Complex & Intelligent Systems (2020) 6:669–679
https://doi.org/10.1007/s40747-020-00165-0

ORIGINAL ARTICLE

Multi-criteria decision-making method with double risk parameters in interval-valued intuitionistic fuzzy environments

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Received: 9 December 2019 / Accepted: 3 June 2020 / Published online: 19 June 2020

Abstract

In the multi-criteria decision-making (MCDM) process, decision-makers with different risk attitudes may have different decision results. To address this issue and present decision-makers' mentality, this paper introduces two mentality parameters. These parameters reflect the decision-makers' risk attitudes in determining the membership and non-membership degrees of the evaluation information. In addition, the parameters demonstrate the risk attitude in terms of the hesitancy degree under interval-valued intuitionistic fuzzy information. Then, a new score function of interval-valued intuitionistic fuzzy numbers (IVIFNs) is proposed that uses the introduced mentality parameters. Meanwhile, certain properties of the proposed score function are discussed. Furthermore, the weighted comprehensive score value of IVIFNs is introduced, and an MCDM method is developed in an interval-valued intuitionistic fuzzy environment. Finally, a numerical example and comparative analyses are provided to illustrate the feasibility and effectiveness of the proposed method.

Keywords Multi-criteria decision-making · Risk attitudes · Interval-valued intuitionistic fuzzy numbers · Score function · Mentality parameter

Introduction

In 1986, Atanassov [1] proposed intuitionistic fuzzy sets (IFSs), which are an extension of fuzzy sets [2]. In IFSs, the membership, non-membership, and hesitancy degrees of an element that belongs to a set are considered. These new sets are more flexible and practical than traditional fuzzy sets in dealing with the uncertainty of the objectives [3–9]. For example, Melliani and Castillo [7] introduced recent advances in intuitionistic fuzzy logic systems, Roeva and Michalikova [8] proposed a generalized net model based on intuitionistic fuzzy logic control, and Atanassov and Sotirov [9] discussed neural networks with interval valued intuitionistic fuzzy conditions. However, in some complex decision-making situations, decision-makers may not have sufficient knowledge to provide crisp values of membership and non-membership degrees. Nonetheless, their ranges can be indicated. Therefore, in 1989, Atanassov and Gargov [10] introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs), whose membership and non-membership are closed intervals instead of crisp values. Because the membership degree and non-membership degree of IVIFSs are described by intervals, scholars have considered IVIFSs as a suitable tool to express uncertain and vague information for practical issues [11–14].

Over the last few decades, IVIFSs have been widely applied in multi-criteria decision-making (MCDM) [15–21], such as the evaluation of service quality in public bus transportation [22], healthcare evaluation in hazardous waste recycling [18], multi-perspective collaborative scheduling [23], evaluation of the risk of failure modes [24], photovoltaic module selection [25], measurement of the service quality of urban rail transit [20], and evaluation of the effectiveness of knowledge transfer [21]. The main goal in solving MCDM problems is to rank the admissible alternatives that have been evaluated based on the given attributes and

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to choose the best one. Thus, with respect to the abovementioned MCDM problems which involve many interval-valued intuitionistic fuzzy number (IVIFN) methods, how to select a reasonable approach to rank IVIFNs becomes an important research topic.

A considerable number of studies have been conducted on ranking and comparing IVIFNs [26–32]. For example, some ranking methods are based on the possibility degree and divergence degree [17], geometric aspect [30, 33], complex proportional assessment method [18], distance-based approach [31, 34–36], score function [26, 37], generalized exponential entropy [32], and probability density functions and variances of IVIFNs [38]. Numerous methods have been developed to rank and compare IVIFNs in the abovementioned literature. These methods can be roughly classified into three categories: (1) independent ranking [17, 26, 32, 37, 39], (2) pairwise comparison-based ranking [40, 41], and (3) reference based ranking [34–36]. The processes have been developed from various perspectives to rank and compare IVIFNs with applications to solve real-life decision making problems.

Among these approaches to rank IVIFNs, the score function is an effective method. Xu and Chen [42] introduced a ranking method based on the score function and accuracy function of IVIFVs. Ye [29] proposed a novel accuracy function for IVIFNs by taking into account the hesitancy degree of IVIFNs. Lakshmana Gomathi Nayagam et al. [43] provided a novel accuracy function for IVIFNs to overcome the difficult decision-making process of existing accuracy functions in some cases. Wang and Chen [26] proposed a new score function of IVIFNs to handle the drawback of Chen and Huang [44] method. In another study, Wang and Chen [27] further introduced the new score function and accuracy function of IVIFNs to deal with the drawback of Wang and Chen [26] method. Nguyen [45] defined a generalized p-norm knowledge-based score function for IVIFNs, which is a generalization of the score function of IFSs. Additional score functions of IVIFNs can be found in [46, 47].

These methods are effective in solving decision-making problems in interval-valued intuitionistic fuzzy environments. However, the methods suffer from several shortcomings. (1) The ranking method cannot distinguish whether two IVIFNs have equal middles of membership degree and non-membership intervals [42] or have equal accuracy functions [29, 43]. (2) In the decision-making process, the decision result depends on the risk attitude of decision-makers. For example, risk-averse decision-makers are inclined to select low-risk alternatives, whereas risk-seeking decision-makers tend to select high-risk options. However, there are few studies on these aspects among most of the abovementioned ranking methods. (3) Some methods neglect the discussion of a reasonable score function, which can generate unreasonable and unreliable decision results [17].

To overcome these shortcomings, the objectives of this study are summarized as follows:

1. To show that the ranking results are strongly affected by the decision-makers' risk attitudes, this study introduces two mentality parameters.
2. A new score function of IVIFNs is proposed by introducing mentality parameters and integrating the membership, non-membership, and hesitancy degrees of IVIFNs.
3. Some basic properties of the proposed score function of IVIFNs are discussed to verify its reasonability and effectiveness, which can make the decision results believable and trustworthy.

The remainder of the paper is organized as follows. In "Preliminaries", we briefly review basic concepts such as IFSs and IVIFSs. In "New score function of IVIFNs", a new score function is proposed based on the discussion of the hesitation of the IVIFNs to properly reflect the decision-makers' risk attitudes. Several illustrative examples are provided to show the feasibility of the proposed method. In "MCDM in interval-valued intuitionistic fuzzy environments", we apply the proposed score function to MCDM with IVIFNs. In "Illustrative example", an illustrative example and comparative analyses are provided to demonstrate the practicality and effectiveness of our method. The conclusions are presented in "Conclusions".

Preliminaries

This section presents the basic concepts of IFSs and IVIFSs.

Definition 1 [1] Let X be a fixed set with a non-empty universe. An IFS A in X is denoted as $A = \left\{ \langle x,\ \mu_A(x),\ v_A(x)\rangle \middle|\ x \in X \right\}$, where $\mu_A(x) \in [0, 1]$ and $v_A(x) \in [0, 1]$ represent the membership and non-membership degrees of element x to set A, respectively, satisfying $0 \le \mu_A(x) + v_A(x) \le 1$ for all $x \in X$. The pair $\left(\mu_A(x),\ v_A(x)\right)$ is called an intuitionistic fuzzy number.

The concept of IFSs is further extended to IVIFSs.

Definition 2 [10] Let X be a finite and non-empty set. An IVIFS $\tilde{A}$ in X is an object with the following form.

$$\tilde{A} = \left\{ \langle x,\ \mu_{\tilde{A}}(x),\ v_{\tilde{A}}(x)\rangle \middle|\ x \in X \right\}, \tag{1}$$

where the functions $\mu_{\tilde{A}}(x)$: $X \to D[0, 1]$ and $v_{\tilde{A}}(x)$: $X \to D[0, 1]$ denote the membership degree and non-membership degree of element $x \in X$ in $\tilde{A}$, respectively. The pair $\left(\mu_{\tilde{A}}(x),\ v_{\tilde{A}}(x)\right)$ is called an IVIFN. $\mu_{\tilde{A}}(x)$ and $v_{\tilde{A}}(x)$ are two closed intervals, and their lower and upper boundaries are denoted by $\mu_{\tilde{A}L}(x)$, $\mu_{\tilde{A}U}(x)$, $v_{\tilde{A}L}(x)$, and $v_{\tilde{A}U}(x)$, respectively. IVIFS $\tilde{A}$ is denoted as

$$\tilde{A} = \left\{ \langle x,\ [\mu_{\tilde{A}L}(x),\ \mu_{\tilde{A}U}(x)], [v_{\tilde{A}L}(x), v_{\tilde{A}U}(x)]\rangle \middle| \ x \in X \right\},$$

where $0 \le \mu_{\tilde{A}U}(x) + v_{\tilde{A}U}(x) \le 1$, $\mu_{\tilde{A}L}(x) \ge 0$, and $v_{\tilde{A}L}(x) \ge 0$. Specifically, if $\mu_{\tilde{A}L}(x) = \mu_{\tilde{A}U}(x)$ and $v_{\tilde{A}L}(x) = v_{\tilde{A}U}(x)$, then IVIFS $\tilde{A}$ reduces to an IFS.

For simplicity, IVIFN $\tilde{\alpha} = \left(\mu_{\tilde{\alpha}}(x),\ v_{\tilde{\alpha}}(x)\right)$ is usually denoted as $\tilde{\alpha} = ([a,b],[c,d])$, where $[a,b] \subset D[0,1]$, $[c,d] \subset D[0,1]$, and $b + d \le 1$.

Example 1 Let $\tilde{\alpha} = ([0.6, 0.7], [0.1, 0.2])$ be an IVIFN; its meaning can be explained as follows: for an election with 100 voters, the value $[0.6, 0.7]$ implies that 60–70 are expected to be in favor, the value $[0.1, 0.2]$ implies that 10–20 are expected to be against, and the value $[0.1, 0.3]$ implies that 10–30 are expected to abstain.

Definition 3 [10] Let $\tilde{\alpha} = \left(\mu_{\tilde{\alpha}}(x),\ v_{\tilde{\alpha}}(x)\right)$ be an IVIFN.

$$\begin{aligned} \pi_{\tilde{\alpha}}(x) &= 1 - \mu_{\tilde{\alpha}}(x) - v_{\tilde{\alpha}}(x) \\ &= \left[1 - \mu_{\tilde{\alpha}U}(x) - v_{\tilde{\alpha}U}(x), 1 - \mu_{\tilde{\alpha}L}(x) - v_{\tilde{\alpha}L}(x)\right], \end{aligned} \tag{2}$$

then $\pi_{\tilde{\alpha}}(x)$ is called the hesitancy degree of $\tilde{\alpha}$.

Xu [48] introduced some basic arithmetical operations of IVIFNs as follows.

Definition 4 [48] Let $\tilde{\alpha} = \left(\left[a_1, b_1\right], \left[c_1, d_1\right]\right)$ and $\tilde{\beta} = \left(\left[a_2, b_2\right], \left[c_2, d_2\right]\right)$ be two IVIFNs. Then, their operational laws are defined as:

$$\tilde{\alpha} \succ \tilde{\beta} \text{ if } a_1 > a_2,\ b_1 > b_2,\ c_1 < c_2 \text{ and } d_1 < d_2; \tag{3}$$

$$\tilde{\alpha} \sim \tilde{\beta} \text{ if } a_1 = a_2,\ b_1 = b_2,\ c_1 = c_2, \text{ and } d_1 = d_2. \tag{4}$$

New score function of IVIFNs

In this section, we review several representative score functions of IVIFNs and discuss their shortcomings. This is our motivation for proposing a new score function of IVIFNs, which is provided in "New score function of IVIFNs".

For simplicity, we use the following symbols: let $\tilde{\alpha} = ([a,b],[c,d])$ be an IVIFN, $M\left(\mu_{\tilde{\alpha}}\right) = \frac{a+b}{2}$ denotes the middle value of the membership interval, $M\left(v_{\tilde{\alpha}}\right) = \frac{c+d}{2}$ is the non-membership interval, and $M\left(\pi_{\tilde{\alpha}}\right) = \frac{(1-b-d)+(1-a-c)}{2}$ is the hesitancy interval, where $M\left(u_{\tilde{\alpha}}\right) + M\left(v_{\tilde{\alpha}}\right) + M\left(\pi_{\tilde{\alpha}}\right) = 1$.

Some representative score functions of IVIFNs

Xu and Chen [42] introduced the score function and accuracy function of IVIFNs as follows.

Definition 5 [42] For an IVIFN $\tilde{\alpha} = ([a,b],[c,d])$, the score function and accuracy function can be denoted as:

$$S(\tilde{\alpha}) = \frac{a - c + b - d}{2}, \tag{5}$$

$$h(\tilde{\alpha}) = \frac{a + b + c + d}{2}. \tag{6}$$

Obviously, $S(\tilde{\alpha}) = M\left(u_{\tilde{\alpha}}\right) - M\left(v_{\tilde{\alpha}}\right) \in [-1, 1]$, and $h(\tilde{\alpha}) = M\left(u_{\tilde{\alpha}}\right) + M\left(v_{\tilde{\alpha}}\right) \in [0, 1]$. The larger the score value of $\tilde{\alpha}$ is, the larger IVIFN $\tilde{\alpha}$ is. Based on Definition 5, a prioritized comparison method of IVIFNs is introduced as follows.

Definition 6 [42] For any two IVIFNs $\tilde{\alpha}$ and $\tilde{\beta}$,

1. If $S(\tilde{\alpha}) < S\left(\tilde{\beta}\right)$, then $\tilde{\alpha}$ is smaller than $\tilde{\beta}$, denoted as $\tilde{\alpha} \prec \tilde{\beta}$;
2. If $S(\tilde{\alpha}) > S\left(\tilde{\beta}\right)$, then $\tilde{\alpha}$ is larger than $\tilde{\beta}$, denoted as $\tilde{\alpha} \succ \tilde{\beta}$;
3. If $S(\tilde{\alpha}) = S\left(\tilde{\beta}\right)$, then

(1) if $h(\tilde{\alpha}) < h\left(\tilde{\beta}\right)$, then $\tilde{\alpha}$ is smaller than $\tilde{\beta}$, denoted as $\tilde{\alpha} \prec \tilde{\beta}$;
(2) if $h(\tilde{\alpha}) > h\left(\tilde{\beta}\right)$, then $\tilde{\alpha}$ is larger than $\tilde{\beta}$, denoted as $\tilde{\alpha} \succ \tilde{\beta}$;
(3) if $h(\tilde{\alpha}) = h\left(\tilde{\beta}\right)$, then $\tilde{\alpha}$ is equivalent $\tilde{\beta}$, denoted as $\tilde{\alpha} \sim \tilde{\beta}$.

The score function and accuracy function are considered at the midpoint of the membership interval and non-membership interval of the IVIFN. Most of the order relationship of IVIFNs can be distinguished by using this method. Unfortunately, we cannot identify the optimal method when the midpoints are equal by applying Definition 6. This shortcoming is shown in Example 2.

Example 2 Let $\tilde{\alpha}_1 = ([0.3, 0.5],\ [0.1, 0.3])$ and $\tilde{\alpha}_2 = ([0.4, 0.4],\ [0.2, 0.2])$ be two IVIFNs for two alternatives.

Via Eq. (5), we obtain $S\left(\tilde{\alpha}_1\right) = S\left(\tilde{\alpha}_2\right) = 0.2$. Then, we obtain $h\left(\tilde{\alpha}_1\right) = h\left(\tilde{\alpha}_2\right) = 0.6$ by applying Eq. (6). In this case, the order relationship of $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ cannot be distinguished.

Considering the limitations of Xu's method, Ye [29] and Lakshmana Gomathi Nayagam et al. [43] introduced new accuracy functions by considering the hesitancy degree of the IVIFN.

Definition 7 [29] Let $\tilde{\alpha} = ([a,b],[c,d])$ be an IVIFN. Its accuracy function is denoted as:

$$J(\tilde{\alpha}) = a + b - 1 + \frac{c+d}{2}. \tag{7}$$

Definition 8 [43] Let $\tilde{\alpha} = ([a,b],[c,d])$ be an IVIFN. Its accuracy function is denoted as:

$$L(\tilde{\alpha}) = \frac{1}{2}[a + b - d(1-b) - c(1-a)]. \tag{8}$$

Ye [29] and Nayagam et al. [43] have asserted that the order relationship of IVIFNs can be obtained using the value of the accuracy function. Unfortunately, these methods have some shortcomings.

Example 3 Let $\tilde{\alpha}_3 = ([0.1,0.2],[0.6,0.8])$ and $\tilde{\alpha}_4 = ([0.4,0.5],[0,0.1])$ be two IVIFNs for two alternatives.

Since the membership degree interval of $\tilde{\alpha}_3$ is smaller than that of $\tilde{\alpha}_4$, the non-membership degree interval of $\tilde{\alpha}_3$ is larger than that of $\tilde{\alpha}_4$. According to Eq. (3), we can obtain $\tilde{\alpha}_3 \prec \tilde{\alpha}_4$. This conclusion is intuitive.

In addition, by applying Eq. (7), we obtain $J(\tilde{\alpha}_3) = 0$ and $J(\tilde{\alpha}_4) = -0.05$. Since $J(\tilde{\alpha}_3) > J(\tilde{\alpha}_4)$, we have $\tilde{\alpha}_3 \succ \tilde{\alpha}_4$. The above discussion demonstrates that the order relationship of $\tilde{\alpha}_3 \succ \tilde{\alpha}_4$ is unreasonable and counterintuitive.

Example 4 Let $\tilde{\alpha}_5 = ([0.3,0.3],[0.2,0.2])$ and $\tilde{\alpha}_6 = ([0.2,0.4],[0.2,0.2])$ be two IVIFNs for two alternatives.

By applying Eq. (8), we obtain $L(\tilde{\alpha}_5) = L(\tilde{\alpha}_6) = 0.4$. In this case, the order relationship of alternatives cannot be distinguished. Since the range of the membership degree interval $\tilde{\alpha}_5$ is smaller than that of $\tilde{\alpha}_6$ and their non-membership degree intervals are equivalent, the order relationship $\tilde{\alpha}_5 \succ \tilde{\alpha}_6$ can be easily obtained. This result indicates that the method proposed by Lakshmana Gomathi Nayagam et al. [43] has some shortcomings.

The score function and accuracy function mentioned above are used to obtain the order relationship of IVIFNs. However, these ranking methods do not give sufficient attention to the score function and accuracy function. To overcome these shortcomings, a new method to rank IVIFNs considering the risk attitude of the decision-makers is proposed in the following section.

New score function of IVIFNs

In the decision-making process, the decision result depends on the risk attitude of the decision-makers. In other words, the risk attitudes of the decision-makers affect the decision results. In this paper, the risk parameter with a new score function of IVIFNs is proposed. Some related properties are discussed to verify the reasonability and effectiveness of the proposed score function.

Before introducing the new score function of IVIFN, we first introduce $\lambda \in [0,1]$ to reflect the decision-makers' risk attitude in determining the membership degree and the non-membership degree. For an IVIFN $\tilde{\alpha} = ([a,b],[c,d])$, let $M^\lambda(u_{\tilde{\alpha}}) = a + \lambda(b-a)$ and $M^\lambda(v_{\tilde{\alpha}}) = c + (1-\lambda)(d-c)$. Then, the hesitancy degree integrated with decision-makers' risk attitudes can be obtained as follows: $M^\lambda(\pi_{\tilde{\alpha}}) = 1 - M^\lambda(u_{\tilde{\alpha}}) - M^\lambda(v_{\tilde{\alpha}}) = \lambda(1-b-c) + (1-\lambda)(1-a-d)$. Clearly, we have $M^\lambda(u_{\tilde{\alpha}}) \in [a,b]$, $M^\lambda(v_{\tilde{\alpha}}) \in [c,d]$, and $M^\lambda(\pi_{\tilde{\alpha}}) \in [1-b-d, 1-a-c]$. The value of λ depends on the risk attitude of the decision-makers. If $0 \le \lambda < 1/2$, the decision-makers are risk-averse. If $\lambda = 1/2$, the decision-makers are risk-neutral. If $1/2 < \lambda \le 1$, the decision-makers are risk-seeking.

For the hesitancy interval according to the vote model, supposing that people with hesitancy are always affected by supporters and opponents, which lead to support and opposition, respectively, we introduce the second attitude parameter $\theta \in [0,1]$. The attitude parameter θ expresses the proportion of the supporters in the original group of people with hesitancy. Among those with hesitancy, $\theta M^\lambda(\pi_{\tilde{\alpha}})$ tend to support, whereas $(1-\theta)M^\lambda(\pi_{\tilde{\alpha}})$ tend to oppose. If $0 \le \theta < 1/2$, the decision-makers are risk-averse. The smaller the value θ is, the more risk-averse the decision-makers are. If $1/2 < \theta \le 1$, the decision-makers are risk-seeking. The larger the value θ is, the more risk-seeking the decision-makers are. If $\theta = 1/2$, the decision-makers are risk-neutral.

Combining these demonstrations, the new score function of IVIFNs is defined as follows.

Definition 9 Let $\tilde{\alpha} = ([a,b],[c,d])$ be an IVIFN. Its score function can be denoted as:

$$S_{New}(\tilde{\alpha}) = \left[M^\lambda(u_{\tilde{\alpha}}) + \theta M^\lambda(\pi_{\tilde{\alpha}})\right] - \left[M^\lambda(v_{\tilde{\alpha}}) + (1-\theta)M^\lambda(\pi_{\tilde{\alpha}})\right], \tag{9}$$

where,

$$M^\lambda(u_{\tilde{\alpha}}) = a + \lambda(b-a),\ M^\lambda(v_{\tilde{\alpha}}) = c + (1-\lambda)(d-c),$$
$$M^\lambda(\pi_{\tilde{\alpha}}) = \lambda(1-b-c) + (1-\lambda)(1-a-d),\ \lambda \in [0,1],\ \theta \in [0,1].$$

This score function can be further simplified as follows:

$$S_{\text{New}}(\tilde{\alpha}) = M^\lambda(u_{\tilde{\alpha}}) - M^\lambda(v_{\tilde{\alpha}}) + (2\theta - 1)M^\lambda(\pi_{\tilde{\alpha}}). \tag{10}$$

Evidently, if we include $\lambda = \theta = 1/2$ in Eq. (10), we obtain $S_{\text{New}}(\tilde{\alpha}) = S(\tilde{\alpha})$. In this case, the new score function $S_{\text{New}}(\tilde{\alpha})$ proposed in Eq. (10) is equivalent to Xu score function $S(\tilde{\alpha})$ introduced in Eq. (5). This result confirms that

Xu and Chen [42] method is a special case of the proposed method.

Example 5 The IVIFNs are identical to those provided in Example 2.

According to Eq. (10), we obtain:

$S_{\text{New}}(\tilde{\alpha}_1) = 0.4(2\theta - 1 + \lambda),$
$S_{\text{New}}(\tilde{\alpha}_2) = 0.4(2\theta - 1 + 1/2).$

Then, $S_{\text{New}}(\tilde{\alpha}_1) - S_{\text{New}}(\tilde{\alpha}_2) = 0.4(\lambda - 1/2)$. Therefore, for any $\theta \in [0, 1]$, we have the following:

1. If $\lambda < 1/2$, then $S^{\lambda}(\tilde{\alpha}_1) < S^{\lambda}(\tilde{\alpha}_2)$. Risk-averse decision-makers believe that $\tilde{\alpha}_1 \prec \tilde{\alpha}_2$;
2. If $\lambda > 1/2$, then $S^{\lambda}(\tilde{\alpha}_1) > S^{\lambda}(\tilde{\alpha}_2)$. Risk-seeking decision-makers believe that $\tilde{\alpha}_1 \succ \tilde{\alpha}_2$;
3. If $\lambda = 1/2$, then $S^{\lambda}(\tilde{\alpha}_1) = S^{\lambda}(\tilde{\alpha}_2)$. Risk-neutral decision-makers believe that $\tilde{\alpha}_1 \sim \tilde{\alpha}_2$.

The derived rankings may be different due to different risk attitudes of the decision-makers. This result is consistent with the real situation. By using Eq. (7), we obtain that $J(\tilde{\alpha}_1) = J(\tilde{\alpha}_2) = 0$. Therefore, we cannot obtain the order relationship of these two alternatives. By applying Eq. (8), we obtain $L(\tilde{\alpha}_1) = 0.29$ and $L(\tilde{\alpha}_2) = 0.28$. Hence, $\tilde{\alpha}_1 \succ \tilde{\alpha}_2$, which is consistent with the result if the decision-makers are risk-seeking.

Example 6 The IVIFNs are identical to those provided in Example 3.

According to Eq. (10), we obtain

$S_{\text{New}}(\tilde{\alpha}_3) = -0.7 + 0.3\lambda + (2\theta - 1)(0.1 + 0.1\lambda),$
$S_{\text{New}}(\tilde{\alpha}_4) = 0.3 + 0.2\lambda + 0.5(2\theta - 1).$

Then, $S_{\text{New}}(\tilde{\alpha}_4) - S_{\text{New}}(\tilde{\alpha}_3) = 0.6 + 0.8\theta - 0.2\lambda\theta > 0$. For any $\lambda, \theta \in [0, 1]$, we have $S_{\text{New}}(\tilde{\alpha}_4) > S_{\text{New}}(\tilde{\alpha}_3)$, which implies that alternative $\tilde{\alpha}_4$ is better than alternative $\tilde{\alpha}_3$. This finding is intuitive.

Example 7 The IVIFNs are identical to those provided in Example 4.

According to Eq. (10), we obtain,

$S_{\text{New}}(\tilde{\alpha}_5) = -0.4 + \theta,$
$S_{\text{New}}(\tilde{\alpha}_6) = -0.4 + \theta + 0.4(1 - \theta)(\lambda - 1/2).$

Then, $S_{\text{New}}(\tilde{\alpha}_6) - S_{\text{New}}(\tilde{\alpha}_5) = 0.4(1 - \theta)(\lambda - 1/2)$. Therefore, we obtain the following:

1. If $0 \le \lambda < 1/2$ and $0 \le \theta < 1/2$, then $S_{\text{New}}(\tilde{\alpha}_5) > S_{\text{New}}(\tilde{\alpha}_6)$. Risk-averse decision-makers believe that $\tilde{\alpha}_5 \succ \tilde{\alpha}_6$;
2. If $1 \ge \lambda > 1/2$ and $1 > \theta > 1/2$, then $S_{\text{New}}(\tilde{\alpha}_5) < S_{\text{New}}(\tilde{\alpha}_6)$. Risk-seeking decision-makers believe that $\tilde{\alpha}_5 \prec \tilde{\alpha}_6$;
3. If $\lambda = 1/2$ and $\theta = 1/2$ then, $S_{\text{New}}(\tilde{\alpha}_5) = S_{\text{New}}(\tilde{\alpha}_6)$. Risk-neutral decision-makers believe that $\tilde{\alpha}_5 \sim \tilde{\alpha}_6$.

By using Eqs. (7) and (8), we obtain $\tilde{\alpha}_5$~$\tilde{\alpha}_6$. The order relationship of these two alternatives cannot be obtained. However, if we use the proposed method, their order relationship can be distinguished when $0 \le \lambda < 1/2$ or $1 \ge \lambda > 1/2$. This finding confirms the effectiveness of the proposed score function.

Example 8 Let $\tilde{\alpha}_7 = ([0.5, 0.5], [0.4, 0.5])$ and $\tilde{\alpha}_8 = ([0.4, 0.4], [0.3, 0.4])$ be two IVIFNs for two alternatives.

According to Eq. (10), we obtain,

$S_{\text{New}}(\tilde{\alpha}_7) = 0.2\lambda\theta,$
$S_{\text{New}}(\tilde{\alpha}_8) = 0.2\lambda\theta + 0.2(2\theta - 1).$

Then, $S_{\text{New}}(\tilde{\alpha}_8) - S_{\text{New}}(\tilde{\alpha}_7) = 0.2(2\theta - 1)$. For any $\lambda \in [0, 1]$, we have the following:

1. If $\theta < 1/2$, then $S_{New}(\tilde{\alpha}_7) > S_{New}(\tilde{\alpha}_8)$. Risk-averse decision-makers believe that $\tilde{\alpha}_7 \succ \tilde{\alpha}_8$;
2. If $\theta > 1/2$, then $S_{New}(\tilde{\alpha}_7) < S_{New}(\tilde{\alpha}_8)$. Risk-seeking decision-makers believe that $\tilde{\alpha}_7 \prec \tilde{\alpha}_8$;
3. If $\theta = 1/2$, then $S_{New}(\tilde{\alpha}_7) = S_{New}(\tilde{\alpha}_8)$. Risk-neutral decision-makers believe that $\tilde{\alpha}_7 \sim \tilde{\alpha}_8$.

By using Eqs. (7) and (8), we obtain $\tilde{\alpha}_7 \succ \tilde{\alpha}_8$, which is consistent with the result of risk-seeking decision-makers.

Some properties of the proposed score function of IVIFN are discussed in the following section to confirm its reasonability and effectiveness.

Property 1. *Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. If the score function $S_{New}(\tilde{\alpha})$ is defined by Definition 9, then.*

1. $-1 \le S_{\text{New}}(\tilde{\alpha}) \le 1$, for any $\lambda, \theta \in [0, 1]$;
2. For any λ, $\theta \in [0, 1]$, $S_{\text{New}}(\tilde{\alpha}) = 1$ if and only if $\tilde{\alpha} = ([1, 1], [0, 0])$;

3. For any $\lambda,\ \theta \in [0,1]$, $S_{\text{New}}(\tilde{\alpha}) = -1$ if and only if $\tilde{\alpha} = ([0,0],[1,1])$.

Proof

1. Combining Eq. (10) with $M^{\lambda}(\pi_{\tilde{\alpha}}) = 1 - M^{\lambda}(u_{\tilde{\alpha}}) - M^{\lambda}(v_{\tilde{\alpha}})$, we observe that,

$$S_{\text{New}}(\tilde{\alpha}) = M^{\lambda}(u_{\tilde{\alpha}}) - M^{\lambda}(v_{\tilde{\alpha}}) + (2\theta - 1)\left(1 - M^{\lambda}(u_{\tilde{\alpha}}) - M^{\lambda}(v_{\tilde{\alpha}})\right). \tag{11}$$

Notably,

$$0 \le \theta \le 1,\ 1 - M^{\lambda}(u_{\tilde{\alpha}}) - M^{\lambda}(v_{\tilde{\alpha}}) \ge 0,$$
$$M^{\lambda}(v_{\tilde{\alpha}}) \ge 0.$$

Hence,

$$S_{\text{New}}(\tilde{\alpha}) \le M^{\lambda}(u_{\tilde{\alpha}}) - M^{\lambda}(v_{\tilde{\alpha}}) + \left(1 - M^{\lambda}(u_{\tilde{\alpha}}) - M^{\lambda}(v_{\tilde{\alpha}})\right) = 1 - 2M^{\lambda}(v_{\tilde{\alpha}}) \le 1.$$

However,

$$S_{\text{New}}(\tilde{\alpha}) = M^{\lambda}(u_{\tilde{\alpha}}) - M^{\lambda}(v_{\tilde{\alpha}}) + (2\theta - 1)\left(1 - M^{\lambda}(u_{\tilde{\alpha}}) - M^{\lambda}(v_{\tilde{\alpha}})\right) = 2M^{\lambda}(u_{\tilde{\alpha}}) + 2\theta M^{\lambda}(\pi_{\tilde{\alpha}}) - 1.$$

Considering that,

$$M^{\lambda}(u_{\tilde{\alpha}}) \ge 0,\ M^{\lambda}(\pi_{\tilde{\alpha}}) \ge 0,$$
$$\theta \ge 0,$$

we obtain,

$$S_{\text{New}}(\tilde{\alpha}) \ge -1.$$

This result establishes

$$-1 \le S_{\text{New}}(\tilde{\alpha}) \le 1 \text{ for any } \lambda, \theta \in [0,1].$$

2. Combining Eq. (10) with

$$M^{\lambda}(u_{\tilde{\alpha}}) = a + \lambda(b-a),\ M^{\lambda}(v_{\tilde{\alpha}}) = c + (1-\lambda)(d-c),$$
$$M^{\lambda}(\pi_{\tilde{\alpha}}) = \lambda(1-b-c) + (1-\lambda)(1-a-d),$$

we can observe

$$S_{\text{New}}(\tilde{\alpha}) = 2a - 1 + 2\lambda(b-a) + 2\theta(1-a-d) + 2\theta\lambda(a+d-b-c). \tag{12}$$

If $S_{\text{New}}(\tilde{\alpha}) = 1$ for any $\lambda,\ \theta \in [0,1]$ according to Eq. (12), we can obtain

$$2a - 1 = 1,\ 2(b-a) = 0,$$
$$2(1-a-d) = 0,$$
$$2(a+d-b-c) = 0.$$

Then,

$$\tilde{\alpha} = ([1,1],[0,0]).$$

However, if $\tilde{\alpha} = ([1,1],[0,0])$, then we can easily observe $S_{\text{New}}(\tilde{\alpha}) = 1$ for any $\lambda, \theta \in [0,1]$.
Hence,

$$S_{\text{New}}(\tilde{\alpha}) = 1 \Leftrightarrow \tilde{\alpha} = ([1,1],[0,0]) \text{ for any } \lambda,\ \theta \in [0,1].$$

3. If $S_{\text{New}}(\tilde{\alpha}) = -1$ for any $\lambda, \theta \in [0,1]$. Using Eq. (12), we can obtain

$$2a - 1 = -1,\ 2(b-a) = 0,$$
$$2(1-a-d) = 0,$$
$$2(a+d-b-c) = 0.$$

Thus,

$$\tilde{\alpha} = ([0,0],[1,1]).$$

However, if $\tilde{\alpha} = ([0,0],[1,1])$, then we can easily observe $S_{\text{New}}(\tilde{\alpha}) = -1$ for any $\lambda, \theta \in [0,1]$.
Hence,

$$S_{\text{New}}(\tilde{\alpha}) = -1 \Leftrightarrow \tilde{\alpha} = ([0,0],[1,1]) \text{ for any } \lambda,\ \theta \in [0,1].$$

Property 2 *Let* $\tilde{\alpha} = \left([a_1,b_1],[c_1,d_1]\right)$ *and* $\tilde{\beta} = \left([a_2,b_2],[c_2,d_2]\right)$ *be two IVIFNs. If* $a_1 \ge a_2$, $b_1 \ge b_2$ *and* $c_1 \le c_2$, $d_1 \le d_2$, *then* $S_{\text{New}}(\tilde{\alpha}) \ge S_{\text{New}}(\tilde{\beta})$.

Proof According to Eq. (10), by taking the partial derivative of $S_{\text{New}}(\tilde{\alpha})$ with respect to a, b, c, and d, we can obtain,

$$\frac{\partial S_{\text{New}}(\tilde{\alpha})}{\partial a} = 2(1-\lambda)(1-\theta),\quad \frac{\partial S_{\text{New}}(\tilde{\alpha})}{\partial b} = 2\lambda(1-\theta),$$
$$\frac{\partial S_{\text{New}}(\tilde{\alpha})}{\partial c} = -2\lambda\theta,\quad \text{and}\quad \frac{\partial S_{\text{New}}(\tilde{\alpha})}{\partial d} = -2(1-\lambda)\theta.$$

Notably, $0 \le \lambda \le 1$, and $0 \le \theta \le 1$. Hence,

$$\frac{\partial S_{\text{New}}(\tilde{\alpha})}{\partial a} \ge 0,$$
$$\frac{\partial S_{\text{New}}(\tilde{\alpha})}{\partial b} \ge 0,$$
$$\frac{\partial S_{\text{New}}(\tilde{\alpha})}{\partial c} \le 0,$$
$$\frac{\partial S_{\text{New}}(\tilde{\alpha})}{\partial d} \le 0.$$

Then, $S_{\text{New}}(\tilde{\alpha})$ on a and b monotonously increases; $S_{\text{New}}(\tilde{\alpha})$ on c and d monotonously decreases.
Therefore, if

$$a_1 \ge a_2,\ b_1 \ge b_2,\ c_1 \le c_2,\ \text{and } d_1 \le d_2,$$

then,

$S_{\text{New}}(\tilde{\alpha}) \geq S_{\text{New}}(\tilde{\beta})$.

Property 3 *Let* $\tilde{\alpha} = ([a_1, b_1], [c_1, d_1])$ *and* $\tilde{\beta} = ([a_2, b_2], [c_2, d_2])$ *be two IVIFNs. For any* $\lambda, \theta \in [0, 1]$, $S_{\text{New}}(\tilde{\alpha}) = S_{\text{New}}(\tilde{\beta})$ *if and only if* $\tilde{\alpha} = \tilde{\beta}$.

Proof According to Eq.(10), we obtain,

$$S_{\text{New}}(\tilde{\alpha}) = 2a_1 - 1 + 2\lambda(b_1 - a_1) + 2\theta(1 - a_1 - d_1) + 2\theta\lambda(a_1 + d_1 - b_1 - c_1),$$
$$S_{\text{New}}(\tilde{\beta}) = 2a_2 - 1 + 2\lambda(b_2 - a_2) + 2\theta(1 - a_2 - d_2) + 2\theta\lambda(a_2 + d_2 - b_2 - c_2).$$

If $S_{\text{New}}(\tilde{\alpha}) = S_{\text{New}}(\tilde{\beta})$ for any $\lambda, \theta \in [0, 1]$, then, we obtain

$$2a_1 - 1 = 2a_2 - 1,\ 2(b_1 - a_1) = 2(b_2 - a_2),$$
$$2(1 - a_1 - d_1) = 2(1 - a_2 - d_2),$$
$$2(a_1 + d_1 - b_1 - c_1) = 2(a_2 + d_2 - b_2 - c_2).$$

Evidently,

$$a_1 = a_2,\ b_1 = b_2,\ c_1 = c_2,$$
$$d_1 = d_2.$$

Then,

$$\tilde{\alpha}_1 = \tilde{\alpha}_2.$$

However, if $\tilde{\alpha} = \tilde{\beta}$, then evidently, $S_{New}(\tilde{\alpha}) = S_{New}(\tilde{\beta})$ for any $\lambda, \theta \in [0, 1]$. Hence,

$$S_{\text{New}}(\tilde{\alpha}) = S_{\text{New}}(\tilde{\beta}) \Leftrightarrow \tilde{\alpha} = \tilde{\beta} \quad \text{for any } \lambda, \theta \in [0, 1].$$

MCDM in interval-valued intuitionistic fuzzy environments

In this section, an MCDM problem in an interval-valued intuitionistic fuzzy environment is presented to illustrate the feasibility and effectiveness of the proposed method.

Let $A = \{A_1, A_2, \ldots, A_n\}$ be a set of alternatives, and let $G = \{G_1, G_2, \ldots, G_m\}$ be a set of criteria. Assume that the weight of criteria G_j, $j = 1, 2, \ldots, m$ is provided by the decision-makers and denoted as w_j, $w_j \in [0, 1]$, and $\sum_{j=1}^{m} w_j = 1$. The evaluation of alternatives A_i with respect to criterion G_j is provided by IVIFNs and denoted as $\tilde{r}_{ij} = \left(\left[a'_{ij}, b'_{ij}\right], \left[c'_{ij}, d'_{ij}\right]\right)$, $i = 1, 2, \ldots, n$. $j = 1, 2, \ldots, m$, which indicates the degree of alternative A_i that satisfies or does not satisfy criterion G_j. Therefore, the decision matrix $D = (\tilde{r}_{ij})_{n \times m}$ is obtained, which is expressed by IVIFNs.

In summary, the MCDM procedure is designed to determine the best alternative. This procedure is discussed in the following steps.

Step 1: Normalize the IVIFN decision matrix

Using the formulae introduced in Ye [29], the normalized IVIFN decision matrix $\overline{D} = (\tilde{\alpha}_{ij})_{n \times m}$ is obtained, where $\tilde{\alpha}_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$ and $w = (w_1, w_2, \ldots, w_m)^T$ are the weight vector of the criteria.

Step 2: Calculate the weighted comprehensive score values

Choose the attitude parameter values λ and θ according to the decision-makers' risk attitudes. Then, the weighted comprehensive score value S_i of alternatives A_i, $i = 1, 2, \ldots, n$ can be obtained by the following formula:

$$S_i(\tilde{\alpha}_{i1}, \tilde{\alpha}_{i2}, \ldots, \tilde{\alpha}_{im}) = w_1 S_{\text{New}}(\tilde{\alpha}_{i1}) + w_2 S_{\text{New}}(\tilde{\alpha}_{i2}) + \cdots + w_m S_{\text{New}}(\tilde{\alpha}_{im}). \tag{13}$$

Step 3: Obtain the ranking of the alternatives

The ranking of alternatives A_i, $i = 1, 2, \ldots, n$ can be obtained based on the weighted comprehensive score values S_i, $i = 1, 2, \ldots, n$.

Illustrative example

In this section, we present the application of the proposed decision-making method through a practical example introduced in [49]. Comparative analyses of this method are also conducted to show the effectiveness of the proposed method.

A panel has four possible alternatives for company investment: (1) A_1 is a car company, (2) A_2 is a food company, (3) A_3 is a computer company, and (4) A_4 is an arms company. The investment company will make a decision according to five criteria: (1) G_1 is the productivity, (2) G_2 is the technological innovation capability, (3) G_3 is the marketing capability, (4) G_4 is the management, and (5) G_5 is the risk avoidance. The criteria are independent, and the criterion weights are provided by the decision-makers as follows: $w = (0.2, 0.3, 0.15, 0.1, 0.25)^T$. Since the available investment companies involve several different industries, the decision-makers may not have sufficient knowledge to provide the membership degree and non-membership degree of the admissible alternatives evaluated under the given several attributes as crisp values. Therefore, their ranges can be provided. Using IVIFNs to express decision-makers' evaluation information is considered the most effective method. The evaluation information is included in the IVIFN decision matrix $\overline{D} = (\tilde{\alpha}_{ij})_{4 \times 5}$, which is a normalized one.

$$\overline{D}=\begin{pmatrix} ([0.4,0.5],[0.1,0.3]) & ([0.5,0.6],[0.1,0.2]) & ([0.3,0.4],[0.2,0.3]) & ([0.7,0.8],[0.1,0.2]) & ([0.5,0.6],[0.1,0.2]) \\ ([0.5,0.6],[0.1,0.2]) & ([0.3,0.4],[0.1,0.3]) & ([0.7,0.8],[0.1,0.2]) & ([0.3,0.4],[0.3,0.4]) & ([0.4,0.5],[0.1,0.2]) \\ ([0.6,0.7],[0.1,0.2]) & ([0.7,0.8],[0.1,0.2]) & ([0.5,0.6],[0.3,0.4]) & ([0.4,0.5],[0.3,0.4]) & ([0.3,0.5],[0.3,0.4]) \\ ([0.5,0.6],[0.2,0.3]) & ([0.4,0.5],[0.3,0.4]) & ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.1,0.3]) \end{pmatrix}$$

.

Process developed using the proposed method

To select the best potential company in which to invest, the decision-making process is developed through the following steps.

Step 1: Normalize the IVIFN decision matrix

The normalized IVIFN decision matrix is provided by the decision-makers and shown in decision matrix $\overline{D}=\left(\tilde{\alpha}_{ij}\right)_{4\times 5}$.

Step 2: Calculate the weighted comprehensive score values

Select the attitude parameter values $\lambda,\theta\in[0,1]$ according to the decision-makers' risk attitudes. Then, the weighted comprehensive score values of alternatives A_i, $i=1,2,3,4$ can be obtained according to Eq. (13). The computing results are shown in Table 1.

Step 3: Obtain the ranking of the alternatives

According to the weighted comprehensive score values shown in Step 2, the alternatives are ranked as follows: (1) if $\lambda=\theta=\frac{1}{3}$, then $A_3\succ A_4\succ A_1\succ A_2$, which implies that a computer company is the best alternative for the company to invest in; (2) if $\lambda=\theta=\frac{1}{2}$, then $A_1\succ A_3\succ A_4\succ A_2$, which implies that a car company is the best alternative for the company to invest in; and (3) if $\lambda=\theta=\frac{2}{3}$, then $A_1\succ A_2\succ A_3\succ A_4$, and a car company is the best alternative for the company to invest in.

Table 1 The weighted comprehensive score values with different attitude parameters

Attitude parameters	Weighted comprehensive score value
$\lambda=\theta=\frac{1}{3}$	$S_{\mathrm{New}}(\tilde{\alpha}_1)=0.2078, S_{\mathrm{New}}(\tilde{\alpha}_2)=0.14$ $S_{\mathrm{New}}(\tilde{\alpha}_3)=0.2378, S_{\mathrm{New}}(\tilde{\alpha}_4)=0.2122$
$\lambda=\theta=\frac{1}{2}$	$S_{\mathrm{New}}(\tilde{\alpha}_1)=0.345, S_{\mathrm{New}}(\tilde{\alpha}_2)=0.29$ $S_{\mathrm{New}}(\tilde{\alpha}_3)=0.3325, S_{\mathrm{New}}(\tilde{\alpha}_4)=0.3025$
$\lambda=\theta=\frac{2}{3}$	$S_{\mathrm{New}}(\tilde{\alpha}_1)=0.4844, S_{\mathrm{New}}(\tilde{\alpha}_2)=0.4433$ $S_{\mathrm{New}}(\tilde{\alpha}_3)=0.4244, S_{\mathrm{New}}(\tilde{\alpha}_4)=0.3956$

Table 2 Ranking results obtained from different methods

Methods		Score values	Ranking results
Xu and Chen [42] method $S_1(\tilde{\alpha})=\frac{a-c+b-d}{2}$		$S^1_{\tilde{\alpha}_1}=0.3450, S^1_{\tilde{\alpha}_2}=0.2800$ $S^1_{\tilde{\alpha}_3}=0.3325, S^1_{\tilde{\alpha}_4}=0.3025$	$A_1\succ A_3\succ A_4\succ A_2$
Ye [29] method $S_2(\tilde{\alpha})=a+b-1+\frac{1}{2}(c+d)$		$S^2_{\tilde{\alpha}_1}=0.2150, S^2_{\tilde{\alpha}_2}=0.1150$ $S^2_{\tilde{\alpha}_3}=0.4150, S^2_{\tilde{\alpha}_4}=0.4075$	$A_3\succ A_4\succ A_1\succ A_2$
Lakshmana Gomathi Nayagam et al. [43] method $S_3(\tilde{\alpha})=\frac{1}{2}[a+b-d(1-b)-c(1-a)]$		$S^3_{\tilde{\alpha}_1}=0.4358, S^3_{\tilde{\alpha}_2}=0.3658$ $S^3_{\tilde{\alpha}_3}=0.4685, S^3_{\tilde{\alpha}_4}=0.4535$	$A_3\succ A_4\succ A_1\succ A_2$
Wang and Chen [26] method $S_4(\tilde{\alpha})=$ $\frac{a+b+\sqrt{bd}(1-a-c)+\sqrt{ac}(1-b-d)}{2}$		$S^4_{\tilde{\alpha}_1}=0.6163, S^4_{\tilde{\alpha}_2}=0.5696$ $S^4_{\tilde{\alpha}_3}=0.6501, S^4_{\tilde{\alpha}_4}=0.6399$	$A_3\succ A_4\succ A_1\succ A_2$
Wang and Chen [27] method $S_5(\tilde{\alpha})=\frac{(a+b)(a+c)-(c+d)(b+d)}{2}$		$S^5_{\tilde{\alpha}_1}=0.1735, S^5_{\tilde{\alpha}_2}=0.1165$ $S^5_{\tilde{\alpha}_3}=0.1938, S^5_{\tilde{\alpha}_4}=0.1633$	$A_3\succ A_1\succ A_4\succ A_2$
Lakshmana Gomathi Nayagam et al. [47] method $S_6(\tilde{\alpha})=\frac{a+b+c-d+ab+cd}{3}$		$S^6_{\tilde{\alpha}_1}=0.4090, S^6_{\tilde{\alpha}_2}=0.3543$ $S^6_{\tilde{\alpha}_3}=0.4965, S^6_{\tilde{\alpha}_4}=0.4718$	$A_3\succ A_4\succ A_1\succ A_2$
Proposed method: $S_{\mathrm{New}}(\tilde{\alpha})$	$\lambda=\theta=\frac{1}{3}$	$S_{\tilde{\alpha}_1}=0.2078, S_{\tilde{\alpha}_2}=0.1400$ $S_{\tilde{\alpha}_3}=0.2378, S_{\tilde{\alpha}_4}=0.2122$	$A_3\succ A_4\succ A_1\succ A_2$
	$\lambda=\theta=\frac{1}{2}$	$S_{\tilde{\alpha}_1}=0.3450, S_{\tilde{\alpha}_2}=0.2900$ $S_{\tilde{\alpha}_3}=0.3325, S_{\tilde{\alpha}_4}=0.3025$	$A_1\succ A_3\succ A_4\succ A_2$
	$\lambda=\theta=\frac{2}{3}$	$S_{\tilde{\alpha}_1}=0.4844, S_{\tilde{\alpha}_2}=0.4433$ $S_{\tilde{\alpha}_3}=0.4244, S_{\tilde{\alpha}_4}=0.3956$	$A_1\succ A_2\succ A_3\succ A_4$

Comparative study and discussion

To validate the feasibility of the proposed approach, a comparative study with other approaches is conducted on the basis of the identical illustrative example introduced before "Process developed using the proposed method". To compare these approaches, the ranking results obtained from the different methods are listed in Table 2.

Table 2 shows that the ranking results derived from the approach of Xu and Chen [42] are identical to the ranking results of the proposed approach when $\lambda = \theta = \frac{1}{2}$. The ranking results derived from the approaches of Ye [29], Lakshmana Gomathi Nayagam et al. [43], Wang and Chen [26], and Lakshmana Gomathi Nayagam et al. [47] are identical to the proposed approach when $\lambda = \theta = \frac{1}{3}$. When $\lambda = \theta = \frac{2}{3}$, the method of Wang and Chen [27] and the proposed method are different from the other methods. However, the best alternative derived from Wang and Chen [27] is identical to those from Ye [29], Lakshmana Gomathi Nayagam et al. [43], Wang and Chen [26], and Nayagam et al. [47]. The best alternative is derived from Xu and Chen [42] method and the proposed approach when $\lambda = \theta = \frac{1}{2}$ and $\lambda = \theta = \frac{1}{3}$ are the same, thereby indicating that the proposed approach is effective. However, the results derived from the proposed method vary when decision-makers have different risk attitudes. Identical rankings may be derived using different approaches because when the value is set to $\lambda = \theta = \frac{1}{2}$ in the proposed method, the ranking index is equivalent to that in Xu and Chen [42] method. The same ranking index results in identical outcomes. The ranking indexes are different when the value is set to $\lambda = \theta = \frac{1}{3}$ in the proposed method compared with the ranking indexes of Ye [29], Lakshmana Gomathi Nayagam et al. [43], Wang and Chen [26], and Lakshmana Gomathi Nayagam et al. [47]. These approaches derive the rankings of the alternatives on the basis of the score values of IVIFNs. Therefore, the index of the score values of IVIFNs may result in identical ranking results.

In addition, Table 2 shows that the ranking results are different depending on decision-makers' risk attitudes. Risk-averse decision-makers believe that $A_3 \succ A_4 \succ A_1 \succ A_2$. Risk-neutral decision-makers believe that $A_1 \succ A_3 \succ A_4 \succ A_2$, while risk-seeking decision-makers believe that $A_1 \succ A_2 \succ A_3 \succ A_4$. The risk attitudes of the decision-makers are confirmed to affect the decision results in the decision-making process. Based on this fact, the risk parameter and score function must be combined, and the proposed method in this paper has a wide practical application.

The comparative analysis indicates that the MCDM approach proposed in this study has the following advantages over the other approaches,

1. Based on the membership degree, non-membership degree, hesitancy degree, and decision-makers' risk attitudes, a new score function has been proposed to reflect that the decision results are highly affected by the decision-makers' risk attitudes.
2. The proposed new score function is equivalent to Xu and Chen [42] method when the decision-makers' risk attitudes are set to certain values. This finding confirms that Xu and Chen [42] method is a special case of the proposed method, and the proposed method has a wide practical application.

Conclusions

To examine decision-makers' mentality, two mentality parameters are introduced in this paper. First, a new score function of IVIFNs is proposed that uses the introduced mentality parameters. Second, some properties of the proposed score function of IVIFNs are discussed to confirm the score function's reasonability and effectiveness. Furthermore, the weighted comprehensive score value of IVIFNs is proposed, and an MCDM problem is developed in an interval-valued intuitionistic fuzzy environment. Finally, a numerical example and comparative analyses are provided to illustrate the feasibility and effectiveness of the proposed method.

This study provides several substantial contributions to MCDM problems, which are summarized as follows. (1) A new score function has been proposed to reflect decision-makers' risk attitudes. The risk parameters of decision-makers are associated with the membership degree, non-membership degree, and hesitancy degree of IVIFNs. (2) The proposed MCDM method is more widely used than Xu and Chen [42] method in practice, as the latter is a special case of the proposed method. In the future, MCDM with large-scale social networks is worth examining, and the complex practical problems should be further studied.

Acknowledgements The authors thank the anonymous reviewers and the editor for their insightful and constructive comments and suggestions that have led to an improved version of this paper. This work was supported by the National Natural Science Foundation of China (No. 61866006), Guangxi innovation-driven development of special funds project (gui ke AA17204091), the Natural Science Foundation of Guangxi (No. AB17292095), the Research Funds for the Guangxi University Xingjian College of Science and Liberal Arts (No. Y2018ZKT01) and Promotion project of Middle-aged and Young Teachers' Basic Scientific Research Ability in Universities of Guangxi (No. 2019KY0963).

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