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ORIGINAL ARTICLE

Application of the Dupuit–Forchheimer model to groundwater flow into a well

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Abstract

Existing models are built upon to develop new ones. As a foundational model in porous media flow, the Darcy flow model has been built upon by many researchers. The groundwater flow equation evolved from the Darcy equation. Dupuit–Forchheimer built upon it to develop the simplified forms for both the confined and unconfined aquifers flow, usable for studying groundwater flow into wells. Aside from height, permeability, and availability of water in the aquifers, other factors influence groundwater flow into wells, as enshrined in the flow equation. This paper investigates the roles of storability, hydraulic conductivity, and source/sink strength in both confined and unconfined groundwater flow into wells using the Dupuit–Forchheimer assumption. In this model, the Dupuit–Forchheimer pressure assumption is substituted into the groundwater flow equations and solved using the Bessel form for separation of variable technique, and Mathematica 11.2 computational software to obtain the expressions for the pressure, which are computed and presented quantitatively. The results show that an increase in the hydraulic conductivity and storability have no effect on the flow pressures in the confined and unconfined aquifers but cause fluctuation in the pressure structure in the unconfined aquifer; the source/sink strength factor causes fluctuation in the pressure structures in both confined and unconfined aquifers flow. However, in both confined/unconfined aquifers the pressures increase as the radii of the wells increase. Importantly, the fluctuation in the pressure structures causes a loss of energy for groundwater flow into the wells.

Keywords Storability · Source/sink strength · Hydraulic conductivity · Groundwater · Dupuit–Forcheheimer

Introduction

Modelling of groundwater flow has application in drainage, dam stability, management of landslides, etc.

Depending on where and how it is formed, groundwater has many facets. Some from meteoric and snow melts, and these penetrate the soil, cracks and faults to form saturated water zones called aquifers/water tables; some called connate waters are associated with granulated sand/sediments in petroleum reservoirs, and some called magmatic/juvenile water, released into the atmosphere during a volcanic eruption is associated magma of the Earth crust.

Specifically, the aquifers are geologic formations and are classified into: unconfined and confined. While the unconfined aquifers lie below permeable layers of soil, the confined lie below impermeable layers of rock or clay. The water in the confined aquifers is under high pressure. In the presence of leakages, semi-confined aquifers may be formed. More so, there are other offshoots of the unconfined and confined aquifers and are called perched aquifers. They are separated from the main aquifers by unsaturated rocks called aquicludes and are non-rechargeable. Aside from this, the perched aquifers are characterized by interconnected spaces by which water moves through the formations. Furthermore, water tables may be deep or shallow; rise or fall, depending on the amount of recharge or discharge/usage. Similarly, their rates of flow depending on the size and connectivity of the pores (porosity), hydraulic conductivity (a measure

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of the ability to transmit fluid through pores, spaces and fractures in the presence of hydraulic gradient), hydraulic gradient (a change in the hydraulic head) and strength of the hydraulic head**.** Groundwater flows from the region of higher hydraulic head to a lower one; from the region of higher elevation to the lower one, and from the region of high pressure to a lower one.

Being in pores, cracks and fractures the flow rate of groundwater is slow compared to surface flow. Therefore, its Reynolds number is prescribed to be less than unity or not above 10. However, in limestone karst formations, where groundwater flows through caves and large solution channels, its flow is not slow. The flow is either steady, where the hydraulic head is independent of time or transient, when the hydraulic head and aquifer boundary conditions are dependent on time, as in changing aquifer recharge.

Importantly, when wells are borne into the aquifers, water flows from the aquifers to fill them to points or levels where the water in the two regions is in equilibrium. With the discharge of water from the wells, the one in the aquifers flows into them to strike equilibrium. Therefore, there exists a discharge-recharge relation in groundwater flow. The recharge of aquifers is dependent on the amount of rain and snow-falls, which are seasonal. So, in the dry season, the aquifers are not recharged, and with constant usage of the water the depths of the water tables drop, and the water level of the wells goes down.

Geophysically, before water bore-holes (wells) are drilled, aquifers are delineated to locate the water-saturated ones using up-hole, down-hole, seismic reflection and refraction approaches. Aquifer delineation is cost-effective, as it saves efforts and resources that would have been exerted on drilling non-water yielding and sustaining wells. After delineation, bore-holes/wells of different shapes and sizes; depending on what one's choice are drilled into prospective aquifers. The commonest one is the cylindrical type. A bore casing made of PVC pipes and a screen (that filters out unwanted particles around the pipe) are installed. Water in a well can be brought to the surface through a pipe and a pump. However, wells bore into confined aquifers (or artesian wells) do not need a pump; as they have natural pressures for that.

This work hinges on the groundwater flow into a drilled well to study the factors accounting for the sustainability of the well using a modelling approach. Models are used to predict the state of any system and the effects of malfunctioning due to some existing situations in the system. In particular, groundwater modelling enables us to determine heads, the maximal values for the hydraulic conductivity and recharge values for different platforms in an environment; simulates responses of the aquifer under hypothetical situations. Many models and software packages exist on groundwater modelling. For example, for models, we have chemical, which predicts the quality and movement of pollutants in the groundwater in an environment; analogue and mathematical models. More so, there are modelling/simulation packages. Among these are the MODFLOW, which uses finite difference, and FEFLOW, which uses the finite element method. As a mathematical representation of flow through an aquifer under hypothetical situations, groundwater flow modelling approaches involve numerical and analytical techniques.

Studies have been carried out on groundwater flow. Some on simulation using software, others on modelling. For example, Philips (1970) numerically studied groundwater flow; Verma and Mishra (1973) analytically investigated a one-dimensional groundwater recharge flow using similarity transformation approach; Prickett (1975) gave an in-depth discussion on the approaches for groundwater evaluation; Swaroop and Mehta (2001), considered a one-dimensional flow in unsaturated porous media using finite element method; Patel et al. (2012) investigated used the method of series solutions in vertical groundwater flow through an unsaturated heterogeneous porous medium. Similarly, Mehdinejadiani et al. (2013) examined analytically the unsteady one-dimensional model of water-table profile between two parallel sub-surfaces using the Glover–Dumm's model; using conformal mapping, David et al. (2015) investigated the groundwater flow in an aquifer containing an extraction or recharge cavity of arbitrary shape; Vázquez-Báez et al. (2019) numerically studied the Ayamonte-Huelva aquifer bringing to bare the homogenous and isotropic, and inhomogeneous and anisotropic characteristics. More so, while Thangarajan (2007) and Szczepinski (2019) dwelt on the relevance of modelling groundwater flow and its environmental impact, Zhou and Li (2011) and Mussa et al. (2020) gave elaborate state-of-the-art reviews on the modelling of groundwater flow. Similarly, using the Dupuit assumption, Mehta (1975) examined groundwater flow by the method of singular perturbation technique; Shreekant and Twinkle (2014) considered a one-dimensional groundwater recharge flow in an inclined porous media. Also, by the Forchheimer model approach, Shi et al. (2018) numerically studied the groundwater flow through a fault; Mwetulundila and Atangana (2020) investigated the flow recharge through a fractured aquifer. Employing the Dupuit–Forchheimer model of groundwater flow, Strack (1995) investigated a 3-D flow under a non-constant density; Haitjema et al. (2011) examined the flow into a horizontal well; Srinivasan (2016) investigated the relationship through porous media between flux (velocity) and the driving using the Barus formula for viscosity; Zerihun (2018) studied the effects of streamlined curvature on both homogeneous and isotropic flow and non-homogeneous phreatic flow in an unconfined aquifer, but failed to consider the problem when the conductivities of the layers are different.

Haitjema et al. (2011) examined the flow into a horizontal well using Dupuit–Forchheimer model. In their model, the well sustainability situation was neglected. Therefore, the objective of the paper is to investigate the flow of groundwater in relation to the factors accounting for the stainability of wells in aquifers, both confined and unconfined using the Dupuit–Forchheimer assumption, as a simplifier. In effect, we shall examine the roles of hydraulic conductivity, storability and source/sink strength on the groundwater flow into wells.

Mathematical foundation of groundwater flow modeling

With modelling as an integral art, models are built upon one another in the quest for simplicity, improvement and finding solutions to physical problems in similar domains. For example, Darcy, by his experimental law, on which the foundation of porous media flow and hydrogeology is laid, prescribed the flow rate q (m^3/s) of fluid in porous media as,

$$q = \frac{-\kappa A}{\mu L}\nabla p,$$

where κ is the permeability (m^2) of the porous medium, A is the cross-sectional area of the porous media; μ is the fluid viscosity ($Pa.s$), L is the length of the porous medium sample (m), p is the pressure (Paschal or N/m^2), and ∇p is the pressure drop or change in pressure over a distance. By the hydrostatic pressure assumption (Steven's law), we have

$$p = \rho g d,$$

such that the Darcy law becomes,

$$q = \frac{-\kappa A g}{\upsilon L}\nabla d,$$

where ρ is the fluid density, g is the acceleration due to gravity and d is the altitude.

The common form of the Darcy law is given as,

$$q = -\frac{\kappa}{\mu}\nabla p,$$

and is derived from the Navier–Stokes equation. More so, the hydraulic conductivity κ (or Darcy flux or Darcy velocity (not the velocity of the fluid in the pores)) is defined as

$$K = \frac{\upsilon L}{\kappa A g},$$

where υ is the kinematic viscosity.

Also, the fluid velocity is prescribed as

$$u = \frac{q}{\chi^2},$$

where χ^2 is the porosity of the porous medium.

The propelling forces in the Darcy law are gravity and pressure and are influenced by the viscous resistance of the porous media. Since the pressure gradient in the flow occurs from high pressure towards a lower one in the opposite direction, a negative sign is associated with the Darcy law. It is applicable where the flow is laminar and Reynolds number is less than or equal to unity. The Darcy law has relevance in many engineering situations that may not agree with its original assumptions. This calls for the modification of the law. Several models have been built upon it for an extension. For example, Brinkman (1949) introduced the Brinkman terms into it to obtain

$$\mu_1\nabla^2 q + q = -\frac{\kappa}{\mu}\nabla p,$$

where μ_1 is the effective viscosity. The Brinkman terms cater for transitional flow at the boundaries, where the grains of the media are porous. Studies have shown that the model is difficult to apply.

Importantly, by the equation of the fluid velocity, the Darcy equation, in its commonest form is

$$-\nabla p = \frac{\mu}{\kappa}u.$$

It is neither applicable to the flow into a well at a high pumping rate nor to flow in aquifers and faults (Mathias and Todman 2010). Attempting to cater for the flow in aquifers and fault, Forchheimer extended the Darcy equation by introducing $\gamma\rho u^2$ (the Forchheimer term) to obtain the Darcy-Forchheimer equation

$$-\nabla p = \frac{\mu}{\kappa}u + \gamma\rho u^2.$$

where γ is the non-Darcy coefficient, which describes the nature of the porous media, and is prescribed as a function of porosity $\left(\chi^2 = \frac{\mu}{\kappa}\right)$ and permeability κ (Yao et al. 2015). Its basic assumptions are: that the flow is macroscopic and one-dimensional; the aquifer is homogeneous; the porous medium is isotropic; the groundwater is incompressible with constant density and velocity. More so, the non-Darcy coefficient is prescribed as

$$\gamma = \omega\kappa^n,$$

where ω is a constant and n is an index (Chen et al. 2015). For engineering purposes, this is empirically given as

$$\gamma = \frac{m}{\sqrt{\kappa}},$$

where m is called the Forchheimer coefficient (or drag coefficient of the fluid flowing past the porous media) depends on the porous media internal structure (Dukhan and Patel 2011). By a further extension, Dupuit–Forchheimer modified the Darcy law as

$$-\left(m\|u\| + \frac{\mu}{\kappa}u\right) = \nabla p,$$

where m (the Forchheimer coefficient) is a positive constant (see, Srinivasan 2016).

Similarly, the groundwater flow equation is prescribed as

$$S\frac{\partial h}{\partial t} = -\nabla \cdot q - G,$$

where h is the hydraulic head, S is the storability of the aquifer and G the source/sink term. This is similar to the heat conduction in the solids equation. With the hydraulic head given by Darcy's law as

$$q = -\kappa \nabla h,$$

the groundwater equation becomes

$$\frac{\partial h}{\partial t} = \beta \nabla^2 h - N,$$

where $\beta = \frac{\kappa}{S}$ is called the hydraulic diffusivity; $N = \frac{G}{S}$ is the source/sink term. Its steady case is similar to the potential or Poisson equation. The assumptions bounding the use of the groundwater equation are: that the aquifer materials are incompressible (no change of matrix with respect to pressure change-subsidence); the external load on the aquifer such as overburden and atmospheric pressure are constant; the aquifer is non-leaky; groundwater is incompressible, its hydraulic conductivity is an isotropic scalar, and its flow is slow with a Reynolds number less than unity, and in a horizontal direction.

As in the case of Darcy law, the groundwater equation is for flow in rock matrixes where the Reynolds number is less than or equal to unity, as such the advection term is zero and flow is slow. Therefore, for application in other areas within the domain, as in the flow in faults and fractures (non-Darcy), where $1 \leq$ Reynolds number < 10, the groundwater equation has to be modified to cater for such situations (Mwetulundila and Atangana 2020; Skejetne and Auriault 1996).

The groundwater flow equation has been modified and prescribed for the flow in the rock matrix as

$$S_m\frac{\partial h}{\partial t} = DK_m\frac{\partial^2 h_m}{\partial x^2} + V_r,$$

where,

$$V_r = \xi T_m\left(h_f - h_m\right),$$

is the fluid exchange rate between the porous media and the fractures, D is the depth of the elementary volume, K_m is the hydraulic conductivity, S_m is the storage coefficient in matrix rocks; h as the hydraulic head, f represents the fracture, m represents the rock matrix,$T_m = DK_m$ is the transmissivity of the rock matrix, ξ is the fluid transfer parameter (see, Mwetulundila and Atangana 2020).

As a further extension, the Forchheimer equation, prescribed as

$$F = (\alpha + \eta q)q = F(q)q,$$

where α and η are Forchheimer linear and non-linear parameters, with α depending on the fluid properties, and η on the media properties like porosity (Venkataraman and Rao 1996) was modelled into the groundwater equation to obtain,

$$S_f\frac{\partial h}{\partial t} = \frac{D}{F(q)}\frac{\partial^2 h_f}{\partial x^2} + V_r,$$

flow equation for impermeable fault, where S_f is the storage coefficient in the fault (see, Mwetulundila and Atangana 2020).

In a similar development, when the rock matrix is impermeable, there will be no groundwater within the matrix; therefore, no storage exists. As such, the linear parameter of the Forchheimer equation becomes zero and the Forchheimer equation reduces to $F = \eta q$. More so, with no water exchange between the impermeable rock matrix and the fracture, the fluid transfer rate V_r becomes zero, and the governing groundwater flow equation for the fracture reduces to,

$$\frac{\partial h_f}{\partial t} = \frac{D}{\eta q S_f}\frac{\partial^2 h_f}{\partial x^2}.$$

Still on the furtherance of the modification and extension of the groundwater flow equation, Dupuit–Forchheimer, taking pressure as the hydraulic head, assumed it as a function of height d, density and gravity such that,

$$p(z) = -\rho g d.$$

The duo assumed that the water table is relatively flat, groundwater is hydrostatic (i.e. equipotential lines are vertical), flow is under gravity influence, and the vertical curvature of the streamline is negligible.

Physics of problem and mathematical formulation

A mathematical model of groundwater flow into a well using the Dupuit–Forchheimer assumption is now considered for a two-dimensional flow. We assumed the pores of the aquifer into which the well is bored are cylindrical and the flow velocity is symmetrical about the θ-axis. Then, if (r, z, t) are the spatial radial and z-coordinates, p is the pressure (hydraulic head), the governing equations of mass balance and groundwater flow into a cylindrical well are prescribed as

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial p}{\partial t} = \alpha H\left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2}\right) - N, \tag{2}$$

for the confined aquifers, and,

$$\frac{\partial p^2}{\partial t} = \alpha H\left(\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r}\frac{\partial p^2}{\partial r} + \frac{\partial^2 p^2}{\partial z^2}\right) - N, \tag{3}$$

for the unconfined aquifers with the boundary conditions,

$$t > 0: \quad u = o, w = 0, p = p_\infty \text{ at } r = 0, \tag{4}$$

$$p = p_w \text{ at } r = 1, \tag{5}$$

where $p_\infty < p_w$, $\alpha = \frac{K}{S}$, $N = \frac{G}{S}$, $p_\infty < p_w$, p_∞ is the equilibrium pressure and p_w pressure at the wall, K is the hydraulic conductivity (m/s), G is the source/sink strength, S is the storability/specific yield of the aquifer, H is the height of the aquifer, A is the area, $g$is the gravity force, L is the channel length, H is the height difference between the aquifer and the well.

By the Dupuit–Forchheimer assumption, the pressure/head depends on the height/elevation (the distance between the bottom and the top of the well, and is taken as d or z). Therefore,

$$p(z) = -\rho g d, \; p^2(z) = \rho^2 g^2 d^2, \tag{6}$$

where ρ is the density of groundwater.

By (6) Eqs. (2) and (3) reduces to a one-dimensional unsteady radial flow of the form

$$\frac{\partial p}{\partial t} = \alpha H\left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r}\right) - N, \tag{7}$$

for the confined aquifers, and,

$$\frac{\partial p^2}{\partial t} = \alpha H\left(\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r}\frac{\partial p^2}{\partial r} + 2\rho^2 g^2\right) - N, \tag{8}$$

for the unconfined aquifers,

where,

$$p = p(r, t). \tag{9}$$

Also, Eq. (1) is satisfied by the boundary condition.

Methodology

Solving Eqs. (7) and (8) under the specified boundary conditions, Eqs. (4) and (5), we seek for Bessel technique method of separation of variables of the form:

$$p = W(r)T(t), p^2 = W^2(r)T^2(t). \tag{10}$$

Substituting Eq. (9) appropriately into Eqs. (7) and (8), respectively and equating the result to $-\lambda^2$ (a constant), we have

$$T' + T\lambda^2 = 0, \tag{11}$$

$$W'' + \frac{1}{r}W' + \left(\frac{\lambda^2}{\alpha H}\right)W = \frac{N}{\alpha H}, \tag{12}$$

with the boundary conditions,

$$T = 0 \text{ at } r = 0, \tag{13}$$

$$W = p_\infty \text{ at } r = 0, \tag{14}$$

$$W = p_w \text{ at } r = 1, \tag{15}$$

for the confined aquifer, and,

$$T = -\frac{2}{\lambda^2}, \tag{16}$$

$$\begin{aligned}&\left(2\alpha H T^2\right)W' + \left(2\alpha H T^2\right)W + \left(T^2\lambda^2\right)W^2 \\ &= -\left(2\alpha H \rho^2 g^2 - N\right).\end{aligned} \tag{17}$$

Furthermore, substituting Eq. (15) into Eq. (16) gives

$$\left(\frac{8\alpha H}{\lambda^4}\right)W' + \frac{1}{r}\left(\frac{8\alpha H}{\lambda^4}\right)W + \left(\frac{4}{\lambda^2}\right)W^2 = N - 2\alpha H \rho^2 g^2, \tag{18}$$

with the boundary condition

$$W = p_w \text{ at } r = 1, \tag{19}$$

for the unconfined aquifer.

Equations (10)–(14), (17) and (18) are solved using Mathematica 11.2 computational software.

Results

Studies have shown that the flow of water from the aquifer into wells and the sustainability of the wells depend on the hydraulic conductivity (K), storability (S) and source/sink strength (G) of the aquifer. Also, it is prescribed that the range of values for the storability of an aquifer is 10^{-5} to 10^{-3} units for a confined aquifer and 0.01–0.03 units for an unconfined aquifer. Transmissivity, which is the product of the hydraulic conductivity (K) and thickness (b) of the aquifer, is dependent on the porosity of the aquifer rocks and is prescribed as < 1000 gallons/day/ft for domestic/low yielding wells, and > 1000 gallons/day/ft for industrial/agricultural producing wells. The dependency of the well sustainability on those factors is investigated. For constant values of $H=10$, $\lambda^2=0.3$, $\rho=0.9$, $g=10$, $p_w=2$, and for varied values of $K=0.2, 0.4, 0.6, 0.8, 1.0, 1.2$; $G=0.1, 0.3, 0.5, 1.0, 1.5, 2.0$; $S=0.00001, 0.00003, 0.0001, 0.0003, 0.001, 0.003$ for confined aquifer; $S=0.01, 0.015, 0.02, 0.025, 0.03$ for unconfined aquifers, Tables 1, 2, 3, 4, 5 and 6 are obtained.

Discussion

Table 1 shows that an increase in the hydraulic conductivity does not affect the flow pressure in the confined aquifers. Naturally, the hydraulic conductivity of water-bearing layers is a proportionality constant, influenced by the variation in density and viscosity of the groundwater. Groundwater density is affected by the content of the water-layer materials. While density decreases with the increase in pressure, temperature and concentration of the dissolved solids, viscosity decrease with the increase in the water temperature. The lower the density, the easier the water flows through the pores. As seen, the pressure is constant, despite the variation in the hydraulic conductivities; however, it increases with

Table 1 Pressure-hydraulic conductivity for a confined aquifer

p	$p(K=0.2)$ Indeterminate	$p(K=0.4)$ Indeterminate	$p(K=0.6)$ Indeterminate	$p(K=0.8)$ Indeterminate	$p(K=1.0)$ Indeterminate	$p(K=1.2)$ Indeterminate
0.5	0.5495	0.5495	0.5495	0.5495	0.5495	0.5495
1.0	0.7358	0.7358	0.7358	0.7358	0.7358	0.7358
1.5	1.0312	1.0312	1.0312	1.0312	1.0312	1.0312
2.0	1.4391	1.4391	1.4391	1.4391	1.4391	1.4391
2.5	1.9606	1.9606	1.9606	1.9606	1.9606	1.9606
3.0	2.5960	2.5960	2.5960	2.5960	2.5960	2.5960

Table 2 Pressure–hydraulic conductivity (K) relation for the unconfined aquifers

p	$p(K=0.2)$ Indeterminate	$p(K=0.4)$ Indeterminate	$p(K=0.6)$ Indeterminate	$p(K=0.8)$ Indeterminate	$p(K=1.0)$ Indeterminate	$p(K=1.2)$ Indeterminate
0.5	2.3506	2.3506	2.3506	2.3506	2.3506	2.3506
1.0	0.7358	0.7358	0.7358	0.7358	0.7358	0.7358
1.5	0.1400	0.1400	0.1400	0.1400	0.1400	0.1400
2.0	0.2486	0.2486	0.2486	0.2486	0.2486	0.2486
2.5	0.5782	0.5782	0.5782	0.5782	0.5782	0.5782
3.0	0.9032	0.9032	0.9032	0.9032	0.9032	0.9032

Table 3 Pressure-storability (S) relation for a confined aquifer

p	$p(S=0.00001)$ Indeterminate	$p(S=00{,}003)$ Indeterminate	$p(S=0.0001)$ Indeterminate	$p(S=0.0003)$ Indeterminate	$p(S=0.001)$ Indeterminate	$p(S=0.003)$ Indeterminate
0.5	0.5495	0.5495	0.5495	0.5495	0.5495	0.5495
1.0	0.7358	0.7358	0.7358	0.7358	0.7358	0.7358
1.5	1.0312	1.0312	1.0312	1.0312	1.0312	1.0312
2.0	1.4391	1.4391	1.4391	1.4391	1.4391	1.4391
2.5	1.9606	1.9606	1.9606	1.9606	1.9606	1.9606
3.0	2.5960	2.5960	2.5960	2.5960	2.5960	2.5960

Table 4 Pressure–storability relation in an unconfined aquifer

p	$p(S=0.00001)$ Indeterminate	$p(S=00{,}003)$ Indeterminate	$p(S=0.0001)$ Indeterminate	$p(S=0.0003)$ Indeterminate	$p(S=0.001)$ Indeterminate	$p(S=0.003)$ Indeterminate
0.5	2.0759	2.0759	2.0759	2.0759	2.0759	2.0759
1.0	0.7358	0.7358	0.7358	0.7358	0.7358	0.7358
1.5	0.1551	0.1551	0.1551	0.1551	0.1551	0.1551
2.0	0.2357	0.2357	0.2357	0.2357	0.2357	0.2357
2.5	0.5508	0.5508	0.5508	0.5508	0.5508	0.5508
3.0	0.8280	0.8280	0.8280	0.8280	0.8280	0.8280

Table 5 Pressure-source/sink strength (G) relation for the confined aquifer

p	$p(G=0.1)$ Indeterminate	$p(G=0.3)$ Indeterminate	$p(G=0.5)$ Indeterminate	$p(G=1.0)$ Indeterminate	$p(G=1.5)$ Indeterminate	$p(G=2.0)$ Indeterminate
0.5	0.6960	0.6899	0.6840	0.6690	0.6541	0.6391
1.0	0.7358	0.7358	0.7358	0.7358	0.7358	0.7358
1.5	0.7627	0.7737	0.7846	0.8120	0.8394	0.8668
2.0	0.7857	0.7857	0.7857	0.7857	0.7857	0.7857
2.5	0.8076	0.8547	0.9017	1.0194	1.1370	1.2547
3.0	0.8296	0.9017	0.9738	1.1540	1.3343	1.5145

Table 6 Pressure-source/sink strength (G) relation for the unconfined aquifer

p	$p(G=0.1)$ Indeterminate	$p(G=0.3)$ Indeterminate	$p(G=0.5)$ Indeterminate	$p(G=1.0)$ Indeterminate	$p(G=1.5)$ Indeterminate	$p(G=2.0)$ Indeterminate
0.5	2.1604	2.1570	2.1535	2.1449	2.1363	2.1276
1.0	$2/e$	$2/e$	$2/e$	$2/e$	$2/e$	$2/e$
1.5	0.1082	0.1101	0.1120	0.1168	0.1216	0.1264
2.0	0.3202	0.3168	0.3133	0.3047	0.2961	0.2875
2.5	0.6691	0.6642	0.6594	0.6473	0.6353	0.6232
3.0	0.9783	0.9721	0.9660	0.9507	0.9353	0.9200

radial distance, implying that the flow pressure increases with the size of the well bore into the confined aquifers. More so, in a confined aquifer, due to the confinement of water under burdens, the pressure is naturally endowed.

Table 2 depicts that an increase in the hydraulic conductivity does not affect the flow pressure in the unconfined aquifers. Usually, hydraulic conductivity, which is a function of density and viscosity that in turn, depends, amidst others, on the state variables ought to cause a change in the pressure. Therefore, the non-changing pressure effect may mean that the groundwater is at equilibrium with the varying hydraulic conductivity. More so, it is seen that fluctuation/oscillation sets into the pressure structure as the well radius increases. The pressure increases and drops at $r = 1.5$ then rise at $r \geq 2$. The presence of fluctuation in the pressure structure due to the increase in the hydraulic conductivity factor may lead to a loss of strength for the flow.

Table 3 shows that the storability factor does not affect the flow pressure in confined/consolidated aquifers. As the storability of an aquifer of a given thickness accounts for the volume of water released from the aquifer storage in a unit surface in response to a decline in the hydraulic head; therefore, its non-effect on the hydraulic head is an indication that the pressure is at equilibrium with the varying storability factor. However, it is seen that the pressure increases with the radial distance, implying that the flow pressure increases with the size of the well bore into the confined aquifers.

Table 4 show that an increase in the storability factor has no effects on the flow pressure in unconfined aquifers. Being a constant reciprocal term in the groundwater flow equation, storability in its infinitesimal size tends to produce no change in the pressure. However, it is seen that oscillation sets in in the pressure structure as the radius increases. The pressure increases and drops at $r = 1.5$ then rise at $r \geq 2$. As usual, the oscillatory pressure behaviour may lead to a loss of strength for the flow.

Table 5 depicts that the aquifer strength has a fluctuating effect on the flow pressure in confined aquifers. As is seen, at $r = 0.5$ the pressure decreases as the source/sink strength increases; at $r = 1.0$ the pressure is constant despite the increase in the source/sink strength G, and at $r \geq 1.5$ the pressure increases with an increase in the source/sink strength. The source/sink strength accounts for how the aquifer responds to the normal pressure decline in the reservoir. It accounts for the volume of water received by the aquifers and, the sink strength the volume of water leaving the aquifers. An aquifer strength is perfect or strong when the water flow rate is almost at par with the reservoir withdrawal rate under the reservoir conditions. More so, the pressure increases with the increase in the radial distance, depicting that the flow pressure increases with the size of the well in the confined aquifers.

Table 6 depicts that in an unconfined aquifer, the flow pressure is highly fluctuating. At $r = 0.5$, the pressure drops for all values of G; at $r = 1.0$, the pressure is constant for all values of the source/sink G; at $r = 1.5$ the pressure decreases as values of the source/sink G increases; at $r \geq 2$, the pressure decreases with the increase in the source/sink G. However, the pressure increases with the increase in the radial distance, implying that in unconfined aquifers, the flow pressure increases as the radius of the well increases.

From Tables 5 and 6, it is evident that the increase in the aquifer strength causes fluctuation in the flow pressure in both confined and unconfined aquifers.

The flow situations shown by the results have some implications. The fluctuation in the pressure structure leads to a loss of energy for the flow. The non-changing effect on the pressure structures depicts that the pressure is in equilibrium with the varying factors under consideration. It is an indication that the groundwater flows into the well through natural and other causal effects. Therefore, the sustainability of the well does not depend on the hydraulic conductivity of the water, storability of the aquifer nor the source/sink strength but possibly, on the availability of water in the aquifer, porosity/permeability of soil matrixes, and the height difference between the aquifer and the bored well. The great question is of what roles are the investigated parameters playing in the groundwater flow equation?

Conclusion

Apart from the availability of water in the aquifers for replenishment at discharge from the wells, the height difference between the aquifers and the wells, and the permeability of the matrixes of the aquifers, it is claimed that the groundwater flow and the sustainability of wells borne into the heart of aquifers depend on the hydraulic conductivity and storability of the groundwater, and source/sink strength of the aquifers, as enshrined in the flow equations. We investigated the effects of hydraulic conductivity, storability and source/sink strength on the pressure force moving the groundwater into wells to account for the sustainability of wells using the Dupuit–Forchheimer assumption. The analysis of results shows that increase in:

- Hydraulic conductivity has no pressure-increasing effect in both confined and unconfined aquifer; it causes fluctuation in the pressure force in the unconfined aquifer.
- The storability factor has no pressure-increasing effect in both confined and unconfined aquifers; it causes fluctuation in the pressure force in the unconfined aquifer
- Source/sink strength G causes fluctuation in the pressure force in both confined and unconfined aquifers.

Importantly, the fluctuation in the hydraulic head results in a loss of energy for the groundwater flow into wells.

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