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Empirical Evaluations of Skinner's Analysis of Problem Solving

Judah B. Axe[1] · **Stephanie H. Phelan**[2,3,4] · **Caitlin L. Irwin**[2]

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Abstract
We reviewed 12 studies in which the researcher taught problem-solving strategies, such as self-questioning and visual imagining, to children and adolescents with and without disabilities to facilitate the learning of math, spelling, play/social, and communication skills. We analyzed these studies in terms of types of problem-solving strategies, the multiple control involved in problem solving, the extent to which problem solving occurred at the overt or covert level. In addition to suggesting limitations of the literature, we recommend areas for future research and practice.

Keywords Problem solving · Precurrent behavior · Verbal behavior · Multiple control

Skinner (1957, 1968) defined a *problem* as a situation in which there is no behavior immediately available to the individual that will reduce deprivation or provide escape from aversive stimulation. In current terms, a problem exists when there is an establishing operation (EO) but the conditions do not evoke behaviors that have produced the reinforcer in the past. Donahoe and Palmer (2004) expanded on Skinner's definition, suggesting a problem exists if three criteria are met: (a) the target response is in the individual's repertoire (i.e., has a history of reinforcement), (b) the target response that will solve the problem is scheduled for reinforcement, and (c) stimuli in the current environment do not directly evoke the target response. Nonexamples are situations in which (a) the response is not in the repertoire (e.g., demonstrating a double backflip),

✉ Judah B. Axe
judah.axe@simmons.edu

1 Department of Behavior Analysis, Department of Education, Simmons University, Boston, MA 02115, USA

2 Department of Behavior Analysis, Simmons University, Boston, MA 02115, USA

3 ABA Consultation and Services, Woburn, MA, USA

4 Anuenue Behavior Analysts, Haleiwa, HI, USA

(b) the response is not scheduled for reinforcement (e.g., a rhetorical question), and (c) the behavior is readily available (e.g., you have your keys).

Problem solving is behavior evoked by a problem in which an individual manipulates, supplements, and generates discriminative stimuli (S^D) to which he or she subsequently responds (Donahoe & Palmer, 2004; Skinner, 1957, 1968). This manipulating of stimuli has been termed "precurrent," or "mediating," behavior. Therefore, a problem can be conceptualized as an EO but with insufficient S^Ds—a situation that evokes precurrent behaviors in which the individual manipulates stimuli to produce S^Ds that evoke the current behavior and produce the solution. For example, the problem of being on time for work and having no keys (EO/no S^D) evokes searching behavior; a question about a past event evokes verbal responses about the event and/or visualizations of the event that ultimately evoke a reinforceable answer; and a two-by-two-digit math problem with no calculator evokes smaller scale operations until the numbers evoke the answer.

A stimulus control analysis is required to distinguish behaviors in everyday life that do and do not require problem solving. One distinction could be that behaviors emitted in everyday routines do not require problem solving. For example, you sit at your desk, push the power button on your computer, enter your username and password, and wait until the home screen shows. This requires very little problem solving—you do it every day. But, then you may have a problem: do you check e-mail, work on your article, or prepare for class? That is a problem that evokes manipulating verbal stimuli in the sequence of, for example, considering each option and the potential consequences of each, then picking one. A second distinction can be made between rote responses and responses that require problem solving. For most adults, answering the following questions does not require problem solving: What's your name? What's your address? What number comes after two? However, answering the following questions usually requires problem solving: What did you eat for dinner last Tuesday? What is 495 plus 257? What will it take to have peace in the Middle East? (see Palmer, 2016). Discriminating mundane versus complex stimulus arrangements is a first step in determining if problem solving is necessary.

An additional level of complexity is that problem solving occurs at both the overt and covert levels. Manipulating stimuli in one's home to search for one's keys may be considered overt problem solving, and silently stating names corresponding to each successive letter of the alphabet to try to recall someone's name is best considered covert problem solving. Even though from a radical behaviorism perspective, overt and covert behaviors are controlled by the same types of variables based on the basic principles of behavior, covert behavior is more difficult to define, observe, measure, and teach.

Despite the limitations of the privacy of much of our problem solving, the analysis of problem solving is critical for education in general (see Robbins, 2011) and for teaching complex skills to children with autism. The hallmark of discrete-trial teaching with children with autism is breaking skills down into individual parts so that children learn particular responses to simple stimulus presentations. This approach is often needed to establish initial verbal and academic repertoires with children with significant delays. However, this approach has been criticized. For example, Schreibman (2000) characterized common criticisms of discrete-trial teaching:

> Despite its impressive effects in terms of teaching important behaviors to children with autism, the highly structured discrete trial model encountered problems with

> generality. Specifically, some of the problems noted included cue dependency, lack of spontaneity and self-initiated behavior, rote responding, and failure to generalize behavioral gains across settings and responses. (p. 374)

In addition, Butler, Miller, Lee, and Pierce (2001, p. 20; cited in Neef, Nelles, Iwata, & Page, 2003) stated that current problem-solving standards for math curricula demonstrate “a shift from a behaviorist approach of teaching rote learning of facts and procedures to a constructivist approach” (see Bernstein & Chase, 2013, for a discussion of this distinction at the college level). Some teaching by rote is needed in early stages of learning, but once children have foundational language, academic, and social skills, behavior analysts must teach problem-solving behaviors to help children succeed in complex stimulus arrangements, such as those problems mentioned previously. General education and the complex social world are full of problems, and behavior analysts must prepare children with autism, as well as others in general, to solve these problems.

The purpose of this article is to review applied behavior-analytic studies on problem solving to examine the problems that have been addressed and the problem-solving strategies that have been taught. The second purpose is to discuss themes that emerge from the literature, including the analysis of multiple control in problem solving and the extent to which the problem solving occurred at the overt or covert level. The third purpose is to recommend future research and ways to incorporate problem solving into the education of children with and without autism.

Method

We searched the PsycINFO database for “problem solving” and “behavior analysis,” and limited the results to peer-reviewed articles published in English, yielding 123 articles. We reviewed the titles to identify articles presenting empirical data and then reviewed the reference lists of the selected articles to limit them further to only those citing one of Skinner’s (1957, 1968) analyses of problem solving. This limited the article pool to 10 studies. To ensure all relevant studies were identified, we reviewed the reference lists of the articles, repeating the strategy of reviewing the titles for empirical studies and the reference lists for a Skinner citation. This search yielded an additional 2 studies, resulting in 12 total studies. To determine the extent to which the search could be replicated, a doctoral student in behavior analysis repeated the search using the previous criteria. In comparing the resulting lists, the replication yielded no additional studies, and there was agreement on 11 of the 12 studies identified, yielding 92% interobserver agreement.

Results and Discussion

The results and discussion consist of the following. First is a description of the demographics of the participants, prerequisite skills, target skills, and experimental designs in the 12 studies. Second is a description of the 12 studies categorized by type of precurrent repertoire. Third is a discussion of multiple control in problem solving. Fourth is a discussion of whether the problem-solving behaviors occurred at the overt or covert level. Fifth is a discussion of limitations of the literature. The final section is applications and areas for future research.

Demographics of Participants, Prerequisite Skills, Target Skills, and Experimental Designs

Across the 12 studies, there was a total of 68 participants with a mean age of 14 years (range 3–20; *SD* = 7); however, excluding one study with thirty-three 20-year-old students (Mayfield & Chase, 2002), the mean age was 8 (range 3–23; *SD* = 6; Table 1). Eight studies included children with autism or other developmental delays, and four studies included only typically developing individuals. The prerequisite skills of math, spelling, social skills, and manding were likely necessary for the efficacy of the problem-solving strategies.

There were six categories of skills targeted in the 12 studies: math (Grimm, Bijou, & Parsons, 1973; Levingston, Neef, & Cihon, 2009; Mayfield & Chase, 2002; Neef et al., 2003; Parsons, 1976), spelling (Aguirre & Rehfeldt, 2015), play (Parsonson & Baer, 1978), social skills (Park & Gaylord-Ross, 1989), mands (Marckel, Neef, & Ferreri, 2006), and intraverbals (Frampton & Shillingsburg, 2018; Kisamore, Carr, & LeBlanc, 2011; Sautter, LeBlanc, Jay, Goldsmith, & Carr, 2011; Table 2). Two studies used reversal designs (Grimm et al., 1973; Parsons, 1976), one study presented individual data within a group design (Mayfield & Chase, 2002), one study used a multiple-probe across-participants design (Aguirre & Rehfeldt, 2015), and the remaining eight studies used a multiple-baseline or multiple-probe across-behaviors design.

Precurrent repertoires

The precurrent repertoires taught in these 12 studies may be categorized as (a) behavior chains, (b) rules/self-questioning/self-prompting, (c) visual imagining, (d) recombining units/improvising, and (e) sorting/sequencing/tacting (Tables 2 and 3).

Behavior chains The precurrent behaviors in the counting (Grimm et al., 1973; Parsons, 1976) and word problem-solving (Levingston et al., 2009; Neef et al., 2003) studies were behavior chains applied to each problem. Grimm et al. (1973) targeted three types of math problems: (a) matching a numeral to a quantity of dots (i.e., “numeral-symbol”), (b) matching a quantity of dots to a numeral (i.e., “symbol-numeral”), and (c) matching a quantity of dots to the same quantity of dots (i.e., “symbol-symbol”). The trained precurrent behaviors were (a) tacting the printed numeral, (b) pointing to each dot and counting aloud, (c) stopping when the count reaches the printed numeral, and (d) circling the set of all the counted dots. Parsons (1976) replicated Grimm et al. (1973) and demonstrated that blocking the precurrent responses degraded correct solutions.

Neef et al. (2003) targeted solving math word problems, such as “If Bob had 2 books and bought 7 more, how many did he have in the end?” (p. 26). Listed below the problem were boxes in which to write (precurrent behavior) the (a) initial set (e.g., 2), (b) change set (e.g., 7 more), (c) operation (e.g., addition), (d)

Table 1 Ages, diagnoses, and prerequisite skills of participants across the problem-solving studies

Study	Participants			
	Number	Age (years)	Diagnoses (if any)	Prerequisite skills
Grimm et al. (1973)	2	7, 9	Cerebral palsy, not identified	Tacting numerals, counting from 1 to 20
Parsons (1976)	5	4–5	Typically developing	Counting from 1 to 20
Mayfield and Chase (2002)	33	College students	Typically developing	Course on finite math (lowest level math available for college credit)
Neef et al. (2003)	2	19, 23	Developmental disability	Computing addition and subtraction problems, reading vocabulary words in the word problems
Levingston et al. (2009)	2	10	Typically developing, autism spectrum disorder	Computing multiplication and division problems, discriminating larger and smaller numbers from pairs between 1 and 100
Aguirre and Rehfeldt (2015)	3	17–18	ADHD, Asperger's syndrome, auditory processing disorder, dyslexia, dyscalculia	Identifying as a listener, tacting, and writing all letters
Parsonson and Baer (1978)	5	3–6	Typically developing, language and behavioral delays	
Park and Gaylord-Ross (1989)	3	18, 16, 18	Mental retardation, cerebral palsy	Stated willingness to improve social behaviors, some reading and math
Marckel et al. (2006)	2	4, 5	Autism spectrum disorder	Color, shape, and action matching to sample; independent manding with PECS
Sautter et al. (2011)	4	3–5	Typically developing	
Kisamore et al. (2011)	4	4–5	Typically developing	
Frampton and Shillingsburg (2018)	3	5–6	Autism spectrum disorder	VB-MAPP Milestones scores (125–143.5) VB-MAPP Intraverbal scores (5) VB-MAPP Tact scores (12–13.5) VB-MAPP Mand scores (10–12)

ADHD attention-deficit hyperactivity disorder, *PECS* picture exchange communication system, *VB-MAPP* Verbal Behavior Milestones Assessment and Placement Program (Sundberg, 2008)

Table 2 Skills, problems, precurrent behaviors, and current behaviors across the problem-solving studies

Skill	Study	Problem	Precurrent behaviors	Current behaviors
Math	Grimm et al. (1973)	Matching sets of symbols to numbers, numbers to sets of symbols, and equal sets of symbols to each other	Follow steps in a matching task analysis.	Circle the matching symbol or number.
	Parsons (1976)	In the presence of a set of symbols, circling the number that corresponds to the symbols in the sample	Follow steps in a counting task analysis.	Circle the number that corresponds to the number of symbols in the left column.
	Mayfield and Chase (2002)	$[(3g^5 \times 8g^9)/4g^8]^2 =?$	Simplify expressions based on taught rules.	Simplify expressions that require combining rules.
	Neef et al. (2003)	"If Sam had 10 pens and then lost 8, how many did he have left?"	Write components.	Write the solution to the word problem.
	Levingston et al. (2009)	"Nathan's wall is 12 ft wide. How many posters can he hang on his wall if each poster is 2 ft wide?"	Write components.	Write the solution to the word problem.
Spelling	Aguirre and Rehfeldt (2015)	"Write ____." (e.g., connoisseur)	Visually imagine.	Write correctly spelled words matched to dictated words.
Play	Parsonson and Baer (1978)	A request to perform a skill (e.g., hammering, filling a container, lacing a shoe) without a conventional tool	Use trained, unconventional tools.	Use untrained, unconventional tools.
Social skills	Park and Gaylord-Ross (1989)	"A client approaches you at work. What are you supposed to say?"	Self-question.	Engage in appropriate social interaction at work.
Mands	Marckel et al. (2006)	EO, no corresponding PECS icon	Describe the missing item.	Mand for the missing item.
Intraverbals	Sautter et al. (2011)	"Tell me some [category]."	State and follow rules.	Emit a list of items within a category.
	Kisamore et al. (2011)	"Tell me some [category]."	Visually imagine.	Emit a list of items within a category.
	Frampton and Shillingsburg (2018)	A request to explain how to complete an activity (e.g., "How do I play bowling?")	Sort, sequence, and tact pictures of the target activity.	Explain how to complete an activity.

PECS picture exchange communication system

Table 3 Types of precurrent repertoires

Precurrent repertoires	Studies	Problems
Behavior chains	Grimm et al. (1973) Parsons (1976) Neef et al. (2003) Levingston et al. (2009)	Matching a numeral to a quantity Matching a numeral to a quantity Completing math word problems Completing math word problems
Rules/self-questioning/self-prompting	Park & Gaylord Ross (1989) Sautter et al. (2011)	Solving social problems at work Listing members of a category
Visual imagining	Kisamore et al. (2011) Aguirre and Rehfeldt (2015)	Listing members of a category Spelling
Recombining units/improvising	Parsonson and Baer (1978) Mayfield and Chase (2002) Marckel et al. (2006)	Playing when a tool is absent Solving algebra problems Manding when the PECS icon is missing
Sorting/sequencing/tacting	Frampton and Shillingsburg (2018)	Explaining how

PECS picture exchange communication system

resulting set (e.g., unknown), and (e) solution (e.g., 9). After each component was mastered, a probe of novel problems tested for the emergence of the remaining steps. Both participants acquired all precurrent behaviors and subsequently solved novel word problems. Levingston et al. (2009) extended Neef et al. (2003) by targeting multiplication and division with two elementary school students, one with autism, using a self-checking consequence procedure.

Strengths of the behavior-chains type of problem solving are that the strategies can be instructed, modeled, and reinforced in a straightforward manner. A potential limitation is that there is no training of verbal behavior used to evoke each precurrent behavior. This is limiting for two reasons. First, behavior chains alone may constrain current behavior; they may be beneficial for math problems but limited for other problems. Second, it is not clear how a participant would discriminate variations in the problem situations. For example, in Grimm et al. (1973), it is not clear how one would decide to apply the numeral-symbol or symbol-numeral set of precurrent behaviors. This could be established through discrimination training or with a "self-rule," such as "If the numeral is on the left, first say its name and then count the dots on the right." Such a rule was not necessary in these four studies, but it could support chain-based problem solving.

Rules/self-questioning/self-prompting Park and Gaylord-Ross (1989) suggested that although instructions, modeling, role-play, and feedback can increase social skills, this training method alone results in limited generalization. Alternatively, they suggested a problem-solving approach that "relies on the person's understanding and acting upon the rules of a social situation" (p. 374). With three adolescents with intellectual disabilities, Park and Gaylord-Ross targeted social initiations, conversational expansions, conversational terminations, and mumbling with coworkers during work (e.g., washing dishes) and breaks. The precurrent behaviors consisted of asking and answering seven rules, or questions:

1. What's happening?
2. What are three behaviors I could emit?
3. What will be the outcome of each?
4. Which is better?
5. Pick one.
6. Emit the behavior.
7. How did I feel?

The training involved showing a picture of a social situation, explaining, modeling, and praise. For two out of three participants, the problem-solving approach resulted in improvements in the target behaviors compared to baseline and compared to instructions, modeling, role-play, and feedback alone.

Sautter et al. (2011) targeted emitting many items when given a category (e.g., "Tell me some animals.") by teaching children to use self-rules and intraverbal chains as precurrent behaviors. Initial training was comprised of teaching (a) tacting the item and the group (e.g., "It's a tiger and zoo animal."), (b) tacting the group and the category (e.g., "It lives in the zoo and it's an animal."), (c) intraverbally responding to groups with items (e.g., "Lion, tiger, monkey, giraffe" as a response to "Tell me some zoo animals."), and (d) intraverbally responding to categories with groups (e.g., "Farm, zoo, and ocean" as a response to "Tell me the groups of animals"; pp. 231–232). The precurrent behaviors were trained in three phases. First was responding to the question "What are your four rules?" with "Say three groups," "pick a group," "pick another group," and "say the last group" (p. 233). Second was responding to the question "What is your *x*th rule?" with the rule and the response (e.g., responding, "Pick a group: farm," to the question "What's your second rule?"). Third was modeling the strategy of using rules to respond to the target intraverbal question. This training, combined with prompts during probes, increased responding to desired levels (i.e., 12 responses to each target question).

The precurrent behaviors trained in these two studies were behavior chains with the added component of asking or saying something and responding to it. The strategy used by Park and Gaylord-Ross (1989) is appealing as the questions are open-ended and not as constrained as behavior chains alone. The self-rules used by Sautter et al. (2011) were not open-ended but also required saying a self-rule and responding to it. The strategy used by Neef et al. (2003) might be enhanced by teaching the self-rules of "Find the initial set," "Find the change set," and so on.

Visual imagining Kisamore et al. (2011) extended Sautter et al. (2011) by teaching visual imagining. First, the experimenter asked the participant to look at a picture of a group with all its targeted members (e.g., a picture of a farm with four animals) on a PowerPoint slide on a laptop. Second, the experimenter closed her eyes and the computer screen went gray to simulate what it looks like when eyes are closed. Third, the scene reappeared on the screen and the experimenter said, "I see an [item]," and that item appeared in the scene. Finally, the experimenter asked the participant to close his eyes, imagine the place, and state what he sees. Responding increased to criterion levels when the experimenter prompted the use of the strategy.

Aguirre and Rehfeldt (2015) taught visual imagining to increase spelling. The experimenter delivered instructions, said a word, showed the printed word for 3 s, removed the printed word, and said,

> See if you can see the written word in your head (3 s pause). Imagine the word on a piece of white paper (3 s pause). Help yourself remember the word by imagining yourself writing over each letter of the word (3 s pause). Write ___. (p. 123)

The experimenter did not deliver consequences or prompts. A final condition included positive (praise) and corrective (modeling) feedback. For two out of three participants, the visual-imagining condition increased correct responses, and the consequences improved responding further.

Visual imagining seems common in everyday problem solving, such as recalling past events, giving geographical directions, and solving math problems. There could be many ways to teach visual imagining to children, such as by briefly holding up a picture of a scene, removing it, and asking questions about it. This could be a precursor to using visual imagining to solve more complex problems.

Recombining units/improvising The problem addressed by Parsonson and Baer (1978) was a request to perform a play skill without conventional tools present. The targeted skills were (a) hammering toy pegs, (b) filling a container with 80 marbles and carrying it around, and (c) lacing a shoe. In baseline, the experimenter demonstrated the skills using a wooden toy claw hammer, a paper bag, and a shoelace, respectively. After each demonstration, the experimenter removed the tool and presented a tray with additional items that could be used to perform the skill, such as a piece of brick, a glove, and a pipe cleaner, respectively. There were also distractor items that could not be used to perform the skill, such as a Styrofoam hammer, a tissue, and a wide ribbon. The participants could use the generalization tools as either "simple improvisations" (e.g., using the brick or the glove) or "complex improvisations" (e.g., combining a rod and a drilled block, combining a bottomless Styrofoam cup and foil). The modeled precurrent behaviors were simple and complex improvisations using tools other than those in the probe. Results showed increased improvisations in the training phases compared to baseline.

Mayfield and Chase (2002) targeted simplifying algebraic expressions (e.g., "$[(3g^5 \times 8g^9)/4g^8]^2 =?$"; p. 109) with college students. The experimenters taught five rules for simplifying algebraic expressions, including how to multiply variables and coefficients with exponents, divide variables and coefficients with exponents, and find the roots of variables and coefficients with exponents. They presented these rules on a worksheet with examples and provided positive and corrective feedback for applying the rules in a set of 25 expressions. For the "problem-solving items" on the tests, the rules had to be combined to simplify the expressions. The rule training increased scores on the problem-solving items on the tests.

Marckel et al. (2006) addressed a problem for children who use the picture exchange communication system (PECS): There is an EO for a reinforcer, but no icon depicting that reinforcer. The precurrent behaviors were combining icons of shapes, colors, and functions depicting the desired items. For example, to request a marshmallow, the trained response was "I want eat white circle," and to request a videotape, the response was "I want watch green rectangle" (p. 100). In addition to acquisition, both children demonstrated within-class generalization (e.g., requesting "green" when "red" and "blue" were taught), but not between-class generalization (e.g., requesting "green" when "eat" and "circle" were taught).

Recombining minimal units may be a useful explanation for the problem solving that occurs when learning language or other skills in everyday life. The research on matrix training and recombinative generalization has shown that individuals can recombine units when learning object-preposition combinations (H. Goldstein, Angelo, & Mousetis, 1987), action-object combinations (Frampton, Wymer, Hansen, & Shillingsburg, 2016), onset-rime combinations (Mueller, Olmi, & Saunders, 2000), and spelling (Kinney, Vedora, & Stromer, 2003). In addition, the concept of recombining minimal units is the foundation of emitting and responding to autoclitic frames (Speckman, Greer, & Rivera-Valdes, 2012). These analyses are paramount in explaining the emergence of untrained behavior and are especially relevant for emitting novel combinations of behavioral units in the face of novel stimulus conditions (i.e., problem solving).

Sorting/sequencing/tacting Frampton and Shillingsburg (2018) addressed the problem of a request to explain how to complete an activity, such as "How do I play bowling?" The target response was explaining how, such as "First set up pins, next pick up the ball" (p. 240), and so on. The researchers ensured the participants could tact the four pictures corresponding to the steps of each activity. The "problem-solving strategy training (PSST)" was presenting the request with the pictures present and prompting sorting, sequencing, and tacting the pictures using "first," "next," and so on. After PSST with two or four activities and tact training with two or four other activities, responding increased with the other activities with materials present and absent.

This type of problem solving differs from the others in that the emission of the trained precurrent behaviors—sorting, sequencing, and tacting pictures—occurred at an earlier point in time relative to the current behavior and not at all for the generalized responses. Frampton and Shillingsburg (2018) hypothesized that covertly sorting, sequencing, and tacting the steps resulted in correct intraverbal responses, but that covert repertoire was not directly trained or measured. Future researchers may combine the "first, next" rule training (i.e., Sautter et al., 2011) with the visual-imagining training (i.e., Aguirre & Rehfeldt, 2015; Kisamore et al., 2011) to teach sorting, sequencing, and tacting to produce "explaining how" and other complex intraverbals.

Overall, these 12 studies demonstrated five effective types of problem solving: (a) behavior chains, (b) rules/self-questioning/self-prompting, (c) visual imagining, (d) recombining units/improvising, and (e) sorting/sequencing/tacting. Future researchers should continue evaluating these strategies, as well as combining them to teach complex repertoires.

Multiple control and problem solving

All five types of problem solving rest on, and require an analysis of, the multiple stimulus control leading to the solution response (Michael, Palmer, & Sundberg, 2011). Analyzing multiple sources of antecedent control often mitigates rote responding, or responding under fewer sources of control than occur under natural conditions. Examples of rote responding are stating memorized answers to math problems; repeatedly stating, "Dog, cat, bird," when asked to "tell me some animals"; and stating, "Fine," independent of a peer's social initiation. On the other hand, through engaging in precurrent behaviors such as following a set of rules, responding in the presence of a math problem comes under the multiple control of the math problem and the verbal stimuli produced by the stated rules. In the same manner, when faced with an intraverbal categorization problem, responding under the multiple control of the verbal stimulus (e.g., "Tell me some [category].") and either a covert, visually imagined stimulus (Kisamore et al., 2011) or the stimuli produced by stated rules (Sautter et al., 2011) may lead to the learner applying the strategy to other categorization problems, resulting in a more functional repertoire. To progress from teaching rote responding, researchers must carefully analyze all relevant controlling variables to develop effective teaching procedures for complex skills.

Another area of research that captures the concept of multiple control and may be considered problem solving is *joint control*. Lowenkron (1998) defined joint control as "the effect of two S^Ds acting jointly to exert stimulus control over a common response topography" (pp. 328–329), where the S^Ds are the products of self-echoic and tact or textual responses. For example, when faced with the problem of a delayed auditory-visual matching-to-sample task, the analysis of joint control is that the subject self-echoes the sample stimulus while tacting (or emitting textual behavior to) the comparison stimuli; when there is a match between the product of the self-echoic and the product of the tact or textual behavior, the selection occurs. This analysis becomes a procedure when, for example, a child is presented with an array of 12 pictures cards; instructed to "give me the ball, cup, and spoon"; and taught to repeat this instruction to him- or herself to facilitate giving the three items in the order instructed (Causin, Albert, Carbone, & Sweeney-Kerwin, 2013). According to the joint control analysis, the correct responses were multiply controlled by the products of the self-echoics and tacts/textual behaviors. This joint control procedure meets the definition of a problem-solving strategy, and the tacting and counting in the Grimm et al. (1973) and Parsons (1976) studies may be considered joint control. Future research could examine other ways the joint control analysis can influence problem solving.

Location of problem solving: overt or covert?

A difficulty in analyzing problem solving is that the precurrent responses are typically covert (Palmer, 2011; Skinner, 1957; e.g., counting, weighing options "in your head"). The researchers in this review addressed this challenge by requiring the overt emission of the precurrent responses during the acquisition phase. For example, in the two counting studies (Grimm et al., 1973; Parsons, 1976), the participants emitted an overt chain of responses including tacting the numeral and touching and tacting each picture before emitting the final response of circling the correct number of stimuli. Parsons (1976) demonstrated that prohibiting these overt responses reduced accuracy, suggesting that the responses did not successfully transfer to the covert level. In the two word-problem studies (Levingston et al., 2009; Neef et al., 2003), the precurrent responses of writing numbers from the word problem into the graphic organizer were taught solely as overt responses. Mayfield and Chase (2002) did not specify if their participants emitted the precurrent behaviors (e.g., adding exponents) at the overt or covert level. Future researchers could train the precurrent responses from these five studies at the covert level.

In the two improvisational studies, the participants were taught the overt responses of selecting PECS icons matching the stimulus properties of the presented items (Marckel et al., 2006) and using untrained tools (Parsonson & Baer, 1978). Given the media, these behaviors would be impossible to teach at the covert level. However, a replication of Marckel et al. (2006) with vocal verbal behavior could allow for a study of this process. Park and Gaylord-Ross (1989) trained participants to emit vocal responses matching a prespecified set of rules with correction for rule omission. However, because precurrent responses were not recorded, it is not possible to determine if they were demonstrated overtly, covertly, or not at all. Precurrent responses could only be inferred based on increases in prosocial responses following training.

The categorization study by Sautter et al. (2011) was unique in that it directly studied the transfer from overt to covert responding. Participants were trained to vocally emit and follow rules for categorization, and data were collected on vocal emission of the rule statements. For three of the four participants, audible rule statements decreased while correct, terminal response emission remained constant, suggesting rule statements faded from the overt to the covert level. The two visual-imagining studies, by definition, targeted covert behaviors impossible to directly observe. To attempt to evaluate the role of covert behavior mediating spelling, Aguirre and Rehfeldt (2015) measured finger spelling, vocal spelling (including moving lips), echoing, and looking away from paper (i.e., collateral responses), but they observed no correlations with correct spelling. In Kisamore et al. (2011), two pieces of evidence suggest visual imagining occurred. First, the participants emitted responses clustered by group (e.g., farm animals) in varied orders each session, suggesting use of the imagining strategy and not simply an intraverbal chain. Second, the participants emitted phrases consistent with imagining (e.g., "I forget what is under the plane."), which would be unlikely in the absence of an imagined scene. Frampton and Shillingsburg (2018) hypothesized sequencing and tacting occurred at the covert level. Future researchers may examine other ways to test the hypothesis of visual imagining, such as by requiring participants to reconstruct the scene using pictures or a computer program.

In summary, even though problem solving in everyday life often occurs at the covert level, few of the reviewed studies established problem solving at the covert level. For maximal outcomes, future researchers teaching overt problem solving should include an extra phase with an attempt to transfer precurrent responses to the covert level. In general, this may be accomplished through practice, verbal report, and public accompaniments. For example, researchers may first teach participants to overtly emit behavior chains to solve word problems; then they may teach the participants to emit the chains covertly and occasionally report on the covertly emitted steps. With self-questioning, a sequence of steps may be (a) bring overt self-questioning to fluency, (b) require portions of self-questions and answers to be emitted covertly, and finally (c) require all self-questioning and answering to be emitted covertly followed by overt responses. Although we may never observe responding at the covert level, the procedures for teaching covert problem solving are overt and measurable, as are the targeted current responses.

Limitations

We raise two important limitations of this literature. First, problem-solving strategies are more complex and time-consuming to teach compared to simple prompting, prompt fading, and reinforcement. For example, Neef et al. (2003) found that many trials to criterion were needed to train their participants. Parsons (1976) suggested that monitoring and reinforcing precurrent behaviors in a classroom may not be practical, and he reported poor social validity. Park and Gaylord-Ross (1989) stated that the precurrent behaviors involved in teaching participants to respond to social problems at work may be too advanced for some learners and teaching these behaviors to fluency may be too time-consuming. Therefore, future researchers should evaluate the duration of problem-solving training to ultimately use efficient procedures.

A second limitation was limited data on maintenance and generalization. Although Park and Gaylord-Ross (1989) conducted maintenance probes, they did not specify the times at which they were conducted. Kisamore et al. (2011) collected weekly maintenance data on prerequisite skills but not on the visual-imagining training responses. Generalization, or perhaps generality, is inherent in problem solving, as problem solving is defined as emitting behaviors under novel stimulus conditions. Nevertheless, only half (six) of the studies used the generalization promotion tactic of multiple-exemplar training (Frampton & Shillingsburg, 2018; Levingston et al., 2009; Marckel et al., 2006; Mayfield & Chase, 2002; Neef et al., 2003; Parsonson & Baer, 1978), and none of the studies addressed generalization across settings or people. Future researchers should more thoroughly evaluate the generalization and maintenance of these skills.

In addition to limitations of the literature, a limitation of this review was the narrow focus on applied studies explicitly informed by Skinner (1957, 1968). There is a wealth of research and writing in behavior analysis addressing problem solving from other points of view. For example, there are many basic research studies on delayed matching to sample (e.g., Arntzen, 2006; Ribeiro, Silveira, Mackay, & de Rose, 2016; Urcuioli & DeMarse, 1997) and derived relational responding (e.g., Fields, Garruto, & Watanabe, 2010; Holth & Arntzen, 2000; Ma, Miguel, & Jennings, 2016; Stewart, Barrett, McHugh, Barnes-Holmes, & O'Hora, 2013) that discuss possible mechanisms of precurrent behavior, problem solving, and memory. As previously mentioned, joint control and rehearsal can be conceptualized as problem solving (Clough, Meyer, & Miguel, 2016; Gutierrez, 2006; Lowenkron, 1998), and there is an emerging applied literature in this area (Causin et al., 2013; Tu, 2006). Behavioral variability may be useful in analyzing problem solving, as problem-solving behaviors may be a function of extinction-induced or reinforced variability (Neuringer, 2004; Shahan & Chase, 2002). It would be beneficial to connect the literature on rule-governed behavior and talk-aloud procedures to problem solving to address the covert behavior challenges

(Arntzen, Halstadtro, & Halstadtro, 2009; Cerutti, 1989; Johnson, 1997; Rehfeldt, Dixon, Hayes, & Steele, 1998). Finally, other applied strategies, such as mands for information (Shillingsburg, Frampton, Wymer, & Bartlett, 2016), may be regarded as problem-solving strategies. The current review offers a first step in providing a review and conceptual analysis of applied behavior-analytic approaches to problem solving. Further analysis is needed for a comprehensive review of problem solving.

Applications and recommendations for future research

Because problem solving is ubiquitous in everyday behavior, the possibilities for applications in practice and directions for future research are endless, and they are particularly salient for teaching communication, social, and academic skills. For example, once an initial verbal repertoire is established in a rote fashion, behavior analysts should analyze the problem-solving behaviors that will lead to complex communication skills, such as answering questions about the past. An effective social skills repertoire requires rapid discriminations and adaptations to ever-changing stimulus conditions. Social skills such as noticing disinterest, initiating and maintaining conversations, and responding to social conflicts all require fluid social repertoires facilitated through problem solving. Finally, although prompting and prompt fading have been effective for teaching many academic skills (Spooner, Knight, Browder, & Smith, 2012), problem-solving strategies could be applied to establish academic repertoires that serve as behavioral cusps and increase generalized responding. Examining and translating published curricula may provide ideas for learning targets and problem-solving approaches.

A. P. Goldstein, McGinnis, Sprafkin, Gershaw, and Klein (1997) presented 50 task analyses of social repertoires for adolescents with social delays ranging from “introducing yourself” to “expressing your feelings” to “dealing with embarrassment.” It could be fruitful to translate the skills into behavior-analytic terms and select problem-solving behaviors inherent in those repertoires. For example, the four steps of Skill 1, listening, are (a) look at the person who is talking, (b) think about what is being said, (c) wait your turn to talk, and (d) say what you want to say. Step 2 could be analyzed as a covert self-echoic problem-solving behavior in which the student covertly repeats what the other person said. Step 3 could be analyzed as discriminating when the person is and is not talking. Step 4 could be analyzed as a covert self-rehearsal prior to overt emission. Another example is Skill 4, asking a question. The four steps are (a) decide what you’d like to know more about, (b) decide whom to ask, (c) think about different ways to ask your question and pick one way, and (d) pick the right time to ask your question. These behaviors require discriminations and the covert problem-solving behaviors of “brainstorm possibilities and pick one.” Because of the complexity of social behavior, researchers and practitioners should teach problem-solving behaviors to establish complex social repertoires.

In addition to social skills, behavior analysts may play a valuable role in more traditional educational environments through applications of problem solving. For example, reading comprehension could be taught using behavior chains (Neef et al., 2003), perhaps by having students fill component information into graphic organizers while reading (e.g., Bethune & Wood, 2013). Counting forward beginning with a given number could be facilitated by teaching students to visually imagine a number line, start with the stated number, and read the subsequent numbers (Aguirre & Rehfeldt, 2015). The studies in this review provide hints at the potential we may be able to achieve if more frontline applications were developed for educators.

Conclusion

The 12 applied studies in this review provide empirical support for Skinner's (1957, 1968) definition of problem solving. Studying and appreciating the analysis of problem solving will help behavior analysts continue moving from teaching rote responses to facilitating the emergence of complex communication, social, and academic repertoires. In the reviewed studies, researchers taught the precurrent repertoires of behavior chains, self-questioning, visual imagining, recombining units, and sequencing to increase math, spelling, play, social, mand, and intraverbal skills. In all 12 studies, the ultimate (i.e., current) behavior was never taught; rather, the researchers taught problem-solving (i.e., precurrent) behaviors whose products evoked the current behaviors. Problem solving requires an appreciation of the multiple control of behavior and may be regarded as a method of programming for generalization. Research on problem solving is challenging, as problem solving often occurs covertly; yet, researchers can observably instruct participants to use problem-solving strategies that may occur at the covert level.

Because problem solving is inherent in countless daily interactions between humans and their environments, there are numerous future directions for studying and analyzing problem solving. We offered a framework for selecting targets for problem solving: examine a written protocol for complex behavior, translate the language into behavior-analytic and problem-solving terms, and teach those problem-solving strategies. When targeting complex repertoires, researchers and practitioners should move beyond prompting, prompt fading, and reinforcement to complete analyses of the multiple variables impacting those repertoires and the problem-solving behaviors that will facilitate their acquisition and generalization.

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Compliance with ethical standards

Conflict of interest Judah B. Axe declares that he has no conflict of interest. Stephanie H. Phelan declares that she has no conflict of interest. Caitlin L. Irwin declares that she has no conflict of interest.

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