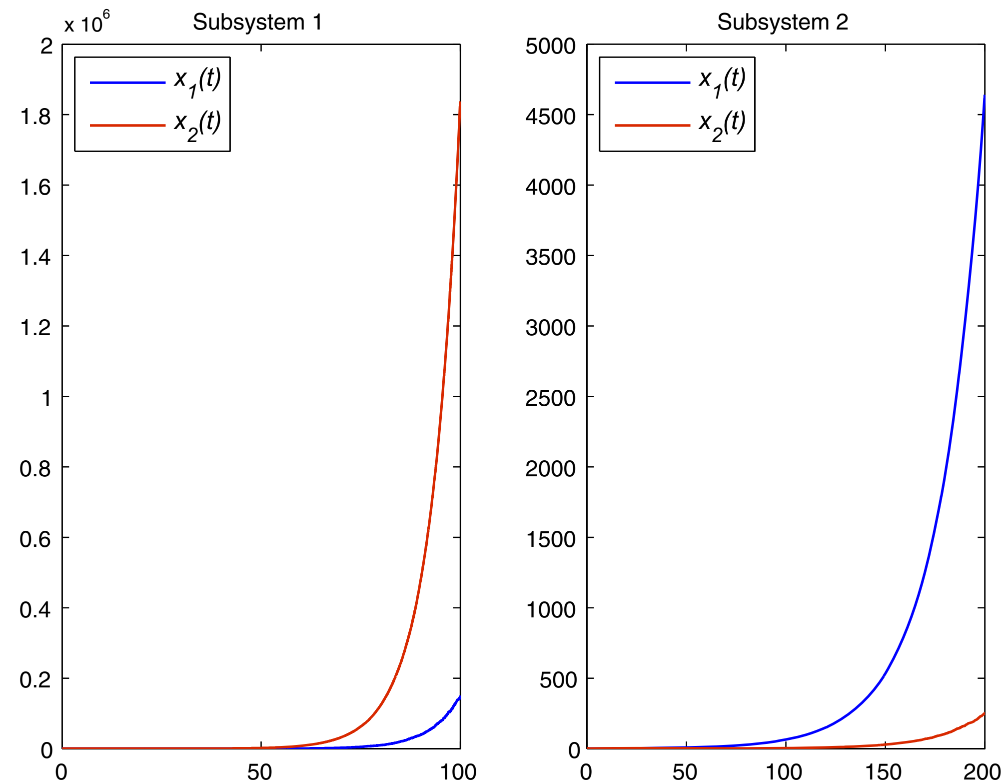


Fig. 1 State response of subsystem 1 and subsystem 2

Then, we investigate the situation that the asynchronous switching between the system models and the controllers, and the controllers are designed by considering the asynchronous conditions. We get $\mu_1 = 9.7281$, $\mu_2 = 13.0302$, $\tau_1^* = 32.2918$ and $\tau_2^* = 22.6363$ with the above parameters. Using the LMI toolbox in Matlab to solve (48), (49), and (50), the controller gains are listed as follows:

$$K_{11} = \begin{bmatrix} 0.1450 & 6.3475 \\ 0.1332 & -0.3805 \end{bmatrix}, \quad K_{12} = \begin{bmatrix} -0.1433 & 4.2655 \\ 0.0626 & -0.5197 \end{bmatrix},$$
$$K_{21} = \begin{bmatrix} -0.6258 & 1.7423 \\ 0.0695 & -0.4502 \end{bmatrix}, \quad K_{22} = \begin{bmatrix} -0.5616 & 3.7148 \\ 0.0886 & -0.2252 \end{bmatrix}.$$

The state response for this situation is shown in Fig. 3. As is shown in Fig. 3,the states of the system converge to zero.

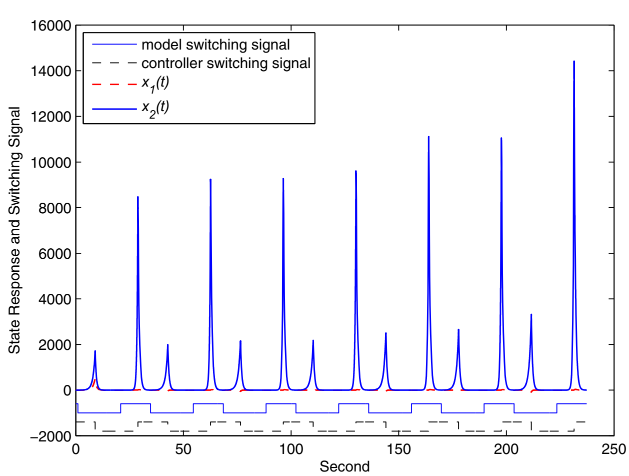


Fig. 2 State response of the closed-loop asynchronous switched system with controllers designed by synchronous conditions

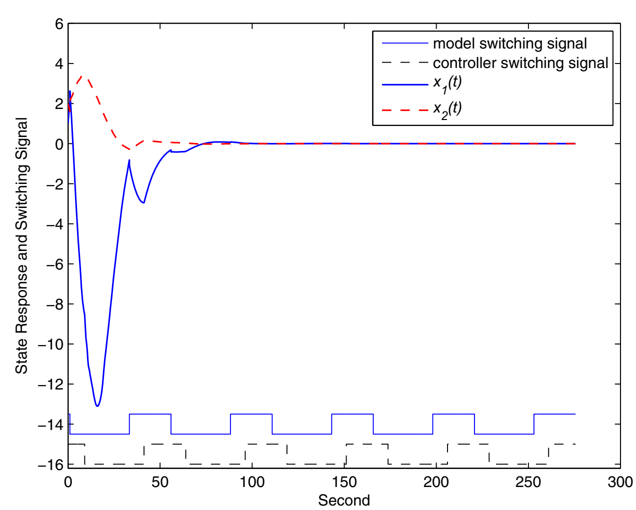


Fig. 3 State response of the closed-loop asynchronous switched system with controllers designed by considering asynchronous conditions

It means that the designed controllers by considering asynchronous conditions are effective. Since the asynchronous switching is very common in real world, it is necessary for us to consider this effect on the controller design. Through comparing Fig. 2 with Fig. 3, we can conclude if we ignore the effects of asynchronous behaviors on the controller design when there exists asynchronous switching, the controllers designed cannot meet the actual requirements.

Next, we give some comparisons to show the advantage of our methods. We firstly compare the closed-loop nonlinear impulsive switched system under two different switching schemes MDADT and ADT. The parameters and computation results are shown in Table 1. It is clearly shown in Table 1 that $\tau^* > \tau_1^*$ and $\tau^* > \tau_2^*$. Therefore, any switching signals which satisfy the ADT of the closed-loop nonlinear impulsive switched system will satisfy the MDADT of all subsystems of that system, that is, $\tau_p^* \leq \tau^*$, $\forall p \in S$.

Secondly, we give a comparison between Theorem 2 and Corollary 2. Given the parameters $T_M = 8$, $\beta_1 = 0.08$ and $\beta_2 = 0.045$ for Theorem 2 and Corollary 2, by changing α_1 with a step of 0.01, the lower bounds of α_2 for Theorem 2 and Corollary 2 are plotted in Fig. 4. As is shown in Fig. 4, the lower bounds of α_2 of Theorem 2 are smaller than those of Corollary 2. As we see from (23) and (24), a smaller α_p leads to a quicker decay of Lyapunov functions and a smaller τ_p^*. It is because that the QLFs method is a special case of the FLFs method.

Example 2 Here, we give a practical example to show the validity of our method. Consider a continuous stirred tank reactor (CSTR) where an exothermic, irreversible reaction of the form $A \to B$ happens. As shown in [36], there are two different feeding streams to feed the reactor, and these two feeding streams are selected by a selector. Source 1 feeds pure species A at the flow rate $F_1 = 50$ L/min, concentration $C_{A_1} = 1.5$ mol/L and temperature $T_{A_1} = 350$ K, and source 2 feeds pure species A at the flow rate $F_2 = 200$ L/min, concentration $C_{A_2} = 0.75$ mol/L and temperature $T_{A_2} = 350$ K. In other words, the reactor has two modes with respect to the feeding stream. For each mode of operation, the mathematical model for the process has the following differential equations [27, 36]:

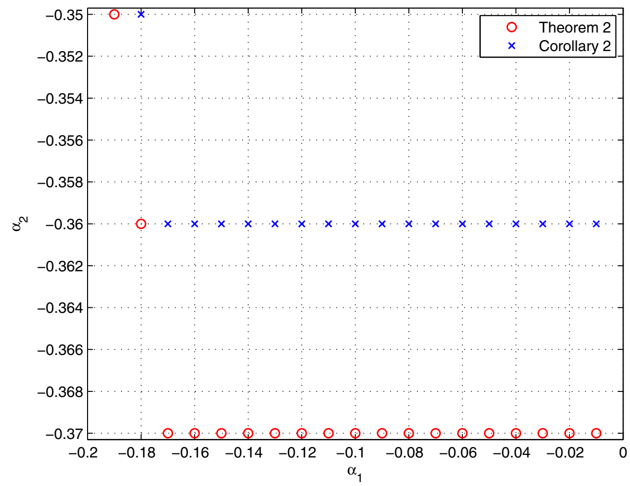


Fig. 4 The lower bounds of α_2 for Theorem 2 and Corollary 2 by changing α_1 with a step of 0.01

$$\begin{aligned} \dot{C_A} &= \frac{F_\sigma}{V}(C_{A\sigma} - C_A) - k_0 e^{-E/RT_R} C_A, \\ \dot{T_R} &= \frac{F_\sigma}{V}(T_{A\sigma} - T_R) + \frac{-\triangle H}{\rho c_p} k_0 e^{-E/RT_R} C_A + \frac{Q_\sigma}{\rho c_p V}, \end{aligned} \tag{53}$$

where C_A represents the concentration of the species A, T_R denotes the temperature of the reactor, Q_σ is the heat removed from the reactor, V is the volume of the reactor, $k_0, E, \triangle H$ are the pre-exponential constant, the activation energy, and enthalpy of the reaction, c_p, ρ are the heat capacity and fluid density in the reactor, and $\sigma(t) \in \{1, 2\}$ is the switching signal which is a discrete variable. The values of all process parameters can be found in [27].

The system (53) is a switched nonlinear system. Substituting all the process parameters into equation (53), we can get the following two subsystems:

Subsystem 1: $(\sigma = 1)$

Table 1 Parameters and computation results for the closed-loop system under two different switching schemes

Switching schemes	MDADT switching	ADT switching
Parameters	$T_M = 8$	$T_M = 8$
	$\alpha_1 = -0.12, \alpha_2 = -0.2$	$\alpha = -0.12$
	$\beta_1 = 0.08, \beta_2 = 0.045$	$\beta = 0.08$
	$\varepsilon_{11} = \varepsilon_{12} = 1$, $\varepsilon_{21} = \varepsilon_{22} = 0.5$	$\varepsilon_{11} = \varepsilon_{12} = \varepsilon_{21} = \varepsilon_{22} = 0.5$
The value of μ	$\mu_1 = 3.7392$, $\mu_2 = 4.4394$	$\mu = \max\{\mu_1, \mu_2\} = 14.3330$
Switching signals	$\tau_1^* = 10.9716$ $\tau_2^* = 7.7666$	$\tau^* = 35.5214$

$$\dot{C_A} = -0.0334C_A - 1.2\times 10^9 e^{-10,000T_R} C_A + 0.026386,$$
$$\dot{T_R} = -0.0334T_R + 2.4\times 10^{11} e^{-10,000T_R} C_A + 11.77684 + \frac{Q_\sigma}{23.9},$$

Subsystem 2: $(\sigma = 2)$

$$\dot{C_A} = -0.0167C_A - 1.2\times 10^9 e^{-10,000T_R} C_A + 0.0167,$$
$$\dot{T_R} = -0.0167T_R + 2.4\times 10^{11} e^{-10,000T_R} C_A + 5.177 + \frac{Q_\sigma}{23.9}.$$

When $Q_\sigma = 0$, the two steady-states can be easily obtained as $(C_A, T_R)_1 = (0.57, 395.3)$ and $(C_A, T_R)_2 = (0.738, 509.12)$.

Using the T-S fuzzy model [19] and from [27], the nonlinear system (53) can be approximated by the following subsystem S_σ:

Subsystem S_σ:

Rule 1: IF the concentration of the species A is $M_{\sigma 1}(x1)$ (i.e., $x_1(t)$ is 0.57). THEN

$$\delta\dot{x}_\sigma(t) = A_{\sigma 1}\delta x(t) + B_{\sigma 1}\delta u(t),$$

Rule 2: IF the concentration of the species A is $M_{\sigma 2}(x1)$ (i.e., $x_1(t)$ is 0.738). THEN

$$\delta\dot{x}_\sigma(t) = A_{\sigma 2}\delta x(t) + B_{\sigma 2}\delta u(t),$$

where $\sigma \in \{1, 2\}$ represents the subsystem subscript, $x_\sigma(t) = [x_{\sigma 1}(t) \quad x_{\sigma 2}(t)]^T = [C_A \quad T_R]^T$, $\delta x_\sigma(t) = x_\sigma(t) - x_\sigma^d$, and x_σ^d is the stationary point of the subsystem σ. It was shown in [27] that the $A_{\sigma 1}$ and $A_{\sigma 2}$ have the following values:

$$A_{11} = \begin{bmatrix} -4.5803\times 10^{-2} & 6.6748\times 10^{-5} \\ 2.4807 & -3.61\times 10^{-3} \end{bmatrix},$$
$$A_{12} = \begin{bmatrix} -3.5728 & 5.1826\times 10^{-5} \\ 707.89 & -0.010268 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} -0.029103 & 5.1833\times 10^{-5} \\ 2.4807 & -0.0036045 \end{bmatrix},$$
$$A_{22} = \begin{bmatrix} -3.564 & 5.1826\times 10^{-5} \\ 706.13 & -0.010265 \end{bmatrix}$$

The parameters $B_{\sigma 1}$ and $B_{\sigma 2}$ are set as $B_{11} = B_{12} = B_{21} = B_{22} = [-0.000005; 0.0030]$. The nonlinear functions $g_\sigma(t, x(t))$ used here are the same as in Example 1. Based on different properties of the source 1 and source 2, the matrices D_σ are given as

$$D_1 = \begin{bmatrix} 0.02 & 0 \\ 0 & -0.02 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & -0.02 \end{bmatrix}.$$

If $g_\sigma(t, x(t))$ and D_σ can be measured in practice, we can use these practical values to replace the values used in this example.

The normalized membership functions for Rule 1 and Rule 2 of the two subsystems are taken as

$$\begin{cases} h_{11}(x_1) = h_{21}(x_1) = \dfrac{\arctan(50\times(x_1 - 0.654)) + \pi/2}{\pi}, \\ h_{12}(x_1) = 1 - h_{11}(x_1), h_{22}(x_1) = 1 - h_{21}(x_1). \end{cases}$$

By the method of [29], the parameters ε_{ps} used in this example are calculated as $\varepsilon_{11} = \varepsilon_{12} = \varepsilon_{21} = \varepsilon_{22} = 2$. Given $\eta = 0.01$, $\varepsilon = 0.01$, $T_M = 200$, $\alpha_1 = -0.02$, $\alpha_2 = -0.015$, $\beta_1 = 0.01$ and $\beta_2 = 0.015$, substituting above parameters into (48), (49), and (50), we find these LMIs are feasible. A set of feasible controller gains can be given as follows:

$$K_{11} = [6.9574 \quad 4.2107], \quad K_{12} = [-18.3518 \quad 2.483],$$
$$K_{21} = [7.9568 \quad -1.2109], \quad K_{22} = [-20.3518 \quad 5.4854].$$

By assuming the initial conditions to be $x(t_0) = [0.5, 404.9]^T$, and setting $\tau_1^* = 765.2733$, $\tau_2^* = 681.3643$, the simulation results are shown in Figs. 5 and 6. As we see from these two figures, although there exist effects of impulsive behaviors and asynchronous switching, the system (53) can still be stable, which illustrates the validity of our method.

5 Conclusion

In this paper, the T-S fuzzy model is first used to study the exponential stability and asynchronous stabilization problem of the continuous-time nonlinear impulsive switched

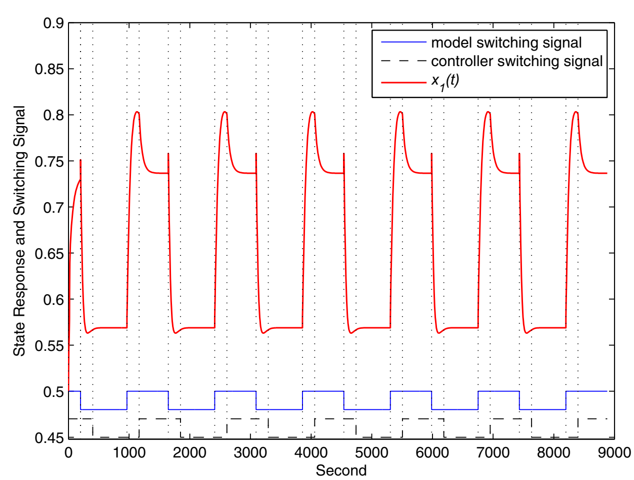


Fig. 5 The state trajectory of $x_1(t)$

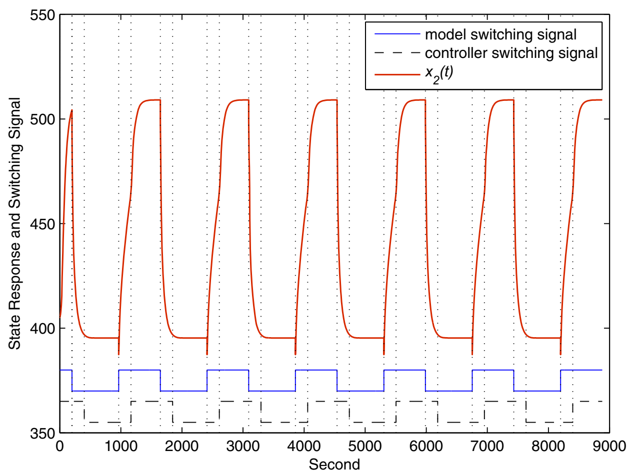


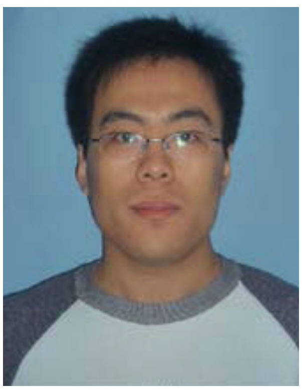
Fig. 6 The state trajectory of $x_2(t)$

systems with asynchronous switching. Based on the switching FLFs approach and the MDADT technique, the stability conditions and asynchronous stabilization conditions are obtained. Moreover, the state-feedback fuzzy controllers are designed to guarantee the closed-loop systems to be exponentially stable. Besides, we remark that the obtained results can also apply to the nonlinear switched systems without the effects of asynchronous switching or impulsive behaviors. Finally, a numerical example and a chemical process example are given to illustrate the advantage and applicability of the results obtained.

References

1. Liberzon, D.: Switching in Systems and Control, Berlin. Birkhauser, Germany (2003)
2. Ye, H., Michel, A.N., Hou, L.: Stability theory for hybrid dynamical systems. IEEE Trans. Autom. Control **43**(4), 461–474 (1998)
3. Hespanha, J. P.,Morse, A.S.: Stability of switched systems with average dwell time. In *Proceedings of the 38th IEEE Conference Decision and Control*, Phoenix, AZ, pp. 2655–2660 (1999)
4. Lee, S.H., Kim, T.H., Lim, J.T.: A new stability analysis of switched systems. Automatica **36**(6), 917–922 (2000)
5. Cheng, D.Z., Guo, L., Lin, Y.D., Wang, Y.: Stabilization of switched linear systems. IEEE Trans. Autom. Control **50**(5), 661–666 (2005)
6. Dayawansa, W.P., Martin, C.F.: A converse Lyapunov theorem for a class of dynamical systems which undergo switching. IEEE Trans. Autom. Control **44**(4), 751–760 (1999)
7. Branicky, M.S.: Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. IEEE Trans. Autom. Control **43**(4), 475–482 (1998)
8. Daafouz, J., Riedinger, P., Iung, C.: Stability analysis and control synthesis for switched systems: A switched Lyapunov function approach. IEEE Trans. Autom. Control **47**(11), 1883–1887 (2002)
9. Chiou, J.S., Wang, C.J., Cheng, C.M., Wang, C.C.: Stability analysis and controller design of the nonlinear switched systems via T-S discrete-time fuzzy model. Int. J. Fuzzy Syst. **11**(4), 213–224 (2009)
10. Wu, L., Qi, T., Feng, Z.: Average dwell time approach to L_2-L_∞ control of switched delay systems via dynamic output feedback. IET Control Theory Appl. **3**(10), 1425–1436 (2009)
11. Zhang, L., Gao, H.: Asynchronously switched control of switched linear systems with average dwell time. Automatica **46**(5), 953–958 (2010)
12. Zhang, L., Shi, P.: Stability, L_2-gain and asynchronous H_∞ control of discrete-time switched systems with average dwell time. IEEE Trans. Autom. Control **54**(9), 2193–2200 (2009)
13. Zhao, X., Zhang, L., Shi, P., Liu, M.: Stability and stabilization of switched linear systems with mode-dependent average dwell time. IEEE Trans. Autom. Control **57**(7), 1809–1815 (2012)
14. Zhao, X., Liu, H., Wang, Z.: Weighted H_∞ performance analysis of switched linear systems with mode-dependent average dwell time. Int. J. Syst. Sci. **44**(11), 2130–2139 (2013)
15. Lu, Q., Zhang, L., Karimi, H.R., Shi, Y.: H_∞ control for asynchronously switched linear parameter-varying systems with mode-dependent average dwell time. IET Control Theory Appl. **7**(5), 673–683 (2013)
16. Guan, Z., Hill, D.J., Shen, X.: On hybrid impulsive and switching systems and application to nonlinear control. IEEE Trans. Autom. Control **50**(7), 1058–1062 (2005)
17. Li, C., Ma, F., Feng, G.: Hybrid impulsive and switching time-delay systems. IET Control Theory Appl. **3**(11), 1487–1498 (2009)
18. Xu, H., Teo, K.L.: Exponential stability with L_2-gain condition of nonlinear impulsive switched systems. IEEE Trans. Autom. Control **55**(10), 2429–2433 (2010)
19. Takagi, T., Sugeno, M.: Fuzzy identification of systems and its application to modeling and control. IEEE Trans. Syst. Man Cybern. **15**(1), 116–132 (1958)
20. Tanaka, K., Wang, H.O.: Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach. Wiley, New York (2001)
21. Li, H., Gao, Y., Wu, L., Lam, H.K.: Fault detection for T-S fuzzy time-delay systems: delta operator and input-output methods. IEEE Trans. Cybern. **45**(2), 229–241 (2015)
22. Li, H., Pan, Y., Zhou, Q.: Filter design for interval type-2 fuzzy systems with D stability constraints under a unified frame. IEEE Trans. Fuzzy Syst. **23**(3), 719–725 (2015)
23. Cao, K., Gao, X.Z., Lam, H.K., Vasilakos, A.V., Pedrycz, W.: A new relaxed stability condition for Takagi-Sugeno fuzzy control systems using quadratic fuzzy Lyapunov functions and staircase membership functions. Int. J. Fuzzy Syst. **16**(3), 327–337 (2014)
24. Lee, C., Chen, H.: Identification and fuzzy controller design for nonlinear uncertain systems with input time-delay. Int. J. Fuzzy Syst. **11**(2), 73–86 (2009)
25. Wang, L., Liu, X.: Robust H_∞ fuzzy output feedback control for uncertain discrete-time nonlinear systems. Int. J. Fuzzy Syst. **12**(3), 218–226 (2010)
26. Sun, C.H., Wang, W.J., Lin, W.W.: Linear control and parallel distributed fuzzy control design for T-S fuzzy time-delay system. Int. J. Fuzzy Syst. **9**(4), 229–235 (2007)
27. Chiou, J.S., Wang, C.J., Cheng, C.M., Wang, C.C.: Analysis and synthesis of switched nonlinear systems using the T-S fuzzy model. Appl. Math. Model. **34**(6), 1467–1481 (2010)
28. Mao, Y., Zhang, H., Xu, S.: Exponential stability and asynchronous stabilization of a class of switched nonlinear system via T-S fuzzy model. IEEE Trans. Fuzzy Syst. **22**(4), 817–828 (2014)
29. Tanaka, K., Hori, T., Wang, H.O.: A fuzzy Lyapunov approach to fuzzy control system design. Proc. Am. Control Conf. **2001**, 4790–4795 (2001)
30. Tanaka, K., Hori, T., Wang, H.O.: A multiple Lyapunov function approach to stabilization of fuzzy control systems. IEEE Trans. Fuzzy Syst. **11**(4), 582–589 (2003)

31. Lee, D.H., Park, J.B., Joo, Y.H.: A new fuzzy Lyapunov function for relaxed stability condition of continuous-time Takagi-Sugeno fuzzy systems. IEEE Trans. Fuzzy Syst. **19**(4), 785–791 (2011)
32. Wang, B., Zhang, H., Xie, D., Wang, G., Dang, C.: Asynchronous stabilisation of impulsive switched systems. IET Control Theory Appl. **7**(16), 2021–2027 (2013)
33. Huang, L.: Linear Algebria in Systems and Control Theory. Science Press, Beijing (1984)
34. Shi, G., Zou, Y., Wang, C.: An algebraic approach to robust H_∞ control via state feedback. Syst. Control Lett. **18**(5), 365–370 (1992)
35. de Oliveira, M.C., Bernussou, J., Geromel, J.C.: A new discrete-time robust stability condition. Syst. Control Lett. **37**(4), 261–265 (1999)
36. Yazdi, M.B., Jahed-Motlagh, M.R., Attia, S.A., Raisch, J.: Modal exact linearization of a class of second-order switched nonlinear systems. Nonlinear Analysis: Real World Applications **11**(4), 2243–2252 (2010)



Qunxian Zheng received the BE degree in biomedical engineering and ME degree in biophysics from the University of Electronic Science and Technology of China, Chengdu, in 2008 and 2011, respectively. He is currently doing his PhD degree in circuits and systems at the University of Electronic Science and Technology of China, Chengdu. His research interests include fuzzy control and switched systems.

Hongbin Zhang received the BE degree in aerocraft design from the Northwestern Polytechnical University, Xi'an, China, in 1999, and the ME and PhD degrees in circuits and systems from the University of Electronic Science and Technology of China, Chengdu, in 2002 and 2006, respectively. He has been with the School of Electrical Engineering, the University of Electronic Science and Technology of China, since 2002, where he is currently a professor. From August 2008 to August 2010, he has served as a research fellow with the Department of Manufacturing Engineering and Engineering Management, the City University of Hong Kong, Kowloon, Hong Kong. His current research interests include intelligent control, autonomous cooperative control, and integrated navigation.