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ORIGINAL ARTICLE

Decision making method based on Pythagorean fuzzy sets and its application to solid waste management

Lazim Abdullah[1] · **Pinxin Goh**[1]

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Abstract

The fuzzy decision-making trial and evaluation laboratory (fuzzy DEMATEL) has been used to solve various multi-criteria group decision-making problems where triangular fuzzy numbers are utilized in defining decision makers' linguistic judgements. Most of the fuzzy DEMATEL modifications are built from linguistic variables based on fuzzy sets. Recent literature suggests that Pythagorean fuzzy sets (PFS) can offer a better alternative particularly when fuzzy sets have some extent of limitations in handling vagueness and uncertainty. This paper proposes a modification fuzzy DEMATEL characterized by PFS for linguistic variables. Differently from the typical fuzzy DEMATEL which directly utilizes triangular fuzzy numbers with a single membership, this modification introduces membership and non-membership of PFS to enhance judgements in the group decision-making environment. The proposed method has a number of attractive features. It includes linguistic variables, expert's weights, and score function, in which all of these features are expressed by PFS. The proposed modification is applied to a case of solid waste management (SWM) where ten criteria are considered in assessment. Six experts in SWM were invited to provide linguistic judgments with respect to the criteria, and the eleven-step computational procedure of the proposed method was implemented without losing the general structure of the DEMATEL method. The results unveiled that four criteria are identified as 'cause group' and six criteria are identified as 'effect group' in SWM. The grouping of criteria would help policy makers in identifying the criteria that could enhance the efficiency of SWM.

Keywords Pythagorean fuzzy set · DEMATEL · Causal diagram · Decision making · Solid waste management

Introduction

The decision-making trial and evaluation laboratory (DEMATEL) is one of the multi-criteria decision making (MCDM) methods that purposely used for building and analysing a structural model. Developing the causal relationships between complex criteria in MCDM problems are the ultimate aim behind the structural model. According to Chang et al. [8] the DEMATEL was developed to explore and solve complex and interrelated criteria in groups of MCDM problems. Fragmented and antagonistic phenomena of MCDM problems are solved using the DEMATEL in an integrated manner. The DEMATEL method can convert the relationship between the causes and effects of criteria in MCDM into an intelligible structural mode. The strength of DEMATEL method lies on the applications of decision matrices and digraphs to consider the structure of complicated causal relationships [24]. The DEMATEL is particularly practical and useful for visualizing the structure of complicated causal relationships between criteria using digraphs. Using the DEMATEL, a visual representation is finally constructed to unravel the relationships between criteria of MCDM problems. In addition, the DEMATEL method can improve understanding of the specific problem antique, the cluster of intertwined problems, and contribute to identification of workable solutions by a hierarchical structure [33]. The causal diagram uses digraphs rather than directionless graphs to portray the basic concept of contextual relationships and the strengths of influence among the elements. Owing to these advantages, the DAMATEL has been applied in many recent MCDM problems (see [10, 21, 34]).

The DEMATEL also has been successfully combined with fuzzy sets as to handle the uncertainties and vagueness in MCDM problems, and also incomplete information about

✉ Lazim Abdullah
lazim_m@umt.edu.my

1 Management Science Research Group, School of Informatics and Applied Mathematics, Universiti Malaysia Terengganu, 21030 Kuala Terengganu, Malaysia

the data of the MCDM problems. The concept of fuzzy set theory was firstly proposed by Zadeh [46] and since then, the sets have been wisely fused into many MCDM methods with no exceptional of the DEMATEL. Likewise the DEMATEL, the fuzzy DEMATEL also has been applied in many MCDM problems in diverse areas. In supply chain research, for example, the fuzzy DEMATEL method was made to identify the critical success criteria [7, 25]. The DEMATEL was applied in marketing resources [3], knowledge management [29], and information support management [23]. Very recently, Chakraborty, et al. [6] developed a causal model to evaluate the critical issues in reverse supply chain implementation using fuzzy DEMATEL. Also in business related research, Mavi and Standing [26] analysed cause and effect of business intelligence benefits with fuzzy DEMATEL. As an extension to fuzzy sets, intuitionistic fuzzy set (IFS) was proposed by Atanassov [4] as to handle the issues of membership, non-memberships and hesitation degrees of decision problems. The IFS was intended to be an extension of single membership of fuzzy set theory. With the ultimate aim to solve the relationships between criteria in MCDM, the fusion of IFS and the DEMATEL was proposed (see [28, 32]). This fusion is normally written as intuitionistic fuzzy DEMATEL (IF-DEMATEL), and has been recently utilised by Zhou et al. [49]. The IF-DEMATEL method is a potent method that helps in gathering group knowledge for forming a structural model under uncertain and incomplete information. The IF-DEMATEL is introduced to represent the correlation among criteria in an intuitionistic fuzzy environment. The proposed IF-DEMATEL uses triangular intuitionistic fuzzy numbers to find weights [32]. In real case applications, the IF-DEMATEL method was used in prioritising the components in insurance industry [28], developed green practices and performances in a green supply chain [22], ranking the risk of construction projects [39], risk analysis of coal combustion [47]. The vagueness of human's subjective judgments are conquered by embracing the two memberships in judgements of the IF- DEMATEL.

However, the two memberships of IFS have some limitations particularly on the arithmetic addition of two memberships and also hesitation degree. To improve hesitation degree, author such as Zeng et al. [47, 48] proposed interval-valued hesitant fuzzy sets and its arithmetic operations. For the two memberships of IFS, the focal point is on its arithmetic addition. It is known that the sum of two memberships of IFS is limited to one. In response to this limitation, Yager [45] introduced Pythagorean fuzzy set (PFS) where the limitation in IFS has been modified. The sum of two memberships in IFS is now substituted with squares of each membership where the sum of these two squares is less or equal to one. In other words, the PFS is characterized by a membership degree and non-membership degree where the square sum of its membership degree and non-membership degree is less than or equal to one. The PFS is one of the most successful sets, in terms of representing comprehensively uncertain and vague information [11]. In this regards, the PFS has been fused to the many MCDM methods particularly in aggregation methods. For example, the new aggregation operators were proposed by combining PFS-Choquet–Frank aggregation operators [44], PFS-Einstein aggregation operator [14, 31], PFS-averaging and geometric operators with logarithmic laws [19], PFS-geometric-Einstein operators [20]. The PFS also was successfully integrated with the concepts of confidence level [15], and decision making with probabilities [17]. The new linguistic and exponential operational laws with PFS were also proposed [16, 18]. About similar with other sets, the PFS was also extended to interval-valued PFS and hesitant PFS. These two sets were successfully used in developing new aggregation operators such as Maclaurin Symmetric Mean Operator [13, 43], averaging and geometric aggregation operators [12].

The PFS fusion based models also has been applied in many real life problems. For example, a three-phase PFS-MCDM method has been applied to haze management [40]. Also very recently, the PFS based analysis model has been applied in solar power plants [9]. The PFS-analytic hierarchy process and PFS-similarity measures were proposed to solve MCDM problems [41, 42]. It can be seen that there were handful of research applied MCDM methods based on PFS. Moreover, so far, there has been little discussion about the applications of DEMATEL based on PFS to solid waste management (SWM). To bridge the literature gap between the DEMATEL based on PFS and other MCDM methods, this paper proposes a modified DEMATEL based on PFS and applies the proposed method to the case of SWM. Differently from the DEMATEL, the proposed method introduces new linguistic variable of influence, experts' weights and score function based on PFS. In short, the objective of this research is to propose the DEMATEL method based on PFS and apply it to the case of SWM. This paper is organised as follows. "Preliminary" presents the definition that related to PFS. The proposed work is presented in "Proposed method". "Empirical case to construct causal diagram" provides the application of the proposed method to the case of SWM. Finally, "Conclusions" concludes.

Preliminary

This section recalls the definitions of PFS and some its algebraic operations. As a basis to the proposed PFS-DEMATEL, the section also provides the basis of compuational steps in the DEMATEL.

Definition 1 PFS [45].

A PFS P in a finite universe of discourse is

$$P = \{< x, \mu_P(x), \upsilon_P(x) > | x \in X\} >$$

where $\mu_P, \upsilon_P : X \rightarrow [0, 1]$ with the condition that the square sum of its membership degree and non-membership degree is less than or equal to 1.

$$[\mu_P(x)]^2 + [\upsilon_P(x)]^2 \leq 1.$$

Definition 2 Degree of Indeterminacy [30].

The degree of indeterminacy of x to P is given by $\pi_P(x) = \sqrt{1 - \mu_P^2(x) - \upsilon_P^2(x)}$. If $B = P(\mu_B, \upsilon_B)$ is a Pythagorean fuzzy number then, degree of indeterminacy of B is given as $\pi_B = \sqrt{1 - \mu_B^2 - \upsilon_B^2}$ where $\mu_B, \upsilon_B \in [0, 1]$ and $(\mu_B)^2 + (\upsilon_B)^2 \leq 1$.

Definition 3 Algebraic Operations of Pythagorean fuzzy number (PFN).

Given two PFNs, $A = P(\mu_A, \upsilon_A)$ and $B = P(\mu_B, \upsilon_B)$, where $\mu_B, \upsilon_B \in [0, 1]$,$\mu_A, \upsilon_a \in [0, 1]$, then some arithmetic operations can be described as follows:

1. $A \cup B = P(\max\{\mu_A, \mu_B\}, \min\{\upsilon_A, \upsilon_B\})$
2. $A \cap B = P(\min\{\mu_A, \mu_B\}, \max\{\upsilon_A, \upsilon_B\})$
3. $A^C = P(\upsilon_A, \mu_A)$
4. $A \oplus B = P\left(\sqrt{\mu_A^2 + \mu_B^2 - \mu_A^2\mu_B^2}, \upsilon_A\upsilon_B\right)$
5. $A \otimes B = P\left(\mu_A\mu_B, \sqrt{\upsilon_A^2 + \upsilon_B^2 - \upsilon_A^2\upsilon_B^2}\right)$
6. $\lambda A = P\left(\sqrt{1 - (1 - \mu_A^2)^\lambda}, (\upsilon_A)^\lambda\right), \lambda > 0$
7. $A^\lambda = P\left((\mu_A)^\lambda, \sqrt{1 - (1 - \upsilon_A^2)^\lambda}\right), \lambda > 0$
8. $\lambda(A \oplus B) = \lambda A \oplus \lambda B, \lambda > 0$

These definitions are relevant and necessitated in the proposed method.

Proposed method

Most studies in the field of decision making, have only focused on specific methods of MCDM such as the DEMATEL method and combined with fuzzy sets and its extensions. For example, the DEMATEL method has been combined with interval type-2 fuzzy sets [1, 5, 27]. To the best of authors knowledge, previous studies of DEMATEL have not been combined with PFS. In this paper, the proposed method is a fusion of the PFS and the DEMATEL method. The PFS is used as a linguistic judgment and substitute it into the decision-making method DEMATEL. In this section, we provide acronym for this combination as PF-DEMATEL and will be used throughout this text. In other words, the PF-DEMATEL is a decision-making method that worked in the framework of DEMATEL with the use of PFS in linguistic judgement. It is anticipated that the proposed PF-DEMATEL method is a potent method that helps in gathering group knowledge for developing a structural model under Pythagorean fuzzy condition. The proposed framework is divided into three phases. In phase 1, a new linguistic variable based on PFS is developed. The integration of PFS with DEMATEL is made in the phase 2. Multiplication with the weights of DMs, aggregation operators and defuzzification are the main mathematical operations in this phase. Finally, the causal diagram is illustrated is phase 3. Figure 1 presents a schematic representation of the proposed method.

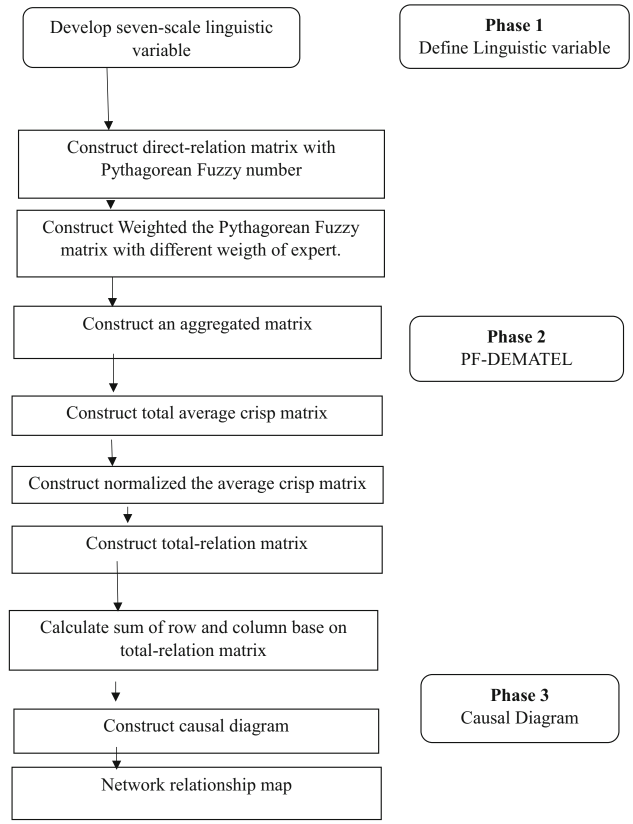

Fig. 1 Schematic representation of the PF-DEMATEL

Based on the framework, the algorithm of PF-DEMATEL is proposed as follows.

Step 1: Define linguistic variables.

Table 1 Linguistic variable in Pythagorean fuzzy sets

Linguistic terms	Rating Scale in crisp number	Rating scale in Pythagorean fuzzy sets
Very low influence	0	⟨0, 0⟩
Low influence	1	⟨0.1, 0.9⟩
Medium low influence	2	⟨0.2, 0.9⟩
Medium influence	3	⟨0.4, 0.6⟩
Medium high influence	4	⟨0.5, 0.7⟩
High influence	5	⟨0.7, 0.2⟩
Very high influence	6	⟨0.9, 0.1⟩

With the linguistic variable 'influence', seven linguistic terms are defined based on the rating scales of DEMATEL. The pair-wise comparison scales are made in seven terms, where the scores of 0, 1, 2, 3, 4, 5 and 6 represent "*Very low influence*", "*Low influence*", "*Medium low influence*", "*Medium influence*", "*Medium high influence*", "*High influence*", and "*Very high influence*", respectively. These scales are now being introduced in PFS instead of crisp number Table 1 shows the linguistic terms used in judgment and its respective scales in PFS.

Step 2: Obtain a $n \times n$ matrix as initial direct-relation matrix Z by pair-wise comparisons in terms of influences and directions between criteria, in which is denoted as the degree to which the criterion i affects the criterion j, i.e., $Z = \left[z_{ij}\right]_{n\times n}$. The value of Z_{ij} is written in PFS.

$$Z_m = \left[z_{ij}\right]_{n\times n} = \begin{matrix} \\ C_1 \\ \vdots \\ C_n \end{matrix}\begin{bmatrix} C_1 & \ldots\ C_n \\ \langle 0,0\rangle & \cdots \left\langle \mu_{m_{1n}},\ \upsilon_{m_{1n}}\right\rangle \\ \vdots & \ddots\ \vdots \\ \left\langle \mu_{m_{n1}},\ \upsilon_{m_{n1}}\right\rangle & \cdots\ \langle 0,0\rangle \end{bmatrix}. \tag{1}$$

Step 3: Calculate the weighted initial direct-relation matrix,

$$\lambda_m Z_m = \begin{bmatrix} \lambda_m\langle 0,0\rangle & \ldots & \lambda_m P_{1n} \\ \vdots & \ddots & \vdots \\ \lambda_m P_{n1} & \cdots & \lambda_m\langle 0,0\rangle \end{bmatrix} \tag{2}$$

where $\lambda_m P_{ij} = \left\langle \sqrt{1-\left(1-\mu_{ij}^2\right)^{\lambda}}, \left(\upsilon_{ij}\right)^{\lambda} \right\rangle$ is the weighted PFS element.

Step 4: Calculate the aggregated matrix using addition operation

$$\lambda_{m_1} Z_{m_1} \oplus \lambda_{m_2} Z_{m_2} = \begin{bmatrix} \lambda_{m_1} P_{Z_{m_1},11} \oplus \lambda_{m_2} P_{Z_{m_2},11} & \ldots & \lambda_{m_1} P_{Z_{m_1},1n} \oplus \lambda_{m_2} P_{Z_{m_2},1n} \\ \vdots & \ddots & \vdots \\ \lambda_{m_1} P_{Z_{m_1},n1} \oplus \lambda_{m_2} P_{Z_{m_2},n1} & \cdots & \lambda_{m_1} P_{Z_{m_1},nn} \oplus \lambda_{m_2} P_{Z_{m_2},nn} \end{bmatrix} \tag{3}$$

where,

$$\lambda_{m_1} P_{Z_{m_1},ij} \oplus \lambda_{m_2} P_{Z_{m_2},ij} = \left\langle \sqrt{\mu^2_{\lambda_{m_1} Z_{m_1}} + \mu^2_{\lambda_{m_2} Z_{m_2}} - \mu^2_{\lambda_{m_1} Z_{m_1}} \mu^2_{\lambda_{m_2} Z_{m_2}}}, \upsilon_{\lambda_{m_1} Z_{m_1}} \upsilon_{\lambda_{m_2} Z_{m_2}} \right\rangle$$

Step 5: Construct total average crisp matrix by using score function as a defuzzification function.

$$A = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \tag{4}$$

where $a_{ij} = \mu^2_{P_z,ij} - \upsilon^2_{P_z,ij}$

Step 6: Construct the normalized average crisp matrix X.

The crisp matrix $X = x_{ij}$ where, $0 \le x_{ij} \le 1$ are obtained using the equation

$$X = s \cdot A \tag{5}$$

where $s = \frac{1}{\max_{1\le i\le n} \sum_{j=1}^{n} a_{ij}}$ $\quad i, j = 1, 2, \cdots, n.$

Step 7: Construct the total-relation matrix T using the Eq. (6).

$$T = X(I - X)^{-1} \tag{6}$$

where I is the identity matrix.

Step 8: The sum of rows and the sum of columns are separately denoted as D and R within the total-relation matrix T through $T = t_{ij}$, where $i, j = 1, 2, \ldots, n$.

Sum of row,

$$r = \sum_{j=1}^{n} t_{ij}, \tag{7}$$

Sum of Column,

$$c = \sum_{i=1}^{n} t_{ij} \tag{8}$$

Step 9: Draw a causal diagram.

Table 2 Personal profiles of experts

Expert	Designation	Company	Experience (years)
E_1	Chief Executive Officer	Eco-ideal Consulting Sdn. Bhd.	5–9
E_2	General Manager	Vision Waste Disposal Sdn. Bhd	15–20
E_3	Operation Manager	WilGreen Recovery Sdn. Bhd	5–9
E_4	Operation Manager	Dynasty Recycling Disposal Services	
E_5	Associate Professor	School of Ocean Enginering of a public universiy	10–12
E_6	Professor	School of Ocean Enginering of a public universiy	25–30

A causal diagram is obtained by mapping the dataset of $(r+c,\ r-c)$, where the horizontal axis $(r+c)$ is made by adding r to c, and the vertical axis $(r-c)$ is made by subtracting r from c.

If $(r-c)$ is positive, then the criteria is under the cause category.

If $(r-c)$ negative, then the criteria is under the effect category.

The nine-step proposed algorithm is implemented to a case of SWM where two causal categories of criteria is determined.

Empirical case to construct causal diagram

One of the most significant current discussions in sustainability and environmental studies is solid waste management (SWM). Recent status of solid waste generation, trends and regulations was comprehensively reviewed by the researcher [37]. The relationship between SWM and fuzzy decision making was discussed in a study [36]. The fuzzy analytic hierarchy process were combined with the fuzzy ideal solution to achieve the integrated SWM decision. This current research mainly discusses about the evaluation of SWM by using the proposed PF-DEMATEL method. Specifically, the experiment attempts to visualise the cause criteria and effect criteria of SWM in Malaysia.

Table 3 Description of criteria

Criteria	Description
Relative Cost (C_1)	Financial and economics will influence the solid waste management. The amount of money provide by the government to manage the solid waste also depends on the financial. The more the money provide, the solid waste management system can be improve more
Environmental Health (C_2)	Environmental health always effect the human health. Government need to take more consider during the process of design solid waste management system
Socio-culture (C_3)	People engage in open dumping on reservations for many reasons that are not directly related to culture. The culture of dirt is a bad culture influence solid waste management so much
Public Awareness (C_4)	Public awareness is the public's level of understanding about the importance and implications of women's and girls' safety in cities and communities. High public awareness occurs when an issue that is of great importance to all citizens happen. Malaysia citizens have low public awareness in solid waste management which make the challenges of designing the system increase
Institutional (C_5)	The responsibilities, authority and revenues between national, regional and local governments must be determined to succeed the waste management programs in the city. The role of government institutions needs to shift from service provision to regulation
Technical (C_6)	The solid waste management is said badly influenced if the technology used in the system are choose wrongly. Wrong technology chosen will also cause the wrong ways used to manage solid waste. This can cause more pollution or produces more waste during the process of solid waste management
Operation & Maintenance Challenges (C_7)	In solid waste management system, operation and maintenance challenges will influence the design of the system and also the outcome
Population Size (C_8)	Population size is the actual number of individuals in a population. It will influence the design of solid waste management a lot. The population size is always concern by the solid waste management system designer in everywhere
Human Health (C_9)	Human health is an importance issue in solid waste management system. Climate change, together with other natural and human-made health stressors, influences human health and disease in numerous ways
Consumption Habits (C_{10})	Consumption habits of citizen always influence the amount of solid waste produce. However, most of Malaysia citizens have bad consumption habits which increase the amount of solid waste. Bad attitude of citizen will deteriorate worsen the waste management because they do not have the awareness about their own health and the effect to the environment

Table 4 Initial direct matrix (E_1)

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$
C_2	$\langle 0.5, 0.5\rangle$	$\langle 0, 0\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.2, 0.9\rangle$
C_3	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.9, 0.1\rangle$
C_4	$\langle 0.1, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$
C_5	$\langle 0.7, 0.2\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0, 0\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.5, 0.5\rangle$
C_6	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.2, 0.9\rangle$
C_7	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.1, 0.9\rangle$
C_8	$\langle 0.4, 0.6\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.1, 0.9\rangle$
C_9	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.2, 0.9\rangle$
C_{10}	$\langle 0.2, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$

Experts and criteria

The evaluation criteria that influence solid waste management are retrieved from literature while the weight and priority of the criteria are provided by a group of experts in the field that related to solid waste management. Personal profiles of the experts are given in Table 2.

Interviews with experts were conducted to collect linguistic evaluation. The interviews mainly aimed at obtaining a pair-wise comparison between criteria based on the developed linguistic terms (see Table 1). The criteria selected for this study are Relative Cost (C_1), Environmental Health (C_2), Socio-culture (C3), Public Awareness(C4), Institutional(C5), Technical (C6), Operation and Maintenance Challenges (C7), Population Size (C8), Human Health (C9), and Consumption Habits (C10). A brief description about the criteria are summarised in Table 3.

The linguistic evaluations provided by experts are then computed using the proposed PF-DEMATEL method (see "Proposed method").

Computation

Computations are executed in accordance with the proposed method (see "Proposed method"). Linguistic judgements are transformed into a 10 × 10 matrix as initial direct-relation matrix Z by pair-wise comparisons in the linguistic terms of influences between criteria. The elements of matrix Z represent the degree to which the criterion i affects the criterion j. Table 4. presents the initial direct matrix, $Z_m = [z_{ij}]_{10\times 10}$ obtained from E_1 (see Eq. (1)).

Table 5 Experts weight

Experts	Experts weight score	Experts weight, λ_m
E_1	0.35	0.0897
E_2	0.40	0.1026
E_3	0.80	0.2051
E_4	0.75	0.1923
E_5	0.75	0.1923
E_6	0.85	0.2179

The five other similar matrices are obtained from E_2, E_3, and E_4 and E_6. Details of the matrices are shown in "Appendix".

In this empirical case, the weights of experts λ_m is determined using the following equation.

Expert weight, $\lambda_m = \frac{\text{Expert weight score}}{\text{Sum of weight score}}$.

The weights of experts are determined and summarised in Table 5.

The weights of experts are multiplied with the initial direct matrix using Eq. (2). The weighted initial direct matrix is

Table 6 Weighted initial direct matrix (E_1)

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	⟨0.0000, 0.0000⟩	⟨0.1596, 0.9397⟩	⟨0.1246, 0.9552⟩	⟨0.1596, 0.9397⟩	⟨0.1246, 0.9552⟩	⟨0.1246, 0.9552⟩	⟨0.3721, 0.8133⟩	⟨0.0300, 0.9906⟩	⟨0.3721, 0.8133⟩	⟨0.1596, 0.9397⟩
C_2	⟨0.1596, 0.9397⟩	⟨0.0000, 0.0000⟩	⟨0.0605, 0.9906⟩	⟨0.2422, 0.8655⟩	⟨0.2422, 0.8655⟩	⟨0.1596, 0.9397⟩	⟨0.3721, 0.8133⟩	⟨0.2422, 0.8655⟩	⟨0.3721, 0.8133⟩	⟨0.0605, 0.9906⟩
C_3	⟨0.2422, 0.8655⟩	⟨0.2422, 0.8655⟩	⟨0.0000, 0.0000⟩	⟨0.1596, 0.9397⟩	⟨0.1596, 0.9397⟩	⟨0.1596, 0.9397⟩	⟨0.2422, 0.8655⟩	⟨0.0300, 0.9906⟩	⟨0.1246, 0.9552⟩	⟨0.3721, 0.8133⟩
C_4	⟨0.0300, 0.9906⟩	⟨0.3721, 0.8133⟩	⟨0.1596, 0.9397⟩	⟨0.0000, 0.0000⟩	⟨0.1596, 0.9397⟩	⟨0.1246, 0.9552⟩	⟨0.1596, 0.9397⟩	⟨0.0300, 0.9906⟩	⟨0.1596, 0.9397⟩	⟨0.3721, 0.8133⟩
C_5	⟨0.2422, 0.8655⟩	⟨0.1246, 0.9552⟩	⟨0.1246, 0.9552⟩	⟨0.2422, 0.8655⟩	⟨0.0000, 0.0000⟩	⟨0.2422, 0.8655⟩	⟨0.2422, 0.8655⟩	⟨0.0300, 0.9906⟩	⟨0.0300, 0.9906⟩	⟨0.1596, 0.9397⟩
C_6	⟨0.2422, 0.8655⟩	⟨0.2422, 0.8655⟩	⟨0.1596, 0.9397⟩	⟨0.1596, 0.9397⟩	⟨0.1246, 0.9552⟩	⟨0.0000, 0.0000⟩	⟨0.3721, 0.8133⟩	⟨0.1246, 0.9552⟩	⟨0.1596, 0.9397⟩	⟨0.0605, 0.9906⟩
C_7	⟨0.1596, 0.9397⟩	⟨0.1596, 0.9397⟩	⟨0.0605, 0.9906⟩	⟨0.1246, 0.9552⟩	⟨0.2422, 0.8655⟩	⟨0.3721, 0.8133⟩	⟨0.0000, 0.0000⟩	⟨0.1596, 0.9397⟩	⟨0.0300, 0.9906⟩	⟨0.0300, 0.9906⟩
C_8	⟨0.1246, 0.9552⟩	⟨0.3721, 0.8133⟩	⟨0.1596, 0.9397⟩	⟨0.0605, 0.9906⟩	⟨0.1596, 0.9397⟩	⟨0.3721, 0.8133⟩	⟨0.3721, 0.8133⟩	⟨0.0000, 0.0000⟩	⟨0.1246, 0.9552⟩	⟨0.0300, 0.9906⟩
C_9	⟨0.1246, 0.9552⟩	⟨0.1596, 0.9397⟩	⟨0.1246, 0.9552⟩	⟨0.3721, 0.8133⟩	⟨0.2422, 0.8655⟩	⟨0.1596, 0.9397⟩	⟨0.3721, 0.8133⟩	⟨0.3721, 0.8133⟩	⟨0.0000, 0.0000⟩	⟨0.0605, 0.9906⟩
C_{10}	⟨0.0605, 0.9906⟩	⟨0.3721, 0.8133⟩	⟨0.1246, 0.9552⟩	⟨0.1246, 0.9552⟩	⟨0.2422, 0.8655⟩	⟨0.2422, 0.8655⟩	⟨0.2422, 0.8655⟩	⟨0.0605, 0.9906⟩	⟨0.3721, 0.8133⟩	⟨0.0000, 0.0000⟩

Table 7 Aggregated matrix

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	⟨0.0000, 0.0000⟩	⟨0.3457, 0.6899⟩	⟨0.4642, 0.5376⟩	⟨0.4336, 0.5731⟩	⟨0.5847, 0.4309⟩	⟨0.5943, 0.3490⟩	⟨0.6441, 0.3158⟩	⟨0.2680, 0.7903⟩	⟨0.6033, 0.3848⟩	⟨0.5517, 0.4675⟩
C_2	⟨0.2805, 0.7897⟩	⟨0.0000, 0.0000⟩	⟨0.4008, 0.6275⟩	⟨0.8346, 0.1389⟩	⟨0.7097, 0.2222⟩	⟨0.6548, 0.3050⟩	⟨0.8065, 0.2086⟩	⟨0.8465, 0.1624⟩	⟨0.8777, 0.1143⟩	⟨0.5774, 0.3643⟩
C_3	⟨0.3513, 0.6813⟩	⟨0.4652, 0.5225⟩	⟨0.0000, 0.0000⟩	⟨0.3105, 0.7435⟩	⟨0.4296, 0.6019⟩	⟨0.5025, 0.4820⟩	⟨0.7344, 0.3006⟩	⟨0.4747, 0.5188⟩	⟨0.3674, 0.6750⟩	⟨0.7505, 0.2206⟩
C_4	⟨0.1000, 0.9000⟩	⟨0.9000, 0.1000⟩	⟨0.3895, 0.6263⟩	⟨0.0000, 0.0000⟩	⟨0.5350, 0.4984⟩	⟨0.4063, 0.6179⟩	⟨0.7381, 0.2928⟩	⟨0.1969, 0.8633⟩	⟨0.4814, 0.6087⟩	⟨0.8435, 0.1341⟩
C_5	⟨0.6714, 0.2385⟩	⟨0.6480, 0.2695⟩	⟨0.4239, 0.5998⟩	⟨0.6803, 0.2681⟩	⟨0.0000, 0.0000⟩	⟨0.5721, 0.3861⟩	⟨0.7249, 0.2327⟩	⟨0.4464, 0.5664⟩	⟨0.5328, 0.4452⟩	⟨0.6642, 0.2945⟩
C_6	⟨0.8902, 0.1064⟩	⟨0.5779, 0.3772⟩	⟨0.3908, 0.6435⟩	⟨0.4764, 0.5295⟩	⟨0.6068, 0.3301⟩	⟨0.0000, 0.0000⟩	⟨0.8777, 0.1143⟩	⟨0.6621, 0.3444⟩	⟨0.7249, 0.2327⟩	⟨0.4193, 0.6141⟩
C_7	⟨0.8603, 0.1320⟩	⟨0.7757, 0.2542⟩	⟨0.4688, 0.5413⟩	⟨0.5389, 0.5058⟩	⟨0.4688, 0.5413⟩	⟨0.8887, 0.1074⟩	⟨0.0000, 0.0000⟩	⟨0.6753, 0.3878⟩	⟨0.6871, 0.3462⟩	⟨0.5558, 0.3946⟩
C_8	⟨0.4726, 0.5289⟩	⟨0.9000, 0.1000⟩	⟨0.8372, 0.1557⟩	⟨0.4673, 0.5286⟩	⟨0.5603, 0.3971⟩	⟨0.7836, 0.1645⟩	⟨0.8456, 0.1329⟩	⟨0.0000, 0.0000⟩	⟨0.8401, 0.1454⟩	⟨0.7064, 0.3102⟩
C_9	⟨0.3677, 0.6554⟩	⟨0.5807, 0.3728⟩	⟨0.7385, 0.2811⟩	⟨0.8478, 0.1317⟩	⟨0.6476, 0.2723⟩	⟨0.6201, 0.3125⟩	⟨0.7736, 0.1807⟩	⟨0.9000, 0.1000⟩	⟨0.0000, 0.0000⟩	⟨0.7644, 0.2324⟩
C_{10}	⟨0.4131, 0.5980⟩	⟨0.8675, 0.1363⟩	⟨0.5532, 0.3985⟩	⟨0.5404, 0.4155⟩	⟨0.5652, 0.3934⟩	⟨0.8332, 0.1613⟩	⟨0.8465, 0.1624⟩	⟨0.8214, 0.1604⟩	⟨0.8760, 0.1153⟩	⟨0.0000, 0.0000⟩

Table 8 Total average crisp matrix

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	0.0000	− 0.3564	− 0.0735	− 0.1404	0.1562	0.2314	0.3152	− 0.5528	0.2159	0.0859
C_2	− 0.5450	0.0000	− 0.2330	0.6772	0.4543	0.3357	0.6069	0.6901	0.7572	0.2006
C_3	− 0.3407	− 0.0566	0.0000	− 0.4564	− 0.1777	0.0202	0.4491	− 0.0438	− 0.3207	0.5145
C_4	− 0.8000	0.8000	− 0.2405	0.0000	0.0378	− 0.2168	0.4590	− 0.7066	− 0.1388	0.6935
C_5	0.3938	0.3472	− 0.1801	0.3909	0.0000	0.1782	0.5242	− 0.1215	0.0857	0.3544
C_6	0.7811	0.1917	− 0.2614	− 0.0535	0.2593	0.0000	0.7572	0.3197	0.4713	− 0.2013
C_7	0.7227	0.5370	− 0.0732	0.0345	0.3821	0.7782	0.0000	0.3056	0.3522	0.1532
C_8	− 0.0563	0.8000	0.6766	− 0.0610	0.1562	0.5869	0.6974	0.0000	0.6847	0.4028
C_9	− 0.2944	0.1983	0.4664	0.7013	0.3452	0.2869	0.5657	0.8000	0.0000	0.5302
C_{10}	− 0.1870	0.7340	0.1472	0.1194	0.1647	0.6683	0.6901	0.6489	0.7541	0.0000

Table 9 Normalized average crisp matrix

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	0.0000	− 0.0704	− 0.0145	− 0.0277	0.0308	0.0457	0.0622	− 0.1091	0.0426	0.0170
C_2	− 0.1076	0.0000	− 0.0460	0.1337	0.0897	0.0663	0.1198	0.1363	0.1495	0.0396
C_3	− 0.0673	− 0.0112	0.0000	− 0.0901	− 0.0351	0.0040	0.0887	− 0.0087	− 0.0633	0.1016
C_4	− 0.1579	0.1579	− 0.0475	0.0000	0.0075	− 0.0428	0.0906	− 0.1395	− 0.0274	0.1369
C_5	0.0778	0.0686	− 0.0356	0.0772	0.0000	0.0352	0.1035	− 0.0240	0.0169	0.0700
C_6	0.1542	0.0378	− 0.0516	− 0.0106	0.0512	0.0000	0.1495	0.0631	0.0930	− 0.0397
C_7	0.1427	0.1060	− 0.0145	0.0068	0.0754	0.1537	0.0000	0.0603	0.0695	0.0302
C_8	− 0.0111	0.1579	0.1336	− 0.0120	0.0308	0.1159	0.1377	0.0000	0.1352	0.0795
C_9	− 0.0581	0.0391	0.0921	0.1385	0.0682	0.0566	0.1117	0.1579	0.0000	0.1047
C_{10}	− 0.0369	0.1449	0.0291	0.0236	0.0325	0.1319	0.1363	0.1281	0.1489	0.0000

Table 10 Total relation matrix

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	0.0302	− 0.0845	− 0.0256	− 0.0303	0.0289	0.0399	0.0462	− 0.1102	0.0267	0.0032
C_2	− 0.1004	0.1466	− 0.0236	0.2050	0.1579	0.1640	0.2629	0.2163	0.2471	0.1249
C_3	− 0.0383	− 0.0004	0.0050	− 0.0950	− 0.0280	0.0308	0.0901	0.0217	− 0.0384	0.0857
C_4	− 0.1706	0.2030	− 0.0629	0.0479	0.0351	− 0.0084	0.1173	− 0.0723	0.0181	0.1447
C_5	0.0829	0.1240	− 0.0453	0.1118	0.0421	0.0883	0.1704	0.0083	0.0756	0.1001
C_6	0.1837	0.0824	− 0.0437	0.0275	0.0990	0.0685	0.2191	0.0898	0.1506	− 0.0003
C_7	0.1662	0.1695	− 0.0139	0.0575	0.1345	0.2260	0.1258	0.1110	0.1596	0.0760
C_8	0.0074	0.2622	0.1413	0.0545	0.1095	0.2289	0.2970	0.1145	0.2465	0.1543
C_9	− 0.0568	0.1799	0.1048	0.1810	0.1283	0.1600	0.2609	0.2259	0.1063	0.1859
C_{10}	− 0.0156	0.2724	0.0498	0.1018	0.1202	0.2508	0.3065	0.2380	0.2745	0.0849

calculated for every expert. Table 6 presents the weighted initial direct matrix of E_1.

The weighted initial direct matrices of E_2, E_3, E_4, E_5 and E_6 are computed similarly.

The aggregated matrix to represent assessments made by six experts are calculated using Eq. (3). The aggregated matrix is shown in Table 7.

The score function [see Eq. (4)] is used to obtain total average crisp matrix. The average crisp matrix is summarised in Table 8.

The average crisp matrix is then normalised using Eq. (5). Table 9 presents the normalised average crisp matrix.

Toward the end of this compuation, the total-relation matrix T is obtained using Eq. (6). Table 10 presents the total relation matrix.

Table 11 Sum of rows, sum of columns and its substraction and addition

Criteria	r	c	$r+c$	$r-c$
C_1	− 0.0755	0.0887	0.0132	− 0.1641
C_2	1.4005	1.3550	2.7555	0.0455
C_3	0.0333	0.0859	0.1192	− 0.0526
C_4	0.2519	0.6617	0.9136	− 0.4098
C_5	0.7582	0.8278	1.5680	− 0.0696
C_6	0.8765	1.2488	2.1253	− 0.3723
C_7	1.2122	1.8961	3.1083	− 0.6839
C_8	1.6163	0.8428	2.4590	0.7733
C_9	0.4761	1.2666	2.7428	0.2095
C_{10}	1.6834	0.9595	2.6429	0.7238

The sum of rows and the sum of columns of total relation matrix are separately denoted as c and r within the total-relation matrix T using Eqs. (6) and (7). The values of c, r. $c+r$ and c-r are shown in Table 11.

Based on the information in Table 11, finally, the causal diagram is drawn. Date set of ($r+c$, $r-c$) are mapped out onto two-dimension plane. Figure 2 shows the coordinates of criteria in two dimesion plane.

It is apparent from this table that the criteria are separated into two categories based on the values of $r-c$. The first category is 'cause group' and the second category is denoted as 'effect group'. The criteria in the 'cause group' are population size, consumption habits, human health and environmental health. On the other hand, the criteria in the 'effect group' are relative cost, socio-culture, public awareness, institutional, technical and operational and maintenance challenges. The causal diagram depicts the coordinates of all ten criteria of SWM where the four criteria in 'cause group' would influence the six criteria in 'effect group'. The results indicate that consumer habit and population size are among the criteria in cause group where as Technical and operational maintainance are placed in effect group. The importance of criteria is determined based on the (r + c) values. It can be seen that 'operational and maintenance challenges' is the most important criteria in the effect group. All in all, the results of the empirical case provide evidence on the application of the proposed method.

The results obtained from the proposed method suggests that 'operational and maintenance challenges' is most influenced criteria. This result is not consistent with an empirical study conducted by Soroudi et al. [35] where 'soil depth' is the most influenced criteria. They utilised the DEMATEL and the analytical network process to determine interaction and weight of criteria of SWM. In another experiment, by

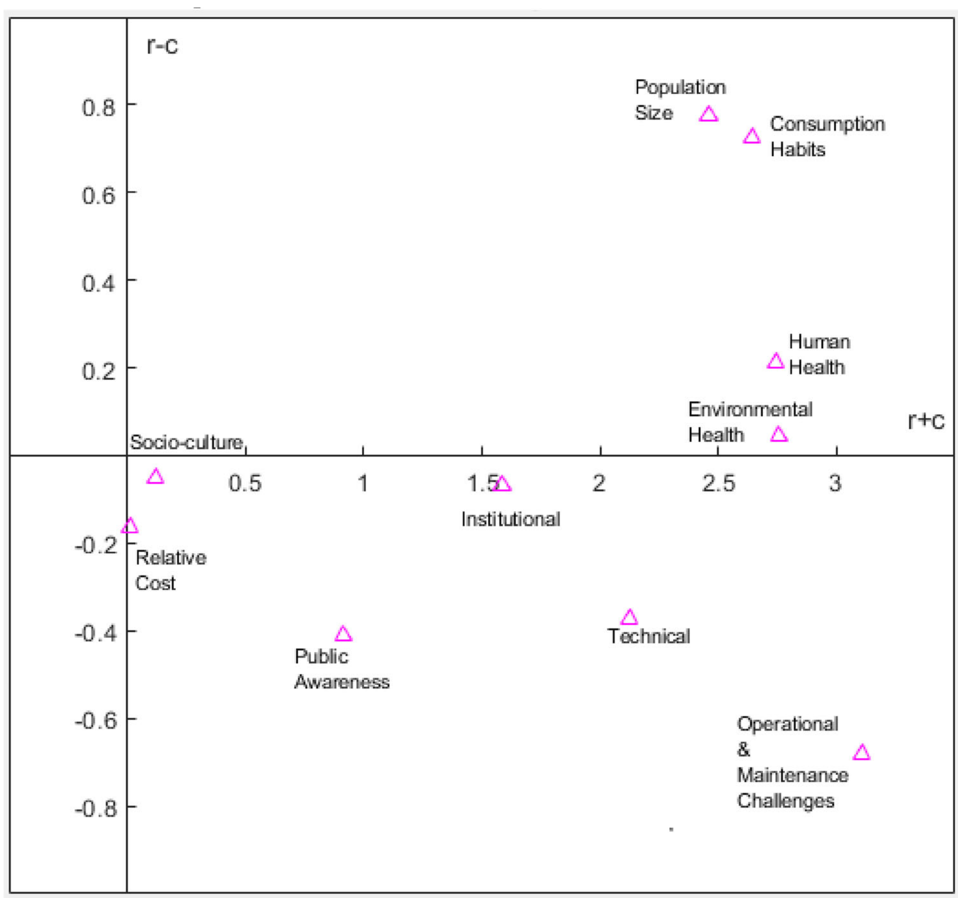

Fig. 2 Causal diagram

using the DEMATEL method, Tseng [38] found that the criteria ‘natural resources’ is considered as the most influenced criteria. This result also differs from that of Abdullah et al. [2] who suggest that the criteria ‘political issues’ was selected as the most important criteria.

Conclusions

The DEMATEL provides a comprehensive tool and computationally feasible way in dealing with relationships between criteria of MCDM problems. The DEMATEL has been combined with IFS to address uncertain and vague information in solving MCDM problems. However, the combined IFS-DEMATEL fails to solve problems particularly on the cases where sum of two memberships is greater than one. To address this limitation, the PFS was proposed with the assumption that the linearity in two membership functions of IFS is now modified to the square functions. In this paper, the combination of DEMATEL and PFS has been proposed with three innovations. Firstly, two memberships of PHS were used to replace two linear membership functions of IFS. This combination entails newly defined linguistic variable, weights of experts and score function. The proposed PFS-DEMATEL shows some extent of advantages as the proposed MCDM method provides more obvious improvement in judgements where uncertainties are now expressed in squares of memberships. In other words, the PFS-DEMATEL creates a new perspective in solving MCDM problems where linguistic variables are defined by considering membership and non-membership of PFS. Another objective of this paper was to apply the proposed method in a SWM problem. Therefore, the proposed eleven-step DEMATEL method was computationally implemented to an empirical study in SWM. A group of experts were invited to assess the degree of influence among ten criteria of SWM using the seven-scale linguistic terms. The computational feasible method was successfully segregated the ten criteria into two categories of which four criteria included in cause group while six criteria are placed in effect group. The current study found that the criteria ‘operational and maintenance challenges’ is the most important criteria in solving SWM problem.

Appendix

See Tables 12, 13, 14, 15 and 16.

Table 12 Initial direct matrix (E_2)

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	$\langle 0, 0\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$
C_2	$\langle 0.2, 0.9\rangle$	$\langle 0, 0\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$
C_3	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$
C_4	$\langle 0.1, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$
C_5	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$
C_6	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$
C_7	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$
C_8	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$
C_9	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$
C_{10}	$\langle 0.4, 0.6\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$

Table 13 Initial direct matrix (E_3)

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	$\langle 0, 0\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$
C_2	$\langle 0.2, 0.9\rangle$	$\langle 0, 0\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.4, 0.6\rangle$
C_3	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0, 0\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.7, 0.2\rangle$
C_4	$\langle 0.1, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.7, 0.2\rangle$
C_5	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$
C_6	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$
C_7	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$
C_8	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.2, 0.9\rangle$
C_9	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$
C_{10}	$\langle 0.4, 0.6\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0, 0\rangle$

Table 14 Initial direct matrix (E_4)

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	$\langle 0, 0\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$
C_2	$\langle 0.4, 0.6\rangle$	$\langle 0, 0\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$
C_3	$\langle 0.1, 0.9\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0, 0\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$
C_4	$\langle 0.1, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0, 0\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$
C_5	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0, 0\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.7, 0.2\rangle$
C_6	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$
C_7	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$
C_8	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$
C_9	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$
C_{10}	$\langle 0.4, 0.6\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$

Table 15 Initial direct matrix (E_5)

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	$\langle 0, 0\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.5, 0.5\rangle$
C_2	$\langle 0.2, 0.9\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$
C_3	$\langle 0.1, 0.9\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0, 0\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$
C_4	$\langle 0.1, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.9, 0.1\rangle$
C_5	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$
C_6	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0, 0\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.2, 0.9\rangle$
C_7	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$
C_8	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$
C_9	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$
C_{10}	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$

Table 16 Initial direct matrix (E_6)

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	$\langle 0, 0\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$
C_2	$\langle 0.1, 0.9\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.4, 0.6\rangle$
C_3	$\langle 0.1, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0, 0\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.5, 0.5\rangle$
C_4	$\langle 0.1, 0.9\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0, 0\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.7, 0.2\rangle$
C_5	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$
C_6	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.4, 0.6\rangle$
C_7	$\langle 0.9, 0.1\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$
C_8	$\langle 0.4, 0.6\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.4, 0.6\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0, 0\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.5, 0.5\rangle$
C_9	$\langle 0.2, 0.9\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$	$\langle 0.7, 0.2\rangle$
C_{10}	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0, 0\rangle$

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