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BRIEF REPORT

Derived Equivalence Relations of Geometry Skills in Students with Autism: an Application of the PEAK-E Curriculum

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Abstract The present study evaluated the efficacy of equivalence-based instruction (EBI) as described in the PEAK-E curriculum (Dixon, 2015) for promoting the emergence of derived geometry skills in two children with high-functioning autism. The results suggested that direct training of shape name (A) to shape property (B) (i.e., A-B relations) was effective for both participants. Following A-B training, both participants demonstrated emergent relations that are consistent with symmetry (B-A), as well as emergent shape name (A) to shape picture (C) relations that are consistent with transitivity (A-C). The results expand on existing literature by demonstrating the emergence of an A-C relation when neither A nor B stimuli were ever trained to C stimuli and illustrate the efficacy of EBI for training geometry skills.

Keywords Autism · Equivalence-based instruction · Geometry · Stimulus equivalence · PEAK

Many studies have sought to teach simple mathematic skills (e.g., number matching, counting) to students with math difficulties (e.g., Lynch & Cuvo, 1995) and intellectual impairments and autism (e.g., Cihak & Grim, 2008). Fewer studies have demonstrated procedures for teaching more complex mathematic skills, such as geometry and algebra (Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman, 2008) with individuals with autism. Given that almost half of individuals with autism are estimated as having average intelligence (Centers for Disease Control and Prevention, 2014), procedures to teach more complex academic skills are needed. The dearth of literature concerning

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instruction of advanced mathematical skills with students with autism could be due to a greater focus being placed on priorities such as social development or due to the symbolic and abstract nature of mathematical operations that may be particularly difficult for children with the disorder. Stimulus equivalence procedures (Sidman & Tailby, 1982) may have utility in overcoming these issues given the symbolic and referential nature of equivalence learning (Horne & Lowe, 1996) and the efficiency of the resulting emergence of untrained relations between stimuli following equivalence procedures. According to Sidman (1994), stimulus equivalence describes the phenomenon in which stimuli come to have identical functions through both training of direct relations and resulting symmetrical and transitive relations. For example, if taught to respond to a picture of a dog (A) by saying the word "dog" (B) and to respond to the picture of a dog (A) by selecting the printed word DOG (C) from an array, one might display not only the trained responses, described as A-B and A-C relations, respectively, but also the untrained symmetry relations of responding to the spoken word "dog" (B) by selecting the picture of the dog (A) and to the printed word DOG (C) by selecting the picture of dog (A) (B-A and C-A relations, respectively), as well as the emergent transitive relations of responding to the spoken word "dog" (B) by selecting the printed word DOG (C) and by responding to the printed word DOG by saying "dog" (B) (B-C and C-B relations, respectively).

Procedures designed to produce untrained relational responses via equivalence class formation have been used to teach mathematical skills to individuals with and without disabilities. Such equivalence-based instruction (EBI) has been used to teach addition and subtraction to typical second through fifth graders (Henklain & Carmo, 2013) and fraction and decimal relations to typical fifth and sixth graders (Lynch & Cuvo, 1995). EBI provided to individuals with moderate intellectual disabilities resulted in the emergence of untrained matching between equal coin combinations after individual coin combinations had been trained to a printed value (McDonagh, McIlvane, & Stoddard, 1984) and between coin combination components and new coin combinations with prices and individual coins (Stoddard, Brown, Hulbert, Manoli, & McIlvane, 1989). In a study conducted by Keintz, Miguel, Kao, and Finn (2011), two 6-year-old boys with autism were taught to respond to a dictated coin name by selecting the actual coin from an array (A-B), to respond to the actual coin by selecting a printed price (B-C), and in the presence of the dictated price to select the printed price (D-C). Following training, all seven of the potential untrained relations were observed for one participant and four of the seven untrained relations were observed for the other, who subsequently learned the remaining relations through direct training.

Although shapes are common stimuli used to teach listener and tact responses, procedures to teach geometric properties of shapes have yet to be explored in children with autism. Teaching the defining features of geometric shapes such as the lengths of sides, angles of corners, or the number of corners may build the foundational skills needed to learn more complex geometry skills. Furthermore, the capacity of such procedures for promoting the emergence of untrained relations requires demonstration, as an instructor may otherwise need to teach all possible relations. The present study extends equivalence research on mathematics and geometrical instruction in two ways. First, the study evaluated direct training of shape name (A) to shape property (B) (i.e., A-B relations) as well as the emergence of symmetrical shape property (B) to shape name (A) (i.e., B-A relations). Second, the study evaluated derived transitive shape name (A) to shape picture (C) (i.e., A-C relations) without direct training of either A or B stimuli to C stimuli.

Method

Participants, Setting, and Materials

Two males with autism who attended a special school for children with autism participated. Josh was 15 years old and Dustin was 13 years old. Autism diagnoses were obtained from the participants' school records. Both participants demonstrated mastery of all items on the *Verbal Behavior Milestones Assessment and Placement Program* (VB-MAPP; Sundberg, 2008) as well as the *PEAK: Direct Training Assessment* (PDA; Dixon, 2014), two behavioral assessments used to evaluate the language skills of individuals with autism. School records indicated that the WISC-IV Short Form intelligence quotient test was completed within 6 months prior to the study. Josh and Dustin had IQ scores of 92 and 104, respectively. Caregivers reported that both participants could follow instructions to identify corners and lines on shapes.

The *Promoting the Emergence of Advanced Knowledge: Equivalence Module* (PEAK-E; Dixon, 2015) assessment and curriculum was conducted to evaluate general equivalence responding. The PEAK-E assessment and curriculum contains 184 items and related programs targeting learning skills, including basic matching and labeling, formal logic and perspective taking, and mathematic skills such as geometric shape properties. PEAK assessment items and programs utilize a train/test methodology (see Green & Saunders, 1998), which involves first teaching necessary relations and then testing for the emergence of untrained relations. A PEAK-E Pre-Assessment Long Form (PEAK-E-PLF) was conducted with each of the participants to evaluate their ability to derive untrained relations. The PEAK-E-PLF consists of 48 assessment items that are organized into four categories: reflexivity, symmetry, transitivity, and equivalence. Each category consists of three levels of complexity (Basic, Intermediate, and Advanced) with two programs for each level. Each program of the PEAK-E-PLF includes an instructional component designed to establish a trained relation and a test for subsequent emergent relations. Each program includes two assessment items thus producing an overall max score of 48. No praise or feedback is provided for correct responding; however, reinforcement is provided following the assessment for compliance. Josh and Dustin had PEAK-E-PLF scores of 41 and 42, respectively, suggesting that they were both able to demonstrate all responses consistent with reflexivity and symmetry relations, as well as most of the tested transitivity and equivalence relations.

Sessions were conducted in the students' home classrooms and lasted between 20 and 60 min. Each classroom contained four to eight students, 1 to 2 paraprofessionals, and a classroom teacher. The materials and procedures used in the study were adapted from the PEAK-E curriculum program, *5E—Symmetry: Shape Names*. Stimuli included vocal shape names (A), written numbers denoting a number of shape sides (B), and pictures of the shapes (C). The five shapes used in the study included a pentagon, hexagon, heptagon, octagon, and decagon. The study was conducted over 2 weeks.

Dependent Variable and Interobserver Agreement

The dependent variable was the percentage of correct trials within a five-trial session. Correct responding was defined as emitting a response consistent with the trial relation within 20 s. For A-B trials, a correct response was touching the related number card. For B-A trials, a correct intraverbal response was vocally stating the shape name. For A-C trials, a correct response was touching the appropriate shape picture.

Interobserver agreement (IOA) was obtained for 32 % of sessions, distributed equally across phases, by two trained research assistants. Interobserver agreement was calculated by dividing the number of trials where both observers recorded the same scores, divided by the total number of trials, and multiplying by 100; IOA was 100 % for both participants.

Design and Procedure

A concurrent multiple-probe design across participants was used to evaluate the effects of training. Following baseline, training for the A-B relation was conducted. Mastery criterion consisted of a minimum of five sessions at 100 % correct responding. Following mastery of the trained A-B relation, probes were conducted for derived B-A and A-C relations in two testing phases.

Baseline and Test Probes During baseline, A-B, B-A, and A-C relations were probed. No reinforcement was provided for correct responses, and no prompting was used following incorrect responses. Each session consisted of five trials. Targets related to each of the five shapes were presented one time in each session. A-B probes were conducted by presenting five picture cards each with a written number (i.e., 5, 6, 7, 8, 10) representing the number of sides of each of the shapes. The experimenter asked, “How many sides does (A) have?” where A was the vocal shape name. The participant then selected one of the written numbers from the array. B-A intraverbal probes were conducted by presenting a written number card (B), and asking “Which shape has this many sides?” where B was the number written on the card. The participant then vocally provided a shape name. A-C probes were conducted by presenting an array of the five shapes (C). The experimenter then asked, “Which is a (A)?” The participant then selected one of the shapes from the array. Test probes of B-A and A-C relations were conducted following mastery of the trained A-B relation and were conducted identically to baseline.

A-B Training (Vocal Name to Property) Training was conducted to establish the selection of a number card (B) when presented with the vocal discriminative stimulus, “How many sides does (A) have?” (A-B). The presentation of stimuli was the same as in baseline except a general praise statement (e.g., “great job” “that’s right”) was provided for correct responses. Specific praise (e.g., “great job, a pentagon does have five sides”) was never provided. No other responses, such as attending, were praised during sessions. If an incorrect response occurred, a least-to-most intrusive prompting procedure was used. Specifically, upon emission of the first incorrect response, the experimenter stated, “not that one, try again.” If the participant then emitted a second incorrect response, the experimenter pointed to the correct card. Contingent upon another incorrect response, the experimenter provided a hand-over-hand prompt to participant.

B-A (Property to Vocal Name) and A-C (Vocal Name to Picture) Testing Following mastery of the A-B relation, B-A and A-C test probes identical to baseline were conducted across five sessions. No feedback or prompts were provided during any trial.

Results and Discussion

The results of the present study are summarized in Fig. 1. During the baseline phase, neither participant showed mastery of any of the relations. In the A-B training phase, both participants met the mastery criterion of five consecutive sessions with 100 % correct responding. Following direct training of the A-B relation, responding during test probes for the B-A relation increased suggesting emergence consistent with symmetry. Responding during the A-C test probes also increased suggesting emergence of the relation consistent with transitivity.

The emergence of the relation between the shape names (A) and shape pictures (C) was particularly interesting given that only the A-B relation was trained. No training to

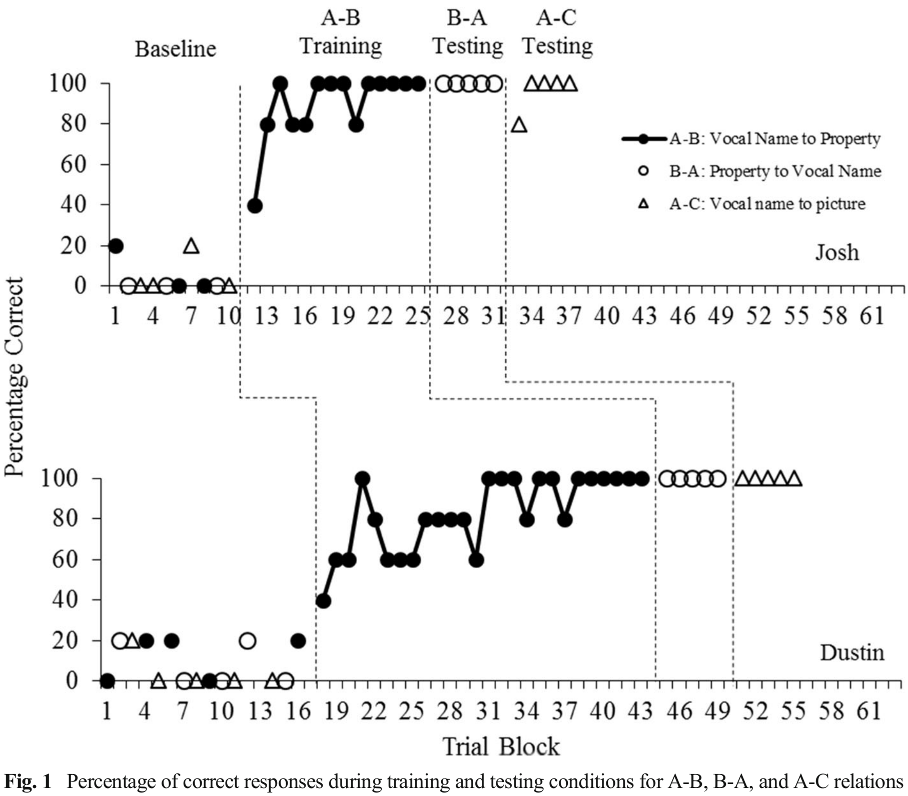

Fig. 1 Percentage of correct responses during training and testing conditions for A-B, B-A, and A-C relations

relate the shape properties (B) to shape pictures (C) was provided. Although the B-C relation was not tested in the current study, it can be hypothesized that the relation was either already in place prior to the study or emerged during the study. Prior to training, the participants were able to identify the location of sides as features on simpler shapes, suggesting that stimulus control existed between the stimulus "sides" and the feature to which "sides" refers. Given this history, the participants could possibly engage in the B-C relation by selecting the correct shape when asked "which shape has this many sides" and shown a number card (B) in the presence of an array of shapes by simply counting the sides of shapes in the array. Thus, after learning the number of sides that a vocally named shape has (A-B), the A-C relation could emerge as a result of the participant counting the number of sides on the picture card (C) and relating it to the name of the shape with that many sides. This does not negate the A-C relation as being consistent with transitivity as the A-B relation is still a required node for relating the name of the shape (A) with the shape picture (C). In other words, the number of sides (B) must be first related to the shape name (A) before any pre-experimental repertoire of side counting can be applied. This hypothesis for the emergence of the transitivity relations includes the possibility of private mediating behaviors (e.g., Palmer, 2009). Under this conceptualization, the participants could be said to have engaged in a series of multiply controlled covert responses in which the discrete trial stimuli evoked the counting of the number of sides on the shape (C), thus allowing for the untaught (B-C) relation, followed by a selection response of the shape based on the count. If this were the case, the data provides a potential demonstration of how mediating covert verbal behavior, or private events, may interact with the formation of equivalence relations in mathematics instruction. Alternatively, the participants may have engaged in relations that were beyond identity matching, i.e., equivalence, but rather were comparative relations (e.g., Hayes, Barnes-Holmes, & Roche, 2001). Under this explanation, based upon the relational frame theory (RFT), the participants responded to a hierarchical sequence of shapes as "greater than" others in the array based on the number of sides each item contained. An explanation of the phenomenon from an RFT perspective may be better suited to the symbolic nature of geometrical reasoning, but its relationship to the obtained data is unknown at this time. Anecdotally, the participants were not observed engaging in any overt mediational or framing behaviors, such as echoing the number of shape sides or the shape names, so future research may consider methods for examining covert behaviors suggested as being important in producing emergent responses. As the purpose of this study was to determine if the instructional procedures utilized were adequate in promoting the emergence of untrained relations between experimental stimuli, praise was necessarily kept non-specific. Statements such as "good job, a pentagon does have five sides" might have resulted in the adventitious reinforcement of relations which were being purposely left untrained. Future research may wish to explore the interaction between verbal-consequence content and the interaction with emerging additional stimulus relations.

Although the extent of the data in this brief report is limited, the results highlight several important implications for practice and future research. First, equivalence relations can easily be identified within the content of geometry and mathematics curricula. Second, the data suggest that the ability of the learner to fluently reverse and extend trained relations may be important in learning from educational strategies that do not include instruction on all typically trained relations. Where curricula such as the PEAK-E explicitly check for the emergence of untrained relations, other curricula may inadvertently ignore deficits in responding to untrained responses. For example, if a teacher never stops to determine if the learner can respond to the number of sides (B) with the shape name (A) (i.e., B-A) even though they were taught to respond to the shape name (A) given by selecting the number of sides it has (B) (i.e., A-B), the learner may be left without the necessary relations for responding to the next stimulus taught, such as the degree of angles, length of sides, etc. In a similar way, curricula that focus exclusively on Skinner's verbal operants as independent repertoires, and thus, teach language skills in relative isolation from each other, may fail to provide adequate instruction for producing untrained relations. The current study presented a training/testing sequence that juxtaposed listener responding (A-B and A-C relations) with intraverbal responding (B-A). In effect, this meant that related stimuli frequently varied in arrangement within the discrete-trial presentations, e.g., the name of the shape (A) was both a sample (A-B) and a comparison (B-A) across trials. Further research should be conducted to determine if these aspects of stimulus presentation indeed promote equivalence responding or enhance the rate of correct responding to mathematic stimuli.

Although these data provide support for the general efficacy of the PEAK-E program *5E—Symmetry: Shape Names* to promote the emergence of untrained relations, care should be exercised when generalizing these findings. As both of the participants in this study demonstrated a relatively high pre-experimental mastery of reflexivity and symmetry repertoires, as indicated by their PEAK-E-PLF scores, it is yet unclear how individuals with weaker equivalence repertoires would benefit from these instructions. As a result, future research should incorporate participants with a wider range of impairments. Likewise, this study does not provide insight into the program modification that is recommended by the PEAK-E curriculum when participants fail to display untrained relations. Another potential limitation was the ability of the participants to identify what a "side" was prior to training, which may have allowed the participants to utilize non-equivalence-based learning histories to respond to the A-C relations. Selecting a novel shape property, such as degree of angles, may help future research to limit the influence of pre-existing histories. Finally, the lack of probes for symmetry and transitivity relations during the A-B training limits the ability of the data to speak to the emergence of untrained relations as a function of the A-B training; therefore, it is unclear if the untrained relations are acquired in proportion to the A-B relations or if they emerge more suddenly once the A-B relations are mastered. Future research may consider conducting test probes throughout the training process.

In conclusion, this study adds support to the evidence that equivalence relations can be promoted in children with autism and in the area of mathematics. Furthermore, the PEAK-E curriculum program *5E—Symmetry: Shape Names* may be an effective guide for providing the necessary trained relations and test opportunities to promote the emergence of equivalence responding in geometry shapes and their properties.

Compliance with Ethical Standards

Conflict of Interest and Disclosures The first author receives small royalties from the sales of the PEAK curriculum. Remaining authors declare that they have no conflict of interest.

Human and Animal Rights All procedures performed involving our human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards.

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