



An integrated group FWA-ELECTRE III approach based on interval type-2 fuzzy sets for solving the MCDM problems using limit distance mean

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Abstract

The material handling equipment (MHE) has a close connection with layout of machinery and plays the important role in productivity of servicing or manufacturing systems. Since each of MHE has distinct characteristics than the others with respect to conflicting criteria and design experts may state the different subjective judgments with respect to qualitative criteria, the material handling equipment selection problem (MHESP) can be taken into account as a group multi-criteria decision-making (GMCDM) problem. In this paper, a version of type-2 fuzzy sets (T2FSs), named Gaussian interval type-2 fuzzy sets (GIT2FSs), is first used as an alternative to the traditional triangular membership functions (MFs) to weight criteria and sub-criteria and also evaluate of alternatives with respect to sub-criteria. The synthetic value method of GIT2FSs is then carried out to convert the assessments stated as GIT2FSs for each alternative with respect to each sub-criterion and also weights of criteria (sub-criteria) into the single fuzzy rating and weight, respectively. Then, the fuzzy weighted average (FWA) approach is adopted to integrate the single fuzzy ratings of each alternative with respect to sub-criteria and the single fuzzy weights of sub-criteria under each criterion with the aggregated weighted ratings. In next stage, ELECTRE III (ELimination Et Choix Traduisant la Realité—elimination and choice translation reality) is generalized with GIT2FSs to select the optimal MHE through a new ranking approach. Moreover, some arithmetic operations and properties are extended to GIT2FSs. In addition, to demonstrate its potential applications, the proposed methodology is implemented in a real case study and an illustrative example, and then, the ranking results are compared with those of the others in the literature. Finally, the sensitivity analysis is carried out to show robustness and stability of the obtained results.

Keywords Material handling equipment selection problem · Gaussian interval type-2 fuzzy sets · FWA · ELECTRE III

Introduction

In the today competition world, one of the managers' important duties for gaining the greater share of product market is to reduce costs. The considerable section of these costs is related to the material handling. After the stages of production and process design when setting up of the manufacturing or servicing system, the design of material handling equipment

(MHE) (including the activities of selection, arrangement, and establishment) should be carried out in stage of operation design. Obviously, the material handling equipment selection problem (MHESP) changes the productivity index, since it depends on the cost of goods manufactured directly. MHE can encompass “30–75% of this cost and the selection of efficient MHE can be responsible for reducing the plant's operating costs by 15–30%” [1]. The choice of unsuitable MHE not only results in the rearranging costs but also imposes reinvestment for purchasing new MHE. There are different models of MHE for transporting goods or materials. Each of them has distinguished properties as compared to the others, such that solution of the MHESP has transformed with a complex and time-consuming process. Complexity of MHESP has some reasons as follows. These properties are usually categorized into two groups of criteria, namely the quantitative criteria (such as measure of investment, cost

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of spare parts, capacity, etc.) and subjective or qualitative criteria (such as attainability of spare parts, convenience, maintainability, etc.). On the other hand, some criteria (along with their corresponding sub-criteria) are of benefit type and some criteria (along with their corresponding sub-criteria) are of cost type. In addition, since each criterion consists of several sub-criteria, it is difficult to apply several times of the decision-making technique to each criterion and then calculate the final score. Furthermore, an expert may apply the different probability density functions such as Gaussian, Beta, or etc. distribution instead of triangular or trapezoidal membership functions (MFs). Hence, such a problem can be considered as a multi-criteria decision-making (MCDM) problem in which the qualitative criteria are expressed based on verbal terms or linguistic variables. An MCDM problem can be stated as the process of determining the best alternative among all possible alternatives with respect to the different criteria. One of the important advantages of MCDM problems is that they can take into account both quantitative and qualitative criteria (including crisp and fuzzy data). There are different techniques for solving the MCDM problems according to appraising style of criteria or alternatives. ELimination Et Choix Traduisant la Realité (ELECTRE) that was first presented by Roy [2] is one of the most popular techniques of MCDM. The case that distinguishes this method than the others is property of its non-compensatory. It means that bad scores of criteria cannot be compensated by good scores of other criteria. It compares different alternatives as pairwise by considering various indices and determining indifference, veto, and preference thresholds. In the classical ELECTRE method, the measures of appraisals and weights of criteria are determined with crisp values. However, the crisp data are not suitable for managers' subjective judgments when faced with the real decision-making problems, because these are often expressed as verbal terms. To express the uncertainty in the real-world problems, fuzzy data instead of crisp data have been adopted in many MCDM techniques including ELECTRE. In ELECTRE III, the ratings and weights can state by means of the fuzzy data. However, a decision-maker may have doubt about the measure of MF. In other words, in type-1 fuzzy sets, it is often difficult for an expert to express his/her notions as a specified number at the interval $[0, 1]$ regarding MF. Hence, type-2 fuzzy sets (T2FSs) were presented by Zadeh [3] as a generalization of the concept of type-1 fuzzy sets. T2FSs are depicted in three-dimensional space including finite non-empty set X , secondary grade $f_x(u)$, and the domain of secondary MF J_x , where MF is defined by a fuzzy set at the interval $[0, 1]$ [4]. Interval type-2 fuzzy sets (IT2FSs) are a special case of T2FSs characterized by an interval MF, such that decision-makers can have more flexible to represent the ambiguity of MF [5]. In other words, IT2FSs are suitable for situations in which decision-maker not only has indecisive standpoints as compared to measure

of a variable on X but also he is uncertain regarding measure of MF. IT2FSs have a greater ability as compared to type-1 fuzzy sets to handle uncertainty and imperfect information coming from judgments of different experts and can motivate more degrees of flexibility to show the vagueness in real applications. Lack of freedom degree for selecting the interval MFs is led to increase the number of experts, such that the solving the MCDM problem will be complicated and time-consuming. Accordingly, many studies have applied IT2FSs to the management and industrial scopes as follows: Hagrass [6], Sepulveda et al. [7], Kumbasar and Hagrass [8], Lynch et al. [9], and Kumbasar [10] adopted T2FSs to optimize and design intelligent controllers. Similarly, Castillo et al. [11] presented a comparative study of type-2 fuzzy logic systems with respect to interval type-2 and type-1 fuzzy logic systems to show the efficiency and performance of a generalized type-2 fuzzy logic controller. Ontiveros-Robles et al. [12] proposed a comparison regarding the robustness of interval type-2 and generalized type-2 fuzzy logic controllers, to generate criteria to decide which type of controller is better in specific applications. Auephanwiriyakul et al. [13] and Liu and Mendel [14] used T2FSs to obtain data from human sources (linguistic responses) in management studies. Bouchachia and Mittermeir [15] described a fuzzy neural approach for information retrieval where the fuzzy representation was used to reflect the hierarchical nature of texts. Wu and Mendel [16] extended antecedent connector word models to the framework of Mamdani type-1 fuzzy logic system where the uncertainties originating from descriptive words, connector words, and data were modeled simultaneously. Castillo and Melin [17] and also Own et al. [18] adopted type-2 fuzzy logic for achieving adaptive noise cancelation. Particularly, Melin et al. [19] used the neural networks to analyze the sound signal of an unknown speaker, and then, a set of type-2 fuzzy rules were applied to decision-making. Moreover, they utilized genetic algorithms to optimize the architecture of the neural networks. Sanchez et al. [20] introduced the generalized type-2 fuzzy control system for a mobile robot regarding three types of external perturbations, namely band-limited white noise, pulse noise, and uniform random number noise. Liang and Wang [21] and also Shu and Liang [22] applied T2FSs to analyze the lifetime of a wireless sensor network and forecast strength of sensed signal in wireless sensors, respectively. Baguley et al. [23] planned a T2FSs-based model to predict time to market from performance measures, which is a potentially valuable tool for decision-making and continuous improvement. Gu and Zhang [24] suggested the web shopping expert system based on the IT2FSs for online users. Rhee [25] introduced interval type-2 clustering method by modifying the prototype-updating and hard-partitioning procedures in the type-1 fuzzy objective function-based clustering method. Linda and Manic [26] extended interval type-2 fuzzy logic

to the fuzzy voting scheme, which constitutes an essential component of many fault tolerant systems. Cervantes and Castillo [27] presented a new approach for complex control combining several simpler individual fuzzy controllers. It is useful when the problem is a multivariable control system. Castillo et al. [28] applied the granular approach for intelligent control using generalized type-2 fuzzy logic in which granularity was used to divide the design of the global controller into several individual simpler controllers. In the scope of MCDM, Rashid et al. [29] extended technique for order preference by similarity to ideal solution (TOPSIS) to generalized interval-valued trapezoidal fuzzy numbers for the selection of a suitable robot. Keshavarz Ghorabae [30] presented a GMCDM for robot selection where vlskriterijumska optimizacija i kompromisno resenje (VIKOR) method with interval type-2 fuzzy numbers was handled. Soner et al. [31] integrated analytic hierarchy process (AHP) and VIKOR with IT2FSs for the hatch cover design selection problem, which has utmost importance in structure of bulk carrier ships to prevent water ingress and protect cargo from the external damages. Zhong and Yao [32] extended the ELECTRE I method to interval type-2 fuzzy numbers for electronic materials supplier selection problem. Abdullah et al. [33] handled interval type-2 fuzzy simple additive weighting (SAW) for ambulance location problem, which has an important role for improving the emergency medical services. Deveci et al. [34] developed weighted aggregated sum product assessment (WASPAS)-based TOPSIS to IT2FSs for the car sharing station selection problem.

There are the known versions for IT2FSs such as trapezoidal interval type-2 fuzzy sets (TraIT2FSs), triangular interval type-2 fuzzy sets (TriIT2FSs), and Gaussian interval type-2 fuzzy sets (GIT2FSs) in the literature. Triangular or trapezoidal *MFs* are the simplest *MFs* formed using straight lines. *MFs* of triangular and trapezoidal fuzzy numbers have steep slopes in their reference points. In the real-life problems, however, the decision-maker may consider smoother slope for the *MFs* in reference points. Hence, “Gaussian *MFs* are suitable for problems requiring continuously differentiable curves, whereas the triangular and trapezoidal *MFs* do not possess these abilities” [35]. In this paper, the performance ratings with respect to sub-criteria and also weights of criteria and sub-criteria are first expressed as linguistic variables and Gaussian interval type-2 fuzzy numbers (GIT2FNs) are then defined for them. GIT2FNs are better selection than the others while the *MFs* are in the curved form.

The goal of this paper is to introduce a generalized hybrid group decision-making methodology based on GIT2FSs for MHESP. Application of fuzzy data as GIT2FNs is more suitable relative to type-1 fuzzy sets according to the reasons described above. The following cases are the main contributions of the present paper:

- Because of MHE’s selection which is a group activity [36], the synthetic value method is implemented in integrating the interval type-2 fuzzy data of an alternative with respect to sub-criteria, interval type-2 fuzzy weights of criteria and sub-criteria, and interval type-2 fuzzy thresholds.
- Several MCDM techniques based on IT2FSs are integrated to solve the MHESP.
- Since each criterion contains a set of sub-criteria with different weights, the authors first utilized the interval type-2 fuzzy weighted average (FWA) technique as a suitable integrating tool for aggregating the synthetic type-2 fuzzy ratings of each alternative with respect to all sub-criteria under each criterion and the synthetic type-2 fuzzy weights of sub-criteria to the aggregated weighted ratings. This approach makes it unnecessary to use several times of the ELECTRE III technique for each criterion.
- The interval type-2 fuzzy ELECTRE III method based on the aggregated weighted ratings is then applied to choose optimal MHE.
- The new proposed methodology is usable to each type of *MF* (both straight and curve lines). Moreover, it can be applied to rank type-1 fuzzy sets and T2FSs.
- To incorporate GIT2FNs into the MCDM techniques, some arithmetic operations of GIT2FNs are presented.
- In addition to the final orders of alternatives, the proposed ranking methodology is applied to determine thresholds and weight criteria and sub-criteria.

The rest of present paper is organized as follows: Sect. 2 presents the literature review regarding the MCDM techniques, ELECTRE III, and MHESP. In Sect. 3, preliminaries (including ELECTRE III and arithmetic operations of IT2FSs) are reviewed. The suggested ranking approach is introduced in Sect. 4. In Sect. 5, the proposed ranking methodology is incorporated into the ELECTRE III framework. Section 6 includes a real case study and two illustrative examples in which our ranking methodology is extended to the ELECTRE III method, and finally, conclusions and comparisons are studied in Sect. 7.

Literature review

Mardani et al. [37] listed the most famous methods for solving the MCDM problems and then categorized the applications of the MCDM problems into 15 fields. In a more general categorization, the MCDM techniques can be divided into two branches: multi-objective decision-making (MODM) and multi-attribute decision-making (MADM). An MADM problem includes small number of pre-specified alternatives assessed with respect to a set of criteria. In the classical MADM methods, evaluations and weights are stated as precise and crisp, whereas in the real world,

decision-making process accommodates with uncertain and doubt. If it is integrated with fuzzy data to deal with the uncertainty, it is named fuzzy MADM (FMADM). Hence, the FMADM-based approaches are applied to resolve the imprecision and vagueness. There are the different FMADM methods [e.g., preference ranking organization method for enrichment evaluations (PROMETHEE), TOPSIS, VIKOR, etc.]. In another comprehensive study, Mardani et al. [38] grouped the applications and methodologies of the fuzzy MCDM (FMCDM) techniques into four main fields as follows: engineering, management and business, science, and technology. Among the classical MCDM methods developed for solving the real-world decision problems, the ELECTRE III method (based on the concept of outranking relations) was introduced to relieve difficulties coming from the existing decision-making solution methods [39, 40].

Lots of applications based on the ELECTRE III method have been introduced in the literature. Roy and Bouyssou [2] compared two decision-aid models for a nuclear power plant siting problem using ELECTRE III. Siskos and Hubert [41] analyzed the energy alternatives from the social and public health point of view. Karagiannidis and Moussiopoulos [42] used ELECTRE III to decisions in the area of municipal solid-waste management in Greece. Alomoush [43] applied the ELECTRE III method to select the best location of thyristor-controlled phase angle regulator in a competitive energy market. Montazer et al. [44] designed a new mixed expert decision-aid system where fuzzy ELECTRE III method was applied to vendor selection. Rogers [45] evaluated complex civil/structural engineering projects using ELECTRE III. Tam et al. [46] implemented ELECTRE III in evaluating performance of construction plants. Leyva Lopez [47] solved the student selection problem using the ELECTRE III methodology under fuzzy outranking relation. Li and Wang [48] developed a new ranking method to the ELECTRE III method. Zak and Fierek [49] implemented ELECTRE III/IV and AHP in evaluating the generated variants of urban transportation system. Papadopoulos and Karagiannidis [50] implemented ELECTRE III in the optimization of decentralized energy systems. Radziszewska-Zielina [51] handled the ELECTRE III to select the best partner construction enterprise in terms of partnering relations. Giannoulis and Ishizaka [52] adopted a web-based decision support system with ELECTRE III for ranking British universities. Cavallo [53] used the ELECTRE III method to select production processes of thin-film solar technology. Marzouk [54] formulated ELECTRE III for value engineering applications. Liu and Zhang [55] proposed the entropy weight and the improved ELECTRE-III method for the supplier selection of a supply chain. Cliville et al. [56] deployed the ELECTRE III and MACBETH multi-criteria ranking methods for small and medium enterprises tactical performance improvements. Fancello et al. [57] planned a decision-aid system based on

ELECTRE III for safety analysis in a suburban road network. Heracles et al. [58] compared the ELECTRE III and PROMETHEE II methods regarding the choice of alternative investment scenarios for a geothermal field. Shafia et al. [59] ranked scenarios based on fuzzy cognitive map through the ELECTRE III where scenarios help to simulate future events to analyze the outcomes of possible courses.

On the other hand, there are the several extensions of the ELECTRE approach in the literature. Vahdani and Hadipour [60] presented the interval-valued fuzzy ELECTRE method for solving the MCDM problems where the weights of criteria were determined using the concept of interval-valued fuzzy sets. Hatami-Marbini and Tavana [61] developed the ELECTRE I method to consider the uncertain and imprecise assessments based on standpoints of a group of DMs. Vahdani et al. [62] extended the group ELECTRE method to intuitionistic fuzzy sets. Chen [63] first extended the interval-valued intuitionistic fuzzy ELECTRE (IVIF-ELECTRE) method. Next, the concordance and discordance indices were obtained using an aggregated importance weight score function and a generalized distance measurement between weighted evaluative ratings, respectively. Finally, two IVIF-ELECTRE ranking procedures were developed for the partial and complete ranking of the alternatives based on the concordance and discordance dominance matrices. Hashemi et al. [64] proposed the extended version of ELECTRE III under the interval-valued intuitionistic fuzzy environment. Wang et al. [65], Juan et al. [66], and Na et al. [67] provided the hesitant fuzzy environment on the different versions of the ELECTRE method.

Many attempts have been carried out for solving the MHESP during the last 3 decades. In a general classification, these studies can be categorized into the following four classes: (1) decision support systems (expert system for the design of repetitive manual materials handling tasks [68], the material handling expert system [69], the prototype expert system for the selection of industrial truck type, an expert consultant for in-plant transportation equipment [70], and a knowledge-based system for the choice of conveyor equipment [71]); (2) hybrid methods (an intelligent knowledge-based expert system, named intelligent consultant system, for selection and evaluation of material handling equipment, which has been composed of the following four models: equipment and their attributes, a rule-based database for selecting type of equipment, an MCDM technique (AHP) for selection of optimal equipment, and simulators to evaluate the performance of the equipment model [72], a decision support system based on axiomatic design principles [73], and a hybrid fuzzy knowledge-based expert system and genetic algorithm for the selection and assignment of the most appropriate MHE [74]); (3) optimization formulations (an integer programming model for minimizing material handling costs in manufacturing systems or warehousing facilities [75],

a hybrid methodology including the integer programming formulation for designing layout and material handling system simultaneously [76], a 0–1 integer programming model to determine operation allocation and selection of material handling system in an flexible manufacturing systems simultaneously [77], a mathematical model to implement problem modeled by radio-frequency identification technology [78], and a multi-objective optimization model for selection of material handling system for large ship [79]); (4) the MCDM problems (fuzzy axiomatic design principles for selecting automated guided vehicles [80], fuzzy AHP (FAHP), and decision-making trial and evaluation laboratory (DEMATEL)-ANP with geographic information system for the selection of the best space for leisure in a blighted urban site [81], FAHP and fuzzy additive ratio assessment for selection of conveyor [82], the fuzzy complex proportional assessment (COPRAS) method for evaluating performance measures of equipment in total productive maintenance [83], the AHP and TOPSIS methods for selecting the most appropriate tomography equipment [84], COPRAS, simple additive weighting, and TOPSIS for analyzing and prioritizing Rotor systems [85], FAHP, fuzzy entropy, fuzzy TOPSIS (FTOPSIS), and multi-objective mixed integer linear programming for the choice of MHE in warehouse [86], FAHP for bulk material handling equipment [87], and comparing the four MCDM methods: combinative distance based assessment, evaluation based on distance from average solution method, weighted aggregated sum product assessment method, and multi-objective optimization on the basis of ratio analysis for selection of automated guided vehicles [88]). Recently, Hadi-Vencheh and Mohamadghasemi [89] introduced an integrated MCDM model for the MHESP. Despite all its advantages, it has some shortcomings as follows: it applies the voting approach to weight criteria and sub-criteria. Unfortunately, the voting method is of compensatory model type, i.e., the value of a significantly weak assessment grade with respect to one criterion could be directly compensated by the measures of other good assessment grades. In addition, note that it uses the linguistic variables as assessment grades, such that it does not consider the shape of their MFs. On the other hand, the application of type-1 fuzzy sets is other limitation of the proposed approach based on the same reasons described in Sect. 1. Lack of more flexibility to choose the interval MFs is caused that cost and time of data collection increase. This methodology adopts the triangular fuzzy numbers with usual arithmetic operations to weight and evaluate. However, other versions such as GIT2FNs are the more suitable than the others as argued in Sect. 1. In addition, the authors obtained a set of compromise solutions. Nevertheless, one may select a unique solution as optimal MHE. Accordingly, the proposed methodology of present paper eliminates all weaknesses stated above.

This paper focuses on the last class where a model among the different commercial models is evaluated based on IT2FSs with respect to a set of criteria and sub-criteria. The constructed MCDM approaches for the MHESP have some shortcomings. Some studies expressed above adopt the traditional AHP with crisp data; the MHESP is solved by one design expert; and/or the equal weights are considered for criteria. On the other hand, type-1 fuzzy sets use a specified *MF* at interval [0, 1]. However, there are many situations in the real-world decision-makings, which due to further ambiguity, it is better to choose an interval for *MF*. Thus, to eliminate the difficulties reviewed in the studies mentioned above, this paper structures a new ranking technique for GIT2FNs. Due to the use of GIT2FNs in the ELECTRE III method, this paper is the first work in scope of the MHESPs.

Preliminaries

Basic principles of the ELECTRE III method

The ELECTRE method was first introduced by Benayoun et al. [90]. Later, this method was generalized in its different versions such as ELECTRE I [91], ELECTRE II [92], ELECTRE III [93], ELECTRE IV [94], ELECTRE IS [95], ELECTRE TRI [96], ELECTREGKMS [97], ELECTRE TRI-C [98], and ELECTRE TRI-NC [99]. Among them, the ELECTRE III method has been widely used to solve MCDM problems. ELECTRE III was adopted to relieve drawbacks of ELECTRE II. It can take into account uncertain data.

The following steps are a summary of the ELECTRE III methodology:

Step 1 Construct the crisp decision matrix $D = [x_{ij}]_{m \times n}$ for the MCDM problem where there are m alternatives $A_i (i = 1, \dots, m)$ under n criteria $C_j (j = 1, \dots, n)$ and x_{ij} represents the assessment measure of equipment i with respect to criterion j . Moreover, let $[w_j]_{1 \times n}$ be the weights of criteria. The above MCDM problem can also be showed in matrix format as follows:

$$D = \begin{matrix} & \begin{matrix} w_1 & w_2 & w_3 & \dots & w_n \\ C_1 & C_2 & C_3 & \dots & C_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \dots & x_{3n} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \dots & x_{mn} \end{bmatrix} \end{matrix}, \tag{1}$$

Step 2 Obtain the outranking relations using the following three thresholds:

- Indifference threshold (q_j): If difference of performance measures between alternatives A_r and A_k under criterion j , namely x_{rj} preferred to the alternative.

- and x_{kj} , is less than or equal q_j (i.e. $x_{rj} - x_{kj} \leq q_j$ and $x_{kj} - x_{rj} \leq q_j$), it is said that alternative A_r and alternative A_k under criterion j are indifference.
- Veto threshold (v_j): If difference of performance measures between alternatives A_k and A_r with respect to criterion j , i.e., x_{kj} and x_{rj} , is more than or equal v_j (or $x_{kj} - x_{rj} \geq v_j$), it is said that the alternative A_r is not better than the alternative A_k .
- Preference threshold (p_j): If difference of performance measures between alternatives A_r and A_k under criterion j , i.e., x_{rj} and x_{kj} , is more than or equal p_j (or equivalently, $x_{rj} - x_{kj} \geq p_j$), the alternative A_r is preferred to the alternative A_k . If $x_{kj} + q_j < x_{rj} \leq x_{kj} + p_j$, the alternative A_r is weakly preferred to the alternative A_k .

Based on the above descriptions, the following three relations are introduced for determining the level of the decision-makers' preferences:

$A_r P_j A_k$	x_{rj} is strictly preferred to x_{kj} under criterion j	\leftrightarrow	$x_{rj} - x_{kj} \geq p_j$
$A_k Q_j A_r$	x_{rj} is weakly preferred to x_{kj} under criterion j	\leftrightarrow	$x_{kj} + q_j < x_{rj} \leq x_{kj} + p_j$
$A_k I_j A_r$	x_{rj} is indifferent to x_{kj} under criterion j ; and x_{kj} is indifferent to x_{rj} under criterion j	\leftrightarrow	$ x_{rj} - x_{kj} \leq q_j$

The thresholds q_j , p_j , and v_j for each criterion are usually determined based on experts' views, nature of decision-making problem, or (maximum measure – minimum measure) * certain coefficient. By using these thresholds, an outranking relation S as $A_r S A_k$ are constructed for comparing pairs of alternatives A_r and A_k . It means that “ A_r is at least as good as A_k ” OR “ A_r is not worse than A_k ” [100].

In order to examine expression $A_r S A_k$, the following two principles should be taken account into:

- The concordance principle: it includes a majority of criteria such that agree with expression $A_r S A_k$, that is the majority principle.
- The non-discordance principle: it includes a minority of criteria such that do not satisfy expression $A_r S A_k$, which is the minority principle.

The final goal is to rank the alternatives based on the outranking relations and the principles described above. For a certain criterion, more concordance and less non-discordance indices indicate stronger superiority.

Step 3 Calculate the concordance index of alternative A_r relative to alternative A_k ($C(A_r, A_k)$) as follows:

$$C(A_r, A_k) = \frac{\sum_{j=1}^n w_j * c_j(A_r, A_k)}{\sum_{j=1}^n w_j}, \quad r, k = 1, \dots, m; \quad r \neq k, \tag{2}$$

where w_j and $c_j(A_r, A_k)$ are weight of criterion j and the superior level of alternative A_r versus alternative A_k . Calculate the discordance index of alternative, respectively. For benefit (B) and cost (C) criteria, $c_j(A_r, A_k)$ is calculated, respectively, by the following relations:

$$c_j(A_r, A_k) = \begin{cases} 0 & x_{kj} - x_{rj} \geq p_j, \\ \frac{x_{rj} - x_{kj} + p_j}{p_j - q_j} & q_j < x_{kj} - x_{rj} \leq p_j, \\ 1 & x_{kj} - x_{rj} \leq q_j, \end{cases} \quad j = 1, \dots, n, \tag{3}$$

and

$$c_j(A_r, A_k) = \begin{cases} 0 & x_{rj} - x_{kj} \geq p_j, \\ \frac{x_{kj} - x_{rj} + p_j}{p_j - q_j} & \text{otherwise,} \\ 1 & x_{rj} - x_{kj} \leq q_j, \end{cases} \quad j = 1, \dots, n. \tag{4}$$

Step 4 Calculate the discordance index of alternative A_r versus alternative A_k for criterion j ($d_j(A_r, A_k)$). It shows lack of superiority an alternative versus other alternative with respect to other criteria. The following two expressions show the discordance indices of alternative A_r versus alternative A_k for B and C criteria, respectively:

$$d_j(A_r, A_k) = \begin{cases} 0 & x_{kj} - x_{rj} \leq p_j, \\ \frac{x_{kj} - x_{rj} - p_j}{v_j - p_j} & p_j < x_{kj} - x_{rj} \leq v_j, \\ 1 & x_{kj} - x_{rj} \geq v_j, \end{cases} \quad j = 1, \dots, n, \tag{5}$$

and

$$d_j(A_r, A_k) = \begin{cases} 0 & x_{rj} - x_{kj} \leq v_j, \\ \frac{x_{rj} - x_{kj} - p_j}{v_j - p_j} & \text{otherwise,} \\ 1 & x_{rj} - x_{kj} \geq v_j, \end{cases} \quad j = 1, \dots, n. \tag{6}$$

Step 5 Determine outranking relation by the following credit degree index:

$$S(A_r, A_k) = \begin{cases} C(A_r, A_k), & \text{if } d_j(A_r, A_k) \leq C(A_r, A_k), \quad \forall j, \\ C(A_r, A_k) \cdot \prod_{j \in J: d_j(A_r, A_k) > C(A_r, A_k)} \frac{1 - d_j(A_r, A_k)}{1 - C(A_r, A_k)} & \text{otherwise,} \end{cases} \tag{7}$$

where $S(A_r, A_k)$ and J are the degree of outranking A_r relative to alternative A_k and the set of criteria for which $d_j(A_r, A_k) > C(A_r, A_k)$, respectively.

Step 6 Calculate the final score and rank alternatives.

There are several approaches to rank alternatives in the ELECTRE III technique. (1) Min max method: in this method, the optimal alternative is the alternative that the largest measure in its corresponding column in credit degree matrix is less than the same measure for other alternatives. (2) Double ranking method: in this method, two distillation procedures are utilized. The first type is descending distillation in which ranking order is from the best to the worst alternatives. In the second type, named ascending distillation, alternatives are prioritized from the worst to the best alternative. The results of above two procedures are then combined to determine the final ranking.

T2FSs and their arithmetic operations

Definition 3.2.1 Let \tilde{A} be the T2FS in the universe of discourse X as follows [101]:

$$\tilde{A} = \left\{ \left((x, u), \mu_{\tilde{A}}(x, u) \right) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1], 0 \leq \mu_{\tilde{A}}(x, u) \leq 1 \right\}, \tag{8}$$

where X denotes the domain of \tilde{A} , $\mu_{\tilde{A}}$ refers to the MF (secondary MF) of \tilde{A} , and J_x is an sub-interval at $[0, 1]$ and denotes the primary MF. \tilde{A} can also be expressed as follows:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u), \tag{9}$$

where $J_x \subseteq [0, 1]$ and $\int \int$ denotes the overall admissible union of x and u .

Definition 3.2.2 For the T2FS \tilde{A} , if all $\mu_{\tilde{A}}(x, u) = 1$, \tilde{A} is named IT2FS. An IT2FS \tilde{A} can be regarded as a special case of a T2FS as follows [101]:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1 / (x, u), \tag{10}$$

where $J_x \subseteq [0, 1]$. It is obvious that the upper and the lower MFs of an IT2FS are both type-1 MFs.

Definition 3.2.3 Footprint of uncertainty (FOU) is derived from the union of all primary memberships:

$$\text{FOU}(\tilde{A}) = \int_{x \in X} J_x. \tag{11}$$

The FOU can also be represented by the lower and upper MFs [102]:

$$\text{FOU}(\tilde{A}) = \int_{x \in X} \left[\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x) \right], \tag{12}$$

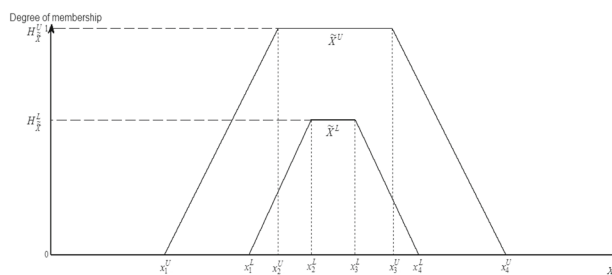


Fig. 1 Subnormal TraIT2FN

where $\underline{\mu}_{\tilde{A}}(x)$ and $\bar{\mu}_{\tilde{A}}(x)$ are the lower and upper MFs of a T2FS. An IT2FS \tilde{A} , is said to be normal if $\underline{\mu}_{\tilde{A}}(x) = \bar{\mu}_{\tilde{A}}(x) = 1$. An IT2FS \tilde{A} , is said to be subnormal if $\underline{\mu}_{\tilde{A}}(x) < 1$ and $\bar{\mu}_{\tilde{A}}(x) = 1$.

Definition 3.2.4. Let \tilde{X}^L and \tilde{X}^U (L and U are equal to the lower and upper MFs) be two non-negative trapezoidal type-1 fuzzy numbers [101]. In addition, let $H_{\tilde{X}^L}$ and $H_{\tilde{X}^U}$ denote the heights of \tilde{X}^L and \tilde{X}^U , respectively. Let $x_1^L, x_2^L, x_3^L, x_4^L, x_1^U, x_2^U, x_3^U, x_4^U$ be non-negative real values. Trapezoidal type-t fuzzy numbers (TraIT2FNs) can be represented by (see Fig. 1):

$$\begin{aligned} \tilde{X} &= [\tilde{X}^L, \tilde{X}^U] \\ &= \left[\left(x_1^L, x_2^L, x_3^L, x_4^L; H_{\tilde{X}^L} \right), \left(x_1^U, x_2^U, x_3^U, x_4^U; H_{\tilde{X}^U} \right) \right]. \end{aligned} \tag{13}$$

Definition 3.2.5. Let \tilde{X}_1 and \tilde{X}_2 be two non-negative TraIT2FNs, where $\tilde{X}_1 = [\tilde{X}_1^L, \tilde{X}_1^U] = \left[\left(x_{11}^L, x_{12}^L, x_{13}^L, x_{14}^L; H_{\tilde{X}_1^L} \right), \left(x_{11}^U, x_{12}^U, x_{13}^U, x_{14}^U; H_{\tilde{X}_1^U} \right) \right]$ and $\tilde{X}_2 = [\tilde{X}_2^L, \tilde{X}_2^U] = \left[\left(x_{21}^L, x_{22}^L, x_{23}^L, x_{24}^L; H_{\tilde{X}_2^L} \right), \left(x_{21}^U, x_{22}^U, x_{23}^U, x_{24}^U; H_{\tilde{X}_2^U} \right) \right]$. Arithmetic operations between \tilde{X}_1 and \tilde{X}_2 are defined as follows:

Addition operation:

$$\begin{aligned} \tilde{X}_1 \oplus \tilde{X}_2 &= \left[\left(x_{11}^L + x_{21}^L, x_{12}^L + x_{22}^L, x_{13}^L + x_{23}^L, x_{14}^L + x_{24}^L; \right. \right. \\ &\quad \left. \left. \min \left\{ H_{\tilde{X}_1^L}^L, H_{\tilde{X}_2^L}^L \right\} \right), \left(x_{11}^U + x_{21}^U, x_{12}^U + x_{22}^U, x_{13}^U + x_{23}^U, x_{14}^U + x_{24}^U; \right. \right. \\ &\quad \left. \left. \min \left\{ H_{\tilde{X}_1^U}^U, H_{\tilde{X}_2^U}^U \right\} \right) \right]. \end{aligned} \tag{14}$$

Subtraction operation:

$$\tilde{X}_1 \ominus \tilde{X}_2 = \left[\left(x_{11}^L - x_{24}^L, x_{12}^L - x_{23}^L, x_{13}^L - x_{22}^L, x_{14}^L - x_{21}^L; \right. \right.$$

$$\min \left\{ H_{\tilde{X}_1}^L, H_{\tilde{X}_2}^L \right\} \Bigg], \left[\left(x_{11}^U - x_{24}^U, x_{12}^U - x_{23}^U, x_{13}^U - x_{22}^U, x_{14}^U - x_{21}^U; \min \left\{ H_{\tilde{X}_1}^U, H_{\tilde{X}_2}^U \right\} \right) \right]. \tag{15}$$

Multiplication operation:

$$\tilde{X}_1 \otimes \tilde{X}_2 = \left[\left(x_{11}^L \cdot x_{21}^L, x_{12}^L \cdot x_{22}^L, x_{13}^L \cdot x_{23}^L, x_{14}^L \cdot x_{24}^L; \min \left\{ H_{\tilde{X}_1}^L, H_{\tilde{X}_2}^L \right\} \right), \left(x_{11}^U \cdot x_{21}^U, x_{12}^U \cdot x_{22}^U, x_{13}^U \cdot x_{23}^U, x_{14}^U \cdot x_{24}^U; \min \left\{ H_{\tilde{X}_1}^U, H_{\tilde{X}_2}^U \right\} \right) \right]. \tag{16}$$

Division operation:

$$\tilde{X}_1 \oslash \tilde{X}_2 = \left[\left(\frac{x_{11}^L}{x_{24}^L}, \frac{x_{12}^L}{x_{23}^L}, \frac{x_{13}^L}{x_{22}^L}, \frac{x_{14}^L}{x_{21}^L}; \min \left\{ H_{\tilde{X}_1}^L, H_{\tilde{X}_2}^L \right\} \right), \left(\frac{x_{11}^U}{x_{24}^U}, \frac{x_{12}^U}{x_{23}^U}, \frac{x_{13}^U}{x_{22}^U}, \frac{x_{14}^U}{x_{21}^U}; \min \left\{ H_{\tilde{X}_1}^U, H_{\tilde{X}_2}^U \right\} \right) \right]. \tag{17}$$

Multiplication by an ordinary number:

$$\tilde{X}_1 \cdot r = r \cdot \tilde{X}_1 = \begin{cases} \left[\left(r \cdot x_{11}^L, r \cdot x_{12}^L, r \cdot x_{13}^L, r \cdot x_{14}^L; H_{\tilde{X}_1}^L \right), \left(r \cdot x_{11}^U, r \cdot x_{12}^U, r \cdot x_{13}^U, r \cdot x_{14}^U; H_{\tilde{X}_1}^U \right) \right] & \text{if } r \geq 0 \\ \left[\left(r \cdot x_{14}^L, r \cdot x_{13}^L, r \cdot x_{12}^L, r \cdot x_{11}^L; H_{\tilde{X}_1}^L \right), \left(r \cdot x_{14}^U, r \cdot x_{13}^U, r \cdot x_{12}^U, r \cdot x_{11}^U; H_{\tilde{X}_1}^U \right) \right] & \text{if } r \leq 0 \end{cases}. \tag{18}$$

$$\tilde{G} = [\tilde{G}^L, \tilde{G}^U] = \left[\left(\mu^L; \sigma^L; H_{\tilde{G}}^L \right), \left(\mu^U; \sigma^U; H_{\tilde{G}}^U \right) \right], \tag{19}$$

where $\mu^L; \sigma^L$ and $\mu^U; \sigma^U$ are the mean and standard deviation of the lower and upper Gaussian MFs, respectively, such that $H_{\tilde{G}}^L = H_{\tilde{G}}^U, \mu^L = \mu^U$ and $\sigma^L < \sigma^U$ [103].

Definition 3.2.7 A set of all GIT2FNs \tilde{G}_i is said symmetric if there is no intersection between beginning left reference limits or final right reference limits (see Fig. 3).

Definition 3.2.8 The α -cut of \tilde{A} is presented as follows [104]:

$$A_\alpha = \{ (x, u) \mid f_x(u) \geq \alpha \}. \tag{20}$$

Definition 3.2.9 The α -cut of \tilde{A} may be represented by the α -cut of its FOU using the extension of α -cut of a type-1 fuzzy set \tilde{A} [105] as follows:

$$A_\alpha = \{ x \mid \underline{\mu}_{\tilde{A}}(x) \geq \alpha, \bar{\mu}_{\tilde{A}}(x) \geq \alpha \}, \tag{21}$$

For a normal GIT2FN, α -cut may be presented as interval as follows (see Fig. 3):

Normal:

$$\hat{G}_{i\alpha} = \left[\left[\bar{x}_{1i\alpha}^l, \underline{x}_{2i\alpha}^l \right], \mu_i, \left[\underline{x}_{1i\alpha}^r, \bar{x}_{2i\alpha}^r \right] \right], \tag{22}$$

Definition 3.2.6 Let \tilde{G} be a normal GIT2FN as follows (see also Fig. 2):

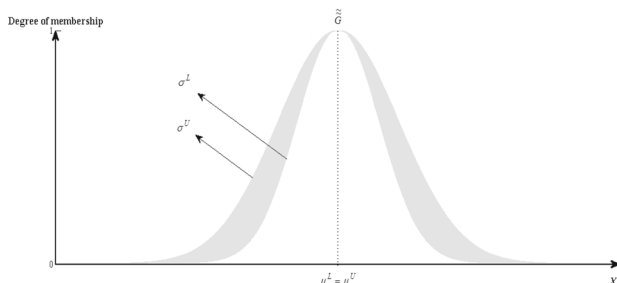


Fig. 2 Normal GIT2FN

where $\bar{x}_{1i\alpha}^l < \underline{x}_{2i\alpha}^l < \mu < \underline{x}_{1i\alpha}^r < \bar{x}_{2i\alpha}^r$, and l and r show the left and right MFs of \tilde{G} , respectively.

Definition 3.2.10 Let $X = [x_1, x_2]$ and $Y = [y_1, y_2]$ be two positive interval numbers, such that $x_1 \leq x \leq x_2$ and $y_1 \leq y \leq y_2$ (x_1, y_1 and x_2, y_2 are the infima and the suprema, respectively). Interval arithmetic operations including addition, subtraction, multiplication, and division are defined, respectively, as follows [106]:

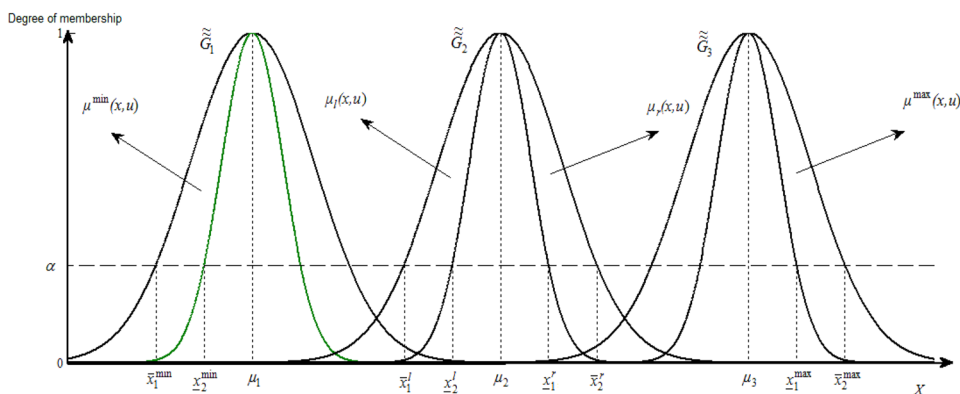
Addition operation:

$$X + Y = [x_1 + y_1, x_2 + y_2]. \tag{23}$$

Subtraction operation:

$$X - Y = [x_1 - y_2, x_2 - y_1]. \tag{24}$$

Fig. 3 Set of symmetric GIT2FNs (the left, right, minimum, and maximum reference limits)



Multiplication operation:

$$X \cdot Y = [\min(x_1 \cdot y_1, x_1 \cdot y_2, x_2 \cdot y_1, x_2 \cdot y_2), \max(x_1 \cdot y_1, x_1 \cdot y_2, x_2 \cdot y_1, x_2 \cdot y_2)]. \tag{25}$$

Division operation:

$$\frac{X}{Y} = [x_1, x_2] \cdot \left(\frac{1}{[y_1, y_2]} \right),$$

where $\frac{1}{[y_1, y_2]} = \left[\frac{1}{y_2}, \frac{1}{y_1} \right]$ if $0 \notin [y_1, y_2]$. (26)

Distance between X and Y:

$$\Delta_{X-Y} = \frac{1}{2} |(x_1 - y_2) + (x_2 - y_1)|. \tag{27}$$

Some arithmetic operations of normal GIT2FNs

Let \tilde{G}_1 and \tilde{G}_2 be two non-negative normal GIT2FNs, where $\tilde{G}_1 = [\tilde{G}_1^L, \tilde{G}_1^U] = \left[\left(\mu_1^L; \sigma_1^L; H_{\tilde{G}_1}^L \right), \left(\mu_1^U; \sigma_1^U; H_{\tilde{G}_1}^U \right) \right]$, $\tilde{G}_2 = [\tilde{G}_2^L, \tilde{G}_2^U] = \left[\left(\mu_2^L; \sigma_2^L; H_{\tilde{G}_2}^L \right), \left(\mu_2^U; \sigma_2^U; H_{\tilde{G}_2}^U \right) \right]$, and $H_{\tilde{G}_1}^L = H_{\tilde{G}_2}^L = H_{\tilde{G}_1}^U = H_{\tilde{G}_2}^U$. The arithmetic operations between \tilde{G}_1 and \tilde{G}_2 at level α ($\alpha = \alpha_1, \dots, \alpha_N$; N is the number of alpha cuts) are calculated using Definition 3.2.9 ($\hat{G}_{i\alpha} = [[\bar{x}_{1i\alpha}^l, \underline{x}_{2i\alpha}^l], \mu_i, [x_{1i\alpha}^r, \bar{x}_{2i\alpha}^r]]$) and extensions of definitions stated by Chen and Lee [101] as follows:

Addition operation:

$$\hat{G}_{1\alpha} \oplus \hat{G}_{2\alpha} = \left[\left(\underline{x}_{21\alpha}^l + \underline{x}_{22\alpha}^l, \mu_1 + \mu_2, \underline{x}_{11\alpha}^r + \underline{x}_{12\alpha}^r; \min\{H_{\tilde{G}_1}^L, H_{\tilde{G}_2}^L\} \right), \left(\bar{x}_{11\alpha}^l + \bar{x}_{12\alpha}^l, \mu_1 + \mu_2, \bar{x}_{21\alpha}^r + \bar{x}_{22\alpha}^r; \min\{H_{\tilde{G}_1}^U, H_{\tilde{G}_2}^U\} \right) \right]. \tag{28}$$

Subtraction operation:

$$\hat{G}_{1\alpha} \ominus \hat{G}_{2\alpha} = \left[\left(\underline{x}_{21\alpha}^l - \underline{x}_{12\alpha}^r, \mu_1 - \mu_2, \underline{x}_{11\alpha}^r - \underline{x}_{22\alpha}^l; \min\{H_{\tilde{G}_1}^L, H_{\tilde{G}_2}^L\} \right), \left(\bar{x}_{11\alpha}^l - \bar{x}_{22\alpha}^r, \mu_1 - \mu_2, \bar{x}_{21\alpha}^r - \bar{x}_{12\alpha}^l; \min\{H_{\tilde{G}_1}^U, H_{\tilde{G}_2}^U\} \right) \right]. \tag{29}$$

Multiplication operation:

$$\hat{G}_{1\alpha} \otimes \hat{G}_{2\alpha} = \left[\left(\underline{x}_{21\alpha}^l \times \underline{x}_{22\alpha}^l, \mu_1 \times \mu_2, \underline{x}_{11\alpha}^r \times \underline{x}_{12\alpha}^r; \min\{H_{\tilde{G}_1}^L, H_{\tilde{G}_2}^L\} \right), \left(\bar{x}_{11\alpha}^l \times \bar{x}_{12\alpha}^l, \mu_1 \times \mu_2, \bar{x}_{21\alpha}^r \times \bar{x}_{22\alpha}^r; \min\{H_{\tilde{G}_1}^U, H_{\tilde{G}_2}^U\} \right) \right]. \tag{30}$$

Division operation:

$$\hat{G}_{1\alpha} / \hat{G}_{2\alpha} = \left[\left(\frac{\underline{x}_{21\alpha}^l}{\underline{x}_{12\alpha}^r}, \frac{\mu_1}{\mu_2}, \frac{\underline{x}_{11\alpha}^r}{\underline{x}_{22\alpha}^l}; \min\{H_{\tilde{G}_1}^L, H_{\tilde{G}_2}^L\} \right), \left(\frac{\bar{x}_{11\alpha}^l}{\bar{x}_{22\alpha}^r}, \frac{\mu_1}{\mu_2}, \frac{\bar{x}_{21\alpha}^r}{\bar{x}_{12\alpha}^l}; \min\{H_{\tilde{G}_1}^U, H_{\tilde{G}_2}^U\} \right) \right]. \tag{31}$$

Multiplication by an ordinary number:

$$\hat{G}_{1\alpha} \cdot r = r \cdot \hat{G}_{1\alpha} = \begin{cases} \left[\left(r \cdot \underline{x}_{21\alpha}^l, r \cdot \mu_1, r \cdot \underline{x}_{11\alpha}^r; H_{\tilde{G}_1}^L \right), \left(r \cdot \bar{x}_{11\alpha}^l, r \cdot \mu_1, r \cdot \bar{x}_{21\alpha}^r; H_{\tilde{G}_1}^U \right) \right] & \text{if } r \geq 0 \\ \left[\left(r \cdot \underline{x}_{11\alpha}^r, r \cdot \mu_1, r \cdot \underline{x}_{21\alpha}^l; H_{\tilde{G}_1}^L \right), \left(r \cdot \bar{x}_{21\alpha}^r, r \cdot \mu_1, r \cdot \bar{x}_{11\alpha}^l; H_{\tilde{G}_1}^U \right) \right] & \text{if } r \leq 0 \end{cases} \tag{32}$$

Average operation:

Let $\tilde{G}_1, \dots, \tilde{G}_U$ be U non-negative normal GIT2FNs, where $\tilde{G}_u = [\tilde{G}_u^L, \tilde{G}_u^U] = \left[\left(\mu_u^L; \sigma_u^L; H_{\tilde{G}_u}^L \right), \left(\mu_u^U; \sigma_u^U; H_{\tilde{G}_u}^U \right) \right]$ and $H_{\tilde{G}_u}^L = H_{\tilde{G}_u}^U$ ($u = 1, \dots, U$). The average operations $\tilde{G}_\alpha = (\tilde{x}_{1\alpha}^l, \tilde{x}_{2\alpha}^l, \tilde{\mu}_\alpha, \tilde{x}_{1\alpha}^r, \tilde{x}_{2\alpha}^r)$ between $\tilde{G}_1, \dots, \tilde{G}_U$ at level α ($\alpha = \alpha_1, \dots, \alpha_N$) are calculated as follows:

$$\tilde{x}_{1\alpha}^l = \left(\sum_{u=1}^U \tilde{x}_{1u\alpha}^l \right) / U, \quad \alpha = \alpha_1, \dots, \alpha_N, \tag{33}$$

$$\tilde{x}_{2\alpha}^l = \left(\sum_{u=1}^U \tilde{x}_{2u\alpha}^l \right) / U, \quad \alpha = \alpha_1, \dots, \alpha_N, \tag{34}$$

$$\tilde{\mu}_\alpha = \left(\sum_{u=1}^U \mu_{u\alpha} \right) / U, \quad \alpha = \alpha_1, \dots, \alpha_N, \tag{35}$$

$$\tilde{x}_{1\alpha}^r = \left(\sum_{u=1}^U \tilde{x}_{1u\alpha}^r \right) / U, \quad \alpha = \alpha_1, \dots, \alpha_N, \tag{36}$$

$$\tilde{x}_{2\alpha}^r = \left(\sum_{u=1}^U \tilde{x}_{2u\alpha}^r \right) / U, \quad \alpha = \alpha_1, \dots, \alpha_N. \tag{37}$$

Weighted average operation:

Let $\tilde{G}_{ij\alpha} = \left[\left[\tilde{x}_{1ij\alpha}^l, \tilde{x}_{2ij\alpha}^l \right], \mu_{ij}, \left[\tilde{x}_{1ij\alpha}^r, \tilde{x}_{2ij\alpha}^r \right] \right]$ and $\tilde{w}_{j\alpha} = \left[\left[\tilde{w}_{1j\alpha}^l, \tilde{w}_{2j\alpha}^l \right], \mu_j, \left[\tilde{w}_{1j\alpha}^r, \tilde{w}_{2j\alpha}^r \right] \right]$ be the evaluation of alternative i with respect to criterion j ($j = 1, \dots, J$) and the weight of criterion j at level α , respectively, which both of them are normal GIT2FNs. Moreover, let $H_{\tilde{G}_{ij}}^L = H_{\tilde{G}_{ij}}^U = 1$ (for $i = 1, \dots, I, j = 1, \dots, J$) and $H_{\tilde{w}_j}^L = H_{\tilde{w}_j}^U = 1$ (for $j = 1, \dots, J$). The weighted average of alternative i ($W\hat{A}_{i\alpha}$) at level α is obtained as follows:

$$\begin{aligned} W\hat{A}_{i\alpha} &= \sum_{j=1}^J (\hat{w}_{j\alpha} \times \hat{G}_{ij\alpha}) / \sum_{j=1}^J \hat{w}_{j\alpha}, \quad i = 1, \dots, I; \alpha = \alpha_1, \dots, \alpha_N, \\ &= \frac{\sum_{j=1}^J \left(\left[\left[\tilde{w}_{1j\alpha}^l, \tilde{w}_{2j\alpha}^l \right], \mu_{j\alpha}, \left[\tilde{w}_{1j\alpha}^r, \tilde{w}_{2j\alpha}^r \right] \right] \times \left[\left[\tilde{x}_{1ij\alpha}^l, \tilde{x}_{2ij\alpha}^l \right], \mu_{ij\alpha}, \left[\tilde{x}_{1ij\alpha}^r, \tilde{x}_{2ij\alpha}^r \right] \right] \right)}{\sum_{j=1}^J \left\{ \left[\tilde{w}_{1j\alpha}^l, \tilde{w}_{2j\alpha}^l \right], \mu_{j\alpha}, \left[\tilde{w}_{1j\alpha}^r, \tilde{w}_{2j\alpha}^r \right] \right\}}, \\ &= \frac{\sum_{j=1}^J \left\{ \left[\tilde{w}_{2j\alpha}^l \times \tilde{x}_{2ij\alpha}^l, \mu_{j\alpha} \times \mu_{ij\alpha}, \tilde{w}_{1j\alpha}^r \times \tilde{x}_{1ij\alpha}^r \right], \left[\tilde{w}_{1j\alpha}^l \times \tilde{x}_{1ij\alpha}^l, \mu_{j\alpha} \times \mu_{ij\alpha}, \tilde{w}_{2j\alpha}^r \times \tilde{x}_{2ij\alpha}^r \right] \right\}}{\sum_{j=1}^J \left\{ \left[\tilde{w}_{1j\alpha}^l, \tilde{w}_{2j\alpha}^l \right], \mu_{j\alpha}, \left[\tilde{w}_{1j\alpha}^r, \tilde{w}_{2j\alpha}^r \right] \right\}}. \end{aligned} \tag{38}$$

Property 1 Let \tilde{A}_1 and \tilde{A}_2 be two non-negative normal TraIT2FNs or triangular interval type-2 fuzzy numbers (TriIT2FNs). Then, all expressions in Sect. 3.3 hold true for them, respectively.

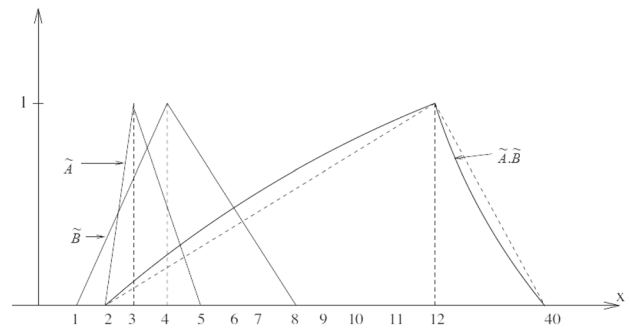


Fig. 4 $\tilde{A} \cdot \tilde{B}$ is not a triangular fuzzy number

Proof Since α -cuts can easily be depicted for normal TraIT2FNs or TriIT2FNs and reference points are generated at each level α , one can calculate the expressions calculated above for them.

A new approach for ranking GIT2FNs

The normal GIT2FNs case

Based on Kaufmann and Gupta [107], although \tilde{A} and \tilde{B} are triangular fuzzy numbers (as shown in Fig. 4), $\tilde{A} \cdot \tilde{B}$ is not a triangular fuzzy number (the left and right MFs are not straight lines). It can challenge the distances between IT2FNs, while the curved MFs are applied to evaluations. The proposed approach is able to calculate distances on the different levels and rank GIT2FNs at interval $[0, 1]$ concurrently. Generally, it considers the maximum and minimum reference limits among all GIT2FNs as reference limits (ideal solutions) and then calculates the limit distance mean (LDM) by subtracting the right and left limits of GIT2FNs from these reference limits.

For this purpose, the left and right MFs of $\mu_{\tilde{G}}(x, u)$, for a GIT2FN, are partitioned into two MFs $\mu_l(x, u)$ (for $x < \mu$) and $\mu_r(x, u)$ (for $x > \mu$), respectively (as shown in Fig. 3).

Moreover, assume that the minimum reference limit $\mu^{\min}(x, u)$ and the maximum reference limit $\mu^{\max}(x, u)$ are $\{\min\{\mu_{l_i}(x, u)\} \mid i \in \text{all GIT2FNs}\}$ and $\{\max\{\mu_{r_i}(x, u)\}, i \in \text{all GIT2FNs}\}$, respectively, as represented in Fig. 3.

sure of desirable alternative has the shortest distance from the positive ideal (PI) solution, the distance of $\mu_l(x, u)$ from $\mu^{\min}(x, u)$ and the distance of $\mu_r(x, u)$ from $\mu^{\max}(x, u)$ should have the shortest distance for C and benefit B criteria, respectively. Hence, LDM can be calculated for the PI solution with respect to C criteria as follows:

$$LDM_{PI,C}(\tilde{A}) = \frac{\sum_{\alpha=0.1}^1 (\mu_{l\alpha}(x, u) - \mu_{\alpha}^{\min}(x, u))\alpha}{\sum_{\alpha=0.1}^1 (\mu_{l\alpha}(x, u) - \mu_{\alpha}^{\min}(x, u)) - \sum_{\alpha=0.1}^1 (\mu_{r\alpha}(x, u) - \mu_{\alpha}^{\max}(x, u))}, \tag{39}$$

Since the intersection of α -cut with a GIT2FN creates the interval numbers, interval arithmetic operations can be applied to them. As demonstrated in Fig. 3, suppose that the α -cut of the minimum and maximum reference limits $\mu_{\alpha}^{\min}(x, u)$ and $\mu_{\alpha}^{\max}(x, u)$ generates intervals $[\underline{x}_1^{\min}, \underline{x}_2^{\min}]_{\alpha}$ and $[\underline{x}_1^{\max}, \bar{x}_2^{\max}]_{\alpha}$ on X , respectively, where \bar{x}_1^{\min}

where $\alpha = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8,$ and 0.9 . Note that $\sum_{\alpha=0.1}^1 (\mu_{l\alpha}(x, u) - \mu_{\alpha}^{\min}(x, u))$ and $\sum_{\alpha=0.1}^1 (\mu_{r\alpha}(x, u) - \mu_{\alpha}^{\max}(x, u))$ are the positive and negative values. Thus, the negative sign was considered in the denominator. To simplify the calculations, Eq. (39) can be converted into the following equation using notation in Definition 3.2.9:

$$LDM_{PI,C}(\tilde{A}) = \frac{\sum_{\alpha=0.1}^1 [\bar{x}_1^l, \underline{x}_2^l]_{\alpha} - [\bar{x}_1^{\min}, \underline{x}_2^{\min}]_{\alpha}}{\sum_{\alpha=0.1}^1 [\bar{x}_1^l, \underline{x}_2^l]_{\alpha} - [\bar{x}_1^{\min}, \underline{x}_2^{\min}]_{\alpha} - \sum_{\alpha=0.1}^1 [\underline{x}_1^r, \bar{x}_2^r]_{\alpha} - [\underline{x}_1^{\max}, \bar{x}_2^{\max}]_{\alpha}}. \tag{40}$$

and \underline{x}_2^{\min} are related to the upper and lower MFs of $\mu^{\min}(x, u)$, respectively, and \underline{x}_1^{\max} and \bar{x}_2^{\max} are equal to the lower and upper MFs of $\mu^{\max}(x, u)$, respectively. In addition, let α -cut of the left and right MFs of a GIT2FN such as \tilde{G}_2 cre-

Obviously, measures $[\bar{x}_1^l, \underline{x}_2^l]_{\alpha} - [\bar{x}_1^{\max}, \bar{x}_2^{\max}]_{\alpha}$ and $[\underline{x}_1^r, \bar{x}_2^r]_{\alpha} - [\underline{x}_1^{\max}, \bar{x}_2^{\max}]_{\alpha}$ equal to the negative measures when $(\bar{x}_1^l < \bar{x}_2^{\max}; \underline{x}_2^l < \underline{x}_1^{\max})$ or $(\underline{x}_1^r < \bar{x}_2^{\max}; \bar{x}_2^r < \underline{x}_1^{\max})$. Instead, $LD_{PI,C}$ can be calculated using Eq. (27) as follows:

$$LDM_{PI,C}(\tilde{A}) = \frac{\sum_{\alpha=0.1}^1 \frac{1}{2} |(\bar{x}_1^l - \underline{x}_2^{\min})_{\alpha} + (\underline{x}_2^l - \bar{x}_1^{\min})_{\alpha}|}{\sum_{\alpha=0.1}^1 \frac{1}{2} |(\bar{x}_1^l - \underline{x}_2^{\min})_{\alpha} + (\underline{x}_2^l - \bar{x}_1^{\min})_{\alpha}| + \sum_{\alpha=0.1}^1 \frac{1}{2} |(x_1^r - \bar{x}_2^{\max})_{\alpha} + (\bar{x}_2^r - \underline{x}_1^{\max})_{\alpha}|} \times \frac{\sum_{\alpha=0.1}^1 |(\bar{x}_1^l - \underline{x}_2^{\min})_{\alpha} + (\underline{x}_2^l - \bar{x}_1^{\min})_{\alpha}|}{\sum_{\alpha=0.1}^1 |(\bar{x}_1^l - \underline{x}_2^{\min})_{\alpha} + (\underline{x}_2^l - \bar{x}_1^{\min})_{\alpha}| + \sum_{\alpha=0.1}^1 |(x_1^r - \bar{x}_2^{\max})_{\alpha} + (\bar{x}_2^r - \underline{x}_1^{\max})_{\alpha}|}. \tag{41}$$

ates the intervals $[\bar{x}_1^l, \underline{x}_2^l]_{\alpha}$ and $[\underline{x}_1^r, \bar{x}_2^r]_{\alpha}$, respectively, where \bar{x}_1^l and \underline{x}_2^l are related to the upper and lower MFs of $\mu_l(x, u)$ and \underline{x}_1^r and \bar{x}_2^r are equal to the lower and upper MFs of $\mu_r(x, u)$. With these assumptions in mind, if the evaluation's mea-

Similarly, the PI solution for the set of B criteria and the negative ideal (NI) solution for the set of C and B criteria are given, respectively, as follows:

$$LDM_{PI,B}(\tilde{A}) = \frac{\sum_{\alpha=0.1}^1 |(x_1^r - \bar{x}_2^{\max})_{\alpha} + (\bar{x}_2^r - \underline{x}_1^{\max})_{\alpha}|}{\sum_{\alpha=0.1}^1 |(x_1^r - \bar{x}_2^{\max})_{\alpha} + (\bar{x}_2^r - \underline{x}_1^{\max})_{\alpha}| + \sum_{\alpha=0.1}^1 |(\bar{x}_1^l - \underline{x}_2^{\min})_{\alpha} + (\underline{x}_2^l - \bar{x}_1^{\min})_{\alpha}|}, \tag{42}$$

$$LDM_{NI,C}(\tilde{A}) = \frac{\sum_{\alpha=0.1}^1 |(\bar{x}_1^l - \bar{x}_2^{\max})_{\alpha} + (\underline{x}_2^l - \underline{x}_1^{\max})_{\alpha}|}{\sum_{\alpha=0.1}^1 |(\bar{x}_1^l - \bar{x}_2^{\max})_{\alpha} + (\underline{x}_2^l - \underline{x}_1^{\max})_{\alpha}| + \sum_{\alpha=0.1}^1 |(x_1^r - \underline{x}_2^{\min})_{\alpha} + (\bar{x}_2^r - \bar{x}_1^{\min})_{\alpha}|}, \tag{43}$$

$$\text{LDM}_{\text{NI},B}(\tilde{A}) = \frac{\sum_{\alpha=0.1}^1 \left| (x_1^r - x_2^{\min})_{\alpha} + (\bar{x}_2^r - \bar{x}_1^{\min})_{\alpha} \right|}{\sum_{\alpha=0.1}^1 \left| (x_1^r - x_2^{\min})_{\alpha} + (\bar{x}_2^r - \bar{x}_1^{\min})_{\alpha} \right| + \sum_{\alpha=0.1}^1 \left| (\bar{x}_1^l - \bar{x}_2^{\max})_{\alpha} + (x_2^l - x_1^{\max})_{\alpha} \right|}. \quad (44)$$

Obviously, the desirable alternative has the shortest distance from the PI solution and the longest distance from the NI solution based on Eqs. (41)–(42) and (43)–(44), respectively. In other words, in case of the NI solution, the distance of $\mu_l(x, u)$ from $\mu^{\max}(x, u)$ and the distance of $\mu_r(x, u)$ from $\mu^{\min}(x, u)$ should have the longest distance with respect to C and B criteria for a desirable alternative, respectively. On the other hand, in the situation of PI solution, the distance of $\mu_l(x, u)$ from $\mu^{\min}(x, u)$ and the distance of $\mu_r(x, u)$ from $\mu^{\max}(x, u)$ should have the shortest distance with respect to C and B criteria, respectively.

Moreover, measures $(x_1^r - \bar{x}_2^{\max}) + (\bar{x}_2^r - x_1^{\max})$ and $(\bar{x}_1^l - x_2^{\min}) + (x_2^l - \bar{x}_1^{\min})$ are equal to zero when $[x_1^r, \bar{x}_2^r]$ and $[\bar{x}_1^l, x_2^l]$ matches $\mu^{\max}(x, u)$ and $\mu^{\min}(x, u)$, respectively. Hence, the measures $\frac{\sum_{\alpha=0.1}^1 \left| (\bar{x}_2^{\max} - x_1^{\max})_{\alpha} \right|}{\alpha}$ and $\frac{\sum_{\alpha=0.1}^1 \left| (x_2^{\min} - \bar{x}_1^{\min})_{\alpha} \right|}{\alpha}$ are used to calculate LDM s when the distances of reference limits $\mu^{\max}(x, u)$ and $\mu^{\min}(x, u)$ are obtained from themselves, respectively.

Some properties of the above LDM s:

Property 2 Neither of the denominators in Eqs. (41)–(44) is zero.

Property 3 All distances obtained by LDM s in Eqs. (41)–(44) include at interval $[0, 1]$.

Property 4 Four LDM s in Eqs. (41)–(44) can be utilized for the MCDM techniques where it is necessary to calculate the PI and NI ideal solutions.

Property 5 Let \tilde{G}_1 and \tilde{G}_2 be two non-negative symmetric normal GIT2FNs in a set of non-negative symmetric normal GIT2FNs, where $\tilde{G}_1 = [\tilde{G}_1^L, \tilde{G}_1^U] = \left[\left(\mu_1^L; \sigma_1^L; H_{\tilde{G}_1}^L \right), \left(\mu_1^U; \sigma_1^U; H_{\tilde{G}_1}^U \right) \right]$ and $\tilde{G}_2 = [\tilde{G}_2^L, \tilde{G}_2^U] = \left[\left(\mu_2^L; \sigma_2^L; H_{\tilde{G}_2}^L \right), \left(\mu_2^U; \sigma_2^U; H_{\tilde{G}_2}^U \right) \right]$. In addition, suppose that distance between \tilde{G}_1 and \tilde{G}_2 is showed as $d(\tilde{G}_1, \tilde{G}_2) = LDM(\tilde{G}_1) - LDM(\tilde{G}_2)$. Then, $\left| d(\tilde{G}_1, \tilde{G}_2) \right| = \left| LDM(\tilde{G}_1) - LDM(\tilde{G}_2) \right| \geq 0$ for all LDM s.

Property 6 Let \tilde{G}_1 and \tilde{G}_2 be two non-negative symmetric normal GIT2FNs where $\tilde{G}_1 = [\tilde{G}_1^L, \tilde{G}_1^U] = \left[\left(\mu_1^L; \sigma_1^L; H_{\tilde{G}_1}^L \right), \left(\mu_1^U; \sigma_1^U; H_{\tilde{G}_1}^U \right) \right]$ and $\tilde{G}_2 = [\tilde{G}_2^L, \tilde{G}_2^U] = \left[\left(\mu_2^L; \sigma_2^L; H_{\tilde{G}_2}^L \right), \left(\mu_2^U; \sigma_2^U; H_{\tilde{G}_2}^U \right) \right]$. Moreover, suppose that distance between \tilde{G}_1 and \tilde{G}_2 is showed as $d(\tilde{G}_1, \tilde{G}_2)$. Then, $\left| d(\tilde{G}_1, \tilde{G}_2) \right| = \left| d(\tilde{G}_2, \tilde{G}_1) \right|$.

Property 7 Let \tilde{G}_1 , \tilde{G}_2 , and \tilde{G}_3 be three non-negative symmetric normal GIT2FNs in a set of non-negative symmetric normal GIT2FNs where $\tilde{G}_1 = [\tilde{G}_1^L, \tilde{G}_1^U] = \left[\left(\mu_1^L; \sigma_1^L; H_{\tilde{G}_1}^L \right), \left(\mu_1^U; \sigma_1^U; H_{\tilde{G}_1}^U \right) \right]$, $\tilde{G}_2 = [\tilde{G}_2^L, \tilde{G}_2^U] = \left[\left(\mu_2^L; \sigma_2^L; H_{\tilde{G}_2}^L \right), \left(\mu_2^U; \sigma_2^U; H_{\tilde{G}_2}^U \right) \right]$, and $\tilde{G}_3 = [\tilde{G}_3^L, \tilde{G}_3^U] = \left[\left(\mu_3^L; \sigma_3^L; H_{\tilde{G}_3}^L \right), \left(\mu_3^U; \sigma_3^U; H_{\tilde{G}_3}^U \right) \right]$. Then, $d(\tilde{G}_3, \tilde{G}_2) + d(\tilde{G}_2, \tilde{G}_1) \geq d(\tilde{G}_3, \tilde{G}_1)$.

The interested reader can easily implement this proof in other LDM s in this section.

Property 8 $LDM_{\text{NI},B}(\tilde{G}) = LDM_W(\tilde{G})$ introduced in Eq. (44) can be utilized to weight criteria and rank GIT2FNs.

- If $LDM_{\text{NI},B}(\tilde{G}_2) > LDM_{\text{NI},B}(\tilde{G}_1)$, then \tilde{G}_2 is better than \tilde{G}_1 .
- If $LDM_{\text{NI},B}(\tilde{G}_2) < LDM_{\text{NI},B}(\tilde{G}_1)$, then \tilde{G}_1 is better than \tilde{G}_2 .
- If $LDM_{\text{NI},B}(\tilde{G}_2) = LDM_{\text{NI},B}(\tilde{G}_1)$, then \tilde{G}_1 and \tilde{G}_2 are the same (indifferent).

IN order to demonstrate the effectiveness of Eqs. (41)–(44), the following two examples are taken into account:

Example 1 Consider the following two GIT2FNs as type-2 fuzzy weights for criteria C_1 and C_2 , respectively:

$$\tilde{G}_1 = \left[\tilde{G}_1^L, \tilde{G}_1^U \right] = [(6, 1.5, ; 1), (6, 2; 1)]$$

$$\text{and } \tilde{G}_2 = \left[\tilde{G}_2^L, \tilde{G}_2^U \right] = [(8, 1.5, ; 1), (8, 2; 1)].$$

As shown in the Fig. 5, C_2 is larger than C_1 . Table 1 represents levels of α -cuts for the above two GIT2FNs. According

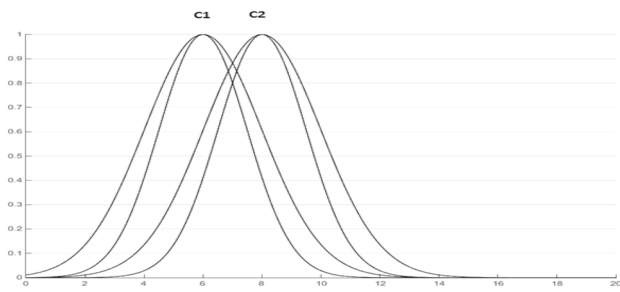


Fig. 5 GIT2FNs of Example 1

to Table 2, measures LDM_W for criteria C_1 and C_2 are 0.405 and 0.594, respectively. It proves the graphical representation.

Example 2 Let two GIT2FNs are as depicted in Fig. 6. Obviously, the lower limits of two GIT2FNs are equal. Note that C_2 is larger than C_1 due to its larger upper limit.

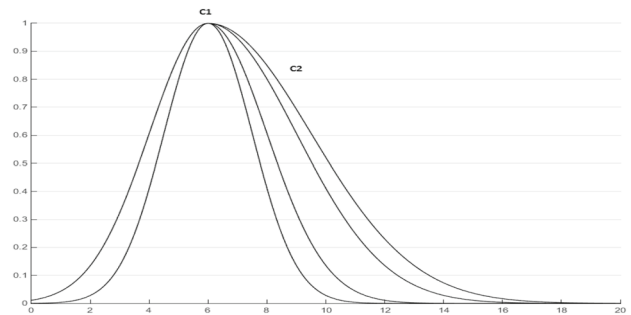


Fig. 6 GIT2FNs of Example 2

Table 3 represents levels of α -cuts for these two GIT2FNs. As shown in last row of Table 4, measures LDM_W [Eq. (44)] for criteria C_1 and C_2 are 0.411 and 0.500, respectively.

Property 9 Properties 2–8 also hold true for non-negative symmetric normal TraIT2FNs and TriIT2FNs.

Table 1 Levels of α -cuts of two GIT2FNs

α -Cuts	C_1				C_2			
	$\bar{x}_{11\alpha}^l$ $\mu^{\min}(x, u)$	$\bar{x}_{21\alpha}^l$	$\bar{x}_{11\alpha}^r$	$\bar{x}_{21\alpha}^r$	$\bar{x}_{12\alpha}^l$	$\bar{x}_{22\alpha}^l$	$\bar{x}_{12\alpha}^r$ $\mu^{\max}(x, u)$	$\bar{x}_{22\alpha}^r$
0.1	1.711	2.791	9.225	10.3	3.712	4.782	11.22	12.29
0.2	2.411	3.312	8.694	9.595	4.412	5.313	10.7	11.59
0.3	2.901	3.672	8.324	9.105	4.892	5.673	10.33	11.1
0.4	3.292	3.972	8.034	8.704	5.293	5.973	10.03	10.71
0.5	3.642	4.232	7.764	8.354	5.643	6.233	9.756	10.36
0.6	3.982	4.492	7.514	8.014	5.983	6.483	9.515	10.02
0.7	4.312	4.732	7.264	7.684	6.313	6.733	9.256	9.685
0.8	4.662	5.003	7.004	7.334	6.663	7.004	8.994	9.335
0.9	5.083	5.313	6.683	6.903	7.094	7.314	8.684	8.914

Table 2 Measures LDM_W

α -Cuts	C_1		C_2	
	$ (\bar{x}_{11}^r - \bar{x}_{21}^{\min})_{\alpha} + (\bar{x}_{21}^r - \bar{x}_{11}^{\min})_{\alpha} $	$ (\bar{x}_{11}^l - \bar{x}_{21}^{\max})_{\alpha} + (\bar{x}_{21}^l - \bar{x}_{11}^{\max})_{\alpha} $	$ (\bar{x}_{12}^r - \bar{x}_{22}^{\min})_{\alpha} + (\bar{x}_{22}^r - \bar{x}_{12}^{\min})_{\alpha} $	$ (\bar{x}_{12}^l - \bar{x}_{22}^{\max})_{\alpha} + (\bar{x}_{22}^l - \bar{x}_{12}^{\max})_{\alpha} $
0.1	15.023	19.008	19.008	15.016
0.2	12.566	16.567	16.567	12.565
0.3	10.856	14.857	14.857	10.865
0.4	9.474	13.476	13.476	9.474
0.5	8.244	12.242	12.242	8.24
0.6	7.054	11.061	11.061	7.069
0.7	5.904	9.897	9.897	5.895
0.8	4.673	8.664	8.664	4.662
0.9	3.19	7.202	7.202	3.19
$LDM_{PI,C}$	0.016		0.983	
$LDM_{PI,B}$	0.983		0.016	
$LDM_{NI,C}$	0.594		0.405	
LDM_W	0.405		0.594	

Table 3 Levels of α -cuts of two GIT2FNs

α -Cuts	C_1				C_2			
	$\bar{x}_{11\alpha}^l$ $\mu^{\min}(x, u)$	$\underline{x}_{21\alpha}^l$	$\bar{x}_{11\alpha}^r$	$\bar{x}_{21\alpha}^r$	$\bar{x}_{12\alpha}^l$	$\underline{x}_{22\alpha}^l$	$\bar{x}_{12\alpha}^r$ $\mu^{\max}(x, u)$	$\bar{x}_{22\alpha}^r$
0.1	1.7110	2.7910	9.2250	10.3000	1.7110	2.7910	12.4300	13.5000
0.2	2.4110	3.3120	8.6940	9.5950	2.4110	3.3120	11.3800	12.2800
0.3	2.9010	3.6720	8.3240	9.1050	2.9010	3.6720	10.6600	11.4300
0.4	3.2920	3.9720	8.0340	8.7040	3.2920	3.9720	10.0600	10.7400
0.5	3.6420	4.2320	7.7640	8.3540	3.6420	4.2320	9.5350	10.1200
0.6	3.9820	4.4920	7.5140	8.0140	3.9820	4.4920	9.0350	9.5350
0.7	4.3120	4.7320	7.2640	7.6840	4.3120	4.7320	8.5240	8.9540
0.8	4.6620	5.0030	7.0040	7.3340	4.6620	5.0030	8.0040	8.3340
0.9	5.0830	5.3130	6.6830	6.9030	5.0830	5.3130	7.3840	7.6040

Table 4 Measures LDM_W

α -Cuts	C_1		C_2	
	$ (\underline{x}_{11}^r - \underline{x}_2^{\min})_\alpha + (\bar{x}_{21}^r - \bar{x}_1^{\min})_\alpha $	$ (\bar{x}_{11}^l - \bar{x}_2^{\max})_\alpha + (\underline{x}_{21}^l - \underline{x}_1^{\max})_\alpha $	$ (\underline{x}_{12}^r - \underline{x}_2^{\min})_\alpha + (\bar{x}_{22}^r - \bar{x}_1^{\min})_\alpha $	$ (\bar{x}_{12}^l - \bar{x}_2^{\max})_\alpha + (\underline{x}_{22}^l - \underline{x}_1^{\max})_\alpha $
0.1	15.023	21.428	21.428	21.428
0.2	12.566	17.937	17.937	17.937
0.3	10.856	15.517	15.517	15.517
0.4	9.474	13.536	13.536	13.536
0.5	8.244	11.781	11.781	11.781
0.6	7.054	10.096	10.096	10.096
0.7	5.904	8.434	8.434	8.434
0.8	4.673	6.673	6.673	6.673
0.9	3.19	4.592	4.592	4.592
$LDM_{PI,C}$	0.018		0.501	
$LDM_{PI,B}$	0.981		0.498	
$LDM_{NI,C}$	0.588		0.500	
LDM_W	0.411		0.500	

Application of the new ranking method to FWA-ELECTRE III with GIT2FNs

Figure 7 shows the framework of the proposed methodology. The first stage is to define criteria, sub-criteria, and alternatives (different MHE) based on experts' standpoints or suggestions. In the second stage, the group multi-criteria decision-making (GMCDM) matrix based on GIT2FSs is constructed with GIT2FSs for the MHESP. The weights of criteria and sub-criteria are determined based on GIT2FSs selected by experts in the next stage. The FWA technique based on alpha cuts is then implemented to obtain the aggregated weighted ratings from the Gaussian interval type-2 fuzzy assessments of each alternative with respect to sub-criteria and the Gaussian interval type-2 fuzzy weights of sub-criteria, and finally, the most suitable MHE is chosen using the ELECTRE III approach.

The following steps are the summary of the integrated algorithm FWA- ELECTRE III based on GIT2FNs for ranking MHE:

Step 1 Assume that I alternatives A_i ($i = 1, \dots, I$) are to be evaluated by L experts with respect to J criteria C_j ($j = 1, \dots, J$). Moreover, let each criterion C_j includes k sub-criteria c_{jk} ($k = 1, 2, \dots, k_j$).

Step 2 Determine two types of linguistic variables for the MHESP. The first type is adopted to evaluate alternatives with respect to sub-criteria (see Table 5) and the second one is applied to weight criteria and sub-criteria (see Table 6).

Step 3 Apply the data in Table 6 to weight criteria and sub-criteria. There are the following two vectors of weights with respect to criteria (W) and their corresponding sub-criteria (W_j), respectively:

$$W = (w_1, w_2, \dots, w_J), \tag{45}$$

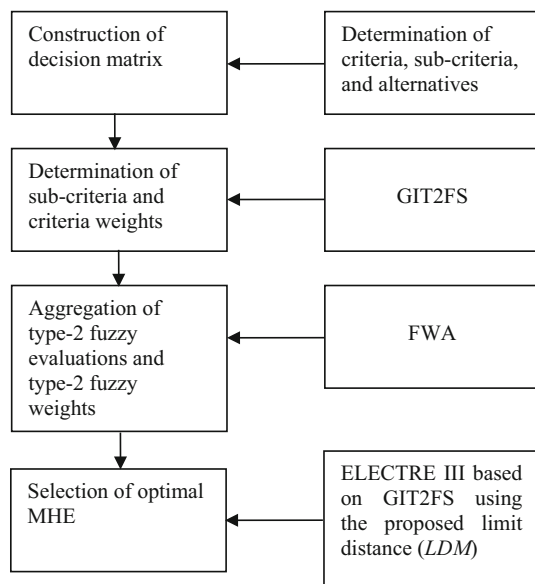


Fig. 7 Framework of the proposed methodology

$$W_j = (w_{j1}, w_{j2}, \dots, w_{jk_j}), \quad j = 1, \dots, J. \quad (46)$$

Step 4 Construct the Gaussian interval type 2 fuzzy MCDM (GIT2FMCDM) matrix (as represented in Table 7) for the MHESP as follows:

where each criterion C_j ($j = 1, \dots, J$) includes k ($k = 1, 2, \dots, k_j$) sub-criteria (c_{j1}, \dots, c_{jk_j}) and \tilde{X}_{ijk}^l is a GIT2FN selected by l th expert assessing alternative i

($i = 1, \dots, I$) with respect to k th ($k = 1, 2, \dots, k_j$) sub-criterion (c_{jk}) regarding criterion C_j ($j = 1, \dots, J$). \tilde{X}_{ijk}^l is a GIT2FN as $\left[\left(\mu^L; \sigma^L; H_{\tilde{G}}^L \right), \left(\mu^U; \sigma^U; H_{\tilde{G}}^U \right) \right]$ (as explained in Definition 3.2.6).

Step 5 Integrate the type-2 fuzzy ratings \tilde{X}_{ijk}^l (for all $l = 1, \dots, L$) for each sub-criterion (c_{jk}) (for each $i = 1, \dots, I, j = 1, \dots, J$ and $k = 1, 2, \dots, k_j$) with the synthetic type-2 fuzzy rating $\tilde{\tilde{X}}_{ijk}$ using the following equation:

$$\tilde{\tilde{X}}_{ijk} = (1/L) \otimes (\tilde{X}_{ijk}^1 \oplus \tilde{X}_{ijk}^2 \oplus \dots \oplus \tilde{X}_{ijk}^L), \quad i = 1, \dots, I; \quad j = 1, \dots, J; \quad k = 1, \dots, k_j. \quad (47)$$

Indeed, it is the average of the type-2 fuzzy ratings \tilde{X}_{ijk}^l ($l = 1, \dots, L$) selected by L experts. Similarly, integrate the type-2 fuzzy weights \tilde{w}_j^l and \tilde{w}_{jk}^l (for all $l = 1, \dots, L$) for each criterion and sub-criterion (c_{jk}) (for each $j = 1, \dots, J$ and $k = 1, 2, \dots, k_j$) with the synthetic type-2 fuzzy weight $\tilde{\tilde{w}}_j$ and $\tilde{\tilde{w}}_{jk}$, respectively, as follows:

$$\tilde{\tilde{w}}_j = (1/L) \otimes (\tilde{w}_j^1 \oplus \tilde{w}_j^2 \oplus \dots \oplus \tilde{w}_j^L), \quad j = 1, \dots, J, \quad (48)$$

$$\tilde{\tilde{w}}_{jk} = (1/L) \otimes (\tilde{w}_{jk}^1 \oplus \tilde{w}_{jk}^2 \oplus \dots \oplus \tilde{w}_{jk}^L), \quad j = 1, \dots, J; \quad k = 1, \dots, k_j. \quad (49)$$

Table 5 Linguistic variables for ratings

Definitions of linguistic variables for the ratings	$\left[\left(\mu^L, \sigma^L; H_{\tilde{G}}^L \right), \left(\mu^U, \sigma^U; H_{\tilde{G}}^U \right) \right]$
Absolutely low (AL)	[(3, 0.5; 1), (3, 1; 1)]
Very low (VL)	[(5, 0.5; 1), (5, 1; 1)]
Low (L)	[(7, 0.5; 1), (7, 1; 1)]
Medium (M)	[(9, 0.5; 1), (9, 1; 1)]
High (H)	[(11, 0.5; 1), (11, 1; 1)]
Very high (VH)	[(13, 0.5; 1), (13, 1; 1)]
Absolutely high (AH)	[(15, 0.5; 1), (15, 1; 1)]

Table 6 Linguistic variables for weights

Definitions of linguistic variables for weighting criteria and sub-criteria	$\left[\left(\mu^L, \sigma^L; H_{\tilde{G}}^L \right), \left(\mu^U, \sigma^U; H_{\tilde{G}}^U \right) \right]$
Absolutely unimportant (AU)	[(3, 0.5; 1), (3, 1; 1)]
Very unimportant (VU)	[(5, 0.5; 1), (5, 1; 1)]
Unimportant (U)	[(7, 0.5; 1), (7, 1; 1)]
Medium (M)	[(9, 0.5; 1), (9, 1; 1)]
Important (I)	[(11, 0.5; 1), (11, 1; 1)]
Very important (VI)	[(13, 0.5; 1), (13, 1; 1)]
Absolutely important (AI)	[(15, 0.5; 1), (15, 1; 1)]

Table 7 GIT2FMCDM matrix for the MHESP

Alternatives	Criteria										
	C ₁			C _j			C _J				
	Sub-criteria			Sub-criteria			Sub-criteria				
	c ₁₁	...	c _{1k₁}	...	c _{j1}	...	c _{jk_j}	...	c _{J1}	...	c _{Jk_J}
A ₁	\tilde{x}_{111}^1	...	$\tilde{x}_{11k_1}^1$...	\tilde{x}_{1j1}^1	...	$\tilde{x}_{1jk_j}^1$...	\tilde{x}_{1J1}^1	...	$\tilde{x}_{1Jk_J}^1$
	⋮	...	⋮	...	⋮	...	⋮	...	⋮	...	⋮
A _i	\tilde{x}_{i11}^L	...	$\tilde{x}_{i1k_1}^L$...	\tilde{x}_{ij1}^L	...	$\tilde{x}_{ijk_j}^L$...	\tilde{x}_{iJ1}^L	...	$\tilde{x}_{iJk_J}^L$
	⋮	...	⋮	...	⋮	...	⋮	...	⋮	...	⋮
A _J	\tilde{x}_{J11}^1	...	$\tilde{x}_{J1k_1}^1$...	\tilde{x}_{Jj1}^1	...	$\tilde{x}_{Jjk_j}^1$...	\tilde{x}_{JJ1}^1	...	$\tilde{x}_{JJk_J}^1$
	⋮	...	⋮	...	⋮	...	⋮	...	⋮	...	⋮

Using the notation described in Definition 3.2.9, GIT2FNs $\tilde{X}_{ijk\alpha}$ at level α (for each $i = 1, \dots, I, j = 1, \dots, J$ and $k = 1, 2, \dots, k_j$) can be represented as follows:

$$\hat{x}_{ijk\alpha} = (\bar{x}_{1ijk\alpha}^l, \underline{x}_{2ijk\alpha}^l, \mu_{ijk\alpha}, \underline{x}_{1ijk\alpha}^r, \bar{x}_{2ijk\alpha}^r),$$

$$i = 1, \dots, I; j = 1, \dots, J; k = 1, \dots, k_j, \quad (50)$$

where $\mu_{ijk\alpha}$ is the mean of GIT2FN when alternative i ($i = 1, \dots, I$) is evaluated with respect to k th ($k = 1, 2, \dots, k_j$) sub-criterion (c_{jk}) regarding criterion C_j ($j = 1, \dots, J$) at level α .

Similarly, GIT2FN $\tilde{X}_{ijk\alpha}$ selected by l th expert at level α can be showed as follows:

$$\hat{x}_{ijk\alpha}^l = (\bar{x}_{1ijk\alpha}^{ll}, \underline{x}_{2ijk\alpha}^{ll}, \mu_{ijk\alpha}, \underline{x}_{1ijk\alpha}^{rl}, \bar{x}_{2ijk\alpha}^{rl}),$$

$$i = 1, \dots, I; j = 1, \dots, J; k = 1, \dots, k_j; l = 1, \dots, L. \quad (51)$$

The average (synthesis) of five reference points is obtained for GIT2FNs chosen by L experts as $\bar{x}_{ijk\alpha} = (\bar{x}_{1ijk\alpha}^l, \underline{x}_{2ijk\alpha}^l, \bar{\mu}_{ijk\alpha}, \underline{x}_{1ijk\alpha}^r, \bar{x}_{2ijk\alpha}^r)$ at level α ($\alpha = \alpha_1, \dots, \alpha_N; i = 1, \dots, I; j = 1, \dots, J$; and $k = 1, 2, \dots, k_j$) using extension of Eqs. (33)–(37) as follows:

$$\bar{x}_{1ijk\alpha}^l = \left(\sum_{l=1}^L \bar{x}_{1ijk\alpha}^{ll} \right) / L, i = 1, \dots, I; j = 1, \dots, J;$$

$$k = 1, \dots, k_j; \alpha = \alpha_1, \dots, \alpha_N, \quad (52)$$

$$\bar{x}_{2ijk\alpha}^l = \left(\sum_{l=1}^L \underline{x}_{2ijk\alpha}^{ll} \right) / L, \quad i = 1, \dots, I;$$

$$j = 1, \dots, J; k = 1, \dots, k_j; \alpha = \alpha_1, \dots, \alpha_N, \quad (53)$$

$$\bar{\mu}_{ijk\alpha} = \left(\sum_{l=1}^L \mu_{ijk\alpha}^l \right) / L, \quad i = 1, \dots, I;$$

$$j = 1, \dots, J; k = 1, \dots, k_j; \alpha = \alpha_1, \dots, \alpha_N, \quad (54)$$

$$\bar{x}_{1ijk\alpha}^r = \left(\sum_{l=1}^L \underline{x}_{1ijk\alpha}^{rl} \right) / L, \quad i = 1, \dots, I;$$

$$j = 1, \dots, J; k = 1, \dots, k_j; \alpha = \alpha_1, \dots, \alpha_N, \quad (55)$$

$$\bar{x}_{2ijk\alpha}^r = \left(\sum_{l=1}^L \bar{x}_{2ijk\alpha}^{rl} \right) / L, \quad i = 1, \dots, I;$$

$$j = 1, \dots, J; k = 1, \dots, k_j; \alpha = \alpha_1, \dots, \alpha_N. \quad (56)$$

Similarly, the average of five reference points for weights of criteria and sub-criteria $\bar{w}_{j\alpha} = (\bar{w}_{1j\alpha}^l, \underline{w}_{2j\alpha}^l, \bar{\mu}_{j\alpha}, \underline{w}_{1j\alpha}^r, \bar{w}_{2j\alpha}^r)$ and $\bar{w}_{jk\alpha} = (\bar{w}_{1jk\alpha}^l, \underline{w}_{2jk\alpha}^l, \bar{\mu}_{jk\alpha}, \underline{w}_{1jk\alpha}^r, \bar{w}_{2jk\alpha}^r)$ are determined at level α , respectively, as follows:

$$\bar{w}_{1j\alpha}^l = \left(\sum_{l=1}^L \bar{w}_{1j\alpha}^{ll} \right) / L, \quad j = 1, \dots, J;$$

$$\alpha = \alpha_1, \dots, \alpha_N, \tag{57}$$

$$\bar{w}_{2j\alpha}^l = \left(\sum_{l=1}^L w_{2j\alpha}^{ll} \right) / L, \quad j = 1, \dots, J;$$

$$\alpha = \alpha_1, \dots, \alpha_N, \tag{58}$$

$$\bar{\mu}_{j\alpha} = \left(\sum_{l=1}^L \mu_{j\alpha}^l \right) / L, \quad j = 1, \dots, J;$$

$$\alpha = \alpha_1, \dots, \alpha_N, \tag{59}$$

$$\bar{w}_{1j\alpha}^r = \left(\sum_{l=1}^L w_{1j\alpha}^{rl} \right) / L, \quad j = 1, \dots, J;$$

$$\alpha = \alpha_1, \dots, \alpha_N, \tag{60}$$

$$\bar{w}_{2j\alpha}^r = \left(\sum_{l=1}^L \bar{w}_{2j\alpha}^{rl} \right) / L, \quad j = 1, \dots, J;$$

$$\alpha = \alpha_1, \dots, \alpha_N, \tag{61}$$

and

$$\bar{w}_{1jk\alpha}^l = \left(\sum_{l=1}^L \bar{w}_{1jk\alpha}^{ll} \right) / L, \quad j = 1, \dots, J;$$

$$k = 1, \dots, k_j; \alpha = \alpha_1, \dots, \alpha_N, \tag{62}$$

$$\bar{w}_{2jk\alpha}^l = \left(\sum_{l=1}^L w_{2jk\alpha}^{ll} \right) / L, \quad j = 1, \dots, J; \quad k = 1, \dots, k_j;$$

$$\alpha = \alpha_1, \dots, \alpha_N, \tag{63}$$

$$\bar{\mu}_{jk\alpha} = \left(\sum_{l=1}^L \mu_{jk\alpha}^l \right) / L, \quad j = 1, \dots, J; \quad k = 1, 2, \dots, k_j;$$

$$\alpha = \alpha_1, \dots, \alpha_N, \tag{64}$$

$$\bar{w}_{1jk\alpha}^r = \left(\sum_{l=1}^L w_{1jk\alpha}^{rl} \right) / L, \quad j = 1, \dots, J; \quad k = 1, 2, \dots, k_j;$$

$$\alpha = \alpha_1, \dots, \alpha_N, \tag{65}$$

$$\bar{w}_{2jk\alpha}^r = \left(\sum_{l=1}^L \bar{w}_{2jk\alpha}^{rl} \right) / L, \quad j = 1, \dots, J; \quad k = 1, 2, \dots, k_j;$$

$$\alpha = \alpha_1, \dots, \alpha_N. \tag{66}$$

Step 6 If number of sub-criteria under one criterion are greater than or equal 2 ($k \geq 2$), integrate the synthetic type-2 fuzzy ratings of each alternative with respect to all sub-criteria under one criterion (calculated by Eqs. (52)–(56)) and

the synthetic type-2 fuzzy weights of sub-criteria (obtained by Eqs. (62)–(66)) with the aggregated weighted ratings $\hat{a}_{ij\alpha}$ at level α using the generalization of the FWA approach introduced in Eq. (38):

$$\hat{a}_{ij\alpha} = \sum_{k=1}^{k_j} (\hat{w}_{jk\alpha} \times \hat{x}_{ijk\alpha}) / \sum_{k=1}^{k_j} \hat{w}_{jk\alpha}, \quad i = 1, \dots, I;$$

$$j = 1, \dots, J; \alpha = \alpha_1, \dots, \alpha_N, \tag{67}$$

where $\hat{a}_{ij\alpha} = \left\{ \left[\bar{a}_{1ij\alpha}^l, \underline{a}_{2ij\alpha}^l \right], \left[\bar{a}_{1ij\alpha}^r, \underline{a}_{2ij\alpha}^r \right] \right\}$.

Thus, the GIT2FMCDM matrix for the MHESP (as presented in Table 7) is transformed with the following aggregated weighted MCDM matrix:

$$C_1 \cdots C_J$$

$$\hat{A}_{I \times J\alpha} = \begin{matrix} A_1 \\ \vdots \\ A_I \end{matrix} \begin{bmatrix} \hat{a}_{11\alpha} & \cdots & \hat{a}_{1J\alpha} \\ \vdots & \dots & \vdots \\ \hat{a}_{I1\alpha} & \cdots & \hat{a}_{IJ\alpha} \end{bmatrix},$$

$$i = 1, \dots, I; \quad j = 1, \dots, J; \alpha = \alpha_1, \dots, \alpha_N, \tag{68}$$

where $\bar{w}_{j\alpha} = (\bar{w}_{1j\alpha}^l, \bar{w}_{2j\alpha}^l, \bar{\mu}_{j\alpha}, \bar{w}_{1j\alpha}^r, \bar{w}_{2j\alpha}^r)$ is the average of weights vector for $\alpha = \alpha_1, \dots, \alpha_N$ and $j = 1, \dots, J$.

Step 7 Calculate three thresholds q_j, p_j , and v_j .

First, the largest and the smallest of evaluations for the quantitative criteria are obtained as follows:

$$A_j^+ = \left\{ \max_i (x_{ij}) \right\}, \quad j = 1, \dots, n', \tag{69}$$

$$A_j^- = \left\{ \max_i (x_{ij}) \right\}, \quad j = 1, \dots, n'. \tag{70}$$

Second, measures of thresholds for the quantitative criteria are calculated as follows:

$$q_j = (A_j^+ - A_j^-) * \beta_{1j}, \quad j = 1, \dots, n', \tag{71}$$

$$p_j = (A_j^+ - A_j^-) * \beta_{2j}, \quad j = 1, \dots, n', \tag{72}$$

$$v_j = (A_j^+ - A_j^-) * \beta_{3j}, \quad j = 1, \dots, n', \tag{73}$$

where β_1, β_2 , and β_3 are scalars that their measures are determined by decision-makers.

On the other hand, \tilde{q}_j, \tilde{p}_j , and \tilde{v}_j are determined using GIT2FNs and then their LDMs are calculated using Eq. (41).

Step 8 Calculate the concordance index of alternative A_r relative to alternative A_k ($C(A_r, A_k)$) by the following formula:

$$\hat{C}(A_r, A_k) = \frac{\sum_{j=1}^{n'} LDM_W(\tilde{w}_j) * c_j(A_r, A_k) + \sum_{j=n'+1}^n LDM_W(\tilde{w}_j) * \hat{c}_j(A_r, A_k)}{\sum_{j=1}^n LDM_W(\tilde{w}_j)}, \quad r, k = 1, \dots, m; \quad r \neq k, \quad (74)$$

where $LDM_W(\tilde{w}_j)$, $c_j(A_r, A_k)$, and $\hat{c}_j(A_r, A_k)$ are weight of criterion j ($j = n' + 1, \dots, n$) by using Eq. (44), the superior level of alternative A_r relative to alternative A_k for quantitative criteria [Eqs. (3) and (4)], and the superior level of alternative A_r relative to alternative A_k for qualitative criteria, respectively. For B and C qualitative criteria, $\hat{c}_j(A_r, A_k)$ is calculated, respectively, by the following relation:

where $LDM'_{PI,B} = 1 - LDM_{PI,B}$ and $LDM'_{PI,C} = 1 - LDM_{PI,C}$.

Step 9 Calculate the discordance index of alternative A_r versus alternative A_k ($\hat{d}_j(A_r, A_k)$) for quantitative criteria ($j = 1, \dots, n'$) using Eqs. (5) and (6) and for B and C qualitative criteria ($j = n' + 1, \dots, n$) as:

$$\hat{c}_j(A_r, A_k) = \begin{cases} 0 & LDM'_{PI,B}(\tilde{X}_{kj}) - LDM'_{PI,B}(\tilde{X}_{rj}) \geq LDM'_{PI,B}(\tilde{p}_j), \\ \frac{LDM'_{PI,B}(\tilde{X}_{rj}) - LDM'_{PI,B}(\tilde{X}_{kj}) + LDM'_{PI,B}(\tilde{p}_j)}{LDM'_{PI,B}(\tilde{p}_j) - LDM'_{PI,B}(\tilde{q}_j)} & LDM'_{PI,B}(\tilde{q}_j) < LDM'_{PI,B}(\tilde{X}_{kj}) - LDM'_{PI,B}(\tilde{X}_{rj}) \\ & \leq LDM'_{PI,B}(\tilde{p}_j), \\ 1 & LDM'_{PI,B}(\tilde{X}_{kj}) - LDM'_{PI,B}(\tilde{X}_{rj}) \leq LDM'_{PI,B}(\tilde{q}_j), \end{cases} \quad (75)$$

and

$$\hat{c}_j(A_r, A_k) = \begin{cases} 0 & LDM_{PI,C}(\tilde{X}_{rj}) - LDM_{PI,C}(\tilde{X}_{kj}) \geq LDM_{PI,C}(\tilde{p}_j), \\ \frac{LDM_{PI,C}(\tilde{X}_{kj}) - LDM_{PI,C}(\tilde{X}_{rj}) + LDM_{PI,C}(\tilde{p}_j)}{LDM_{PI,C}(\tilde{p}_j) - LDM_{PI,C}(\tilde{q}_j)} & \text{otherwise,} \\ 1 & LDM_{PI,C}(\tilde{X}_{rj}) - LDM_{PI,C}(\tilde{X}_{kj}) \leq LDM_{PI,C}(\tilde{q}_j), \end{cases} \quad (76)$$

$$\hat{d}_j(A_r, A_k) = \begin{cases} 0 & LDM'_{PI,B}(\tilde{X}_{kj}) - LDM'_{PI,B}(\tilde{X}_{rj}) \leq LDM'_{PI,B}(\tilde{p}_j), \\ \frac{LDM'_{PI,B}(\tilde{X}_{kj}) - LDM'_{PI,B}(\tilde{X}_{rj}) - LDM'_{PI,B}(\tilde{p}_j)}{LDM'_{PI,B}(\tilde{v}_j) - LDM'_{PI,B}(\tilde{p}_j)}, & LDM'_{PI,B}(\tilde{p}_j) < LDM'_{PI,B}(\tilde{X}_{kj}) - LDM'_{PI,B}(\tilde{X}_{rj}) \leq LDM'_{PI,B}(\tilde{v}_j), \\ 1 & LDM'_{PI,B}(\tilde{X}_{kj}) - LDM'_{PI,B}(\tilde{X}_{rj}) \geq LDM'_{PI,B}(\tilde{v}_j), \end{cases} \quad (77)$$

and

$$\hat{d}_j(A_r, A_k) = \begin{cases} 0 & LDM_{PI,C}(\tilde{X}_{rj}) - LDM_{PI,C}(\tilde{X}_{kj}) \leq LDM_{PI,C}(\tilde{p}_j), \\ \frac{LDM_{PI,C}(\tilde{X}_{rj}) - LDM_{PI,C}(\tilde{X}_{kj}) - LDM_{PI,C}(\tilde{p}_j)}{LDM_{PI,C}(\tilde{v}_j) - LDM_{PI,C}(\tilde{p}_j)} & \text{otherwise,} \\ 1 & LDM_{PI,C}(\tilde{X}_{rj}) - LDM_{PI,C}(\tilde{X}_{kj}) \geq LDM_{PI,C}(\tilde{v}_j). \end{cases} \quad (78)$$

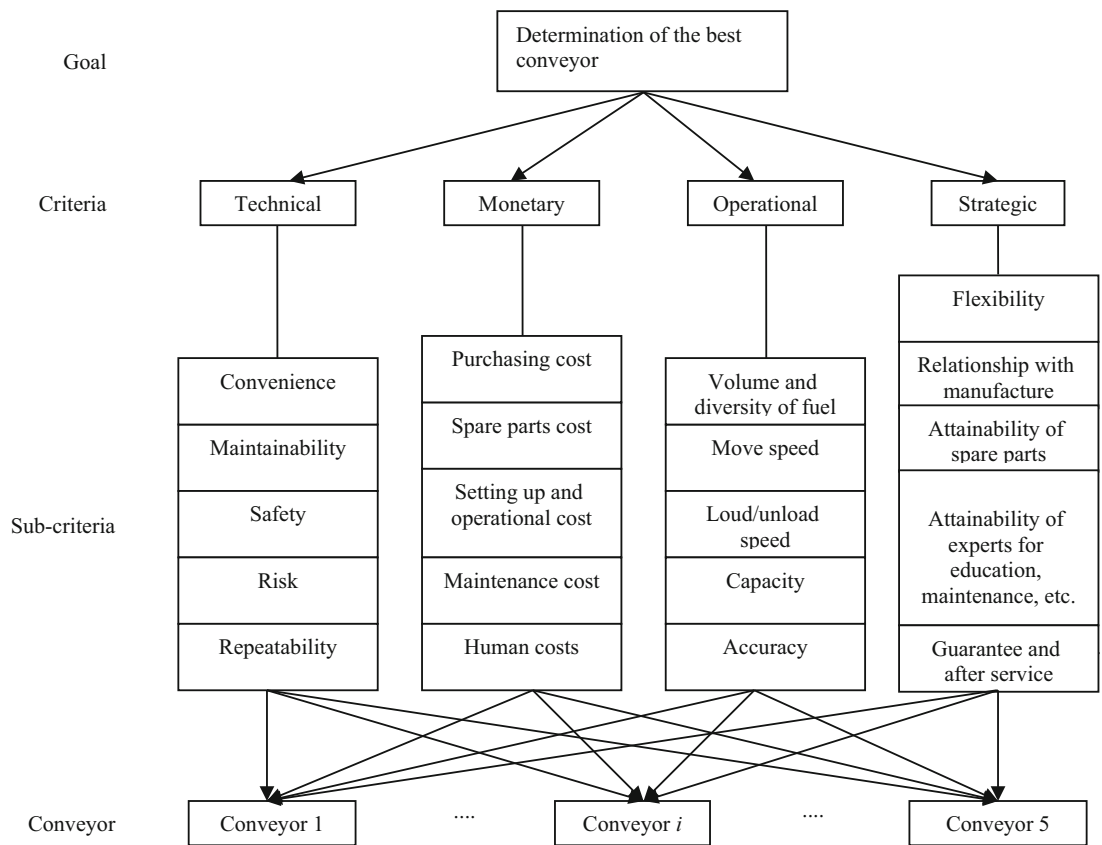


Fig. 8 Hierarchical structure of the GIT2FMCDM for the MHESP

Step 10 Determine credit degree index of alternative A_r versus alternative $A_k(\hat{S}(A_r, A_k))$ for quantitative criteria ($j = 1, \dots, n'$) using Eq. (7) and for qualitative criteria ($j = n' + 1, \dots, n$) by the following formula:

$$\theta_r^c = \sum_{k=1, k \neq r}^m \hat{S}(A_r, A_k), \quad r = 1, \dots, m. \quad (80)$$

$$\hat{S}(A_r, A_k) = \begin{cases} \hat{C}(A_r, A_k), & \text{if } \hat{d}_j(A_r, A_k) \leq \hat{C}(A_r, A_k) \\ \hat{C}(A_r, A_k) \cdot \prod_{j \in J: \hat{d}_j(A_r, A_k) > \hat{C}(A_r, A_k)} \frac{1 - \hat{d}_j(A_r, A_k)}{1 - \hat{C}(A_r, A_k)} & \text{otherwise,} \end{cases} \quad \forall j, \quad (79)$$

where $\hat{S}(A_r, A_k)$ and J are the degree of outranking alternative A_r relative to alternative A_k and the set of criteria for which $\hat{d}_j(A_r, A_k) > \hat{C}(A_r, A_k)$, respectively.

Step 11 Calculate the final scores and rank alternatives:

In addition to cases discussed in Sect. 3.1, in this paper, the net credibility value method [108] is used to determine the final scores. To acquire the net credibility value, the concordance credibility value and the discordance credibility value should be first calculated as follows:

I. The following concordance credibility value is determined for alternative r :

II. The discordance credibility value is determined for alternative r using the following relation:

$$\theta_r^d = \sum_{k=1, k \neq r}^m \hat{S}(A_k, A_r), \quad r = 1, \dots, m. \quad (81)$$

III. The following net credibility value is obtained for alternative r :

$$\theta_r = \theta_r^c - \theta_r^d, \quad r = 1, \dots, m, \quad (82)$$

where θ_r represents the total score of alternative r (the higher the score, the greater the preference level).

Table 8 Determining the weights of criteria

Criteria	Experts		
	1	2	3
Technical (C_1)	I	M	U
Monetary (C_2)	VI	I	M
Operational (C_3)	AI	I	VI
Strategic (C_4)	AU	VU	U

Table 9 Determining the weights of sub-criteria with respect to technical (C_1)

Sub-criteria	Experts		
	1	2	3
Convenience (c_{11})	AI	VI	VI
Maintainability (c_{12})	VI	I	M
Safety (c_{13})	U	VU	U
Risk (c_{14})	AU	AU	U
Repeatability (c_{15})	VU	I	I

Application

Case study

To show the ability of the proposed approach to solve the MHESP, a real case study is considered in this section where five conveyers A_i ($i = 1, 2, 3, 4, 5$) [pneumatic conveyor (alternative 1), chute conveyor (alternative 2), roller conveyor (alternative 3), flat-belt conveyor (alternative 4), and wheel conveyor (alternative 5)] are evaluated with respect to four criteria C_j ($j = 1, 2, 3, 4$) (characterizations of technical, monetary, operational, and strategic).

Design experts are three experts who select the most suitable conveyor among the different alternatives. Obviously, each kind of MHE has different characterizations. Figure 8 shows the hierarchy structure of optimal MHE selection process with respect to four criteria. Each criterion includes five sub-criteria in the next level. In addition, five types of conveyors are in the lowest level.

First, linguistic variables with their GIT2FNs are applied to weight criteria and sub-criteria in the table. Three experts are then asked to select one of them for determining their importance degree. Table 8 and also Tables 9, 10, 11 and 12 show these variables for criteria and sub-criteria, respectively.

Assessments of alternatives with respect to sub-criteria have been also presented in Tables 13 and 14 using the data in Table 5. The linguistic variables located in rows and columns are the preference level of experts when appraising the alternatives with respect to sub-criteria. Then, the synthetic type-2 fuzzy weights of criteria, sub-criteria, and the

Table 10 Determining the weights of sub-criteria with respect to monetary (C_2)

Sub-criteria	Experts		
	1	2	3
Purchasing cost (c_{21})	AI	AI	VI
Spare part cost (c_{22})	M	I	M
Setting up and operational cost (c_{23})	VI	AI	I
Maintenance cost (c_{24})	VI	I	U
Human costs (c_{25})	VU	AU	AU

Table 11 Determining the weights of sub-criteria with respect to operational (C_3)

Sub-criteria	Experts		
	1	2	3
Volume and diversity of fuel (c_{31})	VU	M	M
Move speed (c_{32})	U	VU	VU
Loud/unload speed (c_{33})	AI	AI	VI
Capacity (c_{34})	VI	I	VI
Accuracy (c_{35})	U	AU	AU

Table 12 Determining the weights of sub-criteria with respect to strategic (C_4)

Sub-criteria	Experts		
	1	2	3
Flexibility (c_{41})	VI	I	VI
Relationship with manufacture (c_{42})	U	I	M
Attainability to spare parts (c_{43})	VI	AI	VI
Attainability to experts for education, maintenance, etc. (c_{44})	I	I	M
Guarantee and after service (c_{45})	U	AU	U

synthetic type-2 fuzzy ratings of alternatives with respect to sub-criteria obtained using Eqs. (57)–(61), Eqs. (62)–(66), and Eqs. (52)–(56) for $\alpha = 0.1, 0.2, 0.4, 0.6, 0.8, 0.9$.

Now, the weights of criteria (LDM_W) are calculated for $\alpha = 0.1, 0.2, 0.4, 0.6, 0.8, 0.9$ using Eq. (44). Table 15 represents the weights of criteria, which are 0.5000, 0.6717, 0.8433, and 0.1569, respectively.

Aggregated weighted ratings $\hat{a}_{ij\alpha}$ are also calculated using Eq. (67) for $\alpha = 0.1, 0.2, 0.4, 0.6, 0.8, 0.9$ (see Tables 16 and 17).

Table 18 demonstrates thresholds as the linguistic variables with respect to criteria. The linguistic variables are specified based on the preferences of experts using the data in Table 5.

Based on the data in Tables 18, 19 represents the measures of thresholds for quantitative criteria using Eqs. (71)–(73) and for qualitative criteria using Eq. (41). Table 20 shows

Table 13 Subjective evaluations of three experts for each alternative with respect to sub-criteria under C_1 and C_2

Alternatives	Criteria									
	C_1					C_2				
	Sub-criteria					Sub-criteria				
A_1	VL	L	AL	VH	L	AL	AH	M	AH	AH
	AL	AL	L	VH	VL	L	VH	H	VH	AH
	VL	VL	VL	AH	VL	AL	VH	VH	VH	VH
A_2	AH	VH	AH	VH	AH	H	L	VL	L	L
	AH	VH	VH	VH	VH	L	M	M	M	L
	VH	H	H	H	H	L	M	M	M	VL
A_3	AH	VH	AH	VH	VH	AL	AH	M	AH	AH
	VH	VH	VH	H	AH	L	VH	H	VH	AH
	AH	VH	VH	M	VH	AL	VH	VH	VH	VH
A_4	VH	M	VL	H	L	L	M	H	M	AH
	VH	VL	VH	M	AL	M	M	H	H	H
	H	M	M	M	VL	H	H	M	H	L
A_5	VH	L	M	M	VL	VL	AH	AL	VH	VH
	AH	M	M	H	L	AL	VH	L	AH	H
	H	M	M	M	AL	L	M	M	L	VH

Table 14 Subjective evaluations of three experts for each alternative with respect to sub-criteria under C_3 and C_4

Alternatives	Criteria									
	C_3					C_4				
	Sub-criteria					Sub-criteria				
	c_{35}	c_{34}	c_{33}	c_{32}	c_{31}	c_{45}	c_{44}	c_{43}	c_{42}	c_{41}
A_1	AL	VL	AL	VL	L	H	AH	AH	VH	AL
	VL	M	VL	L	AL	VH	M	AH	H	VL
	VL	M	AL	AL	L	VH	M	VH	H	VL
A_2	L	AH	H	H	M	VL	M	M	VH	VL
	H	H	VH	VH	AH	H	VH	AH	L	L
	M	M	M	L	L	M	AH	AH	M	M
A_3	VH	H	VH	AH	VH	VL	AH	AH	VL	L
	H	AH	AH	M	AH	H	L	VH	VH	VL
	M	AH	AH	AH	AH	M	M	VL	M	AL
A_4	M	VL	VH	VH	L	M	M	AH	VL	AL
	VL	L	M	L	M	AH	AH	H	VL	L
	VL	M	M	M	M	AH	M	M	M	M
A_5	M	M	AL	H	AL	VL	M	M	VL	AL
	L	VH	VL	M	M	L	AH	L	L	VL
	M	AH	AL	M	L	M	M	M	M	AL

Table 15 Weights of criteria

	Weight	Weight	Weight	Weight
C_1	0.5000	C_2 0.6717	C_3 0.8433	C_4 0.1569

measures *LDMs* with respect to criteria based on Eqs. (41) and (42).

Now, the concordance matrix (Table 21) is constructed based on the comparison of the alternatives, according to Eqs. (74)–(76).

After calculating the discordance matrices for each criterion based on Eqs. (77) and (78) (for qualitative criteria), the comparisons between the concordance and discordance matrices are carried out, as shown in Table 22 based on Eq. (79).

Table 16 Aggregated weighted ratings $\hat{a}_{ij\alpha}$ with respect to criteria C_1 and C_2 for $\alpha = 0.1, 0.2, 0.4, 0.6, 0.8, 0.9$

Alternatives	α_n	Criteria							
		C_1				C_2			
		$\bar{a}_{1r1\alpha}^l$	$\underline{a}_{2r1\alpha}^l$	$\underline{a}_{1r1\alpha}^r$	$\bar{a}_{2r1\alpha}^r$	$\bar{a}_{1r2\alpha}^l$	$\underline{a}_{2r2\alpha}^l$	$\underline{a}_{1r2\alpha}^r$	$\bar{a}_{2r2\alpha}^r$
A ₁	0.1	3.2513	4.9527	10.6204	15.3271	6.9687	9.3703	16.6163	22.1178
	0.2	3.7428	5.2847	10.0035	13.5994	7.6798	9.8253	15.8614	20.1429
	0.3	4.1180	5.5298	9.5908	12.5164	8.2125	10.1587	15.3680	18.8817
	0.4	4.4455	5.7385	9.2587	11.7023	8.6730	10.4392	14.9570	17.9171
	0.5	4.7562	5.9240	8.9879	11.0162	9.0981	10.6901	14.6178	17.0916
	0.6	5.0639	6.1070	8.7282	10.4019	9.5284	10.9297	14.2955	16.3531
	0.7	5.3912	6.2934	8.4834	9.8226	9.9699	11.1710	13.9861	15.6434
	0.8	5.7557	6.4990	8.2270	9.2389	10.4688	11.4418	13.6632	14.9265
	0.9	6.2127	6.7470	7.9309	8.5872	11.0618	11.7728	13.2860	14.1121
A ₂	0.1	8.1336	13.6438	12.3757	20.3467	3.5337	5.2769	10.7228	14.9616
	0.2	8.8051	14.1749	11.9321	18.9384	4.04559	5.6090	10.1496	13.4301
	0.3	9.3015	14.5636	11.6405	18.0111	4.4288	5.8542	9.7606	12.4514
	0.4	9.7208	14.8923	11.3898	17.2955	4.7624	6.0604	9.4499	11.7237
	0.5	10.1054	15.1794	11.1881	16.6680	5.0787	6.2444	9.1943	11.0912
	0.6	10.4859	15.4599	10.9850	16.1101	5.3934	6.4250	8.9506	10.5157
	0.7	10.8771	15.7351	10.7957	15.5576	5.7145	6.6067	8.7188	9.9802
	0.8	11.3137	16.0469	10.5940	14.9938	6.0761	6.8089	8.4740	9.4306
	0.9	11.8235	16.4291	10.3559	14.3428	6.5279	7.0477	8.1920	8.8188
A ₃	0.1	6.67810	9.4169	18.0259	24.9094	6.9687	9.3703	16.6163	22.1178
	0.2	7.48341	9.9404	17.1073	22.4030	7.6798	9.8253	15.8615	20.1430
	0.3	8.09065	10.3282	16.5032	20.8218	8.2125	10.1588	15.3680	18.8817
	0.4	8.61594	10.6549	16.0049	19.6244	8.6730	10.4392	14.9570	17.9172
	0.5	9.10547	10.9428	15.6017	18.6068	9.0982	10.6902	14.6179	17.0917
	0.6	9.59624	11.2255	15.2097	17.7067	9.5284	10.9298	14.2956	16.3531
	0.7	10.1086	11.5078	14.8402	16.8402	9.9699	11.1710	13.9861	15.6434
	0.8	10.6849	11.8235	14.4558	15.974	10.4689	11.4419	13.6633	14.9265
	0.9	11.3810	12.2111	14.0059	14.9922	11.0619	11.7729	13.2860	14.1121
A ₄	0.1	3.8248	5.7896	12.2002	17.4530	5.3649	7.4944	13.9904	18.9638
	0.2	4.3980	6.1685	11.5053	15.5294	5.9961	7.8968	13.3094	17.1742
	0.3	4.8310	6.4512	11.0405	14.3154	6.4670	8.1947	12.8559	16.0248
	0.4	5.2079	6.6894	10.6685	13.4134	6.8741	8.4442	12.4904	15.1653
	0.5	5.5648	6.9009	10.3645	12.6425	7.2537	8.6678	12.1865	14.4197
	0.6	5.9212	7.1090	10.0711	11.9513	7.6373	8.8822	11.8961	13.7485
	0.7	6.2910	7.3187	9.7965	11.3026	8.0260	9.0981	11.6201	13.1120
	0.8	6.7086	7.5521	9.5062	10.6443	8.4663	9.3403	11.3264	12.4603
	0.9	7.2270	7.8343	9.1730	9.9131	9.0015	9.6335	10.9922	11.7348
A ₅	0.1	3.9345	5.9399	12.4639	17.8019	5.3444	7.4179	13.7716	18.6461
	0.2	4.5202	6.3268	11.7584	15.8481	5.9560	7.8121	13.1052	16.8913
	0.3	4.9619	6.6147	11.2848	14.6135	6.4150	8.1024	12.6659	15.7713
	0.4	5.3464	6.8574	10.9076	13.6998	6.8126	8.3461	12.3036	14.9219
	0.5	5.7108	7.0727	10.5977	12.9151	7.1829	8.5639	12.0070	14.1924
	0.6	6.0750	7.2850	10.3002	12.2111	7.5552	8.7740	11.7226	13.5365
	0.7	6.4508	7.4980	10.0201	11.5521	7.9376	8.9850	11.4520	12.9124
	0.8	6.8757	7.7357	9.7237	10.8823	8.3699	9.2218	11.1688	12.2793
	0.9	7.4049	8.0226	9.3857	10.1391	8.8911	9.5087	10.8371	11.5644

Table 17 Aggregated weighted ratings $\hat{a}_{ij\alpha}$ with respect to criteria C_3 and C_4 for $\alpha = 0.1, 0.2, 0.4, 0.6, 0.8, 0.9$

Alternatives	α_n	Criteria							
		C_3				C_4			
		$\bar{a}_{1r3\alpha}^l$	$\bar{a}_{2r3\alpha}^l$	$\bar{a}_{1r3\alpha}^r$	$\bar{a}_{2r3\alpha}^r$	$\bar{a}_{1r4\alpha}^l$	$\bar{a}_{2r4\alpha}^l$	$\bar{a}_{1r4\alpha}^r$	$\bar{a}_{2r4\alpha}^r$
A ₁	0.1	1.9853	3.3796	8.1793	12.2422	5.4631	7.6627	14.3543	19.4654
	0.2	2.3853	3.6564	7.6486	10.7452	6.1140	8.0797	13.6575	17.6278
	0.3	2.6908	3.8587	7.2952	9.8090	6.6018	8.3867	13.1910	16.4520
	0.4	2.9598	4.0321	7.0139	9.1110	7.0219	8.6456	12.8114	15.5614
	0.5	3.2184	4.1892	6.7787	8.5223	7.4150	8.8728	12.5009	14.7962
	0.6	3.4745	4.3413	6.5606	7.9852	7.8069	9.0965	12.2004	14.1080
	0.7	3.7424	4.4980	6.3519	7.4964	8.2128	9.3188	11.9167	13.4519
	0.8	4.0460	4.6713	6.1318	6.9946	8.6677	9.5669	11.6176	12.7852
	0.9	4.4330	4.8785	5.8813	6.4419	9.2196	9.8704	11.2707	12.0356
A ₂	0.1	5.3525	7.6655	15.0621	21.0484	5.3810	7.5097	14.0084	18.9850
	0.2	6.0302	8.1092	14.2672	18.8639	6.0101	7.9133	13.3326	17.1950
	0.3	6.5400	8.4385	13.7407	17.4852	6.4808	8.2117	12.8759	16.0488
	0.4	6.9839	8.7159	13.3141	16.4540	6.8873	8.4626	12.5069	15.1870
	0.5	7.4002	8.9641	12.9622	15.5696	7.2690	8.6824	12.2050	14.4406
	0.6	7.8209	9.2040	12.6274	14.7812	7.6508	8.9002	11.9135	13.7683
	0.7	8.2517	9.4467	12.3077	14.0363	8.0425	9.1157	11.6377	13.1309
	0.8	8.7398	9.717	11.9733	13.2842	8.4828	9.3565	11.3470	12.4825
	0.9	9.3385	10.0454	11.5911	12.4403	9.0191	9.6487	11.0109	11.7542
A ₃	0.1	7.1170	9.9177	18.6945	25.7108	4.3352	6.2400	12.1302	16.6804
	0.2	7.9408	10.4551	17.7620	23.1594	4.8958	6.6028	11.5139	15.0401
	0.3	8.5603	10.8464	17.1494	21.5515	5.3173	6.8702	11.0977	13.9929
	0.4	9.0978	11.1792	16.6469	20.3307	5.6814	7.0960	10.7617	13.2028
	0.5	9.5978	11.4770	16.2263	19.2933	6.0244	7.2943	10.4875	12.5239
	0.6	10.1023	11.7625	15.8344	18.3711	6.3646	7.4908	10.2223	11.9110
	0.7	10.6234	12.0524	15.4516	17.4914	6.7187	7.6868	9.9722	11.3308
	0.8	11.2091	12.3753	15.0546	16.6082	7.1141	7.9046	9.7087	10.7403
	0.9	11.9225	12.7691	14.6021	15.6067	7.6004	8.1684	9.4029	10.0785
A ₄	0.1	3.9889	5.9275	12.2635	17.4637	4.7391	6.7705	13.0029	17.7933
	0.2	4.5530	6.3020	11.5787	15.5600	5.3394	7.1569	12.3533	16.0688
	0.3	4.9779	6.5800	11.1195	14.3611	5.7881	7.4413	11.9132	14.9640
	0.4	5.3503	6.8147	10.7515	13.4708	6.1760	7.6811	11.5605	14.1367
	0.5	5.7037	7.0253	10.4466	12.7059	6.5415	7.8917	11.2690	13.4198
	0.6	6.0592	7.2300	10.1604	12.0162	6.9062	8.1000	10.9904	12.7701
	0.7	6.4220	7.4382	9.8864	11.3773	7.2791	8.3066	10.7254	12.1601
	0.8	6.8342	7.6694	9.5996	10.7276	7.6992	8.5373	10.4450	11.5355
	0.9	7.3472	7.9444	9.2719	10.0030	8.2154	8.8167	10.1234	10.8381
A ₅	0.1	3.3771	5.2208	11.2925	16.2964	3.2279	4.8774	10.0748	14.1394
	0.2	3.9132	5.5800	10.6315	14.4627	3.7110	5.1940	9.5273	12.6699
	0.3	4.3186	5.8439	10.1940	13.3099	4.0748	5.4275	9.1539	11.7322
	0.4	4.6733	6.0681	9.8417	12.4492	4.3903	5.6246	8.8560	11.0328
	0.5	5.0096	6.2701	9.5482	11.7165	4.6906	5.7974	8.6143	10.4273
	0.6	5.3458	6.4649	9.2738	11.0533	4.9869	5.9710	8.3793	9.8778
	0.7	5.6927	6.6635	9.0111	10.4417	5.2939	6.1428	8.1585	9.3644
	0.8	6.0874	6.8847	8.7351	9.8170	5.6377	6.3344	7.9253	8.8394
	0.9	6.5786	7.1510	8.4192	9.1224	6.0685	6.5639	7.6549	8.2542

Table 18 Linguistic variables of thresholds with respect to criteria

	C ₁		C ₂		C ₃		C ₄	
\tilde{q}_j	VL	L	AL	L	L	VL	L	VL
\tilde{p}_j	M	M	L	M	M	H	H	L
\tilde{v}_j	H	VH	H	VH	AH	VH	VH	H

Table 19 Thresholds with respect to different criteria

	Criteria			
	C ₁	C ₂	C ₃	C ₄
q_j	0.1667	0.2778	0.2223	0.0036
p_j	0.4445	0.5556	0.5556	0.5000
v_j	0.7222	0.8889	0.7778	0.8333

Table 20 Measures of LDMs of alternatives with respect to criteria

Alternatives	Criteria			
	C ₁	C ₂	C ₃	C ₄
A ₁	0.0738	0.9653	0.0444	0.6955
A ₂	0.8087	0.0109	0.5575	0.8913
A ₃	0.7359	0.9653	0.7996	0.3942
A ₄	0.1352	0.4707	0.2973	0.5984
A ₅	0.1617	0.4421	0.2107	0.1159

Table 21 Concordance matrix

	A ₁	A ₂	A ₃	A ₄	A ₅
A ₁	1.0000	0.4180	0.3815	0.9643	1.0000
A ₂	0.6907	1.0000	0.6676	0.7974	0.8292
A ₃	0.9600	0.9282	1.0000	0.9758	1.0000
A ₄	0.7519	0.6870	0.2022	1.0000	1.0000
A ₅	0.6545	0.6533	0.0721	0.9306	1.0000

Now, the results of credit degree ($S(A_r, A_k)$) between alternatives are presented in Table 23 using Eq. (79) (for qualitative criteria). Then, the net credibility matrix is made using Eqs. (80)–(82), as demonstrated in Table 24. Based on the data of this table, the ranking order is as $A_3 > A_1 > A_4 > A_5 > A_2$.

Table 22 Comparison matrix

	A ₁	A ₂	A ₃	A ₄	A ₅
A ₁	–	$c(1, 2) > d(1, 2)$	$c(1, 3) < d_{1,3}(1, 3)$	$c(1, 4) > d(1, 4)$	$c(1, 5) > d(1, 5)$
A ₂	$c(2, 1) < d_2(2, 1)$	–	$c(2, 3) < d_2(2, 3)$	$c(2, 4) > d(2, 4)$	$c(2, 5) > d(2, 5)$
A ₃	$c(3, 1) > d_1(3, 1)$	$c(3, 2) > d(3, 2)$	–	$c(3, 4) > d(3, 4)$	$c(3, 5) > d(3, 5)$
A ₄	$c(4, 1) > d(4, 1)$	$c(4, 2) < d_1(4, 2)$	$c(4, 3) < d_1(4, 3)$	–	$c(4, 5) > d(4, 5)$
A ₅	$c(5, 1) > d(5, 1)$	$c(5, 2) < d_{1,4}(5, 2)$	$c(5, 3) < d_{1,3}(5, 3)$	$c(5, 4) > d(5, 4)$	–

Table 23 Credit degree matrix

	A ₁	A ₂	A ₃	A ₄	A ₅
A ₁	–	0.4180	0.0573	0.9643	1.0000
A ₂	0.0000	–	0.0000	0.7974	0.8292
A ₃	0.9600	0.9282	–	0.9758	1.0000
A ₄	0.7519	0.5610	0.3489	–	1.0000
A ₅	0.6545	0.3919	0.5262	0.9306	–

Table 24 Net credibility matrix

	A ₁	A ₂	A ₃	A ₄	A ₅
θ_r^c	2.4397	1.6267	3.8641	2.6619	2.5033
θ_r^d	2.3665	2.2992	0.9326	3.6682	3.8292
θ_r	0.0731	–2.2992	2.9315	–1.0062	–1.3259

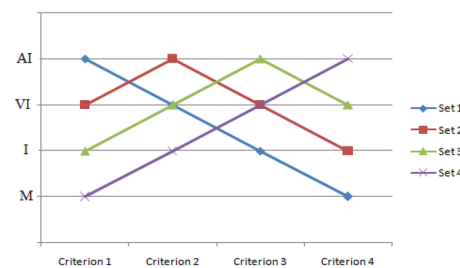


Fig. 9 Sets of criteria weights

Sensitivity analysis

In the ELECTRE III approach, the criteria weights and thresholds affect the stability of the obtained orders within a decision-making process. To demonstrate the reliability of the obtained rankings, it is necessary to analyze the sensitivity analysis with respect to different values of the criteria weights and thresholds measures. Accordingly, the authors considered four sets of criteria weights and three sets of thresholds measures to investigate the use of the different weights and measures in the proposed method, respectively. In each set of the criteria weights, there is one criterion with the highest weight (as shown in Fig. 9). The proposed method is then solved with respect to each set, separately. Orders of each set of criteria are represented in Table 25. To check the similarity of the ranking results for all sets of

Table 25 Ranking results with respect to the different sets of the criteria weights

<i>i</i>	Sets			
	1	2	3	4
1	2	2	2	2
2	4	5	4	4
3	1	1	1	1
4	3	3	3	3
5	5	4	5	5

Table 26 Correlation coefficients between the different sets of criteria weights

	Set 1	Set 2	Set 3	Set 4
Set 1	1.0	0.9	1.0	1.0
Set 2	–	1.0	0.9	0.9
Set 3	–	–	1.0	1.0
Set 4	–	–	–	1.0

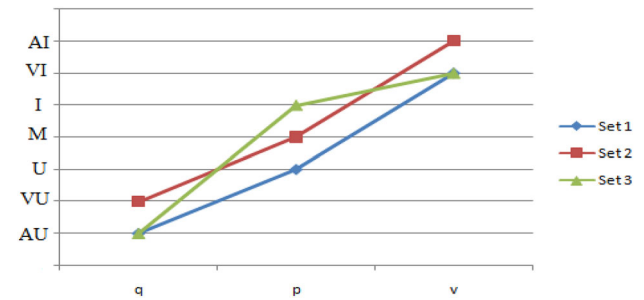


Fig. 10 Sets of thresholds

Table 27 Ranking results with respect to different sets of thresholds with respect to criterion C_1

<i>i</i>	Sets		
	1	2	3
1	2	2	2
2	5	5	5
3	1	1	1
4	3	3	4
5	4	4	3

criteria, the authors utilized the Spearman correlation coefficient. Table 26 represents the correlation coefficients between these sets. According to the data of this table, coefficients are greater than 0.9 when calculating relations between sets. It can be inferred that the proposed method has a fairly good stability with respect to the different weights of criteria. Figure 10 and Tables 27, 28, 29, 30 represent the sets of thresholds and rankings of each set, respectively. In each set of thresholds under a certain criterion, only these thresholds change and the thresholds measures of other criteria are fixed. Table 31 shows the correlation coefficients among these sets for different criteria. As represented in this table, the correlation coefficients are greater than 0.9 when calculating coefficients. It concludes that the obtained orders are stable and reliable based on our proposed approach.

Table 28 Ranking results with respect to different sets of thresholds with respect to criterion C_2

<i>i</i>	Sets		
	1	2	3
1	2	2	2
2	5	5	5
3	1	1	1
4	3	3	3
5	4	4	4

Table 29 Ranking results with respect to different sets of thresholds with respect to criterion C_3

<i>i</i>	Sets		
	1	2	3
1	2	2	2
2	4	4	5
3	1	1	1
4	3	3	3
5	5	5	4

Table 30 Ranking results with respect to different sets of thresholds with respect to criterion C_4

<i>i</i>	Sets		
	1	2	3
1	2	2	2
2	5	5	5
3	1	1	1
4	3	4	4
5	4	3	3

Table 31 Correlation coefficients between the different sets of thresholds with respect to the different criteria

	Set 1	Set 2	Set 3
C_1			
Set 1	1.0	1.0	0.9
Set 2	–	1.0	0.9
Set 3	–	–	1.0
C_2			
Set 1	1.0	1.0	1.0
Set 2	–	1.0	1.0
Set 3	–	–	1.0
C_3			
Set 1	1.0	1.0	0.9
Set 2	–	1.0	0.9
Set 3	–	–	1.0
C_4			
Set 1	1.0	0.9	0.9
Set 2	–	1.0	1.0
Set 3	–	–	1.0

Comparison with other methods

To further illustrate the effectiveness of the proposed methodology, the results obtained by ELECTRE III based on

Table 32 Comparison of ranking results by other techniques

Methods	Ranking
The proposed approach	$A_3 > A_1 > A_4 > A_5 > A_2$
VIKOR with IT2FSs [30]	$A_3 > A_2 > A_4 > A_5 > A_1$
VIKOR with IT2FSs [31]	$A_3 > A_2 > A_4 > A_5 > A_1$
FVIKOR [89]	$(A_3, A_2) > A_4 > A_5 > A_1$
TOPSIS [109] with IT2FSs	$A_3 > A_2 > A_4 > A_5 > A_1$
SAW with IT2FSs [33]	$A_3 > A_2 > A_1 > A_5 > A_4$
WASPAS with IT2FSs [34]	$A_3 > A_2 > A_4 > A_1 > A_5$

Table 33 Ranking results by FVIKOR

	Ranking
By $DF(\tilde{Q}_i)$	$A_3 > A_2 > A_4 > A_5 > A_1$
By $DF(\tilde{S}_r)$	$A_3 > A_2 > A_1 > A_4 > A_5$
By $DF(\tilde{R}_r)$	$A_3 > A_2 > A_4 > A_5 > A_1$

GIT2FSs are compared with some existing approaches in the literature. Table 32 represents the ranking results based on VIKOR with IT2FSs [30], VIKOR with IT2FSs [31], TOPSIS with IT2FSs [109], SAW with IT2FSs [33], and WASPAS with IT2FSs [34]. As represented in this table, A_3 is the optimal alternative in all approaches. It verifies the effectiveness and robustness of the proposed method with the other MCDM techniques introduced above when determining the optimal alternative. But, the authors obtained the different analysis when the use of the fuzzy VIKOR (FVIKOR) method [89]. The ranking priorities obtained by indices

$DF(\tilde{Q}_r)$, $DF(\tilde{S}_r)$, and $DF(\tilde{R}_r)$ have been shown in Table 33 according to descending order, in which $DF(\tilde{Q}_r)$, $DF(\tilde{S}_r)$, and $DF(\tilde{R}_r)$ show the measures of defuzzification \tilde{Q}_r , \tilde{S}_r , and \tilde{R}_r , respectively.

Illustrative example 1

In this section, to show the effectiveness of our ranking method with, the illustrative example adopted by Rashid et al. [29] is taken into account where three robots R_1 , R_2 , and R_3 are assessed with respect to six criteria C_1, C_2, C_3, C_4, C_5 , and C_6 . The authors have decided to eliminate the effect of thresholds using the thresholdless techniques such as TOPSIS and VIKOR. In other words, the authors want to understand that similar results can be achieved among different approaches when thresholds are not used to them. Note that criteria 4 and 6 are of cost type and the other criteria are of benefit type. Tables 34, 35, 36 represent the performance ratings, weights, and normalized measures with respect to different criteria, respectively.

Based on Tables 36, 37 shows the weighted normalized matrix. In addition, Tables 38 and 39 represent PI and NI solutions based on the existence TOPSIS [29] and the interval type-2 fuzzy TOPSIS methods using LDM , respectively.

According to Tables 39, 40 presents LDM_{PI} , LDM_{NI} , ideal separation (\hat{S}_i^+), anti-ideal separation (\hat{S}_i^-), and relative closeness (RC). Finally, Table 41 shows the ranking results of different approaches. Obviously, the most desirable alternative is robot 3 based on the different approaches and the ranking results are as $R_3 > R_2 > R_1$ for all of them. In fact,

Table 34 Performance ratings of robots with respect to different criteria

Criteria	Robots	The performance ratings
C_1	R_1	[(0.5292, 0.5733, 0.6533, 0.6808; 0.8),(0.4533, 0.5333, 0.6933, 0.7567; 1)]
	R_2	[(0.5292, 0.5733, 0.6533, 0.6808; 0.8),(0.4533, 0.5333, 0.6933, 0.7567; 1)]
	R_3	[(0.7108, 0.7508, 0.8050, 0.8225; 0.8),(0.6567, 0.7233, 0.8333, 0.8733; 1)]
C_2	R_1	[(0.5508, 0.5833, 0.6433, 0.6658; 0.8),(0.4933, 0.5533, 0.6733, 0.7233; 1)]
	R_2	[(0.8925, 0.9283, 0.9567, 0.9642; 0.8),(0.8600, 0.9133, 0.9733, 0.9900; 1)]
	R_3	[(0.7108, 0.7508, 0.8050, 0.8225; 0.8),(0.6567, 0.7233, 0.8333, 0.8733; 1)]
C_3	R_1	[(0.5292, 0.5733, 0.6533, 0.6808; 0.8),(0.4533, 0.5333, 0.6933, 0.7567; 1)]
	R_2	[(0.7108, 0.7508, 0.8050, 0.8225; 0.8),(0.6567, 0.7233, 0.8333, 0.8733; 1)]
	R_3	[(0.7825, 0.8150, 0.8850, 0.9075; 0.8),(0.7200, 0.7800, 0.9200, 0.9700; 1)]
C_4	R_1	[(72.25, 72.5, 72.7, 73; 0.8),(72, 72.25, 73.5, 74; 1)]
	R_2	[(69, 69.5, 70, 71; 0.8),(68.5, 68.8, 71.5, 72; 1)]
	R_3	[(67.5, 68, 69, 69.5; 0.8),(67, 67.3, 70, 70.3; 1)]
C_5	R_1	[(48.5, 49, 50, 50.5; 0.8),(48, 48.25, 51, 52; 1)]
	R_2	[(44, 44.5, 45, 45.5; 0.8),(43.5, 43.8, 46, 46.5; 1)]
	R_3	[(43.5, 44, 45, 45.5; 0.8),(43, 43.3, 47, 47.5; 1)]
C_6	R_1	[(0.115, 0.12, 0.13, 0.135; 0.8),(0.1, 0.11, 0.14, 0.142; 1)]
	R_2	[(0.15, 0.157, 0.165, 0.17; 0.8),(0.14, 0.145, 0.18, 0.185; 1)]
	R_3	[(0.162, 0.165, 0.17, 0.175; 0.8),(0.155, 0.16, 0.185, 0.19; 1)]

Table 35 Weights of criteria

Criteria	The weights of criteria
C_1	[(0.8925, 0.9283, 0.9567, 0.9642; 0.8),(0.8600, 0.9133, 0.9733, 0.9900; 1)]
C_2	[(0.8925, 0.9283, 0.9567, 0.9642; 0.8),(0.8600, 0.9133, 0.9733, 0.9900; 1)]
C_3	[(0.2975, 0.3417, 0.4117, 0.4392; 0.8),(0.2267, 0.3067, 0.4467, 0.5100; 1)]
C_4	[(0.4025, 0.4525, 0.5375, 0.5675; 0.8),(0.3200, 0.4100, 0.5800, 0.6500; 1)]
C_5	[(0.8925, 0.9283, 0.9567, 0.9642; 0.8),(0.8600, 0.9133, 0.9733, 0.9900; 1)]
C_6	[(0.8375, 0.8717, 0.9208, 0.9358; 0.8),(0.7900, 0.8467, 0.9467, 0.9800; 1)]

Table 36 Normalized measures

Criteria	Robots	The normalized measures
C_1	R_1	[(0.6059, 0.6565, 0.7481, 0.7796; 0.8),(0.5191, 0.6107, 0.7939, 0.8664; 1)]
	R_2	[(0.6059, 0.6565, 0.7481, 0.7796; 0.8),(0.5191, 0.6107, 0.7939, 0.8664; 1)]
	R_3	[(0.8140, 0.8598, 0.9218, 0.9418; 0.8),(0.7519, 0.8283, 0.9542, 1.0000; 1)]
C_2	R_1	[(0.5564, 0.5892, 0.6498, 0.6726; 0.8),(0.4983, 0.5589, 0.6801, 0.7306; 1)]
	R_2	[(0.9015, 0.9377, 0.9663, 0.9739; 0.8),(0.8687, 0.9226, 0.9832, 1.0000; 1)]
	R_3	[(0.7180, 0.7584, 0.8131, 0.8308; 0.8),(0.6633, 0.7306, 0.8418, 0.8822; 1)]
C_3	R_1	[(0.6653, 0.6934, 0.7901, 0.8560; 0.8),(0.5986, 0.6533, 0.8494, 0.9993; 1)]
	R_2	[(0.5507, 0.5627, 0.6033, 0.6373; 0.8),(0.5187, 0.5436, 0.6262, 0.6898; 1)]
	R_3	[(0.4991, 0.5118, 0.5558, 0.5789; 0.8),(0.4670, 0.4923, 0.5807, 0.6291; 1)]
C_4	R_1	[(0.9178, 0.9215, 0.9241, 0.9273; 0.8),(0.9054, 0.9115, 0.9273, 0.9305; 1)]
	R_2	[(0.9436, 0.9571, 0.9640, 0.9710; 0.8),(0.9305, 0.9370, 0.9738, 0.9781; 1)]
	R_3	[(0.9640, 0.9710, 0.9852, 0.9925; 0.8),(0.9530, 0.9571, 0.9955, 1.0000; 1)]
C_5	R_1	[(0.8514, 0.8600, 0.8775, 0.8865; 0.8),(0.8269, 0.8431, 0.8911, 0.8958; 1)]
	R_2	[(0.9450, 0.9555, 0.9662, 0.9772; 0.8),(0.9247, 0.9347, 0.9817, 0.9885; 1)]
	R_3	[(0.9450, 0.9555, 0.9772, 0.9885; 0.8),(0.9052, 0.9148, 0.9930, 1.0000; 1)]
C_6	R_1	[(0.7407, 0.7692, 0.8333, 0.8695; 0.8),(0.7042, 0.7142, 0.9090, 1.0000; 1)]
	R_2	[(0.5882, 0.6060, 0.6369, 0.6666; 0.8),(0.5405, 0.5555, 0.6896, 0.7142; 1)]
	R_3	[(0.5714, 0.5882, 0.6060, 0.6172; 0.8),(0.5263, 0.5405, 0.6250, 0.6451; 1)]

the obtained results imply that the use of *LDMs* in the thresholdless techniques can show the effectiveness and reliability of the proposed ranking method.

Illustrative Example 2

In this section, a numerical example introduced by Zhong and Yao [32] is presented to compare the ranking results of extended ELECTRE III method with those of the others in the literature. A high-technology company intends to evaluate five qualified suppliers $A_1, A_2, A_3, A_4,$ and A_5 with respect to seven criteria including company reputation (C_1), technical performance (C_2), date of delivery (C_3), service (C_4), cost control (C_5), management performance (C_6), and quality (C_7), which their weights are $w_1 = 0.0562, w_2 = 0.1234, w_3 = 0.0452, w_4 = 0.1245, w_5 = 0.2325, w_6 = 0.1380,$ and $w_7 = 0.2801$ based on the entropy method, respectively. Note that the above criteria are of benefit type. Table 42 represents the evaluated decision matrix of alternatives with

respect to criteria. Based on Tables 42, 43 shows the PI and NI solutions.

Table 44 presents the concordance matrix based on the different approaches. By calculating the discordance matrices for each criterion based on Eqs. (77) and (78), the comparisons between the concordance and discordance matrices are carried out based on Eq. (79), as shown in Table 45.

Now, the results of credit degree ($S(A_r, A_k)$) between alternatives are presented in Table 46 using Eq. (79). In the next step, the net credibility matrix is made using Eqs. (80)–(82), as demonstrated in Table 47. In addition, Table 48 presents the ranking results using the different methods. According to the data of this table, the ranking order of alternatives is as $A_5 > A_4 > A_1 > A_2 > A_3$ using the proposed method. Thus, alternative 5 is selected as the optimal alternative which is also the most important alternative based on the last four approaches. On the other hand, the ranking results obtained by the ELECTRE I method [32] are different

Table 37 Weighted normalized matrix

Criteria	Robots	The weighted normalized measures
C_1	R_1	[(0.5408, 0.6094, 0.7157, 0.7516; 0.8),(0.4464, 0.5577, 0.7727, 0.8577; 1)]
	R_2	[(0.5408, 0.6094, 0.7157, 0.7516; 0.8),(0.4464, 0.5577, 0.7727, 0.8577; 1)]
	R_3	[(0.7264, 0.7981, 0.8818, 0.9080; 0.8),(0.6466, 0.7564, 0.9287, 0.9900; 1)]
C_2	R_1	[(0.4965, 0.5469, 0.6216, 0.6484; 0.8),(0.4285, 0.5104, 0.6619, 0.7233; 1)]
	R_2	[(0.8046, 0.8705, 0.9244, 0.9390; 0.8),(0.7470, 0.8426, 0.9569, 0.9900; 1)]
	R_3	[(0.6408, 0.7040, 0.7778, 0.8010; 0.8),(0.5704, 0.6673, 0.8193, 0.8733; 1)]
C_3	R_1	[(0.1979, 0.2369, 0.3253, 0.3759; 0.8),(0.1357, 0.2003, 0.3794, 0.5096; 1)]
	R_2	[(0.1638, 0.1922, 0.2484, 0.2799; 0.8),(0.1175, 0.1667, 0.2797, 0.3518; 1)]
	R_3	[(0.1485, 0.1749, 0.2288, 0.2542; 0.8),(0.1058, 0.1510, 0.2594, 0.3208; 1)]
C_4	R_1	[(0.3694, 0.4170, 0.4967, 0.5262; 0.8),(0.2897, 0.3737, 0.5378, 0.6048; 1)]
	R_2	[(0.3798, 0.4331, 0.5181, 0.5510; 0.8),(0.2977, 0.3841, 0.5648, 0.6357; 1)]
	R_3	[(0.3880, 0.4393, 0.5295, 0.5632; 0.8),(0.3049, 0.3942, 0.5774, 0.6500; 1)]
C_5	R_1	[(0.7599, 0.7983, 0.8395, 0.8548; 0.8),(0.7111, 0.7700, 0.8673, 0.8868; 1)]
	R_2	[(0.8434, 0.8870, 0.9244, 0.9422; 0.8),(0.7952, 0.8537, 0.9555, 0.9786; 1)]
	R_3	[(0.8434, 0.8870, 0.9349, 0.9531; 0.8),(0.7785, 0.8355, 0.9665, 0.9900; 1)]
C_6	R_1	[(0.6203, 0.6705, 0.7673, 0.8137; 0.8),(0.5563, 0.6047, 0.8606, 0.9800; 1)]
	R_2	[(0.4926, 0.5283, 0.5846, 0.6238; 0.8),(0.4270, 0.4703, 0.6528, 0.7000; 1)]
	R_3	[(0.4785, 0.5127, 0.5580, 0.5776; 0.8),(0.4157, 0.4576, 0.5916, 0.6322; 1)]

Table 38 PI and NI solutions (the existence TOPSIS method [29])

Criteria	Ideal solutions	
C_1	PI	[(0.7264, 0.7980, 0.8818, 0.9081; 0.8),(0.6466, 0.7564, 0.9287, 0.9900; 1)]
	NI	[(0.5408, 0.6094, 0.7157, 0.7516; 0.8),(0.4464, 0.5577, 0.7727, 0.8577; 1)]
C_2	PI	[(0.8046, 0.8705, 0.9244, 0.9390; 0.8),(0.7470, 0.8426, 0.9569, 0.9900; 1)]
	NI	[(0.4965, 0.5469, 0.6216, 0.6484; 0.8),(0.4285, 0.5104, 0.6619, 0.7233; 1)]
C_3	PI	[(0.2399, 0.2870, 0.3755, 0.4108; 0.8),(0.1682, 0.2465, 0.4236, 0.5100; 1)]
	NI	[(0.1622, 0.2019, 0.2772, 0.3082; 0.8),(0.1059, 0.1686, 0.3192, 0.3978; 1)]
C_4	PI	[(0.3694, 0.4170, 0.4967, 0.5262; 0.8),(0.2897, 0.3737, 0.5378, 0.6048; 1)]
	NI	[(0.3880, 0.4393, 0.5295, 0.5632; 0.8),(0.3049, 0.3924, 0.5774, 0.6500; 1)]
C_5	PI	[(0.8324, 0.8747, 0.9198, 0.9363; 0.8),(0.7938, 0.8474, 0.9546, 0.9900; 1)]
	NI	[(0.7466, 0.7855, 0.8278, 0.8436; 0.8),(0.7111, 0.7605, 0.8610, 0.8852; 1)]
C_6	PI	[(0.4785, 0.5127, 0.5580, 0.5776; 0.8),(0.4157, 0.4576, 0.5916, 0.6322; 1)]
	NI	[(0.6203, 0.6705, 0.7673, 0.8137; 0.8),(0.5563, 0.6047, 0.8606, 0.9800; 1)]

from those of the others. It implies that the recent approach cannot be the suitable method for ranking alternatives.

Conclusion and discussion

In the real world, the selection of the best alternative with respect to conflicting criteria is a difficult and complex work when data are vague and inexact. Although type-1 fuzzy sets could greatly resolve ambiguities in decision problems, it takes into account only a specified measure for *MF*. Thus, T2FSs are applied to consider an interval at [0, 1] for *MF*

when decision-maker is uncertain about the value of *MF*. The MFs of a T2FS can take the different versions such as triangular, trapezoidal, and Gaussian. Since GIT2FNs have the smoother *MF*, the authors adopted them for evaluating alternatives with respect to qualitative criteria. This paper presents a new approach for prioritizing GIT2FNs. The proposed method first depicts alpha cuts and then calculates the distances from reference limits. It generates the distances or ranking measures automatically at interval [0, 1]. It is also able to rank TriIT2FNs, TraIT2FNs, and other curved forms. This approach can be applied to the MCDM techniques such as ELECTRE III needing to rank weights of

Table 39 PI and NI solutions (interval type-2 fuzzy TOPSIS using LDM)

Criteria	Robots	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1$
C ₁	PI	(0.9080, 0.9900)	(0.9025, 0.9775)	(0.8974, 0.9650)	(0.8919, 0.9530)	(0.8869, 0.9405)	0.9287
	NI	(0.4464, 0.5408)	(0.4687, 0.5548)	(0.4917, 0.5683)	(0.5133, 0.5823)	(0.5358, 0.5958)	0.5573
C ₂	PI	(0.9390, 0.9900)	(0.936, 0.983)	(0.9330, 0.9765)	(0.9300, 0.9700)	(0.9275, 0.9635)	0.9569
	NI	(0.4285, 0.4965)	(0.4452, 0.5068)	(0.4617, 0.5168)	(0.4777, 0.5268)	(0.4942, 0.5373)	0.5104
C ₃	PI	(0.4108, 0.5100)	(0.4037, 0.4922)	(0.3962, 0.4747)	(0.3897, 0.4577)	(0.3822, 0.4407)	0.4236
	NI	(0.1059, 0.1622)	(0.1186, 0.1701)	(0.1311, 0.1781)	(0.1441, 0.1861)	(0.1566, 0.1941)	0.1686
C ₄	PI	(0.2897, 0.3694)	(0.3067, 0.3792)	(0.3237, 0.3887)	(0.3402, 0.3982)	(0.3577, 0.4077)	0.3747
	NI	(0.5632, 0.6500)	(0.5563, 0.6353)	(0.5498, 0.6208)	(0.5428, 0.6063)	(0.5363, 0.5918)	0.5774
C ₅	PI	(0.9363, 0.9900)	(0.933, 0.9830)	(0.9300, 0.9760)	(0.9265, 0.9685)	(0.9230, 0.9615)	0.9545
	NI	(0.7111, 0.7466)	(0.7214, 0.7544)	(0.7309, 0.7624)	(0.7409, 0.7699)	(0.7509, 0.7779)	0.7605
C ₆	PI	(0.4157, 0.4785)	(0.4242, 0.4852)	(0.4327, 0.4922)	(0.4412, 0.4992)	(0.4492, 0.5058)	0.4576
	NI	(0.8137, 0.9800)	(0.8044, 0.9555)	(0.7949, 0.9315)	(0.7854, 0.9080)	(0.7764, 0.8844)	0.8606

Table 40 Measures LDM_{PI} , LDM_{NI} , \hat{S}_i^+ , \hat{S}_i^- , and RC based on interval type-2 fuzzy TOPSIS using LDM

Robots	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆		RC
<i>R</i> ₁								
LDM_{PI}	0.9628	0.9856	0.9672	0.1114	0.0390	0.9336	$\hat{S}_i^+ = 3.9997$	0.3904
LDM_{NI}	0.3828	0.2754	0.3842	0.4979	0.6324	0.3887	$\hat{S}_i^- = 2.5616$	
<i>R</i> ₂								
LDM_{PI}	0.9628	0.0101	0.4254	0.4556	0.9175	0.0575	$\hat{S}_i^+ = 2.8291$	0.5269
LDM_{NI}	0.3828	0.7637	0.5095	0.5041	0.3565	0.6346	$\hat{S}_i^- = 3.1514$	
<i>R</i> ₃								
LDM_{PI}	0.0258	0.4720	0.0735	0.7721	0.9738	0.0154	$\hat{S}_i^+ = 2.3329$	0.5849
LDM_{NI}	0.6605	0.5122	0.5177	0.4807	0.3630	0.7542	$\hat{S}_i^- = 3.2885$	

Table 41 Ranking results of different approaches

Robots	Interval type-2 fuzzy TOPSIS using LDM	The existence TOPSIS method [29]	WASPAS with IT2FSs [34]	VIKOR with IT2FSs [30]	SAW with IT2FSs [33]
<i>R</i> ₁	3	3	3	3	3
<i>R</i> ₂	2	2	2	2	2
<i>R</i> ₃	1	1	1	1	1

criteria or determine distances of type-2 fuzzy numbers from ideal solutions. To show the effectiveness of the proposed methodology, it was also implemented in a real case study. The authors proposed an integrated GMCDM approach to solve the MHESP where the judgments of experts were represented by GIT2FNs. In real, the experts ranked a kind of equipment named conveyor with respect to a set of criteria and their corresponding sub-criteria where all criteria and sub-criteria weights were expressed as GIT2FNs. In our suggested methodology, the type-2 fuzzy weights of criteria and sub-criteria and also type-2 fuzzy ratings of alternatives with respect to sub-criteria were first integrated with the synthetic type-2 fuzzy weights and ratings, respectively. The FWA method was then adopted by the authors for incorporating

the synthetic ratings of each alternative with respect to all sub-criteria under one criterion and the synthetic weights of sub-criteria to the aggregated weighted ratings and finally, the alternatives were ranked by ELECTRE III. According to the obtained results, the authors have concluded that the alternative 3 is the optimal MHE for all approaches (see Table 32). However, there are a few differences in ranking the other alternatives due to the different nature of compared approaches. In the following, we state them in detailed. As shown in Table 32, there are the similar ranking results as $A_3 > A_2 > A_4 > A_5 > A_1$ between VIKOR with IT2FSs [30], VIKOR with IT2FSs [31], FVIKOR [89], and TOPSIS [109] with IT2FSs approaches, since all of them are based on the closeness to PI solution and the avoidance of NI solution.

Table 42 Evaluated decision matrix of alternatives with respect to criteria

A ₁	C ₁	[(0.3100, 0.5000, 0.5800, 0.7000; 1,1),(0.4000, 0.4500, 0.5000, 0.6000; 0.95, 0.95)]
	C ₂	[(0.8600, 0.9800, 0.9900, 1.0000; 1,1),(0.9200, 0.9700, 0.9800, 0.9900; 0.95, 0.95)]
	C ₃	[(0.5400, 0.7200, 0.7700, 0.8800; 1, 1),(0.6300, 0.6800, 0.7200, 0.8000; 0.95, 0.95)]
	C ₄	[(0.5800, 0.7800, 0.8400, 0.9200; 1,1),(0.6800, 0.7300, 0.7800, 0.8500; 0.95, 0.95)]
	C ₅	[(0.5400, 0.7200, 0.7800, 0.8600; 1,1),(0.6300, 0.6800, 0.7200, 0.7900; 0.95, 0.95)]
	C ₆	[(0.8600, 0.9800, 0.9900, 1.0000; 1,1),(0.9200, 0.9700, 0.9800, 0.9900; 0.95, 0.95)]
	C ₇	[(0.2100, 0.3800, 0.4500, 0.5800; 1,1),(0.2800, 0.3300, 0.3800, 0.4800; 0.95, 0.95)]
A ₂	C ₁	[(0.4200, 0.6200, 0.7000, 0.8000; 1,1),(0.5200, 0.5700, 0.6200, 0.7100; 0.95, 0.95)]
	C ₂	[(0.6200, 0.8200, 0.8700, 0.9600; 1,1),(0.7200, 0.7700, 0.8200, 0.8900; 0.95, 0.95)]
	C ₃	[(0.3800, 0.5800, 0.6600, 0.7800; 1,1),(0.4800, 0.5300, 0.5800, 0.6800; 0.95, 0.95)]
	C ₄	[(0.6600, 0.8400, 0.8900, 0.9400; 1,1),(0.7500, 0.8000, 0.8400, 0.8900; 0.95, 0.95)]
	C ₅	[(0.1800, 0.3400, 0.4000, 0.5400; 1,1),(0.2400, 0.2900, 0.3400, 0.4400; 0.95, 0.95)]
	C ₆	[(0.6200, 0.8200, 0.8700, 0.9600; 1,1),(0.7200, 0.7700, 0.8200, 0.8900; 0.95, 0.95)]
	C ₇	[(0.0600, 0.1800, 0.2300, 0.3800; 1,1),(0.1100, 0.1600, 0.1800, 0.2800; 0.95, 0.95)]
A ₃	C ₁	[(0.3000,0.5000, 0.6000, 0.7000; 1,1),(0.4000, 0.4500, 0.5000, 0.6000; 0.95, 0.95)]
	C ₂	[(0.6200, 0.8200, 0.8700, 0.9600; 1,1),(0.7200, 0.7700, 0.8200, 0.8900; 0.95, 0.95)]
	C ₃	[(0.4600, 0.6600, 0.7200, 0.8600; 1,1),(0.5600, 0.6100, 0.6600, 0.7600; 0.95, 0.95)]
	C ₄	[(0.8200, 0.9600, 0.9800, 1.0000; 1,1),(0.8900, 0.9400, 0.9600, 0.9800; 0.95, 0.95)]
	C ₅	[(0.3500, 0.5400, 0.6200, 0.7200; 1,1),(0.4400, 0.4900, 0.5400, 0.6300; 0.95, 0.95)]
	C ₆	[(0.7800, 0.9000, 0.9200, 0.9400; 1,1),(0.8400, 0.8900, 0.9000, 0.9200; 0.95, 0.95)]
	C ₇	[(0.2700, 0.4600, 0.5500, 0.6600; 1,1),(0.3600, 0.4100, 0.4600, 0.5600; 0.95, 0.95)]
A ₄	C ₁	[(0.5100, 0.7000, 0.7500, 0.8600; 1,1),(0.6000, 0.6500, 0.7000, 0.7800; 0.95, 0.95)]
	C ₂	[(0.7800, 0.9400, 0.9700, 1.0000; 1,1),(0.8600, 0.9100, 0.9400, 0.9700; 0.95, 0.95)]
	C ₃	[(0.5000, 0.6800, 0.7400, 0.8400; 1,1),(0.5900, 0.6400, 0.6800, 0.7600; 0.95, 0.95)]
	C ₄	[(0.6200, 0.8000, 0.8500, 0.9200; 1,1),(0.7100, 0.7600, 0.8000, 0.8600; 0.95, 0.95)]
	C ₅	[(0.0900, 0.2200, 0.2800, 0.4200; 1,1),(0.1500, 0.2000, 0.2200, 0.3200; 0.95, 0.95)]
	C ₆	[(0.9000, 1.0000, 1.0000, 1.0000; 1,1),(0.9500, 1.0000, 1.0000, 1.0000; 0.95, 0.95)]
	C ₇	[(0.2500, 0.4200, 0.4800, 0.6200; 1,1),(0.3200, 0.3700, 0.4200, 0.5200; 0.95, 0.95)]
A ₅	C ₁	[(0.5100, 0.7000, 0.7500, 0.8600; 1,1),(0.6000, 0.6500, 0.7000, 0.7800; 0.95, 0.95)]
	C ₂	[(0.7400, 0.9200, 0.9600, 1.0000; 1,1),(0.8300, 0.8800, 0.9200, 0.9600; 0.95, 0.95)]
	C ₃	[(0.3800, 0.5800, 0.6700, 0.7600; 1,1),(0.4800, 0.5300, 0.5800, 0.6700; 0.95, 0.95)]
	C ₄	[(0.9000, 1.0000, 1.0000, 1.0000; 1,1),(0.9500, 1.0000, 1.0000, 1.0000; 0.95, 0.95)]
	C ₅	[(0.4300, 0.6200, 0.6800, 0.8000; 1,1),(0.5200, 0.5700, 0.6200, 0.7100; 0.95, 0.95)]
	C ₆	[(0.6300, 0.8000, 0.8400, 0.9000; 1,1),(0.7100, 0.7600, 0.8000, 0.8500; 0.95, 0.95)]
	C ₇	[(0.2700, 0.4600, 0.5500, 0.6600; 1,1),(0.3600, 0.4100, 0.4600, 0.5600; 0.95, 0.95)]

Although, in the approaches of VIKOR with IT2FSs [30, 31], both conditions C_1 and C_2 were satisfied and only one alternative was chosen as the best MHE. However, the authors obtained a set of the compromise solutions when FVIKOR [89] was applied. Hence, the conditions C_1 and C_2 should be analyzed for obtaining the final ranking results. Since alternative three has the lowest value regarding the above indices, so alternative three has acceptable stability. In addition, condition C_1 (i.e., $DF(\tilde{Q}_2) - DF(\tilde{Q}_3) = 0.155 < 0.25$) is not satisfied. Hence, alternative 4 that has third situation in the ranking list by $DF(\tilde{Q}_r)$ should be considered for satisfying the condition C_1 . Since the obtained expression (i.e., $DF(\tilde{Q}_4) - DF(\tilde{Q}_3) = 0.583 > 0.25$) holds true, alternatives 2 and 3 are proposed as a set of compromise solutions. More-

over, because of the SAW [33] and WASPAS [34] methods which have the similar ranking programs, the ranking results are almost similar. Note that two last methods have the different ranking structure as compared to the others. As a result, there are the partial ranking differences between them due to the use of weighted product model (WPM) in the WASPAS method. However, they also determined alternative 3 as the most desirable alternative. On the other hand, conveyors 1 and 2 have ranks 2 and 5 based on our proposed method (ELECTRE III with IT2FSs), respectively. In real, lower measure of threshold p_1 as compared to other criteria and also lower measures of thresholds v_j with respect to criteria 1 and 3 as compared to criteria 2 and 4 are the principal reasons of this ranking change. In real-world prob-

Table 43 PI and NI solutions

PI	C_1	[(0.0333, 0.0457, 0.0490, 0.0562; 1,1),(0.0392, 0.0424, 0.0457, 0.0509; 0.95, 0.95)]
	C_2	[(0.1061, 0.1209, 0.1222, 0.1234; 1,1),(0.1135, 0.1196, 0.1209, 0.1222; 0.95, 0.95)]
	C_3	[(0.0277, 0.0369, 0.0395, 0.0452; 1,1),(0.0323, 0.0349, 0.0369, 0.0410; 0.95, 0.95)]
	C_4	[(0.0277, 0.0369, 0.0395, 0.0452; 1,1),(0.1182, 0.1245, 0.1245, 0.1245; 0.95, 0.95)]
	C_5	[(0.1459, 0.1946, 0.2108, 0.2325; 1,1),(0.1703, 0.1838, 0.1946, 0.2135; 0.95, 0.95)]
	C_6	[(0.1242, 0.1380, 0.1380, 0.1380; 1,1),(0.1311, 0.1380, 0.1380, 0.1380; 0.95, 0.95)]
	C_7	[(0.1145, 0.1952, 0.2334, 0.2801; 1,1),(0.1527, 0.1740, 0.1952, 0.2376; 0.95, 0.95)]
NI	C_1	[(0.0196, 0.0326, 0.0379, 0.0457; 1,1),(0.0264, 0.0294, 0.0326, 0.0392; 0.95, 0.95)]
	C_2	[(0.0756, 0.1012, 0.1074, 0.1185; 1,1),(0.0888, 0.0950, 0.1012, 0.1098; 0.95, 0.95)]
	C_3	[(0.0195, 0.0297, 0.0399, 0.0390; 1,1),(0.0246, 0.0272, 0.0297, 0.0344; 0.95, 0.95)]
	C_4	[(0.0722, 0.0971, 0.1045, 0.1145; 1,1),(0.0846, 0.0908, 0.0971, 0.1058; 0.95, 0.95)]
	C_5	[(0.0243, 0.0594, 0.0756, 0.1135; 1,1),(0.0405, 0.0540, 0.0594, 0.0865; 0.95, 0.95)]
	C_6	[(0.0855, 0.1104, 0.1159, 0.1242; 1,1),(0.0979, 0.1048, 0.1104, 0.1173; 0.95, 0.95)]
	C_7	[(0.0254, 0.0763, 0.0976, 0.1612; 1,1),(0.0466, 0.0679, 0.0763, 0.1188; 0.95, 0.95)]

Table 44 Concordance matrix

	A_1	A_2	A_3	A_4	A_5
A_1	1.0000	0.7697	0.8475	0.7478	0.6719
A_2	0.4563	1.0000	0.4494	0.7607	0.2607
A_3	0.7853	0.7404	1.0000	0.5460	0.6380
A_4	0.6618	0.8376	0.5001	1.0000	0.4540
A_5	0.7054	0.7853	0.8324	0.6276	1.0000

Table 45 Comparison matrix

	A_1	A_2	A_3	A_4	A_5
A_1	–	$c(1, 2) < d_1(1, 2)$	$c(1, 3) > d(1, 3)$	$c(1, 4) < d_1(1, 4)$	$c(1, 5) > d(1, 5)$
A_2	$c(2, 1) > d(2, 1)$	–	$c(2, 3) > d(2, 3)$	$c(2, 4) > d(2, 4)$	$c(2, 5) > d(2, 5)$
A_3	$c(3, 1) > d(3, 1)$	$c(3, 2) < d_1(3, 2)$	–	$c(3, 4) < d_1(3, 4)$	$c(3, 5) < d_1(3, 5)$
A_4	$c(4, 1) > d(4, 1)$	$c(4, 2) > d(4, 2)$	$c(4, 3) > d(4, 3)$	–	$c(4, 5) > d(4, 5)$
A_5	$c(5, 1) > d(5, 1)$	$c(5, 2) > d(5, 2)$	$c(5, 3) > d(5, 3)$	$c(5, 4) > d(5, 4)$	–

lems, it may be a situation in which a decision-maker has not any preference with respect to some measures of alternatives. In other words, thresholds can foster better understanding of experts when solving real problems. For example, an investor may have no difference between costs 1000 and 2000 dollar. Thus, the measures of thresholds can help to experts when facing with such situations. These reasons imply that the ELECTRE III method with GIT2FNs is more reliable and more logical for GMCMDM problems.

On the other hand, to show effectiveness of the proposed ranking method, LDMs were applied to the thresholdless techniques such as TOPSIS and VIKOR (see illustrative Example 1). Based on Table 41, the results ranked by our method are similar to those of the others. It proves reliability of the proposed ranking method.

Again, the proposed ranking method was integrated with ELECTRE III in the illustrative Example 2. According to Table 48, several conclusions can be obtained as follows:

- The last four approaches (the thresholdless techniques) in this table have the similar ranking results as: $A_5 > A_1 > A_3 > A_4 > A_2$.
- Alternative 5 was selected as the optimal alternative based on the proposed method (ELECTRE III using LDM), TOPSIS with IT2FSs [29], WASPAS with IT2FSs [34], VIKOR with IT2FSs [30], and SAW with IT2FSs [33].
- The results determined by the ELECTRE I method [32] are quite different from those of the others. Alternative 4 was chosen as the most desirable alternative. It implies that the recent approach cannot be the suitable method for ranking alternatives.

Table 46 Credit degree matrix

	A ₁	A ₂	A ₃	A ₄	A ₅
A ₁	0.0000	0.4820	0.8475	0.5376	0.6719
A ₂	0.4563	0.0000	0.4494	0.7607	0.2607
A ₃	0.7853	0.0000	0.0000	0.0000	0.3381
A ₄	0.6618	0.8376	0.5001	0.0000	0.4540
A ₅	0.7054	0.7853	0.8324	0.6276	0.0000

Table 47 Net credibility matrix

	A ₁	A ₂	A ₃	A ₄	A ₅
θ_r^c	2.5389	1.9271	1.1234	2.4535	2.9507
θ_r^d	2.6088	2.1049	2.6294	1.9259	1.7247
θ_r	− 0.0699	− 0.1778	− 1.5060	0.5276	1.2260

Table 48 Ranking results of different approaches

Robots	The proposed method (ELECTRE III using <i>LDM</i>)	The ELECTRE I method [32]	TOPSIS with IT2FSs [29]	WASPAS with IT2FSs [34]	VIKOR with IT2FSs [30]	SAW with IT2FSs [33]
A ₁	3	3	2	2	2	2
A ₂	4	5	5	5	5	5
A ₃	5	4	3	3	3	3
A ₄	2	1	4	4	4	4
A ₅	1	2	1	1	1	1

Generally, it should be noted that the differences of orders between the ELECTRE III with the others can be arisen from the impact of the threshold values in the ELECTRE III method. In other words, the main reason of these differences is the lack of thresholds in the VIKOR and TOPSIS methods. Moreover, the results obtained by our approach are stable and reliable when changing the criteria weights and thresholds.

Some important directions for further studies are as follows:

1. Our methodology was implemented only in a kind of MHE (named conveyor). However, managers can apply it to the others such as cranes, forklift truck, etc.
2. Other criteria and sub-criteria may be taken into account in the other manufacturing industries or service organizations.
3. In our case study, the distance between alpha cuts was determined 0.1 when *LDMs* were calculated. To obtain more accurate results, one can adopt the smaller values for them such as 0.05 or 0.01.
4. The proposed methodology is efficient for each number of alternatives, qualitative criteria, and quantitative criteria.
5. The proposed approach is applicable to other MCDM techniques, in addition to ELECTRE.

6. The proposed method can be adopted for other mathematical branches such as data envelopment analysis (DEA) and/or statistical science where probability density functions with different curved forms may be applied.
7. The proposed method can be used to other fuzzy environments such as intuitionistic fuzzy sets, spherical fuzzy sets, neutrosophic fuzzy sets, etc.

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References

1. Sule DR (1994) Manufacturing facilities: location, planning and design, 2nd edn. PWS Publishing Company, Boston

2. Roy B, Bouyssou B (1986) Comparison of two decision-aid models applied to a nuclear power plant siting example. *Eur J Oper Res* 25:200–215
3. Zadeh L (1975) The concept of a linguistic variable and its application to approximate reasoning, Part 1'. *Inform Sci* 8:199–249
4. Niewiadomski A (2013) Cylindric extensions of interval-valued fuzzy sets in data linguistic Summaries. *J Ambient Intell Human Comput* 4:369–376
5. Pedrycz W (2010) Human centricity in computing with fuzzy sets: an interpretability quest for higher order granular constructs. *J Ambient Intell Hum Comput* 1:65–74
6. Hagrass H (2007) Type-2 FLCs: a new generation of fuzzy controllers. *IEEE Comput Intell M* 2:30–43
7. Sepulveda R, Castillo O, Melin P, Rodriguez-Diaz A, Montiel O (2007) Experimental study of intelligent controllers under uncertainty using type-1 and type-2 fuzzy logic. *Inform Sci* 177:2023–2048
8. Kumbasar T, Hagrass H (2015) A self-tuning zSlices-based general type-2 fuzzy PI controller. *IEEE Trans Fuzzy Syst* 23:991–1013
9. Lynch C, Hagrass H, Callaghan V (2006) Using uncertainty bounds in the design of an embedded real-time type-2 neuro-fuzzy speed controller for marine diesel engines. In: *Proceedings of the 2006 IEEE International Conference on Fuzzy Systems, Vancouver, Canada*, pp 7217–7224
10. Kumbasar T (2016) Robust stability analysis and systematic design of single input interval type-2 fuzzy logic controllers. *IEEE Trans Fuzzy Syst* 24:675–694
11. Castillo O, Amador-Angulo L, Castro JR, Garcia-Valdez M (2016) A comparative study of type-1 fuzzy logic systems, interval type-2 fuzzy logic systems and generalized type-2 fuzzy logic systems in control problems. *Inform Sci* 354:257–274
12. Ontiveros-Robles E, Melin P, Castillo O (2018) Comparative analysis of noise robustness of type 2 fuzzy logic controllers. *Kybernetika* 54:175–201
13. Auephanwiriyaikul S, Adrian A, Keller JM (2002) Type-2 fuzzy set analysis in management surveys. In: *Proceedings of the 2002 IEEE International Conference on Fuzzy Systems, Honolulu*, pp 1321–1325
14. Liu F, Mendel JM (2007) An interval approach to fuzzistics for interval type-2 fuzzy sets. In: *Proceedings of the 2007 IEEE International Conference on Fuzzy Systems, UK*, pp 1–6
15. Bouchachia A, Mittermeir R (2003) A neural cascade architecture for document retrieval. In: *Proceedings of the International Joint Conference Neural Networks*, vol. 3, Portland, pp 1915–1920
16. Wu H, Mendel JM (2004) Antecedent connector word models for interval type-2 fuzzy logic systems. In: *Proceedings of the 2004 IEEE International Conference on Fuzzy Systems*, vol. 2, Budapest, Hungary, pp 1099–1104
17. Castillo O, Melin P (2004) Adaptive noise cancellation using type-2 fuzzy logic and neural networks. In: *Proceedings of the 2004 IEEE International Conference on Fuzzy Systems*, vol. 2, pp 1093–1098
18. Own CM, Tsai HH, Yu PT, Lee YJ (2006) Adaptive type-2 fuzzy median filter design for removal of impulse noise. *Imaging Sci* 54:3–18
19. Melin P, Urias J, Solano D, Soto M, Lopez M, Castillo O (2006) Voice recognition with neural networks, type-2 fuzzy logic and genetic algorithms. *J Eng Lett* 13:108–116
20. Sanchez MA, Castillo O, Castro JR (2015) Generalized Type-2 Fuzzy Systems for controlling a mobile robot and a performance comparison with Interval Type-2 and Type-1 Fuzzy Systems. *Expert Syst Appl* 42:5904–5914
21. Liang Q, Wang L (2005) Sensed signal strength forecasting for wireless sensors using interval type-2 fuzzy logic systems. In: *Proceedings of the 2005 IEEE International Conference on Fuzzy Systems, Reno*, pp 25–30
22. Shu H, Liang Q (2005) Wireless sensor network lifetime analysis using interval type-2 fuzzy logic systems. In: *Proceedings of the 2005 IEEE International Conference on Fuzzy Systems, Reno*, pp 19–24
23. Baguley P, Page T, Koliza V, Maropoulos P (2006) Time to market prediction using type-2 fuzzy sets. *J Manuf Technol Manag* 17:513–520
24. Gu L, Zhang YQ (2007) Web shopping expert using new interval type-2 fuzzy reasoning. *Soft Comput* 11:741–751
25. Rhee FCH (2007) Uncertainty fuzzy clustering: insights and recommendations. *IEEE Comput Intell M* 2:44–56
26. Linda O, Manic M (2011) Interval type-2 fuzzy voter design for fault tolerant systems. *Inform Sci* 181:2933–2950
27. Cervantes L, Castillo O (2015) Type-2 fuzzy logic aggregation of multiple fuzzy controllers for airplane flight control. *Inform Sci* 324:247–256
28. Castillo O, Cervantes L, Soria J, Sanchez M, Castro JR (2016) A generalized type-2 fuzzy granular approach with applications to aerospace. *Inform Sci* 354:165–177
29. Rashid T, Beg I, Husnine SM (2014) Robot selection by using generalized interval-valued fuzzy numbers with technique for order preference by similarity to ideal solution (TOPSIS). *Appl Soft Comput* 21:462–468
30. Keshavarz Ghorabae M (2016) Developing an MCDM method for robot selection with interval type-2 fuzzy sets. *Robot Comput Integr Manuf* 37:221–232
31. Soner O, Celik E, Akyuz E (2017) Application of AHP and VIKOR methods under interval type 2 fuzzy environment in maritime transportation. *Ocean Eng* 129:107–116
32. Zhong L, Yao L (2017) An ELECTRE I-based multi-criteria group decision making method with interval type-2 fuzzy numbers and its application to supplier selection. *Appl Soft Comput* 57:556–576
33. Abdullah L, Adawiyah CWR, Kamal CW (2018) A decision making method based on interval type-2 fuzzy sets: an approach for ambulance location preference. *Appl Comput Inform* 14:65–72
34. Deveci M, Camtez F, Gökaşar I (2018) WASPAS and TOPSIS based interval type-2 fuzzy MCDM method for a selection of a car sharing station. *Sustain Cities Soc* 41:777–791
35. Hameed IA (2011) Using Gaussian membership functions for improving the reliability and robustness of students' evaluation systems. *Expert Syst Appl* 38:7135–7142
36. Ray PK, Maiti J (2018) Ergonomic design of products and work systems—21st century perspectives of Asia. Springer, Singapore
37. Mardani A, Jusoh A, MD-Nor K, Khalifah Z, Zakwan N, Valipour A, (2015) Multiple criteria decision-making techniques and their applications—a review of the literature from 2000 to 2014. *Econ Res* 28:516–571
38. Mardani A, Jusoh A, Zavadskas EK (2015) Fuzzy multiple criteria decision-making techniques and applications—two decades review from 1994 to 2014. *Expert Syst Appl* 42:4126–4148
39. Roy B (1991) The outranking approach and the foundation of ELECTRE methods. *Theo Decis* 31:49–73
40. Rogers M, Bruen M, Maystre LY (2013) Electre and decision support: methods and applications in engineering and infrastructure investment. Springer Science, Business Media
41. Siskos J, Hubert P (1988) Multi-criteria analysis of the impacts of energy alternatives: a survey and a new approach. *Eur J Oper Res* 13:278–299
42. Karagiannidis A, Moussiopoulos N (1997) Application of ELECTRE III for the integrated management of municipal solid wastes in the Greater Athens Area. In: Fandel G., Gal T. (eds) *Multiple criteria decision making. Lecture notes in economics and mathematical systems*, vol 448, Springer, Berlin, Heidelberg, pp 568–578

43. Alomoush M (2009) Multicriteria optimal location of TCPAR in a competitive energy market using ELECTRE III. Universities Power Engineering Conference (UPEC). In: Proceedings of the 44th International, IEEE, pp 1–5
44. Montazer GA, Qahri Saremi H, Ramezani M (2009) Design a new mixed expert decision aiding system using fuzzy ELECTRE III method for vendor selection. *Expert Syst Appl* 36:10837–10847
45. Rogers M (2000) Using ELECTRE III to aid the choice of housing construction process within structural engineering. *Constr Manage Econ* 18:333–342
46. Tam CM, Tong TKL, Lau CT (2003) ELECTRE III in evaluating performance of construction plants: case study on concrete vibrators. *Constr Innov* 3:45–61
47. Leyva Lopez JC (2005) Multi criteria decision and application to a student selection problem. *Pesquisa Operaciona* 25:45–68
48. Li HF, Wang JJ (2007) An improved ranking method for ELECTREIII. In: International conference on wireless communications, networking and mobile computing
49. Zak J, Fierek S (2007) Design and evaluation of alternative solutions for an integrated urban transportation system. In: 11th World Conference on Transport Research
50. Papadopoulos A, Karagiannidis A (2008) Application of the multi-criteria analysis method ELECTRE III for the optimization of decentralized energy systems. *Omega* 36:766–776
51. Radziszewska-Zielina E (2010) Methods for selecting the best partner construction enterprise in terms of partnering relations. *J Civ Eng Manag* 16:510–520
52. Giannoulis C, Ishizaka A (2010) A Web-based decision support system with ELECTRE III for a personalized ranking of British universities. *Decis Support Syst* 48:488–497
53. Cavallaro F (2010) A comparative assessment of thin-film photovoltaic production processes using the ELECTRE III method. *Energ Pol* 38:463–474
54. Marzouk MM (2011) ELECTRE III model for value engineering applications. *Autom Constr* 20:596–600
55. Liu P, Zhang X (2011) Research on the supplier selection of a supply chain based on entropy weight and improved ELECTRE-III method. *Int J Prod Res* 49:637–646
56. Cliville V, Berrah L, Mauris G (2013) Deploying the ELECTRE III and MACBETH multicriteria ranking methods for SMEs tactical performance improvements. *J Modell Manage* 8:348–370
57. Fancello G, Carta M, Fadda P (2014) A decision support system based on Electre III for safety analysis in a suburban road network. In: 17th Meeting of the EURO Working Group on Transportation, EWGT2014, 2–4 July 2014, Sevilla, Spain, Transportation Research Procedia, vol. 3, pp 175–184
58. Heracles P, Kyriaki H, Dias H (2015) Multi-criteria decision analysis for geothermal energy: a comparison between the ELECTRE III and the PROMETHEE II methods. *Energ Sources Part B Econ Plan Pol* 10:241–249
59. Shafia MA, Moghaddam MR, Teimoury E (2016) Ranking Fuzzy Cognitive Map based scenarios using ELECTRE III: applied on housing market. *Expert Syst* 33:417–431
60. Vahdani B, Hadipour H (2011) Extension of the ELECTRE method based on interval-valued fuzzy sets. *Soft Comput* 15:569–579
61. Hatami-Marbini A, Tavana M (2011) An extension of the ELECTRE I method for group decision-making under a fuzzy environment. *Omega* 39:373–386
62. Vahdani B, Mousavi SM, Tavakkoli-Moghaddam R, Hashemi H (2013) A new design of the elimination and choice translating reality method for multi criteria group decision-making in an intuitionistic fuzzy environment. *Appl Math Model* 37:1781–1799
63. Chen TY (2015) An IVIF-ELECTRE outranking method for multiple criteria decision-making with interval-valued intuitionistic fuzzy sets. *Technol Econ Dev Econ* 22:416–452
64. Hashemi SS, Razavi-Hajiagha SH, Zavadskas EK, Mahdiraji HA (2015) Multicriteria group decision making with ELECTRE III method based on Interval-valued intuitionistic fuzzy information. *Appl Math Model* 40:1554–1564
65. Wang JQ, Wang DD, Zhang HY, Chen XH (2014) Multi-criteria outranking approach with hesitant fuzzy sets. *OR Spectr* 36:1001–1019
66. Juan-Juan P, Jian-qiang W, Jing W, Li-Jun Y, Xiao-Hong C (2015) An extension of ELECTRE to multi-criteria decision-making problems with multi hesitant fuzzy sets. *Inf Sci* 307:113–126
67. Na C, Zeshui X, Meimei X (2015) The ELECTRE I multi-criteria decision-making method based on hesitant fuzzy sets. *Int J Inf Technol Decis Mak* 14:621–657
68. Kabuka M, Genaidyt AM, Asfour SS (1988) A knowledge-based system for the design of manual materials handling. *Appl Ergon* 19:147–155
69. Hosni YA (1989) Inference engine for material handling selection. *Comput Ind Eng* 17:79–84
70. Matson JO, Mellichamp JM, Swaminathan SR (1992) EXCITE: expert consultant for in-plant transportation equipment. *Int J Prod Res* 30:1969–1983
71. Fonseca DJ, Uppal G, Greene TJ (2004) A knowledge-based system for conveyor equipment selection. *Expert Syst Appl* 26:615–623
72. Park YB (1996) ICMES: intelligent consultant system for material handling equipment selection and evaluation. *J Manuf Syst* 15:325–336
73. Kulak O (2005) A decision support system for fuzzy multi-attribute selection of material handling equipment. *Expert Syst Appl* 29:310–319
74. Hamid S, Mirhosseini L, Webb P (2009) A hybrid fuzzy knowledge-based expert system and genetic algorithm for efficient selection and assignment of material handling equipment. *Expert Syst Appl* 36:11875–11887
75. Ioannou G, Kritikos MN (2004) Optimization of material handling in production and warehousing facilities. *Oper Res* 4:317–331
76. Ioannou G (2007) An integrated model and a decomposition-based approach for concurrent layout and material handling system design. *Comput Ind Eng* 52:459–485
77. Sujono S, Lashkari RS (2007) A multi-objective model of operation allocation and material handling system selection in FMS design. *Int J Prod Econ* 105:116–133
78. Poon TC, Choy KL, Cheng CK, Lao SI, Lam HY (2011) Effective selection and allocation of material handling equipment for stochastic production material demand problems using genetic algorithm. *Expert Syst Appl* 38:12497–12505
79. Zhang Y, Huang K, Zhu W (2013) Model of the equipment selection of material handling system for large ship. *Appl Mech Mater* 347:2865–2868
80. Khandekar AV, Chakraborty S (2015) Selection of material handling equipment using fuzzy axiomatic design principles. *Inf* 26:259–282
81. Pourahmad A, Hossinin A, Banaitis A, Nasiri H, Banaitiene N, Tzeng GH (2015) Combination of fuzzy-AHP and DEMATEL-ANP with GIS in a new hybrid MCDM model used for the selection of the best space for leisure in a blighted urban site. *Technol Econ Dev Econ* 21:773–796
82. Nguyen HT, Md Dawal SZ, Nukman YP, Rifai A, Aoyama H (2016) An integrated MCDM model for conveyor equipment evaluation and selection in an FMC based on a fuzzy AHP and fuzzy ARAS in the presence of vagueness. *PLoS One* 11:e01553222
83. Bekar ET, Cakmakci M, Kahraman C (2016) Fuzzy COPRAS method for performance measurement in total productive maintenance: a comparative analysis. *J Bus Econ Manag* 17:663–684
84. Barrios MAO, Felice FD, Negrete KP, Romero BA, Arenas AY, Petrillo A (2016) An AHP-TOPSIS integrated model for selecting

- the most appropriate tomography equipment. *Int J Info Tech Dec Mak* 15:861–885
85. Ceresca A, Podvezko V, Zavadskas EK (2016) Operating characteristics analysis of Rotor systems using MCDM methods. *Stud Inform Control* 25:59–68
 86. Saputro TE, Daneshvar-Rouyendegh B (2016) A hybrid approach for selecting material handling equipment in a warehouse. *Int J Manag Sci Eng Manag* 11:34–48
 87. Sen K, Ghosh S, Sarkar B (2017) Comparison of customer preference for bulk material handling equipment through fuzzy-AHP approach. *J Inst Eng India Ser C* 98:367–377
 88. Mathew M, Sahu S (2018) Comparison of new multi-criteria decision making methods for material handling equipment selection. *Manag Sci Lett* 8:139–150
 89. Hadi-Vencheh A, Mohamadghasemi A (2015) A new hybrid fuzzy multi-criteria decision making model for solving the material handling equipment selection problem. *Int J Comp Integ M* 28:534–550
 90. Benayoun R, Roy B, Sussman N (1966) *Manual de reference du programme electre*. Note de synthese et formation, no. 25, Paris, France
 91. Roy B (1968) Classement et Choix en Presence de Points de vue Multiples (la method Electre). *Revue Francaise d'Informatique et de Recherche Operationnelle* 8:57–75
 92. Roy B (1971) Problems and methods with multiple objective functions. *Math Prog* 1:239–266
 93. Rogers M, Bruen M (1998) A new system for weighting environmental criteria for use within ELECTRE III. *Eur J Oper Res* 107:552–563
 94. Roy B, Hugonnard J (1982) Classement des prolongements de lignes de metro en banlieue parisienne (presentation d'une methode multicritere originale). *Cahiers Du CERO* 24:153–171
 95. Roy B, Skalka J (1984) ELECTRE IS: aspects methodologiques et guide d'utilisation. Document du LAMSADE 30, Universite Paris Dauphine
 96. Yu W (1992) ELECTRE TRI: aspects methodologiques et manuel d'utilisation. Document du LAMSADE 74, Universite Paris-Dauphine
 97. Greco S, Kadzinski M, Mousseau V, Slowinski R (2011) ELECTREGKMS: robust ordinal regression for outranking methods. *Eur J Oper Res* 214:118–135
 98. Almeida-Dias J, Figueira JR, Roy B (2010) ELECTRE TRI-C: a multiple criteria sorting method based on characteristic reference actions. *Eur J Oper Res* 204:565–580
 99. Almeida-Dias J, Figueira JR, Roy B (2012) A multiple criteria sorting method where each category is characterized by several reference actions: the ELECTRE TRI-NC method. *Eur J Oper Res* 217:567–579
 100. Buchanan J, Sheppard P, Vanderpooten D (1999) Project ranking using ELECTRE III. Research report 99-01, University of Waikato, Hamilton, New Zealand
 101. Chen SM, Lee LW (2010) Fuzzy multiple attributes group decision-making based on the ranking values and the arithmetic operations of interval type-2 fuzzy sets. *Expert Syst Appl* 37:824–833
 102. Mendel JM (2001) *Uncertain rule-based fuzzy logic systems: introduction and new directions*. Prentice Hall, Upper Saddle River
 103. Liang Q, Mendel JM (2000) Interval type-2 fuzzy logic systems: theory and design. *IEEE Trans Fuzzy Syst* 8:535–550
 104. Tahayori H, Tettamanzi A, Antoni GD (2006) Approximated type-2 fuzzy set operations. In: *Proceedings of FUZZ-IEEE 2006*, Vancouver, Canada, 9042–9049
 105. Chen SM, Wang CY (2013) Fuzzy decision making systems based on interval type-2 fuzzy sets. *Inform Sci* 242:1–21
 106. Moore RE (1979) *Methods and applications of interval analysis*. SIAM, Philadelphia
 107. Kaufmann A, Gupta MM (1988) *Fuzzy mathematical models in engineering and management science*. North Holland, Amsterdam
 108. Singh S, Olugu EU, Musa SN (2016) Strategy selection for sustainable manufacturing with integrated AHP-VIKOR method under interval-valued fuzzy environment. *Int J Adv Manuf Tech* 84:547–563
 109. Hwang CL, Yoon KP (1981) *Multiple attribute decision making: methods and applications*. Springer, New York

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