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A Fuzzy Set-Based Approach to Multi-objective Multi-item Solid Transportation Problem Under Uncertainty

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Abstract In this paper, a multi-objective multi-item solid transportation problem (MOMISTP) with parameters, e.g., transportation costs, supplies, and demands, as trapezoidal fuzzy variables is formulated. In this MOMISTP, there are limitations on some items and conveyances so that some special items cannot be carried by means of some special conveyances. With the use of the nearest interval approximation of trapezoidal fuzzy numbers, an interval programming model is constructed for the fuzzy MOMISTP and then this model is turned into its deterministic form. Then, a new interval fuzzy programming approach is developed to obtain the optimal solution of the problem. Finally, a numerical example is presented for illustration.

Keywords Multi-objective multi-item solid transportation problem · Fuzzy decision making · Interval programming · Order relations

1 Introduction

The solid transportation problem (STP) is a continuation of the conventional transportation problem in which three-dimensional properties are considered in the objective and constraint set instead of the supply and destination. The necessity of considering this special type of transportation problem appears when heterogeneous conveyances are available for shipment of products. In many industrial problems, a homogeneous product is obtained by its supply to a destination by way of distinct modes of transport called conveyances, such as trucks, cargo flights, goods trains, and ships. These conveyances are taken as the third dimension.

The TP was first developed by Hitchcock [1]. Shell [2] first presented the STP and Haley [3] introduced a solution procedure of the STP.

Because of the difficulty of the real world problems, we may usually encounter uncertain phenomena in constructing the model of realistic applications. For such condition, we generally add the uncertain parameters (fuzzy, interval or stochastic) to the models. Zadeh [4] first presented the concept of fuzzy set theory. Bellman, and Zadeh [5], Zimmermann [6] developed and used the fuzzy set theory. The STP is further discussed and solved by several researchers in crisp as well as uncertain environments. The STP based on fuzzy sets was suggested by Jimenez and Verdegay [7, 8]. A fuzzy programming approach was used to multi-objective STP by Bit and Biswal [9] and an efficient solution was obtained. Li et al. [10] presented a neural network method for multi-criteria STP, and they suggested an improved genetic algorithm to solve multi-objective STPs with fuzzy parameters in [11]. Gen et al., [12] presented a genetic algorithm for solving bi-criteria STP with fuzzy parameters. Liu and Yang [13] discussed the STP with fuzzy parameters. They suggested the fuzzy programming approach for the fixed charge STP by thinking that the direct costs, fixed charges, supplies, demands, and conveyance parameters are fuzzy coefficient (Fig. 1).

Baidya et al. [14] studied the interval multi-item interval STP. Kundu et al. [15, 16] presented fuzzy multi-objective multi-item solid transportation problems (MOMISTPs) with fuzzy coefficients. They used the fuzzy programming

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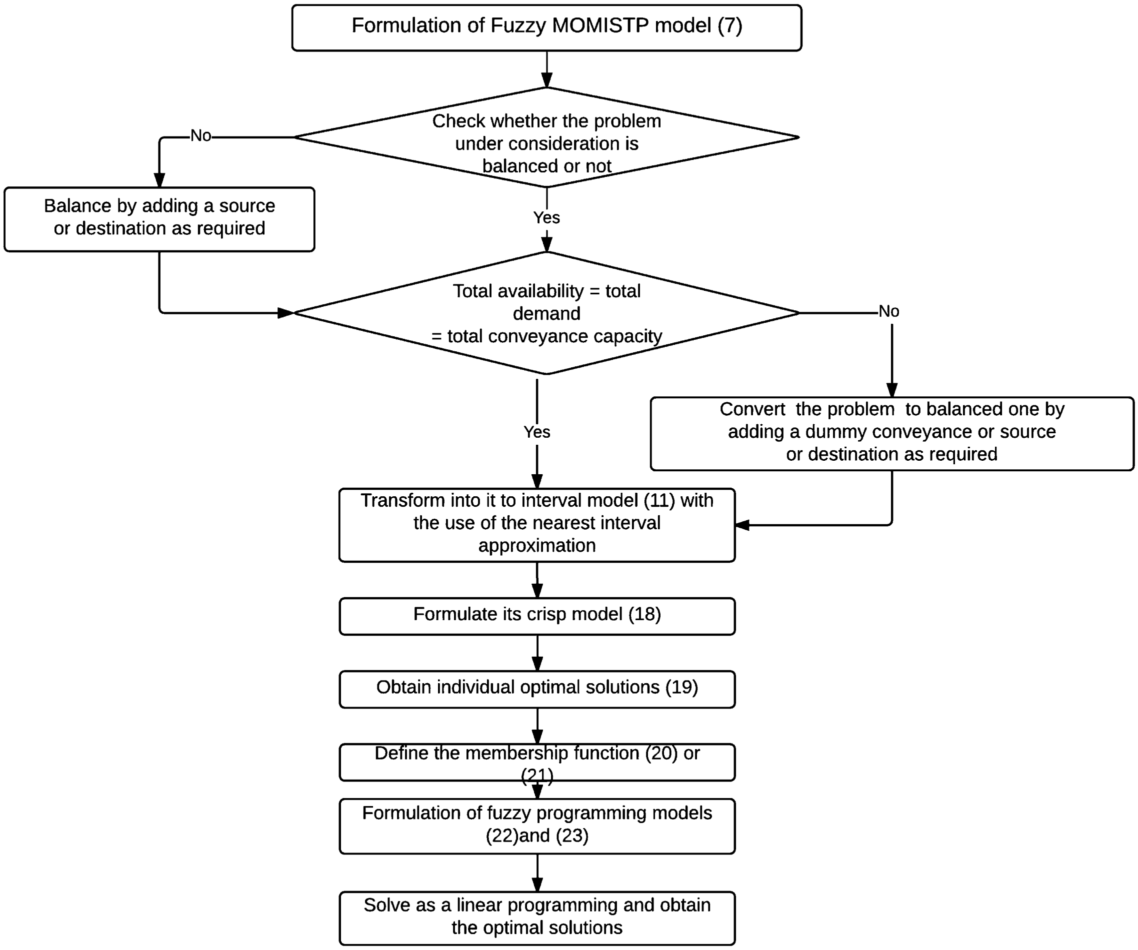

Fig. 1 Application framework of the interval fuzzy programming approach

approach and global criterion method to find the optimal compromise solution. Recently, several kinds of multi-objective solid transportation models are studied by Chakraborty et al. [17], Giri et al. [18], Kaur and Kumar [19], Zavardehi et al. [20], Pramanik et al. [21], Radhakrishnan and Anukokila [22], Tao and Xu [23].

Model parameters in most programming problems require to be addressed as interval parameters, due to poor data for an absolute evaluation but with known extreme bounds of the parameter values. Such interval uncertainty, with partially known and partially unknown components cannot be completely designed by probabilistic or fuzzy logic approach because of deficiency of data to determine probability distribution and shortage of information to exactly determine the membership functions.

The order relation of interval number is a major issue in group decision-making, especially for decision-making with scientific terms, which are generally designed by interval numbers. The interval uncertainty theory was introduced by Moore [24]. Ishibuchi and Tanaka [25] developed the order relation between intervals for linear programming problems with interval objective functions by transforming those into multi-objective programming problem. Several authors such as Chinneck and Ramadan [26], Kuchta [13], Oliveira and Antunes [27], Sengupta and Pal [28], Yu et al. [29] interested in different order relations between intervals to make decisions.

Chanas and Kuchta [30] improved the paper [25] with the aid of a level-cut of the intervals and presented the general method using multi-objective programming for interval linear programming. For interval multi-objective problems, a new approach is developed in [31]. The interval transportation problems were converted into crisp multi-objective problems using the order relation in [32]. The effects of variations on the optimal value and interval solutions are the basic problems investigated in [33].

The interval fuzzy linear programming model is introduced as the basic algorithm of interval programming which is effective for optimization under uncertainty [34]. The interval fuzzy linear programming model is developed by Hajiagha et al. [35, 36] and they obtained Pareto optimal solution of interval multi-objective problems using the different order relation between intervals. Some interactive solution procedure has been developed based on interval and fuzzy programming model in [37, 38].

Here, this paper focuses on MOMISTP under uncertainty and presents a new interval fuzzy programming model for the fuzzy MOMISTP. The application of the suggested method is shown by solving a numerical example in which objective function coefficients, availabilities, and demand parameters are represented by trapezoidal fuzzy numbers. The method converts an unbalanced problem to a balanced one. After constructing that the interval MOMISTP is transformed into a crisp form. Then, based on a compromise solution of deterministic MOMISTP, the best and worst bounds are found and the membership functions are constructed. Using these membership functions, a new interval fuzzy programming model is formulated and easily solved by Maple 18 optimization toolbox.

The main contributions of our work are summarized below;

A nearest interval approximation of fuzzy numbers [39] is used for the fuzzy MOMISTP and an equivalent interval MOMISTP is obtained. Then, a new interval fuzzy programming model is developed to solve the interval MOMISTP. Moreover, while the model concludes infeasible solution by the suggested approach in [35, 36] for the fuzzy MOMISTP, our suggested model gives an optimal solution for the fuzzy MOMISTP.

The rest of this paper is organized as follows. In Sect. 2, we recall some preliminary information about interval and fuzzy arithmetic. We formulate a fuzzy MOMISTP and we convert into a deterministic one. Then provides general information about the fuzzy set-based approach in Sect. 3. A numerical example is solved in Sect. 4.

2 Preliminaries

In this paper, we assumed that all parameters of considered problems are expressed as fuzzy and interval numbers and brief information about the interval arithmetic are presented.

Definition 1 A fuzzy numbers $\tilde{A}$ defined on the universal set of real numbers $\mathbb{R}$, denoted by $\tilde{A} = (a, b, c, d)$, is said to be a trapezoidal fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x < b, \\ 1, & b \le x \le c, \\ \dfrac{x-d}{c-d}, & c < x < d, \\ 0, & \text{otherwise}. \end{cases}.$$

Definition 2 A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be zero trapezoidal fuzzy number if and only if $\tilde{A} = (0, 0, 0, 0)$.

Definition 3 A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be nonnegative trapezoidal fuzzy number if and only if $a \ge 0$.

Definition 4 The support of a fuzzy number $\tilde{A}$ on X is the crisp set of all $x \in X$ such that $\mu_{\tilde{A}}(x) > 0$.

Definition 5 The core of a fuzzy number $\tilde{A}$ on X is the crisp set of all $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

Definition 6 Two trapezoidal fuzzy numbers $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ are said to be equal trapezoidal fuzzy number if $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$ and $d_1 = d_2$.

2.1 Arithmetic Operations

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers and we have

- $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2,\ b_1 + b_2,\ c_1 + c_2,\ d_1 + d_2)$,
- $\tilde{A}_1 \Theta \tilde{A}_2 = (a_1 - d_2,\ b_1 - c_2,\ c_1 - b_2,\ d_1 - a_2)$,
- $k\tilde{A}_1 = \begin{cases} (ka_1, kb_1, kc_1, kd_1), & k \ge 0, \\ (kd_1, kc_1, kb_1, ka_1), & k \le 0, \end{cases}$
- $\tilde{A}_1 \otimes \tilde{A}_2 = (\min(a_1a_2, a_1d_2, d_1a_2, d_1d_2),\ \min(b_1b_2, b_1c_2,\ c_1b_2, c_1c_2),\ \max(b_1b_2,\ b_1c_2, c_1b_2, c_1c_2),\ \max(a_1a_2, a_1d_2, d_1a_2, d_1d_2))$.

2.1.1 Liou and Wang Ranking Approach for Trapezoidal Fuzzy Numbers [40]

Let $F(\mathbb{R})$ be a set of fuzzy numbers defined on the set of real numbers $\mathbb{R}$ and let $\tilde{A} = (a, b, c, d) \in F(\mathbb{R})$. According to the ranking approach suggested by Liou and Wang [40] to find the crisp value of trapezoidal fuzzy numbers,

$$\Re(\tilde{A}) = \frac{(a + b + c + d)}{4}$$

is called a ranking function $\Re : F(\mathbb{R}) \to \mathbb{R}$, which maps each fuzzy number into the real line.

2.2 Comparison of Trapezoidal Fuzzy Numbers

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers and then, we have

- $\tilde{A}_1 \succ \tilde{A}_2$ if $\Re(\tilde{A}_1) > \Re(\tilde{A}_2)$.
- $\tilde{A}_1 \prec \tilde{A}_2$ if $\Re(\tilde{A}_1) < \Re(\tilde{A}_2)$.
- $\tilde{A}_1 \approx \tilde{A}_2$ if $\Re(\tilde{A}_1) = \Re(\tilde{A}_2)$.

2.3 Nearest Interval Approximation of Fuzzy Numbers

According to Grzegorzewski [39], a fuzzy number is approximated to an equal crisp interval. Let $A = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number and its α-cut is defined as $[\underline{A}_\alpha, \bar{A}_\alpha]$. Therefore, we can write $\underline{A}_\alpha = a_1 + \alpha(a_2 - a_1)$ and $\bar{A}_\alpha = a_3 + \alpha(a_4 - a_3)$. Using the definition of nearest interval approximation, we get lower and upper bounds, respectively, as

$$\underline{C} = \int_0^1 \underline{A}_\alpha(\alpha)\mathrm{d}\alpha = \int_0^1 [a_1 + \alpha(a_2 - a_1)]\mathrm{d}\alpha = \frac{1}{2}(a_2 + a_1), \quad 0 \le \alpha \le 1 \tag{1}$$

$$\bar{C} = \int_0^1 \bar{A}_\alpha(\alpha)\mathrm{d}\alpha = \int_0^1 [a_3 + \alpha(a_4 - a_3)]\mathrm{d}\alpha = \frac{1}{2}(a_4 + a_3), \quad 0 \le \alpha \le 1 \tag{2}$$

Here, with the use of the nearest interval approximation, a trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4)$ is converted to a crisp closed interval as

$$[\underline{C}, \bar{C}] = \left[\frac{1}{2}(a_2 + a_1), \frac{1}{2}(a_4 + a_3)\right] \tag{3}$$

2.4 Interval Numbers

The interval uncertainty theory was presented by Moore [24] and defined following concepts in [41].

Definition 7 (Moore et al. [41]) An interval number is a number whose exact value is unknown distribution information, but a range within which the value lies is known. Interval number is a number with both lower and upper bounds.$X \in [\underline{x}, \bar{x}]$. where $\underline{x} \le \bar{x}$. The main arithmetic operations can be expressed in interval numbers.

Definition 8 (Moore et al [41]) Let $\tilde{x}_1 = [\underline{x}_1, \bar{x}_1]$ and $\tilde{x}_2 = [\underline{x}_2, \bar{x}_2]$ be a closed interval number. The following notations can be satisfied:

- $\tilde{x}_1 + \tilde{x}_2 = [\underline{x}_1 + \underline{x}_2, \bar{x}_1 + \bar{x}_2]$
- $\tilde{x}_1 - \tilde{x}_2 = [\underline{x}_1 - \bar{x}_2, \bar{x}_1 - \underline{x}_2]$
  $\tilde{x}_1 * \tilde{x}_2 = [\min(\underline{x}_1\underline{x}_2, \underline{x}_1\bar{x}_2, \bar{x}_1\underline{x}_2, \bar{x}_1\bar{x}_2),\ \max(\underline{x}_1\underline{x}_2, \underline{x}_1\bar{x}_2, \bar{x}_1\underline{x}_2, \bar{x}_1\bar{x}_2)]$
  $\tilde{x}_1 \div \tilde{x}_2 = [\underline{x}_1, \bar{x}_1]\frac{1}{[\bar{x}_2, \underline{x}_2]}$When $X \in [\underline{x}, \bar{x}]$ is an interval number, its absolute value is the maximum of the absolute value of its endpoints:
- $|x| = \max(|\underline{x}|, |\bar{x}|)$

The center, x_c, and width, x_w, of an interval number of $X \in [\underline{x}, \bar{x}]$ are defined as follows:

$$x_c = \frac{1}{2}[\underline{x} + \bar{x}] \text{ and } x_w = \frac{1}{2}[\bar{x} - \underline{x}]$$

It is easily demonstrable that $\bar{x} = x_c + x_w$ and $\underline{x} = x_c - x_w$.

In addition, Ishibuchi and Tanaka [25] defined the following order relations between intervals.

Definition 9 (Ishibuchi and Tanaka [25]) Let $\tilde{x} = [\underline{x}, \bar{x}]$ and $\tilde{y} = [\underline{y}, \bar{y}]$ are two closed interval numbers and then the order relation $\le_{\mathrm{LR}}$ (LR represents the upper and lower bounds of an interval) is defined as

- $\tilde{x} \le_{\mathrm{LR}} \tilde{y} \Leftrightarrow \underline{x} \le \underline{y}$ and $\bar{x} \le \bar{y}$
- $\tilde{x} <_{\mathrm{LR}} \tilde{y} \Leftrightarrow \underline{x} \le_{\mathrm{LR}} \underline{y}$ and $\bar{x} \ne \bar{y}$

Definition 10 (Ishibuchi and Tanaka [25]) The order relation $\le_{\mathrm{CW}}$ between two interval numbers $\tilde{x} = [\underline{x}, \bar{x}]$ and $\tilde{y} = \left[\underline{y}, \bar{y}\right]$ is expressed as

$$\tilde{x} \le_{\mathrm{CW}} \tilde{y} \Leftrightarrow x_{\mathrm{C}} \le y_{\mathrm{C}} \text{ and } x_{\mathrm{W}} \le y_{\mathrm{W}}$$

$$\tilde{x} <_{\mathrm{CW}} \tilde{y} \Leftrightarrow \tilde{x} \le_{\mathrm{CW}} \tilde{y} \text{ and } x \ne y$$

The order relations $\le_{\mathrm{CW}}$ and $\le_{\mathrm{LR}}$ certainly not conflict with each other. Furthermore, the order relations $\le_{\mathrm{CW}}$ and $\le_{\mathrm{LR}}$ are antisymmetric, reflexive, and transitive (Sengupta and Pal [28]).

3 Problem Formulation

In order to construct the model, we use the following notations for the fuzzy MOMISTP;

- i is the $\{1, 2, \ldots, m\}$number of sources.
- j is the $\{1, 2, \ldots, n\}$number of demands.
- k is the $\{1, 2, \ldots, l\}$number of conveyances or different modes of transportation.
- p is the $\{1, 2, \ldots, r\}$ number of fuzzy available for item p at supplies and destinations.
- x_{ijk}^p is the unknown quantity of item p transported from supply i to destination j by conveyance k.
- $\tilde{c}_{ijk}^p$ is the fuzzy penalty for transporting one unit of item p from supply i to destination j by conveyance $k$for the fuzzy objective.
- $\tilde{a}_i^p$ is the fuzzy availability of item p at supply i which can be transported to n destinations.
- $\tilde{b}_j^p$ is the fuzzy demand of item p at destination j.
- $\tilde{e}_k$ is the transportation capacity of conveyance k.

In fact, in a MOMISTP, the total availability of item p from supply i is no more than a_i^p. Thus, the first constraint of the system is obtained as follows:

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}^{p} \le a_i^p, i = 1,2,\ldots,m;\ p = 1,2,\ldots,r \tag{4}$$

The constraint of the system is the total availability of item p transported from supplies should satisfy the demand for destination j. Then, the second constraint of the system is obtained as follows:

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk}^{p} \ge b_j^p, j = 1,2,\ldots,n; p = 1,2,\ldots,r \tag{5}$$

In addition, the total quantity is transported by means of conveyance is no more than its transportation capacity. Thus, the last constraint of the system is obtained as follows:

$$\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{p} \le e_k^p, k = 1,2,\ldots,l \tag{6}$$

Using these system constraints (4), (5), and (6), programming model of a fuzzy MOMISTP can be formulated as follows:

$$(Z_1, Z_2, \ldots, Z_K) \to \min$$
$$\text{s.t.}\begin{cases} \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}^{p} \le \tilde{a}_i^p, i = 1,2,\ldots,m; p = 1,2,\ldots,r, \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk}^{p} \ge \tilde{b}_j^p, j = 1,2,\ldots,n; p = 1,2,\ldots,r, \\ \sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{p} \le \tilde{e}_k^p, k = 1,2,\ldots,l, \\ x_{ijk}^{p} \ge 0, \forall p,i,j,k \end{cases} \tag{7}$$

where

$$\underset{t=1,2,\ldots,K}{Z_t} = \left(\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}^{tp} x_{ijk}^{p}\right).$$

In the fuzzy MOMISTP, all parameters are taken as trapezoidal fuzzy numbers and for the above problem to be balanced, it should satisfy the following conditions.

- Total availability of an item at all sources should be equal to its demand at all the destinations.

$$\Re\left(\sum_{j=1}^{n}\sum_{k=1}^{l} \tilde{a}_i^p\right) = \Re\left(\sum_{i=1}^{m}\sum_{k=1}^{l} \tilde{b}_j^p\right); p = 1,2,\ldots,r, \tag{8}$$

- Total availability of all the items at all the sources, total demand of all the items at all the destinations, and total conveyance capacity should be equal.

$$\Re\left(\sum_{j=1}^{n}\sum_{k=1}^{l} \tilde{a}_i^p\right) = \Re\left(\sum_{i=1}^{m}\sum_{k=1}^{l} \tilde{b}_j^p\right) = \Re\left(\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{e}_k^p\right). \tag{9}$$

Thus, the balanced fuzzy MOMISTP is formulated as

$$(Z_1, Z_2, \ldots, Z_K) \to \min$$
$$\text{s.t.}\begin{cases} \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}^{p} = \tilde{a}_i^p, i = 1,2,\ldots,m; p = 1,2,\ldots,r, \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk}^{p} = \tilde{b}_j^p, j = 1,2,\ldots,n; p = 1,2,\ldots,r, \\ \sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{p} = \tilde{e}_k^p, k = 1,2,\ldots,l, \\ x_{ijk}^{p} \ge 0, \forall p,i,j,k \end{cases} \tag{10}$$

where

$$\underset{t=1,2,\ldots,K}{Z_t} = \left(\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}^{tp} x_{ijk}^{p}\right).$$

Here, with the use of nearest interval approximation (3) for the fuzzy numbers, all fuzzy variables of the above fuzzy MOMISTP can be converted to its equivalent interval variables and the model (10) converted into the interval MOMISTP as follows;

$$\left(\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \left[\underline{c}_{ijk}^{tp}, \bar{c}_{ijk}^{tp}\right]\right)_{t=1,2,\ldots K} \to \min$$
$$\text{s.t.}\begin{cases} \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}^{p} = [\underline{a}_i^p, \bar{a}_i^p], i = 1,2,\ldots,m;\ p = 1,2,\ldots,r, \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk}^{p} = \left[\underline{b}_j^p, \bar{b}_j^p\right], j = 1,2,\ldots,n;\ p = 1,2,\ldots,r, \\ \sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{p} = [\underline{e}_k^p, \bar{e}_k^p], k = 1,2,\ldots,l, \\ x_{ijk} \ge 0, \forall p,i,j,k \end{cases} \tag{11}$$

3.1 Deterministic Model of the Interval MOMISTP

Hajiagha et al. [35, 36] and Huang et al. [34] suggested different interval fuzzy methods to optimize the interval-type problems. These methods transformed an interval linear programming method into two equivalent models for its lower and upper bounds. Then, with the use of the order relation between intervals, deterministic linear programming problems are generated. But, these models generally produce the infeasible solution and fail to provide the most appropriate solutions to interval MOMISTPs.

Therefore, we suggest a different method for solving interval MOMISTP. At first, the supply, demand, and conveyance constraints of the interval MOMISTP (11) are constructed as follows:

$$\underline{a}_i^p \le \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}^p \le \bar{a}_i^p;\ \underline{b}_j^p \le \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk}^p \le \bar{b}_j^p;$$
$$\underline{e}_k^p \le \sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^p \le \bar{e}_k^p. \tag{12}$$

Then, interval MOMISTP (11) is transformed into the following problems

$$\left(\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\left[\underline{c}_{ijk}^{tp}, \bar{c}_{ijk}^{tp}\right]\right)_{t=1,2,\ldots,K} \to \min$$
$$\text{s.t.}\begin{cases} \underline{a}_i^p \le \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}^p \le \bar{a}_i^p,\ i=1,2,\ldots,m;\ p=1,2,\ldots,r, \\ \underline{b}_j^p \le \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk}^p \le \bar{b}_j^p,\ j=1,2,\ldots n;\ p=1,2,\ldots,r, \\ \underline{e}_k^p \le \sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^p \le \bar{e}_k^p,\ k=1,2,\ldots,l, \\ x_{ijk} \ge 0, \forall p,i,j,k \end{cases} \tag{13}$$

If the original objective is to minimize $Z_t(x)$, ($t = 1, 2,\ldots, K$), the solution of the model (13) can be obtained as the set of optimal solutions of the following programming bi-objective problem:

$$(Z_{tC}(x), \bar{Z}_t(x))_{(t=1,2,\ldots,K)} \to \min \tag{14}$$

where $\bar{Z}_t(x)$ is the upper bound of interval objective function and $Z_{tC}(x)$ is its center of interval objective function. Then, the objective function of minimization model (13) is obtained as follows;

$$\left(\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\left(\frac{\underline{c}_{ijk}^{tp}+\bar{c}_{ijk}^{tp}}{2}\right)\right)_{t=1,2,\ldots,K} \to \min$$
$$\left(\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\bar{c}_{ijk}^{tp}\right)_{t=1,2,\ldots,K} \to \min \tag{15}$$

If the original objective is to maximize $Z_t(x)$, ($t = 1, 2, \ldots, K$) the solution of model (13) can be obtained as the set of optimal solutions of the following problem:

$$(\underline{Z}_t(x), Z_{tC}(x))_{(t=1,2,\ldots,K)} \to \max \tag{16}$$

where $\underline{Z}_t(x)$ is the lower bound of interval objective function and $Z_{tC}(x)$ is its center. Then, the objective function of maximization model is obtained as follows;

$$\left(\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\underline{c}_{ijk}^{tp}\right)_{t=1,2,\ldots K} \to \max$$
$$\left(\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\left(\frac{\underline{c}_{ijk}^{tp}+\bar{c}_{ijk}^{tp}}{2}\right)\right)_{t=1,2,\ldots,K} \to \max \tag{17}$$

The above interval MOPMSTP is transformed into the following crisp form:

$$\begin{cases} \left(\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\left(\frac{\underline{c}_{ijk}^{tp}+\bar{c}_{ijk}^{tp}}{2}\right)\right)_{t=1,2,\ldots,K} \to \min \\ \left(\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\bar{c}_{ijk}^{tp}\right)_{t=1,2,\ldots,K} \to \min \end{cases}$$
$$\text{s.t.}\begin{cases} \underline{a}_i^p \le \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}^p \le \bar{a}_i^p,\ i=1,2,\ldots,m;\ p=1,2,\ldots,r, \\ \underline{b}_j^p \le \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk}^p \le \bar{b}_j^p,\ j=1,2,\ldots n;\ p=1,2,\ldots,r, \\ \underline{e}_k^p \le \sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^p \le \bar{e}_k^p,\ k=1,2,\ldots,l, \\ x_{ijk} \ge 0, \forall p,i,j,k \end{cases} \tag{18}$$

In this model, solve each objective function as a single STP ignoring all other objectives and the range of optimal objective functions based on lower and upper bounds are determined as

$$Z_t^{opt} = \left[\underline{Z}_t^{opt}, \bar{Z}_t^{opt}\right], (p = 1, 2, \ldots, r) \tag{19}$$

Definition 11 The function $Z{:}R^n \to I$ ($I \in R$) is called a closed and bounded interval function on the R^n and defined as $Z_t(x) = \left[\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\underline{c}_{ijk}^{tp}, \sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\bar{c}_{ijk}^{tp}\right]$ where $\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\underline{c}_{ijk}^{tp}$ and $\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\bar{c}_{ijk}^{tp}$ are the lower limit and the upper limit of interval respectively. Then, we have $\forall x_{ijk}^p \in X$, (X is the feasible region of problem) $\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\underline{c}_{ijk}^{tp} \le \sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\bar{c}_{ijk}^{tp}$, $(t = 1, 2, \ldots, K)$; $(p = 1, 2, \ldots, r)$.

3.2 An Interval Fuzzy Programming Model to Interval MOMISTP

In order to construct an interval fuzzy programming model, we consider the interval objective function $Z_t\,(x)$ of model (13). After determining the best lower and worst upper bounds (19) of objective function values, then its membership function (for the minimization type objective) can be determined as follows:

$$\mu_t(x) = \begin{cases} 1 & Z_t(x) \le \underline{Z}_t^{opt} \\ \dfrac{\bar{Z}_t^{opt} - Z_t(x)}{\bar{Z}_t^{opt} - \underline{Z}_t^{opt}} & \underline{Z}_t^{opt} \le Z_t(x) \end{cases} \tag{20}$$

where the decreasing of $Z_t\,(x)$ will increase the membership degree $\mu_t\,(x)$.

Lemma 1 *In the membership function (20) it always holds that* $\mu_t \le 1$.

Proof Assume that $\mu^{\mathrm{r}} \ge 1$. According to the membership function (20), we get

$$\mu^t(Z_t(x)) \ge 1 \Rightarrow \frac{\bar{Z}_t^{opt} - Z_t(x)}{\bar{Z}_t^{opt} - \underline{Z}_t^{opt}} \ge 1$$
$$\Rightarrow \bar{Z}_t^{opt} - Z_t(x) \ge \bar{Z}_t^{opt} - \underline{Z}_t^{opt} \Rightarrow Z_t(x) \le \underline{Z}_t^{opt}$$

From membership function (20), $Z_t(x) \le \underline{Z}_t^{opt}$ required that $\underline{Z}_t^{opt} \le Z_t(x)$ which contradicts with the optimality of $\underline{Z}_t^{opt}$ Thus, we obtained as $\mu_t \le 1$. □

Also, for maximization type objective, the membership function can be specified with the use of optimal solution (19) as follows:

$$\mu_t(x) = \begin{cases} 1 & Z_t(x) \ge \bar{Z}_t^{opt} \\ \dfrac{Z_t(x) - \underline{Z}_t^{opt}}{\bar{Z}_t^{opt} - \underline{Z}_t^{opt}}, & Z_t(x) \le \bar{Z}_t^{opt} \end{cases} \tag{21}$$

where the increasing of $Z_t(x)$ will increase the membership degree $\mu_t(x)$.

After constructing membership function (20) and/or (21), using the fuzzy decision of Bellman and Zadeh [5], the interval fuzzy programming model of MOPMSTP (13) can be formulated as follows:

$$\begin{cases} \{\mu_1, \mu_2, \ldots, \mu_K\} \to \max \\ \text{s.t.} \begin{cases} \mu_t \le 1, t = 1, 2, \ldots, K, \\ \underline{a}_i^p \le \displaystyle\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk}^p \le \bar{a}_i^p,\ i = 1, 2, \ldots, m; p = 1, 2, \ldots, r, \\ \underline{b}_j^p \le \displaystyle\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk}^p \le \bar{b}_j^p, j = 1, 2, \ldots n; p = 1, 2, \ldots, r, \\ \underline{e}_k^p \le \displaystyle\sum_{p=1}^{r} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^p \le \bar{e}_k^p,\ k = 1, 2, \ldots, l, \\ x_{ijk} \ge 0, \forall p, i, j, k \end{cases} \end{cases} \tag{22}$$

where μ_t, $(t = 1, 2, \ldots, K)$ is an interval functions.

Here, $\mu_t = [\underline{\mu}_t, \bar{\mu}_t] \le 1$ represents the minimum value of the objective function;

it can be defined as follows based on model (14):$\bar{\mu}_t \le 1$ and $\left(\frac{\underline{\mu}_t + \bar{\mu}_t}{2}\right) \le 1,\ t = 1, 2, \ldots, K$

Thus, this problem is transformed into the following deterministic equivalent problem:

$$\begin{cases} \left\{ \displaystyle\sum_{t=1}^{K} \left(\underline{\mu}_t + \left(\frac{\underline{\mu}_t + \bar{\mu}_t}{2} \right) \right) \right\} \to \max \\ \text{s.t.} \begin{cases} \bar{\mu}_t \le 1, t = 1, 2, \ldots, K, \\ \left(\dfrac{\underline{\mu}_t + \bar{\mu}_t}{2} \right) \le 1, t = 1, 2, \ldots, K, \\ \underline{a}_i^p \le \displaystyle\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk}^p \le \bar{a}_i^p, i = 1, 2, \ldots, m; p = 1, 2, \ldots, r, \\ \underline{b}_j^p \le \displaystyle\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk}^p \le \bar{b}_j^p, j = 1, 2, \ldots n; p = 1, 2, \ldots, r, \\ \underline{e}_k^p \le \displaystyle\sum_{p=1}^{r} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^p \le \bar{e}_k^p, k = 1, 2, \ldots, l, \\ x_{ijk} \ge 0, \forall p, i, j, k \end{cases} \end{cases} \tag{23}$$

The complete solution procedure for the MOPMSTP with interval coefficients can be summarized as follows:

- Develop fuzzy MOMISTP (7).
- Check fuzzy MOMISTP (7) is balanced. If not, convert into its balanced one using the Liou and Wang Ranking approach.
- With the use of nearest interval approximation (3), turn it to interval MOMISTP (11).
- Formulate its crisp model (18).
- Solve the crisp MOMISTP as a linear programming problem ignoring all the other objectives.
- Determine the best lower and worst upper bounds (19).
- Define the membership functions (20) and/or (21) of each objective function (13).
- Formulate, interval fuzzy programming model (21) and (23), respectively.
- Solve problem (23) as a single linear programming with the use of the simple equal weighted method. The flow chart of these solution steps is presented in (Fig. 1).

Definition 12 (Hossein Razavi Hajiagha et al. [35], Jiménez and Bilbao [42]) Assume that the feasible set of a fuzzy programming model (22) is X. A $x^0 \in X$ is an efficient solution to the fuzzy problem (22) if there does not exist other solution $x \in X$ such that $\mu_t(x) \ge \mu_t(x_0)$ and $\mu_t(x) \ge \mu_t(x_0)$ at least one index k.

4 A Numerical Example

Let us consider the following numerical example presented by Kundu et al. [16] to illustrate the application of fuzzy MOMIS TP. Data of the problem are as given in Tables 1, 2, 3, 4, and 5.

Here,

$$\sum_{i=1}^{2} \tilde{a}_i^1 = (49, 56, 61, 65); \sum_{i=1}^{2} \tilde{a}_i^2 = (57, 62, 67, 72),$$

$$\sum_{j=1}^{3} \tilde{b}_j^1 = (43, 51, 59, 68); \sum_{j=1}^{3} \tilde{b}_j^2 = (51, 58, 63, 71),$$

$$\sum_{j=1}^{2} \tilde{e}_k = (97, 102, 107, 112).$$

Before solving FSTP, it is needed providing a balance between total supply, demand, and conveyance capacities. By means of adding dummy supply, demand or conveyance point, the balance equality. Using the Liou and Wang ranking approach, we obtain that $\Re\left(\sum_{i=1}^{2} \tilde{a}_i^p\right) \geq \Re\left(\sum_{j=1}^{3} \tilde{b}_j^p\right)$. So to balance the problem add a dummy destination with demands of items 1 and 2 equal to any fuzzy number whose ranks are 2.5 and 3.75, respectively. Here, we add dummy destinations as $\tilde{b}_4^1 =$

Table 1 Unit penalties of transportation in the first objective

Item 1							
Conveyance $k = 1$				Conveyance $k = 2$			
Source	Destination 1	Destination 2	Destination 3	Source	Destination 1	Destination 2	Destination 3
S1	(5, 8, 9, 11)	(4, 6, 9, 11)	(10, 12, 14, 16)	S1	(9, 11, 13, 15)	(6, 8, 10,12)	(7, 9, 12, 14)
S2	(8, 10, 13, 15)	(6, 7, 8, 9)	(11, 13, 15, 17)	S2	(10, 11, 13,15)	(6, 8, 10,12)	(14, 16, 18, 20)

Table 2 Unit penalties of transportation in the first objective

Item 2							
Conveyance $k = 1$				Conveyance $k = 2$			
Source	Destination 1	Destination 2	Destination 3	Source	Destination 1	Destination 2	Destination 3
S1	(9, 10, 12,13)	(5, 8, 10,12)	(10, 11, 12, 13)	S1	(11, 13, 14,15)	(6, 7, 9, 11)	(8, 10, 11, 13)
S2	(11, 13, 14, 16)	(7, 9, 12,14)	(12, 14, 16, 18)	S2	(14, 16, 18, 20)	(9, 11, 13, 14)	(13, 14, 15, 16)

Table 3 Unit penalties of transportation in the second objective

Item 1							
Conveyance $k = 1$				Conveyance $k = 2$			
Source	Destination 1	Destination 2	Destination 3	Source	Destination 1	Destination 2	Destination 3
S1	(4, 5, 7, 8)	(3, 5, 6, 8)	(7, 9, 10, 12)	S1	(6, 7, 8, 9)	(4, 6, 7, 9)	(5, 7, 9, 11)
S2	(6, 8, 9,11)	(5, 6, 7, 8)	(6, 7, 9, 10)	S2	(4, 6, 8, 10)	(7, 9, 11, 13)	(9, 10, 11, 12)

Table 4 Unit penalties of transportation in the second objective

Item 2							
Conveyance $k = 1$				Conveyance $k = 2$			
Source	Destination 1	Destination 2	Destination 3	Source	Destination 1	Destination 2	Destination 3
S1	(5, 7, 9, 10)	(4, 6, 7, 9)	(9, 11, 12, 13)	S1	(7, 8, 9, 10)	(4, 5, 7, 8)	(8, 10, 11, 12)
S2	(10, 11, 13, 14)	(6, 7, 8, 9)	(7, 9, 11, 12)	S2	(6, 8, 10, 12)	(5, 7, 9, 11)	(9, 10, 12, 14)

Table 5 Fuzzy availability and demand data

Fuzzy availability	Interval demand	Interval conveyance capacities
$\tilde{a}_1^1$ =(21, 24, 26, 28)	$\tilde{b}_1^1 = (14, 16, 19, 22)$	$\tilde{e}_1 = (46, 49, 51, 53)$
$\tilde{a}_2^1 = (28, 32, 35, 37)$	$\tilde{b}_2^1 = (17, 720, 22, 25)$	$\tilde{e}_2 = (51, 53, 56, 59)$
$\tilde{a}_1^2 = (32, 34, 37, 39)$	$\tilde{b}_3^1 = (12, 15, 18, 21)$	
$\tilde{a}_2^2 = (25, 28, 30, 33)$	$\tilde{b}_1^2 = (20, 23, 25, 28)$	
	$\tilde{b}_2^2 = (16, 18, 19, 22)$	
	$\tilde{b}_3^2 = (15, 17, 19, 21)$	

$(1, 2, 3, 4)$, $\tilde{b}_4^2 = (2, 3, 4, 6)$ and this produces $\Re\left(\sum_{i=1}^{2} \tilde{a}_i^p\right) = \Re\left(\sum_{j=1}^{3} \tilde{b}_j^p\right) = 122$. However, $\sum_{j=1}^{2} \tilde{e}_k = 104 < 122$ and insert a dummy conveyance having total fuzzy capacity equal to any fuzzy number with rank 17.75. Here, we add dummy conveyance as $\tilde{e}_3 = (15, 16, 19, 21)$.

Thus, the analyzed problem is balanced and consists of two objectives, two items, two sources, four destinations, and three distinct modes of transportation.

Using the nearest interval approximation as given in (3), all fuzzy numbers in data Tables 1, 2, 3, 4 and 5 can be converted into the interval numbers as in the following data Tables 6,7, 8, 9 and 10.

Table 6 Interval transportation costs in the first objective

Item 1							
Conveyance $k = 1$				Conveyance $k = 2$			
Source	Destination 1	Destination 2	Destination 3	Source	Destination 1	Destination 2	Destination 3
S1	[13/2, 10]	[5, 10]	[11, 15]	S1	[10 14]	[7, 11]	[8, 13]
S2	[9, 14]	[13/2, 17/2]	[12, 16]	S2	[21/2, 14]	[7, 11]	[15, 19]

Table 7 Interval Transportation Costs in the first objective

Item 2							
Conveyance $k = 1$				Conveyance $k = 2$			
Source	Destination 1	Destination 2	Destination 3	Source	Destination 1	Destination 2	Destination 3
S1	[19/2, 25/2]	[13/2, 11]	[21/2, 25/2]	S1	[12, 29/2]	[13/2, 10]	[9 12]
S2	[12, 15]	[8, 13]	[13, 17]	S2	[15, 19]	[10, 27/2]	[27/2, 31/2]

Table 8 Interval transportation costs in the second objective

Item 1							
Conveyance $k = 1$				Conveyance $k = 2$			
Source	Destination 1	Destination 2	Destination 3	Source	Destination 1	Destination 2	Destination 3
S1	[9/2, 15/2]	[4 7]	[8, 11]	S1	[5, 8]	[6, 10]	[5, 9]
S2	[7 10]	[11/2, 15/2]	[13/2, 19/2]	S2	[8, 12]	[19/2,23/2]	[6,19/2]

Consequently, the considered fuzzy problem is transformed as follows based on model (13):

$$
Z_1 = \left(\begin{array}{l}
\left[\frac{13}{2},10\right]x^1_{111}+[5,10]x^1_{121}+[11,15]x^1_{131} \\
\quad +[9,14]x^1_{211}+\left[\frac{13}{2},\frac{17}{2}\right]x^1_{221}+[12,16]x^1_{231} \\
\quad +[10,14]x^1_{112}+[7,11]x^1_{122}+[8,13]x^1_{132} \\
+\left[\frac{21}{2},14\right]x^1_{212}+[7,11]x^1_{222}+[15,19]x^1_{232} \\
\quad +\left[\frac{19}{2},\frac{25}{2}\right]x^2_{111}+\left[\frac{13}{2},11\right]x^2_{121}+\left[\frac{21}{2},\frac{25}{2}\right]x^2_{131} \\
\quad +[12,15]x^2_{211}+[8,13]x^2_{221}+[13,17]x^2_{231} \\
+\left[12,\frac{29}{2}\right]x^2_{112}+\left[\frac{13}{2},10\right]x^2_{122}+[9,12]x^2_{132} \\
\quad +[15,19]x^2_{212}+\left[10,\frac{27}{2}\right]x^2_{222}+\left[\frac{27}{2},\frac{31}{2}\right]x^2_{232}
\end{array}\right) \to \min
$$

$$
Z_2 = \left(\begin{array}{l}
\left[\frac{9}{2},\frac{15}{2}\right]x^1_{111}+[4,7]x^1_{121}+[8,11]x^1_{131}+[7,10]x^1_{211} \\
\quad +\left[\frac{11}{2},\frac{15}{2}\right]x^1_{221}+\left[\frac{13}{2},\frac{19}{2}\right]x^1_{231}+\left[\frac{13}{2},\frac{17}{2}\right]x^1_{112} \\
\quad +[5,8]x^1_{122}+[6,10]x^1_{132} \\
+[5,9]x^1_{212}+[8,12]x^1_{222}+\left[\frac{19}{2},\frac{23}{2}\right]x^1_{232} \\
\quad +\left[6,\frac{19}{2}\right]x^2_{111}+[5,8]x^2_{121}+\left[10,\frac{25}{2}\right]x^2_{131} \\
\quad +\left[\frac{21}{2},\frac{27}{2}\right]x^2_{211}+\left[\frac{13}{2},\frac{17}{2}\right]x^2_{221}+\left[8,\frac{23}{2}\right]x^2_{231} \\
+\left[\frac{15}{2},\frac{19}{2}\right]x^2_{112}+\left[\frac{9}{2},\frac{15}{2}\right]x^2_{122}+\left[9,\frac{23}{2}\right]x^2_{132} \\
\quad +[7,11]x^2_{212}+[6,10]x^2_{222}+\left[\frac{19}{2},13\right]x^2_{232}
\end{array}\right) \to \min
$$

$$
\text{s.t.}\begin{cases}
x^1_{111}+x^1_{121}+x^1_{131}+x^1_{141}+x^1_{112}+x^1_{122}+x^1_{132}+x^1_{142}+x^1_{113} \\
\quad +x^1_{123}+x^1_{133}+x^1_{143}\le\left[\frac{45}{2},27\right]; \\
x^1_{211}+x^1_{221}+x^1_{231}+x^1_{241}+x^1_{212}+x^1_{222}+x^1_{232}+x^1_{242} \\
\quad +x^1_{213}+x^1_{223}+x^1_{233}+x^1_{243}\le[30,36], \\
x^2_{111}+x^2_{121}+x^2_{131}+x^2_{141}+x^2_{112}+x^2_{122}+x^2_{132}+x^2_{142}+x^2_{113} \\
\quad +x^2_{123}+x^2_{133}+x^2_{143}\le[33,38]; \\
x^2_{211}+x^2_{221}+x^2_{231}+x^2_{241}+x^2_{212}+x^2_{222}+x^2_{232}+x^2_{242}+x^2_{213} \\
\quad +x^2_{223}+x^2_{233}+x^2_{243}\le\left[\frac{53}{2},\frac{63}{2}\right], \\
x^1_{111}+x^1_{211}+x^1_{112}+x^1_{212}+x^1_{113}+x^1_{213}\ge\left[15,\frac{45}{2}\right] \\
x^1_{121}+x^1_{221}+x^1_{122}+x^1_{222}+x^1_{123}+x^1_{223}\ge\left[\frac{37}{2},\frac{47}{2}\right], \\
x^1_{131}+x^1_{231}+x^1_{132}+x^1_{232}+x^1_{133}+x^1_{233}\ge\left[\frac{27}{2},\frac{49}{2}\right], \\
x^1_{141}+x^1_{241}+x^1_{142}+x^1_{242}+x^1_{143}+x^1_{243}\ge\left[\frac{3}{2},\frac{7}{2}\right], \\
x^2_{111}+x^2_{211}+x^2_{112}+x^2_{212}+x^2_{113}+x^2_{213}\ge\left[\frac{43}{2},\frac{53}{2}\right], \\
x^2_{121}+x^2_{221}+x^2_{122}+x^2_{222}+x^2_{123}+x^2_{223}\ge\left[17,\frac{41}{2}\right], \\
x^2_{131}+x^2_{231}+x^2_{132}+x^2_{232}+x^2_{133}+x^2_{233}\ge[16,10], \\
x^2_{141}+x^2_{241}+x^2_{142}+x^2_{242}+x^2_{143}+x^2_{243}\ge\left[\frac{5}{2},5\right] \\
x^1_{111}+x^1_{121}+x^1_{131}+x^1_{141}+x^1_{211}+x^1_{221}+x^1_{231}+x^1_{241} \\
\quad +x^2_{111}+x^2_{121}+x^2_{131}+x^2_{141}+x^2_{211}+x^2_{221}+x^2_{231} \\
\quad +x^2_{241}\le\left[\frac{95}{2},52\right], \\
x^1_{112}+x^1_{122}+x^1_{132}+x^1_{142}+x^1_{212}+x^1_{222}+x^1_{232}+x^1_{242} \\
\quad +x^2_{112}+x^2_{122}+x^2_{132}+x^2_{142}+x^2_{212}+x^2_{222}+x^2_{232} \\
\quad +x^2_{242}\le\left[52,\frac{115}{2}\right], \\
x^1_{113}+x^1_{123}+x^1_{133}+x^1_{143}+x^1_{213}+x^1_{223}+x^1_{233}+x^1_{243} \\
\quad +x^2_{113}+x^2_{123}+x^2_{133}+x^2_{143}+x^2_{213}+x^2_{223}+x^2_{233} \\
\quad +x^2_{243}\le\left[\frac{31}{2},20\right] \\
x_{ijk}\ge 0,\forall p,i,j,k
\end{cases} \tag{24}
$$

Table 9 Interval transportation costs in the second objective

Item 2							
Conveyance $k = 1$				Conveyance $k = 2$			
Source	Destination 1	Destination 2	Destination 3	Source	Destination 1	Destination 2	Destination 3
S1	[13/2, 17/2]	[5, 8]	[10, 25/2]	S1	[15/2, 17/2]	[9/2, 15/2]	[9, 23/2]
S2	[21/2, 27/2]	[13/2, 17/2]	[8, 23/2]	S2	[7, 11]	[6, 10]	[19/2, 13]

Table 10 Interval availability and demand data

Interval availability	Interval demand	Interval conveyance capacities
$\tilde{a}_1^1 = [45/2, 27]$	$\tilde{b}_1^1 = [15, 41/2]$	$\tilde{e}_1 = [95/2, 52]$
$\tilde{a}_2^1 = [30, 36]$	$\tilde{b}_2^1 = [37/2, 47/2]$	$\tilde{e}_2 = [52, 115/2]$
$\tilde{a}_1^2 = [33, 38]$	$\tilde{b}_3^1 = [27/2, 39/2]$	$\tilde{e}_3 = [31/2, 20]$
$\tilde{a}_2^2 = [53/2, 63/2]$	$\tilde{b}_4^1 = [3/2, 7/2]$	
	$\tilde{b}_1^2 = [43/2, 53/2]$	
	$\tilde{b}_2^2 = [17, 41/2]$	
	$\tilde{b}_3^2 = [16, 20]$	
	$\tilde{b}_4^2 = [5/2, 5]$	

This problem further is transformed into a multi-objective linear programming model as follows:

$$\underline{Z}_1 = \begin{pmatrix} \frac{13}{2}x_{111}^1 + 5x_{121}^1 + 11x_{131}^1 + 9x_{211}^1 + \frac{13}{2}x_{221}^1 \\ +12x_{231}^1 + 10x_{112}^1 + 7x_{122}^1 + 8x_{132}^1 + \frac{21}{2}x_{212}^1 \\ +7x_{222}^1 + 15x_{232}^1 + \frac{19}{2}x_{111}^2 + \frac{13}{2}x_{121}^2 \\ +\frac{21}{2}x_{131}^2 + 12x_{211}^2 + 8x_{221}^2 + 13x_{231}^2 + 12x_{112}^2 \\ +\frac{13}{2}x_{122}^2 + 9x_{132}^2 + 15x_{212}^2 + 10x_{222}^2 + \frac{27}{2}x_{232}^2 \end{pmatrix} \rightarrow \min$$

$$Z_{1c} = \begin{pmatrix} 10x_{111}^1 + 10x_{121}^1 + 15x_{131}^1 + 14x_{211}^1 + \frac{17}{2}x_{221}^1 + 16x_{231}^1 \\ +14x_{112}^1 + 11x_{122}^1 + 13x_{132}^1 + 14x_{212}^1 + 11x_{222}^1 \\ +19x_{232}^1 + \frac{25}{2}x_{111}^2 + 11x_{121}^2 \\ +\frac{25}{2}x_{131}^2 + 15x_{211}^2 + 13x_{221}^2 + 17x_{231}^2 \\ +\frac{29}{2}x_{112}^2 + 10x_{122}^2 \\ +12x_{132}^2 + 19x_{212}^2 + \frac{27}{2}x_{222}^2 + \frac{31}{2}x_{232}^2 \end{pmatrix} \rightarrow \min$$

$$\underline{Z}_2 = \begin{pmatrix} \frac{9}{2}x_{111}^1 + 4x_{121}^1 + 8x_{131}^1 + 7x_{211}^1 + \frac{11}{2}x_{221}^1 \\ +\frac{13}{2}x_{231}^1 + \frac{13}{2}x_{112}^1 + 5x_{122}^1 + 6x_{132}^1 + 5x_{212}^1 \\ +8x_{222}^1 + \frac{19}{2}x_{232}^1 + 6x_{111}^2 + 5x_{121}^2 + 10x_{131}^2 \\ +\frac{21}{2}x_{211}^2 + \frac{13}{2}x_{221}^2 + 8x_{231}^2 + \frac{15}{2}x_{112}^2 \\ +\frac{9}{2}x_{122}^2 + 9x_{132}^2 + 7x_{212}^2 + 6x_{222}^2 + \frac{19}{2}x_{232}^2 \end{pmatrix} \rightarrow \min$$

$$Z_{2c} = \begin{pmatrix} \frac{15}{2}x_{111}^1 + 7x_{121}^1 + 11x_{131}^1 + 10x_{211}^1 + \frac{15}{2}x_{221}^1 \\ +\frac{19}{2}x_{231}^1 + \frac{17}{2}x_{112}^1 + 8x_{122}^1 + 10x_{132}^1 + 9x_{212}^1 \\ +12x_{222}^1 + \frac{23}{2}x_{232}^1 + \frac{19}{2}x_{111}^2 + 8x_{121}^2 + \\ \frac{25}{2}x_{131}^2 + \frac{27}{2}x_{211}^2 + \frac{17}{2}x_{221}^2 + \frac{23}{2}x_{231}^2 + \frac{19}{2}x_{112}^2 \\ +\frac{15}{2}x_{122}^2 + \frac{23}{2}x_{132}^2 + 11x_{212}^2 + 10x_{222}^2 + 13x_{232}^2 \end{pmatrix} \rightarrow \min$$

$$
\text{s.t.}\left\{\begin{array}{l}
\frac{45}{2} \le x^1_{111} + x^1_{121} + x^1_{131} + x^1_{141} + x^1_{112} + x^1_{122} + x^1_{132} \\
\quad + x^1_{142} + x^1_{113} + x^1_{123} + x^1_{133} + x^1_{143} \le 27, \\
30 \le x^1_{211} + x^1_{221} + x^1_{231} + x^1_{241} + x^1_{212} + x^1_{222} + x^1_{232} + x^1_{242} \\
\quad + x^1_{213} + x^1_{223} + x^1_{233} + x^1_{243} \le 36, \\
33 \le x^2_{111} + x^2_{121} + x^2_{131} + x^2_{141} + x^2_{112} + x^2_{122} + x^2_{132} + x^2_{142} \\
\quad + x^2_{113} + x^2_{123} + x^2_{133} + x^2_{143} \le 38, \\
\frac{53}{2} \le x^2_{211} + x^2_{221} + x^2_{231} + x^2_{241} + x^2_{212} + x^2_{222} + x^2_{232} \\
\quad + x^2_{242} + x^2_{213} + x^2_{223} + x^2_{233} + x^2_{243} \le \frac{63}{2}, \\
15 \le x^1_{111} + x^1_{211} + x^1_{112} + x^1_{212} + x^1_{113} + x^1_{213} \le \frac{45}{2} \\
\frac{37}{2} \le x^1_{121} + x^1_{221} + x^1_{122} + x^1_{222} + x^1_{123} + x^1_{223} \le \frac{47}{2}, \\
\frac{27}{2} \le x^1_{131} + x^1_{231} + x^1_{132} + x^1_{232} + x^1_{133} + x^1_{233} \le \frac{49}{2}, \\
\frac{3}{2} \le x^1_{141} + x^1_{241} + x^1_{142} + x^1_{242} + x^1_{143} + x^1_{243} \le \frac{7}{2}, \\
\frac{43}{2} \le x^2_{111} + x^2_{211} + x^2_{112} + x^2_{212} + x^2_{113} + x^2_{213} \le \frac{53}{2}, \\
17 \le x^2_{121} + x^2_{221} + x^2_{122} + x^2_{222} + x^2_{123} + x^2_{223} \le \frac{41}{2}, \\
16 \le x^2_{131} + x^2_{231} + x^2_{132} + x^2_{232} + x^2_{133} + x^2_{233} \le 20, \\
\frac{5}{2} \le x^2_{141} + x^2_{241} + x^2_{142} + x^2_{242} + x^2_{143} + x^2_{243} \le 5, \\
\frac{95}{2} \le x^1_{111} + x^1_{121} + x^1_{131} + x^1_{141} + x^1_{211} + x^1_{221} + x^1_{231} \\
\quad + x^1_{241} + x^2_{111} + x^2_{121} + x^2_{131} + x^2_{141} + x^2_{211} + x^2_{221} \\
\quad + x^2_{231} + x^2_{241} \le 52, \\
52 \le x^1_{112} + x^1_{122} + x^1_{132} + x^1_{142} + x^1_{212} + x^1_{222} + x^1_{232} \\
\quad + x^1_{242} + x^2_{112} + x^2_{122} + x^2_{132} + x^2_{142} + x^2_{212} + x^2_{222} + x^2_{232} \\
\quad + x^2_{242} \le \frac{115}{2}, \\
\frac{31}{2} \le x^1_{113} + x^1_{123} + x^1_{133} + x^1_{143} + x^1_{213} + x^1_{223} + x^1_{233} \\
\quad + x^1_{243} + x^2_{113} + x^2_{123} + x^2_{133} + x^2_{143} + x^2_{213} + x^2_{223} \\
\quad + x^2_{233} + x^2_{243} \le 20, \\
x_{ijk} \ge 0, \forall p, i, j, k
\end{array}\right. \tag{25}
$$

Solving the $\underline{Z}_1, Z_{1c}, \underline{Z}_2$ and Z_{2c} problems individually, by the suggested method, will result in $Z_1^{\text{opt}} = [815.450, 960.750]$and $Z_2^{\text{opt}} = [611.625, 747.000]$. Now, the membership functions of objective functions are constituted as follows:

$$
\mu_1(x) = \begin{cases} 1 & Z_1(x) \le 815.450 \\ \dfrac{960.750 - Z_1(x)}{960.750 - 815.450}, & 815.450 \le Z_1(x) \end{cases}
$$

3

and $\mu_2(x) = \begin{cases} 1 & Z_2(x) \le 611.625 \\ \dfrac{747 - Z_2(x)}{747 - 611.625}, & 611.625 \le Z_2(x) \end{cases}$

Consequently, the problem is transformed as follows based on model (22):

$$
\left(\frac{960.750 - Z_1(x)}{960.750 - 815.450}, \frac{747 - Z_2(x)}{747 - 611.625} \right) \to \max
$$

$$
\text{s.t.}\left\{\begin{array}{l}
\frac{960.750 - Z_1(x)}{960.750 - 815.450} \le 1; \frac{747 - Z_2(x)}{747 - 611.625} \le 1 \\
\frac{45}{2} \le x^1_{111} + x^1_{121} + x^1_{131} + x^1_{141} + x^1_{112} + x^1_{122} + x^1_{132} \\
\quad + x^1_{142} + x^1_{113} + x^1_{123} + x^1_{133} + x^1_{143} \le 27, \\
30 \le x^1_{211} + x^1_{221} + x^1_{231} + x^1_{241} + x^1_{212} + x^1_{222} + x^1_{232} \\
\quad + x^1_{242} + x^1_{213} + x^1_{223} + x^1_{233} + x^1_{243} \le 36, \\
33 \le x^2_{111} + x^2_{121} + x^2_{131} + x^2_{141} + x^2_{112} + x^2_{122} + x^2_{132} \\
\quad + x^2_{142} + x^2_{113} + x^2_{123} + x^2_{133} + x^2_{143} \le 38, \\
\frac{53}{2} \le x^2_{211} + x^2_{221} + x^2_{231} + x^2_{241} + x^2_{212} + x^2_{222} + x^2_{232} \\
\quad + x^2_{242} + x^2_{213} + x^2_{223} + x^2_{233} + x^2_{243} \le \frac{63}{2}, \\
15 \le x^1_{111} + x^1_{211} + x^1_{112} + x^1_{212} + x^1_{113} + x^1_{213} \le \frac{45}{2} \\
\frac{37}{2} \le x^1_{121} + x^1_{221} + x^1_{122} + x^1_{222} + x^1_{123} + x^1_{223} \le \frac{47}{2}, \\
\frac{27}{2} \le x^1_{131} + x^1_{231} + x^1_{132} + x^1_{232} + x^1_{133} + x^1_{233} \le \frac{49}{2}, \\
\frac{3}{2} \le x^1_{141} + x^1_{241} + x^1_{142} + x^1_{242} + x^1_{143} + x^1_{243} \le \frac{7}{2}, \\
\frac{43}{2} \le x^2_{111} + x^2_{211} + x^2_{112} + x^2_{212} + x^2_{113} + x^2_{213} \le \frac{53}{2}, \\
17 \le x^2_{121} + x^2_{221} + x^2_{122} + x^2_{222} + x^2_{123} + x^2_{223} \le \frac{41}{2}, \\
16 \le x^2_{131} + x^2_{231} + x^2_{132} + x^2_{232} + x^2_{133} + x^2_{233} \le 20, \\
\frac{5}{2} \le x^2_{141} + x^2_{241} + x^2_{142} + x^2_{242} + x^2_{143} + x^2_{243} \le 5, \\
\frac{95}{2} \le x^1_{111} + x^1_{121} + x^1_{131} + x^1_{141} + x^1_{211} + x^1_{221} + x^1_{231} + x^1_{241} \\
\quad + x^2_{111} + x^2_{121} + x^2_{131} + x^2_{141} + x^2_{211} + x^2_{221} + x^2_{231} \\
\quad + x^2_{241} \le 52, \\
52 \le x^1_{112} + x^1_{122} + x^1_{132} + x^1_{142} + x^1_{212} + x^1_{222} + x^1_{232} + x^1_{242} \\
\quad + x^2_{112} + x^2_{122} + x^2_{132} + x^2_{142} + x^2_{212} + x^2_{222} + x^2_{232} \\
\quad + x^2_{242} \le \frac{115}{2}, \\
\frac{31}{2} \le x^1_{113} + x^1_{123} + x^1_{133} + x^1_{143} + x^1_{213} + x^1_{223} + x^1_{233} + x^1_{243} + x^2_{113} + x^2_{123} \\
+ x^2_{133} + x^2_{143} + x^2_{213} + x^2_{223} + x^2_{233} + x^2_{243} \le 20, \\
x_{ijk} \ge 0, \ \forall p, i, j, k
\end{array}\right. \tag{26}
$$

Table 11 Comparison of solutions by different methods

Different methods			
Objective functions	Kundu et al. [16]	Hajiagha et al. [35]	Our suggested method
Z_1	[1043.019, 1233.229]	Infeasible	[682.000, 978.250]
Z_2	[745.5609, 930.706]		[497.500, 768.000]

The model (26) is an interval linear programming model which can be solved by first finding its optimum lower bound and center. Then, a multi-objective function is constructed and is solved with a simple weighted model with equal weights. The optimal solution is obtained as

$$x^1_{111} = 13,\ x^1_{132} = 13.5, x^1_{212} = 3,\ x^1_{221} = 23.5,$$
$$x^1_{242} = 3.5,\ x^2_{111} = 11,$$
$$x^2_{122} = 20.5,\ x^2_{132} = 5, x^2_{133} = 4.,\ x^2_{212} = 1.5,$$
$$x^2_{213} = 9, x^2_{233} = 11, x^2_{242} = 5.$$

$Z_1^{opt} = [682.000, 978.250]$ and $Z_2^{opt} = [497.500, 768.000]$. From Definition 11, the suggested method performed a preferred solution in both objectives.

Kundu et al. [16] obtained the solution of the fuzzy MOMISTP with the use of the minimum fuzzy number approach. Comparative results are given in the following Table 11.

5 Conclusion

In the real-world applications, we are frequently faced with the uncertainty factor due to lack of detecting data about the unknown state of nature. Thus, this paper presented some useful notions to handle the MOMISTP, where the supplies, destinations, conveyance capacities, and transportation costs are supposed to be fuzzy parameters.

At first, using the nearest interval approximation, the fuzzy MOMISTP is transformed to the MOMISTP in which the parameters have interval values. Then the interval MOMISTP is converted into its crisp form, and compromise programming model based on the best and worst solutions is constructed and solved. Finally, the method attempts to reach the better compromise solution which simultaneously satisfied different objectives based on the interval fuzzy programming model.

Consequently, application of the suggested method is discussed with a numerical model and the effectiveness of the solutions obtained by the suggested method is verified. Moreover, from Table 11, our suggested approach gives a more efficient solution comparing to the approaches of Kundu et al., [16]. Furthermore, this method can be modeled as fuzzy goal or fuzzy interactive satisfied method. Hence, this model may be used in various uncertain environments with different factors.

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