

# Waste Load Allocation Using Machine Scheduling: Model Application

S. Mohan<sup>1</sup> · K. Pavan Kumar<sup>1</sup>

Received: 14 May 2015 / Accepted: 4 January 2016 / Published online: 16 February 2016  
© Springer International Publishing Switzerland 2016

**Abstract** A novel approach for waste load allocation, namely, waste load scheduling, following the concept of machine scheduling has been developed and applied to a case study. The conceptual development and mathematical formulation for the waste load scheduling model has been discussed in the companion paper. The focus of this paper is to identify a suitable solution technique for solving the waste load scheduling model and then apply it to a case study river system in Tamil Nadu, India. The mathematical formulation for the waste load scheduling model was developed by treating it in an analogous way to a machine scheduling problem (as explained in the companion paper). Dynamic programming (DP) has been used by several researchers for solving such machine scheduling and sequencing problems and thus the same method is adopted here for solving the proposed waste load scheduling problem. To demonstrate the practical application of the proposed method, the model was applied to a real case study river system, namely, Thambraparani river system, which is located in the southern part of the Tamil Nadu state in India. The results from the waste load scheduling model proved that the method is very effective in reducing the total waste load discharge into the river and thereby achieving a high level of river water quality. Comparing the results of waste load scheduling (WLS) model with SEDP (Seasonal Effluent Discharge program) and EBP (Effluent By-Pass Piping) model shows that the proposed WLS model satisfies all the three water quality management goals (i.e., cost minimization, minimizing overall inequity, and minimizing total waste load discharge into the river) to a greater degree when compared to SEDP and EBP models.

---

**Electronic supplementary material** The online version of this article (doi:10.1007/s40710-016-0122-x) contains supplementary material, which is available to authorized users.

---

✉ K. Pavan Kumar  
pavankumar1978@mail.com

S. Mohan  
smohan1950@gmail.com

<sup>1</sup> Department of Civil Engineering, Indian Institute of Technology, Madras, Chennai 600036, India

**Keywords** Water quality management · Waste load scheduling and sequencing · Dynamic programming · River water quality

## 1 Introduction

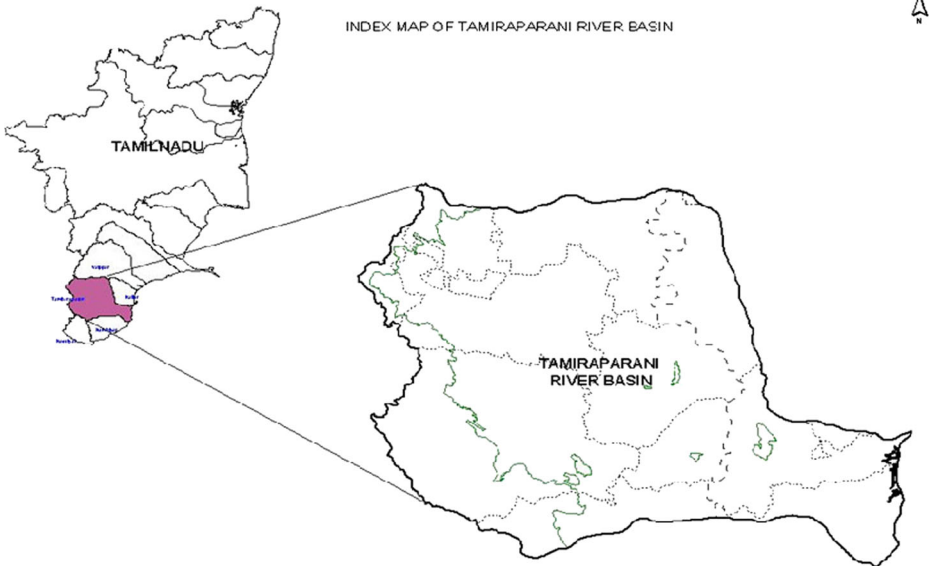
In the companion paper, the conceptual framework and model development for the newly proposed waste load scheduling (WLS) model was discussed. In the present paper, the method for solving the waste load scheduling problem is discussed along with the application of the model to a real life case study area. The mathematical formulation for WLS model was developed by treating it in an analogous way to a machine scheduling problem, thereby converting it to a standard sequencing and scheduling problem. A simple single machine scheduling problem belongs to the class of sequencing and scheduling problems. A typical sequencing and scheduling problem involves stage-wise decision making, and therefore, dynamic programming is more suitable to solve such type of problems. Dynamic programming (DP) has been widely and successfully applied to many machine scheduling problems. A good review of application of DP to various machine scheduling problems can be seen in Cheng and Sin (1990) and McCarthy and Liu (1993).

In water resources engineering, DP has been used to solve a wide variety of problems, such as water resources systems (Larson and Keckler 1969; Heidari et al. 1971), reservoir operation (Young 1967; Hall et al. 1968; Becker and Yeh 1974; Yakowitz 1982), water quality management (Liebmann and Lynn 1966; Shih and Krishnan 1969; Chang and Yeh 1973; Cardwell and Ellis 1993; Mujumdar and Saxena 2004); irrigation scheduling (Yaron et al. 1987; Sunantara and Ramirez 1997). A good review of application of DP to various water resources engineering problems can be found in Yakowitz (1982). A review of literature of similar works revealed that DP can be effectively used for solving various types of water resources engineering problems, when sequential decision making is the major criterion.

The proposed waste load scheduling problem is a typical sequencing and scheduling problem. The objective of the WLS problem is to determine an optimal sequence among the  $n$  number of point source polluters that maximizes the total waste load allocation. The problem therefore involves stage-wise decision making such that at each stage the decision variables will be the optimal sequence of polluters and their corresponding waste load allocation. As the problem requires stage-wise decision making, dynamic programming is possibly the best choice of the available models for solving the waste load scheduling model. The supplementary material consist a hypothetical waste load scheduling problem which is solved using dynamic programming technique (Supplementary Material S1).

## 2 Application: Case Study

Thambraparani River, located in the southern part of Tamil Nadu state of India was selected as a case study area to validate the proposed waste load allocation scheduling model. Figure 1 shows the index map of the case study area. Thambraparani flows for a distance of about 120 km and has a perennial source of water. Irrigation and drinking water supply take up a large portion of river water supply. Most of the towns situated in the catchment area have a partial to no sewerage system. This lack of an organized sewerage system makes the river more vulnerable since most of the sewage generated from various municipalities or towns are



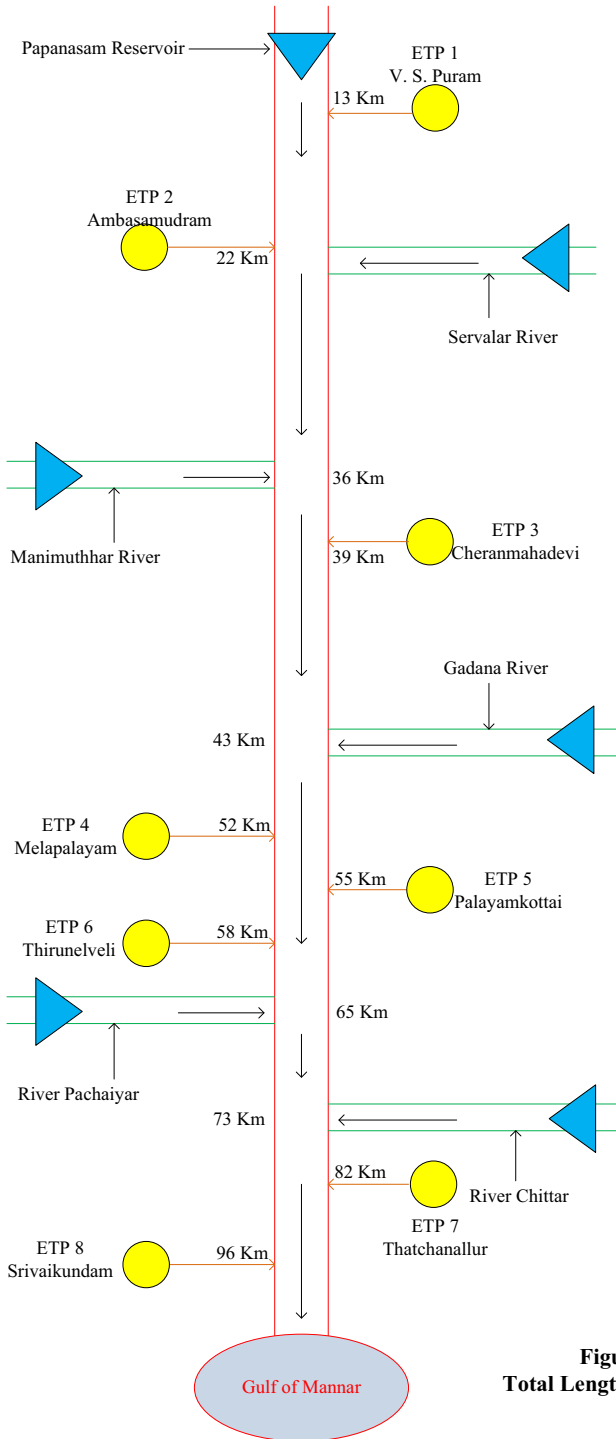
**Fig. 1** Index map of Tambraparani river basin (Source: Tambraparani Microlevel Study, IWS, Chennai)

discharged directly into the river without proper treatment. Although the water quality in the river is within the permissible limits (Annual Report on Air and Water Quality Network 2003) the increase in population and urbanization will cause great stress on both water quantity and quality.

In order to arrest the indiscriminate discharge of effluent into the Tambraparani River, TWAD (Tamil Nadu Water Supply and Drainage) Board has proposed to construct and install an underground sewerage system and sewage treatment plants for some of the major towns in the river basin (Microlevel Study of Tambraparani River 2004). When the system will be in operation the continuous discharge of effluent from waste water treatment plants may discharge directly into the river which may have a severe impact on the river water quality. This necessitates developing a waste load allocation program for the river basin so that the assimilative capacity of the river can be used for effluent discharge while still maintaining the river water quality to the desired standard levels. The only towns which have partial to full sewerage system are Tirunelveli, V.S. Puram, Ambasamudram, Cheranmahadevi, Palayamkottai, Melapalayam and Srivaikundam (Fig. 2). The data pertaining to the waste flow rate and waste load in terms of kg/d of BOD for the towns was determined based on their population size. Data on population for the various towns were collected from census data published by the Government of India. Table 1 gives the details of population, wastewater flow rate and raw waste load quantity for each town.

### 3 Forward Dynamic Programming Algorithm for the Waste Load Scheduling Problem

Extending the analysis of WLS described in the example problem, the DP algorithm for the case study area for a period of one year can be developed as follows. The analysis was carried



**Figure Not To Scale**  
**Total Length of the River = 120 Km**

**Fig. 2** Schematic diagram of Thambraparani River with locations of point source polluters

**Table 1** Population, wastewater flow rate and raw waste load details for the towns which are discharging the effluent directly into the river

Town	Location (km)	Population (2011)	Wastewater flowrate <sup>a</sup> (m <sup>3</sup> /s)	Raw waste load <sup>b</sup> (kg/d of BOD <sub>5</sub> )
V. S. Puram	13	97,275	0.122	3891
Ambasamudram	22	427,557	0.062	1975
Cheranmahadevi	39	129,825	0.162	5193
Melapalayam	52	99,525	0.124	3981
Palayamkottai	55	142,250	0.178	5690
Tirunelveli	58	492,375	0.615	19,695
Thatchanallur	82	97,350	0.122	3894
Srivaikundam	96	25,325	0.032	1013

<sup>a</sup> The per-capita wastewater contribution is 108 lpcd i.e. 80 % of the per-capita domestic water consumption (135 lpcd)

<sup>b</sup> Raw wastewater load was calculated based on a per-capita BOD load of 0.04 kg/pc/d

out by dividing a year into twelve seasons with length of each season in days being equal to the length of the corresponding month in days. Each season was accommodated with two schedules. This was done to ensure that the polluters get greater opportunity for effluent discharge in a season and also to minimize the effluent storage space. Let  $S = \{1, 2, 3, \dots, s\}$  represent the set of seasons within a year and  $R = \{1, 2, 3, \dots, r\}$  represent the set of possible number of schedules in a season. For the waste load scheduling problem, the stage represents the number of days elapsed in a schedule, the state variables at stage  $t$  will be the number of polluters remaining for effluent discharge, possible values of effluent processing time, and the accumulated effluent volume for the polluters. The decision variables at stage  $t$  will be the optimal sequence of polluters and the total waste load allocation for the optimal sequence. Hence the objective function for the DP recursive equation can be expressed as:

$$WL_{t,r,s}^* = \max \sum_{s=1}^S \sum_{r=1}^R \sum_{t=1}^T \left[ W_{t,r,s} \left( P_i^{(t+1)-e_{j,t}}, e_{j,t} \right) + W_{t-1,r,s} (S_{k,r}) \right] \forall i, j, r, s \quad (1)$$

where  $WL_{t,r,s}^*$  is the optimal waste load allocation (in kg/d of BOD<sub>5</sub>) for stage  $t$  in schedule  $r$  and season  $s$ ;  $W_{t,r,s} \left( P_i^{(t+1)-e_{j,t}}, e_{j,t} \right)$  represents the waste load allocation at stage  $t$ , for schedule  $r$  during season  $s$  as a function of the polluter discharging effluent on day  $(t+1) - e_{j,t}$ , possible effluent processing time values for stage  $t$  ( $e_{j,t}$ ).  $W_{t-1,r,s}(S_{k,r})$  represents the waste load allocation for all the possible sequences up to stage  $t-1$ . The recursive Eq. 1 is subjected to the following constraints.

### 3.1 Constraint on Effluent Discharge Time

A polluter is not allowed to discharge the effluent until the effluent discharged by previous polluter is completely assimilated by the river. Let  $d_{i,r,s}^k$  be the day on which polluter  $i$  discharges the effluent in schedule  $r$ , during season  $s$ , with  $k$  being the position of polluter  $i$  in scheduling sequence  $r$ . Let  $d_{j,r,s}^{k-1}$  be the day on which polluter  $j$  discharges the effluent in  $r$ ,

during season  $s$ , with  $k-1$  being the polluter’s position in the scheduling sequence, then the constraint on effluent discharge time can be written as:

$$d_{i,r,s}^k \geq d_{j,r,s}^{k-1} + EPT_{j,r,s} \text{ and } i \neq j \tag{2}$$

where,  $EPT_{j,r,s}$  is the effluent processing time for polluter  $j$ , within the schedule  $r$  and during season  $s$ .

### 3.2 Constraint on Schedule Length

According to this constraint, all polluters need to be accommodated within the predefined schedule length. In other words, the sum of  $EPTs$  for all the polluters should be equal to the total length of the schedule. This can be written as:

$$EPT_{1,r,s} + EPT_{2,r,s} + \dots + EPT_{n,r,s} = SL_r \tag{3}$$

$SL_r$  is the predefined length of a schedule  $r$  (i.e., one week, ten days or 15 days etc.) in days.

### 3.3 Constraint on Effluent Processing Time (EPT)

This constraint stipulates that each polluter will have a maximum and minimum possible  $EPT$  values. The minimum value ( $T_r^{min}$ ) will be one day and the maximum value is given by  $T_r^{max} = SL_r - N \times T_r^{min}$ , where  $SL_r$  is the length of schedule  $r$  in days and  $N$  is the total number of polluters. Mathematically, the constraint can be expressed as:

$$T_r^{min} \leq EPT_{i,r,s} \leq T_r^{max} \tag{4}$$

### 3.4 Constraint on Effluent Storage Space

The constraint on effluent storage space will be applicable to only those polluters for whom there is a limitation for land acquisition. If  $Qe_i$  is the effluent flow rate (in  $m^3/d$ ) for polluter  $i$ ,  $EST_i^{max}$  is the maximum duration for which polluter has to store the effluent,  $Ar_i^{max}$  is the maximum area available (in  $m^2$ ) for polluter  $i$  for effluent storage and  $H$  is the depth of effluent storage unit (in  $m$ ), then the constraint on effluent storage space can be written as:

$$Qe_i \times EST_i^{max} \leq Ar_i^{max} \times H \tag{5}$$

### 3.5 Constraint on Water Quality

It is important that the river water quality meets the water quality standard and this constraint is given in terms of DO. The DO is used as a surrogate for identifying the health of the river water. The effluent discharged by the polluters should meet the DO standards at all the  $m$  number of checkpoints along the river. Let  $[F_{t,r,s}]_{m \times k}$  denote the pollutant transfer coefficient matrix due to  $k$  number of polluters during stage  $t$  in schedule  $r$  and for season  $s$ . Let  $[DO_s^{sat}]_{m \times 1}$  be the saturation DO matrix for season  $s$ . Let  $[DO_s^{des}]_{m \times 1}$  be the desirable DO level matrix

for season  $s$ . Let  $[D_{u,s}]_{m \times 1}$  be the DO deficit matrix due to non-point source pollution for season  $s$ . The constraint on meeting DO standard then can be expressed as:

$$[DO_s^{sat}]_{m \times 1} - [F_{t,r,s}]_{m \times k} \times [WLA_{t,r,s}]_{k \times 1} - [D_{u,s}]_{m \times 1} \geq [DO_s^{des}]_{m \times 1} \quad (6)$$

where  $[WLA_{t,r,s}]_{k \times 1}$  is the waste load allocation matrix for stage  $t$ , for schedule  $r$  during season  $s$  involving pollutants in sequence  $S_k$ .

### 3.6 Constraint on Treatment Level

The upper limit (maximum possible treatment level with available technology) and lower limit (minimum treatment level need to be prescribed by the PCA for all pollutants considering the river condition and the season) on the treatment level. This can be expressed as:

$$x_i^{min} \leq x_{i,r,s} \leq x_i^{max} \quad (7)$$

where  $x_{i,r,s}$  is the treatment level for pollutant  $i$  for schedule  $r$  and during season  $s$ .

## 4 Results and Discussion

To demonstrate the environmental and long-term economic advantage of the proposed waste load scheduling, the results of the model were compared with the traditional water quality management methods, like seasonal effluent discharge program (Boner and Furland 1982; Reheis et al. 1982; Rossmann 1989; Lence et al. 1990; Ng et al. 2006; Jha and Gu 2010) and effluent by-pass piping EBP (Graves et al. 1969). In both of these methods the control is applied directly to the waste water, similar to the WLS model, and hence it was felt appropriate to compare the proposed model with seasonal effluent discharge program (SEDP) and effluent by-pass piping (EBP) model. For all the models the desirable DO level at any checkpoint was taken to be 5 mg/L. For the waste load scheduling model the permissible BOD value for defining effluent processing time was taken to be 5 mg/L. Also for developing the schedule the year was divided into twelve seasons, each season being equivalent to the corresponding month. Again each season was divided into two schedules, the length of each schedule being equivalent to half of the number of days in a season. After solving the waste load scheduling optimization model, the results thus obtained were compared with other two models, i.e., seasonal effluent discharge program and effluent by-pass piping model. The models were compared for the total system cost, total waste load discharge into the river and resulting water quality (measured in terms of DO level) of the river, and the overall equity achieved in total cost allocation among the pollutants.

### 4.1 Waste Load Allocation and River Water Quality

Upon solving the waste load scheduling optimization model, the season-wise waste load allocation for a typical year for the pollutants were obtained which is shown in Table 2. The annual waste load allocation for the pollutants under the three different water quality management programs is given in Table 3. For the EBP model it was found that for pollutants P4 and P5, the cost of by-passing the entire effluent to reach R8 will be more economical when compared to constructing an effluent treatment plant. Similarly, P1 can save some money through by-passing some part of the effluent to reach R2. From the results in Table 3, it can be

**Table 2** Optimal waste load scheduling for a typical year

Month	P1		P2		P3		P4		P5		P6		P7		P8	
	Day	WLA <sup>1</sup>	Day	WLA	Day	WLA	Day	WLA	Day	WLA	Day	WLA	Day	WLA	Day	WLA
Jan	3	3184	1	1383	6	2752	7	8881	10	3557	11	14,191	14	10,257	5	2205
	17	3888	16	1643	20	8382	23	10,098	28	4126	29	18,533	26	10,000	19	2755
Feb	2	3183	1	811	5	2014	7	9680	4	1313	12	16,655	10	8561	6	2760
	16	3066	15	765	19	1894	18	1332	21	10,220	26	16,655	24	8426	20	2677
Mar	3	788	1	498	14	1904	7	2629	11	3007	15	4573	4	1775	2	548
	25	1381	18	994	21	1919	28	1384	30	1677	31	4109	29	1579	16	1203
Apr	2	420	3	472	14	1717	7	1664	11	2355	15	3916	4	1228	1	402
	27	1194	16	598	23	2436	20	1920	29	1484	30	3916	28	1391	18	1070
May	6	214	8	444	2	3722	11	2407	14	1741	15	5353	12	1438	1	894
	17	868	23	418	29	3592	20	1969	28	2820	30	6308	26	2224	16	1290
Jun	6	3819	8	6921	2	6828	11	3015	14	3401	15	5955	12	6623	1	2022
	17	4617	23	6479	29	3615	20	4940	28	2967	30	5705	26	5967	16	2052
Jul	1	11,404	3	5082	13	12,531	10	9998	4	13,992	8	30,180	2	8543	6	2501
	16	11,173	18	5534	29	13,677	26	10,161	20	14,523	24	32,209	17	9620	22	2501
Aug	2	4401	4	5827	13	5156	6	9781	9	11,962	14	13,477	12	5089	1	2384
	17	2890	18	3457	19	7595	23	9781	26	12,396	29	16,249	21	8477	16	2368
Sep	1	2937	8	3479	13	13,863	4	7585	6	7507	10	16,563	3	11,091	2	2844
	17	2841	27	5341	29	12,511	21	4752	20	11,984	24	17,092	18	11,157	16	2844
Oct	1	8179	11	5186	13	9544	8	6232	7	15,573	5	21,037	2	10,982	3	2855
	16	10,250	19	5341	29	6748	28	11,196	24	6885	27	22,243	17	9912	21	2883
Nov	2	5631	4	5557	9	4342	6	9599	12	5302	13	15,750	10	11,116	1	2844
	18	5923	17	2618	22	11,491	20	7113	27	5302	28	15,750	25	10,334	16	2820
Dec	1	3599	4	3000	10	14,465	7	8483	9	4984	13	16,229	5	10,497	3	2865
	17	3815	19	5341	26	14,268	23	8398	25	5298	29	16,746	21	10,936	16	2764

<sup>1</sup> WLA –Waste load allocation (kg/d of BOD<sub>5</sub>)



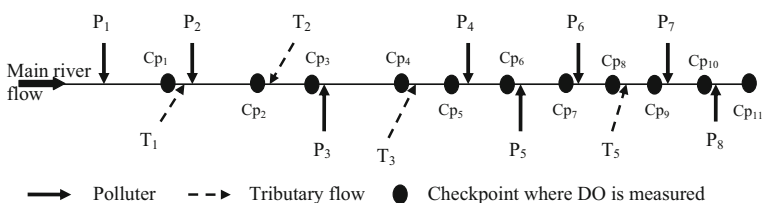
**Table 3** Annual waste load allocation (tons of BOD<sub>5</sub>) under SEDP, EBP and WLS program

Polluter	SEDP	EBP	WLS
P1	536	592	100
P2	498	181	113
P3	171	380	150
P4	662		186
P5	131		194
P6	144	728	276
P7	809	895	167
P8	259	259	129
Annual waste load discharge into the river (tons of BOD <sub>5</sub> )	3210	3035	1315

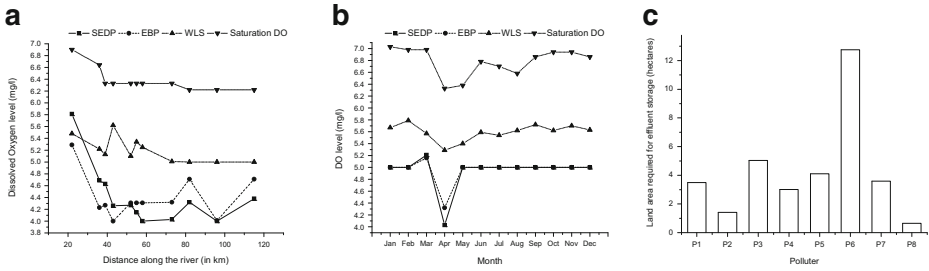
inferred that the proposed WLS model is more stringent in waste load allocation to the polluters when compared to SEDP and EBP models. A stringent waste load allocation for the polluters means that the total waste load discharge into the river will be considerably less, thereby having a relatively less negative impact on the river water quality.

Often in river water quality management problems, the pollution level of a river is measured in terms of available dissolved oxygen (DO). The DO is measured at various checkpoints in the river, to ensure that the DO level is above or equal to the desired level. For the case study river, the DO level is measured at eleven checkpoints along the river, whose locations are shown in Fig. 3. Among all the checkpoints, c8 (at mileage of 72 km) is the critical checkpoint, as the maximum DO deficit occurs at this point when all the polluters discharge their untreated waste load. Also the lowest stream discharge was observed during the month of April. If DO level at the critical checkpoint is satisfied, then it definitely will be satisfied at all the other checkpoints. Similarly, if DO level at all the checkpoints is satisfied during the critical month, then it can be safely assumed that it will be satisfied for the rest of the months. Considering these points, a plot of DO level at critical checkpoint for different months under different water quality management programs is shown in Fig. 4 (a) and (b) respectively. The variation of DO across the length of river for the critical month (i.e., April) is shown in Fig. 4 (b).

From Fig. 4 (b), it can be observed that the DO level under SEDP and EBP models, at critical checkpoint during the critical month is well below the desired DO level of 5 mg/L. This means the pollution control agency will have to employ some alternate water quality control measure (for example low flow augmentation) to bring the DO level to the desired level, which will incur extra cost to the polluters and pollution control agency. On the other hand, under the WLS model, the DO level is satisfied for all the months at critical checkpoint and also the DO level during the critical period is satisfied at all the checkpoints, thereby negating the necessity



**Fig. 3** A schematic diagram of the river with point source polluters, tributary flows and details of checkpoints where DO is measured



**Fig. 4** (a) DO level at various checkpoints for the critical month of April (b) DO level at critical checkpoint (c8) for different months (c) Effluent storage space required for the polluters (assuming a depth of 5 m)

of employing any alternate water quality control measure. Hence from river water quality consideration it can be argued that the proposed WLS model performs relatively better compared to the SEDP model and EBP model. Also, as DO level at the critical checkpoint is well above the desired DO level of 5 mg/L for all the seasons, it can be surmised that slight variations in river discharge or waste load quantity will not have drastic reduction in DO level. The same cannot be said for the SEDP and EBP models. Therefore, WLS model can be considered as more robust from river water quality consideration.

### 4.2 Cost Comparison for the SEDP, EBP and WLS Model

Cost minimization is an essential objective for any water quality management program. The total cost for any polluter can be expressed as the sum of capital cost (which includes construction of effluent treatment plant, and land acquisition) and operation and maintenance (O&M) cost. For the present research work, the cost data for the effluent treatment plants were taken from Sato et al. (2007). To compare and contrast the three methods, all the costs were annualized using the annual discounting technique, by adapting a design period of 30 years for the ETPs and discounting rate of 10 %. The cost comparison for the three models is shown in Table 4. At first

**Table 4** Total annual cost under SEDP, EBP and WLS model

Polluter (1)	Annual system cost (Rs Million) (2)			Excess cost of treating effluent to the comparable level of WLS (Rs Million) (3)		Total cost including excess cost of effluent treatment (Rs Million) (4)	
	SEDP	EBP	WLS	SEDP	EBP	SEDP	EBP
P1	8.84	9.00	10.81	2.98	2.26	11.82	11.26
P2	5.05	5.54	6.25	1.75	2.44	6.80	7.98
P3	11.29	11.06	13.08	4.16	3.91	15.45	14.98
P4	8.31	3.51	10.29	2.98		11.29	3.51
P5	11.83	1.75	13.62			11.83	1.75
P6	29.33	39.72	35.72		9.87	29.33	49.60
P7	8.17	8.04	10.29	2.68	2.53	10.85	10.57
P8	2.98	2.98	0.84	1.45	1.45	4.43	4.43
Total annual cost (Rs Million)	85.80	81.60	100.91	16.00	22.46	101.79	104.05

glance it would appear that the total system cost is higher for the proposed WLS model compared to the SEDP and EBP model. However, it should be noted that the annual waste load reduction (total annual untreated waste load minus total annual waste load discharge) will be significantly greater under WLS program. Therefore, if same level of waste load reduction is to be achieved under SEDP and EBP models, then a capital of Rs 16 million and Rs 22.6 million in excess will have to be invested under SEDP and EBP models, respectively. If this amount is added to the initially estimated annual cost, then the total system cost will come out as Rs 101.79 million for the SEDP model and Rs 104.05 million for the EBP model. Based on this observation it can be argued that the proposed WLS model is overall slightly more economical when compared to SEDP and EBP models. Finally, Fig. 4 (c) shows the land required by the polluters for effluent storage.

### 4.3 Comparison of Equity Level Under SEDP, EBP and WLS Model

Equity refers to the concept of fairness as perceived by the end users in any allocation problem. If the perception of fairness among the end users is uneven, then the overall equity level will be less, which eventually will give rise to conflicts. In water quality management problems conflicts can arise among the polluters if any individual or group of polluters feel that their overall cost allocation is proportionately greater. This level of conflict among the polluters can be assessed indirectly through inequity measures. The lesser the inequity measure, the greater will be the fairness in cost allocation among the polluters and the corresponding water quality management program will be preferred by both polluters and the pollution control agency (PCA). For the present research work therefore, the equity measure for the polluters was calculated using the method proposed by Burn and Yulianti (2001) and Murty et al. (2006), which is based on the principle that increased fairness occurs when the ratio of removal fraction at a source to the average removal is as close as possible to the ratio of the pollutant input at a source to the average pollutant input. Mathematically it can be expressed as:

$$IEM = \sum_{i=1}^n \left| \frac{x_i}{x} - \frac{W_i}{W} \right| \quad (8)$$

where,  $IEM$  is the inequity measure;  $x$  is the average removal level for  $n$  sources;  $x_i$  is the treatment removal level for polluter  $i$ ;  $W_i$  is the waste input (or discharge) for polluter  $i$ ;  $W$  is the average waste input. From Eq. (8) it can be seen that complete equity will occur when  $IEM$  is equal to zero. In other words, a water quality management program for which  $IEM$  is closer to zero can be considered as comparatively more equitable. For the present work Eq. (8) was slightly modified by substituting individual polluter's cost and average cost in place of  $x_i$  and  $x$ , respectively. Table 5 gives the summary of results for overall inequity level under each of the water quality management programs. From the tabulated results it can be surmised that the proposed waste load scheduling model gives comparatively a fairer cost allocation among the polluters.

## 5 Conclusions

From the results obtained, it can be concluded that the proposed waste load scheduling model can be considered one of the viable water quality control options. Comparing the results of the

**Table 5** Inequity measure under different waste load allocation program

Polluter	Annual waste load allocation			Annual effluent treatment cost			Inequity measure		
	SEDP	EBP	WLS	SEDP	EBP	WLS	SEDP	EBP	WLS
P1	536	592	100	2.95	2.02	2.65	0.41	0.58	0.29
P2	498	181	113	1.64	2.13	1.49	0.73	0.26	0.17
P3	171	380	150	3.89	3.66	2.84	0.79	0.31	0.06
P4	662		186	2.34		2.11	0.92		0.40
P5	131		194	3.84		2.77	0.88		0.22
P6	144	728	276	7.61	9.73	7.02	2.03	1.38	0.71
P7	809	895	167	2.28	2.15	2.12	1.30	1.15	0.28
P8	259	259	129	0.98	0.98		0.34	0.23	
Overall equity measure	7.40	3.91	2.13						

three models, it was found that the river water quality (measured in terms of DO level at a checkpoint) under the proposed method was relatively better when compared to SEDP and EBP models. Also, the overall inequity level under the proposed model was found to be less compared to SEDP and EBP models. The overall system cost was slightly higher under the proposed method when compared to under SEDP and EBP model. The reason for this is due to the extra cost incurred for purchasing land for effluent storage. This cost difference between the proposed model and the other two models, however, is offset by the greater quantity of waste load treated under the proposed model when compared to SEDP and EBP models. The conservative approach adopted by the proposed method towards waste load discharge into the river will have an overall positive impact on the river water quality in the long run. The proposed method can be considered very suitable when there is sufficient space available for effluent storage. Although in the present research paper a deterministic approach has been adopted, the method can be extended without much difficulty for stochastic conditions. Also, in the present research paper, the waste load scheduling model was solved by making an assumption that only one polluter can discharge effluent on any given day. This is, however not a limiting condition for the model and the condition can be relaxed, i.e., more than one polluters can discharge the effluent on same day, when (i) the number of polluters is such that the condition of one-day-one-polluter will result in longer schedule lengths, (ii) the river assimilative capacity is sufficiently greater, and (iii) there is a restriction on the storage space availability for the polluters. If the polluters are grouped, it will reduce the schedule length, which in turn will minimize the required effluent storage space.

## References

- Annual Report on Air and Water Quality Network (2003), Central Pollution Control Board, New Delhi, India
- Becker L, Yeh WW-G (1974) Optimization of real-time operation of a multiple reservoir system. *Water Resour Res* 10(6):1107–1112
- Boner MC, Furland LP (1982) Seasonal treatment and variable effluent quality based on assimilative capacity. *J Water Pollut Control Fed* 54(10):1408–1416
- Burn DH, Yulianti JS (2001) Waste load allocation using genetic algorithm. *J Water Resour Plan Manag* 127(2): 121–129

- Cardwell H, Ellis H (1993) Stochastic dynamic programming models for water quality management. *Water Resour Res* 29(4):803–813
- Chang S, Yeh WW-G (1973) Optimal allocation of artificial aeration along a polluted stream using dynamic programming. *J Am Water Resour Assoc* 9(5):985–997
- Cheng TCE, Sin CCS (1990) A state of the art review of parallel machine scheduling research. *Eur J Oper Res* 47(3):271–292
- Graves GW, Hatfield GB, Whinston AB (1969) Water pollution control using by-pass piping. *Water Resour Res* 5:13–47
- Hall WA, Butcher WS, Esogbue A (1968) Optimization of the operation of a multi-purpose reservoir by dynamic programming. *Water Resour Res* 4(3):471–477. doi:10.1029/WR004i003p00471
- Heidari M, Chow VT, Kokotovic PV, Meredith DD (1971) Discrete differential dynamic programming approach to water resources systems optimization. *Water Resour Res* 7(2):273–282. doi:10.1029/WR007i002p00273
- Jha M, Gu R (2010) Risk analysis of seasonal stream water quality management. *Water Sci Technol* 62(9):2075–2082
- Larson RE, Keckler WG (1969) Applications of dynamic programming to the control of water resources systems. *Automatica* 5(1):15–26
- Lence BJ, Eheart JW, Brill Jr ED (1990) Risk equivalent seasonal discharge programs for multidischarge streams. *J Water Resour Plan Manag* 116(2):170–186
- Liebmann JC, Lynn WR (1966) The optimal allocation of stream dissolved oxygen. *Water Resour Res* 2(3):581–591. doi:10.1029/WR002i003p00581
- McCarthy BL, Liu J (1993) Addressing the gap in scheduling research: a review of optimization and heuristic methods in production scheduling. *Int J Prod Res* 33(1):59–79
- Mujumdar PP, Saxena P (2004) A stochastic dynamic programming model for stream water quality management. *Sadhana* 29(5):477–497
- Murty YSR, Sreenivasan K, Murthy BS (2006) Multiobjective optimal waste load allocation models for rivers using non-dominated sorting genetic algorithm-II. *J Water Resour Plan Manag* 132(3):133–143
- Ng AWM, Perera BJC, Tran DH (2006) Improvement of river water quality through a seasonal effluent discharge program (SEDP). *Water Air Soil Pollut* 176:113–137
- Reheis HF, Dozier J, Word D, Holland J (1982) Treatment costs savings through monthly variable effluent limits. *J Water Pollut Control Fed* 54(8):1224–1230
- Rossmann LA (1989) Risk equivalent seasonal waste load allocation. *Water Resour Res* 25(10):2083–2090
- Sato N, Okubo T, Onodera T, Agarwal LK, Ohashi A, Harada H (2007) Economic evaluation of sewage treatment processes in India. *J Environ Manag* 84(4):447–460
- Shih CS, Krishnan P (1969) Dynamic optimization for industrial waste treatment design. *J Water Pollut Control Fed* 41(10):1787–1802
- Sunantara JD, Ramirez JA (1997) Optimal stochastic multicrop seasonal and intraseasonal irrigation control. *J Water Resour Plan Manag* 123(1):39–48
- Tamiraparani river basin: Microlevel study Vol I (2004) A report published by Institute for Water Studies, Public Works Department, Govt. of Tamil Nadu, Tharamani, Chennai -113
- Yakowitz S (1982) Dynamic programming applications in water resources. *Water Resour Res* 18(3):673–696
- Yaron D, Dinar A, Meyers S (1987) Irrigation scheduling – theoretical approach and application problems. *Water Resour Manag* 1(1):17–31
- Young G (1967) Finding reservoir operating rules. *J Hydraul Div* 93(6):297–321