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Parametric and Interval Estimation Under Step-Stress Partially Accelerated Life Tests Using Adaptive Type-II Progressive Hybrid Censoring

Intekhab Alam[1] · Aquil Ahmed[1]

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Abstract

In this paper, the likelihood estimation of model parameters and acceleration factor are considered under step-stress partially accelerated life test using adaptive type-II progressive hybrid censoring scheme, when the lifetime of the test units follows Exponentiated Pareto distribution. The numerical values of Maximum likelihood estimators are obtained using the Newton–Raphson technique. The performance of model parameters and acceleration factor in terms of mean square errors and biases are evaluated using the Monte-Carlo simulation technique.

Keywords Step-stress partially accelerated life test · Adaptive type-II progressive hybrid censoring · Exponentiated pareto distribution · Newton–raphson technique · Monte-carlo simulation

1 Introduction

In the present time, technology is improved day by day. The lifetime of products/items is improved due to this change in technology and the reliability of the products also become high. If a reliability practitioner tests the lifetime of these types of products under normal operating conditions, he receives no or very small number of failures at the end of the experiment. So, the reliability practitioners introduced a test for testing the lifetime of products in a short period and shortage of cost, called Accelerated life tests (ALTs) or partially accelerated life tests (PALTs). To obtain the information on the lifetime of the products, the reliability practitioner prefers this test because the test saves time and money and gives a good and appropriate result at the end of the test. There is a little difference in ALT and PALT; the test units are tested at both use and accelerated conditions in PALT, while in ALT, all test units are subjected to higher

✉ Intekhab Alam
intekhab.pasha54@gmail.com

1 Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh, India

usual stress levels. The failure behavior of the products at normal use conditions is estimated with the use of information data collected from the experiment in the ALTs or PALTs.

According to Nelson [1], the stress can be applied in many ways. The most common used methods are constant-stress PALTs (CSPALTs) and step-stress PALTs (SSPALTs). In this paper, we focus only on SSPALTs. In SSPALTs, the test units are run first at use (normal) condition, and after this, the units are run at accelerated condition until the test is ended if they do not fail for a specified duration of time. While in CSPALTs, each test unit is run at constant stress until either failure occurs or the test is ended.

In many circumstances, reliability practitioner may not be able to gather the exact information about the failure time for the testing units (data). This type of data is known as censored data, and the scheme is known as censoring. Many types of censoring occur in reliability theory, but type-I and type-II are the two most common and important censoring schemes. These two schemes have a major drawback. These schemes may not allow drawing the units at any point except the ending point of the test. To remove this type of complication, Balakrishnan and Aggarwala [2] introduced a new censoring scheme, which is known as progressive censoring type-II or progressively type-II hybrid censoring. This scheme allows to reliability practitioner to drawing the units at any point in the test. The progressive type-II hybrid censoring is defined as.

If we put n independent and identically test units on the testing and the observed failures to be m. $t_{(1)}$ is the time when the first failure occurs, R_1 test units randomly withdraw from the experiment at that time from the $n - 1$ remaining (surviving) units. $t_{(2)}$ is the time when the second failure occurs, R_2 test units randomly withdraw from the experiment at that time from the $n - 2 - R_1$ remaining (surviving) units. This process continues until the mth failure occurs. All the remaining units $R_m = n - m - R_1 - R_2 - \cdots - R_{m-1}$ are taken off from the experiment at this point. Where $R_1, R_2, \ldots, R_m$ are the whole numbers which are pre-fixed. These whole numbers may also occur randomly in some practical situations. Such as Yuen and Tse [3] presented a study on the number of patients taken from the clinical test at each phase is random and also can not be prefixed. Many authors presented a statistical inference on lifetime distribution using a progressive type-II censoring scheme for random removals. For example, Wu et al. [4], Yan et al. [5], Tse et al. [6], and Dey and Dey [7]. If we are talking about especially for progressive type-II censoring scheme, Kundu and Joarder [8], Ng et al. [9], Lin et al. [10], Mokhtari et al. [11] and Alma and Belaghi [12] are presented some work on ordinary life testing or PALT. PALTs are studied under type-I and type-II censoring schemes by several authors, such as Goel [13], DeGroot and Goel [14], Bhattacharyya and Soejoeti [15], Bai and Chung [16], Abdel-Ghani [17], Abd-Elfattah et al. [18], Aly and Ismail [19], Hassan and Thobety [20]. Ismail and Sarhan [21], Srivastava and Mittal [22], and Mohie EL-Din et al. [23] are presented studies on PALTs using progressive censoring (Tables 1, 2).

A vast literature is dedicated to the SSPALT research domain under the different censoring schemes. Ismail [24] presented a study first time on SSPALT with the use of an adaptive progressively type-II hybrid censoring scheme. Ismail [25] has been also done a work on SSPALT using adaptive type-I progressively hybrid censoring scheme for Weibull distribution. Alam et al. [26] has been done a study on SSPALT using

Table 1 Mean values of RABs, MSEs and MLEs, when the parameters λ, γ, τ, β and δ are fixed at 1, 1.2, 1.4, 2.2 and 4

(n, m)	Schemes	Estimates of λ			Estimates of γ			Estimates of β		
		MLE	RAB	MSE	MLE	RAB	MSE	MLE	RAB	MSE
(30, 10)	1	0.884	0.552	0.664	1.316	0.733	0.874	1.553	0.511	0.609
	2	1.221	0.631	0.713	0.993	0.862	0.912	1.442	0.573	0.676
	3	0.665	0.583	0.687	1.265	0.798	0.894	1.453	0.533	0.632
(40, 10)	1	1.342	0.503	0.612	0.997	0.688	0.764	1.523	0.430	0.521
	2	0.887	0.586	0.665	1.119	0.797	0.831	1.338	0.518	0.598
	3	0.778	0.544	0.633	1.331	0.734	0.809	1.556	0.466	0.544
(60, 10)	1	1.009	0.455	0.563	1.271	0.608	0.674	1.667	0.312	0.443
	2	0.554	0.538	0.599	1.331	0.721	0.764	1.487	0.462	0.533
	3	0.996	0.481	0.588	0.989	0.664	0.710	1.551	0.377	0.476
(30, 16)	1	0.775	0.408	0.510	1.441	0.536	0.593	1.563	0.233	0.365
	2	0.574	0.471	0.555	1.226	0.655	0.684	1.333	0.389	0.449
	3	1.286	0.438	0.539	0.992	0.589	0.611	1.442	0.278	0.397
(40, 16)	1	1.223	0.355	0.427	1.336	0.480	0.532	1.662	0.156	0.286
	2	0.889	0.412	0.488	1.454	0.548	0.617	1.553	0.300	0.365
	3	0.598	0.382	0.467	1.187	0.520	0.587	1.338	0.211	0.312
(60, 16)	1	0.776	0.299	0.317	1.442	0.398	0.453	1.432	0.108	0.199
	2	0.751	0.351	0.387	0.909	0.473	0.520	1.229	0.223	0.276
	3	1.334	0.316	0.349	1.227	0.432	0.477	1.343	0.187	0.221

adaptive type-I progressively hybrid censoring for Inverse Rayleigh distribution. Lone et al. [27] has been done a study on SSPALT using adaptive type-II progressively hybrid censoring scheme for Rayleigh distribution. Lone et al. [28] has been also done a study on SSPALT with competing risk when the lifetime of units follows Weibull distribution using adaptive type-I progressively hybrid censoring scheme. Mazen and Abu [29] proposed a work on Burr type-XII distribution using adaptive type-II progressively hybrid censoring scheme with approximation form of Lindley distribution. Alam et al. [30] presented a study on CSPALT using Exponentiated Exponential distribution for multiply censoring scheme.

Our study is based on estimating failure information on SSPALT when the lifetime of units follows Exponentiated Pareto distribution with the use of an adaptive progressively type-II hybrid censoring scheme.

The rest of the paper is organized as follows. The Exponentiated Pareto distbution and adaptive type-II progressive hybrid censoring scheme are introduced in Sect. 2. The test method is also described in this section. The maximum likelihood estimation method under used censoring scheme is described in Sect. 3. The point and interval estimation are also described in this section. In Sect. 4, the simulation study and results are presented. Finally, the conclusion is made in Sect. 5.

Table 2 Mean values of RABs, MSEs and MLEs, when the parameters λ, γ, τ, β and δ are fixed at 1.1, 1.4, 1.6, 2.3 and 6

(n,m)	Schemes	Estimates of λ			Estimates of γ			Estimates of β		
		MLE	RAB	MSE	MLE	RAB	MSE	MLE	RAB	MSE
(30, 10)	1	0.743	0.398	0.489	0.664	0.332	0.421	1.443	0.770	0.712
	2	0.993	0.440	0.554	0.993	0.390	0.467	1.654	0.821	0.764
	3	1.223	0.421	0.512	0.882	0.354	0.440	1.221	0.800	0.721
(40, 10)	1	1.372	0.307	0.400	1.020	0.287	0.341	1.748	0.698	0.632
	2	1.009	0.354	0.476	1.290	0.342	0.409	1.192	0.761	0.709
	3	0.673	0.320	0.431	0.993	0.315	0.383	1.332	0.724	0.685
(60 ,10)	1	0.783	0.223	0.322	1.293	0.234	0.265	1.354	0.608	0.543
	2	1.332	0.276	0.387	0.775	0.322	0.331	1.443	0.687	0.632
	3	0.983	0.243	0.356	0.993	0.242	0.321	1.754	0.643	0.589
(30, 16)	1	1.329	0.134	0.259	1.453	0.155	0.187	1.002	0.519	0.476
	2	1.002	0.190	0.301	1.002	0.243	0.263	1.121	0.609	0.556
	3	0.837	0.154	0.278	1.234	0.178	0.242	0.993	0.547	0.525
(40, 16)	1	1.309	0.080	0.167	0.995	0.119	0.044	1.029	0.433	0.388
	2	1.212	0.130	0.232	1.121	0.165	0.182	1.563	0.529	0.476
	3	1.008	0.118	0.209	1.442	0.121	0.103	1.774	0.477	0.429
(60, 16)	1	0.773	0.030	0.109	1.220	0.091	0.009	1.282	0.353	0.277
	2	0.929	0.102	0.142	1.009	0.110	0.110	1.632	0.443	0.380
	3	1.372	0.021	0.122	0.987	0.077	0.056	1.222	0.378	0.344

2 Description of Model and Test Method

The Pareto distribution plays an important role in analyzing skewed data and real-world situations, not only the field of statistics or economics. This distribution was mainly introduced to model the uneven distribution of wealth since it was recognized the way that a superior part of the wealth of any community is owned by a lesser percentage of the public. Sizes of sand molecules and groups of Bose–Einstein condensate are the closed examples of Pareto distributed Phenomenon. The Exponentiated Pareto (EP) distribution plays an important role in reliability analysis and life testing because the failure rate of EP is decreasing and upside-down bathtub form. The EP distribution also plays an important role in the field of medical, biological sciences and engineering, etc. for modeling and analyzing the data. Gupta et al. [31] introduced a two parameters distribution called EP distribution. If Y is Pareto distributed random variable, then $X = \log Y$ is called corresponding EP distribution.

The probability density function (pdf) of two parameters EP distribution is given as

$$f(t) = \lambda\gamma(1+t)^{-(\gamma+1)}[1-(1+t)^{-\gamma}]^{\lambda-1}, \ t, \lambda, \gamma > 0 \tag{1}$$

where λ, γ are the shape parameters.

The cumulative density function (cdf) of two parameters EP distribution is given as

$$F(t) = [1 - (1 + t)^{-\gamma}]^{\lambda} \tag{2}$$

The Reliability function of two parameters EP distribution is given as

$$R(t) = 1 - [1 - (1 + t)^{-\gamma}]^{\lambda} \tag{3}$$

The Hazard function of two parameters EP distribution is given as

$$H(t) = \frac{\lambda\gamma(1 + t)^{-(\gamma+1)}[1 - (1 + t)^{-\gamma}]^{\lambda-1}}{1 - [1 - (1 + t)^{-\gamma}]^{\lambda}} \tag{4}$$

As we know, mostly Pareto distributions are defined over one side of the real line. So, five EP distributions and their sveral properties are introduced by Saralees [32] for wider use in the field of statistics as well as economics, medical, and engineering also.

Ali et al. [33, 34] considered some exponentiated distributions including EP distribution and talked about their important properties. They showed that EP distribution provides a good and better fit for the tail-distribution of NASDAQ data. Shawky and Hanaa Abu-Zinadah [35] proposed how the different estimators of the unknown parameters of EP distribution can perform for different sample sizes and different parameter values. They considered the maximum likelihood estimation of the different parameters of EP distribution. Gupta et al. [31] showed the most important property of EP distribution. They studied that this distribution is used quite successfully in analyzing many lifetime data. The failure rate of this distribution depends on the shape parameter, it has decreasing and upside-down bathtub shaped failure rates. Kumar [36] estimated EP distribution parameters for Progressive type-II censored data using Random Removals scheme, while Singh et al. [37] estimated EP distribution parameters for Progressive type-II censored with Binomial Random Removals scheme. Mahmoud et al. [38] estimated EP distribution parameters under progressively type-II right censored data.

The lifetime, say, Y of an item under SSPALT is given as

$$Y = \begin{cases} T & \text{if } T \le \tau \\ \tau + \beta^{-1}(T - \tau) & \text{if } T > \tau \end{cases} \tag{5}$$

where the lifetime of an unit under normal operating conditions is denoted by T. β is acceleration factor and τ is stress change time.

The above technique has been proposed by DeGroot and Goal [39] and is known as the variable-transformation technique. The pdf under SSPALT is given as

$$f(y) = \begin{cases} 0, & y \le 0 \\ f_1(y), & 0 < y \le \tau \\ f_2(y), & y > \tau \end{cases} \tag{6}$$

where $f_1(y) = \lambda\gamma(1 + y)^{-(\gamma+1)}[1 - (1 + y)^{-\gamma}]^{\lambda-1}$ and $f_2(y) = \lambda\gamma(1 + \tau + \beta(y - \tau))^{-(\gamma+1)}[1 - (1 + \tau + \beta(y - \tau))^{-\gamma}]^{\lambda-1}$, $f_2(y)$ is obtained by applying the variable transformation technique on Eq. (1).

Kundu and Joarder [8] introduced a new censoring scheme which is called the progressive type-II hybrid censoring (PHCT-II). This scheme is described as follows.

If a life testing experimentation ends at a random time $\min(X_{m:m:n}, \delta)$. $1 \leq m \leq n$ and $0 < \delta < \infty$ are set to prior and $X_{1:m:n} \leq X_{2:m:n} \leq \cdots \leq X_{m:m:n}$ are the ordered failure times resultant from the experimentation, then $(R_1, R_2, \ldots, R_m)$ is described the progressive hybrid censoring (PHC) scheme. If the mth progressively censored observed will take place prior to time $\delta(\delta > X_{m:m:n})$, then the life testing experimentation terminates at the time $X_{m:m:n}$.. Otherwise, the experimentation will terminate at the time δ with $X_{j:m:n} < \delta < Y_{j+1:m:n}$, and all the remaining $(n - \sum_{i=1}^{j} R_i - j)$ existing units are censored at δ. Here, the number of failed units up to δ is j and it is a random variable.

The reliability practitioner (engineer) faces a difficulty if he uses the above discussed censoring schemes, the engineer may come out with a tiny sample size (even it is equal to zero), and so, this is impossible to come with usual inference procedures to get positive and exact outcomes. To remove this difficulty, the adaptive censoring scheme is introduced by Ng et al. [9]. Mohamed et al. [40] has been done an estimation procedure for Generalized Pareto distribution using adaptive type-II progressive hybrid censoring. The observed number of failures m is fixed to prior in this scheme, and the experimentation time is open to run overtime δ. The experiment will be in process along with pre-specified progressive censoring schemes $(R_1, R_2, R_3, \ldots, R_m)$ if $X_{m:m:} < \delta$, else, the surviving units (existing units), which following the $(j + 1)th$ to $(m - 1)th$ observed failures, are not removed from the experiment. All the existing units $R_m = n - m - \sum_{i=1}^{j} R_i$ are taken back from the experimant at the moment point $X_{m:m:n}$ if m observed failure attained, i.e. $R_{j+1} = \cdots = R_{m-1} = 0$. The progressive type-II censoring is attained if $n \to \infty$ and the traditional type-II censoring is attained, if $n \to 0$. If the engineer have no boundation to change the value of δ, then this type of scheme is called an adaptive progressive type-II hybrid censoring scheme (APHCT-II). This change in δ is done to adjust the optimum of lessened testing time and a greater possibility of monitoring many failures.

3 Maximum Likelihood Estimation

If $Y_1, Y_2, \ldots, Y_n$ be the n independent and identically distributed (i.i.d.) lifetime of test units which are following the EPD and $y_{1:m:n} < \cdots < y_{n_u+1:m:n} < \cdots < y_{J:m:n} \leq \delta < y_{J+1:m:n} < \cdots < y_{m:m:n}$ are the m completely (ordered) lifetimes.

3.1 Point Estimation

The likelihood function under SSPALT for m ordered lifetime data set using the APHC scheme is given as

$$L(\lambda, \gamma, \beta) \propto \prod_{i=1}^{m} f_1(y_{i:m:n}) f_2(y_{i:m:n}) \prod_{i=1}^{J} [S_1(\tau)]^{R_i} [S_2(y_{m:m:n})]^{n-m-\sum_{i=1}^{J} R_i} \tag{7}$$

where, $f_1(y_{i:m:n}) = \lambda\gamma(1 + y_i)^{-(\gamma+1)}[1 - (1 + y_i)^{-\gamma}]^{\lambda-1}$

$$\begin{aligned}
&f(y_{i:m:n}) = \lambda\gamma(1 + \tau + \beta(y_i - \tau))^{-(\gamma+1)}[1 - (1 + \tau + \beta(y_i - \tau))^{-\gamma}]^{\lambda-1}\\
&S_1(\tau) = 1 - [1 - (1 + \tau)^{-\gamma}]^{\lambda}\\
&S_2(y_{m:m:n}) = 1 - [1 - (1 + \tau + \beta(y_{m:m:n} - \tau))^{-\gamma}]\\
&\chi_i = \tau + \beta(y_i - \tau), \;\; \chi_m = \tau + \beta(y_{m:m:n} - \tau)\\
&J = n_u + n_a
\end{aligned}$$

n_u is the number of units which are failed on the usual condition and n_a is the number of units which are failed in accelerated condition.

The log-likelihood function is obtained by taking the natural logarithm of Eq. (7) and is given as

$$\begin{aligned}
\ln L(\lambda, \gamma, \beta) =& m \ln(\lambda\gamma) - (\gamma + 1) \sum_{i=1}^{m} \ln(1 + y_i) + (\lambda - 1)\\
&\times \sum_{i=1}^{m} \ln[1 - (1 + y_i)^{-\gamma}] + m \ln(\beta\lambda\gamma) - (\gamma + 1)\\
&\times \sum_{i=1}^{m} \ln(1 + 1 + \tau + \beta(y_i - \tau)) + (\lambda - 1)\\
&\times \sum_{i=1}^{m} \ln[1 - (1 + (1 + \tau + \beta(y_i - \tau))^{-\gamma}]\\
&+ \sum_{i=1}^{J} R_i \ln[1 - (1 - (1 + \tau)^{-\gamma})^{\lambda}]\\
&+ \sum_{i=1}^{J} (n - m - \sum_{i=1}^{J} R_i) \ln[1 - (1 - (1 + \tau + \beta(y_{m:m:n} - \tau))^{-\gamma})^{\lambda}]
\end{aligned}$$

$$\begin{aligned}
\ln L =& m \ln(\lambda\gamma) - (\gamma + 1) \sum_{i=1}^{m} \ln(1 + y_i) + (\lambda - 1) \sum_{i=1}^{m} \ln[1 - (1 + y_i)^{-\gamma}]\\
&+ m \ln(\beta\lambda\gamma) - (\gamma + 1) \sum_{i=1}^{m} \ln(1 + 1 + \chi_i) + (\lambda - 1)\\
&\times \sum_{i=1}^{m} \ln[1 - (1 + (1 + \chi_i)^{-\gamma}]
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{J} R_i \ln[1-(1-(1+\tau)^{-\gamma})^{\lambda}] \\
&+\sum_{i=1}^{J} (n-m-\sum_{i=1}^{J} R_i) \ln[1-(1-(1+\chi_m)^{-\gamma})^{\lambda}] \qquad (8)
\end{aligned}
$$

where $\ln L = \ln L(\lambda, \gamma, \beta)$.

Differentiate the above log-likelihood equation with respect to parameters λ, γ and β, and equating to zero to get Maximum likelihood (ML) estimates of λ, γ and β, respectively.

$$
\begin{aligned}
\frac{\partial \ln L}{\partial \lambda} = {} & 2m\lambda^{-1} + \sum_{i=1}^{m} \ln[1-(1+y_i)^{-\gamma}] + \sum_{i=1}^{m} \ln[1-(1+(1+\chi_i)^{-\gamma}] \\
& - \sum_{i=1}^{J} R_i \frac{\lambda(1-(1+\tau)^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau)^{-\gamma})^{\lambda}} \\
& - \sum_{i=1}^{J} (n-m-\sum_{i=1}^{J} R_i) \frac{\lambda(1-(1+\chi_m)^{-\gamma})^{\lambda-1}}{1-(1-(1+\chi_m)^{-\gamma})^{\lambda}} = 0 \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \ln L}{\partial \gamma} = {} & 2m\gamma^{-1} - \sum_{i=1}^{m} \ln[(1+y_i) - \sum_{i=1}^{m} \frac{\gamma(1+y_i)^{-\gamma-1}}{1-(1+y_i)^{-\gamma}} - \sum_{i=1}^{m} \ln[1+\tau+\beta(y_i-\tau)] \\
& - \sum_{i=1}^{m} \frac{\gamma(1+\chi_i)^{-\gamma-1}}{1-(1+\chi_i)^{-\gamma}} \\
& - \sum_{i=1}^{J} (n-m-\sum_{i=1}^{J} R_i) \frac{\lambda\gamma(1-(1+\chi_m)^{-\gamma})^{\lambda-1}(1+\chi_m)^{-\gamma-1}}{1-(1-(1+\chi_m)^{-\gamma})^{\lambda}} \\
& - \sum_{i=1}^{J} R_i \frac{\lambda\gamma(1+\tau)^{-\gamma-1}(1-(1+\tau)^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau)^{-\gamma})^{\lambda}} = 0 \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \ln L}{\partial \beta} = {} & m\beta^{-1} - \sum_{i=1}^{m} \ln[(1+y_i) - (1+\gamma) \sum_{i=1}^{m} \frac{(y_i-\tau)}{(1+\chi_i)} \\
& - \gamma(\lambda-1) \sum_{i=1}^{m} \frac{(y_i-\tau)(1+\chi_i)^{-\gamma-1}}{1-(1+\chi_i)^{-\gamma}} + \sum_{i=1}^{J} (n-m-\sum_{i=1}^{J} R_i) \\
& \times \frac{\lambda\gamma(1-(1+1+\chi_m)^{-\gamma})^{\lambda-1}(1+\chi_m)^{-\gamma-1}(y_{m:m:n}-\tau)}{1-(1-(1+\chi_m)^{-\gamma})^{\lambda}} = 0 \qquad (11)
\end{aligned}
$$

It looks impossible to obtain an exact solution of above non-linear Eqs. (9), (10), and (11). So, an iterative technique (Newton–Raphson Method) is applied to obtain ML estimates of λ, γ and β.

3.2 Interval Estimation

Using the data obtained from the APHC scheme, the interval ML estimates of λ, γ and β are estimated here. The asymptotic distribution of ML estimates is suggested by Miller [41] and this distribution for λ, γ and β is given as

$$((\hat{\lambda}-\lambda),(\hat{\gamma}-\gamma),(\hat{\beta}-\beta)) \rightarrow N(0, I^{-1}(\lambda,\gamma,\beta))$$

where $I^{-1}(\lambda,\gamma,\beta)$ is the variance–covariance matrix of unknown model parameters λ, γ and β.

The 3×3 matrix $I^{-1}(\lambda,\gamma,\beta)$, which is approximated equal to $I(\lambda,\gamma,\beta)$ based on used censoring scheme is given as

$$I=\begin{bmatrix} -\frac{\partial^2 \ln L}{\partial\lambda^2} & -\frac{\partial^2 \ln L}{\partial\lambda\partial\gamma} & -\frac{\partial^2 \ln L}{\partial\lambda\partial\beta} \\ -\frac{\partial^2 \ln L}{\partial\gamma\partial\lambda} & -\frac{\partial^2 \ln L}{\partial\gamma^2} & -\frac{\partial^2 \ln L}{\partial\gamma\partial\beta} \\ -\frac{\partial^2 \ln L}{\partial\beta\partial\lambda} & -\frac{\partial^2 \ln L}{\partial\beta\partial\gamma} & -\frac{\partial^2 \ln L}{\partial\beta^2} \end{bmatrix} \tag{12}$$

The elements of the matrix are

$$\begin{aligned}
-\frac{\partial^2 \ln L}{\partial\lambda^2} &= 2m\lambda^{-2}+\sum_{i=1}^{J} R_i \frac{\lambda(1-(1+\tau)^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau)^{-\gamma})^{\lambda}} \\
&\quad\times\left[-\lambda^{-1}+\ln(1-(1+\tau)^{-\gamma})+\frac{\lambda(1+(1+\tau)^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau)^{-\gamma})^{\lambda}}\right] \\
&\quad+\sum_{i=1}^{J}(n-m-\sum_{i=1}^{J}R_i)\frac{\lambda(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda}} \\
&\quad\times\left[-\lambda^{-1}+\ln(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})\right. \\
&\quad\left.+\frac{\lambda(1+(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda}}\right]
\end{aligned}$$

$$\begin{aligned}
-\frac{\partial^2 \ln L}{\partial\beta^2} &= m\beta^{-2}-\sum_{i=1}^{m}(\gamma+1)(y_i-\tau)^2(1+\tau+\beta(y_i-\tau))^{-2}+\sum_{i=1}^{m}(\lambda-1)\gamma(y_i-\tau) \\
&\quad\times\left[-\frac{(\gamma+1)(y_i-\tau)}{1+\tau+\beta(y_i-\tau)}+\frac{\lambda(y_i-\tau)(1+\tau+\beta(y_i-\tau))^{-\gamma-1}}{1-(1+\tau+\beta(y_i-\tau))^{-\gamma}}\right]-\sum_{i=1}^{J}\lambda\gamma(y_{m:m:n}-\tau) \\
&\quad\times\left[\frac{(\lambda-1)\gamma(y_{m:m:n}-\tau)(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma-1}}{1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma}}-\frac{(\gamma+1)(y_{m:m:n}-\tau)}{1+\tau+\beta(y_{m:m:n}-\tau)}\right. \\
&\quad\left.-\frac{\lambda\gamma(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma-1}(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda}}\right]
\end{aligned}$$

$$
\begin{aligned}
-\frac{\partial^2 \ln L}{\partial\lambda\partial\gamma} &= -\frac{\partial^2 \ln L}{\partial\gamma\partial\lambda}\\
-\frac{\partial^2 \ln L}{\partial\lambda\partial\gamma} &= -\sum_{i=1}^{m}\frac{\gamma(1+y_i)^{-\gamma-1}}{1-(1+y_i)^{-\gamma}} - \sum_{i=1}^{m}\frac{\gamma(1+\tau+\beta(y_i-\tau))^{-\gamma-1}}{1-(1+\tau+\beta(y_i-\tau))^{-\gamma}}\\
&\quad + \sum_{i=1}^{J}\lambda R_i\frac{(1-(1+\tau)^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau)^{-\gamma})^{\lambda}}\\
&\quad \times\left[\frac{(\lambda-1)\gamma(1+\tau)^{-\gamma-1}}{1-(1+\tau)^{-\gamma}} + \frac{\lambda\gamma(1+\tau)^{-\gamma-1}(1+\tau)^{\lambda-1}}{1-(1-(1+\tau)^{-\gamma})^{\lambda}}\right]\\
&\quad + \sum_{i=1}^{J}\lambda(n-m-\sum_{i=1}^{J}R_i)\frac{(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda}}\\
&\quad \times\left[\frac{(\lambda-1)\gamma(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma-1}}{1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma}}\right.\\
&\quad \left.+\frac{\lambda\gamma(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma-1}(1+\tau+\beta(y_{m:m:n}-\tau))^{\lambda-1}}{1-(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda}}\right]
\end{aligned}
$$

$$
\begin{aligned}
-\frac{\partial^2 \ln L}{\partial\lambda\partial\beta} &= -\frac{\partial^2 \ln L}{\partial\beta\partial\lambda}\\
-\frac{\partial^2 \ln L}{\partial\lambda\partial\beta} &= \sum_{i=1}^{m}\frac{(y_i-\tau)\gamma(1+\tau+\beta(y_i-\tau))^{-\gamma-1}}{1-(1+\tau+\beta(y_i-\tau))^{-\gamma}}\\
&\quad + \sum_{i=1}^{J}(n-m-\sum_{i=1}^{J}R_i)\frac{\lambda(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda}}\\
&\quad \times\left[\frac{(\lambda-1)\gamma(y_{m:m:n}-\tau)(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma-1}}{1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma}}\right.\\
&\quad +\frac{\lambda\gamma(y_{m:m:n}-\tau)(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda}}\\
&\quad \left.\times(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma-1}\right]
\end{aligned}
$$

$$
\begin{aligned}
-\frac{\partial^2 \ln L}{\partial\gamma\partial\beta} &= -\frac{\partial^2 \ln L}{\partial\beta\partial\gamma}\\
-\frac{\partial^2 \ln L}{\partial\beta\partial\gamma} &= \sum_{i=1}^{m}\frac{(y_i-\tau)}{1+\tau+\beta(y_i-\tau)} - \sum_{i=1}^{m}(\lambda-1)(y_i-\tau)\frac{(1+\tau+\beta(y_i-\tau))^{-\gamma-1}}{1-(1+\tau+\beta(y_i-\tau))^{-\gamma}}\\
&\quad \times\left[\ln(1+\tau+\beta(y_i-\tau)) + \frac{\gamma(1+\tau+\beta(y_i-\tau))^{-\gamma-1}}{1-(1+\tau+\beta(y_i-\tau))^{-\gamma}}\right]\\
&\quad - \sum_{i=1}^{J}(n-m-\sum_{i=1}^{J}R_i)\lambda\gamma(y_{m:m:n}-\tau)
\end{aligned}
$$

$$\times \frac{(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma-1}(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda}}$$
$$\times \Bigg[(\lambda-1)\gamma(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma-1}+\gamma^{-1}$$
$$+\ln(1+\tau+\beta(y_{m:m:n}-\tau))$$
$$+\frac{\lambda(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma-1}(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda-1}}{1-(1-(1+\tau+\beta(y_{m:m:n}-\tau))^{-\gamma})^{\lambda}}\Bigg]$$

The $100(1-\alpha)\%$ approximated two-sided limits of confidence for parameters λ, γ and $\beta$are given as

$$\hat{\lambda} \pm Z_{\gamma/2}\sqrt[-1]{I_{11}^{-1}(\hat{\lambda},\hat{\gamma},\hat{\beta})},\ \hat{\gamma} \pm Z_{\gamma/2}\sqrt[-1]{I_{22}^{-1}(\hat{\lambda},\hat{\gamma},\hat{\beta})} \text{ and } \hat{\beta} \pm Z_{\gamma/2}\sqrt[-1]{I_{33}^{-1}(\hat{\lambda},\hat{\gamma},\hat{\beta})}.$$

4 Simulation Study and Results

The comparison of unlike censoring schemes is probably impossible for unlike values of parameters. In this situation, reliability praticioner used different type of techniques or softwares. The Monte-Carlo simulation technique along with R-Software is a useful technique in this type of situation. So, we are using this technique in this section for comparision of performance of MLEs values. This performance of MLEs is evaluated in terms of mean squared errors (MSEs) and Relative absolute biases (RABs). The three censoring schemes are considered for this study and the schemes are.

Scheme (I)$R_1 = R_2 = R_3 = \cdots = R_{m-1},\ R_m = n-m.$
Scheme (II)$R_1 = n-m,\ R_2 = R_3 = R_4 \cdots = 0.$
Scheme (III) $R_1 = R_2 = R_3 = \cdots = R_{m-1},\ R_m = n-2m+1.$

1000 simulations based MSEs and RABs are estimated for this study. The basic steps for this study are given as.

i. First, choose the values of parameters $n,\ m,\ \tau,\ \delta,\ \lambda,\ \gamma$ and β.
ii. Generate a random sample from Exponentiated Pareto distribution with size n by the Inverse CDF method in both cases (normal and accelerated condition).
iii. Generate the progressive hybrid censored sample for the parameters $n,\ m,\ \tau,\ \delta,\ \lambda,\ \gamma$ and β using the technique presented in Eq. (6).
iv. The sample data set used in this study for APHC scheme is given as

$$y_{1:m:n} < y_{2:m:n} < \ldots y_{n_u:m:n} \le \tau < y_{n_u+1:m:n} \le \delta < y_{J+1:m:n} < \ldots < y_{m:n:n}$$

v. Compute the MSEs and RABs correlated with MLEs of parameters.

5 Conclusion

The likelihood estimation of Exponentiated Pareto distribution under step-stress partially accelerated life test models using an adaptive type-II progressive hybrid censoring scheme is considered in this paper. The Fisher Information matrix under this censoring scheme is also estimated. The numerical values of MLEs of parameters are obtained using an iterative technique (Newton–Raphson technique), and their characteristics are measured and recorded in terms of RABs and MSEs. We have seen that if the sample size increases, then the values of RABs and MESs decreases. So, the maximum likelihood estimators are consistent as well as asymptotically normally distributed also. Bayesian inference using step stress PALT under the same censoring scheme will be considered for future work.

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