

# Advanced spectral analysis of sea water level changes

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**Abstract** Navigation in coastal areas requires accurate water levels modeling and prediction. Basically, tides are generated as a response to the attraction forces exerted by the moon and the sun. However, such attraction forces are not the only factors affecting water levels. The shape of bays, local wind and weather patterns also can affect tides. In this paper, the least-squares spectral analysis (LSSA) approach is used to analyze long series of tidal data, atmospheric pressure, and wind speed extended more than 9 years. The tide prediction model is developed by determining the harmonic constituents of the tidal data using LSSA approach. It is found that the resultant spectrum still contains different peaks after forcing all tidal constituents. The water level response to atmospheric pressure is also investigated. The amplitude and phase response of tidal data to atmospheric pressure are determined. It is shown that the response of water level to the atmospheric pressure has an average of about 4.5 mm/millibar. Moreover, the amplitude and phase response of tidal data to wind speed is also investigated. It is found that the power ratio of pressure effect to wind effect is about  $1.64 \times 10^6$ . That means the effect of wind is too small compared to the effect of atmospheric pressure, which can be considered a special case for this location as it is surrounded by mountains that affect the wind speed and its variation.

**Keywords** Water level · Atmospheric pressure · Spectral analysis · Least-squares

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## Introduction

The word “tides” is the term used to define the response of the ocean to the periodic fluctuations of the gravitational attraction of the moon and the sun. This response is in the form of long waves that are generated throughout the ocean. They propagate from place to place, are reflected, refracted, and dissipated just as other long waves. Like the open oceans, lakes are also affected by tides, and the solid Earth crust is affected by the same gravitational force of the sun and the moon. Moreover, other factors such as the coastline shape hydrographic conditions can play an essential role in the tidal range, the difference tidal range, and the duration and occurrence time of the tides. As the tides rise and fall, it produces flood and ebb currents, respectively.

Gravity is the main force that creates tides. Tides are the results of the gravitational force of the moon and the sun as explained by Isaac Newton in 1687. Newton’s universal gravitation law states that the gravitational attraction between two objects has a directly proportional to the corresponding masses, and inversely proportional to the square of the distance between the bodies (Sumich and Morrissey 2004). The gravitational attraction force  $F$  between any two masses,  $m_1$  and  $m_2$  apart with distance  $d$  is given by:

$$F = \frac{Gm_1m_2}{d^2}, \quad (1)$$

where  $G$  is constant depends upon the units employed.

Therefore, the gravitational attraction force between two objects depends mainly on the mass of the objects and the distance between them. So, tidal forces are the result of the gravitational force. In case of the objects on the Earth’s surface, the distance between the two objects is more important than their masses. If we consider the Earth-moon

system, the mass of the Earth and the moon is  $m_e$  and  $m_m$  respectively. The radius of the Earth is  $r$  and the distance between the center of the Earth and the center of the moon is  $d$ , then from Eq. (1) the force towards the moon can be expressed as:

$$F = G \frac{m_e m_m}{(d - r)^2}. \tag{2}$$

But the force required for the moon rotation about the Earth can be given as follows:

$$F = G \frac{m_e m_m}{r^2}. \tag{3}$$

So, the tide-producing force  $F_P$  at point  $P$  is the different between these two forces, then:

$$F_P = G m_e m_m \left( \frac{1}{(d - r)^2} - \frac{1}{r^2} \right) = \frac{G m_e m_m}{r^2} \left( \frac{1}{(1 - r/d)^2} - 1 \right). \tag{4}$$

The term  $1/(1 - r/d)^2$  can be expanded using the following mathematical approximation, for small values of  $x$ :  $\left( \frac{1}{(1-x)^2} \right) = 1 + 2x$ . so, the tide force at point  $P$  can be written as follows:

$$F_P = \frac{2G m_e m_m}{d^3}. \tag{5}$$

From Eq. (5) we can notice that there is inversely relationship between the tidal forces and the distance between objects. If we consider this rule for the relationship between the Earth, the sun, and the moon, the sun is larger than the moon by about 27 million times. If we consider the mass, the attraction force between the sun and the Earth is greater than the attraction force between the Earth and the moon by about 178 times. However, because the sun is 390 times away from the Earth than the moon. Thus, its tidal attraction force is reduced by 3903, or less than the attraction force of the moon by about 59 million times. Because of such situation, the sun’s attraction force is about  $178/390=0.46$  times that of the moon (Forrester 1983).

### Spectral analysis

The idea of spectral analysis is to transform the overall spectrum of the time series into the summation of the spectrum of many sine and cosine waves. Every periodic wave has its specific contribution in the overall spectrum. There are two common approaches of spectral analysis. The first is the classical Fourier methods and the second is the Least Squares Spectral Analysis (LSSA). The LSSA method has been investigated and tested by Vaníček 1969 and 1971. Taylor et al. (1972) introduced a verification for

Vaníček method. The statistical aspects of spectral analysis of unevenly data is introduced by Scargle (1982), Lomb (1976), and Press et al. (1992). Moreover, the stochastic significance of LSSA peaks is investigated by Pagiatakis (1999) and its applications are introduced in Pagiatakis et al. (1985).

Least-squares harmonic analysis can be used to model water level data by estimating the tidal constituents embedded in the water level measurements as follows (El-Diasty and Al-Harbi 2015; Forrester 1983; Hou and Vanicek 1994):

$$y(t_i) = a_0 + \sum_{j=1}^m a_j \cos(\omega_j t) + b_j \sin(\omega_j t), \tag{6}$$

where,  $y(t_i)$  is the series of measured water levels ( $i=1, 2, \dots, n$ );  $a_0$  is the constant term,  $m$  is the number of expected constituents;  $a_j$ , and  $b_j$  are the cosine and sine parameters, respectively;  $\omega_j$  is the frequency of the expected constituent. Equation 6 can be written in matrix notation as follows:

$$Y = AX, \tag{7}$$

where  $Y = [y(t_1), y(t_2), \dots, y(t_n)]^T$  is the observation vector;  $X = [a_0, a_1, b_1, \dots, a_m, b_m]^T$  is the vector of the unknown parameters; and the design matrix  $A$  can be written as:

$$A = \begin{bmatrix} 1 & \cos(\omega_1 t_1) & \dots & \sin(\omega_n t_1) \\ 1 & \cos(\omega_1 t_2) & \dots & \sin(\omega_m t_2) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \cos(\omega_1 t_n) & \dots & \sin(\omega_m t_n) \end{bmatrix}. \tag{8}$$

For an of over determined equations ( $n > 2m + 1$ ), the least-squares solution of the above system and the associated covariance matrix are given by (Vanicek and Krakiwsky 1986) as follows:

$$\hat{X} = (A^T A)^{-1} A^T Y \tag{9}$$

$$\hat{C}_{\hat{X}} = \frac{(Y - A\hat{X})^T (Y - A\hat{X})(A^T A)^{-1}}{n - (2m + 1)}. \tag{10}$$

In which the diagonal elements represent the variances  $\sigma_{\hat{x}_i}^2 (i = 1, 2, \dots, 2m + 1)$  of the estimated parameters and the off-diagonal elements are the covariances  $\sigma_{\hat{x}_i \hat{x}_j}^2 (i, j = 1, 2, \dots, 2m + 1)$  between pairs of parameters.

### Data description

The area of study is Hudson bay, the eastern shore where Inukjauk station lies, Ottawa, Canada. The available data set is the hourly sea level (m) for about 9 years. Also, the average daily atmospheric barometric pressure and wind

speed is available in the period (1939–2005) with percentage of gaps about 29%. Figure 1 shows the plots of the available data, respectively. These data will be analyzed using least-squares spectral analysis which will be described in the next sections.

The LSSA method is used to determine the spectrum and harmonic constituents in the tidal data. After forcing all known tidal constituents from the water level, the spectrum is determined again. Figures 2 and 3 show the whole spectrum of these data and the spectrum after forcing all significant constituents, respectively.

It is clear from Fig. 3 that after forcing all known tidal constituents, there are still some peaks of high power at long periods (low frequencies). In the next sections, we will investigate the rule of atmospheric pressure and wind speed to form such peaks. In other words, the response of water level to atmospheric pressure and wind speed will be determined.

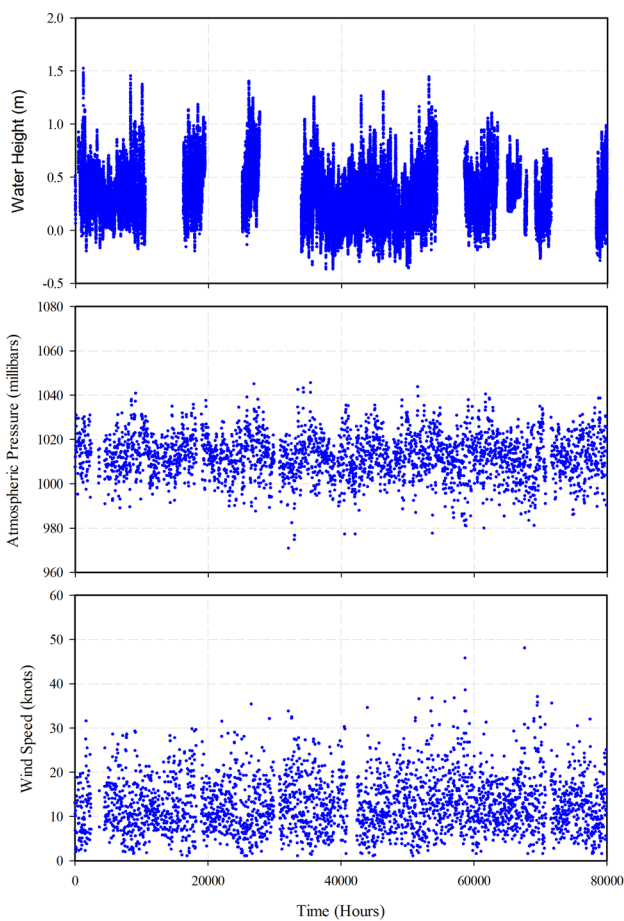


Fig. 1 Water level, atmospheric pressure, and wind speed records

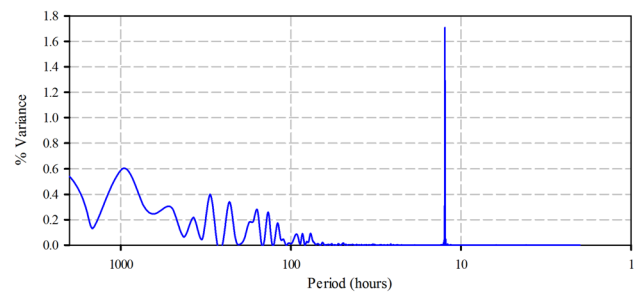


Fig. 2 LSSA spectrum of tidal data

### Ocean response to atmospheric pressure

To investigate the ocean response to the atmospheric pressure first, the least-squares spectra of both tidal and pressure data is computed. Second, the summation of the natural logarithm of both spectra is computed to obtain the product spectrum, Fig. 4. Third, the amplitudes and phases of all significant peaks are computed for both series. Finally, the amplitude and phase response are computed as described in Pagiatakis et al. (2007). Moreover, Fig. 5 shows the tide amplitude and phase response of water level to atmospheric pressure.

As shown in Figure 5, weighted least-squares can be used to fit the best model that describes the amplitude response of water level to the atmospheric pressure as follows:

$$G(T) = 3.115 \times 10^{-13} \times T^{2.516} + 0.002058, \tag{11}$$

where  $T$  is the period, which of course can be replaced by frequency. This response shows that 1 millibar atmospheric pressure corresponds to change of water level by an average as 4.5 mm. weighted least-squares can also be used to determine the best fit equation that represents the phase response of water level to atmospheric pressure as follows:

$$\Delta\phi(T) = 273.2858 - 0.0139 \times T. \tag{12}$$

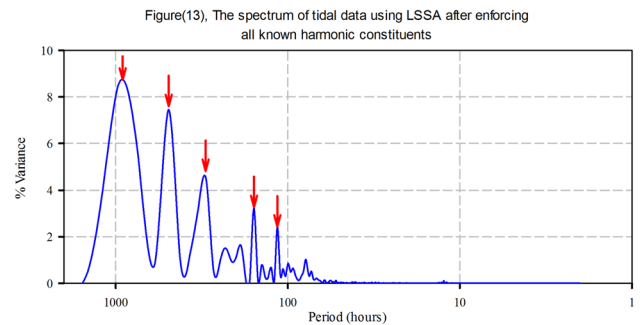
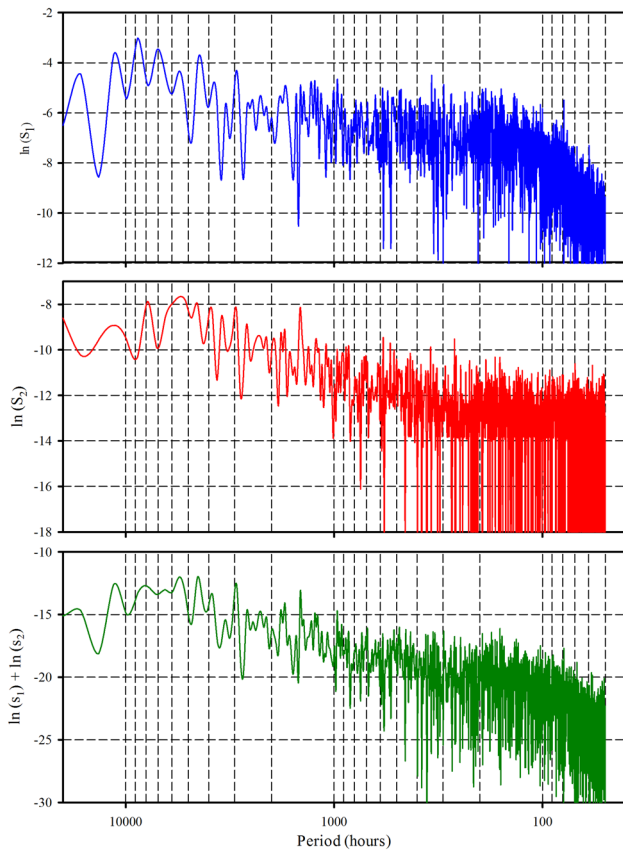
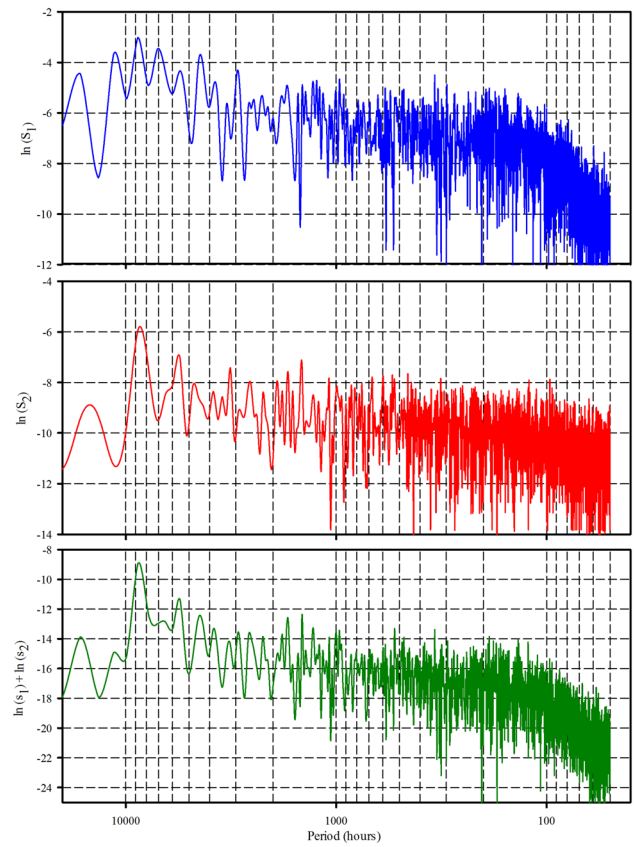


Fig. 3 LSSA spectrum after forcing all known constituents



**Fig. 4** Tide least-squares spectrum (*top*), pressure least-squares spectrum (*middle*), and product least-squares spectrum (*bottom*)

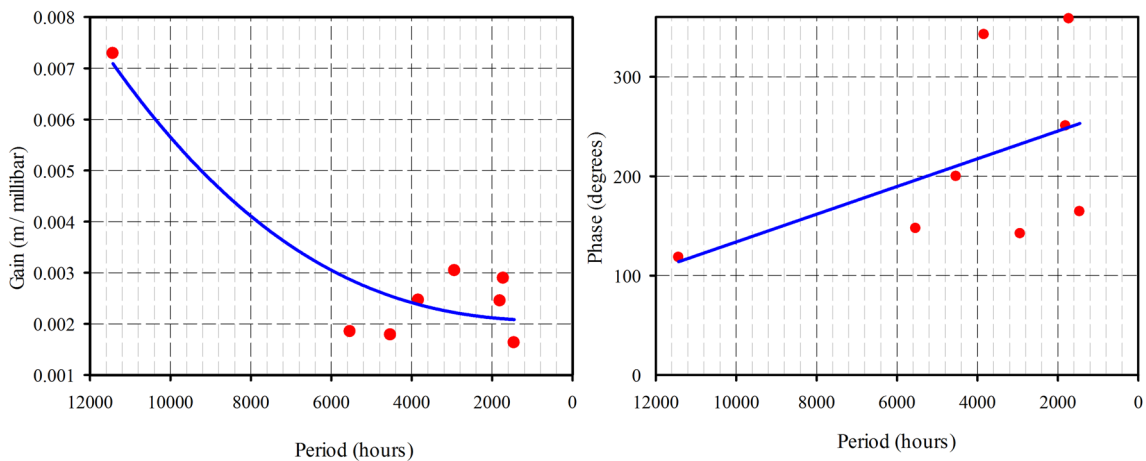


**Fig. 6** Tide least-squares spectrum (*top*), Wind speed least-squares spectrum (*middle*), and product least-squares spectrum (*bottom*)

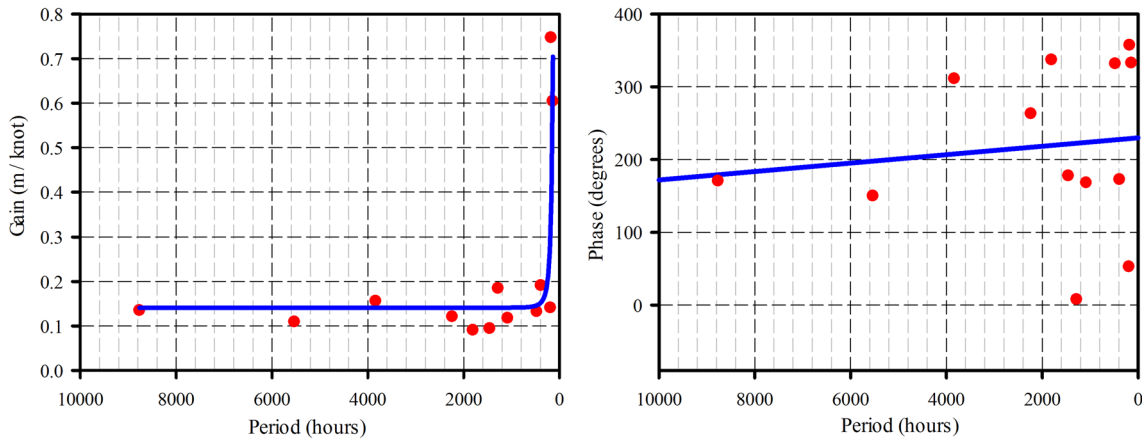
### Ocean response to wind speed

The same steps used in the previous section are applied to calculate the ocean response to the wind speed. In this stage, we will consider the water level data and the wind

speed data. Figure 6 shows the tide least-squares spectrum, the wind least-squares spectrum, and the product least-squares spectrum, respectively. In addition, Fig. 7 shows the tide amplitude and phase response of water level to wind speed.



**Fig. 5** Water level amplitude response (*left*) and phase response (*right*) to atmospheric pressure



**Fig. 7** Amplitude (*left*) and phase (*right*) response to wind speed

Weighted least-squares method is used to determine the best equation that describes the amplitude response of water level to the wind speed as follows:

$$G(T) = 3.698 \times 10^7 \times T^{-3.684} + 0.1282, \tag{13}$$

and the best fit equation to the phase response is found to be as follows:

$$\Delta\phi(T) = 229.8 - 0.005786 \times T \tag{14}$$

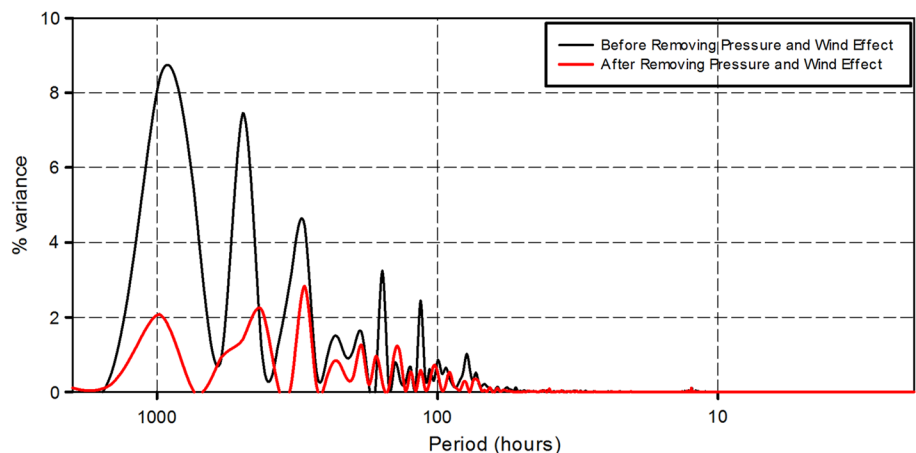
From these results we can see that the water level response to the wind is less than the corresponding response to atmospheric pressure (the power of pressure effect to the power of wind effect =  $1.64 \times 10^6$ ). Recall that in Fig. 3, after all known tidal constituents are forced, spectrum still includes several peaks. After properly modelling the ocean response to atmospheric pressure and wind speed, the spectrum is recalculated. As shown in Fig. 8, the remaining spectrum after forcing all known tidal constituents and considering atmospheric pressure and wind speed is greatly reduced showing noise at low frequency.

### Conclusions

In this paper, the least-squares spectral analysis approach is used to analyze long series of water level, atmospheric pressure, and wind speed data extended more than 9 years. The tide prediction model is developed by determining the harmonic constituents of the tidal data using LSSA approach. It is found that the resultant spectrum still contains different peaks after forcing all tidal constituents.

The water level response to atmospheric pressure is investigated. The amplitude and phase response of water level to atmospheric pressure are determined. It is shown that the response of water level to the atmospheric pressure has an average of about 4.5 mm/millibar. Moreover, the amplitude and phase response of water level to wind speed are also investigated. It is found that the power ratio of pressure effect to wind effect is about  $1.64 \times 10^6$ . That means the effect of wind is too small compared with the effect of atmospheric pressure which can be considered a

**Fig. 8** Power spectrum after removing atmospheric pressure and wind speed effect



special case for this location as it is surrounded by mountains that affect the wind speed and its variation. After removing the effect of atmospheric pressure and wind speed from the water level, it is found that the resultant power spectrum is greatly reduced to show mostly noise at high frequency only.

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