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Separation Axioms on Spatial Topological Space and Spatial Data Analysis

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Abstract
The geographical information system has been generally used for analyzing different types of data. The notions and results of topology have been applied in this connection, known as a spatial topological relation. In this article, we have studied the different layers of geographical data and their intersection property, separation axioms on spatial topological space, spatial analysis.

Keywords Geographical information system · Spatial topology · Separation axioms · Spatial point · Spatial data analysis

Mathematics Subject Classification 03E72 · 90B50 · 03E75 · 14P25 · 62M30 · 91B72

1 Introduction

The entire situation of GIS geographical data offers an understanding of the location of different items in the real world, and creation is a proper layout of geometry. The information can be represented as discrete data (referred to as "feature data") or continuous data (referred to as "raster data"). The nature of examining the influences on how it is best portrayed is self-evident. Raster layers are used to describe the physical environment (mountains, temperatures, and rainfall), whereas vector data is used to describe the built environment (roads, buildings), and organizational data (countries, census areas). Each dataset is organized as a layer in GIS, and analytical engineers can connect them graphically (called overlay review). In the foundational theory of spatial analysis, the idea of accumulating layers containing diverse descriptions of data and associating them with one another based on where things are placed.

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The layers are linked by the fact that they are all geo-referenced to a specific location.

Geometric and topological qualities are essentially included in spatial data as a result of which the extension of locational and characteristic information. Geometric features, such as length, direction, area, and volume, integrate location and dimensions. Connectivity, composition, and adjacency are all topological qualities that characterize spatial interactions. Using these spatial attributes, one can analysis the data from different aspects to acquire deeper insights about it.

GIS reports can be used to answer questions such as where are the best places to built? Home protection, relative slope, distance to existing streets and rivers, and clay formation are just a few of the seemingly unrelated aspects that can be produced as layers and then compared using weighted overlay.

GIS has the true power, which rests in its ability to deliver an inquiry. Spatial analysis is a way for geographical modelling problems, obtaining conclusions through machine processing, and then searching for and analyzing those decisions.

The ultimate goal is to understand how to overcome spatial problems. The center of spatial analysis is determined by many essential spatial analysis workflows: spatial data exploration, modelling with GIS devices, and solving spatial problems.

Data analysis is applied in business related problems to understand the challenges that a company faces, to resolve decision-making issues, and to study data in meaningful way.

Data consists solely of statistics and facts. The process of organizing, interpreting, arranging, and presenting data into useful information is known as data analysis.

Data analysis depicts the situation in a more appealing and understandable manner.

Spatial data analysis allows to tackle complicated location-based challenges and gain a deeper understanding of what's going on in your world. It goes beyond simple mapping to allow you to investigate the properties of places and their interactions. Your decision-making will be enriched by spatial analysis.

If one studies a crime map in the city to see in which regions have the highest crime rates, will require and look the other forms of data, such as school locations, parks, and demographics. This is required to look for the best place to buy a house.

We automatically begin turning a map into information by analyzing its contents—finding patterns, identifying trends, or making decisions, every time one will look at.

This is known as "spatial analysis," and this is what one does automatically when he/she will look at a map.

Here's a quick rundown of Adams and Franzosa's work in [1] "Introduction to Pure and Applied Topology." In 1996, Felice [2] published "A paradigm for describing topological links between complicated geometric features in spatial databases." Point-set topological spatial relations and various models are created by Egenhofer and Franzosa [3–6]. A large number of researchers are working on multidisciplinary set theory and spatial set topology [7–10]. Marshall represents "Line Structure Representation for Road Network Analysis" in 2016 [11]. "Graph Theory" by Ducruet and Rodrigue [12, 13] Alvi Geographical Statistics. In 2017, Tien [14] used artificial intelligence to work on decision-making. "Joint Modeling of Longitudinal CD4 Count and Time-to-Death of HIV/TB Co-infected Patients: A Case of Jimma University Specialized Hospital," Temesgen et al. [15]. Mitra et al. [16] worked on the "Road Network

System in Agartala Municipal Corporation," and we gathered the rest of the data from the internet [17]. In the year 2021 Das et al. worked on data analysis [18].

Data analytics and applications are not only promoted how to use interdisciplinary techniques, but also to interpret and analysis the data. It includes statistics, artificial intelligence, and optimization, to process Big Data and conduct data mining, but also how to use the knowledge gleaned from Big Data for real-life applications. AIDS accepts high-quality contributions on the foundations of data science, technical papers on various challenging problems in Big Data, and meaningful case studies concerning business analytics in the context of Big Data. Qualitative and quantitative analysis are two fundamental methods of collecting and interpreting data in research. The methods can be used independently or concurrently in since they all have the same objectives. They may have some errors, and so using them concurrently can compensate for the errors each has and then produce quality results. In spatial topology qualitative data analysis and quantitative data analysis both have important mining and role. Many researchers have worked on data analysis in various methods. In 2022 Shi [19] studied on Advances in Big Data Analytics: Theory, Algorithm and Practice. In 2021 Thakkar and Shah [20] worked on the topic "An Assessment of Football Through the Lens of Data Science". In 2020 Liu and Shi [21] studied on "Investigating Laws of Intelligence Based on AI IQ Research". Olson and Shi [22] on "Introduction to business data mining". Shi et.al. [23] have widely studied on "Optimization based data mining: theory and applications". Tien [14] in 2017 discuss the idea of "Internet of things, real-time decision making, and artificial intelligence". Das et.al [24] worked on "Multi-criteria group decision making model using single-valued neutrosophic set". In 2020 Mukherjee and Das [25] worked on "Neutrosophic bipolar vague soft set and its application to decision making problems".

In this investigation, we use the topological property to analyze spatial data, which may be used to tackle a variety of essential problems such as health difficulties, criminality, and economic issues.

2 Materials and Methods

For our investigation in this article, we need some definitions and the preliminary idea of GIS data analysis and spatial topological connection for this one may refer to the articles Adams and Franzosa [1], Clementini and Felice [2], Egenhofer and Franzosa [3–5].

We study GIS data for the clarification of confusion and to find the answer mainly to the following questions, such as,

- What are the basic geometric characteristics of geographic objects to represent their relationships?
- How can those relationships be formally described in terms of fundamental geometric characteristics?
- What is an insignificant set of spatial relationships?
- What should be the topological relation between the spatial data?
- How to separate spatial data?

3 Analysis in GIS

Input/updating, data conversion, storage/organization, manipulation, geographical division, and output (performance/display) are the four essential roles that geographic knowledge approaches are thought to play.

4 Geometric Networks

Geometric systems are continuous networks of objects that can be used to describe and analyze interrelated properties. A geographic network is made up of edges that meet at intersection sites, similar to mathematical and computer science architectures. The network, like graphs, can have weight and flow assigned to its edges, which can be used to better accurately depict many interrelated features. Road channels and public service networks, such as electric, gas and river systems, are typically built using geometric networks. Transportation planning, hydrology modeling, and foundation modeling are all examples of when network modeling is used.

We refer to Egenhofer and Franzosa [3] for definitions of spatial data, non-spatial data, interior, closure, and boundary of geographical region, all of which are related to the notion of topology.

5 Topological Modeling and Formal Description of Topological Relations

A GIS can classify and investigate the spatial relationships that exist within digitally collected spatial data. These topological connections allow complex spatial modeling and analysis to be performed. Topological relationships between geometric objects traditionally hold adjacency, containment, and closeness. The topological similarity R^n between two point sets, U and V, is described by the nine set intersections In of U's boundary, interior, and complement with the boundary, interior, and complement of V called the 9-intersection, one may refer to Egenhofer [5]. This is an extensive description of the originally proposed 4-intersection consisting of the four-set intersections of boundaries and interiors, one may refer to Egenhofer and Franzosa [3], Egenhofer and Mark [4].

The tabulated form of various types of spatial connections based on the set $\{\partial U, U^{\circ}, U^{-1}\}$ and the matrix $I_n(A,B) = \begin{pmatrix} \partial U \cap \partial V & \partial U \cap V^0 & \partial U \cap V^{-1} \\ U^0 \cap \partial B & U^0 \cap V^0 & U^0 \cap V^{-1} \\ U^{-1} \cap \partial B & U^{-1} \cap V^0 & U^{-1} \cap V^{-1} \end{pmatrix}$ are the following (Table 1).

6 Results and Discussion

The main aim of this article is to define the separation axioms on the spatial data set and analysis of GIS data based on separation axioms.

Table 1 The specification of then eight topological relationships between the two-point sets in 2-D

$$I_0(U,V)=\begin{pmatrix}\varnothing & \varnothing & \neg\varnothing\\ \varnothing & \varnothing & \neg\varnothing\\ \neg\varnothing & \neg\varnothing & \neg\varnothing\end{pmatrix}$$

(a) $R_{\text{Disjoint}}(U,V)$

$$I_1(U,V)=\begin{pmatrix}\neg\varnothing & \varnothing & \neg\varnothing\\ \varnothing & \varnothing & \neg\varnothing\\ \neg\varnothing & \neg\varnothing & \neg\varnothing\end{pmatrix}$$

(b) $R_{\text{Meet}}(U,V)$

$$I_2(U,V)=\begin{pmatrix}\neg\varnothing & \varnothing & \varnothing\\ \varnothing & \neg\varnothing & \varnothing\\ \varnothing & \varnothing & \neg\varnothing\end{pmatrix}$$

(c) $R_{\text{Equals}}(A,B)$

$$I_3(U,V)=\begin{pmatrix}\varnothing & \neg\varnothing & \varnothing\\ \varnothing & \neg\varnothing & \varnothing\\ \neg\varnothing & \neg\varnothing & \neg\varnothing\end{pmatrix}$$

(d) $R_{\text{Inside}}(U,V)$

$$I_4(U,V)=\begin{pmatrix}\neg\varnothing & \neg\varnothing & \varnothing\\ \varnothing & \neg\varnothing & \varnothing\\ \neg\varnothing & \neg\varnothing & \neg\varnothing\end{pmatrix}$$

(e) $R_{\text{Covered By}}(U,V)$

$$I_5(U,V)=\begin{pmatrix}\varnothing & \varnothing & \neg\varnothing\\ \neg\varnothing & \neg\varnothing & \neg\varnothing\\ \varnothing & \varnothing & \neg\varnothing\end{pmatrix}$$

(f) $R_{\text{Contains}}(U,V)$

$$I_6(U,V)=\begin{pmatrix}\neg\varnothing & \varnothing & \neg\varnothing\\ \neg\varnothing & \neg\varnothing & \neg\varnothing\\ \neg\varnothing & \varnothing & \neg\varnothing\end{pmatrix}$$

(g) $R_{\text{Covers}}\ (U,V)$

$$\mathrm{I}_7(U,V)=\begin{pmatrix}\neg\varnothing & \neg\varnothing & \neg\varnothing\\ \neg\varnothing & \neg\varnothing & \neg\varnothing\\ \neg\varnothing & \neg\varnothing & \neg\varnothing\end{pmatrix}$$

(h) $R_{\text{Overlap}}(U,V)$

Based on GIS data characteristics we have thought some definitions and analyzed its properties by this definition. We have studied, how GIS data can be analyzed in topology and separated based on the geographical situation as well as on problem-based topological formation. For understanding the overlay analysis of GIS data and point, line, and polygon data analysis and how the raster data transfer into vector data, etc. using topological property we establish some result which can help us to analyze spatial data.

6.1 Separation

Our design of topological spatial relationships is based on the point-set topological thoughts of the interior, exterior, and boundary. The concepts of separation and connectedness are necessary for establishing future topological spatial relations among sets. Let $Y \subset X$. a separation of Y is a pair U, V of subsets of X satisfying the following three conditions:

- $U \neq \emptyset$ and $V \neq \emptyset$;
- $U \cup V = Y$; and
- $\overline{U} \cap V = .\emptyset$ and $\overline{V} \cap U = .\emptyset$

Based on GIS data and binary topological relation of spatial relations we have introduced some definitions on separation, connectedness, and compactness. One may refer to Egenhofer and Franzosa [3].

Any single data of spatial analysis from a GIS spatial data set is called **a spatial point.** Spatial point is denoted by $\dot{x}$

Definition 3.1 A spatial topological space (X, τ) is said to be completely spatial connected if every spatial point of spatial sets has the same property with the other i.e., every two spatial sets A and B follow the relation $I_2(A, B)$, i.e. $R_{\text{Equals}}(A, B)$ and for each point of a set A and B are connected.

The connectedness and separation of spatial data is based on their internal and geometrical property.

Example 3.1 When two or more GIS layer of a reason have the same property then this type of sets are the example of completely spatial connected space.

Definition 3.2 A spatial topological space (X, τ) is said to be completely T_0-spatial connected if there exist a spatial point $\dot{x}$ such that $\dot{x} \in A$ and $\dot{x} \in B$, also follows the relation I_3 or I_4 or I_5 or I_6, i.e. one of the relations $\mathrm{R}_{\text{Inside}}(A, B)$, $\mathrm{R}_{\text{Covered By}}\ (A, B)$, $\mathrm{R}_{\text{Contains}}(A, B)$ and $\mathrm{R}_{\text{Covers}}(A, B)$.

Mathematically spatial T_0-connected sets follow the relation $A \cap B \neq \emptyset$ for all cases of sets A and B.

Example 3.2 In a city, the spots of all garbage or pollution sectors come under the completely T_0-spatial connected.

Definition 3.3 A spatial topological space (X, τ) is said to be completely T_1-spatial connected if every two spatial set A and B follow the relation I_3 or I_4 or I_5 or I_6 or I_7 i.e. follow the relation among the relations $R_{\text{Inside}}(A, B)$ or $R_{\text{Covered By}}\ (A, B)$ or $R_{\text{Contains}}(A, B)$ or $R_{\text{Covers}}(A, B)$ and $R_{\text{Overlap}}(A, B)$.

Example 3.3 The example of the above spatial topological space like example 3.2 but in this case spatial data sometimes may coincide.

Definition 3.4 A spatial topological space (X,τ) is said to be completely T_0-spatial disconnected if for any two spatial point $\dot{x}$ and $\dot{y}$ of two spatial set A and B follows the relation $I_1(A, B)$, i.e. $R_{\text{Meet}}(A, B)$ and $\exists$ two open U and V such that $\dot{x} \in U \subseteq A$ and $\dot{y} \in V \subseteq B$ but $\dot{y} \notin U$ and $\dot{x} \notin V$.

Example 3.4 The natural example of completely T_0-spatial disconnected space is the spatial topological relation between insect live in water and outside the water.

Definition 3.5 A spatial topological space (X,τ) is said to be completely T_1-spatial disconnected if for any two spatial point $\dot{x}$ and $\dot{y}$ of two spatial set A and B follows the relation $I_0(A, B)$ i.e. $R_{\text{Disjoint}}(A, B)$ and for every $\dot{x} \in A$, $\dot{y} \in B$ there exist two open sets $U \subseteq A$ and $V \subseteq B$ and $U \cap V = \emptyset$.

Definition 3.6 A spatial topological space (X,τ) is said to be compact if every spatial data covered by finite spatial data.

Example 3.5 All tower of capital is a model of a compact spatial set. Because in every city there will be a calculable number of towers and that can be covered by finite spatial extension data.

Note: The above 5 definitions of separation on spatial data sets are divided into four brilliant way to analysis GIS data set.

Proposition 3.1 Every spatial T_1-disconnected space is spatial T_0-disconnected space but not necessarily conversely.

Proof: From the definition it is obvious that every spatial T_1-disconnected space is spatial T_0-disconnected space.

For the converse part we take an example which is spatial T_0-disconnected space.

Let $X = \{\dot{a}, \dot{b}, \dot{c}, \dot{d}\}$ and $\tau = \{\emptyset, X, \{\dot{a}\}, \{\dot{a}, \dot{b}\}, \{\dot{b}, \dot{c}\}, \{\dot{b}\}\{\dot{a}, \dot{b}, \dot{c}\}\}$ here (X,τ) is a spatial T_0-disconnected space but not a spatial T_1-disconnected space.

Proposition 3.2 Every spatial T_1-connected space is spatial T_0-connected space but not necessarily conversely in general.

Proof: The first part of the statement follows from the Definition 3.2 and Definition 3.3 the proof is obvious. For converse part of the theorem we provide an example.

Example 3.6 Consider a set of spatial plane data of a factory and its different layers. Consider a spatial topology by considering the spatial plane data of factory then the data set follows spatial T_0-connected space but not spatial T_1-connected because when we take the spatial data as a different factory which are connected to another factory, in that case, some data will cover by, some data contains but no data will project to other data. So in such cases, the data set follows the spatial T_0-connected space but not spatial T_1-connected.

Theorem 3.1 Every spatial T_1-disconnected space is T_1-space in general topological space but not necessarily conversely in general.

Proof: Let (X, τ) be a T_1-spatial disconnected topological space, so by the definition of spatial T_1-disconnected space follows the conditions $I_0(A, B)$ i.e. $R_{\text{Disjoint}}(A, B)$ and for every $\dot{x} \in A$, $\dot{y} \in B$ $\exists$ two open sets $U \subseteq A$, $V \subseteq B$ and $U \cap V = \emptyset$ where $\dot{x}$, $\dot{y}$ are spatial points.

Here from the assumption $\dot{x} \in A$, $\dot{y} \in B$, there exist two open sets U, V such that $\dot{x} \in U \subseteq A$, $\dot{y} \in V \subseteq B$ and $U \cap V = \emptyset$ shows that the T_1-spatial disconnected space is a T_1 space in general.

For the converse part of the theorem we can say that if the data sets are not spatial disjoint then it cannot be a spatial T_1-disconnected even though the set can form a T_1 space.

Theorem 3.2 Every spatial T_0-disconnected space is T_0-space in general topological space but not necessarily conversely.

Proof: Let (X, τ) be a T_0-spatial disconnected topological space so by the definition of spatial T_0-disconnected space the conditions $I_1(A, B)$, i.e. $R_{\text{Meet}}(A, B)$ follows and there exist two open U and V such that $\dot{x} \in U \subseteq A$ and $\dot{y} \in V \subseteq B$ but $\dot{y} \notin U$ and $\dot{x} \notin V$ where $\dot{x}$, $\dot{y}$ are spatial point.

Since (X, τ) satisfied the condition $\dot{x} \in U \subseteq A$ and $\dot{y} \in V \subseteq B$ but $\dot{y} \notin U$ and $\dot{x} \notin V$ so (X, τ) is a T_0 space in general topological space.

For the converse part we provide an example.

Let $X = \{0, 1\}$ and $\tau = \{\emptyset, \{0\}, \{0, 1\}\}$ is called Sierpinski space which is T_0 also but not a spatial T_0-disconnected.

Theorem 3.3 Every spatial T_1 disconnected space is T_0 space in general topological space but not necessarily conversely in general.

Proof: the proof is obvious using the theorem 3.1.

Definition 3.7 A spatial topological space (X, τ) is said to be planner spatial topology if all elements of X are plane data set.

Example 3.7 Consider X a different spatial plane data with 1. Input layer, 2. Splitting layer and 3 Output layer. Then Output layer makes a spatial topological relation set. We make a spatial topology by taking spatial plane data which is the planner spatial topology.

Definition 3.7 A spatial topological space (X, τ) is said to be Discrete spatial topology if all the elements of X are point data set.

Example 3.8 Let us consider X be a Mandelbrot set as a spatial data set in which each similar figure consider as a spatial point data. We can make a spatial topology by taking spatial points data which will form the discrete spatial topology. Here the discrete spatial topology is also a spatial T_1-connected space as all spatial point like to be similar. Here we consider each spatial data as a spatial point data set.

Result 3.1 Every planner spatial topological space is spatial compact space.

Result 3.2 Every discrete spatial topological space may not be spatial compact space. Here the example 3.7 is not a spatial compact space.

Result 3.3 Every spatial planner T_1 space is spatial T_1-compact.

7 Application of Spatial Data Analysis with the Help of the Above Methods

7.1 The Geographical Location of the Area of Data Collected

Agartala, the capital city of Tripura is located in between 23° 45' to 23° 55' N latitudes and 91°15' to 91°20' E longitudes. Physiographically, the city is located in the flood plain of the River Haora and Kata Khal. The physiographic structure of Agartala City is saucer shape and characterized with Tilla (Relatively High land) and Lunga (Low land) topography. The Agartala Municipal Council was established in 1874 and the city has become the nerve center of all administrative, political, cultural and commercial activities of the state. The city has emerged as an important border-trading center with international trading linkage with Bangladesh. National Highway-8 is passing through the heart of the City. Maharaja Bir Bikram Airport [23°53′33.96" N and 91°14′37.81" E] is located about 11.75 km north-west from the Central Business District (CBD) of Agartala City. The city is broadly divided into four planning zones (North, Central, East and South) and shared international border with Bangladesh on the western side. Wherever Jirania Rural Development (R.D) Block, Mohanpur R.D. Block and Dukli R.D. Block are in the east, north–south of the city, respectively. The total area of AMC is almost 76.150 Km^2 with 526,292 of the population (AMC, 2018).

Road network has been increased rapidly in Agartala City. In 2016, road density was 7.96 km/km^2 and in 2018, it was 11.512 km/km^2 (Mitra et al., 2018). The total road length of AMC is 678.445 km, including all major and minor roads. The maximum length of the road is found in ward number 5 (31.599 km) and lowest road length has been observed in ward number 36 (5.723 km), with only 24 route (edges) and 25 nodes (vertices). In Agartala, total edges and vertices are 4252 and 3840, respectively (Fig. 2). But edges and vertices of Agartala City not equally distributed among the 49 wards. The average edges and vertices are 87 and 78, respectively. 30 (61%) wards of AMC have below-average edges number. Those wards are 36, 42, 30, 35, 37, 3, 6, 10, 28, 13, 17, 33, 8, 27, 14, 1, 18, 44, 29, 20, 15, 47, 12, 26, 31, 38, 46, 19, 22 and 2. Remaining 19 (49%) wards of AMC have observed with above mean edges number. Similarly it has been found that 27 (55%) and 22 (45%) wards have vertices with below and above average, respectively. Maximum vertices (190) found in ward number 5.

Data of road network has been collected from the field by using handheld Global Positioning System (GPS) receiver. We consider an arbitrary data set. The ward level road network map has been prepared by using Global Mapper v.20 and Arc GIS v.10.7.1. Graph theoretical techniques like alpha (α), beta (β) and gamma (γ) indices have been calculated by using the following formulas:

Alpha Index$(\alpha) = \frac{e-v+1}{2v-5}$, Beta Index$(\beta) = \frac{e}{v}$ and Gamma Index$(\gamma) = \frac{e}{3(v-2)}$.

where, e = number of edges or routes and v = number of vertices or nodes.

In this paper, we will analysis only β-index for the spatial connectivity analysis.

Ward No	Edge	Node	Alpha	Beta	Gamma
1	63	65	− 0.008	0.969	0.333
2	86	87	0.000	0.989	0.332
3	48	47	0.022	1.021	0.345
4	128	122	0.029	1.049	0.352
5	218	190	0.077	1.147	0.384
6	53	71	− 0.124	0.746	0.251
7	115	133	− 0.065	0.865	0.290
8	60	70	− 0.067	0.857	0.288
9	107	106	0.010	1.009	0.339
10	54	59	− 0.035	0.915	0.309
11	103	90	0.080	1.144	0.384
12	79	68	0.092	1.162	0.391
13	57	57	0.009	1.000	0.337
14	62	49	0.151	1.265	0.428
15	72	72	0.007	1.000	0.336
16	102	76	0.184	1.342	0.451
17	58	48	0.121	1.208	0.408
18	65	56	0.093	1.161	0.392
19	84	81	0.025	1.037	0.349
20	68	49	0.215	1.388	0.469
21	155	130	0.102	1.192	0.399
22	84	74	0.077	1.135	0.382
23	123	100	0.123	1.230	0.413
24	128	107	0.105	1.196	0.401
25	99	81	0.121	1.222	0.411
26	80	67	0.109	1.194	0.402
27	61	59	0.027	1.034	0.349
28	55	53	0.030	1.038	0.350
29	67	39	0.397	1.718	0.583
30	39	40	0.000	0.975	0.331
31	81	80	0.013	1.013	0.340
32	140	102	0.196	1.373	0.461
33	58	45	0.165	1.289	0.436
34	104	87	0.107	1.195	0.402
35	44	42	0.038	1.048	0.355
36	24	25	0.000	0.960	0.329
37	47	55	− 0.067	0.855	0.288
38	82	72	0.079	1.139	0.383
39	110	99	0.062	1.111	0.373
40	109	99	0.057	1.101	0.369
41	116	106	0.053	1.094	0.367

Ward No	Edge	Node	Alpha	Beta	Gamma
42	38	35	0.062	1.086	0.369
43	109	95	0.081	1.147	0.385
44	66	64	0.024	1.031	0.347
45	132	131	0.008	1.008	0.341
46	82	86	– 0.018	0.953	0.325
47	75	84	– 0.048	0.893	0.300
48	145	112	0.155	1.295	1.295
49	117	75	0.297	1.560	0.534

One of the relevant metrics of road connectivity is the beta (β) index [11]. The beta index is calculated by dividing the total number of edges by the total number of vertices in a network. The relationship between the number of links (e) and the number of nodes (n) is used to indicate the level of connectivity in a graph (v). A value larger than one indicates a more sophisticated network. The greater the number of links in a network with a fixed number of nodes, the greater the number of pathways feasible in the network. The Beta value of complex networks is very high. The rich-club coefficient is a Beta index that is used to relationships between nodes of greater order (degree); it determines whether connectivity is higher among larger d nodes.

Out of 49 wards in Agartala City, 13 (26.53%) were found to have very high road connectivity and the most complicated road network, with a β-value more than 1.20. 29, 49, 20, 32, 16, 48, 33, 14, 23, 25, 17, 24, and 34 are the wards in question. The majority of the city's high-connectivity wards are located in the city's core. Ward 29 has the highest -value (1.718). High road connection was discovered in 13 of AMC's wards (26.53 percent), namely 26, 21, 12, 18, 43, 5, 11, 38, 22, 39, 40, 41, and 42, where the β-value ranged from 1.09 to 1.20.

With a beta-value of 1.00 to 1.09, it was discovered that 12 wards (24.49 percent) have poor road connectivity. 4, 35, 28, 19, 27, 44, 3, 31, 9, 45, 13, and 15 are the wards. Those wards are near to wards with a lot of traffic. Wards 2, 30, 1, 36, 46, 10, 47, 7, 8, 37, and 6 have very low road connectivity, with a beta-value of less than 1.00. The city's northwestern wards have the highest concentration of very poor connectivity wards (6 wards, 54.54 percent). In the southern section of the city, about 4 (36.36 percent) wards have very poor connectivity. Ward number 30 (Subhash Nagar area) has very low connectivity (β=0.975), but ward number 6 has the worst connectivity ($\beta = 0.746$). (Indranagar).

The ultimate decision has been made, as well as the spatial topological relationship between spatial data, which is as follows:

Class	Class range	Characteristic of class	Number of the ward
I	$\beta_i \geq 1.20$	Very high connectivity or completely spatial connected	29, 49, 20, 32, 16, 48, 33, 14, 23, 25, 17, 24 and 34
II	$1.09 \leq \beta_i \leq 1.20$	High connectivity or completely T_0-spatial disconnected	26, 21, 12, 18, 43, 5, 11, 38, 22, 39, 40, 41 and 42

Class	Class range	Characteristic of class	Number of the ward
III	$1.00 \leq \beta_i \leq 1.09$	Low connectivity or spatial disconnected	4, 35, 28, 19, 27, 44, 3, 31, 9, 45, 13 and 15
IV	$\beta_i \leq 1$	Very low connectivity or completely T_1-spatial disconnected	2, 30, 1, 36, 46, 10, 47, 7, 8, 37 and 6

Result of Alpha, Beta and Gamma Index varying ward to ward. All the indices represent the degree of connectivity with distinctive results which is not conclusive.

For resource allocation, allotment, and apportionment, as well as policy decisions, road connectivity is the most important component. For a more requested decision assistance system, effective metrics of road connectivity were necessary. Over existing Alpha, Beta, and Gamma indies, the Suggested Composite Weightage Dimension Index (CWDI) is a more efficient measure of population dispersion. The Composite Weightage Dimension Index (CWDI) is a decision-making tool that helps with resource allocation and policy decisions in transportation planning and development. It also aids in the classification of the direct influence area in terms of connectivity and infrastructure for future development.

8 Conclusions

In this article, we have some results on spatial data set using the topological property. The GIS data and have separated the data as much as possible for suitable for making use in new setting in formation on separation axioms, so that it can get more profit from GIS spatial data analysis. On using this result one can analyze the GIS data. This model can further be applied under other situations. This article will help for the solving the different social problems like controlling crimes and for development by the spatial topology.

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Data Availability GPS.

Code Availability Not Applicable.

Declarations

Conflict of interest Authors declare that they have no conflict of interest.

Ethical statements We hereby declare that this manuscript is the result of our independent creation under the reviewers' comments. Except for the quoted contents, this manuscript does not contain any research achievements that have been published or written by other individuals or groups. We are the authors of this manuscript. The legal responsibility of this statement shall be borne by us.

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