Economic Theory Bulletin
https://doi.org/10.1007/s40505-024-00275-9

RESEARCH ARTICLE

The existence of Walrasian equilibrium: infinitely many commodities, measure space of agents, and discontinuous preferences

Motoki Otsuka[1]

Received: 22 November 2023 / Accepted: 5 August 2024

Abstract
This study establishes two equilibrium existence results for large economies with infinitely many commodities. The novel results allow for nontransitive, incomplete, discontinuous, and price-dependent preferences and do not require an interiority condition on initial endowments. The first result is an existence result when the positive cone of the commodity space has a nonempty interior. The second result is an existence result under a nonsatiation condition, including the case of the empty interior of the positive cone. The second result covers infinite-dimensional commodity spaces which could not be covered before due to the interiority condition, such as the space of square integrable functions. Specifically, we employ a saturated measure space of agents to appeal to the convexifying effect of aggregation. The notion of the continuous inclusion property introduced for finite-agent economies is applied to large economies, enabling us to dispense with the continuity assumption regarding preferences. In addition, we provide examples of Walrasian equilibrium and infinite-dimensional commodity spaces newly covered by our results.

Keywords Infinite-dimensional commodity space · Measure space of agents · Discontinuous preference · Saturation property · Continuous inclusion property

JEL Classification C02 · C62 · D51

1 Introduction

Large economies were first introduced by Aumann (1964) as an exact model of perfect competition, and he proved the existence of a Walrasian equilibrium in a setting with a finite-dimensional commodity space and an atomless finite measure space of agents without any convexity assumption on preferences (Aumann 1966). In a

✉ Motoki Otsuka
s2220082@u.tsukuba.ac.jp

[1] Graduate School of University of Tsukuba, 1-1-1, Tennodai, Tsukuba, Ibaraki 305-8577, Japan

finite-dimensional commodity space, the application of Lyapunov convexity theorem ensures the convexity of the integrals of multifunctions defined on an atomless measure space. By appealing to this convexifying effect, Aumann (1966) demonstrated that the convexity assumption on preferences is unnecessary for the convexity of aggregate demand in an atomless measure space of agents.

Several attempts have been made to extend the result of Aumann (1966) to infinite-dimensional commodity space. Because Lyapunov's theorem does not generally hold in infinite-dimensional topological vector spaces, it is necessary to assume convexity of preferences to establish the existence of a Walrasian equilibrium. (see Khan and Yannelis 1991). Rustichini and Yannelis (1991) suggest the “more agents than commodities” assumption and successfully dispense with the convexity assumption on preferences in the setting of infinite-dimensional commodity spaces. Podczeck (1997) similarly removes convexity of preferences by assuming that “there exist many agents of every type." Recent studies have established that if the space of agents satisfies the "saturation" condition, Lyapunov's convexity theorem holds and, as a result, the integrals of multifunctions are also convex even in infinite-dimensional Banach space. Additionally, the saturation condition is both necessary and sufficient for the convexity of integrals of multifunctions in infinite-dimensional Banach spaces. The Lebesgue interval, which was initially adopted as a space of agents for large economies by Aumann (1964), can be converted into a saturated measure space by extending the σ-algebra of Lebesgue measurable sets and the Lebesgue measure (Podczeck 2008). Therefore, it is natural to employ a saturated measure space as a space of agents and attempt to extend Aumann's result to infinite-dimensional commodity spaces. Remarkable progress was made in this area in Lee (2013), Khan and Sagara (2016), Khan and Suzuki (2016) and Jang and Lee (2020).

However, previous results for large economies with infinitely many commodities and nonconvex preferences impose strong conditions on preferences, such as transitivity, completeness, and continuity. Lee (2013) imposes the transitivity and continuity assumptions, while Khan and Sagara (2016) and Jang and Lee (2020) impose transitivity, completeness, and continuity. The need to drop the transitivity and completeness assumptions is motivated by experimental economics and psychology, which show that individuals do not always behave this way due to various factors such as errors in decision making (Birnbaum and Schmidt 2008), variability of preferences over time (Regenwetter 2011), and uncertainty (Cettolin 2019). Dasgupta and Maskin (1986) and Reny (1999) emphasize the need to drop the continuity assumption in the game-theoretical setting. Their research is driven by important economic problems modeled as games with discontinuous payoffs, such as Hotelling models and Bertrand competition. Since then, much progress has been made regarding the existence of equilibrium in games and economies with discontinuous preferences (see, Reny 2016; Carmona and Podczeck 2014, 2016; He and Yannelis 2015, 2016, 2017; Cornet 2020; Podczeck and Yannelis 2022, 2024; Anderson et al. 2022).

This study establishes two existence results of economies with an infinite-dimensional commodity space and a saturated measure space of agents whose preferences are nontransitive, incomplete, discontinuous, and price-dependent as well as nonconvex. The proofs proceed in the same manner for both theorems. We exploit the "continuous inclusion property" condition on the intersection of the preference and budget correspondences, ensuring that each agent has a maximal point of the preference in their budget set at any positive price (He and Yannelis 2016). Since the demand correspondence of each agent is not necessarily upper hemicontinuous in the weak topology of the commodity space, we consider the “enlarged” demand correspondence, adopting the technique used in Podczeck (1997) and Lee (2013). By applying the infinite-dimensional version of Gale-Nikaido-Debreu lemma, the existence of Walrasian equilibrium is established.

The first result is an existence result when the positive cone of the commodity space has a nonempty interior. This type of commodity space is employed in the existence results of Khan and Yannelis (1991), Rustichini and Yannelis (1991), Podczeck (1997), Lee (2013), Khan and Sagara (2016) and Jang and Lee (2020). By contrast, the first result allows for nontransitive, incomplete, discontinuous, and price-dependent preferences. Additionally, it does not require an interiority condition under which each agent has an available consumption bundle whose value is strictly smaller than that of his/her initial endowment for any positive price. We provide an example of Walrasian equilibrium covered by the first result, in contrast to these previous results. Note that since the first result requires a stronger version of irreflexivity, referred to as quasi-convexity by some authors, for preferences, it is not directly comparable with the previous existence results.

The second result is an existence result under an additional condition that works like the nonsatiation condition, including the case in which the positive cone of the commodity space has an empty interior. The interiors of the positive cones are empty in many of the most important commodity spaces. For example, in allocation problems under uncertainty, natural commodity bundles are consumption patterns that depend on the state of the world, that is, random variables in some probability space (Ω, Σ, P). If, as in many financial applications, we require that consumption patterns have finite means and variances, we consider the space $L^2(\Omega, \Sigma, P)$, whose positive cone generally has an empty interior (For more details, see, Mas-Colell and Zame 1991). Another example is the space of bounded signed Borel measures on an infinite compact metric space to represent differentiated commodity spaces (Mas-Colell 1975; Jones 1984; Ostroy and Zame 1994; Greinecker and Podczeck 2017).[1] To prove the second result, we modify the infinite-dimensional Gale-Nikaido-Debreu lemma of Yannelis (1985) and Cornet et al. (2023) to allow for positive cones with empty interiors. We present an existence result in L^p spaces, illustrating how infinite-dimensional commodity spaces, previously not covered due to the interiority condition, are now covered by the second result ($1 \leq p < \infty$).

Our work is also in line with studies on abstract and exchange economies with general preferences such as He and Yannelis (2016), Podczeck and Yannelis (2022, 2024), and Anderson et al. (2022). He and Yannelis (2016) prove an existence result in finite-agent exchange economies with finitely many commodities. It allows for nontransitive, incomplete, discontinuous, interdependent, and price-dependent preferences and does not require the interiority condition of initial endowments. The issue is that it assumes the compactness of consumption sets. Podczeck and Yannelis

[1] The space $M(K)$ of bounded signed Borel measures on a compact metric space K is separable if and only if K is countable. The second result includes the case where K is countably infinite.

(2022) replace this assumption with the standard assumption that consumption sets are closed, convex, and bounded from below. Podczeck and Yannelis (2024) further extend these results to infinite-dimensional commodity spaces for price-independent preferences, including infinite-dimensional existence results from Mas-Colell (1986), Yannelis and Zame (1986), Araujo and Monteiro (1989) and Mas-Colell and Richard (1991). Meanwhile, Anderson et al. (2022) prove an equilibrium existence theorem in abstract economies, allowing for arbitrary cardinality of agents, action sets that lie in different locally convex topological vector spaces, and general preferences including discontinuous ones. However, these results do not apply to an economy in which the influence of each agent on the economy is negligible, that is, a large economy. One of our contributions is establishing an existence result for large exchange economies in the same settings, except for interdependence. In particular, to the best of our knowledge, our results are the first to establish the existence of an equilibrium in large economies with discontinuous preferences.[2]

The remainder of this paper is organized as follows. Section 2 introduces the notations and definitions. Section 3 describes our model. Section 4 provides the main results and an example of Walrasian equilibrium newly covered by the results. As this example (Example 1) effectively illustrates our motivation, the reader may first refer to it. Section 5 concludes the paper. The proofs of our results are in Appendix A.

2 Notation and definitions

2.1 Bochner integration of functions and multifunctions

Let $(T, \mathcal{T}, \mu)$ be a measure space and E a Banach space. A function $\phi : T \to E$ that takes only a finite number of values, say $x_1, x_2, \cdots x_n$, is called an E-simple function if $A_i = \phi^{-1}(\{x_i\}) \in \mathcal{T}$ for all i. The formula $\phi = \sum_{i=1}^{n} x_i \chi_{A_i}$ is called the standard representation of ϕ. If $\mu(A_i) < \infty$ for each nonzero x_i, then ϕ is called an E-step function. The integral of an E-step function ϕ is the vector $\int_T \phi d\mu$ in E defined by $\int_T \phi d\mu = \sum_{i=1}^{n} \mu(A_i)x_i$. As in the case of Lebesgue integration, if $\phi = \sum_{j=1}^{m} y_j \chi_{B_j}$ is another representation of ϕ with $\mu(B_j) < \infty$ for each nonzero y_j, then $\int_T \phi d\mu = \sum_{j=1}^{m} \mu(B_j)y_j$. The integral of ϕ over $S \in \mathcal{T}$ is defined by $\int_S \phi d\mu = \int_T \phi \chi_S d\mu$. A function $f : T \to E$ is *strongly measurable* if there exists a sequence $\{\phi_n\}$ of E-simple functions such that $\lim_{n\to\infty} ||f(t) - \phi_n(t)|| = 0$ a.e. $t \in T$. If $f : T \to E$ is strongly measurable, then its norm function $||f||$ is also measurable. Additionally, the space of strongly measurable functions is a vector space containing all E-step functions (see Aliprantis and Border 2006). Thus, we can extend the notion of the integral from E-step functions to strongly measurable functions.

[2] Recently and independently from this study, Bhowmik and Yannelis (2024) have obtained related results, which include existence results in exchange economies with a finite-dimensional commodity space and a measure space of agents whose preferences may be nontransitive, incomplete, discontinuous, interdependent, and price-dependent. The author learned about these results from Nicholas C. Yannelis after the submission of this paper.

Definition 1 A strongly measurable function $f : T \to E$ is Bochner integrable if there exists a sequence $\{\phi_n\}$ of E-step functions such that the measurable function $||f - \phi_n||$ is Lebesgue integrable for each n and

$$\lim_{n\to\infty} \int_T ||f - \phi_n|| d\mu = 0.$$

In this case, for each $S \in \mathcal{T}$ the Bochner integral of f over S is defined by

$$\int_S f d\mu = \lim_{n\to\infty} \int_S \phi_n d\mu,$$

where the last limit is in the norm topology on E.

Denote by $L^p(\mu, E)$ the space of the equivalence classes of strongly measurable functions $f : T \to E$ such that $\int_T ||f||^p d\mu < \infty$. The space $L^p(\mu, E)$ is a Banach space equipped with the norm $||f||_p = (\int_T ||f||^p d\mu)^{\frac{1}{p}} (1 \leq p < \infty)$. Denote by $L^\infty(\mu, E)$ the space of essentially bounded functions normed by the usual essential supremum norm. For $S \in \mathcal{T}$, denote by $L^p_S(\mu, E)$ the subspace of $L^p(\mu, E)$ consisting of strongly measurable functions vanishing outside of S. We simplify the notation $L^p(\mu, \mathbf{R})$ and $L^p_S(\mu, \mathbf{R})$ to $L^p(\mu)$ and $L^p_S(\mu)$, respectively, for all $1 \leq p \leq \infty$.

A mapping from T to the family of (possibly empty) subsets of E is called a *multifunction* or *correspondence*. A multifunction $\Gamma : T \twoheadrightarrow E$ is said to be *measurable* if the set $\{t \in T | \Gamma(t) \cap U \neq \emptyset\}$ is in $\mathcal{T}$ for every open subset U of E. It is *graph measurable* if its graph $\{(t, x) \in T \times E | x \in \Gamma(t)\}$ belongs to $\mathcal{T} \otimes \mathcal{B}_E$, where $\mathcal{B}_E$ is the Borel σ-algebra of E generated by the norm topology. For nonempty closed valued multifunctions, measurability and graph measurability coincide whenever $(T, \mathcal{T}, \mu)$ is complete and E is separable (Castaing and Valadier 1977). A function $f : T \to E$ is a *selection* of Γ if $f(t) \in \Gamma(t)$ a.e. $t \in T$. If E is separable, then any nonempty valued multifunction Γ with a measurable graph admits a measurable selection (Aumann 1969; Castaing and Valadier 1977).

Let B be the closed unit ball in E. A multifunction $\Gamma : T \twoheadrightarrow E$ is *integrably bounded* if there exists a function $\phi \in L^1(\mu)$ such that $\Gamma(t) \subset \phi(t)B$ a.e. $t \in T$. Denote by $\mathcal{S}^1_\Gamma$ the set of Bochner integrable selections of Γ. The integral of Γ is defined as $\int_T \Gamma d\mu = \{\int_T f d\mu | f \in \mathcal{S}^1_\Gamma\}$. If a nonempty valued multifunction Γ is graph measurable and integrably bounded, then it admits a Bochner integrable selection whenever E is separable. Thus, the integral $\int_T \Gamma d\mu$ is nonempty.

Let E be an ordered Banach space. Denote by E^* the dual space of E, i.e. the space of all bounded linear functionals from E to $\mathbf{R}$. A subset A of E is *solid* if the norm interior of A is nonempty. The *polar set* A° of A is defined as $A^\circ = \{p \in E^* : |p \cdot x| \leq 1$ for all $x \in A\}$. Denote by E_+ the positive cone of E, i.e. $E_+ = \{x \in E : x \geq 0\}$. Note that E_+ is closed by definition. For $p \in E^*$ and $x \in E$, we denote by $p \cdot x$ the value of p at x. Denote by E^*_+ the dual cone of E_+, i.e. $E^*_+ = \{p \in E^* : p \cdot x \geq 0$ for all $x \in E_+\}$.

2.2 Saturated measure space and convexifying effect

Let $(T, \mathcal{T}, \mu)$ be a measure space. Here, we denote the complement of any measurable subset $A \in \mathcal{T}$ by $-A$. Denote by $\mathcal{T}_A$ the trace σ-algebra on A, i.e. $\mathcal{T}_A = \{E \subset A : E \in \mathcal{T}\}$. Denote by μ_A the subspace measure on A, i.e. $\mu_A(E) = \mu(E)$ for every $E \in \mathcal{T}_A$. Let $\mathcal{N}(\mu)$ denote the family of null sets in $\mathcal{T}$. It is a σ-ideal of the σ-algebra $\mathcal{T}$.

We define the *measure algebra* $(\mathfrak{A}, \hat{\mu})$ of $(T, \mathcal{T}, \mu)$. Denote by $\sim$ the equivalence relation on $\mathcal{T}$ given by $E \sim F$ if and only if $E \triangle F \in \mathcal{N}(\mu)$. Then, $\mathfrak{A}$ is the set of equivalence classes in $\mathcal{T}$ defined by the relation $\sim$. The set $\mathfrak{A}$ is a Boolean algebra under the natural Boolean operations. Indeed, if (E_1, E_2) and (F_1, F_2) are pairs of equivalent sets, then $E_1 \cup F_1$ is equivalent to $E_2 \cup F_2$. Consequently, the union $\cup^\bullet$ of two equivalence classes is well-defined by selecting representatives from each and forming the equivalence class of their union. The same is true for the intersection $\cap^\bullet$ and the complement $-^\bullet$. A partial order $\subset^\bullet$ on $\mathfrak{A}$ is defined as follows: if $E^\bullet$ and $F^\bullet \in \mathfrak{A}$ are equivalence classes of E and F respectively, then $E^\bullet \subset^\bullet F^\bullet$ if and only if $E^\bullet \cup^\bullet F^\bullet = F^\bullet$. The last condition is equivalent to $E \setminus F \in \mathcal{N}(\mu)$. For a subset $A \subset \mathfrak{A}$, we denote by $\sup A$ the supremum of A with respect to $\subset^\bullet$, if it exists.

A *subalgebra* of $\mathfrak{A}$ is a subset of $\mathfrak{A}$ that contains $\emptyset^\bullet$ and $T^\bullet$, and is closed under the operations $\cup^\bullet$, $\cap^\bullet$, $-^\bullet$. A subalgebra $\mathfrak{B}$ of $\mathfrak{A}$ is *order-closed* if $\sup B$ belongs to $\mathfrak{B}$ whenever $B \subset \mathfrak{B}$ is non-empty upwards directed, and $\sup B$ is defined in $\mathfrak{A}$. A subset A is said to *completely generate* $\mathfrak{A}$ if the smallest order-closed subalgebra of $\mathfrak{A}$ that contains A is $\mathfrak{A}$ itself. The *Maharam type* of μ is the least cardinal number of any subset $A \subset \mathfrak{A}$ that completely generates $\mathfrak{A}$.

It is known that a measure space $(T, \mathcal{T}, \mu)$ is atomless if and only if, for every $E \in \mathcal{T}$ with $\mu(E) > 0$, the Maharam type of μ_E is infinite (Podczeck 2008). The following definition of saturation strengthens the condition of atomlessness.

Definition 2 A measure space $(T, \mathcal{T}, \mu)$ is *saturated* if for every $E \in \mathcal{T}$ with $\mu(E) > 0$, the Maharam type of μ_E is uncountable.

By definition, the saturation condition implies nonatomicity. Several equivalent definitions of the saturation are available [see Fajardo and Keisler (2002), Podczeck (2008), Keisler and Sun (2009) and Fremlin (2011)]. A simple characterization of the saturation property is as follows: a measure space is saturated if and only if $L^1_E(\mu)$ is nonseparable for every $E \in \mathcal{T}$ with $\mu(E) > 0$. The significance of the saturation property lies in the fact that it is both necessary and sufficient for weak compactness and convexity of the Bochner integral of a multifunction, as well as the Lyapunov convexity theorem in Banach spaces (see Podczeck 2008; Sun and Yannelis 2008; Khan and Sagara 2013, 2014a).

2.3 Continuous inclusion property

Let X and Y be topological spaces. A multifunction $\psi : X \twoheadrightarrow Y$ is *upper hemicontinuous* if the set $\{x \in X : \psi(x) \subset U\}$ is open in X for every open subset U of Y. It is *lower hemicontinuous* if the set $\{x \in X : \psi(x) \cap U \neq \emptyset\}$ is open in X for every open subset

U of Y. A multifunction ψ *has open lower sections* if $\phi^l(y) = \{x \in X : y \in \psi(x)\}$ is open in X for any $y \in Y$. Following He and Yannelis (2016, 2017), we weaken these notions as follows:

Definition 3 Let X be a topological space and Y a convex subset of a topological vector space. A multifunction ψ from X to Y is said to *have the continuous inclusion property at* $x \in X$ if there exists an open neighborhood O_x of x and a nonempty valued multifunction $F_x : O_x \twoheadrightarrow Y$ such that $F_x(z) \subset \psi(z)$ for any $z \in O_x$, and $\text{co}F_x$ has a closed graph.[3] The multifunction ψ is said to *have the continuous inclusion property* if it has the continuous inclusion property at every $x \in X$.

Adding additional assumptions typical of fixed-point theorems, various continuity conditions on multifunctions imply the continuous inclusion property for topological vector spaces. For example, multifunctions that are convex valued and have a closed graph clearly have the continuous inclusion property. Multifunctions with open lower sections also have this property (He and Yannelis 2016). He and Yannelis (2017) unify the fixed point theorems of Browder and Kakutani-Fan-Glicksberg for Hausdorff locally convex topological vector spaces as follows.

Theorem 1 (He and Yannelis (2017)) *Let K be a nonempty, convex, and compact subset of a Hausdorff locally convex topological vector space, and let $T : K \twoheadrightarrow K$ be a multifunction with nonempty, convex values that has the continuous inclusion property. Then there exists an element $x^* \in K$ such that $x^* \in T(x^*)$.*

The notion of continuous inclusion properties and this fixed point theorem have been applied to studies on the existence of an equilibrium in economies and games with discontinuous preferences (see, He and Yannelis 2016, 2017; Khan and Uyanik 2021; Podczeck and Yannelis 2022, 2024; Anderson et al. 2022).

3 The model

Our exchange economy model is based on Khan and Yannelis (1991), Lee (2013), and Khan and Sagara (2016). We adopt preference correspondences instead of preference relations to address nontransitive, incomplete, discontinuous, and price-dependent preferences. A key step in our setup is the continuous inclusion property of the intersection between preference and budget correspondence.

The commodity space is an ordered separable Banach space E for which the dual cone of E_+ is non-degenerate (i.e. $E_+^* \neq \{0\}$). The price space is a nonempty subset $\Delta \neq \{0\}$ of E_+^*.

Definition 4 An *economy* $\mathcal{E}$ is a tuple $((T, \mathcal{T}, \mu), (X(t), P_t, e(t))_{t \in T})$ where

- $(T, \mathcal{T}, \mu)$ is a saturated finite measure space of agents;
- $X(t) \subset E$ is a consumption set of agent t;
- $P_t : \Delta \times X(t) \twoheadrightarrow X(t)$ is the preference correspondence of agent t;

[3] $\text{co}F_x$ is defined as $\text{co}F_x(z) = \text{co}(F_x(z))$, that is, the convex hull of $F_x(z)$.

- $e(t) \in X(t)$ is the initial endowment of agent t, and $e : T \to E$ is a Bochner integrable function.

We define the concept of Walrasian equilibrium with free disposal. Let $\mathcal{E}$ be an economy. Given price $p \in \Delta$, the *budget set* of agent t is $B(t, p) = \{x \in X(t) : p \cdot x \leq p \cdot e(t)\}$. The *demand set* of agent t is $D(t, p) = \{x \in B(t, p) : B(t, p) \cap P_t(p, x) = \emptyset\}$. An *allocation* is a Bochner integrable function $x : T \to E$ such that $x(t) \in X(t)$ a.e. t and $\int x(t)d\mu \leq \int e(t)d\mu$.

Definition 5 Let $\mathcal{E}$ be an economy. A *Walrasian equilibrium with free disposal* for $\mathcal{E}$ is a pair (p, x) of a price $p \in \Delta$ and an allocation x such that

1. $p \neq 0$;
2. $x(t) \in D(t, p)$ a.e. $t \in T$.

We consider the following assumptions for an economy $\mathcal{E}$. For each $t \in T$, let $\psi_t : \Delta \times X(t) \twoheadrightarrow X(t)$ be the correspondence defined by $\psi_t(p, x) = B(t, p) \cap P_t(p, x)$ for $(p, x) \in \Delta \times X(t)$.

A.1 $X(t)$ is nonempty, convex and weakly compact for all t, and the correspondence X is integrably bounded.
A.2 For each $p \in \Delta$, the demand correspondence $D(\cdot, p) : T \twoheadrightarrow E$ has a measurable graph in $\mathcal{T} \otimes \mathcal{B}_E$.
A.3 For each $p \in \Delta$ and each x in $X(t)$, it holds that $x \notin \text{co}\psi_t(p, x)$.
A.4 For all t, the correspondence ψ_t has the continuous inclusion property at $(p, x) \in \Delta \times X(t)$ with $\psi_t(p, x) \neq \emptyset$, where Δ is endowed with the weak* topology and $X(t)$ with the weak topology.
A.5 For each $p \in \Delta$, if $x(t)$ is a satiation point for agent t under p, then $x(t) \geq e(t)$.
A.6 For each $p \in \Delta$, if $x(t)$ is not a satiation point for agent t under p, then $x(t)$ belongs to the weak closure of $P_t(p, x(t))$.

In A.1, we assume that consumption sets are weakly compact. It is employed in Khan and Yannelis (1991), Podczeck (1997), Lee (2013), Khan and Sagara (2016), and Jang and Lee (2020). With this assumption, Podczeck (1997), Lee (2013), and Khan and Sagara (2016) derive the compact-valueness of the "enlarged" demand correspondences, and we apply the same technique. A.2 is of technical significance only. This assumption may be mathematically strong, but non-measurable sets are highly "pathological" as Aumann (1964) argues. They are unlikely to occur in the context of economics and thus do not impose a real economic restriction (for further discussion, see the concluding remarks in section 5). A.3 and A.4 are introduced by He and Yannelis (2016) for finite-agent economies. Similar assumptions are later adopted in He and Yannelis (2017), Podczeck and Yannelis (2022, 2024), and Anderson et al. (2022). These two assumptions ensure that the demand set of each agent is nonempty for any positive price. A.5 and A.6 are introduced by Podczeck (1997) and also imposed by Lee (2013) and Khan and Sagara (2016). Analogous assumptions for production economies are adopted in Jang and Lee (2020). A.6 works similarly to the local nonsatiation assumption.

4 Results

4.1 Commodity spaces with solid positive cones

Here, we present the initial results of this study.

Theorem 2 *Let commodity space E be an ordered separable Banach space whose positive cone is solid. Define the price space $\Delta = \{p \in E_+^* : p \cdot v = 1\}$, where v is a norm interior point of E_+. If economy $\mathcal{E}$ satisfies A.1-A.6, then it has a Walrasian equilibrium with free disposal.*

Remark 1 There are three noticeable differences from the previous literature on large economies such as Aumann (1966), Khan and Yannelis (1991), Noguchi (1997), Podczeck (1997), Lee (2013), Khan and Sagara (2016), and Jang and Lee (2020). First, Theorem 2 does not require transitivity, completeness, continuity, or convexity of preferences. Second, it does not require an interiority condition for the initial endowments. Khan and Yannelis (1991), Noguchi (1997), Podczeck (1997), Lee (2013), and Khan and Sagara (2016) impose the following interiority condition:

- For every $t \in T$, there exists an element $z(t)$ in $X(t)$ such that $e(t) - z(t)$ belongs to the norm interior of E_+.

The interiority condition is unnecessary for the upper hemicontinuity of the enlarged demand correspondence under the continuous inclusion property condition. Finally, Theorem 2 requires the stronger irrefrexivity of preferences as described in A.3. Thus, Theorem 2 is not comparable with the previous existence results.

Remark 2 The graph measurability of the demand correspondence and the continuous inclusion property of ψ_t are derived from the standard conditions of the literature of large economies with infinitely many commodities. Consider the following conditions which are standard in the literature.

- $X : T \twoheadrightarrow E$ has a measurable graph in $\mathcal{T} \otimes \mathcal{B}_E$.
- For all $p \in \Delta$, the set $\{(t, x, x') \in T \times E \times E : x \in P_t(p, x')\}$ belongs to $\mathcal{T} \otimes \mathcal{B}_E \otimes \mathcal{B}_E$.
- For all $t \in T$, $p \in \Delta$, and $x \in X(t)$, the sets $\{x' \in X(t) : x' \in P_t(p, x)\}$ and $\{x' \in X(t) : x \in P_t(p, x')\}$ are weakly open subset of $X(t)$.

The graph measurability of the demand correspondence follows from these three conditions (see Lee 2013, Lemma 4). The continuous inclusion property of ψ_t follows from the interiority condition, the last condition (continuity), and convexity of $X(t)$.

Next, we provide an example of Walrasian equilibrium newly covered by Theorem 2. The following example is based on Example 3 of He and Yannelis (2017).

Example 1 Consider the following 2-goods exchange economy with a measure space of agents.

- The space of agents is a Lebesgue open interval $((0, 1), \mathcal{T}, \mu)$ with saturation property.[4]
- The commodity space is $\mathbf{R}^2$ and the price space is $\Delta = \{p \in \mathbf{R}^2 : p_1 + p_2 = 1 \text{ and } p_1, p_2 \geq 0\}$.
- The consumption set of agent t is $X_t = [0, 1] \times [0, 1]$.
- The initial endowment of agent t is $e(t) = (t, 1 - t)$.
- Define $u = (1, 1) \in \mathbf{R}^2$. For all $x, y \in \mathbf{R}^2$, let $(x, y]$ denote the set $\{\lambda x + (1-\lambda)y : \lambda \in [0, 1)\}$ if $x \neq y$, and the empty set if $x = y$. The preference correspondence of agent t is defined by, for $x = (x_1, x_2) \in X_t$,

$$P_t(x) = \begin{cases} \{(y, z) \in X_t : z > y \text{ and } y + z > 1\} \cup (x, u] & (x_1 > x_2) \\ (x, u] & (x_1 = x_2) \\ \{(y, z) \in X_t : y > z \text{ and } y + z > 1\} \cup (x, u] & (x_2 > x_1). \end{cases}$$

Because the agents have nontransitive, incomplete, and discontinuous preferences, previous results such as Lee (2013), Khan and Sagara (2016) and Jang and Lee (2020) do not ensure the existence of equilibrium in the economy, whereas Theorem 2 demonstrate this.

Proposition 3 *The economy described in Example 1 satisfies all the assumptions of Theorem 2.*

Therefore, this exchange economy has a Walrasian equilibrium with free disposal. Indeed, the economy has an equilibrium of $p^* = (\frac{1}{2}, \frac{1}{2})$ and $x^*(t) = (\frac{1}{2}, \frac{1}{2})$ for all $t \in T$. Other Walrasian allocations under the price p^* include $e(t)$ and $\hat{x}(t) = (1-t, t)$.

4.2 Commodity spaces with nonsolid positive cones

The second result of this study includes the case in which the positive cone of the commodity space has an empty interior. Under the continuous inclusion property, there is no need for an interiority condition, which leads to the existence result for the case of nonsolid positive cones. We add a condition that works like the nonsatiation condition to guarantee that the price obtained in our version of the Gale-Nikaido-Debreu lemma (GND lemma) is not zero:

First, we provide an infinite-dimensional GND lemma that is applicable regardless of the topological structure of the positive cone.

Proposition 4 *Let E be a Hausdorff locally convex vector space, C be a nonempty closed convex cone of E, C^* be the dual cone of C, W be a norm neighborhood of* 0 *bounded in the weak topology of E,*[5] *and $\Delta = C^* \cap W^\circ$. Let $\xi : \Delta \twoheadrightarrow E$ be a correspondence such that:*

[4] There exists a Lebesgue closed interval $([0, 1], \mathcal{T}, \mu)$ with saturation property (Podczeck 2008). By restricting it to the open interval $(0, 1)$, we obtain a Lebesgue open interval with saturation property.

[5] This means that for any weak neighborhood V of 0 in E, there exists some $\alpha > 0$ such that $W \subset \alpha V$.

1. *ξ is upper demicontinuous in the weak* topology of Δ and the weak topology of E, i.e. the set $\{p \in \Delta : \xi(p) \subset V\}$ is weakly* open in Δ for every open half space V of E;*
2. *$\xi(p)$ is nonempty, closed and convex for all $p \in \Delta$;*
3. *for all $p \in \Delta$ there exists an element $x \in \xi(p)$ such that $p \cdot x \leq 0$.*

Then, there exists an element $p^* \in \Delta$ such that $0 \in \mathrm{cl}[\xi(p^*) + C]$.[6] *Moreover, if we additionally assume that the correspondence ξ is compact valued, then there exists an element $p^* \in \Delta$ such that $\xi(p^*) \cap (-C) \neq \emptyset$.*

Remark 3 There are two differences between Proposition 4 and the infinite-dimensional version of GND lemma proposed by Yannelis (1985) and Cornet et al. (2023). First, the GND lemma of Yannelis (1985) and Cornet et al. (2023) addresses only solid cones, whereas Proposition 4 includes both solid and nonsolid cones. Second, the GND lemma of Yannelis (1985) and Cornet et al. (2023) ensures that the price is not zero, although Proposition 4 permits the price to be zero.

The second result requires the following nonsatiation assumption[7] for a non-negligible set of agents:

A.7 For any pair of $p \in \Delta$ and an allocation $x : T \rightarrow E$ such that $x(t) \in B(t, p)$ a.e. $t \in T$, there exists a measurable set $S \in \mathcal{T}$ with $\mu(S) > 0$ such that $P_t(p, x(t)) \neq \emptyset$ for all $t \in S$.

He and Yannelis (2017, Theorem 4) and Podczeck and Yannelis (2022) impose the nonsatiation condition for one agent only in the context of finite-agent economies. A.7 is a large-economy version of this assumption.

We are ready to present the second result of this study.

Theorem 5 *Let commodity space E be an ordered separable Banach space. We define price space $\Delta = \{p \in E^*_+ : ||p|| \leq 1\}$. If economy $\mathcal{E}$ satisfies A.1-A.7, then it has a Walrasian equilibrium with free disposal.*

Theorem 5 covers infinite-dimensional commodity spaces which are important in economics but could not be covered due to the interiority condition on the initial endowments.

We present the existence result of L^p spaces, as an application of Theorem 5. Let the commodity space be $L^p(\mathcal{F}, \nu)$ $(1 \leq p < \infty)$ for a measure space $(\Omega, \mathcal{F}, \nu)$. As mentioned in the Introduction, this space is important for the allocation problem under uncertainty. The positive cone of $L^p(\mathcal{F}, \nu)$ generally has no interior points. On the other hand, if the underlying measure space is σ-finite, the dual space coincides with $L^q(\mathcal{F}, \nu)$, where $\frac{1}{p} + \frac{1}{q} = 1$. Hence, the following corollary is an immediate consequence of Theorem 5. Recall that a measure space $(\Omega, \mathcal{F}, \nu)$ is *countably generated* if its σ-algebra can be generated by a countable number of measurable subsets. $(\Omega, \mathcal{F}, \nu)$ is *essentially countably generated* if its σ-algebra can be generated by a countable number of measurable subsets together with the null sets $\mathcal{N}(\nu)$.

[6] $\mathrm{cl}[\xi(p^*) + C]$ denotes the closure of $\xi(p^*) + C$.

[7] If the preference of each agent is price-independent, then this assumption is unnecessary.

Corollary 6 *Let $(\Omega, \mathcal{F}, \nu)$ be an essentially countably generated σ-finite measure space which is nontrivial in the sense that $\nu(\Omega) > 0$. Let the commodity space be $L^p(\mathcal{F}, \nu)$ $(1 \leq p < \infty)$. We define the price space $\Delta = \{f \in L^q(\mathcal{F}, \nu)_+ : ||f||_q \leq 1\}$, where $\frac{1}{p} + \frac{1}{q} = 1$. If economy $\mathcal{E}$ satisfies A.1-A.7, then it has a Walrasian equilibrium with free disposal and with a positive price in $L^q(\mathcal{F}, \nu)$.*

Corollary 6 is an analogy of the existence results of L^∞ spaces (Bewley 1972, 1991; Khan and Sagara 2016) to L^p spaces $(1 \leq p < \infty)$. In particular, when $(\Omega, \mathcal{F}, \nu)$ is a probability space and $p = 2$, there exists a competitive price in $L^2(\mathcal{F}, \nu)$. Thus, the price system has a finite mean and variance.

5 Concluding remarks

We have established two existence results for large economies with an infinite-dimensional commodity space, which involve the following new aspects. The first result allows for nontransitive, incomplete, discontinuous, and price-dependent preferences. It does not require an interiority condition for the initial endowments, which suggests the existence result including the case of a commodity space with a nonsolid positive cone. This is the second result. Note that the second result imposes a nonsatiation condition; thus, the two results are incomparable. Our two results are analogous results in large economies to the existence results in He and Yannelis (2016), Podczeck and Yannelis (2022, 2024), and Anderson et al. (2022) under the framework of inter-independent preferences.

We provide examples of Walrasian equilibrium and commodity spaces covered by our results, in contrast to previous results, including Lee (2013) and Khan and Sagara (2016). In the example of Walrasian equilibrium, the agents' preferences are nontransitive, incomplete, and discontinuous. When we consider an agent as a group whose members have discontinuous preferences and decisions are made by voting, agents with nontransitive, incomplete, and discontinuous preferences naturally emerge. In addition, commodity space L^2 is used in the analysis of financial markets, but it was not covered previously due to the interiority condition. As Corollary 6 shows, Theorem 5 covers this commodity space.

Finally, we describe two open problems and one direction of extension.[8] The first problem is that the graph measurability of the demand correspondence could be replaced with some weaker conditions. What is essential for the existence of Walrasian equilibrium is that the demand correspondence has a measurable selection for each price. Assumption A.2 can be replaced by the two conditions of the graph measurability of $X : T \twoheadrightarrow E$ and the measurability of $\psi : T \times \Delta \times E \twoheadrightarrow E$. It is unclear whether this latter condition could be replaced with the standard assumption of the graph measurability of the preference correspondence.

The second problem is whether the weak compactness of consumption sets could be replaced by the standard assumption that these sets are closed, convex, and bounded from below. The approach of Podczeck and Yannelis (2024), who deal with discontinuous preferences without the compactness of consumption sets in infinite-dimensional

[8] For the first open problem and the direction of extension, see also Bhowmik and Yannelis (2024).

commodity spaces, is insightful. They assume the compactness of feasible allocations and construct subeconomies with finite-dimensional commodity spaces. Using their finite-dimensional existence theorem, they construct a net of equilibria for the subeconomies. By effectively using cone conditions and the continuous inclusion property, they construct an equilibrium of the original economy as a limit of this net. It is also reasonable in our model to assume the weak compactness of feasible allocations and approximate the infinite-dimensional commodity space as a sequence of finite-dimensional spaces (see Khan and Sagara 2017, Section 6.2). However, some obstacles arise in the limiting arguments because the infinite-dimensional Fatou's lemma cannot be applied without the weak compactness of consumption sets. Thus, even if a sequence of allocations weakly converges to some allocation, it is difficult to find an allocation that is a limit point of the sequence for almost every agent. Considering these and other obstacles, it remains uncertain whether this compactness assumption could be removed.

One direction of extension is to impose interdependence on the agents' preferences, as in He and Yannelis (2016), Podczeck and Yannelis (2022, 2024), and Anderson et al. (2022). We can then formulate a more realistic situation in which the impact of each agent on price formation is negligible, but each agent can influence the choice of other agents. One approach to formulating the interdependence of preferences in a large economy is to model the preference of each agent as affected by price, his/her consumption, and statistics determined by allocation, as in Carmona and Podczeck (2014). If we assume an appropriate topological structure in the space of statistics, our results may be extended to large economies where agents' preferences are interdependent.

Appendix A Proofs

A.1 Proof of Theorem 2

The proof follows the arguments in Podczeck (1997) and Lee (2013). Under the continuous inclusion property condition, the "enlarged" demand correspondence is defined more naturally and its weakly upper hemicontinuity is derived immediately.

Lemma 1 Δ *is nonempty and weakly* compact.*

Proof We prove the lemma in two steps.

Step 1. Δ is nonempty.

Since E_+^* is non-degenerate, there exists an element $q \in E_+^*$ such that $q \neq 0$. Then, there exists an element $e \in E$ such that $q \cdot e \neq 0$. From the linearity of q, we may assume that $q \cdot e < 0$. Since v is a norm interior of E_+, there exists a neighborhood W of 0 such that $v + W \subset E_+$. Since W is absorbing, there exists some $\lambda > 0$ such that $\lambda e \in W$. Hence, it follows $q \cdot (v + \lambda e) \geq 0$. Thus, $q \cdot v \geq -\lambda q \cdot e > 0$. Then, we have $\frac{1}{q \cdot v} q \in \Delta$.

Step 2. Δ is weakly* compact (Mas-Colell and Zame 1991).

Take a balanced neighborhood W of 0 such that $v + W \subset E_+$. Fix any $p \in \Delta$. For any $w \in W$, we have $p \cdot v + p \cdot w = p \cdot (v + w) \geq 0$. Thus, it follows from

$p \cdot v = 1$ that $p \cdot w \geq -1$. Since $-w$ also belongs to W, we have $p \cdot w \leq 1$. Thus, Δ is a weakly* closed subset of the polar set W°. Since W° is weakly* compact from Alaogle's theorem, Δ is weakly* compact. □

Lemma 2 *Fix an agent $t \in T$. The demand set $D(t, p)$ is nonempty for all $p \in \Delta$.*

Proof (He and Yannelis 2017) Suppose $D(t, p) = \emptyset$ for some $p \in \Delta$. Then, $\psi_t(p, x) = B(t, p) \cap P_t(p, x) \neq \emptyset$ for all $x \in B(t, p)$. Hence, the correspondence $\text{co}\psi_t(p, \cdot) : B(t, p) \twoheadrightarrow B(t, p)$ is nonempty and convex valued. Additionally, $\text{co}\psi_t(p, \cdot)$ has the continuous inclusion property from A.4. Since $B(t, p)$ is nonempty, convex, weakly compact, it follows from Theorem 1 that there exists an element $x \in B(t, p)$ such that $x \in \text{co}\psi_t(p, x)$. This is a contradiction to A.3. □

Since the evaluation map $(p, x) \mapsto p \cdot x$ is not jointly continuous if E is equipped with the weak topology and Δ with the weak* topology, we construct the enlarged demand set of agent t for a given $p \in \Delta$ as follows:

$$C(t, p) = \{x \in X(t) : \psi_t(p, x) = \emptyset\}.$$

It is clear that $D(t, p) \subset C(t, p)$.

Lemma 3 *For each $t \in T$, the correspondence $C(t, \cdot) : \Delta \twoheadrightarrow X(t)$ is weakly compact valued, and weakly upper hemicontinuous with respect to the weak* topology of Δ.*

Proof Since $X(t)$ is Hausdorff and weakly compact, it suffices to show that $C(t, \cdot)$ has a closed graph in $\Delta \times X(t)$. Let $A = \{(p, x) \in \Delta \times X(t)\ \psi_t(p, x) \neq \emptyset\}$. Then, A is a open subset of $\Delta \times X(t)$ from A.4. Since the graph of $C(t, \cdot)$ is equal to $(\Delta \times X(t))\backslash A$, the graph is a closed subset of $\Delta \times X(t)$. □

We denote by $w\text{-lim}_{n\to\infty} x_n$ the weak limit of a sequence $\{x_n\}_{n\in\mathbf{N}}$ in E. The *weak upper limit* of a sequence of subsets $\{A_n\}_{n\in N}$ in E is defined by

$$w\text{-Ls}A_n = \{x \in E : \exists\{x_{n_i}\}_{i\in\mathbf{N}}, x = w\text{-lim}_{i\to\infty}x_{n_i} \text{ and } x_{n_i} \in A_{n_i} \text{ for all } i \in \mathbf{N}\}.$$

Lemma 4 *$\int C(t, \cdot)d\mu : \Delta \twoheadrightarrow E$ is nonempty, convex, weakly compact valued, and weakly upper hemicontinuous with respect to the weak* topology of Δ.*

Proof We prove the lemma in three steps.

Step 1. $\int C(t, \cdot)d\mu : \Delta \twoheadrightarrow E$ is nonempty valued.

Fix any $p \in \Delta$. From Lemma 2, $D(\cdot, p)$ is nonempty valued. From A.2, $D(\cdot, p)$ has a measurable graph in $\mathcal{T} \otimes \mathcal{B}_E$. Since E is a complete separable metric space, there exists a measurable function $g^p : T \to E$ such that $g^p(t) \in D(t, p)$ a.e. $t \in T$. Since $g^p(t) \in X(t)$ a.e. $t \in T$, and X is integrably bounded, g^p is Bochner integrable. Hence, $\int g^p d\mu \in \int D(t, p)d\mu \subset \int C(t, p)d\mu$.

Step 2. $\int C(t, \cdot)d\mu : \Delta \twoheadrightarrow E$ is convex valued.

Since $(T, \mathcal{T}, \mu)$ is saturated, it follows from Theorem 1 of Podczeck (2008) that $\int C(t, p)d\mu$ is convex for all $p \in \Delta$.

Step 3. $\int C(t,\cdot)d\mu : \Delta \twoheadrightarrow E$ is weakly compact valued and weakly upper hemicontinuous.

Since $X : T \twoheadrightarrow E$ is weakly compact valued and integrably bounded, it follows from Theorem 2 of Podczeck (2008) that $\int X(t)d\mu$ is weakly compact. Hence, it suffices to prove that $\int C(t,\cdot)d\mu$ has a closed graph in $\Delta \times \int X(t)d\mu$. Since a weakly* compact subset of the dual space of a separable Banach space is metrizable, Δ is metrizable. Similarly, since a weakly compact subset of a separable Banach space is metrizable, $\int X(t)d\mu$ is metrizable. Hence, it suffices to prove that if (p_n, z_n) is a sequence of $\Delta \times \int X(t)d\mu$ such that

$$p_n \to p_0 \in \Delta \text{ in the weak* topology of } \Delta,$$
$$z_n \to z_0 \in \int X(t)d\mu \text{ in the weak topology of } E, \text{ and}$$
$$z_n \in \int C(t, p_n)d\mu \text{ for all } n \in \mathbf{N},$$

then $z_0 \in \int C(t, p_0)d\mu$.

It is clear that $z_0 \in w\text{-Ls} \int C(t, p_n)d\mu$. It follows from Theorem 4.5 of Khan and Sagara (2014b) that

$$w\text{-Ls} \int C(t, p_n)d\mu \subset \int w\text{-Ls}C(t, p_n)d\mu.$$

For any $t \in T$, since $C(t,\cdot) : \Delta \twoheadrightarrow X(t)$ has a closed graph, we have

$$w\text{-Ls}C(t, p_n) \subset C(t, p_0).$$

Hence, it follows that

$$z_0 \in w\text{-Ls} \int C(t, p_n)d\mu \subset \int w\text{-Ls}C(t, p_n)d\mu \subset \int C(t, p_0)d\mu.$$

□

We consider the correspondence $\xi : \Delta \twoheadrightarrow E$ defined by

$$\xi(p) = \int C(t, p)d\mu - \int e(t)d\mu.$$

ξ is nonempty, convex, weakly compact valued, and weakly upper hemicontinuous from Lemma 4.

The next lemma is the infinite-dimensional Gale-Nikaido-Debreu lemma by Yannelis (1985).

Lemma 5 *Let E be a Hausdorff locally convex vector space and C a closed convex cone of E such that the dual cone C^* of C is non-degenerate and the norm interior of C is nonempty. Define $\Delta = \{p \in C^* : p \cdot e = 1\}$, where e is a norm interior point of C. Let $\xi : \Delta \twoheadrightarrow E$ be a correspondence such that:*

1. *ξ is upper demicontinuous in the weak* topology of Δ and the weak topology of E, i.e. the set $\{p \in \Delta : \xi(p) \subset V\}$ is weakly* open in Δ for every open half space V of E;*
2. *$\xi(p)$ is nonempty, convex, and compact for all $p \in \Delta$;*
3. *for all $p \in \Delta$ there exists an element $x \in \xi(p)$ such that $p \cdot x \leq 0$.*

Then, there exists an element $p^* \in \Delta$ such that $\xi(p^*) \cap (-C) \neq \emptyset$.

We are now ready to prove Theorem 2.

Proof of Theorem 2 We prove in three steps.

Step 1. For any $p \in \Delta$, there exists some $z \in \xi(p)$ such that $p \cdot z \leq 0$.

Since $D(\cdot, p)$ is nonempty valued and has a measurable graph, there exists a measurable function $g^p : T \to E$ such that $g^p(t) \in D(t, p)$ a.e. $t \in T$. Then, it follows that

$$\int g^p(t)d\mu - \int e(t)d\mu \in \int D(t, p)d\mu - \int e(t)d\mu \subset \xi(p).$$

Additionally, since $p \cdot g^p(t) \leq p \cdot e(t)$ a.e. $t \in T$, it follows that

$$\begin{aligned} p \cdot \left(\int g^p(t)d\mu - \int e(t)d\mu \right) &= p \cdot \int (g^p(t) - e(t))d\mu \\ &= \int p \cdot (g^p(t) - e(t))d\mu \leq 0. \end{aligned}$$

This completes the proof of Step 1.

It follows from Step 1 and Lemma 5 that there exists an element $p^* \in \Delta$ such that $\xi(p^*) \cap (-E_+) \neq \emptyset$. Take an element $z^* \in \xi(p^*) \cap (-E_+)$. Then, there exists a measurable function $x^* : T \to E$ such that

$$x^*(t) \in C(t, p^*) \ a.e. \ t \in T, \text{ and} \tag{A1}$$

$$\int x^*(t)d\mu - \int e(t)d\mu = z^* \leq 0. \tag{A2}$$

We prove (p^*, x^*) is a Walrasian equilibrium with free disposal.

Step 2. $p^* \cdot x^*(t) \geq p^* \cdot e(t)$ a.e. $t \in T$.

It suffices to prove that $p^* \cdot x^*(t) \geq p^* \cdot e(t)$ for all $t \in T$ satisfying $x^*(t) \in C(t, p)$. If $x^*(t)$ is a satiation point of agent t under p^*, then it follows from A.5 that $x^*(t) \geq e(t)$. Since p^* is positive, we have $p^* \cdot x^*(t) \geq p^* \cdot e(t)$. Suppose that $x^*(t)$ is not a satiation point of agent t under p^* and $p^* \cdot x^*(t) < p^* \cdot e(t)$. Then, there exists a weak neighborhood $V_{x^*(t)}$ of $x^*(t)$ such that $p^* \cdot x' < p^* \cdot e(t)$ for all $x' \in V_{x^*(t)}$. Since $x^*(t)$ belongs to the weak closure of $P_t(p^*, x^*(t))$ from A.7, there exists an element $x' \in V_{x^*(t)}$ such that $x' \in P_t(p^*, x^*(t))$. Since $p^* \cdot x' < p^* \cdot e(t)$, this is a contradiction to $x^*(t) \in C(t, p^*)$.

Step 3. $p^* \cdot x^*(t) = p^* \cdot e(t)$ a.e. $t \in T$.

From Step 2, we have $p^* \cdot x^*(t) \geq p^* \cdot e(t)$ a.e. $t \in T$. By integrating both sides, we have $\int p^* \cdot x^*(t) d\mu \geq \int p^* \cdot e(t) d\mu$. On the other hand, since $\int x^*(t) d\mu \leq \int e(t) d\mu$, and p^* is positive, we have $\int p^* \cdot x^*(t) d\mu \leq \int p^* \cdot e(t) d\mu$. Therefore, it follows that

$$\int p^* \cdot x^*(t) d\mu = \int p^* \cdot e(t) d\mu. \tag{A3}$$

Combining Step 2 and (A3), it follows that $p^* \cdot x^*(t) = p^* \cdot e(t)$ a.e. $t \in T$.

From (A1) and Step 3, it follows that $x^*(t) \in D(t, p^*)$ a.e. $t \in T$. Since $\int x^*(t) d\mu \leq \int e(t) d\mu$, the pair (p^*, x^*) is a Walrasian equilibrium with free disposal. □

A.2 Proof of Proposition 3

Proof We only prove A.4 because it is clear that the other assumptions hold. Fix agent t arbitrarily. We prove that ψ_t has the continuous inclusion property at $(p, x) \in \Delta \times X_t$ with $\psi_t(p, x) \neq \emptyset$. We prove this only in the case where $x_2 \leq x_1$.

From the definition of the preference correspondence, there are the following two cases:

1. $\psi_t(p, x) \cap (x, u] \neq \emptyset$.
2. $\psi_t(p, x) \cap (A =)\{(y, z) \in X_t : z > y \text{ and } y + z > 1\} \neq \emptyset$.

Case 1. $\psi_t(p, x) \cap (x, u] \neq \emptyset$.

Step 1. $p \cdot x < p \cdot e(t)$.

If $p_1, p_2 \neq 0$, take $x' \in \psi_t(p, x) \cap (x, u]$ arbitrarily. Since $x \leq x'$ and $x' \neq x$, it follows $p \cdot x < p \cdot x' \leq p \cdot e(t)$. If $p_1 = 1$ and $p_2 = 0$, then it must be that $x_1 \leq t$. Take $x' \in \psi_t(p, x) \cap (x, u]$ arbitrarily. Then, $x_1' = p \cdot x' \leq p \cdot e(t) = t$. Since $x_1 \leq t < 1$ and x_1' can be written as $x_1' = \lambda x_1 + (1 - \lambda) \cdot 1$ for some $\lambda \in [0, 1)$, it follows that $x_1 < x_1'$. Hence, $p \cdot x = x_1 < x_1' \leq t = p \cdot e(t)$. The same arguments hold when $p_1 = 0$ and $p_2 = 1$.

Step 2. ψ_t has the continuous inclusion property at (p, x).

Since the evaluation map $(p, x) \mapsto p \cdot x$ is jointly continuous, there exists a neighborhood O_x of x and a neighborhood O_p of p such that $p' \cdot x' < p' \cdot e(t)$ for all $(p', x') \in O_p \times O_x$. Define the multifunction $F^t_{(p,x)} : O_p \times O_x \twoheadrightarrow X_t$ as

$$\begin{aligned} F^t_{(p,x)}(p', x') &= (x', u] \cap \{y \in X_t : p' \cdot y = p' \cdot e(t)\} \\ &= \{x' + \frac{p' \cdot x' - p' \cdot e(t)}{p' \cdot x' - p' \cdot u}(u - x')\}. \end{aligned}$$

Then, $F^t_{(p,x)}$ is a continuous function on $O_p \times O_x$ such that $F^t_{(p,x)}(p', x') \subset \psi_t(p', x')$ for all $(p', x') \in O_p \times O_x$.

Case 2. $\psi_t(p, x) \cap A \neq \emptyset$.

It follows from the definition of the preference that $x_2 < x_1$. Take a neighborhood O_x of x such that $x_2' < x_1'$ for any $x' \in O_x$. Take $(y, z) \in \psi_t(p, x) \cap A$ arbitrarily. For a sufficiently small $\epsilon > 0$, we have $(y-\epsilon, z-\epsilon) \in \psi_t(p, x) \cap A$. Thus, we may assume $p_1 y + p_2 z < p \cdot e(t)$. Take a neighborhood O_p of p such that $p'_1 y + p'_2 z < p' \cdot e(t)$ for any $p' \in O_p$. Define $F^t_{(p,x)} : O_p \times O_x \twoheadrightarrow X_t$ as

$$F^t_{(p,x)}(p', x') = \{(y, z)\} \subset \psi^t(p', x').$$

Then, $F^t_{(p,x)}$ is a continuous function on $O_p \times O_x$. Thus, ψ_t has the continuous inclusion property at (p, x). □

A.3 Proof of Proposition 4

Proof We prove the proposition in four steps. The proof is almost identical to that of Theorem 3.1 in Cornet et al. (2023).

Step 1. Δ is nonempty, convex, and weakly* compact.

The polar set W° is convex and contains 0. From Alaoglu's theorem, W° is also weakly* compact. Since C^* is weakly* closed, convex, and contains 0, $\Delta = C^* \cap W^\circ$ is convex, weakly* compact, and contains 0.

Step 2. Define the correspondence $F : \Delta \twoheadrightarrow \Delta$ as $F(p) = \{q \in \Delta : q \cdot z > 0$ for all $z \in \xi(p)\}$. Then there exists some $p^* \in \Delta$ such that $F(p^*) = \emptyset$.

Suppose not. Since ξ is upper demicontinuous, the correspondence F has open lower sections. It is clear that F is convex valued. Δ is nonempty, convex, weakly* compact, and F has nonempty convex values and open lower sections. Hence, it follows from Browder fixed point theorem that there exists some $p^* \in \Delta$ such that $p^* \in F(p^*)$. This means that $p^* \cdot z > 0$ for all $z \in \xi(p^*)$. This is a contradiction to 3.

Step 3. There exists some $p^* \in \Delta$ such that $0 \in \text{cl}[\xi(p^*) + C]$.

From Step 2, there exists some $p^* \in \Delta$ such that $F(p^*) = \emptyset$. We prove that $0 \in \text{cl}[\xi(p^*) + C]$ by contradiction. Suppose not. Since $\text{cl}[\xi(p^*) + C]$ is closed convex, the point 0 and $\text{cl}[\xi(p^*) + C]$ can be strictly separated by a continuous linear functional, that is, there exists a continuous linear functional $q \neq 0$ such that

$$0 = q \cdot 0 < \inf_{x \in \text{cl}[\xi(p^*)+C]} q \cdot x \leq \inf_{z \in \xi(p^*), c \in C} q \cdot (z + c). \tag{A4}$$

Since $\xi(p^*)$ and C are nonempty, it follows from (A4) that $\inf_{z \in \xi(p^*)}$ is finite. Hence, we have

$$-\inf_{z \in \xi(p^*)} q \cdot z < \inf_{c \in C} q \cdot c.$$

Let $a = -\inf_{z \in \xi(p^*)} q \cdot z$ and $b = \inf_{c \in C} q \cdot c$. Since C is a nonempty closed convex cone, b must be 0. Thus, $q \in C^*$.

There exists some $\lambda > 0$ such that $\lambda q \in \Delta$. Indeed, the set $N = \{x \in E : |q \cdot x| \leq 1\}$ is a weak neighborhood of 0. Since W is bounded in the weak topology, there exists some $\alpha > 0$ such that $W \subset \alpha N = \{x \in E : |q \cdot x| \leq \alpha\}$. Hence, we have $|q \cdot x| \leq \alpha$ for all $x \in W$. Then, we have $\alpha^{-1} q \in W^\circ$. Since $q \in C^*$ and $\alpha > 0$, if follows that $\alpha^{-1} q \in C^*$. Thus, we have $\alpha^{-1} q \in \Delta$. Define $\lambda = \alpha^{-1}$.

Since $\lambda q \in \Delta$ and $\lambda q \notin F(p^*)$, there exists some $z^* \in \xi(p^*)$ such that $(\lambda q)\cdot z^* \leq 0$. Since $\lambda > 0$, it follows that $q \cdot z^* \leq 0$. Therefore, $a = -\inf_{z \in \xi(p^*)} q \cdot z \geq -q \cdot z^* \geq 0$. This is a contradiction to $a < b = 0$.

Step 4. If ξ is additionally assumed to be compact valued, then:

$$0 \in \text{cl}[\xi(p^*) + C] \Leftrightarrow 0 \in \xi(p^*) + C \Leftrightarrow \xi(p^*) \cap (-C) \neq \emptyset.$$

Since $\xi(p^*)$ is compact and C is closed, $\xi(p^*) + C$. is also closed. Thus, the first equivalence follows. If $0 \in \xi(p^*) + C$, there exist some $z \in \xi(p^*)$ and $c \in C$ such that $z + c = 0$. Then, $z = -c \in \xi(p^*) \cap (-C)$. The converse direction of the second equivalence similarly follows. □

A.4 Proof of Theorem 5

Proof First, note that $\Delta \neq \{0\}$ because E_+ is non-degenerate. Define $C(t, p)$ and $\xi(p)$ as in the proof of Theorem 2. Then, ξ is nonempty, convex, weakly compact valued and weakly upper hemicontinuous as in Theorem 2. Let B and B' be the closed unit ball of E and E^*, respectively. Then, from the definition of the operator norm, it follows that $B^\circ = B'$. Since B is bounded in the weak topology, and $\Delta = E^*_+ \cap B' = E^*_+ \cap B^\circ$, the correspondence ξ satisfies all the assumptions of Proposition 4. Hence, there exists an element $p^* \in \Delta$ such that $\xi(p^*) \cap (-E_+) \neq \emptyset$.

Take an element $z^* \in \xi(p^*) \cap (-E_+)$. Then, there exists a measurable function $x^* : T \to E$ such that

$$x^*(t) \in C(t, p^*) \;\text{ a.e. }\; t \in T, \;\text{ and} \tag{A5}$$

$$\int x^*(t) d\mu - \int e(t) d\mu = z^* \leq 0. \tag{A6}$$

We prove (p^*, x^*) is a Walrasian equilibrium with free disposal.

We prove that $p^* \neq 0$ by contradiction.[9] If $p^* = 0$, then $B(t, p^*) = X(t)$ for all $t \in T$. Since $x^*(t) \in X(t) = B(t, p^*)$ a.e. $t \in T$ and $\int x^*(t) d\mu \leq \int e(t) d\mu$, it follows from A.7 that there exists a measurable set $S \in \mathcal{T}$ with $\mu(S) > 0$ such that $P_t(p, x^*(t)) \neq \emptyset$ for all $t \in S$. This is a contradiction to $x^*(t) \in C(t, p^*)$ a.e. $t \in T$.

From the same arguments as in the proof of Theorem 2, we have

$$p^* \cdot x^*(t) = p^* \cdot e(t) \text{ a.e. } t \in T. \tag{A7}$$

Combining (A5), (A6) and (A7), it follows that (p^*, x^*) is a Walrasian equilibrium with free disposal. □

[9] When the preference of each agent is independent from prices, it is irrelevant whether $p^* = 0$ or not. If $p^* = 0$, then $B(t, p^*) = X(t)$ for all $t \in T$. Since $x^*(t) \in C(t, p^*)$ a.e. $t \in T$, it follows that $x^*(t)$ is a satiation point of agent t for almost all $t \in T$. Hence we have $e(t) \leq x^*(t)$ a.e. $t \in T$. Take $p^{**} \neq 0$ arbitrarily. Given that $e(t) \leq x^*(t)$ a.e. $t \in T$ and $\int x^*(t) d\mu \leq \int e(t) d\mu$, we have $p^{**} \cdot x^*(t) = p^{**} \cdot e(t)$ a.e. $t \in T$. Then, it follows that (p^{**}, x^*) is a Walrasian equilibrium with free disposal.

A.5 Proof of Corollary 6

Proof Since $(\Omega, \mathcal{F}, \nu)$ is nontrivial, $L^q(\mathcal{F}, \nu)_+$ is non-degenerate. Thus, all that remains is to prove that $L^p(\mathcal{F}, \nu)$ is separable. Then, applying Theorem 5, the conclusion follows.

Since $(\Omega, \mathcal{F}, \nu)$ is σ-finite, there exists a sequence $\{\Omega_n\}_{n\in\mathbf{N}}$ such that $\Omega = \bigcup \Omega_n$ and $\nu(\Omega_n) < \infty$ for all $n \in \mathbf{N}$. Suppose $\mathcal{F}$ is generated by $\mathcal{A} \subset \mathcal{F}$ and the null sets $\mathcal{N}(\nu)$, where $\mathcal{A}$ is countable and $\mathcal{A} \cap \mathcal{N}(\nu) = \emptyset$. We may assume $\mathcal{A}$ contains Ω_n for all $n \in \mathbf{N}$. Let $\sigma(\mathcal{A})$ be the σ-algebra generated by $\mathcal{A}$. Then, the measure space $(\Omega, \sigma(\mathcal{A}), \nu|_{\sigma(\mathcal{A})})$ is σ-finite and countably generated. Therefore, $L^p(\sigma(\mathcal{A}), \nu|_{\sigma(\mathcal{A})})$ is separable (Cohn 2013, Proposition 3.4.5).

We show that $L^p(\sigma(\mathcal{A}), \nu|_{\sigma(\mathcal{A})})$ is dense in $L^p(\mathcal{F}, \nu)$. Since the space of $\mathcal{F}$-measurable step-functions is dense in $L^p(\mathcal{F}, \nu)$, it suffice to show that for any χ_A ($A \in \mathcal{F}$ and $\nu(A) < \infty$) and $\epsilon > 0$, there exists a set $A' \in \sigma(\mathcal{A})$ such that $\nu(A') < \infty$ and $||\chi_A - \chi_{A'}||_p < \epsilon$. More strongly, it holds that for any χ_A ($A \in \mathcal{F}$ and $\nu(A) < \infty$), there exists a set $A' \in \sigma(\mathcal{A})$ such that $\nu(A') < \infty$ and $||\chi_A - \chi_{A'}||_p = 0$. In fact, define $\mathcal{G} = \{A \in \mathcal{F} : \exists A' \in \sigma(\mathcal{A}), \nu(A \triangle A') = 0\}$. Then $\mathcal{G}$ is a σ-algebra, and contains $\mathcal{A}$ and $\mathcal{N}(\nu)$. Therefore $\mathcal{G} = \mathcal{F}$. Consider χ_A for any $A \in \mathcal{F}$ with $\nu(A) < \infty$. There exists a set $A' \in \sigma(\mathcal{A})$ such that $\nu(A \triangle A') = 0$. Then, $\nu(A') < \infty$ and $(||\chi_A - \chi_{A'}||_p)^p = \int_\Omega |\chi_A(\omega) - \chi_{A'}(\omega)|^p d\nu = \nu(A \triangle A') = 0$. This completes the proof. □

Acknowledgements The author is grateful to Masakazu Fukuzumi and anonymous referees for their invaluable guidance and insightful feedback throughout the development of this study. The author also expresses gratitude to Nicholas C. Yannelis for his helpful comments and for introducing the works by Cornet et al. (2023), Podczeck and Yannelis (2024), and Bhowmik and Yannelis (2024). Any remaining errors are solely the author's.

Data availability We do not analyse or generate any datasets, because our work proceeds within a theoretical and mathematical approach.

Declarations

Conflict of interest The author has no conflict of interest to declare that are relevant to the content of this article.

References

Aliprantis, C.D., Border, K.C.: Infinite Dimensional Analysis: A Hitchhiker's Guide. Springer, Berlin (2006)

Anderson, R.M., Duanmu, H., Khan, M.A., Uyanik, M.: On abstract economies with an arbitrary set of players and action sets in locally-convex topological vector spaces. J. Math. Econ. **98**, 102581 (2022). https://doi.org/10.1016/j.jmateco.2021.102581

Araujo, A., Monteiro, P.K.: Equilibrium without uniform conditions. J. Econ. Theory (1989). https://doi.org/10.1016/0022-0531(89)90035-5

Aumann, R.J.: Markets with a continuum of traders. Econometrica **32**(1), 39–50 (1964). https://doi.org/10.2307/1913732

Aumann, R.J.: Existence of competitive equilibria in markets with a continuum of traders. Econometrica **34**(1), 1–17 (1966). https://doi.org/10.2307/1909854

Aumann, R.J.: Measurable utility and the measurable choice theorem. In: La décision, Actes Coll. Int., CNRS, Aix-en-Provence, Paris, pp. 15–26 (1969)

Bewley, T.F.: Existence of equilibria in economies with infinitely many commodities. J. Econ. Theory **4**(3), 514–540 (1972). https://doi.org/10.1016/0022-0531(72)90136-6

Bewley, T.F.: A Very Weak Theorem on the Existence of Equilibria in Atomless Economies with Infinitely Many Commodities, In: Equilibrium Theory in Infinite Dimensional Spaces (M. A. Khan and N. C. Yannelis, Eds.), Studies in Economic Theory, Vol. 1, 224–232. Springer-Verlag, New York (1991). https://doi.org/10.1007/978-3-662-07071-0_9

Bhowmik, A., Yannelis, N.C.: Equilibria in abstract economies with a continuum of agents with discontinuous and non-ordered preferences. J. Math. Econ. (2024) **(forthcoming)**

Birnbaum, M.H., Schmidt, U.: An experimental investigation of violations of transitivity in choice under uncertainty. J. Risk Uncert. **37**(1), 77–91 (2008)

Carmona, G., Podczeck, K.: Existence of Nash equilibrium in games with a measure space of players and discontinuous payoff functions. J. Econ. Theory **152**, 130–178 (2014). https://doi.org/10.1016/j.jet.2014.04.002

Carmona, G., Podczeck, K.: Existence of Nash equilibrium in ordinal games with discontinuous preferences. Econ. Theory **61**(3), 457–478 (2016)

Castaing, C., Valadier, M.: Convex Analysis and Measurable Multifunctions. Springer, Berlin (1977)

Cettolin, E.: Revealed preferences under uncertainty: incomplete preferences and preferences for randomization. J. Econ. Theory **181**, 547–585 (2019)

Cohn, D.L.: Measure Theory. Springer, Berlin (2013)

Cornet, B.: The Gale–Nikaido–Debreu lemma with discontinuous excess demand. Econ. Theory Bull. **8**(2), 169–180 (2020). https://doi.org/10.1007/s40505-019-00181-5

Cornet, B., Guo, H., Yannelis, N.C.: On an infinite dimensional generalization of the excess demand theorem of David Gale. Pure Appl. Funct. Anal. **8**(5), 1333–1340 (2023)

Dasgupta, P., Maskin, E.: The existence of equilibrium in discontinuous economic games, I: theory. Rev. Econ. Stud. **53**(1), 1–26 (1986). https://doi.org/10.2307/2297588

Fajardo, S., Keisler, H.J.: Model Theory of Stochastic Processes: Lecture Notes in Logic 14. CRC Press, Boca Raton (2002)

Fremlin, D.: Measure Theory, Vol. 3: Measure Algebras. Torres Fremlin, Colchester (2011)

Greinecker, M., Podczeck, K.: Core equivalence with differentiated commodities. J. Math. Econ. **73**, 54–67 (2017). https://doi.org/10.1016/j.jmateco.2017.08.005

He, W., Yannelis, N.C.: Discontinuous games with asymmetric information: an extension of Reny's existence theorem. Games Econ. Behav. **91**, 26–35 (2015). https://doi.org/10.1016/j.geb.2015.03.013

He, W., Yannelis, N.C.: Existence of Walrasian equilibria with discontinuous, non-ordered, interdependent and price-dependent preferences. Econ. Theory **61**(3), 497–513 (2016). https://doi.org/10.1007/s00199-015-0875-x

He, W., Yannelis, N.C.: Equilibria with discontinuous preferences: new fixed point theorems. J. Math. Anal. Appl. **450**(2), 1421–1433 (2017). https://doi.org/10.1016/j.jmaa.2017.01.089

Jang, H.S., Lee, S.: Equilibria in a large production economy with an infinite dimensional commodity space and price dependent preferences. J. Math. Econ. **90**, 57–64 (2020). https://doi.org/10.1016/j.jmateco.2020.05.005

Jones, L.E.: A competitive model of commodity differentiation. Econometrica **52**(2), 507–530 (1984). https://doi.org/10.2307/1911501

Keisler, H.J., Sun, Y.: Why saturated probability spaces are necessary. Adv. Math. **221**(5), 1584–1607 (2009). https://doi.org/10.1016/j.aim.2009.03.003

Khan, M.A., Sagara, N.: Maharam-types and Lyapunov's theorem for vector measures on Banach spaces. Ill. J. Math. **57**(1), 145–169 (2013)

Khan, M.A., Sagara, N.: The Bang-Bang, purification and convexity principles in infinite dimensions. Set Valued Variat. Anal. **22**(4), 721–746 (2014). https://doi.org/10.1007/s11228-014-0282-7

Khan, M.A., Sagara, N.: Weak sequential convergence in $L^1(\mu, X)$ and an exact version of Fatou's lemma. J. Math. Anal. Appl. **412**(1), 554–563 (2014). https://doi.org/10.1016/j.jmaa.2013.10.082

Khan, M.A., Sagara, N.: Relaxed large economies with infinite-dimensional commodity spaces: the existence of Walrasian equilibria. J. Math. Econ. **67**, 95–107 (2016). https://doi.org/10.1016/J.JMATECO.2016.09.004

Khan, M.A., Sagara, N.: Fatou's lemma, Galerkin approximations and the existence of Walrasian equilibria in infinite dimensions. Pure Appl. Funct. Anal. **2**(2), 317–355 (2017)

Khan, M.A. and T. Suzuki. On Differentiated and Indivisible Commodities: An Expository Re-framing of Mas-Colell's 1975 Model, In: Advances in Mathematical Economics Volume 20 (S. Kusuoka and

T. Maruyama, Eds.), 103–128. Springer, Singapore (2016). https://doi.org/10.1007/978-981-10-0476-6_5
Khan, M.A., Uyanik, M.: The Yannelis-Prabhakar theorem on upper semi-continuous selections in paracompact spaces: extensions and applications. Econ. Theory **71**(3), 799–840 (2021). https://doi.org/10.1007/s00199-021-01359-4
Khan, M.A., Yannelis, N.C.: Equilibria in Markets with a Continuum of Agents and Commodities, In: Equilibrium Theory in Infinite Dimensional Spaces (M. A. Khan and N. C. Yannelis, Eds.), Studies in Economic Theory, Vol. 1, 233–248. Springer-Verlag, New York (1991). https://doi.org/10.1007/978-3-662-07071-0_10
Lee, S.: Competitive equilibrium with an atomless measure space of agents and infinite dimensional commodity spaces without convex and complete preferences. Hitotsubashi J. Econ. **54**(2), 221–230 (2013)
Mas-Colell, A.: A model of equilibrium with differentiated commodities. J. Math. Econ. **2**(2), 263–295 (1975). https://doi.org/10.1016/0304-4068(75)90028-2
Mas-Colell, A.: The price equilibrium existence problem in topological vector lattices. Econometrica (1986). https://doi.org/10.2307/1912321
Mas-Colell, A., Richard, S.F.: A new approach to the existence of equilibria in vector lattices. J. Econ. Theory (1991). https://doi.org/10.1016/0022-0531(91)90140-Y
Mas-Colell, A., Zame, W.R.: Chapter 34 equilibrium theory in infinite dimensional spaces. Handb. Math. Econ. **4**, 1835–1898 (1991). https://doi.org/10.1016/S1573-4382(05)80009-8
Noguchi, M.: Economies with a continuum of consumers, a continuum of suppliers and an infinite dimensional commodity space. J. Math. Econ. **27**(1), 1–21 (1997). https://doi.org/10.1016/0304-4068(95)00759-8
Ostroy, J.M., Zame, W.R.: Nonatomic economies and the boundaries of perfect competition. Econometrica **62**(3), 593–633 (1994). https://doi.org/10.2307/2951660
Podczeck, K.: Markets with infinitely many commodities and a continuum of agents with non-convex preferences. Econ. Theory **9**(3), 385–426 (1997). https://doi.org/10.1007/BF01213846
Podczeck, K.: On the convexity and compactness of the integral of a Banach space valued correspondence. J. Math. Econ. **44**(7–8), 836–852 (2008). https://doi.org/10.1016/J.JMATECO.2007.03.003
Podczeck, K., Yannelis, N.C.: Existence of Walrasian equilibria with discontinuous, non-ordered, interdependent and price-dependent preferences, without free disposal, and without compact consumption sets. Econ. Theory **73**(2), 413–420 (2022). https://doi.org/10.1007/s00199-021-01400-6
Podczeck, K., Yannelis, N.C.: Existence of Walrasian equilibria with discontinuous, non-ordered, interdependent preferences, without free disposal, and with an infinite-dimensional commodity space. Econ. Theory (2024). https://doi.org/10.1007/s00199-024-01553-0
Regenwetter, M.: Transitivity of preferences. Psychol. Rev. **118**(1), 42–56 (2011)
Reny, P.: On the existence of pure and mixed strategy nash equilibria in discontinuous games. Econometrica **67**(5), 1029–1056 (1999)
Reny, P.J.: Equilibrium in discontinuous games without complete or transitive preferences. Econ. Theory Bull. **4**(1), 1–4 (2016). https://doi.org/10.1007/s40505-015-0087-3
Rustichini, A., Yannelis, N.C.: What is Perfect Competition?, In: Equilibrium Theory in Infinite Dimensional Spaces (M. A. Khan and N. C. Yannelis, Eds.), Studies in Economic Theory, Vol. 1, 249–265. Springer-Verlag, New York (1991). https://doi.org/10.1007/978-3-662-07071-0_11
Sun, Y., Yannelis, N.C.: Saturation and the integration of Banach valued correspondences. J. Math. Econ. **44**(7), 861–865 (2008). https://doi.org/10.1016/j.jmateco.2007.07.005
Yannelis, N.C.: On a market equilibrium theorem with an infinite number of commodities. J. Math. Anal. Appl. **108**(2), 595–599 (1985). https://doi.org/10.1016/0022-247X(85)90047-2
Yannelis, N.C., Zame, W.R.: Equilibria in Banach lattices without ordered preferences. J. Math. Econ. (1986). https://doi.org/10.1016/0304-4068(86)90002-9