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Undergraduate Students' Perceptions of Features of Active Learning Models for Teaching and Learning to Teach Mathematics

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Abstract

The recent push toward active learning – engaging students in the learning process – is meant to benefit students. Yet there is still much to learn about students' perceptions of this phenomenon. We share results from an interview study of students' perceptions of features of two active learning models institutionalized at a large doctoral-granting university – a model for teaching foundational mathematics courses and an early field experience model for teaching preservice secondary students to teach mathematics. These models were implemented simultaneously in a single precalculus course. Interviews were conducted with both student populations (i.e., precalculus students and preservice teachers) to understand which in-class features of the models students noticed and identified as beneficial to their learning. Precalculus students identified specific opportunities related to active learning in the undergraduate mathematics teaching model – working in groups on mathematics tasks that engaged students in sensemaking and interacting with their instructor around mathematics. Preservice teachers identified specific opportunities related to three features of the university field experience model – observing a mathematics instructor enacting ambitious instructional practices, planning and teaching a "real" lesson, and observing student thinking and practicing teaching moves during groupwork. We conclude with pedagogical recommendations about particular features of the models that may help mitigate student resistance to active learning.

Keywords Active learning · Undergraduate mathematics education · Students' perceptions · Groupwork · Teacher preparation

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Introduction

The literature is replete with recommendations related to active learning in mathematics and other STEM disciplines (e.g., Freeman et al., 2014; Rasmussen et al., 2019) and, to a lesser extent, how to prepare mathematics instructors to facilitate active learning (e.g., Abell et al., 2018; Lampert et al., 2013; Stein & Smith, 2011). However, research suggests that lecture is still the dominant mode of instruction in undergraduate mathematics courses (Laursen et al., 2019). Deterrents to active learning include instructor concerns about the time and effort needed to learn how to facilitate active learning, concerns about time needed during class to provide active learning opportunities, and concerns about student resistance and negative student evaluations (Conference Board of the Mathematical Sciences (CBMS), 2016; Deslauriers et al., 2019). Concerns about how students might respond to active learning opportunities highlight an issue that we know little about – students' perceptions of active learning in undergraduate mathematics courses. This study attempts to address this gap in the literature.

Specifically, the focus of this paper is on undergraduate students' perceptions of features of two active learning models of instruction institutionalized at a large doctoral-granting university – a model for teaching foundational mathematics courses and an early field experience model for teaching preservice secondary students to teach mathematics. Since students are recipients of these models, we argue that gaining insight into their perceptions is important. Understanding students' perceptions and experiences better enables instructors to anticipate students' concerns and preferences and thus better support students in becoming active participants in their own learning (Uhing et al., 2021). Thus, our goal in this study was not to test the effectiveness of the models, but to investigate which features of the models students noticed and identified as beneficial to their learning.

Literature Review

In the following sections, we define active learning and then review literature on teaching foundational mathematics courses, giving special attention to active learning and classroom tasks implemented for active learning. We conclude by reviewing literature on early field experiences for prospective secondary mathematics teachers, discussing various ways in which they have been carried out with some experiences being more active than others.

Defining Active Learning

There is no single, common definition of active learning in the literature (CBMS, 2016). Defined broadly, active learning opportunities are opportunities that engage students actively in their own learning – giving them opportunities to think, do, and/or discuss the relevant content. For this study, we adopt Felder and Brent (2009) definition:

"Active learning is anything course-related that all students in a class session are called upon to do other than simply watching, listening, and taking notes" (p. 2). Examples of active learning opportunities include clicker surveys, think-pair-share activities, whole-class discussions, groupwork, and problem-based or inquiry-based learning (CBMS, 2016; Felder & Brent, 2009; Rasmussen et al., 2019; Saxe & Braddy, 2015). Common across these opportunities is a change in students' role from passive observer and note-taker to active participant.

Teaching Foundational Mathematics Courses

The vast majority of students in first-year mathematics courses are not math majors (Laursen et al., 2019), and, unfortunately, many of these students are unsuccessful in these courses, earning a D or F, or withdrawing (Ganter & Haver, 2011). High failure rates in such courses lead to high rates of attrition from STEM majors and from college altogether, a situation that has been described as "an embarrassment to our profession" (Saxe & Braddy, 2015, p. 28). Rethinking the pedagogy in undergraduate mathematics courses has been posited as a promising approach to improving student success (Rasmussen et al, 2019; Rocard et al., 2007; Saxe & Braddy, 2015). Specifically, providing students with opportunities for active learning and engaging students in tasks that promote conceptual understanding have been recommended (Abell et al., 2018; Rasmussen et al., 2019).

Active Learning in STEM Undergraduate Education There is considerable evidence that providing active learning opportunities can improve students' learning and increase retention (e.g., Freeman et al., 2014; Watkins & Mazur, 2013) as well as reduce achievement gaps between male and female students and for underrepresented students (Kogan & Laursen, 2014; Laursen et al., 2014; Theobald et al., 2020). Despite substantial evidence in favor of active learning, most STEM instructors still use traditional lecture-based methods (Stains et al., 2018), and mathematics instructors are no exception (Laursen et al., 2019). For example, lecturing while answering students' questions was the most commonly reported instructional format in precalculus; in contrast, only 18.4% of courses were categorized as lecture with some active learning and another 3.9% as minimal lecture (Laursen et al., 2019). This is true even though 44% of respondents in Rasmussen et al. (2019) nationwide study indicated that active learning is "very important" in mathematics instruction. Furthermore, only 15% of respondents reported that implementation of active learning was successful.

Several barriers to adopting active learning methods have been identified, including "concerns about being able to cover required course content, difficulty in implementing active learning in large classes, and increased preparation time" (CBMS, 2016, p. 7). Instructors have also expressed concerns about negative student evaluations or complaints about their teaching (Deslauriers et al., 2019). Indeed, student resistance has been identified as a prominent barrier to the adoption and sustainability of active learning in STEM education (Deslauriers et al., 2019; Shekhar et al., 2020). Shekhar et al. (2020) found that student resistance typically comes in the form of negative affective reactions, disengagement from the lesson, or negative teacher evaluations. Reasons underlying student resistance include perceptions that active learning is ineffective or too time consuming; feelings of resentment related to the increased responsibility for one's own learning; and complaints that "the blind can't lead the blind" (Deslauriers et al., 2019, p. 19251; Shekhar et al., 2020).

Beyond evidence of student resistance to active learning opportunities, we know little about how students experience active learning and in what ways and contexts they might regard it as helpful. Our review of the literature indicates that most of what we do know on this topic has been investigated using surveys rather than probing students' perceptions through interviews or other means. For example, Vroom et al. (2019) analyzed survey responses from 4,969 students and found that most students believed that listening to their instructor lecture about major topics was helpful for their learning. Although the authors acknowledge that they are unable to claim these students would not find a more active approach helpful, they concluded that students seemed to "value a passive experience" (p. 1059). In another survey study, Uhing et al. (2021) investigated students' reported experiences in active learning mathematics courses. Students frequently commented on student-to-student interactions that occurred in the classroom, mainly within groupwork. While the majority of students who mentioned this theme found working in groups to be helpful for their learning, some students reported mixed or negative experiences with groupwork. A key takeaway was that while well-facilitated groupwork can provide important advantages such as productive student-to-student interactions, poorly-facilitated groupwork can be frustrating to students. A second takeaway was that students' relationships with their instructors can greatly impact their experiences in the course in either a positive or negative direction (Uhing et al., 2021).

Implementing Tasks that Promote Conceptual Understanding Although the importance and benefits of developing students' conceptual understanding of mathematics are well-recognized, many foundational undergraduate mathematics courses fail to provide students with the kinds of learning opportunities that best support conceptual understanding (Saxe & Braddy, 2015; National Council of Teachers of Mathematics, 2014). Hiebert and Grouws (2007) identified two learning opportunities that best support students' conceptual understanding: teachers and students attend explicitly to mathematical concepts (connections among mathematical facts, procedures, and ideas) and students engage in productive struggle with these concepts. Engaging students with high-level cognitively demanding tasks can provide both kinds of opportunities and thus supports students' development of conceptual understanding (Stein & Smith, 2011).

Stein and Smith (2011) identified Procedures with Connections tasks as one type of mathematics task with potential for high cognitive demand. These kinds of tasks engage students in making sense of a procedure and connecting the procedure to its key underlying mathematical concepts. But the task itself is not enough – the task must be implemented in a way that maintains the cognitive demand for students. In other words, students must be provided with opportunities to struggle productively with the task. *Struggle* refers to the notion that "students expend effort to make sense of mathematics, to figure something out that is not immediately apparent…[the struggle] comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed" (Hiebert & Grouws, 2007, p. 387). Unfortunately, when engaging students in high cognitive demand tasks, there is a risk that the associated struggle could become unproductive, or even destructive (DiNapoli & Miller, 2020; Warshauer, 2015). Ideally, mathematics instructors work to develop strategies to mitigate such risk.

Preparing Secondary Mathematics Teachers Through Early Field Experiences

The importance of field experiences in learning to teach is well-recognized (e.g., Caena, 2014). The *Standards for the Preparation of Teachers of Mathematics* (Association of Mathematics Teacher Educators, 2017) highlighted the importance of carefully sequencing field experiences so that preservice teachers (PSTs) have opportunities to engage in "increasingly comprehensive acts of teaching" (p. 38). Policy documents about learning to teach typically emphasize the necessity of closely linking theory and practice in field experiences (AMTE, 2017; Caena, 2014).

Here we focus on *early* field experiences (EFEs) – field experiences that occur prior to student teaching – which can take many forms and have unique affordances and limitations (Allsopp et al., 2006; Zeichner, 2010). For example, observation-based field experiences in K-12 schools allow PSTs to observe authentic learning environments, but they do not usually give PSTs opportunities to assume some responsibility for teaching students. Moreover, the instruction that PSTs observe in schools does not always align with the student-centered pedagogy that is taught in university methods courses (Allsopp et al., 2006; Zeichner, 2010). EFEs that provide PSTs with greater responsibility for teaching students, but without sufficient mentorship, run the risk of negatively impacting the math students' learning. Rehearsals and micro-teaching (see Cruickshank & Metcalf, 1993; Lampert et al., 2013) in methods courses have the potential to address these issues by providing PSTs with appropriate mentorship and teaching opportunities in ways that align with what is taught in methods courses. However, rehearsals and micro-teaching can feel inauthentic to PSTs because the "students" are their peers, all of whom already have significant mathematical knowledge. An effective EFE must consider and negotiate these trade-offs.

Despite these inherent challenges, at some point, PSTs need structured opportunities to gain experience with actual teaching practice in authentic settings (Grossman, 2010). Ideally, they will have opportunities to learn about, practice, and get feedback on enacting *ambitious instruction* which requires that teachers teach in response to what students do as they engage in problem solving "while holding students accountable to learning goals that include procedural fluency, strategic competence, adaptive reasoning, and productive dispositions" (Kazemi et al., 2009, p. 11). The National Council of Teachers of Mathematics' (NCTM, 2014) Mathematics Teaching Practices (e.g., implementing tasks that promote reasoning and problem solving, supporting productive struggle in learning mathematics, and eliciting and using evidence of students thinking) are a useful resource for methods instructors supporting PSTs to enact ambitious instruction in the field. The literature reviewed above serves to frame this study, which reports on students' perceptions of important features of two active learning models implemented to improve student outcomes.

Two Active Learning Models

The active learning models described in this study aim to address two seemingly different problems. First, how can foundational undergraduate mathematics courses be designed to better support student learning and success? Driven by concerns about unsatisfactory success rates in their foundational mathematics courses and guided by research on the effectiveness of active learning, a core group of faculty in the Department of Mathematical Sciences at the University of Delaware designed an active learning model and implemented it in four foundational mathematics courses. This model has been employed and continuously improved since 2015.

The second problem is: How can early field experiences be designed to provide preservice teachers with authentic opportunities to learn to teach mathematics in ambitious ways aligned with calls from professional organizations (e.g., NCTM, 2014)? In response to the need to add an additional early field experience to their secondary mathematics teacher preparation program, and guided by the literature on early field experiences, the second focal model in this study was selected and implemented in the Department of Mathematical Sciences at the University of Delaware since 2018. This effort was supported through a grant-funded collaboration between the first, fourth, and fifth authors working to implement, study, and continuously improve an innovative model of early field experience for preservice secondary mathematics teachers at their respective institutions[1]. In the following sections, we describe each model in more detail and the relationship between the two models.

The Mathematical Sciences Learning Laboratory Model

The *Mathematical Sciences Learning Laboratory* (MSLL) model is a model for teaching foundational college mathematics courses that has been implemented, studied, and continuously improved at the University of Delaware over the last seven years. The MSLL model is currently employed in four courses, including the precalculus course which is the focus of this study. Multiple sections of each of these courses are offered each semester. The model is based on best practices and research and is characterized by four key features: 1) opportunities for active learning; 2) the

[1] This model of early field experience was initially developed and piloted at Michigan State University by the fourth author (see Bieda et al., 2019).

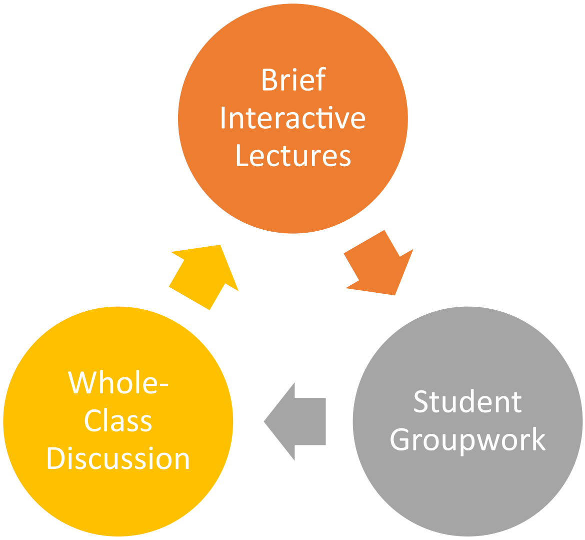

Fig. 1 The instructional cycle executed during MSLL courses

use of a shared curriculum; 3) leveraging adaptive learning technology outside of class; and 4) flexible assessment approaches. For this study, we focus on the first two features, as they are in-class features experienced by both undergraduate populations under study.

Active Learning Opportunities Courses employing the MSLL model provide substantial opportunities for students to engage in active learning. While there are many ways to provide active learning opportunities (e.g., use of Clicker items or inquiry-based learning), MSLL courses were designed to provide two particular types – groupwork and whole-class discussions. In each class session, a cycle of instruction is employed that begins with a brief lecture, is followed by groupwork, and concludes with a whole-class discussion of the task(s) students just worked on (See Fig. 1). This cycle repeats itself multiple times during each class session.

To facilitate groupwork, MSLL classes are taught in rooms furnished with tables seating up to six students per table. This seating arrangement not only facilitates students' collaboration during groupwork but also enables them to chat with each other during the short lectures and whole-class discussions (e.g., asking quick clarification questions of each other). During groupwork, which lasts about 10 to 15 min per cycle, the instructor and one or more undergraduate classroom assistant(s) circulate from group to group, "monitoring" student sensemaking (Stein & Smith, 2011). Thus, the groupwork provides students with opportunities to not only work closely with classmates, but also to interact with their instructor and the classroom assistant(s), sharing their thinking and asking and answering questions.

After each groupwork session, the instructor facilitates a whole-class discussion, engaging students to share their group's findings, compare and contrast different groups' methods, and pose and answer questions about the mathematics. Thus, the whole-class discussions also provide opportunities for students to interact with their peers, the instructor, and the classroom assistant(s) around the mathematics. Importantly, given the repetition of the teaching cycle during each class session, where students move in and out of groupwork, students are not left to struggle *unproductively* for too long, if at all.

Use of a Shared Curriculum For each MSLL course, a shared curriculum has been collaboratively developed and refined by a core group of math faculty who teach the courses regularly. The curriculum consists of two components: student materials and instructor lesson plans for each class session. The student materials consist of all of the mathematical tasks that will be worked on in each class session. While some of the tasks focus solely on procedural knowledge, the majority were designed to support students' conceptual understanding and problem-solving skills and can be classified as high cognitive demand (Procedures with Connections). The tasks are compiled into a coursepack that students in all sections of the course are required to purchase and bring to each class session. (See Appendix A for a sample task.)

The instructor lesson plans specify the learning goal(s), instructional activities, and recommended pacing for each class session. The instructional activities specify which mathematical tasks will be worked on and how they should be implemented to maintain the cognitive demand. The lesson plans indicate when and for how long students should be engaged in each segment of the instructional cycle (Fig. 1) during each class session. All instructors teach from these lesson plans and participate in weekly instructor meetings facilitated by a course coordinator. The shared curriculum and weekly instructor meetings help ensure that consistent active learning opportunities are provided across all sections of each course. The use of a shared curriculum allows instructors to determine what works and what should be revised, thus supporting instructors to continuously improve the course (Berk & Hiebert, 2009; Hiebert et al., 2017).

The University Teaching Experience Model

It is well-recognized that undergraduate students learning to teach should participate in EFEs that support their acquisition of teaching competencies prior to student teaching. However, many EFEs lack some desired qualities such as good alignment between theory and practice, opportunities to practice teaching with real learners, or productive mentoring and support systems (Allsopp et al., 2006; Zeichner, 2010). The model described here was developed to provide PSTs with an EFE that incorporated these desired qualities.

The *University Teaching Experience* (UTE) model is an EFE for a mathematics methods course taken prior to student teaching. This model is situated in a foundational undergraduate mathematics course at a level below calculus, ensuring that the mathematics aligns with mathematics PSTs are likely to teach in high school. The UTE is designed to help PSTs (a) understand the nature and complexities of teaching mathematics in an authentic context, and (b) learn to enact specific practices in a supportive environment. The mathematics teacher educator (MTE) instructing the methods course takes on the role of coordinating and facilitating the UTE (i.e., serves as the field supervisor).

The UTE is an active-learning model in that it provides PSTs with active-learning opportunities to practice teaching with real students. The model is characterized by four key features - opportunities for PSTs to 1) observe a mathematics instructor engaging students in ambitious instruction; 2) plan and teach "real" lessons that provide active learning opportunities under close supervision of the mathematics and methods instructors; 3) observe student thinking and practice teaching moves during groupwork; and 4) debrief with peers and course instructors after they teach. For this study, we focused on the first three features, as they are the in-class features of the UTE.

Observe Ambitious Instruction Importantly, the UTE setting – the undergraduate mathematics course – is taught in a way that aligns with visions of ambitious instruction advocated by professional organizations (e.g., NCTM, 2014). This allows PSTs to observe, in an authentic setting, the teaching practices that they learn about in the methods course. The undergraduate mathematics course instructor for the UTE class is therefore carefully selected as a partner for the UTE because they must not only provide effective active learning opportunities, but also be willing to coordinate with the MTE and support the PSTs' learning.

Plan and Teach a Lesson In the UTE, PSTs assume responsibility for some of the instruction in the course by teaching lessons in the undergraduate mathematics class at least twice during the semester. Depending on the size of the cohort, PSTs may teach individually or in pairs. Before they teach, PSTs receive extensive support from the MTE and the mathematics course instructor as they plan their lessons. This support, coupled with close supervision by both the methods and mathematics course instructors during the lesson, works to minimize the risks to mathematics students that are sometimes associated with receiving instruction from inexperienced PSTs.

Observe Student Thinking and Practice Teaching Moves During Groupwork During the UTE, PSTs have opportunities to observe and support students during groupwork because they sit at tables with the math students (see Fig. 2). As they support groupwork, PSTs have dual opportunities to observe student thinking as well as practice facilitating teaching moves while monitoring student sensemaking. Primary directives for PSTs are to observe PCSs' thinking and support group discussions through monitoring and facilitation rather than direct instruction (e.g., facilitating meaningful discourse and eliciting and responding to student thinking; NCTM, 2014).

Distinction Between the Two Models

It is important to note that the MSLL and the UTE models are distinct – they can be carried out independently of one another. For example, the MSLL model is currently employed in three other multi-section mathematics courses at the University

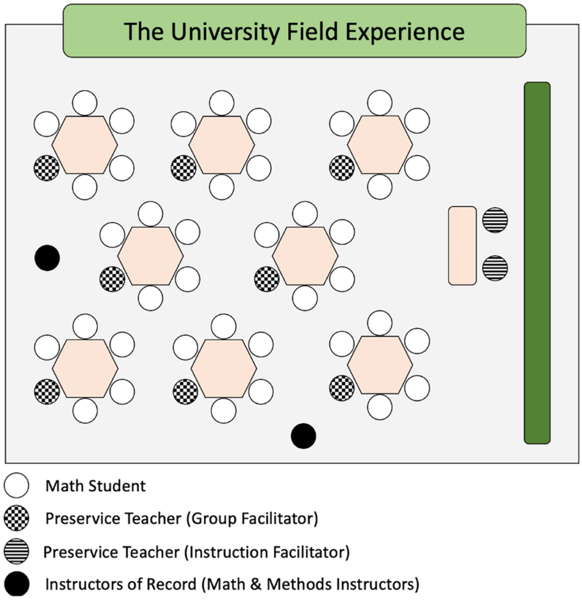

Fig. 2 A diagram of the UTE classroom (Cirillo et al., 2020). (Image republished with permission of the *Journal of Technology and Teacher Education*)

of Delaware not associated with the UTE model. In each section of these courses, an undergraduate classroom assistant (rather than PSTs) assists the instructor in facilitating groupwork during class. Similarly, while the UTE model requires a hosting undergraduate mathematics course that provides active learning opportunities, this course need not be a MSLL-model course. In fact, the UTE model began at another institution, where there is no MSLL model (Bieda et al., 2019), and was later piloted at additional institutions through grant funding.

An Overview of the Study

This study was prompted by the COVID-19 pandemic but is not explicitly focused on its impact. More specifically, in response to calls for research "on the impact of the COVID-19 outbreak on undergraduate education" including "forcing unplanned switching to online classes" (email communication from NSF program director on March 12, 2020), we leveraged the COVID-19 disruption to the Spring 2020 semester to investigate which features of the two models became salient to two populations of undergraduate students after courses unexpectedly shifted online in the middle of the semester. More specifically, the pandemic created a unique situation that researchers could not have built into a research study design because under normal circumstances ethical considerations typically would not allow researchers to withhold or abruptly eliminate treatments with known efficacy (Harris et al., 2006). Here when we say "treatments with known efficacy," we are referring to in-class active learning activities (see Freeman et al., 2014), rather than the models themselves.

Because students experienced these models as intended for the first part of the Spring 2020 semester and then had a very different (we would argue worse) experience for the second part of the semester, we hypothesized that students would be poised to reflect on the models in unique ways under conditions that would have differed if the pandemic-disruption had not occurred. In other words, the abrupt shift to online instruction provided students with a stark contrast between the active learning models they were experiencing and the instruction they experienced online. We chose to study these particular models because they (a) are active learning models that have been institutionalized in the department of the first three authors and (b) are both being continuously analyzed and improved by groups of faculty working together, including the authors of this paper. Although there may be other individual or smaller-scale active learning efforts occurring in the department, each of the authors, has been involved in collaborating on continuous improvement of one of the two models, which in the focal class, were concurrently enacted. Furthermore, the online shift provided a unique opportunity to elicit students' perceptions about the active learning they were experiencing through the two models prior to the shift. To be clear, this study is not about how the affected courses were conducted online but, rather, which features of the models the students claimed to benefit from before the shift occurred.

Thus, this study explores students' perceptions of the in-class features[2] of two innovative models that aspire to enact aspects of the visions described in policy documents for teaching undergraduate foundational mathematics courses and for learning to teach mathematics. Better understanding students' perceptions is important because students' perceptions of their learning and the helpfulness of the instructor's actions can affect their experiences in the course and their willingness to persist in mathematics beyond that course. The utility of such studies is not to determine what teaching practices *should* be used because students believe they are helpful, but rather to identify where there might be agreements or discrepancies between students' perceptions and recommendations from education research literature (Vroom et al., 2019). Thus, the research questions for this study are:

1. What opportunities, created by features of the Mathematical Sciences Learning Laboratory model, did precalculus students report as beneficial to their learning?
2. What opportunities, created by features of the University Teaching Experience model, did secondary mathematics education students report as beneficial to their learning?

Methods

This study was an interview study supplemented with survey data. Specifically, when the pandemic shifted U.S. university courses online, we quickly mobilized to acquire human-subjects study approval and design interview protocols

[2] Due to space constraints, we focused on in-class features only (in contrast to out-of-class features, such as the UMT model's use of adaptive technology for homework assignments or the UFE model lesson debrief that occurred after the precalculus class ended).

to administer to the two study populations (i.e., precalculus students (PCSs) and preservice secondary mathematics teachers (PSTs)) right after the shift online and again at the end of the semester. Following similar timing, two surveys were administered to the PCSs.

Context

This study was conducted at a large PhD-granting institution in the United States during the Spring 2020 semester. The study context was a precalculus course taught using the MSLL model. Five sections of this course were being offered; one of these sections (44 students enrolled) hosted the UTE for a cohort of PSTs (12 students enrolled). Prior to the online shift, the precalculus course met twice a week for 75 min. Each PST attended the precalculus class once a week (i.e., six PSTs on Tuesdays and six on Thursdays) and sat at or circulated amongst the tables of PCSs (See Fig. 2). Before classes moved online, each pair of PSTs had begun the lesson-planning process, but only one pair of PSTs had the opportunity to teach a 15-min whole-class activity.

Due to the COVID-19 outbreak, in-person classes ended in mid-March. After a two-week hiatus, classes resumed online. During the two-week hiatus, the team of precalculus instructors collaborated to convert the course into what was essentially an asynchronous course. The instructors video-recorded and posted lectures based on the shared curriculum that PCSs in all course sections were expected to watch asynchronously each week. In an attempt to provide some active learning opportunities to students, instructors also decided to offer synchronous "workshops" twice a week on Zoom, a cloud platform for video and audio conferencing (Zoom Video Communications Inc., 2016). During the workshops, PCSs were given opportunities to work in groups on mathematics problems via Zoom breakout rooms and to participate in whole-group discussions facilitated by the instructor. Instructors were worried that not all PCSs would have the necessary resources (e.g., equipment, internet access) at home to participate in the workshops. As a result, they agreed to make the workshops optional – PCSs were encouraged, but not required, to attend.

Unsurprisingly, PCS attendance at the Zoom workshops dropped off as the semester progressed. Decreased attendance at the workshops impacted the instructor's ability to maintain consistent student groups. To ensure that there were always at least two PCSs in each Zoom breakout room, the instructor frequently had to switch up the group composition. Moreover, many PCSs who did attend the workshops admitted that they had not watched the instructor-created videos beforehand, limiting their ability to make progress on the problems in their groups. Due to these factors, PCS-to-PCS discussions in the small groups were more challenging and less rich.

The changes to instruction made by the precalculus instructors necessitated some changes in the learning opportunities available to the PSTs. Because the primary mode of instruction shifted to asynchronous videos, PSTs were no longer able to plan and teach a "real" lesson. They were still expected to attend class (now, workshops) and support PCSs' groupwork, but the groupwork was now taking place online in Zoom breakout rooms, rather than in-person. (For more information about how the UTE shifted, see Cirillo et al., 2020; LaRochelle et al., 2021.)

Participants

Participants for this study were recruited from the special section of precalculus that hosted the UTE for the secondary mathematics teachers. Precalculus students (PCSs; $n=44$) were learning precalculus content through the MSLL model. Secondary mathematics preservice teachers (PSTs; $n=12$) were learning to teach mathematics through the UTE model, which was situated within the special section of the MSLL-model-precalculus course. To recruit both PCS and PST participants for the study, one researcher attended an online precalculus and a methods class session, invited students to participate in an interview, and then followed up by email. Interviews occurred in two phases. Phase 1 began shortly after classes shifted online[3]; Phase 2 occurred after classes ended for the semester. Eight PCSs were interviewed in each phase, with two of eight students participating in both phases ($n=14$ PCSs). Six PSTs were interviewed for each phase. All interview participants received monetary compensation, with the exception of the PCSs in Phase 2 who could choose between monetary compensation or earning 10 course points (1% of their total course grade)[4]. The goal was to interview a representative sample of students from each population. For PCSs, this meant selecting PCSs based on anticipated course grades, number of online workshops attended, and other demographics. For PSTs, this meant selecting one PST from each teaching pair because the PSTs were at different stages in the lesson-planning process when classes shifted online. Finally, all PCSs were invited by email to complete an online survey in Phase 1 and again in Phase 2. Students earned five course points (0.5% of their total course grade) for each survey completed.

Data Collection

Data collection occurred in two phases. All interviews were semi-structured and conducted and recorded over Zoom, which research suggests is a viable tool for qualitative data collection (Archibald et al., 2019). In Phase 1, questions were focused on concerns about the online shift. For example, PSTs were asked about the online shift of the UTE, including impacts on PCSs' learning and their own opportunities to learn teaching, as well as general questions about teaching and learning online (e.g., Are there aspects of the [in-person] field experience in [precalculus] that you will miss? If so, what aspects, and why?). In Phase 2, PSTs were asked to

[3] Through an exempt-status study, we were able to fast-track IRB approval.

[4] All enrolled students had the opportunity to earn these points by choosing from a range of activities. Participating in the interviews was one option for earning points.

compare their in-person and online experiences in the UTE. Similarly, PCS interviews focused on comparing the in-person and online versions of their precalculus course (e.g., Which platform, in-person or online, did you prefer, and why?).

All PCSs were invited to complete two surveys administered via Qualtrics. Each survey consisted of Likert-scale and open-ended items. Survey 1, administered about one week after online instruction began, sought to capture students' initial concerns about the online shift and reveal what features of the MSLL model were initially most salient to them. For example, an open-ended item asked, "What concerns do you have about finishing Math 115 online this semester? Please be as specific as possible." A Likert-scale item presented a list of several features of the in-person version of the course that may be lost or changed after moving online, and asked students to indicate their level of concern about each feature changing or being lost (response options were "Not at all concerned", "Somewhat concerned", "Very concerned", and "Unsure."). Survey 2, administered during final exam week, was designed to reveal what features of the model became, or remained, salient to students over the course of the semester. Survey 2 was very similar to Survey 1, with the exception that some items were revised to reflect that the course was over. For example, instead of asking students to indicate the extent to which they were concerned about certain features being lost or changed due to the online shift, they were asked to indicate the extent to which they missed each course feature after the online shift. (Response options were "Did not miss at all", "Somewhat missed", "Missed a lot", and "Unsure.") The assumption was that features students reported missing after the online shift were features that they had found to be beneficial before the shift. Of the 44 PCSs enrolled in the UTE section, 30 students (68%) completed the first survey, and 33 students (75%) completed the second survey.

Data Analysis

Although conducted separately, analyses of the PST and PCS interview data followed the same open coding process. Interview data analyses occurred in phases similar to those outlined by Creswell and Poth (2016). Interviews were transcribed, and the videos and transcripts were uploaded into Transana 3.32d (Woods, 2020), a software application for qualitative data analysis. Authors 1–3 watched a subset of the interviews, following along with the transcripts, and took individual notes on patterns and themes in the data related to features of the models that PCSs and PSTs reported as beneficial to their learning. Codes were drawn from student descriptions of what they appreciated, rather than our preconceived understandings of features of the models. We then met to propose codes and develop a codebook. Using the initial codebook, we worked to develop a shared understanding of the codes and refined them and introduced new codes, as needed. Next, Authors 1 and 3 independently coded additional interviews achieving an 80% inter-coder reliability for the double-coded PCS interviews ($n=3$), and an 83% inter-coder reliability for the double-coded PST interviews ($n=2$) with all disagreements reconciled. Units of analysis were the interviewed students' talk turns (i.e., responses to the interviewer's questions), and a talk turn could receive more than one code. Transana was then used to code the full data set. A feature was deemed "salient" if at least two-thirds of the PCSs or PSTs reported missing something about that feature after the online shift. For each feature, we used the clip-making and keyword tools of Transana to complete the coding. Finally, survey data were analyzed through simple frequency counts. Students' responses indicating they were "Very Concerned" or "Somewhat Concerned" were aggregated.

Results

After the shift to online instruction, we conducted interviews and administered surveys to collect data on the PCSs' and PSTs' perceptions about the two active learning models. In this paper, we focus on in-class features of the models that students identified as beneficial to their learning. We report on features of the MSLL model first, followed by features of the UTE model.

Mathematical Sciences Learning Laboratory (MSLL) Model

Opportunities to engage in active learning were discussed in some way during every interview. Analyses indicated that the opportunity for PCSs to work on mathematics in groups (i.e., a specific active learning opportunity of the MSLL model) was a well-regarded and highly salient feature for both PCSs and PSTs. Furthermore, both PCSs and PSTs lamented the ways in which PCSs' opportunities to do mathematics in groups changed once the precalculus course shifted online. In addition, some PCSs reported missing opportunities to interact directly with their instructor.

Opportunities to Engage in Active Learning Via Groupwork Every PCS and PST who was interviewed identified benefits of the PCSs doing mathematics in-person with peers in groups. According to the PCSs, groupwork was appreciated because it provided opportunities for them to: ask questions of their group members; "bounce ideas off each other" and collaborate to solve mathematics problems; see different ways to solve the same problem; help their peers which, in turn, benefited their own learning; and engage in student-to-student discussions in a comfortable setting. For example, PCS-07 (i.e., Precalculus Student #7) reported appreciating opportunities to both give and receive help from peers[5]:

> I enjoy the peer interaction where, if you don't understand something, you can have your classmates or teachers or the TAs [PSTs] help you out with something. And then from the other perspective, if you know [the math] and your classmates don't, you can teach it to them…I think teaching a subject can help you grasp the concept even more.

[5] Filler speech such as "like" and "um" has been removed from quotations for readability.

Table 1 Percent of PCSs reporting that they were concerned about, or missed, particular opportunities afforded by the MSLL model

Opportunity Afforded by the MSLL Model	Survey 1 Very / Somewhat Concerned	Survey 2 Missed a Lot / Somewhat Missed
Working with other students in groups during class	56.7%	75.8%
Asking my group members questions during class	53.3%	75.8%

In-person groupwork was especially missed by PCSs after the shift online. PCS-06 noted, "I really liked working in a group [when we were in-person] and being able to talk to the TA [PST] and ask questions and work through problems, not just with myself but with other people." When comparing their in-person and online experiences, many PCSs reported that they were unable to interact with their peers as much online, even with the use of Zoom breakout rooms.

Analyses of the survey data also indicated that the PCSs appreciated opportunities to work in groups and missed these opportunities once the course shifted online. In response to an open-ended item asking them to identify their single biggest concern about the precalculus course moving online, several PCSs cited losing groupwork as their top concern. Often, they would link losing opportunities to work in groups with negative impacts on their ability to learn. For example, one PCS wrote, "I am concerned that I am going to do much worse on the exams because I won't get to work with my group members I am used to working with in class." Another PCS reported, "I enjoyed having the group setting and being able to ask questions and learn from my peers; it was a big advantage that I liked better than a lecture style class." A third PCS reported their top concern as "Not being able to work with other students to understand concepts in a comfortable setting." Thus, these data indicate that PCSs felt that the opportunities to work in groups during in-person instruction supported their learning, and they were concerned that the shift online would impact these opportunities in ways that would be less supportive of their learning. Recall that after the online shift, PCSs still had opportunities to work in groups in Zoom breakout rooms during the workshops. However, their responses indicated a sense of anticipated or actual loss regarding the change in dynamic from doing in-person groupwork to online groupwork.

Appreciation for groupwork was also evident in PCSs' responses to the Likert survey items. On Survey 1, more than half of the PCSs reported being very concerned or somewhat concerned about losing the ability to work with students in groups (56.7%) and about losing the ability to ask their group members questions during class (53.3%). Students' concern about losing the groupwork indicates that they found this opportunity beneficial. On Survey 2, 75.8% of the PCSs reported missing both the ability to work with other students in groups and the ability to ask group members questions during class (See Table 1). Although the scales were different on the two surveys in a temporal sense (extent to which students *were concerned* about losing a particular opportunity vs. extent to which students *missed* that same opportunity), we argue that both scales provided a measure of the extent to which PCSs found the opportunity, in this case groupwork, beneficial. Thus, changes in responses from Survey 1 to Survey 2 suggest the perceived benefits of the in-person groupwork component of the MSLL model became more salient to the PCSs as the semester progressed.

PSTs also noted differences in active learning opportunities for PCSs during in-person versus online groupwork. For example, PST-04 (i.e., Preservice Teacher #4) articulated many affordances of in-person groupwork for the PCSs and reported that he had to do more prompting to get discussions started online. This resulted in the PCSs talking more to him than to their peers.

> I think that [PCSs] taking responsibility for their own learning [during in-person groupwork] is way better…I think they were able to explain their math to someone else when someone else wasn't quite sure what it meant. So even building that tool of communicating stuff better is another benefit that they would have in the in-person…Online, I felt like they didn't really discuss with each other. It really went through me, and then I filtered it to the next person. So, I kind of acted as the middle ground of the discussion. I don't think that there really was much discussion when we were online, [but] there was a lot of it in person.

Indeed, many PSTs reported having to do more telling than facilitating and questioning during groupwork after the shift online, and they viewed this as unideal.

Finally, PCSs and PSTs also commented on non-mathematical aspects of groupwork. PCSs made comments about how "fun" in-person groupwork was and that they had "liked" their tablemates. This became salient because the online workshops were voluntary, and so attendance was inconsistent. Consequently, PCSs often worked with different group members each time and the group sizes could be small. For example, one PCS reported: "I really found being in a group helpful during [in-person] classes but now that most people do not attend the lectures, the groups are about two people [which] makes it hard to discuss ideas or ask questions" (Survey 1). PCSs reported that they missed working with their tablemates from the in-person setting and felt uncomfortable working with peers they did not know well. Altogether, both PSTs and PCSs valued the opportunities for the PCSs to engage in authentic groupwork activities, which is a fundamental feature of the MSLL model.

Opportunities to Engage in Active Learning Via Interactions with Instructor In addition to working with peers in small groups, the MSLL model provides PCSs with numerous opportunities to interact with their instructor in class. During groupwork, as the instructor circulates around the room visiting the groups, students have opportunities to show their work to their instructor, share their thinking, answer the instructor's questions about their work, and ask the instructor questions. During whole-class discussions after each groupwork session, students again have opportunities to share their thinking with the instructor, ask questions of the instructor,

Table 2 Percent of PCSs reporting that they were concerned about, or missed, particular opportunities afforded by the MSLL model

Opportunity Afforded by the MSLL Model	Survey 1 Very / Somewhat Concerned	Survey 2 Missed a Lot / Somewhat Missed
Interacting with my instructor during class	60.0%	75.8%
Asking my instructor questions during class	60.0%	75.8%

and answer the instructor's questions. Analyses of the survey data indicated that PCSs appreciated these opportunities to interact with the instructor and were worried about losing them once the course shifted online. In response to an open-ended item asking them to identify their single biggest concern about the online shift, several PCSs cited losing the interactions with the instructor as their top concern. For example, one PCS wrote, "I will feel uncomfortable asking questions directly to the teacher in Zoom." Another responded, "I also am concerned…[that] I may not be able to do as well as I might in-person, especially with asking questions to the professor." A third PCS reported their top concern as "Having opportunities to ask questions that require real dialogue compared to just an email."

Appreciation for opportunities to interact with the instructor was also evidenced in PCSs' responses to the Likert items. On Survey 1, 60% of the PCSs reported being very or somewhat concerned about losing the ability to interact with the instructor during class and about the ability to ask the instructor questions during class. On Survey 2, 75.8% of the PCSs reported missing both the ability to interact with the instructor and ask the instructor questions during class (See Table 2). These data indicate that the PCSs' initial concerns about interactions with the instructor being lost or changed did play out, and that benefits of instructor interactions became more salient to PCSs as the semester progressed.

University Teaching Experience (UTE) Model

Recall that students were asked to discuss aspects of the UTE model that they found to be beneficial to their learning before the online shift. Analyses indicated that three features of the UTE model were identified and appreciated by students. Two features were identified by just the PSTs: opportunities to observe a mathematics instructor enacting ambitious instructional practices (i.e., active learning) and opportunities for PSTs to plan and teach a "real" lesson. One additional feature – opportunities for PSTs to observe student thinking and practice teaching moves during groupwork – was identified by both PCSs and PSTs.

Opportunities for PSTs to Observe a Mathematics Instructor Enacting Ambitious Instructional Practices (i.e., Active Learning) Four of the six PSTs remarked that they appreciated the opportunity to watch an expert instructor teach a live lesson, which was no longer possible after the online shift. Recall that the precalculus instructors had decided to create and share videos of themselves lecturing on the content after the online shift. This meant PSTs could no longer observe in-person live lessons as the instructor introduced new material and engaged students in the MSLL teaching cycle. The PSTs reported that they gained valuable insights into working with students by observing the course instructor interact with students during groupwork, a dynamic which changed significantly in Zoom breakout rooms. For example, PST-05 commented "I really liked [the mathematics instructor's] teaching style…and I feel like I learn a lot from that, like the way she would interact with the students, the way she handled misconceptions. I enjoyed watching that."

Opportunities for PSTs to Plan and Teach Actual Lessons to "Real" Learners One of the most important features of the UTE model, according to PSTs, was the opportunity to teach a "real" lesson. Every PST lamented the missed opportunities to lead a portion of a lesson. Even the pair of PSTs who did have an opportunity to teach reported being very disappointed that they did not get a second chance to teach. For example, PST-01 said:

> I had so much fun, and I am really disappointed that I can't teach again…I think it was the first time that I've ever been able to legitimately teach. So, I've taught lessons before, but to an actual class who is learning this material for the first time of a size more than 20, I haven't - that's not something I've been able to do before. And being able to have all the theory that we've accumulated in our classes and what we've been learning and being able to put that into action. I taught a lesson in the History of Math class I took in sophomore year, and I didn't really know what I was doing and that's okay. But now I have a lot more tools that I would've gotten to use, in a safe setting where I had a safety net with [the methods instructor] and in an environment where I knew the students and that I felt comfortable in.

The PSTs identified several reasons they wanted this opportunity to teach, including opportunities to: plan and teach a "real" lesson to "real" learners, in most cases, for the first time; teach learners they were getting to know; practice the ambitious instructional practices that they were learning in their methods course; and teach in a "safe" environment. Several PSTs also commented that they felt they would enjoy teaching in a real classroom and thought the experience would have been fun. Additionally, the elimination of in-class teaching opportunities affected their motivation to complete their lesson plans, which they had already started working on prior to the online shift. For example, PST-05 said that not being able to teach "made the [lesson planning] process a lot harder." She felt that anticipating the teaching of the lesson made the lesson planning process "more personal" and noted that she "wanted it to be the best it [could] be" when she believed that she was actually going to teach the lesson. Several other PSTs commented that they were less motivated to complete their lesson plans once it became clear that they were no longer teaching their lessons.

Opportunities for PSTs to Observe Student Thinking and Practice Teaching Moves During Groupwork Each PST interviewed made numerous comments about how they missed being able to observe student thinking and practice facilitating groupwork. They noticed that they had to play a much more central role in the Zoom breakout rooms because PCSs rarely, if ever, initiated student-to-student discussions. In contrast, in the in-person group settings, student-to-student discussions were much more common, allowing the PSTs to observe emergent student thinking and practice ambitious teaching practices that aim to support discussions, such as eliciting and responding to student thinking rather than directly explaining. For example, PST-04 commented "I definitely liked interacting with them…I would get to know them as an individual and [I would get to know] the way the group functions." PST-04 appreciated monitoring the student-to-student discussions that occurred in the in-person environment and felt that these discussions had important implications for PCSs' learning. PSTs also missed being able to engage in small talk with the PCSs at their tables before class started, and they felt that doing so helped foster relationships that created a safer and more comfortable environment for PCSs to work together on mathematics.

Additionally, all 14 PCSs reported that they appreciated the support they received from PSTs during groupwork. Specifically, PCSs discussed: the ability to receive extra help when they were stuck (either through direct instruction or questioning); the ability to ask PSTs questions when the instructor was busy elsewhere; and the fact that PSTs were peers so they could relate to them better. For example, PCS-01 talked about how the questions asked by PSTs would not only push his and his groupmates' thinking but also support discussions within their small groups:

> The way that the student teachers would ask questions to force people to interact, and those questions definitely help…I think it was really helpful in terms of thinking outside the box, thinking critically, and it got your mind going and thinking about math in a different way.

The ability to interact with PSTs sitting at their tables during in-person instruction before the online shift was perceived as an important resource to the PCSs. In transitioning to online instruction, this feature was not totally lost, but it was modified. PSTs continued to support PCSs in small groups during the voluntary online workshops. However, because many fewer PCSs attended the online workshops (compared to attendance when class met in-person), the groups were much smaller in size (sometimes as small as two PCSs and one PST) and were unstable in terms of group make-up. As a result, PCSs found online groupwork to be more challenging and less enjoyable online. Hence, many PCSs missed how PSTs would help facilitate student-to-student discussions during groupwork in the in-person version of the class.

Table 3 Opportunities created by features of the MSLL and UTE models as reported by PCSs and PSTs

Model	Feature of the Model	Reported By?	Who Benefits?	Opportunities Created by this Feature
MSLL	PCSs engage in active learning opportunities – groupwork	PCSs and PSTs	PCSs	PCSs were able to: • Ask questions of their group members • Bounce ideas off each other and collaborate to solve mathematics problems • See different ways to solve the same problem • Help their peers which, in turn, benefited their own learning • Engage in student-to-student discussions in a comfortable setting
MSLL	PCSs engage in active learning opportunities – interactions with instructor around mathematics	PCSs	PCSs	PCSs were able to: • Have meaningful interactions with their instructor to support their mathematical understanding during class • Ask questions of their instructor during class
UTE	PSTs observe a mathematics instructor enacting ambitious instructional practices (i.e., active learning)	PSTs	PSTs	PSTs were able to: • Observe an expert instructor teach new material • Observe an expert instructor interact with learners during groupwork • Observe an expert instructor handle learners' misconceptions
UTE	PSTs plan and teach "real" lessons	PSTs	PSTs	PSTs were able to: • Plan and teach a lesson to "real" learners for the first time • Teach learners they are getting to know • Practice ambitious instructional practices they are learning in their methods course • Teach in a "safe" environment
UTE	PSTs observe student thinking and practice teaching moves during groupwork	PCSs and PSTs	PCSs and PSTs	PSTs were able to: • Observe emergent student thinking • Observe how learners worked together on mathematics in groups • Practice teaching moves that support student discussions • Practice fostering relationships with learners PCSs were able to: • Ask PSTs questions or receive help when their instructor was busy elsewhere • Benefit from having the PSTs foster student-to-student discussions during groupwork • Think more critically as they responded to PSTs' questions

Summary of Results

We end by summarizing the opportunities PCSs and PSTs reported as beneficial to their learning and connect these opportunities to the features of the two models (see Table 3). When asked about their experiences related to the online shift, students frequently compared in-class to online experiences and often identified opportunities created by features of the models that they claimed were beneficial to their learning. To be clear, although interviewees were not explicitly asked about any particular model features, with one exception (i.e.., MSLL feature – use of the shared curriculum), students not only discussed opportunities created by each in-class feature, but they mentioned these opportunities frequently enough for the feature to meet the established “salient” threshold.

With two exceptions, opportunities afforded by the model features were raised by the student population each model primarily benefited. Both PCSs and PSTs identified opportunities created for the PSTs by the MSLL feature: PCSs engage in active learning via groupwork. Second, and related, both PCSs and PSTs reported opportunities created for the PSTs and the PCSs by the UTE feature: PSTs observe student thinking and practice teaching moves during groupwork. In this way, the PSTs’ presence in the classroom supported the active learning of both undergraduate student populations: PCSs benefitted from the support provided to them during the groupwork portion of the lesson and PSTs benefitted from opportunities to facilitate ambitious instructional practices.

Discussion

Although much has been written about the kinds of teaching and learning opportunities that undergraduate mathematics students and mathematics education majors *should* be experiencing, research suggests that students are not always provided with such opportunities. This was not the case for students in this study who experienced, during the first part of the target semester, the kinds of active learning opportunities the literature suggests undergraduate students (learning mathematics or learning to teach mathematics) should be experiencing. Our study investigated students’ perceptions on these opportunities – specifically, which features they found beneficial to their learning of mathematics or learning to teach mathematics. Precalculus students (PCSs) identified several affordances of the active-learning feature of the MSLL model. In particular, they reported finding both the opportunities to engage in groupwork with peers and the opportunities to interact with their instructor around mathematics supported their learning of the mathematics. Similarly, PSTs identified several affordances of the UTE model. PSTs indicated that before the online shift, they were benefitting from observing the mathematics course instructor teach the lessons. They also noted a decline in motivation to continue planning their assigned lessons after it became evident that teaching the lessons to PCSs was no longer possible, indicating that they saw great benefit in the opportunity to teach a “real” lesson to “real” students. Finally, both PCSs and PSTs identified as beneficial the opportunity for PSTs to facilitate groupwork, observe student thinking, and practice teaching moves during groupwork. This finding is consistent with findings from Henning et al. (2012) study in which they noted that providing PSTs with opportunities to work with smaller groups of students exposes them to more student talk than in the whole-class setting and consequently increases opportunities for PSTs to reflect on and practice facilitating student discussions.

At first glance the findings may not seem altogether surprising – the development of the two active-learning models was informed by best practice and research, and we learned that students identified ways in which features of the models benefited their learning. Yet, students do not always react favorably to active learning strategies such as groupwork. And while well-facilitated groupwork can evoke positive responses from students (Uhing et al., 2021), we do not yet know enough about what well-facilitated groupwork looks like, especially from students' perspectives. This study sheds light on this and other issues related to active learning.

The study method (i.e., studying the UTE-section of the MSLL-model precalculus course) helped to illuminate salient features of both models in the sense that the PSTs were able report on what occurred in the in-person groups at a level of detail that would be hard to capture had they not been sitting with the groups for several weeks. At the same time, the PCSs were able to identify some benefits of the UTE model because they were able to describe the teaching moves made by the PSTs. These findings have potential implications related to teaching foundational mathematics courses and teacher preparation. We acknowledge that a potential limitation of this study was that we only asked participants what they were concerned about missing after the online shift or what they did miss once the semester was over. However, students were not precluded from commenting on features they did not like or did not miss during the interviews. Yet, no participants reported disliking or not missing in-class features of the two models.

Pedagogical Suggestions

Next, we consider the findings, features of the models, and associated literature to suggest pedagogical implications for supporting undergraduate students to learn mathematics or learn to teach mathematics. We offer three suggestions based on what was learned from the students' responses and our knowledge of the models. We believe these recommendations can be implemented even if one does not adopt these particular active-learning models wholesale.

Assign Groupworthy Tasks When Implementing Active Learning Researchers have identified multiple examples of and reasons for both instructor and student resistance to active learning. Although this study was not specifically about resistance, we hypothesize that the shared curriculum feature of the MSLL model may help minimize student resistance. Specifically, the instructor-designed coursepack consisting of the mathematical tasks to be used in each class session, and associated lesson plans, likely play a critical role.

Importantly, most of the mathematical tasks are at a level of cognitive demand that makes them "groupworthy" (Cohen & Lotan, 2014). Tasks about important content designed at the appropriate level of challenge make the tasks groupworthy in the sense that they create and support interdependence among the group members. This interdependence was evident in much of the data when students discussed the benefits of talking to their peers during groupwork. In particular, PCSs reported that the groupwork segment of the instructional cycle provided opportunities to ask group members for help, "bounce ideas off each other," see different ways to solve problems, and help one another. Furthermore, the requirement that all students buy the coursepack to work in during class supports individual accountability. That is, course instructors can observe student progress on and completion of the tasks as they monitor the groupwork. This visual information supports formative assessment and allows instructors to engage in some of the *facilitation* strategies identified by Andrews et al. (2021) that have been shown to reduce student resistance (e.g., assisting students as needed, encouraging students to engage, and inviting students to ask questions). In turn, this type of instructor support has the potential to keep students in the productive struggle zone.

Engage Students in Short Cycles of Groupwork A second feature of the MSLL model that we hypothesize reduces unproductive struggle and student resistance is the instructional cycle outlined in the lesson plans. Because instructors move students through several rounds of short lecture, followed by approximately 10–15 min of groupwork, and then whole-class discussion, students are not sitting in class floundering in unproductive struggle for too long, if at all. This is important because evidence suggests that students spending "long stretches of class time working in groups" is less productive than breaking up groupwork into shorter segments (Webel, 2010, p. 317). We suspect this recommendation may be more pertinent for teaching students in foundational mathematics courses than for teaching mathematics majors.

Actively Monitor Groupwork There is actually a lack of agreement about if, when, and how instructors should intervene during groupwork (Ehrenfeld & Horn, 2020). In fact, researchers' guidance is downright contradictory. For example, Cohen and Lotan (2014) suggested that teachers only intervene when a group is "hopelessly off task" to support the goal of increasing student autonomy. In contrast, other researchers recommend instructors assume a more active role in facilitating groupwork. Specifically, some researchers promote active monitoring to encourage explanation, foster peer discussion, promote equitable participation, uncover what students understand to move their thinking forward and inform the upcoming whole-class discussion (e.g., Andrews et al., 2021; Ding et al., 2007; Stein & Smith, 2011; Webb et al., 2009; Wegerif et al., 2017). While research on the instructor's role in groupwork raises complex issues and offers little consensus (Ehrenfeld & Horn, 2020), findings from the current study support a more active instructor role during monitoring.

PCSs reported benefiting from opportunities to have one-on-one interactions with their instructor (which typically occurred during groupwork). They also benefitted from PSTs' actions in the form of providing help when they were stuck, answering questions when the instructor was busy, and asking their group questions to facilitate more student-to-student interaction. It is noteworthy that we saw few instances in the data of PCSs reporting that instructors or PSTs *explained* how to do the math. Instead, PCSs claimed to benefit from the active monitoring that occurred during small groupwork rather than direct explanation (i.e., telling). This recommendation is relevant to any course "instructors," whether they be the instructor of record or PSTs or any other classroom assistants supporting groupwork. The findings from this study suggest that undergraduate students in teacher preparation programs make prime candidates for classroom assistants since the arrangement is symbiotic for both the mathematics learners and the PSTs.

Implications for Facilitating Active Learning in the Online Environment

Finally, the data from this study suggest implications related to efforts to facilitate online mathematics instruction, which was still in the developmental phase before the pandemic (Radmehr & Goodchild, 2022). First, both PSTs and PCSs noted that when they were in Zoom breakout rooms, student-to-student interactions declined as compared to when they were working in groups in-person. PSTs specifically noted that the discourse "went through me and then I filtered it to the next person." In LaRochelle et al. (2021), we identified this and other challenges related to communicating mathematics online. Strategies for combatting these challenges include asking students to have cameras on during groupwork when possible, providing students with digital versions of the coursepack pages so that they can share screen and annotate them collaboratively, and proactively teaching students how to use the whiteboard and other features in Zoom.

Another noted online challenge was PCSs' perceived opportunities to interact with and ask questions of their instructor. Clearly the nature of instructor monitoring is significantly modified when instructors need to move in and out of Zoom breakout rooms as compared to in-person monitoring where instructors can casually glance over students' shoulders or listen to group discussions and go somewhat unnoticed. Interesting questions were also raised by participants' comments about the functionality of fixed versus randomized groups in Zoom. Both PCSs and PSTs described how groups seemed to function better when they were working in stable groups in-person because group members got to know and became comfortable working on with each other. However, research by Liljedahl (2021) supports visibly randomized heterogenous grouping, which is exactly the kind of groupwork arrangement students experienced on Zoom. However, interviewees for this study seemed to have strong feelings about how the randomized grouping strategy executed through Zoom was not beneficial for their learning. More research is needed to explore this and other issues about how to effectively conduct active learning in mathematics, both in-person and online.

Appendix A. Sample Task from the Precalculus Coursepack

Problem 4 – Math Scores: In the fall semester, college students in a math class took a final exam in December. Each month afterwards, for the next year, they agreed to take equivalent forms of the exam so that their instructor could investigate how much they retained over time.

(a) First, make a prediction: What do you think will happen to the students' scores on the exam as time passes? In other words, how do you think students' scores in December will compare to their scores when they take the exam the following March? June?

(b) What are the two variables involved in this context? Which is the independent variable, and which is the dependent variable, and how do you know?

(c) After collecting a great deal of data, the instructor found that the students' average percent score $f(x)$ after x months was found to be given by the function below:

$$f(x) = 78 - 20\log(x + 1)$$

What is the y-intercept, and what does it represent in this context?

(d) Find $f(9)$. What does it represent in this context?

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