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Results from a Survey of Abstract Algebra Instructors across the United States: Understanding the Choice to (Not) Lecture

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Abstract In the United States, there is significant interest from policy boards and funding agencies to change students' experiences in undergraduate mathematics classes. Abstract algebra, an upper division undergraduate course typically required for mathematics majors, has been the subject of reform initiatives since at least the 1960s; yet there is little evidence as to whether these change initiatives have influenced the way abstract algebra is taught. We conducted a national survey of abstract algebra instructors at Master's- and Doctorate-granting institutions in the United States to investigate teaching practices, to identify beliefs and contextual factors that support/constrain non-lecture teaching practices, and to identify commonalities and differences between those who do and do not lecture. This work provides insight into how abstract algebra is taught in the United States, factors that influence pedagogical decisions, and avenues for how to approach and better support those are interested in implementing non-lecture teaching approaches.

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Introduction

Teaching Matters. It is arguably the single most important factor affecting student learning. In addition to being the moderator of content, teachers make pedagogical decisions that position them as the greatest influence on student interest, motivation, and confidence – more so than any other individual source. In the United States (US), lecture is the most frequently reported instructional approach in undergraduate mathematics – with about 65% of mathematics faculty reporting extensive lecturing in all or most of their courses (Eagan, 2016). Rasmussen and Marrongelle (2006) described a scale of teaching that ranges along a continuum from "pure telling" to "pure investigation," in which lecturing – an instructional format in which the instructor takes the role of orally disseminating content while students take notes and record the transmitted content (e.g. Eagan, 2016; Freeman et al. 2014) – is typically situated closer to 'telling'.

Advisory panels within the US, along with most mathematics education researchers, have argued that lecture-based pedagogy is problematic for student success, learning, and persistence. Alternatively, researchers recommend pedagogical reforms that are more reflective of how people learn and better reflect the nature of doing mathematics (Kyle 1997; National Academy of Sciences, National Academy of Engineering, and Institute of Medicine 2007; National Research Council, NRC 1996; National Science Foundation 1992, 1996). Indeed, the research community is fairly resolute in the position that active learning benefits students. For instance, a meta-analysis by Freeman et al. (2014) found that in undergraduate STEM courses "active learning leads to increases in examination performance that would raise average grades by a half a letter" (p. 8410), and that students in lecture classes are 1.5 times more likely to fail than those in classes where active learning methods are used. Reaction to this work included articles in *Science, Nature*, and *Chemistry World* that called into question how ethical it was to lecture in the face of such evidence (e.g., Trager 2014).

Specifically in the US mathematics community, beyond the increasing awareness of non-lecture pedagogy, there appears to be a growing collective in support of its adoption. For instance, there have been articles published in the journals of the AMS (American Mathematical Society) about reforming teaching (e.g., Leron and Dubinsky 1995; Halmos et al. 1975; Jones 1977). Outreach efforts at the Joint Mathematics Meetings by the proponents of the Moore Method (e.g., Coppin et al. 2009) give reason to believe that its basic precepts are well-known. Additionally, the National Science Foundation has allocated a large amount of money to support mathematicians and mathematics education researchers in designing new curricula for the undergraduate curriculum, and many more instructors have developed their own materials, some via participation in Project NExT, the Academy of Inquiry-Based Learning, or Moore-Method conferences. This policy push for pedagogical reform and growing awareness of potential innovative teaching practices is not limited to the US. For instance, Nardi et al. (2005) note that in both the US and the United Kingdom, "relevant working groups, activities, publications and conferences provide evidence of a rising willingness within the community of mathematics teaching in higher education to explore the potential of innovative practices" (p. 285).

Even with this growing community and awareness, there are many mathematical instructors who believe that a well-organized lecture is the best way to learn. Burgan (2006), as an example, argued that, “I believe students benefit from seeing education embodied in a master learner who teaches what she learned” (p. 32). She further suggested that it was valuable for students to see a master modeling ways of thinking and questioning. Even early proponents of non-lecture pedagogy, in their writing, acknowledged that *some* lectures might inspire listeners:

> When given by such legendary outstanding speakers as Emil Artin and John von Neumann, even a lecture can be a useful tool—their charisma and enthusiasm come through enough to inspire the listener to go forth and do something—it looks like such fun. (Halmos et al. 1975; p. 466)

Moreover, the distinction between lecture and non-lecture pedagogy is not clear-cut, especially when looking at how instructors self-identify. Evidence from individual observations of instructors who self-reportedly use lecture-based pedagogy suggests this label might encompass a wide variety of practices – the range of which includes Dr. T (Weber 2004) who gave a monologue when lecturing to Dr. Tripp (Fukawa-Connelly 2012) who asked questions and expected answers and even had students present at the board while lecturing. Thus, even those who self-report as lecturers may be incorporating active learning pedagogical techniques.

In this report, we focus on a specific mathematics course to better understand instructors’ pedagogical practices and their choice to engage in (non) lecture pedagogy. To this end, we designed a survey to gather information about typical teaching practices, beliefs about teaching and learning, and contextual affordances and constraints. The 126 completed surveys have been analyzed to better understand the nature of instruction and factors that influence pedagogical decisions.

Literature Review

It has been argued that mathematicians use traditional methods of instruction merely out of habit or apathy (for a review of such claims, see Weber 2004). However, case studies of mathematics instructors find that their instruction is based on a rich belief system and a good deal of thought. In particular, some leverage their beliefs about mathematics and students to justify pedagogical actions that they chose (e.g. Fukawa-Connelly and Newton 2014; Lew et al. 2016; Weber 2012). Other studies, where mathematicians were asked to respond to pedagogical situations, have led to rich discussions about the relationships between their pedagogical goals and actions (e.g., Jaworski et al. 2009; Lai and Weber 2014; Nardi 2007). These studies speak to the multifaceted beliefs and goals that inform instructional practices. Indeed, there is research support for the idea that knowledge, beliefs and goals about teaching and learning are significant in terms of shaping instruction (e.g., Schoenfeld 1998; Calderhead 1996). Additionally, it appears that mathematicians are not completely rigid in how they teach. For instance, mathematics education researchers have observed a variety of practices – including questioning and student presentations – on the part of instructors who self-identify as using lecture-based pedagogy (e.g., Fukawa-Connelly 2012)

Within this growing body of research into pedagogical decision-making and practitioner reflections on their own teaching, two factors arise again and again when discussing the choice to lecture: a belief that lecture is best and coverage concerns. Presumably, instructors are not going to choose to abandon the lecture style if they believe it to be the best way to teach. For instance in Wu's (1999) defense of lecture, he expresses this belief ardently, touting two main strengths: 1) lecture allows the professor's understanding and vision of the subject to provide proper guidance to students, and 2) lecture enables the teacher to cover all of the material needed in the amount of time available.

Such concerns about coverage are ubiquitous when discussing decisions about instructional approaches (e.g., Johnson et al. 2013; Roth McDuffie and Graeber 2003; Wagner et al. 2007). For instance, when discussing his experience implementing non-lecture abstract algebra curriculum materials, Caughman (in Johnson et al. 2013) expressed concerns about content coverage at the level of daily practice and overall course content and was apprehensive about his inability to control the amount of daily progress. Similarly, in a case study of two mathematicians attempting to implement reform curriculum in mathematics courses for pre-service teachers, Roth McDuffie and Graeber (2003) documented that even when teachers *want* to adopt non-lecture approaches, it is difficult to escape the limitations that coverage concerns present. As stated by one of the mathematicians:

> If you've got courses that link together, as most of the math curriculum does… there's an expectation that a certain amount of material be covered…It means that you're limited on how much time you can spend to do real constructivist activities where the depth of knowledge is really greater. (p. 336)

These studies illustrate that it is not simply internal beliefs about teaching that are guiding decisions about instructional approaches, but that perceived external factors exert considerable influence as well. Apart from coverage concerns, perceived notions concerning departmental expectations, lack of support from colleagues or supervisors, and a lack of common vision for reform among the faculty (Roth McDuffie and Graeber 2003; Henderson and Dancy 2007) collectively factor significantly when instructors plan courses. For instance, while the professors in Roth McDuffie and Graeber's (2003) study believed they could make changes to some courses, there was no pressure from the administration to do so and none of their colleagues voiced active support.

This literature identifies the importance of instructors' beliefs and institutional/departmental context – both of which were highlighted by Henderson et al. (2011) as central to understanding pedagogical decision-making and instructional change. Hence, this study explores beliefs and goals for instruction, perceived constraints, and the types of practices that instructors claim to use, as well as their self-identification as a lecturer (or not). By understanding these factors, we hope to better explain instructors' decision making and describe any differences between those who lecture and those who do not. For those who advocate for reform, better understanding of the extant practice and beliefs can be used to identify key factors that can inform reform and propagation efforts.

Specifically, the present report is based on a survey of US abstract algebra instructors designed to document their typical practices, to analyze their beliefs about teaching and learning, and to explore the contextual affordances and constraints they experience. We investigate the following research questions:

1) What pedagogical practices characterize abstract algebra instruction?
2) For abstract algebra professors who self-identify as "Lecturers", what affordances and constraints on their use of non-lecture practices do they perceive and how do these align with those described in the research literature?
3) What commonalities/differences are there among faculty who choose to lecture and those who do not?

Study Context, Data, and Methods

Study Context

To study these research questions, we focused on a particular subset of mathematics instructors – those who teach abstract algebra in US universities that offer either a PhD or Master's degree in mathematics. Here we will provide some contextual information and rationale for those choices.

Course Context and Rationale We decided to focus on abstract algebra for the following three reasons. First, as part of a joint initiative between five US professional associations in the mathematical sciences: the American Mathematics Association for Two Year Colleges, the AMS, the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM), a course guide for abstract algebra was recently released that called for an increase in active student engagement.

> Although we refrain from specifying pedagogical practices, we do feel that active student engagement is necessary for a mastery of algebraic ideas. In particular, it is essential that students should wrestle with hard problems and communicate their solutions with care, in writing and in speaking. In addition, problem-based, inquiry-based and collaborative learning activities are appropriate means of maintaining student engagement. (The Common Vision Committee on the Undergraduate Program in Mathematics 2015)

This statement suggests that there is support from the broader US mathematics community for a decreased proportion of lecture-based pedagogy.

Second, in terms of reform initiatives targeting the upper-division mathematics courses, it is quite possible that none has received a comparable amount of attention as has abstract algebra. The Inquiry Based Learning community and the MAA's Project NeXT are active in abstract algebra (e.g. Ernst 2016; Gallian et al. 2000; Hodge et al. 2013) and three sets of curricular innovations have been developed by mathematics education researchers (Cook 2014; Dubinsky and Leron 1994; Larsen et al. 2013). All of these initiatives include efforts for changing the undergraduate abstract algebra experience; namely, with more student-centered pedagogy.

Finally, in many ways abstract algebra instructors appear to be relatively well-positioned to adopt non-lecture practices. An introductory abstract algebra course is normally taught to upper division mathematics majors in small classes, typically less than 40 students, for whom only a handful will subsequently matriculate into a follow-up course. It is common for at most a handful of sections to run per year and thus issues of coordination across multiple sections and/or instructors are few. These courses, more often than not, are taught by tenured or tenure-track professors who may have greater job security and more autonomy over course material as compared with instructors and itinerant faculty members.

With all of these logistical obstacles addressed, and the curricular innovations and supports available, one could expect to see a significant portion of abstract algebra instructors free to adopt non-lecture pedagogy. Indeed one might expect abstract algebra to be leading a non-lecture revolution across mathematics departments. However, a lack of widespread adoption of non-lecture techniques would suggest the presence of beliefs and institutional-specific constraints that are influencing pedagogical decision-making. The aim of our research is to better understand the nature of instruction in abstract algebra courses across the nation and, if we are correct in our supposition that reform efforts have done little to steer instruction away from traditional lecture-based delivery methods, explore why there has been little change.

If we are correct in assuming that abstract algebra courses are still overwhelmingly lecture-based, it would suggest that the field of mathematics education research (in addition to the national professional organizations) might have misplaced beliefs about what change is possible, or more likely, that we are missing or misunderstanding something fundamental about the class, the instructors, or the instructors' beliefs about the class, students, and learning. Such knowledge is vital to enacting real change in instruction. As Henderson et al. (2011) found, developing and disseminating "best practices" curricular materials and "top down" policy changes are highly ineffective change strategies at the undergraduate level. Instead, we need change strategies that meaningfully take into account the instructors, their beliefs, and their institutional context. The aim of this paper is contribute to such a knowledge base.

University Context and Rationale We chose to focus this study on universities that offer a graduate degree in mathematics (either Master's or PhD) for several reasons. These universities: employ 57% of the full-time mathematics faculty, award 64% of the undergraduate mathematics degrees, and account for 69% of the enrollment in advanced mathematics courses (Blair et al. 2013). Thus, these universities capture both the majority of mathematics faculty and mathematics students. They also offer a nice range of faculty positions and responsibilities. This includes tenured and tenure-track faculty at research-intensive universities who will have heavy research expectations and lower teaching loads (typically two courses a term), tenured and tenure-track faculty at "teaching" universities with fewer research expectations but heavier teaching loads (three or four courses a term), and instructors whose main responsibility is teaching (typically four courses a term). Regardless of the position, 88% of these faculty members hold a PhD in mathematics (Blair et al. 2013).

In regard to teacher preparation, new collegiate faculty members typically have little pedagogical training. What training they did receive would have primarily occurred when they were new graduate students learning how to become graduate teaching assistants (GTAs). In one report of 23 PhD-granting institutions (which the majority of mathematics faculty would have attended), 35.4% of GTAs taught their own introductory mathematics courses, 39.1% lead recitation sections for an introductory mathematics course, and 24.5% were tutors and/or graders (Benlap and Allread, 2009). Prior to taking on these roles, most students have no prior teaching experience and the GTA training provided usually consists of a brief (ranging from a few hours to few days) orientation sessions. During these sessions:

> They may receive information about the specific course they are teaching and a list of tasks they are expected to perform (such as grading homework, administering quizzes, holding exam review sessions, and so on). They might also receive information about teaching, learning, and interacting with students. In some cases, new TAs have the opportunity to practice teaching (often briefly) and receive feedback (often superficial) from their peers or instructors running the orientation sessions" (Speer, Gutmann, and Murphy, 2005).

After these students complete their graduate programs, there are some limited avenues to participate in professional development opportunities. One such example includes the MAA's Project NExT, which works with 80–100 new faculty members a year during a 3-day workshop focusing on all aspects of an academic career (including teaching, research, and service).

Survey Design

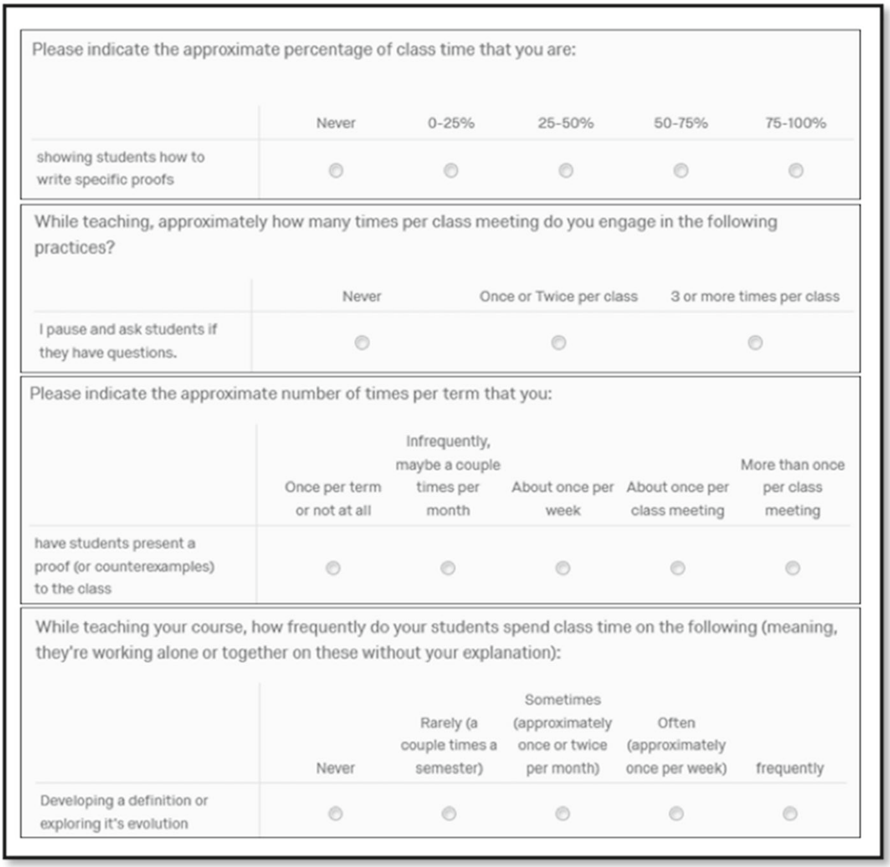
We developed a survey[1] designed to solicit information about the teaching practices, beliefs, and situational context of abstract algebra instructors. This survey was informed in part by both Henderson and Dancy's physics-education survey (Henderson and Dancy 2009) and the Characteristics of Successful Programs in College Calculus surveys.[2] Our survey had sections to assess each of the following types of information: basic demographics and course context, teaching practices, beliefs and influences (including perceived supports/constraints), and knowledge of/openness to non-lecture practices. The survey was designed to solicit similar information in multiple items and the collection of questions was selected with the intention of highlighting differences between lecture and non-lecture modes of instruction. We anticipated differences in the aggregate responses of the "Lecturers" and "Non-Lecturers" in terms of behaviors reported, and furthermore, that the participants' responses within each group might be quite disparate.

In the basic demographics and course context section, the participants were asked to indicate their experience, both with teaching in general and abstract algebra specifically. They were asked to describe the nature of their course (i.e. "rings first", "groups first", etc.) as well as the structure (i.e. "capstone course", "math majors only", etc.). We also asked for approximate grade distributions.

To determine teaching practices, we created five questions – each of which was composed of a list of items to be rated on a frequency scale. The respondents were asked to reflect on their teaching practice by describing classroom activities, pedagogical choices, and level of student engagement in terms of frequency per class meeting, percentage of time use, and/or frequency per semester. A sample of these items can be seen in Fig. 1.

[1] Interested readers can see a version of the survey at: pcrg.gse.rutgers.edu/algebra-survey

[2] See www.maa.org/cspcc for more information about the CSPCC project and a copy of the surveys



Please indicate the approximate percentage of class time that you are:

	Never	0-25%	25-50%	50-75%	75-100%
showing students how to write specific proofs	○	○	○	○	○

While teaching, approximately how many times per class meeting do you engage in the following practices?

	Never	Once or Twice per class	3 or more times per class
I pause and ask students if they have questions.	○	○	○

Please indicate the approximate number of times per term that you:

	Once per term or not at all	Infrequently, maybe a couple times per month	About once per week	About once per class meeting	More than once per class meeting
have students present a proof (or counterexamples) to the class	○	○	○	○	○

While teaching your course, how frequently do your students spend class time on the following (meaning, they're working alone or together on these without your explanation):

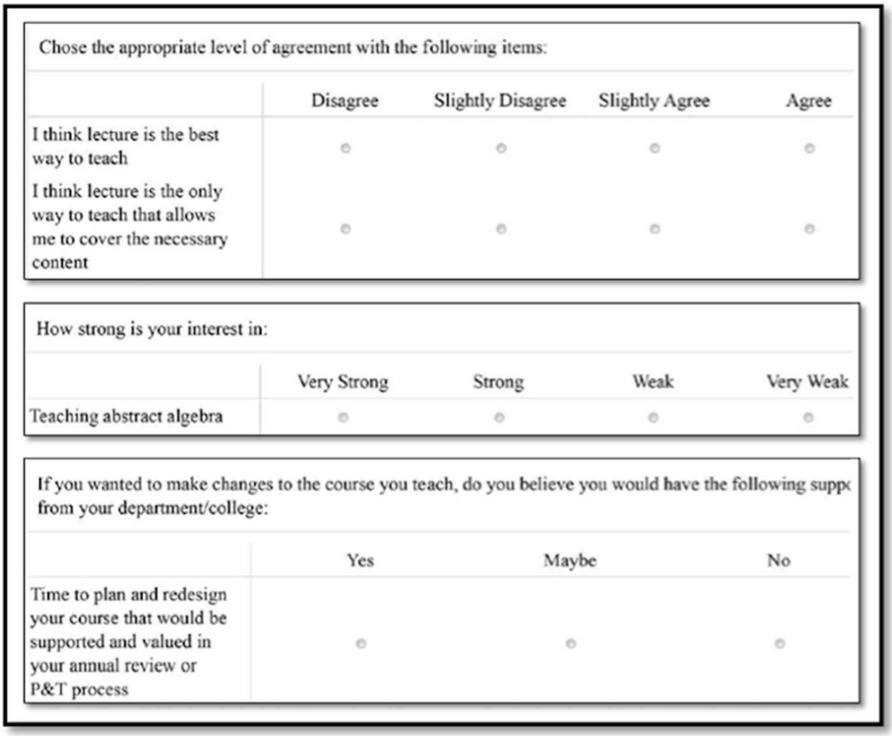
	Never	Rarely (a couple times a semester)	Sometimes (approximately once or twice per month)	Often (approximately once per week)	frequently
Developing a definition or exploring it's evolution	○	○	○	○	○

In the instructor survey they were asked to respond to "Developing a definition or explaining it's [sic] evolution"

Fig. 1 Sample of teaching practices items

To elicit information regarding their beliefs about teaching and learning, we used a combination of Likert scale and open response items. We sought to investigate instructors' beliefs about what students should be doing inside and outside of class as well as the 'best' ways to teach, and, indirectly, the speed at which students can learn as ascertained through responses concerning the appropriateness of the amount of content that they cover. Also included in the beliefs section were items asking how influential a number of different resources were on pedagogical decision-making (e.g., reading PRIMUS and MAA notes volumes, having conversations with colleagues about course content, and personal experiences as a student). To investigate perceived supports and constraints, we included items about commonly cited barriers to instructional change. A sample of these items can be seen in Fig. 2.

In the final section of the instrument, we hoped to classify instructors' existing knowledge of non-lecture pedagogy and gauge their willingness to adopt such practices. Based on their response to the following prompt: *Have you ever taught abstract algebra in a non-lecture format?*, participants were directed to appropriate follow-up questions that either asked them



Chose the appropriate level of agreement with the following items:

	Disagree	Slightly Disagree	Slightly Agree	Agree
I think lecture is the best way to teach				
I think lecture is the only way to teach that allows me to cover the necessary content				

How strong is your interest in:

	Very Strong	Strong	Weak	Very Weak
Teaching abstract algebra				

If you wanted to make changes to the course you teach, do you believe you would have the following suppo from your department/college:

	Yes	Maybe	No
Time to plan and redesign your course that would be supported and valued in your annual review or P&T process			

Fig. 2 Sample of beliefs items

to categorize their non-lecture approach (i.e. "Moore Method", "Inquiry-Based", etc.) or asked them to provide rationale as to why they haven't, or why they wouldn't, consider teaching in a non-lecture format. All participants, regardless of their lecture/non-lecture identification, were also asked to describe their level of satisfaction with their students' learning as a write-in response.

Participants

Survey requests were sent to departmental administrators at approximately 200 institutions, targeting instructors who teach undergraduate abstract algebra. Our intention was to survey instructors at Master's- and PhD-granting institutions; however, a small portion of our respondents (9%) did come from schools that only offered a Bachelor's degree in Mathematics. In total, 129 participants completed[3] the survey. On the whole, the respondents (92% tenure-stream faculty) had significant experience, both with teaching in general (81% reporting 6+ years) and abstract algebra specifically, and were most likely to be teaching an undergraduate groups-first course designed for a mixed (i.e. education, physics, engineering majors commingled with pure math majors) audience (see Fig. 3).

[3] Here completed means that a participant viewed, but not necessarily responded to, each survey item. When looking at individual survey items, the number of responses varies between 110 and 129. For each survey item in the results, we report the number of responses analyzed.

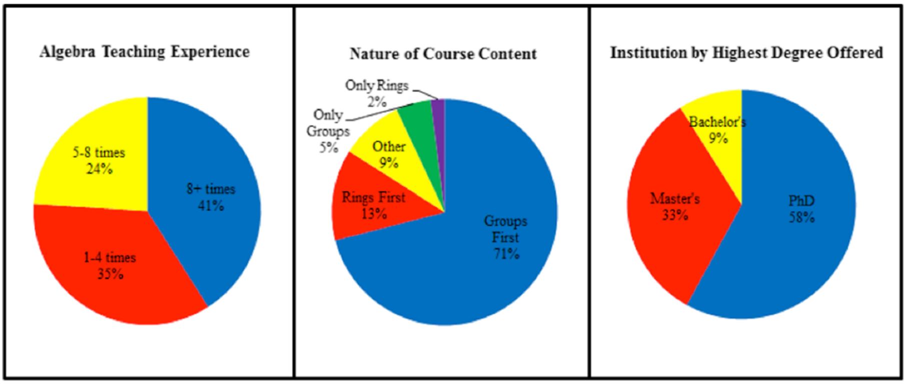


Fig. 3 Information about survey respondents

Data

Due to the length of our instrument and the complexity and variation in the responses, there was far too much data to be properly analyzed in a single paper. In the *Methods* section, we have reported on the particular analyses and subset of the data we felt most closely aligned with our theoretical framework and our research questions. Nonetheless, we feel there is value added by providing additional information related to interesting data that was otherwise unanalyzed by this report. For instance, through this survey we have been able to determine that the mean rate for students receiving a grade of "D", "F", or withdrawing from the course is 12.53% and over 78% of instructors teach groups first then rings, or only groups, in their abstract algebra course.

Methods of Analysis

Research Question 1 What pedagogical practices characterize abstract algebra instruction? To answer this question, we consider three sub-questions: What kinds of pedagogical practices do abstract algebra instructors report using? What range of practices do lecturers claim to use? How are these similar to, and different than, those used by non-lecture instructors? This allowed us to investigate the propensity for instructors to engage in different pedagogical practices, but more importantly, if differences existed between those who lecture and those who do not.

The distinction between those who lectured and those who did not was made using the prompt, *Have you ever taught abstract algebra in a non-lecture format?*, for which we coded the respondents as either self-identified lecturers (hereafter referred to as "Lecturers") for responding *No* or *I have in the past, but I currently lecture*; or as self-identified non-lecturers (hereafter referred to as "Non-Lecturers") for responding *I currently do*. A couple of notes about the interpretation of this prompt are needed. First, given the course in question, we assume that instructors answered this question in regards to their own teaching practice as opposed to the course structure or label (e.g., a course designated as a lecture, a seminar, or a recitation). While some high enrollment lower-division mathematics courses (e.g., calculus) are taught in a lecture/recitation structure, in the US this is highly uncommon for upper division courses, where typical class sizes average between 12 and 20 students (Blair et al. 2013). Second, we want to note that this question asks the instructors to characterize their teaching as either lecture or not and we did not provide a definition for what is a "lecture" or "non-lecture" format. Rather, we left this open for the instructors to self-identify with however they chose to characterize their approach on this dichotomy.

In many ways this is a false dichotomy, one that we were able to investigate by analyzing the reported teaching practices of the Lecturers and the Non-Lecturers. Thus, as researchers, we are not making claims about the teaching practices of those who identified as Lecturers and Non-Lecturers; we are rather using these self-identified labels as a way to talk about the variability between and within these two groups of instructors. Due to the fact that the majority of our participants identified as Lecturers (107/126 = 85%) and therefore their responses would dominate the aggregate data, we chose to report instructors' claimed pedagogical practices only in groups. Independent samples *t*-tests were used to test for significance on each of the pair-wise comparisons (controlling the family-wise error rate for multiple comparisons using the Holm-Bonferroni method).

Research Question 2 For survey participants who self-identified as "Lecturers", what affordances and constraints on their use of non-lecture practices do they perceive and how do these align with those described in the research literature?

Based on our literature review of common reasons and impediments offered as explanations for why instructors do not attempt or maintain the use of non-lecture pedagogical techniques, we designed and analyzed two survey items. The respondents were asked: *Would you ever consider teaching abstract algebra in a non-lecture format?* and, based on their response, were directed to one of two follow-up questions asking them to explain either why they have not or why they would not. For this data, only basic descriptive statistics were computed; however, suspicions about the veracity of these claims called for the computation of cross-tabulations and conditional frequencies/probabilities as a means of identifying contradictory response patterns. Investigating survey questions that were designed to elicit similar information across multiple items allowed us to identify these contradictory response patterns. For instance, we analyzed various levels of perceived departmental supports when a lack of support was cited as a reason for not engaging in particular pedagogical practices.

Research Question 3 What commonalities/differences are there among faculty who choose to lecture and those who do not?

The primary methodological objective in this stage of the analysis was to build a predictive model for classifying respondents as Lecturers or Non-Lecturers based on various beliefs, influences, demographics, and contextual factors. As a precursor to variable selection, we ran an intermediate analysis on the beliefs and professional interest items. The differences in the group means (Lecturers vs. Non-Lecturers) were compared on strength of belief or level of interest reported for each item using independent samples t-tests to evaluate significance.

Then, to ready the data for model-building, we had to sort, organize, and recode our candidate predictors. Some items were left unaltered (e.g. *amount of teaching experience)*; however, for most items, that meant dimension reduction to binary classification (i.e. 4 levels of a Likert scale were reduced to simply *Agree/Disagree* or *Weak/Strong*, etc.). Write-in responses were not considered, the one exception being satisfaction with student learning. This exception was made because we believed dissatisfaction could represent impetus to change practice. We had 110 write-in responses to the prompt "How satisfied are you with your students' learning? Please give some explanation". The comments were organized by domain and level of satisfaction to allow us to observe common themes and assign an overall score to each response. Of the 110 responses, we were able to characterize 25 as *Very Satisfied* (e.g., my students work hard, my students produce high quality projects, my students leave my class prepared for future advanced coursework), 48 as *Moderately Satisfied* (e.g., most of my students have weak proof backgrounds but develop this over the course, my students get decent grades on exams, but not as good as I would like), and 24 as *Dissatisfied* (e.g., a large portion of my students fail or withdraw, my students don't appreciate the material)*;* the remaining 13 were too ambiguous to warrant categorization.

Ultimately, 26 candidate predictors were selected from amongst the survey items to represent a range of categories: demographics, beliefs, interests, resources, constraints, influences, and contextual factors. Please see Appendix 1 for a description of the variable codes and reference categories. Two binomial logistic regression models were developed using the responses to *Have you ever taught AA in a non-lecture format?* and *Would you consider teaching in a non-lecture format?* as the dependent variables. A forward stepwise likelihood ratio (LR) regression method was employed here because he had a large potential list of predictors from which we wished to extract a few. Simple coding was used for the categorical covariates so that each level could be compared to the reference level.

Owing to the large subset of the survey items for which personal beliefs and/or interests were evaluated, we felt that it was reasonable to assume that some underlying latent constructs might exist that would help us to understand the similarities and differences among our participants. To explore that hypothesis, and with the aim of dimension reduction, as a secondary analysis, we conducted an exploratory factor analysis (EFA) using all ten beliefs items (See Figs. 10 and 11) and three items that reflected interest in professional activities. After checking that our data satisfactorily met the underlying assumptions for EFA, we ran a parallel analysis – the result of which was to indicate a five-factor solution. Using SPSS, five factors were extracted using the maximum-likelihood (ML) approach with an oblimin rotation method. Analysis of the factor correlation matrix indicated that an orthogonal rotation was justified (all correlations $< .30$) and thus subsequent analysis led to a set of three extractions (4, 5, and 6-factor solutions) again using the ML approach but with a varimax rotation method this time. Based on the total variance explained and the loading structure of the items, we determined that the six-factor solution was preferred since there was no double-loading in this extraction (loadings $< .5$ were suppressed). Interpretive labels were assigned to each factor and internal reliability estimates were computed where appropriate.

Based on the results thus far, it was our hypothesis that the richest source of data, and possibly the key to characterizing the differences between Lecturers and Non-Lecturers, was in the interest and beliefs items. Additional logistic regression models were computed for the same two response variables as indicated previously, but this time limiting the list of candidate predictors to those factors extracted in the EFA. The intention was to compare the performance of the original models, based primarily on demographic characteristics of the participants and externally situated factors (e.g. resources, influences, constraints), with that of the updated models which were based primarily on internally situated beliefs and interests.

Results and Discussion

Research Question 1 – Pedagogical Practices

The majority of respondents (107/126) self-identified as Lecturers and this result was independent of teaching experience, nature of course, or nature of institution. As expected, there were differences in the teaching practices reported between the Lecturers and Non-Lecturers. When given a choice of six pedagogical activities (see Fig. 4) Non-Lecturers are, on average, more likely than Lecturers to report engaging in every single one of the behaviors; however, only two of these comparisons were statistically significant: *Have students engage in small group discussions* and *Have students ask each other questions* (See Appendix Tables 1 and 2 for statistical details).

Similar analysis of the next set of items revealed a familiar pattern. Here the participants were asked to report the percentage of class time (in 25% increments) that was spent doing seven instructional practices (see Fig. 5). Non-Lecturers are less likely to report engaging in the teacher-centric activities (*Lecturing, Showing students how to write proofs*) and more likely to report engaging in the student-centric activities (*Having students explain thinking, Have students work in small groups,* etc.). (See Appendix Tables 1 and 2 for statistical details.)

Interesting to note is that, on average, Lecturers report that they are not talking the whole class period. Instead, Lecturers report incorporating a wide variety of teaching practices to supplement their lecture: students explaining their thinking, small group

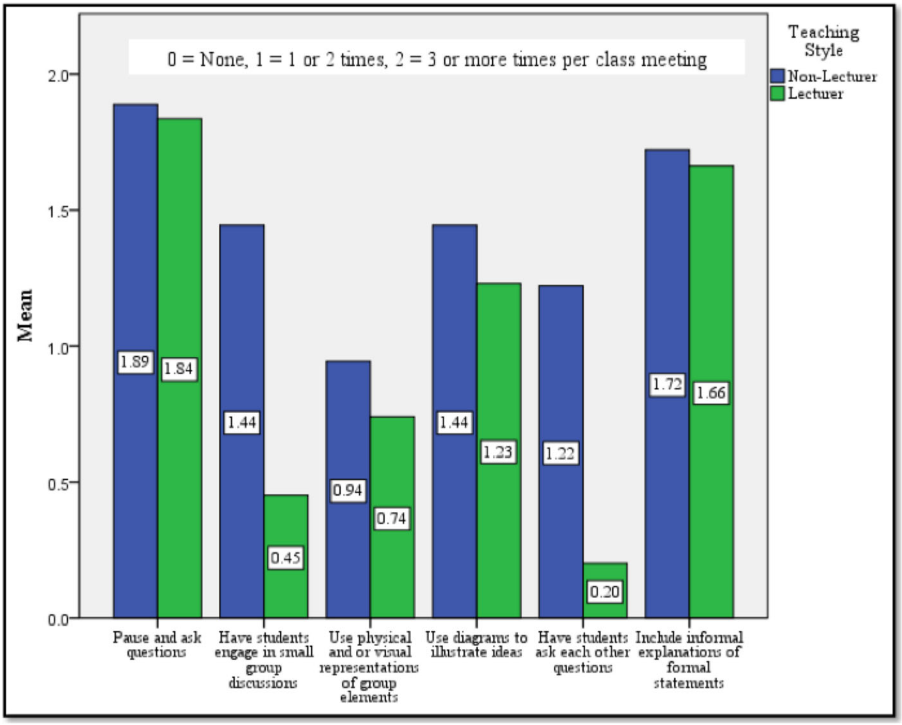


Fig. 4 While teaching, approximately how many times per class meeting do you

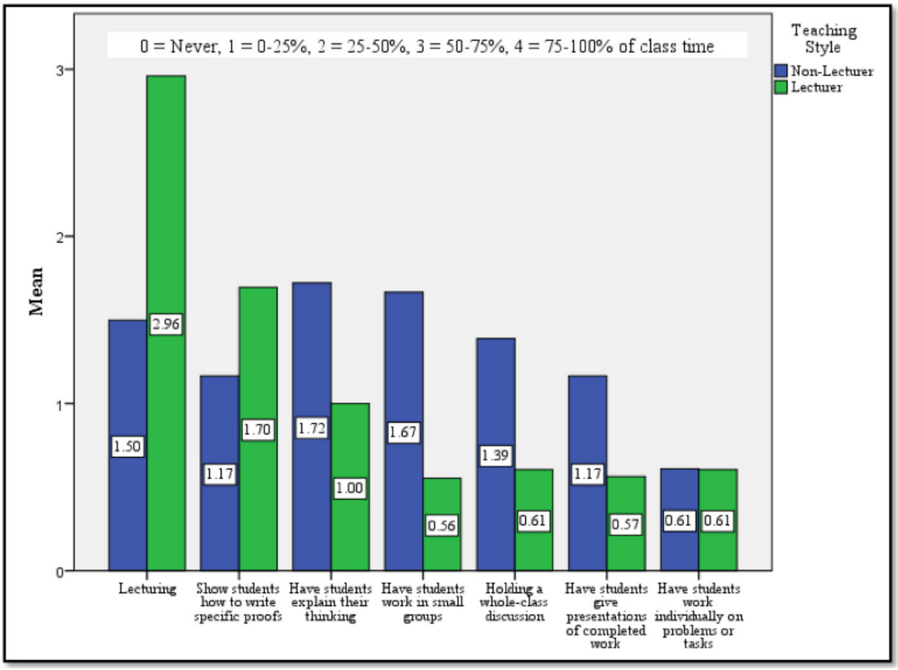


Fig. 5 While teaching, what is the approximate amount of time per class that you

work, whole-class discussions, student presentations, and individual work are all being reported with some frequency. Another important feature of this analysis is that even Non-Lecturers still report the use of lecturing as a teaching practice. Thus, it appears that lecture is nonetheless an important pedagogical tool for our non-lecture participants; however, the average time spent lecturing is reduced to allow for the use of other instructional practices.

For the final set of items analyzed (Fig. 6), the prompt instructed respondents to report on the frequency in which their *students engaged* in particular mathematical activities or tasks. Participants were asked to report frequency of 8 activities on a 5-point scale (see Fig. 6). The analysis showed that again there were differences in what was reported between the Lecturers and Non-Lecturers on nearly every item, particular significance seen in *Developing or explaining a theorem, Developing or critiquing a proof,* and *Developing examples and/or counterexamples of a construct.* (See Appendix Tables 1 and 2 for statistical details.)

In addition to showing that the reported *quantity* of classroom time spent on teacher-centric versus student-centric activities is quite different for that of Lecturers and Non-Lecturers, our data also indicate that there are differences in the nature of the students' activities. Not only are Non-Lecturers more likely to report allowing the students a chance to work individually and collectively, but the nature of the assigned tasks is different as well. The evidence shows that while there is no difference in computational-type exercises (i.e. *Doing Calculations*), Non-Lecturers report providing the students more opportunities to explore and develop the ideas and concepts for themselves than their Lecturer counterparts.

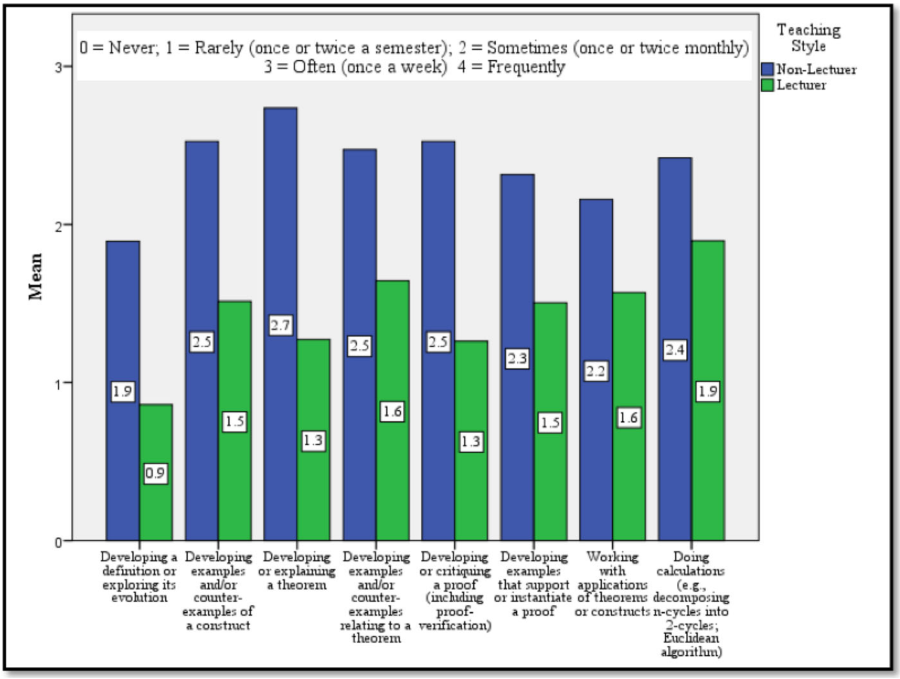


Fig. 6 While teaching, how frequently do your students spend class time

Interestingly, even given these differences, when we look at the participants as a whole (i.e., not separating Lecturers from Non-Lecturers) we do get remarkable agreement on some items. For instance, faculty overwhelmingly reported that definitions, theorems and proofs are presented at least once a week (97%). They report that examples were shown at only slightly lower rates: 93% reported that examples were used to demonstrate constructs at least once a week, 98% reported that examples were used to demonstrate or explain a theorem at least once per week, 90% reported that examples were used to support a proof at least once per week, and 78% reported counter-examples were used at least once per week. Finally, contrary to some claims in the literature (e.g., Davis and Hersh 1981; Dreyfus 1991), faculty reported that alternative and informal representations were used frequently: 96% reported that informal explanations of formal statements were given at least once per class period, 92% reported that diagrams were used to illustrate ideas at least once per class period, and 64% reported that visual or physical representations of group elements were used at least once per class period.

Research Question 2 – Constraints on non-Lecture Approaches

The research literature has identified a number of departmental constraints that impede instructors from transitioning to a non-lecture pedagogical approach. Commonly cited constraints include: lack of release time for course redesign, lack of support for attending professional development opportunities, and departmental expectations about content coverage (Henderson et al. 2011; Henderson and Dancy 2007; Roth McDuffie and Graeber 2003; Wagner et al. 2007). In order to investigate the extent to which the Lecturers in our study perceived such constraints, we asked the following two questions:

1) For Lecturers who said that they would consider teaching in a non-lecture format, we asked: "*Why haven't you*"*?*
2) For Lecturers who said that they would not consider teaching in a non-lecture format, we asked: "*Why wouldn't you*"*?*

Frequency tabulations (Fig. 7) indicate there are a variety of reasons for why Lecturers continue to lecture. However, given the options provided, the majority of instructors cite time – time outside of class to redesign their courses, or time inside of class to cover all the content – as the most significant impediment.

In designing the survey, we purposely asked similar questions in multiple formats. This afforded us the opportunity to probe the complexity of these beliefs and investigate contradictory response patterns. After tabulating the most frequently cited constraints, we cross-referenced this with analogous items from other sections of the survey. In doing so, we discovered several interesting paradoxes. For instance, 14 participants selected, "I don't have the support of my department needed to implement that sort of change", and another 30 selected, "I haven't had time to redesign my course". However, when asked directly about departmental supports, only a very small proportion of our total respondents (See Fig. 8) reported that they *did not* believe departmental supports existed. This inconsistency in the data has two possible explanations. One, the subset of Lecturers answering our survey disproportionately reported a lack of departmental supports; or two, the subset of Lecturers provided conflicting responses. We investigated the former of these possibilities (See Fig. 9), and finding no evidentiary support, turned our attention to trying to understand the conflicting and contradictory responses we obtained about constraints for implementing non-lecture teaching practices.

Lack of time is the leading perceived external constraint for not attempting non-lecture pedagogy with 30/129 respondents listing this as a reason. When those 30 instructors were asked if they feel like their "job requirements allow [them] to spend as

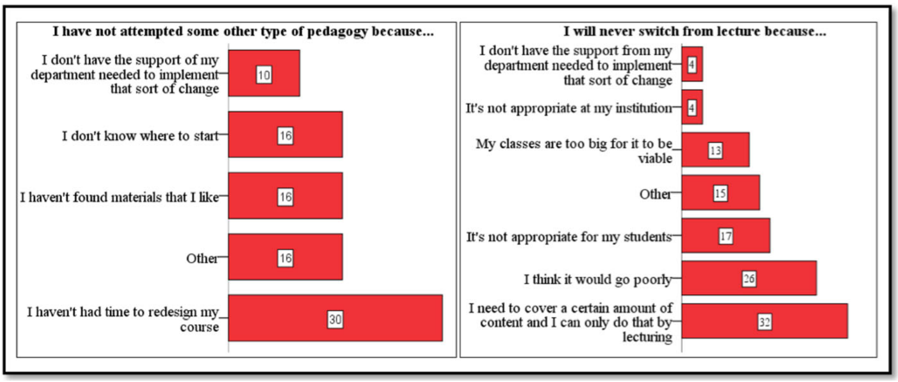


Fig. 7 Reasons selected when asked Why haven't you? or Why wouldn't you?

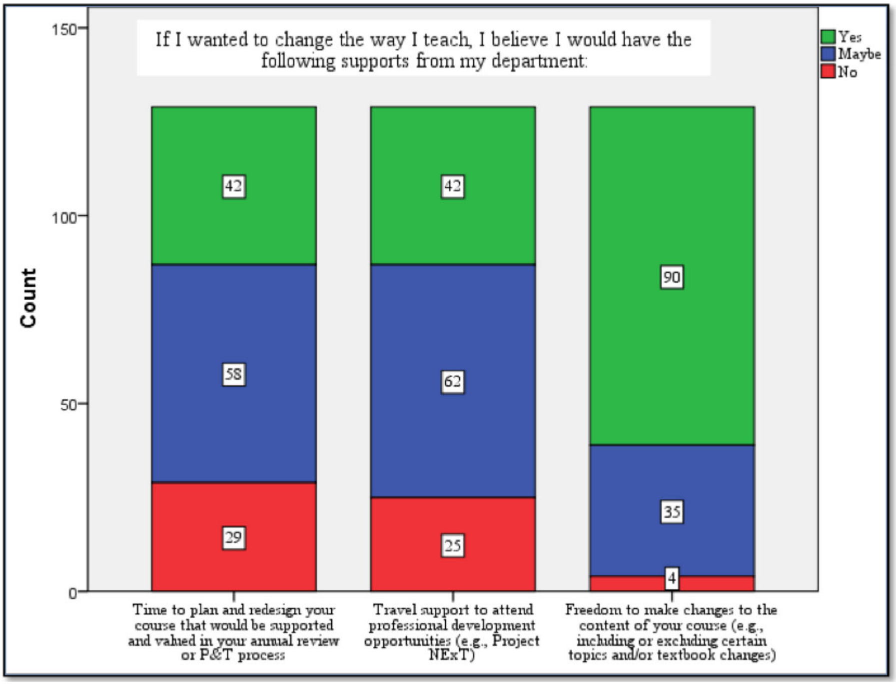


Fig. 8 Belief in departmental support, all participants

much time as [they] would like on teaching and preparing for class (including improving courses)", 19/30 said "no"; but, when we asked those 30 respondents if they believed their department/college would provide support to change their teaching in the form of "time to plan and redesign [their] course that would be supported and valued in [their] annual review or P&T process" only 6 of the 30 disagreed, with 15 saying "maybe" and 9 saying "yes". Taken together, it appears that instructors do not feel like they have as much time as they want for teaching (or for redesigning their courses). However, this constraint does not appear to be directly originating from their departments.

Similarly, perceived content pressure is the leading external constraint cited for unwillingness to consider adopting a non-lecture approach. We asked our participants "Would you ever consider teaching abstract algebra in a non-lecture format? Or, if you used to but no longer do, would you ever consider doing so again?", 47 respondents said "no". When asked, "Why wouldn't you?", 32/47 selected "I need to cover a certain amount of material and I can only do that by lecturing". However, of these 32 respondents, 23 said that they *did not* feel pressure from their department to cover a fixed set of material in their abstract algebra course. Again, this constraint does not appear to be directly originating from their departments.

Finally, there were several reasons for why instructors lecture that we believe could be ameliorated with professional development opportunities. These include the "I don't know where to start", "I haven't found materials that I like", and "I think it would go poorly" responses. 51 participants selected at least one of these reasons. Of those 51 participants, 16 said "yes" their department would provide travel support for professional

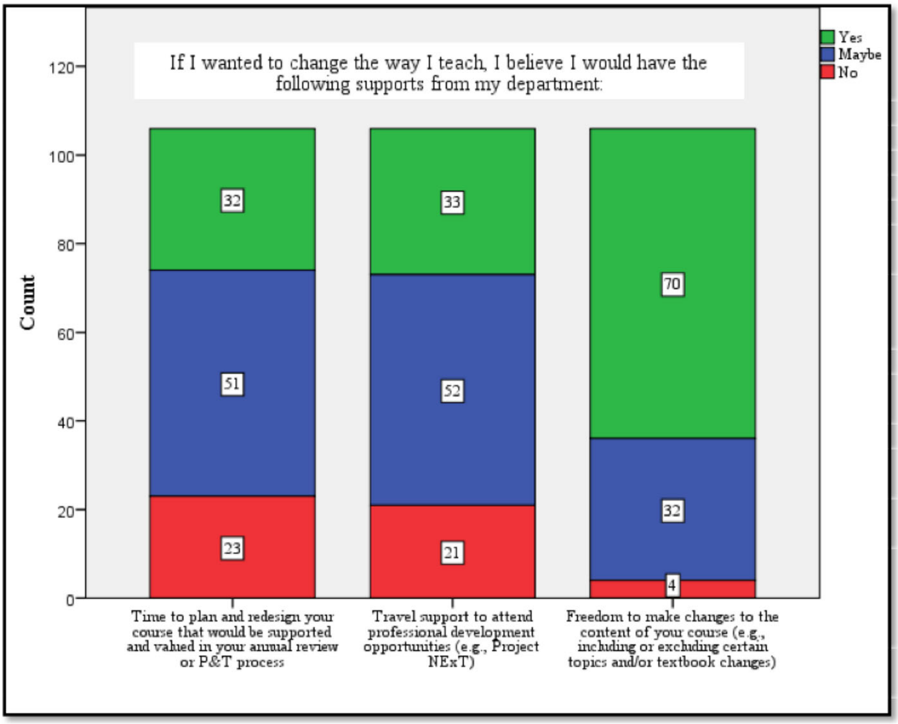


Fig. 9 Belief in departmental support, lecturers

development opportunities, and another 24 said "maybe". We argue that these 51 participants could benefit from professional development, and 40 of them believe that they might have the financial support needed to attend. However, there again appears to be something other than departmental constraints inhibiting their participation.

Collectively, these results suggest that instructors *could* have support to attend professional development opportunities; they *could* have the time and departmental approval to redesign their courses; and they *are not* constrained by departmental-level coverage pressure. It may be the case that these commonly cited reasons might simply be post-hoc justifications used to justify the teaching practice they believe is best. However, it may also be the case that these Lecturers *feel* like they do not have the time, freedom, or supports needed to make changes. In that case, these commonly cited external constraints might actually be internalized constraints.

There is research to support the idea that concerns about coverage, for instance, are influenced by more than external departmental expectations. For instance, Johnson et al. (2015) found no association between teachers' expected pacing rates (sections per week) and their reported coverage concerns. Additionally, there has been some research indicating that coverage concerns may be influenced by instructors' beliefs about their students' more general mathematics education. As discussed by Caughman (in Johnson et al. 2013), his coverage concerns were based on his belief that the point of group theory was to develop a "certain level of sophistication" (p. 749). This sophistication, for example, would be reflected with addressing and understanding the First Isomorphism Theorem. Thus, it was not that the department was requiring him to teach that theorem; instead it was

Caughman's beliefs about mathematics and the role of abstract algebra in his students' mathematical development that were major factors for his coverage concerns.

Research Question 3 – Commonalities/Differences between Lecturers and non-Lecturers

In order to understand the differences between our three groups of participants (those who do not lecture, those who lecture and would *not* consider changing, and those who lecture and *would* consider changing), we conducted a multi-stage analysis as outlined in the methods section. Preliminary descriptive analysis of beliefs/professional interest items was performed, followed by the first round of logistic regression model building, exploratory factory analysis, and finally, the second round of logistic regression model building. We present the results in sub-sections so titled.

Descriptive Analysis of Belief Items Our initial analysis focused on the instructors' reported beliefs. Beliefs were singled out for a number of reasons. First, research indicates that beliefs about teaching and learning are significant in terms of shaping instruction (e.g., Schoenfeld 1998; Calderhead 1996). Second, our analysis of our second research question suggests that internalized *beliefs* about constraints are more influential than actual departmental constraints. We begin with beliefs about teaching and then move to beliefs about students.

Figure 10 displays the mean response of Lecturers and Non-Lecturers to five beliefs about teaching statements. These questions were asked on a 4-point Likert scale, ranging from "strongly agree" to "strongly disagree". The magnitude of the bar reflects

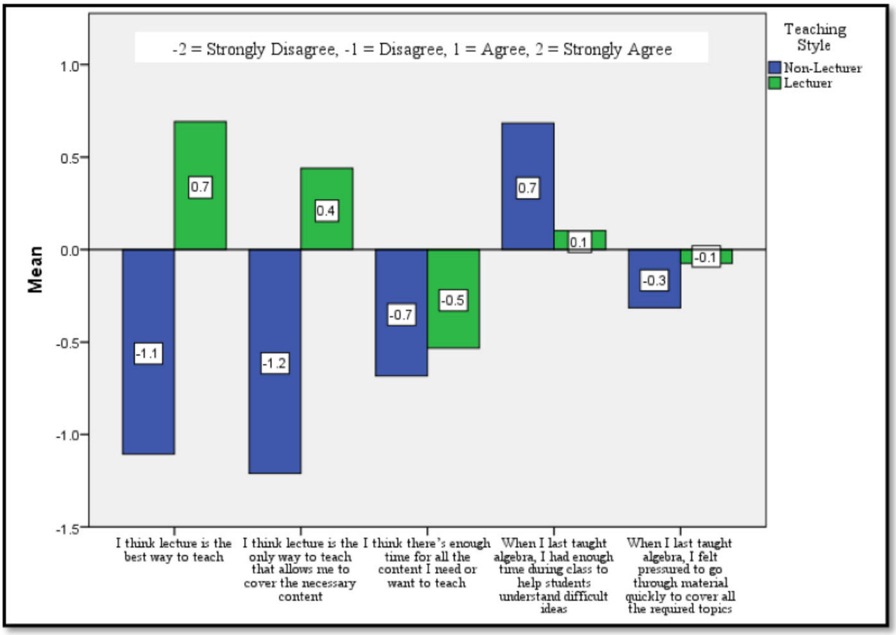


Fig. 10 Beliefs about teaching

the strength of agreement (positive y-axis) or disagreement (negative y-axis). Unsurprisingly, positive beliefs about lecturing were held much more strongly by Lecturers than Non-Lecturers, with particular significance for both *I think lecture is the best way to teach* and *I think lecture is the only way to teach that allows me to cover the necessary content*. As none of the other pairwise comparisons were found to be significant (See Appendix Tables 1 and 2 for details), it would appear that both Lecturers and Non-Lecturers are generally in agreement that *there wasn't enough time for all the content* and that they *felt pressured to cover topics quickly without enough time to help students understand difficult ideas*.

In Fig. 11 we display the mean response of Lecturers and Non-Lecturers to five belief statements about students and learning. Again, these questions were asked on a 4-point Likert scale, ranging from "strongly agree" to "strongly disagree" and the magnitude of the bar reflects the strength of agreement (positive y-axis) or disagreement (negative y-axis). The level of agreement with the following statement: *I think that students learn better when they do mathematical work (in addition to taking notes and attending to the lecture) in class* was stronger for Non-Lecturers, and was the only significant difference between the two groups. Interestingly, nearly 50% of all faculty disagreed that *students can learn advanced mathematics* in general (61/125) and *abstract algebra* (60/125) specifically. While we were unable to find for statistically significant differences between the groups on these beliefs, it is interesting that they trend in opposite directions.

First Round of Model Building In light of all the variance in teaching practices reported and dissimilarities in beliefs expressed, we utilized our logistic regression models to test if any of these distinctions were strong enough to be predictive.[4] We created two models, one trying to identify factors that would predict whether a participant was a Lecturer or a Non-Lecturer, and a second trying to identify factors that would predict if a Lecturer would or would not consider teaching in a non-lecture format. On the whole, both logistic regression models were satisfactory (based on Nagelkerke R-square values of .342 and .406 respectively), although somewhat limited in their applicability.

The first model was able to correctly predict whether a participant identified as a Lecturer or a Non-Lecturer 78.3% of the time. However, the model was much better at accurately predicting Lecturers, doing so 86% of the time; whereas, it only correctly predicted Non-Lecturers 57.9% of the time. Of the 26 factors we included in the model, the Wald criterion demonstrated that only two factors made a significant contribution to prediction: a strong belief that lecture is the best way to teach ($\chi^2(1) = 9.763$; $p = .002$) and overall satisfaction with student learning ($\chi^2(1) = 7.246$, $p = .027$). In terms of predicting Lecturers, belief that lecture is best increased these odds and satisfaction with student learning decreased these odds. For respondents who disagreed with the belief that lecture is best, the model was 8.432 times more likely to predict that they were a Non-Lecturer; for respondents who were moderately satisfied or very satisfied with their student's learning, the model was 8.896 and 27.369 times (respectively) more

[4] These models were conducted on a reduced data set ($n = 69/55$ for *Have you ever…?* and *Would you ever…?* respectively), as only respondents that answered each survey item under consideration could be included.

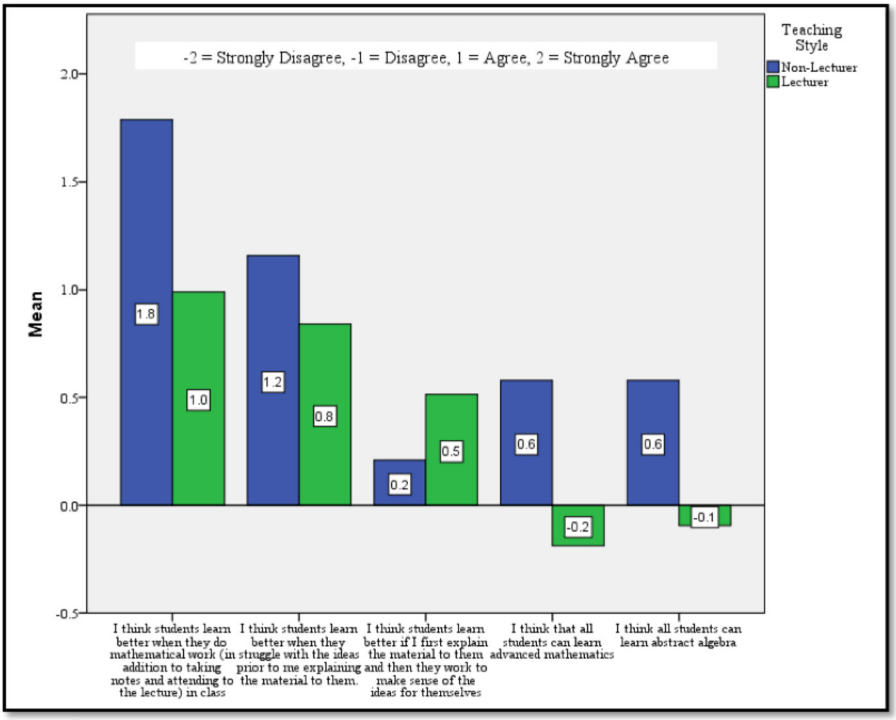


Fig. 11 Beliefs about students and learning

likely to classify them as a Non-Lecturer (as compared with the reference group of dissatisfied respondents).[5]

In the second model, the goal was not to predict those who are (not) Lecturers, but those who would *consider* teaching in a non-lecture format. The overall model accuracy (73.7%) was not quite as high as in the first model, but it should be noted that though this model was slightly less accurate (84%) at predicting those who would not consider switching (proxy for Lecturers), the model had better predictive accuracy (65.6%) at predicting the potential Non-Lecturers – those who would consider switching to a non-lecture format. Again, a strong belief that lecture is best factored significantly in the model ($\chi^2(1) = 5.183$, $p = .023$), but this time, the Wald criterion demonstrated that two additional predictors were significant as well: interest in abstract algebra research ($\chi^2(1) = 6.236$, $p = .013$) and interest in research on learning ($\chi^2(1) = 4.232$, $p = .040$). For respondents with a weak interest in abstract algebra research, the model was 6.084 times more likely to predict that the respondent would consider not lecturing (as compared with the strong interest reference group); for those respondents with a strong interest in learning research, the model was 10.577 times more likely to predict that the respondent would consider not lecturing (as compared with the weak interest reference group); and for respondents that disagreed that lecture

[5] The correct interpretation of these odd rations would be to infer, for instance, that the model was 27 times more likely to predict Non-Lecturer status for a very satisfied instructor as compared with a dissatisfied instructor, assuming other factors held constant – a prediction the model accurately made 57.9% of the time.

was best, the model was 7.482 times more likely to predict that the respondent would consider not lecturing.

The fact that both models were much less accurate at correctly predicting Non-Lecturers or those who would consider switching (potential Non-Lecturers) begs an obvious question: why is it so hard to sort (potential) Non-Lecturers from Lecturers? The limited amount of significant variables suggests that (potential) Non-Lecturers and Lecturers are indistinguishable with regard to many factors, including instructor characteristics (e.g., years of experience); contextual factors of their institution (e.g., terminal degree, the existence of subsequent abstract algebra courses); and departmental constraints/supports (e.g., support for travel, departmental content coverage pressure). That these two groups appear to be indistinguishable could be because the groups are in fact very similar to each other, or that the total population is so divergent on these factors that no trends were observable. It appears that the former best describes our data. For example, we can infer from the similarities between Figs. 9 and 10 (belief in departmental support for the group as a whole and for Lecturers, respectively), that Lecturers and Non-Lecturers experience very similar perceptions of departmental supports. Additionally, if we look at the distribution of teaching experience, we see that these are quite similar for the two groups.

Thus, the logistic models had to make the predictions based on a fairly limited amount of information. For instance, in the first model, the only significant variables were a strong belief that lecture was best and satisfaction with student learning. When we look at the data, we get a better sense for why the model struggled to identify Non-Lecturers based on these two variables. Of the 45 participants who disagreed with the statement *I think lecture is the best way to teach*, only 16 did not lecture. Thus, 64% of our respondents that think lecture is not the best way to teach are lecturing anyway. This is compared to the 88% of the respondents that think lecture is the best way to teach that are lecturing. Using this factor alone, the model would only be able to correctly identify Non-Lecturers 38% of the time. This was improved to 57.9% accuracy when satisfaction with student learning was included. Unfortunately, none of the other candidate predictors were able to significantly improve upon this level of predictive accuracy in a meaningful way, and it is for that reason that they failed to load into the model.

Our purpose for creating this model was to identify high leverage factors for discrimination. A lack of significant predictors led us to conclude that perhaps our candidate predictors are truly ineffective at explaining the variation in our response (in which case, an alternative set of predictors must exist that explains the phenomena); while it is possible that a highly discriminatory predictor was omitted from the survey, we did not want to arrive at that conclusion without first exploring the predictive power of potential underlying latent constructs.

Factor Analysis As our sample size was too small to permit factor analysis with the original number of variables we wished to consider (i.e., nearly all of the survey questions), we were forced to select a subset. Based on the results thus far, it was our determination that the richest source of data, and possibly the key to characterizing the difference between Lecturers and Non-Lecturers, was in the interest and beliefs items. As described in the Methods section, we opted to use a six-factor solution. The Kaiser-

Meyer-Olkin Measure of Sampling Adequacy was .64 (above the recommended value of .6) and Bartlett's Test of Sphericity was significant (χ^2 (78) = 603.545, $p < .001$); collectively, these measures suggest reasonable factorability was demonstrated. The variables (i.e., a subset of survey items) were generally well-defined by the six-factor solution. Communality values were generally reasonable (> .30) and in many cases quite high; furthermore, none of our variables failed to load onto any factor (i.e. loadings < .5). A clean, simple structure was observed with the six-factor solution. Total variance explained by this solution was 62.63%. Information regarding factor loadings and reliability estimates can be seen in Fig. 12.[6]

Second Round of Model Building Using the six extracted factors as candidate predictors, we ran logistic regression models[7] using the same two response variables as in the previous models (i.e., one for Lecturer vs. Non-Lecturer and one for those who would consider not lecturing vs. those who would not consider not lecturing). Both models showed satisfactory performance (based on Nagelkerke R-square values of .415 and .419 respectively) and improved predictive accuracy over the non-factor models.

The first model, composed of Factors 2, 3, and 6, was 88.9% accurate in predicting pedagogical style. This factor model showed an overall improvement in predictive accuracy (88.9% versus 78.3%) as compared with the non-factor model; however, this is primarily due to the increase in predictive accuracy for the Lecturer group (96.9% versus 86%). This model is not only much poorer at predicting Non-Lecturers than Lecturers, but also less accurate than the non-factor model as well (47.4% versus 57.9%).

In logistic regression, the null model assumes that all participants are members of the reference category and the model-building procedure seeks to gather evidence for re-classification. Negative regression coefficients (See Fig. 13) for all three factors indicated that as factor scores decreased, logit values increased; thus, effectively moving participants towards the category of Non-Lecturers. In a practical interpretation:

- the weaker an instructor's agreement with the lecture belief statements (Factor 2) or interest in abstract algebra research (Factor 6), the greater the odds that the model predicted that instructor to be a Non-Lecturer;
- the stronger an instructor's interest in education research (Factor 3), the greater the odds that the model predicted that instructor to be a Non-Lecturer;

[6] Three of the Cronbach's Alphas are below the oft-cited .70 threshold (Nunnally 1978). However, one would expect a deflated alpha due to the small number of items per factor. Additionally, as this is an exploratory factor analysis where we explored our responses ex post facto with an eye towards dimension reduction for our regression model (as opposed to a confirmatory factor analysis in a psychometric situation where prudent instrument design dictates assessment building around multiple items per construct), we decided these levels were satisfactory enough for us to proceed.

[7] Missing cases were handled using listwise deletion when performing the factor analysis. This resulted in a sample size (n = 78) smaller than expected. Model predictions however can be made for any participant who answered the response items, even if some individual predictor items were missing. In the case of these models, predictive accuracy was based on sample sizes of n = 117/97 for the *Have you ever…?* and *Would you consider…?* items, respectively.

Factor	Factor Loadings	Communality	Cronbach's Alpha
Factor 1 – Student Potential			0.973
I think all students can learn advanced mathematics.	0.952	0.961	
I think all students can learn abstract algebra.	0.935	0.944	
Factor 2 – Lecture Beliefs			0.675
I think lecture is the best way to teach.	0.599	0.675	
I think lecture is in the only way to teach that allows me to cover the necessary content.	0.561	0.651	
I think students learn better when they struggle with the ideas prior to me explaining…	0.508[1]	0.303	
I think students learn better if I first explain the material…	0.569	0.374	
Factor 3 – Education Research			0.587
Interest in discussing/reading about how students learn abstract algebra	0.82	0.924	
Interest in research that is scholarship of teaching & learning	0.634	0.464	
Factor 4 – Time			0.671
I think there's enough time for all the content I need or want to teach.	0.569	0.395	
I had enough time during class to help students understand difficult ideas.	0.877	0.849	
Factor 5 – Pressure			N/A
I felt pressured to go through material quickly to cover all the required topics.	0.927	0.999	
Factor 6 – Algebra Research			N/A
Interest in abstract algebra research.	0.526[1]	0.317	

[1]Items were reverse-coded to reverse negative factor loadings.

Fig. 12 Factor analysis results

The second model, composed of Factors 1, 2, 3, 5, and 6, was 75.3% accurate in predicting those who would/would not consider a switch in pedagogical style. This factor model showed a slight overall improvement in predictive accuracy (75.3% versus 73.7%) as compared with the non-factor model; however, this time, the model was actually more accurate at predicting the (potential) Non-Lecturers than the Lecturers. The model correctly classified those who would be willing to consider a non-lecture approach 81.8% of the time as compared with a 66.7% success rate in classifying those who would not.

For this model, the reference category was the potential Non-Lecturers (i.e. those who indicated that they would consider a switch) and therefore negative regression coefficients were indicative of factors that decreased the odds of being a potential Non-Lecturer and positive regression coefficients were indicative of factors that increased those odds (See Fig. 14). In a practical interpretation:

- the weaker an instructor's belief in student potential (Factor 1), interest in education research (Factor 3), or pressure constraint (Factor 5), the greater the odds that the model predicted that instructor to be unwilling to switch;

	B	S.E.	Wald	df	Sig.	Exp(B)
Factor 2	-0.857	0.44	3.792	1	0.052	0.424
Factor 3	-1.398	0.44	10.092	1	0.001	0.247
Factor 6	-1.275	0.423	9.075	1	0.003	0.279
Constant	-2.559	0.459	31.114	1	<.001	0.077

Fig. 13 Regression coefficients – lecturer v. non-lecturer classification model

	B	S.E.	Wald	df	Sig.	Exp(B)
Factor 1	0.599	0.265	5.105	1	0.024	1.82
Factor 2	-1.223	0.364	11.288	1	0.001	0.294
Factor 3	-0.956	0.32	8.895	1	0.003	0.385
Factor 5	0.689	0.27	6.523	1	0.011	1.992
Factor 6	-0.66	0.361	3.346	1	0.067	0.517
Constant	0.662	0.271	5.974	1	0.015	1.939

Fig. 14 Regression coefficients – potential non-lecturer classification model

- the stronger an instructor's agreement with the lecture belief statements (Factor 2) or interest in abstract algebra research (Factor 6), the greater the odds that the model predicted that instructor to be unwilling to switch;

Considering collectively the results of our models, we can say with conviction that if you believe lecture is best then you are going to lecture. We were also able to identify a list of beliefs and interests that can help us classify those who would consider not lecturing. This information has several possible uses. First, it describes a list of characteristics that could be used to identify populations to focus dissemination efforts and professional development opportunities. Second, it provides a list of high leverage beliefs that can be targeted by policy-makers and change agents hoping to convert Lecturers. Finally, the existence of factors that distinguish those who would and would not consider non-lecture pedagogical approaches provides some evidence that these populations are actually distinct. This implies that homogeneity of Lecturers is an unreasonable assumption to make and, in fact, there is a significant proportion of them that may be willing to try something new.

Conclusions

In order to gain information about the nature of undergraduate mathematics instruction and beliefs and institutional/departmental context that influences instructional decision-making, we surveyed abstract algebra instructors at Master's- and PhD-granting institutions. Our decision to survey abstract algebra instructors (as opposed to say calculus or real analysis instructors) was an effort to explore instructional practice in a "best case scenario". This course has relatively few constraints (as opposed to Calculus I, for instance) in that it is normally taught by tenured or tenure-track faculty members to upper division mathematics majors in small classes. There are few concerns about coordination between multiple sections, and only a handful of these students will subsequently matriculate into a follow-up course. Additionally, The Inquiry Based Learning community and Project NeXT are active in abstract algebra (e.g. Ernst 2016; Gallian et al. 2000) and (as opposed to other upper division mathematics courses), there have been a number of research-based curricular innovations in abstract algebra (Dubinsky and Leron 1994; Larsen et al. 2013; and Cook 2014). Thus, this is a course with relatively limited constraints and widely available supports. As such, this course provided a rich context for really focusing on instruction and the beliefs and institutional/departmental context that influences instructional decision-making. For each of our three research questions, we wish to draw attention to our primary findings. We then discuss some implications and avenues for future research.

In terms of our first research question, we can now definitively report that lecture is the predominant mode of instruction. 79 of our 126 participants reported lecturing for at least half of each class period. Further, 84% of our instructors rejected the description of their instruction as "non-lecture", with 97 answering "no" when asked if they had "ever taught abstract algebra in a non-lecture format" and 10 answering, "I have in the past but I currently lecture". However, it is not the case that Lecturers are heterogeneous in their teaching practices; nor is it that their practice is strictly disjoint from the teaching practices of the Non-Lecturers in our study. For instance, Lecturers reported a wide range of percentages of class time actually lecturing, with approximately 30% reporting that they lecture less than 50% of the class time. Interestingly, more than a third of our Non-Lecturers reported that they lectured more than 25% of the time in every class. Further, about 50% of the Lecturers reported that they sometimes ask students to work individually on problems, have students give presentations of completed work, and hold whole-class discussions. Similarly, 11/19 of the Non-Lecturers sometimes ask students to work individually on problems and give presentations of completed work (with 16/19 reporting that they sometimes hold whole class discussions). The wide range of practices reinforces the notion that select lecturers might be quite good at inspiring students and promoting active engagement with the content, although the broad trends suggest some conformity of practice.

This heterogeneity within both the Lecture group and the Non-Lecture group, and the overlap in practices between the two groups, suggests that factors influencing how instructors self-identify and what importance (if any) this identification has on their professional activities and student learning gains warrants further investigation. Why did 10 participants who lecture for less than 25% of the class time not consider themselves to be teaching in a non-lecture format while another 3 who lecture for more than half the class time did consider their instruction to be non-lecture? And, more importantly, does the language one uses to describe his/her instruction matter?

We would also like to point out that we found little to no uptake of any specific research-based curricular innovations. The few participants that are using non-traditional materials are far more likely to have developed their own materials than to have adopted NSF-supported curricula. This is problematic for mathematics education researchers for a number of reasons, but most importantly because it indicates that efforts by mathematics education researchers to develop and disseminate curriculum is having little to no effect, and should cause us to question the design-dissemination model. This aligns with Henderson et al. (2011) meta-analysis. We second their call for new models of design and dissemination, and more importantly, studies that investigate how to best promote and support pedagogical change.

Our investigation into the second research question, about the departmental affordances and constraints that faculty perceive, uncovered a number of contradictions. On the one hand, instructors are citing factors like constraints on their time, content pressure, and lack of curricular resources, knowledge, and supports as reasons for why they lecture. However, on the other hand, many of these same instructors are reporting that their departments might provide time and tenure consideration for redesigning their courses, that they did not feel pressure from their departments to cover a set amount of material, and that there are department funds available for professional development opportunities. Research on K-12 teachers has found that they face a plethora of obligations that they perceive to constrain their practice. These include obligations to the discipline of mathematics, to their schools, their departments, to the students who may take subsequent mathematics courses, and to the instructional settings of their institution (Herbst et al. 2011). Our research suggests that undergraduate instructors, even those who perceive few (external) institutional and departmental constraints, carry similar obligations that inform their teaching decisions. More research is needed to better understand why instructors *feel* like they do not have the time, freedom, or supports needed to make changes to their instruction – especially when they are in departments where they believe time, freedom, and financial support may be available. Moreover, we note that there may be institutional pressures related to pass-rates, research output, and promotion that would inform the amount of time which instructors feel able to devote to certain activities. Our survey did not investigate these institution-level constraints, and further investigation of them would be warranted.

Finally, with our third research question, we sought to understand differences between those who do not lecture, those who lecture and would not consider changing, and those who lecture and would consider changing. Here we want to highlight two main results. First, because we were able to identify factors that distinguish those who would and would not consider non-lecture pedagogical approaches, we have evidence that these are in fact distinct populations. Those who are willing to consider non-lecture pedagogy, are of particular importance for change agents and professional development organizations. These instructors have positive beliefs about their students' capability to learn, negative views of lecture, and a strong interest in educational research. We also have a list of reasons as to why these instructors are still lecturing. They do not feel like they have the time to redesign their course, they do not have materials they like, and they do not know where to start. Now that we have found ways to identify these individuals, we can target these populations and provide the supports they need. Additionally, if we want to move the field as a whole, we now know which belief we need to target: the belief that lecture is the best way to teach. This opens up a wide range of future research implications, including: research into the range of instructional practices instructors would consider using, understanding what types of evidence that instructors hold as dispositive, and investigating how instructors' goals for instruction relate to the type(s) of evidence they find most persuasive.

Second, while our models did allow us to generate some useful conclusions, the percentage of variation being explained and the predictive accuracy, especially in terms of Non-Lecturers, left something to be desired. Despite the inclusion of a large majority of our survey items – all of which were theoretically and/or research-based – the results suggest that an important predictor (or set of predictors) has been omitted. We, as a field, are missing something important about what makes an instructor decide to not lecture. One factor suggested by our data, is the importance of the type of mathematics instruction that the instructors experienced as students. We found in Fukawa-Connelly et al. (2016) that the most significant influence on the pedagogical decision-making of instructors was "their experiences as a teacher (84%) and their experiences as a student (64%)". A limitation of the current research is our failure to ask those questions that could have explicated the relationship between how the classroom is perceived when one is a student and how that informs the decisions one subsequently makes as an instructor. Further, our analysis suggests that perceived and/or internalized constraints may be much more influential than external departmental constraints. More research needs to be done to posit and investigate such factors that may be better able to differentiate between those who choose to lecture and those who do not. This could include interview studies with selected instructors that correspond to the different profiles (e.g., non-lecturers, lecturers who would consider non-lecture pedagogy, and those lecturers who would not consider non-lecture pedagogy).

In total, our study suggests that there is room for impacting and changing undergraduate mathematics education. While lecture is the most common form of instruction, we found evidence that: many Lecturers are incorporating a range of instructional techniques, there is a significant proportion of instructors who do not believe that lecture is the best way to teach and who would be willing to do something different, and that mathematics departments appear to be making supports available for those who are interested in changing their instruction. As a field, we now need to better understand how to support change. To that end, we echo Alcock's (2010) call for a respectful relationship between mathematics educators and mathematicians – that is, a relationship that values the collective experience of both communities and in which mathematics educators seriously consider the needs that mathematicians find most pressing.

Appendix 1 – Logistic Regression Details

Response 1: Have you ever taught in non-lecture format? [0 = no, 1 = yes]

Response 2: Would you ever consider teaching in a non-lecture format? [0 = no, 1 = yes]

TeachingExp [3 levels]
AAExp [3 levels]
For these 2 variables, the reference group was the last (most experience)
Satisfaction [3 levels] *Removed 'other' responses
Reference group was first (dissatisfied)
FollowUpcourse [3 levels]
Reference group was 'yes - required for math majors'
TerminalDegree [**0 = Phd**, 1 = Masters]
WorkInGroups [**0 = Never**, 1 = Sometimes]
GivePresentations [**0 = Never,** 1 = Sometimes]
LectureisBest [0 = Disagree, **1 = Agree**]
LectureIsOnly [0 = Disagree, **1 = Agree**]
MathWork [**0 = Disagree**, 1 = Agree]
EnoughTime [0 = Disagree, **1 = Agree**]
AATeachingInterest [**0 = weak**, 1 = strong]
AALearningResearch [**0 = weak**, 1 = strong]
AAResearchInterest [0 = weak, **1 = strong**]
TandLResearch [**0 = weak**, 1 = strong]
PandTSupport [**0 = no**, 1 = yes]
TravelSupport [**0 = no**, 1 = yes]
CourseFreedom [**0 = no**, 1 = yes]

DeptPressure [0 = no, **1 = yes**]
TimeforClassPrep [**0 = no**, 1 = yes]
CoveragePressure [**0 = Disagree**, 1 = Agree]
FailRate [0 = <20%, **1 = >20%**]
ExtInf [**0 = weak**, 1 = strong] (cutpoint is 5)
StudentExperience [0 = weak, **1 = strong**]
TeacherExperience [0 = weak, **1 = strong**]
TalktoColleagues [0 = weak, **1 = strong**]
(Weak combines 'not at all' and 'somewhat', Strong = 'very')

Appendix 2 – Details on Statistical Conclusions

Table 1 Reported Teaching Practices Pairwise Comparisons

Figure	Hypothesis (Mean for Lecturers = Mean for Non-Lecturers)	*p* - value	Threshold[a]	Decision
4	**Have students engage in small group discussions**	**<.001**	**.002778**	**Reject**
4	**Have students ask each other questions**	**<.001**	**.002941**	**Reject**
4	Use diagrams to illustrate ideas	.147	.010000	Fail to Reject
4	Use physical and/or visual representations of group elements	.372	.016667	Fail to Reject
4	Include informal explanations of formal statements	.538	.025000	Fail to Reject
4	Pause and ask questions	.594	.050000	Fail to Reject
5	**Have students work in small groups**	**<.001**	**.002381**	**Reject**
5	**Lecturing**	**<.001.**	**.002500**	**Reject**
5	Holding a whole-class discussion	.005	.003571	Fail to Reject
5	Have students explain their thinking	.012	.004545	Fail to Reject
5	Show students how to write specific proofs	.027	.005556	Fail to Reject
5	Have students give presentations of completed work	.074	.006250	Fail to Reject
5	Have students work individually on problems or tasks	.304	.012500	Fail to Reject
6	**Developing or explaining a theorem**	**<.001**	**.002632**	**Reject**
6	**Developing or critiquing a proof (including proof-verification)**	**.002**	**.003125**	**Reject**
6	**Developing examples and/or counter-examples of a construct**	**.001**	**.003333**	**Reject**
6	Developing a definition or exploring its evolution	.007	.003846	Fail to Reject
6	Developing examples and/or counter-examples relating to a theorem	.006	.004167	Fail to Reject
6	Developing examples that support or insatiate a proof	.01	.005000	Fail to Reject
6	Working with applications of theorems or constructs	.085	.007143	Fail to Reject
6	Doing calculations (e.g., decomposing n-cycles into 2-cycles, Euclidean algorithm)	.133	.008333	Fail to Reject

[a] Family-wise Error Rate controlled at the .05 level using the Holm-Bonferroni Method

Table 2 Reported Beliefs Pairwise Comparisons

Figure	Hypothesis (Mean for Lecturers = Mean for Non-Lecturers)	*p* - value	Threshold[a]	Decision
10	**I think lecture is the best way to teach.**	**<.001**	**.005000**	**Reject**
10	**I think lecture is the only way to teach that allows me to cover the necessary content.**	**<.001**	**.005556**	**Reject**
10	When I last taught algebra, I hade enough time during class to help students understand difficult ideas.	.078	.010000	Fail to Reject
10	When I last taught algebra, I felt pressured to go through the material quickly to cover all the required topics.	.494	.025000	Fail to Reject
10	I think there's enough time for all the content I need or want to teach.	.696	.050000	Fail to Reject
11	**I think students learn better when they do mathematical work (in addition to taking notes and attending lecture) in class.**	**<.001**	**.006250**	**Reject**
11	I think that all students can learn advanced mathematics.	.03	.007143	Fail to Reject
11	I think all students can learn abstract algebra.	.076	.008333	Fail to Reject
11	I think students learn better when they struggle with the ideas prior to me explaining the material to them.	.0291	.012500	Fail to Reject
11	I think students learn better if I first explain the material to them and then they work to make sense of the ideas for themselves.	.0353	.016667	Fail to Reject

[a] Family-wise Error Rate controlled at the .05 level using the Holm-Bonferroni Method

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