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ORIGINAL ARTICLE

Waste Load allocation Using Machine Scheduling: Model Formulation

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Abstract In this paper a novel approach for effective utilization of river assimilative capacity has been proposed. The method, referred to as waste load scheduling (WLS) is based on the principle that by restricting the effluent discharge into the river to only one polluter at any given day will allow us to utilize the available river assimilative capacity in a more efficient manner. This is achieved by scheduling the dischargeable waste load among the polluters, such that a waste load schedule once developed will specify two things: (1) which polluter has to discharge his/her effluent on a given day; and (2) what is the quantity of effluent that he/she can discharge. By scheduling the waste load discharge into the river thus, will considerably reduce the total effluent discharge into the river and hence a greater degree of water quality level can be achieved when compared to traditional waste load allocation methods. For the mathematical development of the model, the WLS problem was envisaged as analogous to a machine scheduling problem. In a simple single MS problem *n* number of jobs are required to be scheduled on a single machine to minimize/maximize a pre-defined performance measure. In a WLS problem, the river can be treated as a machine and the polluters discharging effluent directly into the river are analogous to the jobs to be scheduled. Treating the waste load scheduling problem in an analogous way to a MS problem enables us to apply the solution methods used for solving standard sequencing and scheduling problems to the proposed waste load scheduling problem. Although the present paper discusses the special case of waste load scheduling in which only one polluter can discharge effluent at any given day (suitable when the number of point load sources is small), it is however, possible to extend it to a more general case involving a large number of polluters as easily. In the accompanying paper, the application of the developed model to a case study has been explained in detail. The proposed model and its application proved that the model is highly efficient in solving the waste load allocation problem in a more comprehensive way.

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1 Introduction

The primary goal of any water quality management program is to effectively utilize the available river assimilative capacity without compromising on the desired water quality standards. This can be achieved by applying directly or indirectly suitable water quality control measures either to the effluent or to the effluent receiving medium, i.e., river. Loucks (1976) gave a comprehensive list of various water quality management methods that can be considered singly or in combination for achieving better river water quality level. Among the measures listed, effluent treatment, process changes, by-pass piping of effluent, flow augmentation, artificial aeration and institutional methods (like effluent charges, taxes etc.) can be considered as being used more often. Each of the methods listed have their own advantages and limitations. For example, a good level of river water quality can be achieved by effluent treatment, although when not combined with other alternatives like flow augmentation or controlled discharge it can be a very costly alternative especially during dry river flow periods (Boner and Furland 1982; Rossman 1989; Takyi and Lence 1995; Ng et al. 2006). Institutional methods can be very effective water quality control measures if not for the difficulty in determining the exact effluent charges or taxes to be levied on the polluters (Boyd 2003). Methods like by-pass piping (Graves et al. 1969) and artificial aeration (Hunter and Whipple 1970; Olgac et al. 1976) would require extra installation equipment and hence their effectiveness will depend upon the comparison of benefits achieved in terms of improved water quality to the cost incurred in equipment installation and maintenance. Among the methods listed by Loucks (1976), flow augmentation has been widely used for river water quality management. The identification of water impoundment for improving river water quality has given rise to many research works concerning reservoir operation for downstream water quality control. Notable among them are Jaworski et al. (1970); Loucks and Jacoby (1972); Ikebuchi et al. (1982); Simonovic and Orlob (1984); Yeh (1985); Willey et al. (1996); Dai and Labadie (2001); Chaves et al. (2003); Dhar and Datta (2008). The method of low flow augmentation for improving river water quality, however, is not short of limitations. As early as 1960s, Symons et al. (1965) questioned the effect of impounded water on the downstream water quality which was also noted by Rinaldi et al. (1979) and Kneese and Bower (1984). Apart from the effect of impounded water quality on downstream water quality, Rinaldi et al. (1979) also noted that the effectiveness of LFA strongly depends on the type of waste, on the conditions of the receiving water, and on the location of the pollution sources and of the reservoir. Apart from the technical limitations of the flow augmentation, Thomann and Muller (1987) noted some practical difficulties in implementation of flow augmentation. For instance, often the authority in control of water quality and the authority for reservoir operation are different, thus it is difficult to make them work together for a common goal. Also, the most critical period from the water quality point of view is the summer season, but this is also the time when the demand for industrial and domestic needs is the greatest. In such situations, conserving some water for relatively less important need like water quality control by sacrificing demand for primary uses like industrial and domestic water demand may not be a suitable approach. A second, less involved, approach to more efficient use of the waste assimilation potential is the storage of waste and regulated release of effluent tailored to the natural variation in waste assimilation capacity (Velz 1970). The method of regulated effluent discharge involves the temporary storage of waste effluents for discharge at times when more water is available for dilution. Often it involves storage of waste during low flow periods in the summer and discharge during winter and/or spring when stream flow is higher (Kneese and Bower 1984). There are, however, few difficulties with the conventional approach of controlled effluent discharge, such as correct estimation of required effluent storage space and storage space minimization. Hence in the present paper, a novel approach towards waste load allocation, namely, waste load scheduling is being proposed which will overcome effluent storage issues to some extent. The method is based on the concept of scheduling wherein the polluters will be allotted a time and quantity for effluent discharge. As the schedule will be deterministic, determining effluent storage space will not pose any computational difficulties. Also, the schedule can be designed in such a way as to minimize the effluent storage space.

2 Machine Scheduling Problem

Since the mathematical approach for the WLS problem was developed by treating it in an analogous way to a machine scheduling (MS) problem, a brief overview of a basic machine scheduling problem is given first, so that it is easier to understand how a waste load allocation problem can be treated similarly to a machine scheduling problem. A general MS problem can be stated as follows:

"There are **n** numbers of jobs available that are to be processed on a single machine or **m** number of machines. Find an optimal schedule for the **n** jobs such that the schedule maximizes/minimizes a pre-defined performance measure."

A general MS problem can vary from a simple *single-machine-n-jobs* model to a complex *m-machines-n-jobs model*. A simplest scheduling problem is one in which there is a single resource or machine, and all processing times are deterministic. As simple as a single MS problem may be, it is still very important as it illustrates a variety of scheduling strategies in a tractable model and also it provides a context in which to investigate many different performance measures and several solution techniques (Baker and Trietsch 2008). A single machine scheduling problem can be characterized by the following conditions (Baker and Trietsch 2008):

C1: There are *n* single-operation jobs simultaneously available for processing (at time zero).
C2: Machines can process at most one job at a time.
C3: Setup times for the jobs are independent of job sequence and are included in the processing times.
C4: Job descriptors are deterministic and known in advance.
C5: Machines are continuously available (no breakdowns occur).
C6: Machines are never kept idle while work is waiting.
C7: Once an operation begins, it proceeds without interruption.

In the above listed conditions, when C2 is relaxed, i.e., when a machine can process more than one job at a time then the problem becomes a *batch scheduling* problem. For some problems, the job processing time is not known in advance and it depends on the nature of the job; in such case C4 becomes invalid. Such problems are classified as machine scheduling with *job-dependent* processing times.

A single machine scheduling problem generally contains a single machine, and n number of jobs (j), such that the processing time (p) for each job is deterministic. If $J = \{j_1, j_2, \ldots.. j_i, \ldots. j_n\}$ is the set of jobs to be scheduled and $P = \{p_1, p_2, \ldots.. p_i, \ldots. p_n\}$ is the set of processing times for the jobs in the set J, then a MS problem constitutes of finding a sequence of the jobs in set J such that the total processing time (or any other such performance measure) is minimized. Although minimizing total flow time (i.e. sum of all the job processing times) is a primary objective in many scheduling problems, a MS problem can also be solved for many other objectives like to minimize total resources used, to minimize the overall cost of operation etc. Figure 1 shows a schematic representation of a *single-machine-n-jobs* scheduling problem. In Fig. 1, the optimal sequence is given by $j_3^1 \rightarrow j_4^2 \rightarrow j_i^3 \rightarrow \ldots j_1^k \rightarrow j_n^{k+1} \rightarrow \ldots j_2^n$, which implies that the sequence should start with j_3^1 next in the order should be j_4^2, and next j_i^3 and so on, up to j_2^n, which is the last job to be processed in the sequence.

3 Analogy Between Machine Scheduling and Waste Load Allocation

A general waste load allocation (WLA) problem comprises of finding optimal dischargeable waste load for a set of point load sources such that the total cost of effluent treatment is minimized, simultaneously satisfying the constraints on river water quality standards. The general assumption in any WLA model is that all the

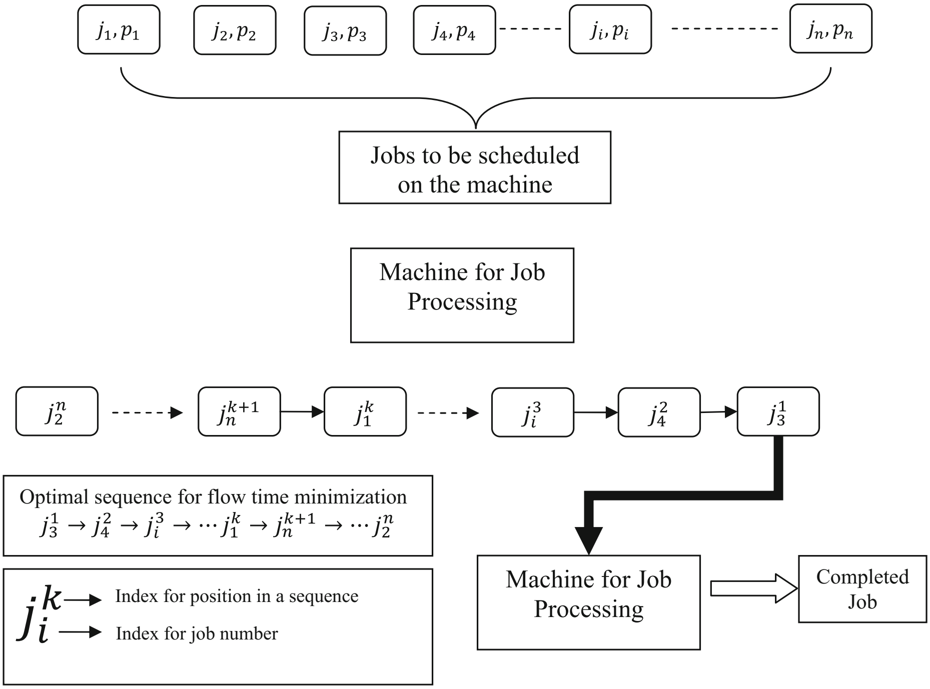


Fig. 1 A schematic representation of a *single-machine-n-jobs* scheduling problem

polluters are discharging the effluent directly into the stream simultaneously. This implies that as one moves downstream in a river, there will be a cumulative increase in waste load discharged into the river due to point loads joining the river at various points. When the river assimilative capacity is high, the river can take up large quantities of waste and can render them harmless in a short period of time through the process of '*self-purification*'. However, when the assimilative capacity is very low (for example during dry seasons), the river may take more time to '*purify*' the waste load discharged, as well as handle less quantity of waste from the point sources. Hence, when the river assimilative capacity is low, discharging large quantities of waste load into the river can severely affect the overall river water quality. In such case, it would be prudent to either increase the river flow (through flow augmentation) or regulate the effluent discharge into the river. The focus of the present research, however, is to develop a methodology for regulating effluent discharge into the river in such a way that the river has enough time and capacity to assimilate waste load and regain its self purification capacity before receiving waste load from another point source. It is towards this approach that a scheduling model for the dischargeable waste load has been developed such that the river will have enough time for assimilating the waste load well within its capacity to purify thus avoiding critical conditions.

Every river has its own assimilative capacity, which can be considered as a river's ability to assimilate waste without deteriorating its water quality. In this respect, the river can be considered similar to a machine which has certain capacity for performing jobs. As a machine can breakdown if jobs exceed its capacity, so can a river if the pollutant load is more than its assimilative capacity. Therefore, an analogy can be drawn between a machine and river where they play a role of a processing unit in their respective systems. Similarly, as both machine and river receive some input upon which they have to act, an analogy can be drawn between input to both systems i.e. jobs/tasks in case of MS problem and point loads in case of WLA problem respectively. Table 1 shows the components of analogy between a MS problem and a WLA problem. Figure 2a, b show a schematic representation of the analogy between MS problem and WLA problem. As shown in Fig. 2, the machine/river receives inputs in the form of jobs/waste-load, and acts upon them to produce a final product in the form of completed job/less polluted river.

Table 1 Analogy between a MS problem and WLA problem

Component of Analogy	Machine scheduling (MS)	Waste load allocation (WLA)
The processor	The machine acts as a processor which accepts an input performs operation on it and produces an output	The river can be considered as a machine which accepts an input (waste load) performs operation (self purification) on it and gives output
Input variable	Job/Task	Effluent waste load discharged by a polluter
Processing time	Time taken for a machine to complete a job	Time taken for complete oxidation of the effluent discharged by a polluter

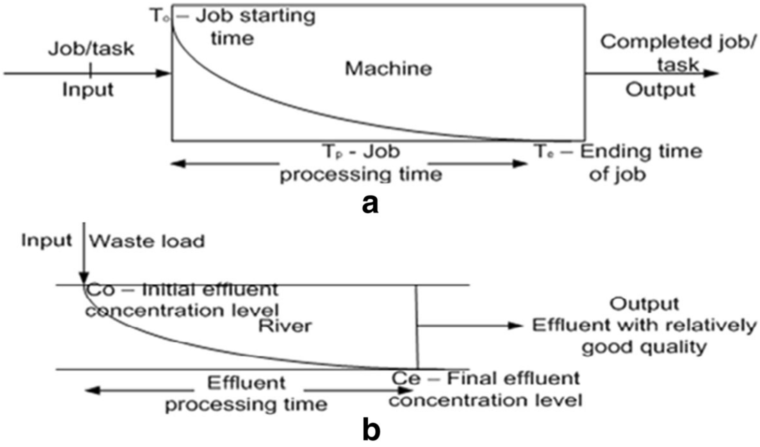
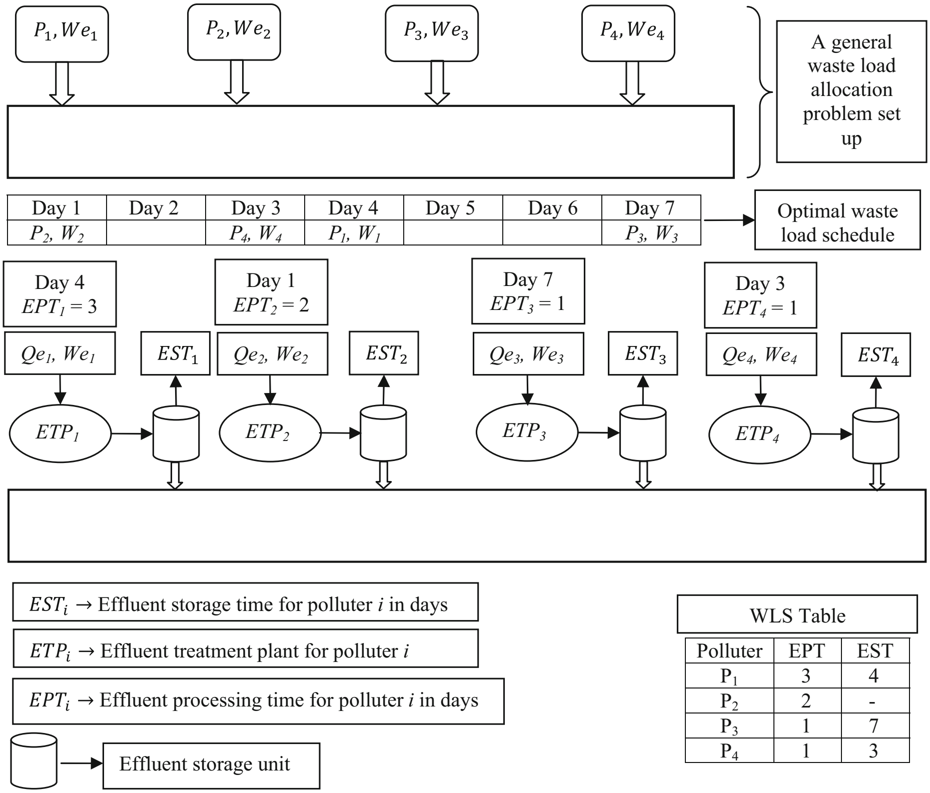


Fig. 2 **a** Machine scheduling process **b** Waste load scheduling process

4 Waste Load Scheduling

Figure 3 schematically explains a waste load scheduling problem. As shown in Fig. 3, the basic concept of a WLS problem is to schedule the dischargeable waste load



Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
P_2, W_2		P_4, W_4	P_1, W_1			P_3, W_3

EST_i → Effluent storage time for polluter *i* in days

ETP_i → Effluent treatment plant for polluter *i*

EPT_i → Effluent processing time for polluter *i* in days

WLS Table

Polluter	EPT	EST
P_1	3	4
P_2	2	-
P_3	1	7
P_4	1	3

Fig. 3 A schematic representation of waste load scheduling problem

among the polluters, such that the river has enough time to assimilate the waste load before accepting the effluent from another point source. The definition of WLS fits the general definition of any scheduling problems, which basically amounts to answering two types of questions (Baker and Trietsch 2008): Which resources should be allocated to perform each task? In the context of WLS problem, this represents the quantity of effluent to be discharged and the portion of the river to be allocated for assimilation. When should each task be performed? In the context of WLS problem, this corresponds to determining the time at which a polluter should discharge his/her effluent.

4.1 Effluent Processing Time (EPT)

Effluent processing time (EPT) can be considered as the fundamental concept of the proposed WLS model. Similar to a job processing time in machine scheduling problems, EPT can be defined as the time taken by a river to completely assimilate the waste load discharged by a polluter within his/her reach, such that the quality of river is maintained as per stipulated standards. When a polluter discharges effluent in a river, the effect of the discharge on the BOD level in the river can be expressed using first order decay rate equation:

$$b_k = L_{mi} e^{-k_d t_k} \tag{1}$$

where b_k is the BOD level at point k in the river mg/L; k_d is the de-oxygenation coefficient in d^{-1}; t_k is the reach travel time expressed in days, which is measured from the point of effluent discharge to the point k where BOD is measured; L_{mi} is the BOD mix at the beginning of reach i in mg/L, which is determined by applying the mass balance equation at the confluence of river and pollutant discharge point. Mathematically, L_{mi} is given by:

$$L_{mi} = \frac{Q_r L_r + 0.0116 W_i}{Q_r + Q_{ei}} \tag{2}$$

where Q_r and Q_{ei} are the river discharge and effluent discharge for polluter i in m^3/s respectively; L_r is the river BOD in mg/L; W_i is the waste load discharge by polluter i in kg/d of BOD_5; and 0.0116 is a conversion factor. By definition, the reach travel time from the point of effluent discharge to the point in the river where $b_k \cong B_k^p$(where B_k^p is the permissible BOD level at point k) will be equivalent to the effluent processing time. Combining Eq. (1) and Eq. (2), substituting B_k^p for b_k and EPT for t_k, the effluent processing time for polluter i can be expressed as:

$$EPT_{\mathrm{i}} = \frac{1}{k_d} ln \left[\frac{\mathrm{Q_r L_r} + 0.0116 W_i}{(Qr + Qe_i) \times \mathrm{B_k^p}} \right] \tag{3}$$

Using Eq. (3) for a specific EPT value and a fixed $\mathrm{B_k^p}$ value, the allowable waste load for a polluter can be determined. Fig. 4(a) shows dischargeable waste load (W_i) as a function of EPT and $\mathrm{B_k^p}$ for constant Q_r and k_d value, and Fig. 4(b) shows dischargeable waste load (W_i) as a function of k_d and EPT for a constant Q_r and $\mathrm{B_k^p}$. From Fig. 4(a) it can be inferred that increasing the acceptable BOD limit will increase the dischargeable waste load for the same processing time. Similarly, a higher

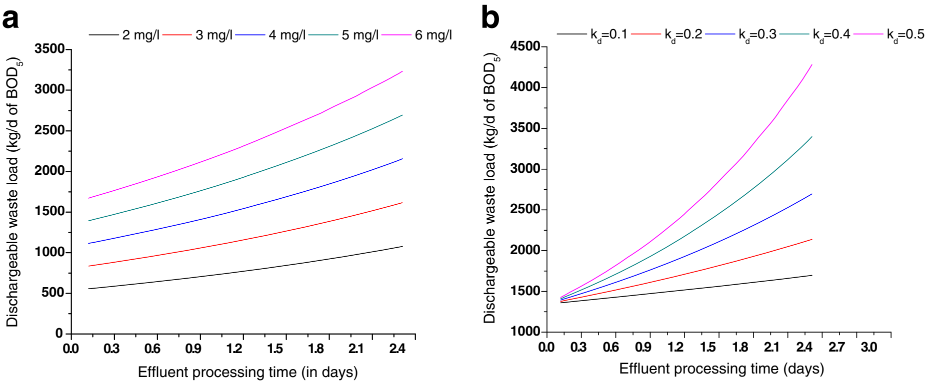


Fig. 4 **a** Dischargeable waste load as a function of permissible BOD level and effluent processing time (EPT) **b** Dischargeable waste load as a function of *kd* and effluent processing time (EPT)

k_d (de-oxygenation rate) value means that more waste load can be discharged for the same effluent processing time.

4.2 Effluent Discharge Time (EDT)

Effluent discharge time may be defined as the time (in day and hour) on which a polluter has to discharge the effluent. In a given schedule, a polluter will be allowed to discharge effluent only once, i.e. a polluter will have only one *EDT* in a schedule. The *EDT* for the polluter will be specified as days (DD) hours (HH) in a day (e.g., 01:13) or simply in DD (01).

4.3 Effluent Storage Time (EST)

The effluent storage time (EST) for any polluter will be equivalent to the time for which the effluent has to be stored before discharged into the river. For an individual polluter, it will be the time elapsed between his/her successive discharge schedules.

4.4 Effluent Storage Volume (ESV)

Effluent storage volume will be the volume of effluent stored during a schedule. It will be a function of EST and will be the product of daily effluent flow rate and maximum EST for the period of analysis (one year).

5 Waste Load Scheduling Optimization Model

Since the WLS problem was treated in an analogous way to a MS problem, some assumptions similar to a MS problem, in addition to the assumptions unique to a water quality management problem need to be considered. The assumptions for WLS model as a single machine approach are as follows:

1. There are *n* number of polluters simultaneously available along the river
2. River can assimilate waste load from at most one polluter at a time

3. The time taken for the polluter to discharge the effluent is included in the effluent processing time
4. The effluent processing time is deterministic and only discrete values for *EPT* are considered
5. The river is available continuously for waste assimilation
6. The river is never kept idle while a polluter's turn for effluent discharge comes

Apart from the assumptions made above, there are some assumptions which are specific to waste load allocation problem. They are:

7. There is a Pollution Control Agency (PCA), which is entrusted with maintenance of river water quality and once a WLA schedule is developed it will be enforced by the PCA
8. All the polluters will adhere to the prescribed schedule
9. The effluent processing time can be adequately described using a steady-state first-order reaction rate equation

With the stated assumptions, the objective function and constraints for the WLS model are formulated as follows. As the discharge in many rivers in general varies seasonally, hence seasonal schedules were developed for the WLS model. Let $\mathcal{S} = \{1, 2, 3, \ldots\ldots., s\}$ be the set of seasons within the period of analysis (i.e., one year). The set will have at least one season to at most twelve seasons in it. In the present study, length of each season was taken equal to one month. Therefore, the set of seasons was: $\mathcal{S} = \{1, 2, 3, \ldots., 11, 12\}$. In general, a schedule can be of varying length, like one week, or ten days or a fortnight. Depending on the length of schedule chosen, each season will have one or more schedules. Let $\mathscr{R} = \{1, 2, 3, \ldots.., r\}$ be the set of schedules within a season and finally let $\mathcal{N} = \{1, 2, 3, \ldots., n\}$ be the set of point source polluters discharging effluent directly into the river.

5.1 Objective Function

The objective function for the WLS model will have two components, namely, the effluent treatment cost at the treatment plants and benefits accrued (if any) by recycling or diverting the effluent from storage unit. The mathematical formulation for each objective is explained below.

5.1.1 Effluent Treatment Cost

The effluent treatment cost for a polluter in general can be expressed as a function of quantity of waste load treated. If TW_i is the quantity of waste load (in kg/d of BOD_5) treated by polluter i, and C_i is the cost of treating one unit of waste load (in [Rs or \$] per kg/d of BOD_5 removed), then the cost incurred by polluter i in treating waste load TW_i will be $C_i \times TW_i$. However, the quantity of waste load to be treated by a polluter will vary from schedule to schedule and season to season. Therefore, the total annual effluent treatment cost for polluter i will be $C_i \times \sum_{s=1}^{S} \sum_{r=1}^{R} TW_{i,r,s}$. It is assumed that the cost of treatment does not vary within a year. Apart from effluent treatment cost, there will be capital cost which includes cost of constructing an effluent treatment plant and land acquisition cost. If CP_i is the capital cost for polluter *i*, then the total system cost (i.e. capital cost and effluent treatment cost for all the polluters) can be expressed as:

$$TC = \min\left\{\sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{i=1}^{N}\left(C_i \times TW_{i,r,s}\right) + \sum_{\mathrm{i}=1}^{\mathrm{N}}\mathrm{CP_i}\right\} \tag{4}$$

5.1.2 Benefits Due to Effluent Diversion and Recycling

The volume of effluent that can be diverted/recycled will depend on end users demand and hence it will vary from day to day. Let V_{it} be the volume of effluent diverted by polluter *i* during day *t* and let B_i be the benefits accrued by polluter *i* for diverting unit volume of effluent (in Rs/m^3). Thus, the total annual benefits accrued by polluter *i* can be given by $B_i \times \sum_{t=1}^{365} V_{it}$. Using similar analysis for all the polluters the objective function related to total annual benefits accrued in diverting/recycling the effluent can be expressed as:

$$TB = \max\sum_{t=1}^{365}\sum_{i=1}^{N} B_i \times V_{it} \tag{5}$$

It is also assumed that the cost of effluent treatment will always be higher than the potential benefits expected from recycling or reuse. The overall objective function therefore is to minimize the total system cost, which can be expressed mathematically as:

$$Z = \min(TC - TB) \tag{6}$$

The objective function is subjected to the following constraints.

5.2 Constraints

The first constraint deals with the effluent discharge time. A polluter will not be allowed to discharge the effluent until the effluent discharged by previous polluter is completely assimilated by the river. If $EDT^{k}_{i,r,s}$ is the effluent discharge time for polluter *i*, within the schedule *r*, during season *s*, whose position in scheduling sequence is *k* and $EDT^{k-1}_{j,r,s}$ is the effluent discharge time for polluter *j*, within the schedule *r*, during season *s*, whose position in scheduling sequence is *k-1*, then the constraint on *EDT* can be written as:

$$EDT^{k}_{i,r,s} \geq EDT^{k-1}_{j,r,s} + EPT_{j,r,s} \text{ and } i \neq j \tag{7}$$

$EPT_{j,r,s}$ is the effluent processing time for polluter *j*, within the schedule *r* and during season *s*. The second constraint is concerned with the schedule length. All polluters should be accommodated within the predefined schedule length. In other words, the sum of *EPT*s for all the polluters should be equal to the total length of the schedule. This can be written as:

$$EPT_{1,r,s} + EPT_{2,r,s} + \ldots + EPT_{n,r,s} = SL \tag{8}$$

where *SL* is the predefined length of a schedule (one week, ten days or 15 days etc.) in days. The third constraint is based on the effluent processing time. Theoretically, *EPT* can take any positive continuous value. Practically however, there will be limits on each polluter's possible *EPT* values, which will depend on polluter's effluent discharge position and length of the river. Hence the lower limit on *EPT* for all the polluters will be one day. This limit is to ensure that no two polluters can discharge effluent on the same day. The upper limit on the *EPT* will be Tr_i^{max}, which is the travel time for effluent from polluter's discharge point to the end of the stream. This can be written as:

$$1 \le EPT_{i,r,s} \le Tr_i^{max} \tag{9}$$

The fourth constraint deals with effluent storage space. The scheduling process proposed in the present research will require the polluters to store their effluent for certain period until their effluent discharge time arrives. This will necessitate acquiring land for effluent storage. Since land acquisition can be difficult (especially in cities) for some polluters, it will be appropriate to have a constraint such that the required land for effluent storage is less than maximum available land. Let Qe_i be the effluent flow rate for polluter i (in m^3/d), Qr_i^{min} be the lowest amount of effluent a polluter has to divert for recycling at any given day (in m^3/d), EST_i^{max} be the maximum duration for which polluter has to store the effluent (in days), Ar_i^{max} be the maximum area available for polluter i for effluent storage (in m^2), and H be the depth of effluent storage unit (in m), then the constraint on effluent storage space can be expressed as:

$$\left(Qe_i - Qr_i^{min}\right) \times EST_i^{max} \le Ar_i^{max} \times H \tag{10}$$

The fifth constraint deals with the volume of effluent a polluter can divert from the storage unit. Let V_{it}^{req} be the effluent volume required by the end users near polluter i during any day t. The volume of effluent to be diverted at any given day (V_{it}) then should be less than or equal to the required effluent volume, i.e.,

$$V_{it} \le V_{it}^{req} \tag{11}$$

$$\text{also } V_{it} + V_{it}^{dis} = V_{it}^{tot} \tag{12}$$

Eq. 12 states the continuity constraint, i.e., the total effluent volume of polluter i during any day t $\left(V_{it}^{tot}\right)$ should be equal to the sum of effluent volume diverted (V_{it}) and effluent volume discharged into the river $\left(V_{it}^{dis}\right)$. In addition, it is important to ensure that the river water quality meets the water quality standard which are measured in terms of available DO in the river. The effluent discharged by the polluters should meet the DO standards at all the m number of checkpoints along the river. Let $[A_{i,s}]_{m\times 1}$ denote the pollutant transfer coefficient matrix for polluter i for season s which in general is a function of river characteristics and water quality model parameters. The pollutant transfer coefficient indicates the impact on DO concentration at a downstream location due to the effluent discharged at some upstream location. In the present research, the well known steady-state one-dimensional Streeter-Phelps BOD-DO equation (Streeter and Phelps 1925) was used for modeling the effect on DO deficit at any downstream checkpoint due to a unit discharge of waste load at some upstream point. Let $\left[DO_s^{sat}\right]_{m\times 1}$ be the saturation DO level matrix for season s. Let $\left[DO_s^{des}\right]_{m\times 1}$ be the desirable DO level matrix for season s. The constraint on meeting DO standard then can be expressed as:

$$\left[DO_s^{sat}\right]_{m\times 1} - \left[A_{i,s}\right]_{m\times 1} \times \left[WL_{i,r,s}\right]_{1\times 1} \geq \left[DO_s^{des}\right]_{m\times 1} \tag{13}$$

where $WL_{i,r,s}$ is the waste load discharged by polluter i, during his turn in schedule r within season s which is equal to $W_i \times (1 - x_{i,r,s})$, in which W_i is the untreated waste load for polluter i, and $x_{i,r,s}$ is the treatment level to be provided by polluter i during his turn in schedule r within season s. Also $W_i = WL_{i,r,s} + TW_{i,r,s}$.

Finally, there will be constraints representing upper limit (maximum possible treatment level with available technology) and lower limit (minimum treatment level to be provided by any polluter as prescribed by the PCA) on the treatment level. This can be expressed as:

$$x^{min} \leq x_{i,r,s} \leq x^{max} \tag{14}$$

The decision variables during any schedule will be the dischargeable waste load for the polluter and the day of effluent discharge in the schedule.

6 Summary and Conclusions

This paper presents a novel approach towards deviation of waste load allocation schedules with an effective utilization of river assimilative capacity. The central idea of the proposed waste load scheduling model is that by scheduling the dischargeable waste load into the river, the total quantity of waste load discharged into the river can be reduced significantly when compared to the traditional approach of waste load allocation problems. The principle of waste load scheduling is that by limiting the effluent discharge into the river to one polluter at any given day will give a river sufficient time to self-purify before receiving waste from next point load source. Therefore, the scheduling of waste load will ensure that the river is never under great ‘strain’ to assimilate the discharged waste load. This approach of scheduling the waste load discharge into the river can be effective, especially during low flow seasons when the stream flow is so low that the river may not have enough assimilative capacity. The method, therefore, will ensure that a river has better water quality even during low flow periods. Also, as the schedules developed are deterministic, the effluent storage space for each polluter can be determined with greater certainty. Another aspect of the proposed waste load scheduling method is the opportunity available for any polluter to divert or recycle his/her effluent. Since in the proposed method of waste load scheduling a polluter has to store his/her effluent, it is possible therefore for a polluter to divert or recycle his/her effluent and accrue some monetary benefits, which will offset the effluent treatment cost to some extent.

It should be noted however, that the method of waste load scheduling proposed here, which follows a single machine scheduling approach, will be suitable when the number of point load sources is less (say less than ten). If the number of polluters is large, the length of schedule will be longer which will lead to practical difficulties in effluent storage. However, in such cases (i.e., when the number of polluters is large) the polluters can be grouped among themselves based on some suitable criterion, and then, the waste load scheduling procedure can be applied to the polluter groups, treating each group as an individual polluter. The accompanying paper titled, “Waste load allocation using machine scheduling approach – Model application”, provides the researchers with an application of the proposed waste load scheduling model in an actual case study.

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