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ORIGINAL ARTICLE

A sustainable generalization of inverse Lindley distribution for wind speed analysis in certain regions of Pakistan

Muhammad Shoaib[1] · Irum Sajjad Dar[1] · Muhammad Ahsan-ul-Haq[1,2] · Rana Muhammad Usman[1]

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Abstract
Information on wind speed and wind power distribution is significant for a few reasons, for example, surveying wind assets, arranging wind cultivates, and limiting the liabilities for wind power improvement. This study provided an application of a new generalization of two-parameter generalized inverse Lindley distribution using the Marshall–Olkin family for analyzing wind speed and wind power characteristics. Some mathematical properties of the new distribution were studied. We had observed the suitability of new distribution as compared to the other well-known wind speed distributions such as Weibull, inverse exponential, inverted Kumaraswamy, inverse Weibull, inverse Lindley, and generalized inverse Lindley distribution. For this purpose, the time-based wind speed data is taken from the four stations of Pakistan as a case study. We conclude based on certain goodness of fit criteria that the newly developed distribution has a better fit as compared to the other wind speed distributions. Therefore, the new model can be used as an alternative distribution for the assessment of wind speed energy potential.

Keywords Marshall-olkin family · Generalized inverse lindley · Weibull · Wind speed analysis · Goodness-of-fit tests

Introduction

Energy is an essential aspect of every single human action and supportable enhancements of the nations. The financial development and social success of any nation typically rely upon the non-sustainable power sources of that nation. Enhanced use of energy and its applications will be the key problem of the real world (Valasai et al. 2017). Among all renewable energy sources, the most dominant, rapidly creating, and extensively utilized wellspring of sustainable power source is wind energy, which is perfect and practical for the ecosystem (Alavi et al. 2016). Wind energy is considered an environment-friendly source all over the world and an increase in its use has been most important in late years (Kantar et al. 2018). In contrast with petroleum derivatives, the breeze vitality has not unfriendly impact on the environment. In recent years, in both public and private sectors, the investment and research on wind resources are increased (Arslan et al. 2017). Two basic segments to use the wind energy successfully given as (i) the decision of the zone for setting up a wind energy farm and (ii) For better modeling execution, the determination of the best statistical model. In determining districts, wind energy potential is checked by modeling the wind speed information. In this way, wind speed data is a fundamental parameter for the advancement of wind energy.

The usage of probabilistic models for the classification of wind speed is common in literature. Numerous authors had recent work on the wind speed analysis with different composite and mixture models such as Abbas et al. (2012), Arslan et al. (2017), Jung et al. (2017), Muhammandi et al. (2017), Kantar et al. (2018), Dey et al. (2019), Khan et al. (2019), Haq et al. (2020a), Kaseem et al. (2020), and Haq et al. (2020b). According to the previous studies, Weibull distribution (WD) is considered the best probabilistic distribution for wind speed modeling (Soukissian 2013). However, in some situations, WD does not provide sufficient information about the modeling of wind speed (Akgul et al.

✉ Muhammad Ahsan-ul-Haq
ahsanshani36@gmail.com

Muhammad Shoaib
shabialvi1@gmail.com

Irum Sajjad Dar
imirumamir@hotmail.com

Rana Muhammad Usman
usmanrana0331@gmail.com

1 College of Statistical and Actuarial Sciences, University of the Punjab, Lahore, Pakistan

2 Quality Enhancement Cell, National College of Arts, Lahore, Pakistan

2016). In other words, WD has inadequate proficiency to deal with bimodal, multimodal, and highly skewed wind speed data. Therefore, it is need of the hour to develop new extensions of probabilistic models for the betterment of wind speed analysis.

Lindley (1958) had suggested a model in the context of the Bayes theorem and named it Lindley distribution (LD). Various applications of Lindley distribution were described by Ghitny et al. (2008). Inverse Lindley (IL) distribution was developed by Sharma et al. (2015) for survival analysis and stress-strength reliability analysis of cancer patients. The density function (pdf) and distribution function (cdf) of IL distribution was given as $f(x) = \frac{\beta^2}{1+\beta}\left(\frac{1+x}{x^3}\right)e^{\left(-\frac{\beta}{x}\right)}$ and $F(x) = \left(1 + \frac{\beta}{(1+\beta)x}\right)e^{\left(-\frac{\beta}{x}\right)}$ respectively, where $x > 0$ and $\beta > 0$ is the only scale parameter. The generalization and applications of IL distribution were discussed by numerous authors especially in hydrology such as Alkarni (2015), Sharma et al. (2016), and Eltehiwy (2020). Here, Barco et al. (2017) proposed an Inverse Lindley (IL) distribution and its generalized form called generalized inverse Lindley (GIL) by using the power transformation $y = z^{-\frac{1}{\varphi}}$ with the pdf and the cdf is given as, respectively.

$$g(y) = \frac{\varphi\beta^2}{1+\beta}\left(\frac{1+y^{\varphi}}{y^{2\varphi+1}}\right)e^{-\frac{\beta}{y^{\varphi}}}, \quad y, \beta, \varphi > 0, \tag{1}$$

$$G(y) = \left(1 + \frac{\beta}{(1+\beta)y^{\varphi}}\right)e^{-\frac{\beta}{y^{\varphi}}}. \tag{2}$$

It is found that the GIL distribution is more skewed to right than that of WD. Let Y follows the GIL distribution and X follows the Weibull distribution, then the upper tail probabilities at similar parameter combinations for both models is computed at $\varphi = 3.5$ and $\beta = 1$. As $P(Y > 2.5) = 0.02023$ and $P(X > 2.5) = 1.86 \times 10^{-11}$, while $P(Y > 2.11) = 0.03661$ and $P(X > 2.11) = 1.18 \times 10^{-6}$. Therefore, it is justified now from the above probabilities at considered percentiles that the GIL distribution has longer tails than that of the Weibull distribution. Therefore, GIL distribution and its generalizations are useful for the analysis of wind speed and these right-tailed probabilities can help the researchers who work in this area.

The main aim of this paper is to develop a new extension of GIL distribution as a wind speed distribution. Therefore, "Development of new model" represented the development of the new model and studied its failure and survival curves. "Mathematical properties of MOGIL distribution" investigated some mathematical properties of the new model. "Wind speed data" had explored the wind speed data and some descriptive measures of data sets. "Results" provided the results and discussion about wind speed analysis and the next section concluded the study.

Methodology

Development of new model

Marshall and Olkin (1997) proposed a new family named as Marshall–Olkin G (MO–G) family of distribution with an additional shape parameter $\psi > 0$. Let $\overline{G(z)} = 1 - G(z)$ be an arbitrary baseline survival function for a continuous random variable X, then the MO-G family has survival function given as

$$\overline{F(z,\gamma)} = \frac{\psi\overline{G(z)}}{1 - \overline{(\psi)}L(z)}, \quad -\infty < z < \infty, \tag{3}$$

where $\overline{\psi} = 1 - \psi$ is a tilt parameter. The cdf and the pdf of the MO-G family are given as

$$F(z) = \frac{G(z)}{1 - (1-\psi)(1 - G(z))}, \tag{4}$$

$$f(z) = \frac{\psi g(z)}{[1 - (1-\psi)(1 - G(z))]^2}, \tag{5}$$

respectively. By substituting (2) in (4), the cdf of Marshall–Olkin generalized inverse Lindley (MOGIL) distribution is obtained as

$$F(y) = \frac{\left(1 + \frac{\beta}{(1+\beta)y^{\varphi}}\right)e^{-\frac{\beta}{y^{\varphi}}}}{1 - (1-\psi)\left(1 - \left(1 + \frac{\beta}{(1+\beta)y^{\varphi}}\right)e^{-\frac{\beta}{y^{\varphi}}}\right)} \quad \text{for } y \geq 0, \ \beta, \psi, \varphi > 0. \tag{6}$$

The corresponding pdf of Eq. (6) is defined as

$$f(y) = \frac{\psi\left(\frac{\varphi\beta^2}{1+\beta}\left(\frac{1+y^{\varphi}}{y^{2\varphi+1}}\right)e^{-\frac{\beta}{y^{\varphi}}}\right)}{\left[1 - (1-\psi)\left\{1 - \left(1 + \frac{\beta}{(1+\beta)y^{\varphi}}\right)e^{-\frac{\beta}{y^{\varphi}}}\right\}\right]^2}. \tag{7}$$

The reliability of any system is checked by the survival function. For MOGIL, the survival rate function is

$$S(y) = \frac{\psi\left(1 - \left(1 + \frac{\beta}{(1+\beta)y^{\varphi}}\right)e^{-\frac{\beta}{y^{\varphi}}}\right)}{1 - (1-\psi)\left(1 - \left(1 + \frac{\beta}{(1+\beta)y^{\varphi}}\right)e^{-\frac{\theta}{y^{\varphi}}}\right)}. \tag{8}$$

Hazard rate or failure rate function has many demographics uses in many research areas such as actuarial sciences and reliability analysis. The hazard rate function of MOGIL distribution from Eq. (7) is defined as

$$H(y)=\frac{\left(\frac{\varphi\beta^2}{1+\beta}\left(\frac{1+x^{\varphi}}{y^{2\varphi+1}}\right)e^{-\frac{\beta}{y^{\varphi}}}\right)}{\left(1-(1-\psi)\left(1-\left(1+\frac{\beta}{(1+\beta)y^{\varphi}}\right)e^{-\frac{\beta}{y^{\varphi}}}\right)*\left(1-\left(1+\frac{\beta}{(1+\beta)y^{\varphi}}\right)e^{-\frac{\beta}{y^{\varphi}}}\right)\right)}. \tag{9}$$

The shapes of MOGIL distribution

The density curves of MOGIL distribution have unimodal behavior and the tails of the distribution curves become heavier by an increase in shape parameters as represented in Fig. 1.

Mathematical properties of MOGIL distribution

Some mathematical properties are derived in this section those are essential to explain a distribution and useful to explain real-life applications. The rth moment of the MOGIL distribution is formulated as

$$\mu_r'=\frac{\psi}{\varphi}\sum_{k,m,\mathcal{T}=0}^{\infty}\tau_{k,m,\mathcal{T}}(\beta(m+1))^{\frac{r-\varphi-\mathcal{T}}{\varphi}}$$
$$\times\left[(\beta(m+1))^{\varphi}\Gamma\left(\frac{-r+2\varphi+\mathcal{T}}{\varphi}\right)+\Gamma\left(\frac{-r+\varphi+\mathcal{T}}{\varphi}\right)\right]\quad \varphi>r. \tag{10}$$

The moment generating function of the MOGIL distribution is given as

$$M_y(t)=\frac{\psi}{\varphi}\sum_{k,m,\mathcal{T},q=0}^{\infty}\frac{(t)^q}{q!}\tau_{k,m,\mathcal{T}}(\beta(m+1))^{\frac{q-\varphi-\mathcal{T}}{\varphi}}$$
$$\times\left[(\beta(m+1))^{\varphi}\Gamma\left(\frac{-q+2\varphi+\mathcal{T}}{\varphi}\right)+\Gamma\left(\frac{-q+\varphi+\mathcal{T}}{\varphi}\right)\right]. \tag{11}$$

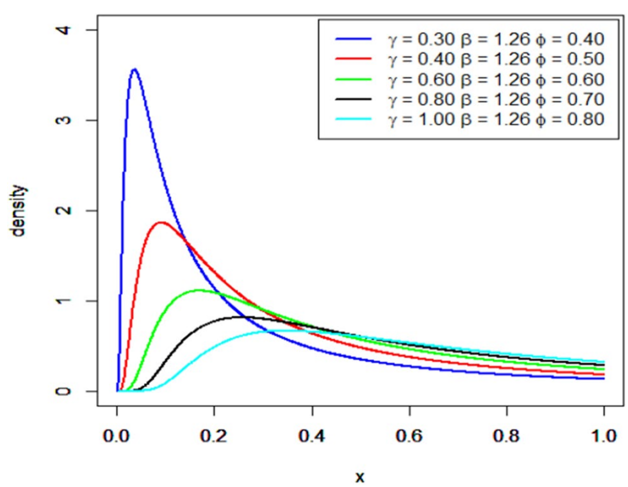

Fig. 1 pdf curves for MOGIL distribution

The explicit expression for the incomplete moment of the MOGIL distribution is given as

$$M_r(\mathcal{G})=\frac{\psi}{\varphi}\sum_{k,m,\mathcal{T}=0}^{\infty}\tau_{k,m,\mathcal{T}}(\beta(m+1))^{\frac{r-\varphi-\mathcal{T}}{\varphi}}$$
$$\times\left[(\beta(m+1))^{\varphi}\Gamma\left(\frac{-\mathrm{r}+2\varphi+\mathcal{T}}{\varphi}\right),\frac{\beta}{\mathcal{G}^{\varphi}}(m+1)\right.$$
$$\left.+\Gamma\left(\frac{-\mathrm{r}+\varphi+\mathcal{T}}{\varphi}\right),\frac{\beta}{\mathcal{G}^{\varphi}}(m+1)\right]. \tag{12}$$

The stress-strength reliability of the MOGIL distribution is given as

$$R=\frac{\psi}{\varphi}\sum_{k,m,\mathcal{T}=0}^{\infty}\tau_{k,m,\mathcal{T}}(\beta(m+2))^{\frac{-\varphi-\mathcal{T}-1}{\varphi}}$$
$$\times\left[(\beta(m+2))^{\varphi}\Gamma\left(\frac{3\varphi+\mathcal{T}+1}{\varphi}\right),\frac{\beta}{z^{\varphi}}(m+2)\right.$$
$$\left.+\Gamma\left(\frac{2\varphi+\mathcal{T}+1}{\varphi}\right),\frac{\beta}{z^{\varphi}}(m+2)\right]. \tag{13}$$

The density of rth order statistics for the MOGIL distribution is:

$$\mu_r'=\frac{\rho\psi}{\varphi}\sum_{z,k,m,\mathcal{T}=0}^{\infty}\tau_{k,m,\mathcal{T}}(\beta(z+\mathcal{T}+1))^{\frac{r-2\varphi-z\varphi-\mathcal{T}\varphi}{\varphi}}$$
$$\times\left[(\beta(z+\mathcal{T}+1))^{\varphi}\Gamma\left(\frac{-r+3\varphi+z\varphi+\mathcal{T}\varphi}{\varphi}\right)\right.$$
$$\left.+\Gamma\left(\frac{-r+2\varphi+z\varphi+j\varphi}{\varphi}\right)\right]. \tag{14}$$

Let $X_1, X_2, X_3, \dots, X_n$ is a random sample that follows MOGIL distribution, and then the log-likelihood function of the distribution is

$$n(L) = n\ln(\psi) + n\ln\left(\varphi\beta^2\right) - n\ln(1+\beta) + \sum_{i=1}^{n}\ln(1+y^{\varphi}) - (2\varphi+1)\sum_{i=1}^{n}\ln(y) - \sum_{i=1}^{n}\frac{\beta}{y^{\varphi}} - 2\sum_{i=1}^{n}\ln\left[1-(1-\psi)\left\{1-\left(1+\frac{\beta}{(1+\beta)y^{\varphi}}\right)e^{-\frac{\beta}{y^{\varphi}}}\right\}\right]. \tag{15}$$

The components of the score vectors can be obtained by taking the partial derivatives of $n(L)$ for ψ, φ and β, and equate them to zero. The observed likelihood equations are

$$\frac{d\ln(L)}{d\psi} = \frac{n}{\psi} - \frac{2\sum_{i=1}^{n}\left(1-e^{-\frac{\beta}{y^{\varphi}}}\left(1+\frac{\beta y^{-c}}{1+\beta}\right)\right)}{1-(1-\psi)\left\{1-\left(1+\frac{\beta}{(1+\beta)y^{\varphi}}\right)e^{-\frac{\beta}{y^{\varphi}}}\right\}}, \tag{16}$$

$$\frac{d\ln(L)}{d\varphi} = \frac{n}{\varphi} + \beta\sum_{i=1}^{n}x^{-\varphi}\ln(y) + \sum_{i=1}^{n}\left(-2\ln(y) + \frac{y^{\varphi}\ln(y)}{1+y^{\varphi}}\right) + \frac{2(1-\psi)\sum_{i=1}^{n}\left(\frac{\beta e^{-\frac{\beta}{x^{\varphi}}}x^{-\varphi}\ln(x)}{1+\beta} - \beta e^{-\frac{\beta}{y^{\varphi}}}y^{-\varphi}\left(1+\frac{\beta y^{-\varphi}}{1+\beta}\right)\ln(y)\right)}{1-(1-\psi)\left(1-e^{-\frac{\beta}{y^{\varphi}}}\left(1+\frac{\beta y^{-\varphi}}{1+\beta}\right)\right)}, \tag{17}$$

$$\frac{d\ln(L)}{d\beta} = \left(\frac{2}{\beta} - \frac{1}{1+\beta}\right)n - \sum_{i=1}^{n}y^{-\varphi} + \frac{2(1-\psi)\sum_{i=1}^{n}\left(-e^{-\frac{\beta}{y^{\varphi}}}\left(-\frac{\beta y^{-\varphi}}{(1+\beta)^2} + \frac{y^{-\varphi}}{1+\beta}\right) + e^{-\frac{\beta}{y^{\varphi}}}y^{-\varphi}\left(1+\frac{\beta y^{-\varphi}}{1+\beta}\right)\right)}{1-(1-\psi)\left(1-e^{-\frac{\beta}{y^{\varphi}}}\left(1+\frac{\beta y^{-\varphi}}{1+\beta}\right)\right)}. \tag{18}$$

Since the derived equations have complex structures, therefore, it is hard to get a closed-form of the estimated parameters. Therefore, we use these equations by iterative procedures such as the Newton–Raphson algorithm for the estimation of parameters during wind speed analysis.

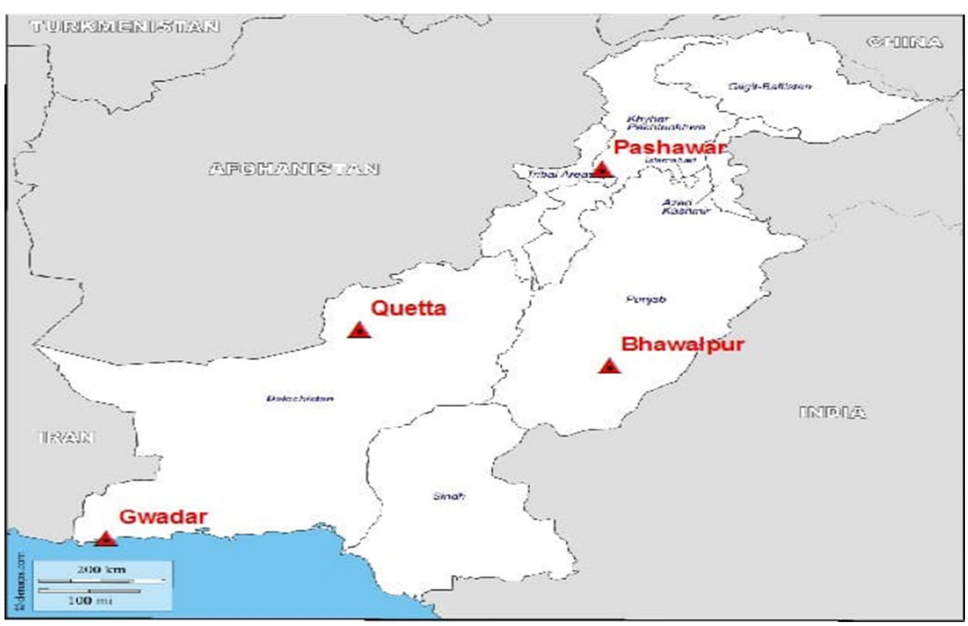

Fig. 2 Considered wind stations of Pakistan

Table 1 Descriptive statistics for the wind speed data at four stations in Pakistan

Stations	Seasons	Min (m/s)	Max (m/s)	Mean (m/s)	Variance	Skewness	Kurtosis	N
A	Winter	0.0044	8.8079	2.9498	1.5282	0.4122	0.1661	12,960
	Spring	0.0894	12.217	3.2619	2.4940	0.5271	0.1012	13,248
	Summer	0.0008	20.073	4.2019	5.1913	0.6745	1.0370	13,248
	Autumn	0.0102	11.716	3.0383	1.9783	1.2542	3.5138	13,403
B	Winter	0.0072	13.563	3.5878	5.1344	0.9851	0.7174	12,960
	Spring	0.0410	13.784	4.5162	7.7266	0.7800	0.1905	13,248
	Summer	0.0320	12.274	4.1283	4.7592	0.5813	0.3315	13,248
	Autumn	0.0059	12.170	3.3974	4.5938	1.0051	0.4050	13,099
C	Winter	0.0284	8.5428	2.4011	1.3139	0.6072	0.0983	12,960
	Spring	0.1021	17.909	3.0853	2.0942	1.4791	5.3696	13,248
	Summer	0.0353	16.148	3.1825	3.3970	1.4299	3.3970	13,248
	Autumn	0.0102	13.418	2.6400	1.4803	1.1029	4.0095	13,104
D	Winter	0.0072	13.527	3.0612	3.8841	1.2789	2.1836	12,960
	Spring	0.0518	12.845	3.4861	4.3396	0.8352	0.1544	13,248
	Summer	0.0588	15.930	3.7796	3.3832	0.6157	0.6792	13,245
	Autumn	0.0025	10.220	2.7589	2.2337	0.9534	0.9860	13,104

Table 2 The pdf and cdf of the corresponding distributions

Distributions	pdf	cdf
Weibull distribution (W)	$f(y) = \left(\frac{\beta}{\varphi}\right)\left(\frac{y}{\varphi}\right)^{\beta-1} e^{\left(-\frac{y}{\varphi}\right)^{\beta}}$	$F(y) = 1 - e^{\left(-\frac{y}{\varphi}\right)^{\beta}}$
Inverse exponential (IE)	$f(y) = \frac{\beta}{y^2} e^{\left(-\frac{\beta}{y}\right)}$	$F(y) = e^{\left(-\frac{\beta}{y}\right)}$
Inverted kumaraswamy (IK)	$f(y) = \varphi\beta(1+y)^{-(\varphi+1)}(1-(1+y)^{-\varphi})^{\beta-1}$	$F(y) = (1-(1+y)^{-\varphi})^{\beta}$
Inverse Weibull (IW)	$f(y) = \left(\frac{\beta}{\varphi}\right)\left(\frac{y}{\varphi}\right)^{-(\beta-1)} e^{\left(-\frac{y}{\varphi}\right)^{-\beta}}$	$F(y) = e^{\left(-\frac{x}{\varphi}\right)^{-\beta}}$
Inverse Lindley (IL)	$f(y) = \frac{\beta^2}{1+\beta}\left(\frac{1+y}{y^3}\right)e^{-\frac{\beta}{y}}$	$F(y) = \left(1 + \frac{\beta}{y(1+\beta)}\right)e^{-\frac{\beta}{y}}$
Generalized inverse Lindley (GIL)	$f(y) = \frac{\varphi\beta^2}{1+\beta}\left(\frac{1+y^{\varphi}}{y^{2\varphi+1}}\right)e^{-\frac{\beta}{y^{\varphi}}}$	$F(y) = \left(1 + \frac{\beta}{(1+\beta)y^{\varphi}}\right)e^{-\frac{\beta}{y^{\varphi}}}$

Table 3 The formulas of criteria for model evolution

Akaike information criteria	$\text{AIC} = -2\log(L) + 2k$
Bayesian information criteria	$\text{BIC} = -2\log(L) + k\log(n)$
Kolmogorov–Smirnov test	$\text{KS} = \max_{1\le i\le N}\left(F(v_i) - \frac{(i-1)}{N}, \frac{i}{N} - F(v_i)\right)$

Wind speed data

Pakistan is the second biggest nation in South Asia and it is the 33rd biggest nation on the planet. Geographically, Pakistan is separated into three unique regions, for example, the Northern Mountains, the Indus River plain, and Baluchistan level.

For this study, data of wind speed measured at 20 m height with the 10-min interval from four distinct stations (Bahawalpur. Gwadar, Peshawar and Quetta) from different geographical locations in Pakistan. The data is taken online from the world bank data catalog. The map of Pakistan shows the locations of selected distinct stations in Fig. 2. Station A (Bahawalpur) is situated between the Indus River plains and south of the Sutlej River. It has 214 m altitude and is situated at 29.3544° N and 71.6911° E. Station B (Gwadar) has a hot desert climate and it also has a high variation between summer and winter temperatures. It has 15 m altitude and situated at 25.2460° N and 62.2861° E. Station C (Peshawar) lies in the Northern Highlands region.

Peshawar has the same behavior as that of the previous two stations and it has 331 m altitude and situated at 34.0151° N and 71.5249° E. In the last, Station D (Quetta) has diversity in its climate with respect to summer and

Table 4 ML estimates with the goodness of fit measures for Station A

Seasons	Model	φ	β	ψ	$-2\log L$	AIC	BIC	KS
Winter	W	2.54	3.32	–	46,638.6	46,644.6	50,832.9	0.1181
	IE	2.12	–	–	56,980.3	56,982.3	56,989.8	0.2394
	IK	2.57	12.1	–	47,387.2	47,391.1	47,406.1	0.1122
	IW	2.03	1.07	–	56,701.1	56,705.1	56,720.0	0.2610
	IL	–	2.69	–	56,984.2	56,986.2	289,154	0.2695
	GIL	1.03	2.70	–	56,933.6	56,937.6	56,952.5	0.2649
	MOGIL	0.43	11.0	0.001	41,841.9	41,845.9	41,860.9	0.0148
Spring	W	2.42	4.30	–	52,437.8	52,443.8	52,466.2	0.0973
	IE	2.37	–	–	58,987.5	58,989.5	58,997.0	0.2912
	IK	2.44	16.9	–	52,067.5	52,071.4	52,086.4	0.1003
	IW	2.71	1.35	–	60,182.3	60,186.3	60,201.2	0.2026
	IL	–	3.64	–	63,053.9	63,055.9	63,063.4	0.2992
	GIL	1.32	4.50	–	60,466.8	60,470.8	60,485.8	0.2073
	MOGIL	0.50	12.8	0.002	49,893.8	49,897.8	49,912.8	0.0642
Summer	W	1.77	4.67	–	71,601.9	71,607.8	71,630.3	0.2095
	IE	2.52	–	–	134,535	134,537	134,545	0.8020
	IK	1.23	3.54	–	71,033.7	71,037.7	71,052.6	0.2168
	IW	1.69	0.49	–	84,636.1	84,640.1	84,655.1	0.3267
	IL	–	3.02	–	86,088.4	86,092.4	86,107.3	0.4337
	GIL	0.45	1.80	–	85,088.4	85,092.4	85,107.3	0.3337
	MOGIL	0.21	9.15	0.002	59,182.2	59,186.2	59,201.2	0.0693
Autumn	W	2.29	3.33	–	60,280.3	60,286.3	60,309.5	0.1122
	IE	2.22	–	–	72,339.9	72,341.9	72,349.7	0.3014
	IK	2.65	19.7	–	61,034.4	61,038.4	61,053.9	0.1068
	IW	2.08	1.22	–	70,573.7	52,577.7	52,592.6	0.2231
	IL	–	2.81	–	71,889.5	71,891.5	71,899.2	0.2879
	GIL	1.17	3.03	–	69,842.1	52,846.2	52,861.0	0.2277
	MOGIL	0.48	11.4	0.002	56,481.7	56,485.7	56,501.2	0.0532

winter temperatures. It has 1679 m altitude and situated at 30.1798° N and 66.9750° E.

The results of Table 1 showed the maximum and minimum wind speed for all stations in different seasons. Moreover, variance, skewness and kurtosis are also calculated for all seasons of all selected stations. As a matter of fact, that all the stations at all seasons have positively skewed behavior, therefore, a positively skewed distribution can significantly explore the aspects.

Table 2 enlightened the pdf and cdf of the existing wind speed models such as Weibull, inverse exponential, inverted Kumaraswamy, inverse Weibull, inverse Lindley, and generalized inverse Lindley distribution.

We have estimated the parameters by using MOGIL distribution and considered wind speed distributions using the maximum likelihood estimation method. We have fitted the observed data sets of wind speed season wise for four stations of Pakistan. The performance and enactment of the models are evaluated based on certain criteria such as Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC) and Kolmogorov–Smirnov test (KS) given in Table 3.

Results

Table 4 showed the estimated parameters, $-2\ln L$, AIC, BIC and KS statistic for the MOGIL distribution and the other well-known wind speed distributions for the different seasons at station A.

Since the MOGIL model has minimum values of $-2\ln L$, AIC, BIC, and KS values for all four seasons of Bahawalpur while IL and IW distributions have maximum values for these goodness-of-fit criteria. Therefore, the MOGIL is considered as the best-fitted model for station A. Figure 3 represented the fitted curves of the pdf and it is also could easily be seen that the MOGIL distribution has the best fit for the heavy-tailed data sets.

See Fig. 4, it could be seen from the histogram of the wind speed data at Gwadar station that the data is more

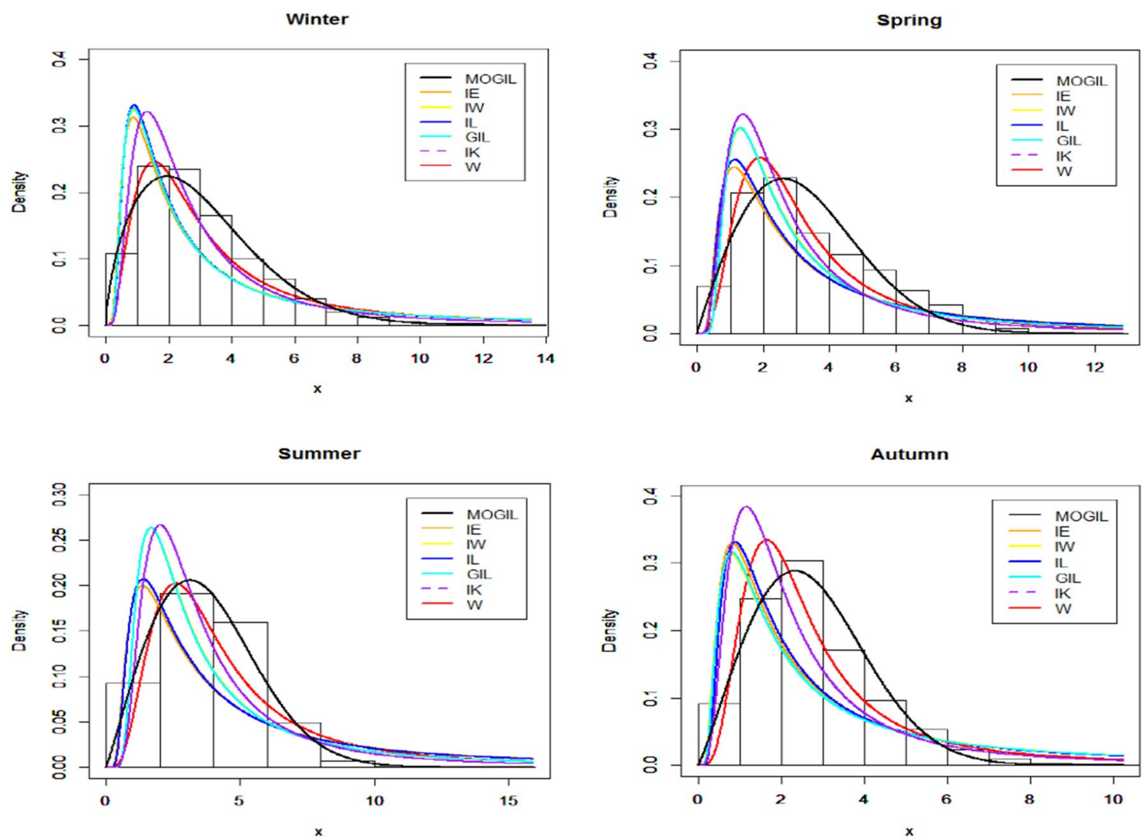

Fig. 3 Graphs for fitted pdf for all seasons at Station A

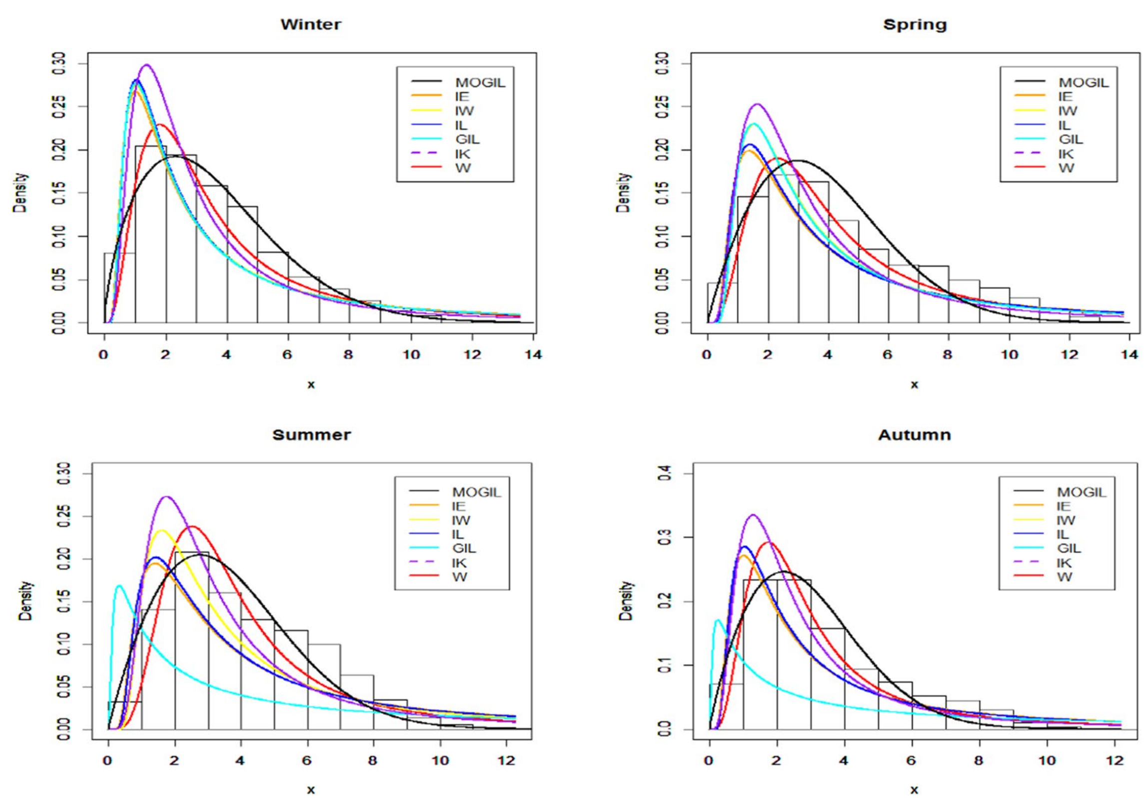

Fig. 4 Graphs for fitted pdf for all seasons at Station B

Table 5 ML estimates with the goodness of fit measures for the Station B

Seasons	Model	φ	β	ψ	$-2\log L$	AIC	BIC	KS
Winter	W	1.6581	4.0266	–	57,399.4	57,405.4	50,832.8	0.0804
	IE	2.0246	–	–	62,360	62,362.6	62,370.1	0.1738
	IK	1.9423	7.5394	–	59,222.1	59,226.1	59,241.1	0.1144
	IW	1.9877	1.0324	–	62,319.5	62,323.5	62,338.4	0.1619
	IL	–	2.5888	–	62,626.8	62,628.8	62,636.3	0.1621
	GIL	0.9836	2.5826	–	62,615.6	62,619.6	62,634.6	0.1679
	MOGIL	0.3195	9.1419	0.0031	54,664.4	54,668.6	54,683.5	0.0216
Spring	W	1.7000	5.0783	–	63,136.9	63,142.9	63,165.4	0.0605
	IE	2.7223	–	–	68,485.6	68,487.6	68,495.1	0.1979
	IK	1.8664	8.7921	–	67,431.2	67,435.2	67,450.2	0.1098
	IW	2.5324	1.1925	–	67,594.4	67,598.4	67,613.4	0.1387
	IL	–	3.3486	–	68,434.8	68,436.8	68,444.3	0.1896
	GIL	1.1527	3.6155	–	67,862.2	67,866.2	67,881.2	0.1438
	MOGIL	0.3518	10.0393	0.0032	61,640.4	61,644.4	61,659.4	0.0399
Summer	W	1.9997	4.6691	–	60,792.0	59,123.5	59,146.0	0.0641
	IE	2.7866	–	–	65,899.8	65,901.8	65,909.3	0.2469
	IK	2.1082	11.9557	–	63,166.8	63,170.8	63,185.8	0.1006
	IW	2.5571	1.2524	–	64,333.5	64,337.5	64,352.5	0.1670
	IL	–	3.4175	–	65,749.9	65,751.9	65,759.5	0.2377
	GIL	1.2131	3.8376	–	64,613.3	64,617.3	64,632.3	0.1722
	MOGIL	0.4978	9.8793	0.0073	56,519.1	56,523.1	56,538.1	0.0422
Autumn	W	1.6764	3.8214	–	55,798.4	55,804.4	55,826.9	0.0825
	IE	1.9949	–	–	61,185.7	60,798.0	60,820.5	0.1085
	IK	2.0822	6.7032	–	59,322.5	59,326.5	59,341.4	0.1134
	IW	1.9456	1.0446	–	61,106.7	61,121.6	61,121.6	0.1692
	IL	–	2.5559	–	61,377	61,379	61,386.5	0.1735
	GIL	0.9962	2.5546	–	61,376.4	61,380.4	61,395.3	0.1748
	MOGIL	0.3682	9.4971	0.0022	53,588.5	53,592.5	53,607.5	0.0531

skewed and have a long tail towards the right. Therefore, graphically it is feasible to fit MOGIL distribution.

Table 5 justified the betterment of the MOGIL distribution as compared to existing wind speed models for Gwadar station. Based on all goodness of fit criteria such as $-2\ln L$, AIC, BIC, and KS we could see that MOGIL distribution has minimum values. Moreover, it seems a good competitor of Weibull distribution to deal with heavy-tailed data. The goodness of fit measures shows that the new proposed model has the best fit for all seasons of Gwadar station.

On the same lines, Table 6 showed the estimated parameters as well as the goodness of fit measures for MOGIL and other wind speed distributions for all seasons of Peshawar station. The minimum values for all goodness of fit measures demonstrated that the MOGIL distribution has the least values for all goodness of fit measures. Therefore, MOGIL distribution seems the best competitor of all existing wind speed distributions for heavy-tailed data sets at station C.

We justified these measures graphically in Fig. 5 and provided evidence about the superiority of MOGIL distribution.

Table 7 shows that the developed model has a better fit as compared to considered inverse models for all seasons at Quetta station. Based on all goodness of fit criteria, MOGIL distribution seems a good competitor of Weibull and other wind speed distributions to deal with heavy-tailed data. It is justified from Fig. 6 that the new proposed model is best fitted than other considered inverted distributions for Station D.

Table 6 ML estimates with the goodness of fit measures for the Station C

Seasons	Model	φ	β	ψ	$-2\log L$	AIC	BIC	KS
Winter	W	2.2279	2.7159	–	42,278.9	42,284.9	42,307.4	0.1021
	IE	1.7675	–	–	49,864.1	49,866.1	49,873.5	0.2805
	IK	2.7786	12.823	–	48,092.9	48,096.9	48,111.9	0.1394
	IW	1.5908	1.4076	–	46,714.8	46,718.8	46,733.7	0.1631
	IL	–	2.6943	–	56,984.2	56,986.2	56,993.6	0.2765
	GIL	1.3354	2.4645	–	47,044.4	47,048.4	47,063.3	0.1698
	MOGIL	0.4131	9.7857	0.0013	39,089.7	39,093.7	39,108.6	0.0224
Spring	W	2.2263	3.4842	–	47,448.2	47,454.2	47,476.6	0.0919
	IE	2.4173	–	–	57,023.7	57,025.7	57,033.2	0.3083
	IK	2.7975	25.954	–	50,742.9	50,746.9	50,761.9	0.1181
	IW	2.1454	1.6650	–	51,152.5	51,156.5	51,171.5	0.1413
	IL	–	3.0188	–	56,506.4	56,508.4	56,515.9	0.2961
	GIL	1.6176	4.1683	–	51,419.1	51,423.1	51,438.1	0.1461
	MOGIL	0.5422	11.959	0.0029	45,863.6	45,867.6	45,882.5	0.0505
Summer	W	1.8876	3.5991	–	51,404.6	51,410.7	51,433.1	0.0675
	IE	2.2183	–	–	58,467.6	58,469.6	58,477.1	0.2389
	IK	2.4126	14.962	–	54,631.8	54,635.8	54,650.8	0.0901
	IW	2.0163	1.3155	–	56,242.7	56,246.7	56,261.6	0.1420
	IL	–	2.8018	–	58,149.4	58,151.4	58,158.9	0.2260
	GIL	1.2617	3.0857	–	56,559.9	56,563.9	56,578.9	0.1481
	MOGIL	0.3977	10.678	0.0012	50,157.2	50,161.3	50,176.3	0.0478
Autumn	W	2.2706	2.9788	–	44,726.2	44,732.2	44,754.7	0.1023
	IE	1.9649	–	–	53,300.9	53,302.9	53,310.4	0.2984
	IK	2.7271	17.282	–	47,600.5	47,604.5	47,619.5	0.1165
	IW	1.8023	1.2543	–	51,396.6	51,400.6	51,415.6	0.2163
	IL	–	2.5227	–	52,906.9	52,908.9	52,916.4	0.2830
	GIL	1.1997	2.6515	–	51,670.8	51,674.7	51,689.7	0.2208
	MOGIL	0.4818	12.488	0.0011	41,207.7	41,211.7	41,226.7	0.0260

Conclusion

It is justified in the study that the well-known Weibull distribution is not always be used as a wind speed distribution to deal with heavy-tailed data sets. In this research, we propose MOGIL distribution as a new heavy-tailed distribution for the modeling of wind speed for all seasons at selected stations of Pakistan. However, some mathematical properties of new models are also evaluated theoretically for a better understanding of the model. We had estimated the parameters by using the ML approach and evaluated some goodness of fit criteria such as $-2\ln L$, AIC, BIC, and KS test. The MLE method is used for the estimation of parameters. We compare the performance of the given model with well-known wind speed models such as Weibull, inverse exponential, inverted Kumaraswamy, inverse Weibull, inverse Lindley, and generalized inverse Lindley distribution. It is justified that the MOGIL distribution has favorable performance for all four regions of Pakistan as compared to other heavy-tailed distributions. It could also be noted that the addition of one shape parameter can increase the flexibility of existing models and improve the potentiality.

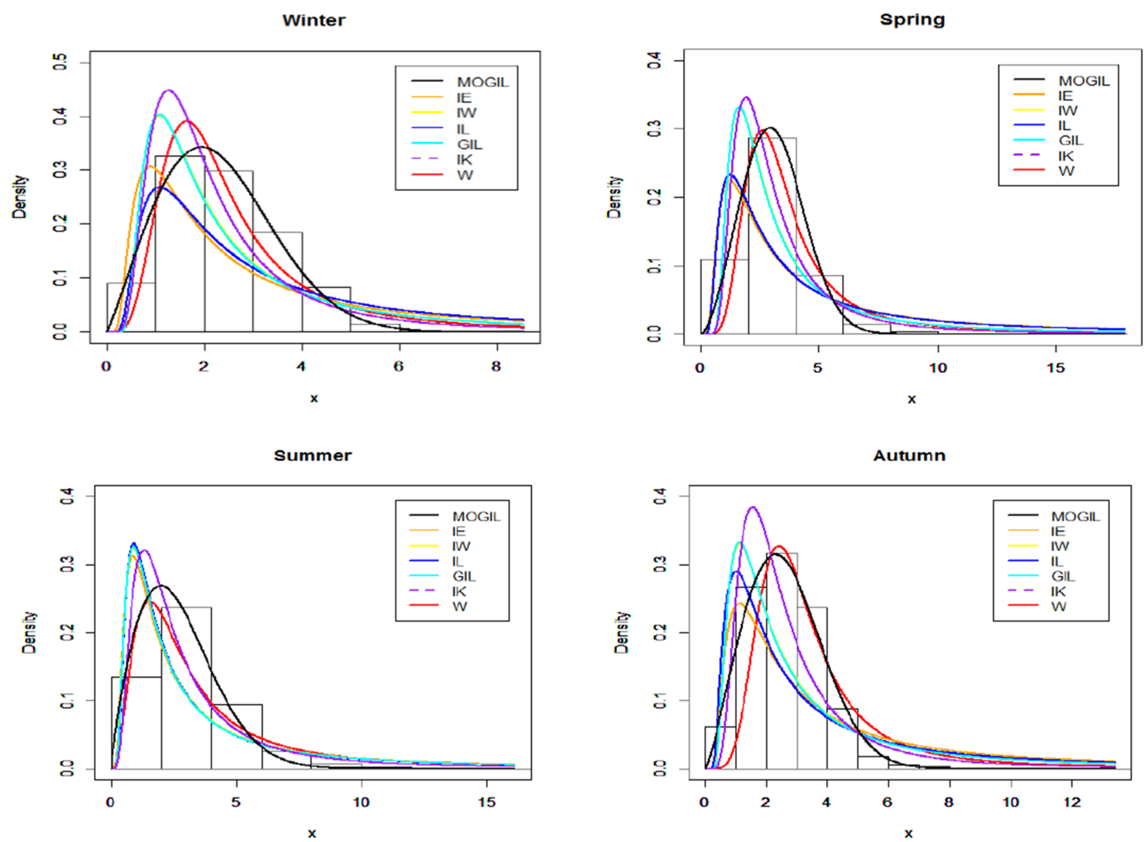

Fig. 5 Graphs for fitted pdf for all seasons at Station C

Table 7 ML estimates with the goodness of fit measures for the Station D

Seasons	Model	φ	β	ψ	$-2\log L$	AIC	BIC	KS
Winter	W	1.6388	3.4342	–	53,545.3	53,551.3	53,573.7	0.0912
	IE	1.7293	–	–	58,303.9	58,305.9	58,313.3	0.1745
	IK	2.0364	7.7248	–	56,433.8	56,437.8	56,452.8	0.0689
	IW	1.6910	1.0409	–	58,242.9	58,246.9	58,261.9	0.1592
	IL	-	2.2598	–	58,590.3	58,592.3	58,599.8	0.1597
	GIL	0.9826	2.2588	–	58,578.4	58,582.4	58,597.3	0.1658
	MOGIL	0.3559	8.9197	0.0026	50,629.7	50,633.7	50,648.6	0.0303
Spring	W	1.7615	3.9301	–	56,904.9	56,910.9	56,933.4	0.0873
	IE	2.2169	–	–	61,103.5	61,105.5	61,112.9	0.2104
	IK	2.1069	8.6389	–	59,433.8	59,437.8	59,452.8	0.1089
	IW	2.0217	1.2987	–	59,368.2	59,372.2	59,387.1	0.1234
	IL	–	2.8003	–	60,871.2	60,873.2	60,880.7	0.1989
	GIL	1.2458	3.0599	–	59,658.3	59,662.3	59,677.2	0.1292
	MOGIL	0.3569	9.8092	0.0022	54,159.1	54,163.1	54,178.0	0.0444
Summer	W	2.1739	4.2717	–	56,504.7	56,510.7	56,533.2	0.1092
	IE	2.7126	–	–	63,119.0	63,121.0	63,128.5	0.2775
	IK	2.2707	17.146	–	58,727.9	58,731.9	58,746.9	0.1183
	IW	2.4360	1.4217	–	59,997.8	60,001.8	60,016.7	0.1562
	IL	–	3.3381	–	62,827.9	62,829.9	62,837.5	0.2673
	GIL	1.3822	4.1514	–	60,253.6	60,257.6	60,272.5	0.1607
	MOGIL	0.4098	11.227	0.0016	52,521.8	52,525.8	59,201.2	0.0157
Autumn	W	1.9441	3.1173	–	48,930.9	48,936.9	48,959.4	0.0915
	IE	1.6392	–	–	57,927.8	57,929.8	57,937.2	0.2032
	IK	2.3408	7.6323	–	51,759.9	51,774.9	51,774.9	0.1127
	IW	1.7059	0.9518	–	57,774.3	57,778.3	57,793.2	0.2248
	IL	–	–	–	58,727.9	58,731.9	58,746.9	0.1183
	GIL	0.9047	2.1949	–	57,963.2	57,967.2	57,982.2	0.2281
	MOGIL	0.3662	9.7007	0.0016	45,620.7	45,624.7	45,639.7	0.0405

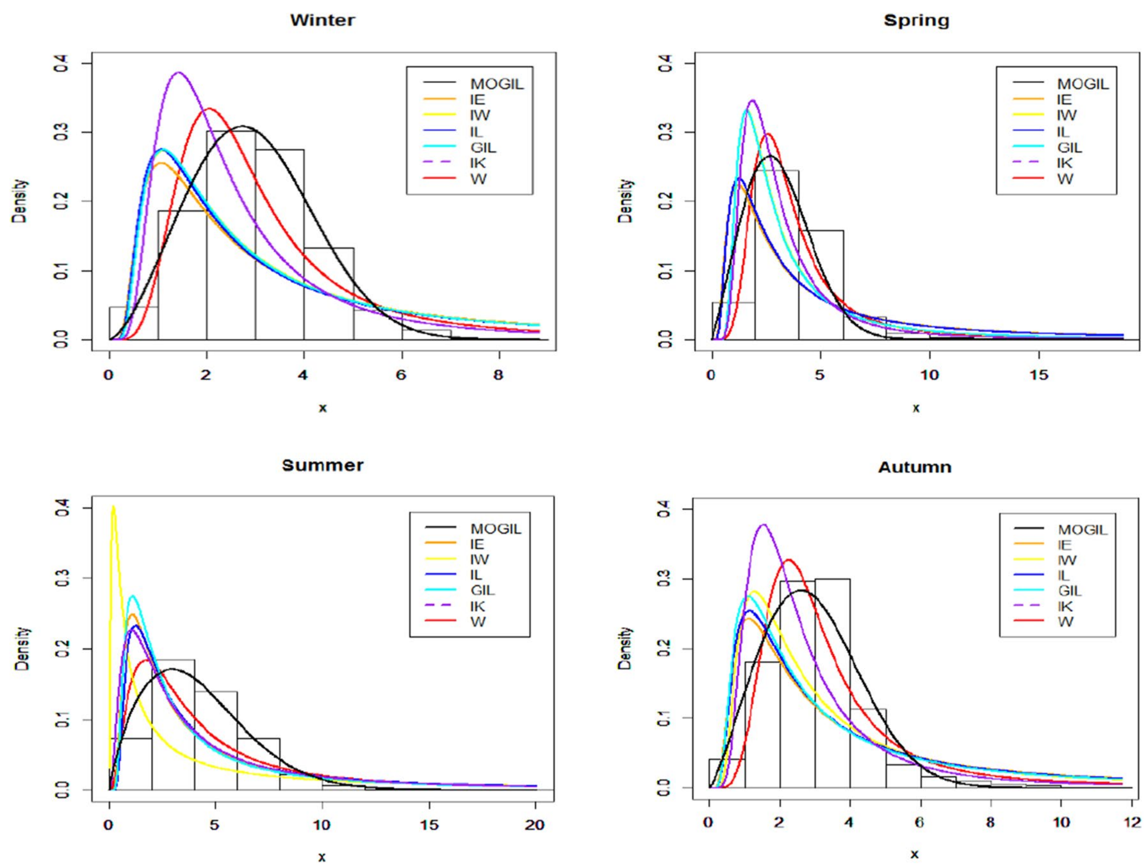

Fig. 6 Graphs for fitted pdf for all seasons at Station D

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