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RESEARCH ARTICLE

Obstacles to redistribution through markets and one solution

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Abstract

Dworczak et al. (Econometrica 89:1665–1698, 2021) study when certain market structures are optimal for agents with linear preferences and bivariate preference heterogeneity. The optimal market structure requires the social planner to know the joint distribution of the value of the good and marginal value of money. We show that the features of the distribution needed to characterize optimal market structure cannot be identified from standard demand data where probability of purchase depends only on observed price. While this is a negative result, we show that the distribution for the value of the good and marginal utility of money can be fully identified when there is an observed measure of quality that can serve as a benchmark to make utility comparisons.

Keywords Demand · Identification · Revealed preference

JEL Classification C00 · D01 · D11

1 Introduction

Dworczak et al. (2021) ask several important economic questions such as: When are price regulations in a market optimal? What is the structure of optimal price regulation? Can redistributive policies improve social welfare? Dworczak et al. (2021) elegantly

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provides answers to each question. The main primitives needed to characterize the optimal market structure are certain features of the joint distribution of the value of the good and marginal value of money. In their analysis, these are assumed known by the social planner a priori.[1] Thus, to apply these results in practice, it is crucial to know whether relevant features of the joint distribution on preferences can be identified. This paper studies identification of this joint distribution from choice data.

From our reading of Dworczak et al. (2021), the results can be viewed in two ways. The first way we view the results are from a social choice perspective. Here, for a given distribution of Pareto weights, a social planner designs a mechanism so that individuals have an incentive to truthfully reveal their willingness-to-pay for the good. We highlight that here the Pareto weights are not endogenously derived. However, the measures of inequality that need to be verified depend on features of the distribution of Pareto weights. Thus, whether redistribution or a competitive market leads to higher welfare depends on which weights the planner uses, and on the distribution of agents' willingness to pay. In this case, the optimal market structure is heavily influenced by the Pareto weights chosen ex ante.[2]

The second interpretation of the model is structural. Here, the Pareto weights are structurally determined by individuals' ratio of the value of the good and marginal utility of money, which are assumed to have economic content. In this case, it is important to identify these parameters since they provide information on when redistributive policies might be useful in practice after measuring the economically relevant terms. In detail, what policy is optimal depends on whether there is low or high same-side inequality which relates the average value of money to the average value of money for the lowest willingness-to-pay agents. For example, when demand side inequality is low the competitive pricing is optimal. We take the second perspective throughout this paper, since the first perspective only depends on the social planner's exogenous Pareto weights.

We present two results. First, even when demand is well-behaved and observed without error for all prices, this is not enough to recover the features needed to characterize optimal market structure. While this is a negative result, our second result shows that we can identify the entire joint distribution of preferences when there is an observed measure of the quality of the good. Importantly, this quality measure sets the units of utilities and so we can separately measure the values for money and a good, relative to a common benchmark quality that has a marginal utility of 1. Thus, while Dworczak et al. (2021) abstract from the quality of goods present in real-world markets, we show that quality information is crucial for the planner to learn the distribution of preferences. Fortunately, such quality data is present for all markets mentioned in

[1] In this paper, we focus only on identification of the demand side since the supply side has the same underlying structure in Dworczak et al. (2021).

[2] In more detail, our comment is essentially on Theorem 8 in the supplemental material to Dworczak et al. (2021). That theorem shows for every set of Pareto weights and distribution of willingness-to-pay parameters, that there is a bivariate distribution for the structural model where the mechanism from the one dimensional case is optimal when reporting the ratio of the bivariate preference parameters. However, this is not the only distribution consistent with the choices which leads to the identification problem we discuss in this paper. If Pareto weights are defined ex-ante by the function $\lambda(r)$, then the distribution of willingness-to-pay (denoted $g(r)$) is identified from demand and the analogue to average value of money used to determine same side inequality is $\int \lambda(r)g(r)dr$.

Dworczak et al. (2021). In the Iranian kidney market, the quality of kidneys can be measured by individual health histories. In the rental real estate market, apartments differ on various measures of quality such as location or square footage. Finally, in the labor market, jobs that have similar pay may have different company culture or benefits.

The mathematical techniques used to show these results are straightforward. However, they provide a first step to take the results from Dworczak et al. (2021) to data. An alternative approach to make predictions without knowledge of the distribution of preferences could follow the approach of Bergemann and Morris (2005).[3] However, this is beyond the scope of this paper.

2 Model

We consider the framework of Dworczak et al. (2021) and examine when choice data can or cannot identify the joint distribution of good value and marginal value of money. Understanding this feature of the model is crucial to apply the results from Dworczak et al. (2021) since the conditions that characterize the optimal mechanism depend on features of this distribution. We briefly summarize the model for the non-owners of the good (demand side) from Dworczak et al. (2021). Analogous results hold for the sellers (supply side) in the market.

Here there exists a unit mass of *non-owners* who have preferences for a good (K) and money (M). The demand side is assumed to have no units of the good K and have unit demand for the good. Each individual has a value for the good (v^K) and a value for money (v^M). Let $(x^K, x^M) \in \{0, 1\} \times \mathbb{R}$ where x^K describes whether the individual purchases the good and x^M is the amount of money the person holds.

Each individual is assumed to receive utility

$$v^K x^K + v^M x^M.$$

The value vector (v^K, v^M) is assumed to be distributed according to a joint distribution $F(v^K, v^M)$. For simplicity, we suppose that the joint distribution has a probability density function given by $f(v^K, v^M)$. We also suppose that v^K and v^M have non-negative and bounded support. These are stronger assumptions than Dworczak et al. (2021), but seem reasonable for application. Moreover, since our first result is a non-identification result, we show that even under additional structure, we cannot recover relevant features of the distribution needed for the characterization of the optimal mechanism.

In the framework of Dworczak et al. (2021), the characterization of the optimal mechanism depends on the magnitude of certain moments of the distribution $F(v^K, v^M)$. In particular, Dworczak et al. (2021) shows that the optimal market struc-

[3] In detail, one could assume that the distribution of preference parameters (v^M, v^K) are drawn from a distribution that is not known to the planner and the planner wants to maximize total surplus for the worst case distribution subject to some natural moment constraints such as a fixed mean.

ture depends on the values

$$\mathbb{E}[v^M] \quad \text{and} \quad \mathbb{E}[v^M \mid \underline{r}]$$

where $\underline{r}$ is the minimum of $\frac{v^K}{v^M}$ over its support. Similarly, $\overline{r}$ is the maximum of $\frac{v^K}{v^M}$ over its support. Many of the propositions depend on whether there is low or high inequality of valuations. Inequality on the demand side of the market is said to be low when $\mathbb{E}[v^M \mid \underline{r}] \leq 2\mathbb{E}[v^M]$ and high when the opposite strict inequality holds.

3 Non-identification with demand

We first focus on what can be identified with demand data. We show that even when the demand function is known ex-ante, we cannot recover the features of the distribution needed to characterize the optimal mechanism. Let $p \in \mathbb{R}_+$ be the price of the good.

When individuals have preferences from Sect. 2, each individual chooses to buy one unit of good K when $\frac{v^K}{v^M} > p$, regardless of their initial monetary holdings. Thus, demand is given by $D(p) = 1 - \int\int 1\left\{\frac{v^K}{v^M} \leq p\right\} f(v^K, v^M) dv^K dv^M$ where $1\{\cdot\}$ is an indicator function.

We note that knowledge of $F(v^K, v^M)$ is equivalent to knowing the joint distribution over r and v^M given by $G(r, v^M)$ where $r = \frac{v^K}{v^M}$, since this gives a one-to-one transformation of random variables under the maintained assumption that $v^M \neq 0$ almost surely. We denote the joint probability density function of $G(r, v^M)$ by $g(r, v^M)$, which exists since our distribution is continuous and we have an almost everywhere invertible Jacobian for the change of variables.[4]

As in Dworczak et al. (2021), it will be useful to reference the marginal cumulative density function of $r = \frac{v^K}{v^M}$. Here, we abuse notation so that the cumulative distribution over r is given by $G(r) = \int\int 1\{\frac{v^K}{v^M} \leq r\} f(v^K, v^M) dv^K dv^M$. Similarly, we denote the probability density function over r as $g(r)$. Thus, it follows that knowing the market demand for good K is equivalent to knowing the cumulative density distribution of r since

$$D(p) = 1 - G(p).$$

Since demand and the marginal distribution of r are directly related, knowledge of demand gives $G(r)$, which we record below.

Lemma 1 *If $D(p)$ is known for all prices, then $G(r)$ is known.*

Under the assumptions on the joint distribution $F(v^K, v^M)$, the demand function will satisfy several properties. For example, demand is monotone decreasing in price.

[4] To see this, note for the relevant change of variables where $(y_1, y_2) = \left(\frac{v^K}{v^M}, v^M\right)$, the Jacobian is $\begin{bmatrix} \frac{1}{v^M} & -\frac{v^K}{(v^M)^2} \\ 0 & 1 \end{bmatrix}$ which is almost everywhere invertible since $v^M \neq 0$ almost surely.

Also, we know that $D(p) = 1$ for all $p \leq \underline{p}$ and $D(p) = 0$ for all $p \geq \overline{p}$ where $\underline{r} = \underline{p}$ and $\overline{r} = \overline{p}$. Since $G(p)$ has a probability density function, we know that $D(p)$ is differentiable on $(\underline{p}, \overline{p})$.

We show that when demand is known, one cannot identify whether the demand side inequality is low or high. The reason for this is that demand only gives us information on the marginal distribution of $G(r, v^M)$ with respect to r but does not restrict conditional moments of v^M. To see this, assuming that conditional moments exist and using the law of iterated expectations, we see that

$$\begin{aligned}\mathbb{E}[v^M] &= \int \int v^M g(r, v^M) dv^M dr \\ &= \int \mathbb{E}[v^M \mid r] g(r) dr.\end{aligned}$$

Thus, even though $g(r)$ is identified from demand, there is limited information about the conditional expectation of v^M given r. We show in the proof of the following proposition that even when demand is known there can be low or high inequality on the demand side of the market.

Proposition 1 *If a researcher knows $D(p)$, then it is not possible to identify whether there is low or high same side inequality of demand.*

Proof We look for an arbitrary function $h(r)$ that maps to finite non-negative numbers to get

$$\mathbb{E}[v^M \mid r] = \frac{h(r)}{g(r)}$$

where $g(r)$ is the probability density identified from demand. Note that this expectation can be generated by letting the distribution of v^M conditional on r be a truncated normal distribution with mean $\frac{h(r)}{g(r)}$ and truncated at $\frac{h(r)}{g(r)} - \varepsilon$ and $\frac{h(r)}{g(r)} + \varepsilon$ where $\varepsilon \in \left(0, \frac{h(r)}{g(r)}\right)$, so the marginal value of money is non-negative.

Note that when the conditional mean satisfies

$$\mathbb{E}[v^M \mid r] = \frac{h(r)}{g(r)}$$

it follows that

$$\mathbb{E}[v^M] = \int_{\underline{r}}^{\overline{r}} h(r) dr.$$

Thus, we see that the condition to determine market inequality reduces to

$$h(\underline{r}) \gtreqless 2g(\underline{r}) \int_{\underline{r}}^{\overline{r}} h(r) dr \tag{1}$$

For example, inequality is low when the scaled area under $h(r)$ exceeds $h\left(\underline{r}\right)$ as stated below

$$h(\underline{r}) \leq 2g(\underline{r}) \int_{\underline{r}}^{\overline{r}} h(r)dr. \tag{2}$$

We show one function that gives low market inequality. Let $h(r) = r - \underline{r} + \delta$ where $\delta > 0$ so that integrating Eq. 2 gives

$$\delta \leq 2g(\underline{r}) \int_{\underline{r}}^{\overline{r}} (r - \underline{r})dr + 2\delta g(\underline{r})(\overline{r} - \underline{r})$$

which is true when $\delta \leq 2\, g(\underline{r}) \int_{\underline{r}}^{\overline{r}} (r - \underline{r})dr$.

Similarly, high same side inequality of demand occurs when

$$h(\underline{r}) > 2g(\underline{r}) \int_{\underline{r}}^{\overline{r}} h(r)dr. \tag{3}$$

One function that gives high same-side inequality is $h(r) = \frac{1}{\sqrt{r - \underline{r} + \delta}}$. Integrating Eq. 3 gives

$$\frac{1}{\sqrt{\delta}} > 4g(\underline{r}) \left(\sqrt{\overline{r} - \underline{r} + \delta} - \sqrt{\delta} \right)$$

which is implied when

$$\delta^2 + (\overline{r} - \underline{r})\delta - \frac{1}{16g^2(\underline{r})} < 0. \tag{4}$$

Finding the roots of this, we find that Eq. 4 holds when $\delta < \frac{1}{2} \left(\sqrt{(\overline{r} - \underline{r})^2 + \frac{1}{4\, g^2(\underline{r})}} - (\overline{r} - \underline{r}) \right)$.

This shows that information about $\mathbb{E}[v^M]$ and $\mathbb{E}[v^M \mid \underline{r}]$ needed to determine low or high inequality cannot be recovered from demand data. □

4 Identification with homogeneous value for quality

The previous section shows how demand data with only prices cannot identify the key features needed to characterize the optimal mechanism from Dworczak et al. (2021). This section shows that if the analyst also observes variation in *quality* of the good, then the previous non-identification result is overturned.[5] To formalize this, we consider buyers with preferences over $(x^K, x^Q, x^M) \in \{0, 1\} \times \mathbb{R}^2$, where x^Q is a measure of quality. Here we imagine quality is constant within a market, but varies exogenously across markets. Covariates that vary across markets have been regularly used in industrial organization. For example, the presence of an airline at an airport

[5] The way we introduce quality differs from Akbarpour et al. (2022) who look at optimal allocation policies that depend on continuous good quality with transfers.

and information on flight details are used in Berry (1992); Ciliberto and Tamer (2009) and distance to a brewery is used in Miller and Weinberg (2017).[6]

Now, the utility of individuals on the demand side is given by

$$(v^K + v^Q x^Q)x^K + v^M x^M.$$

Here, v^Q is the value of quality. When quality is fixed in a market, this is exactly the setup of Dworczak et al. (2021) once we define $\tilde{v}^K = v^K + v^Q x^Q$ as the value of the good. We will provide conditions under which the joint distribution of (v^K, v^Q, v^M) is identified. This in turn identifies the joint distribution of $(\tilde{v}^K, v^M)$. Thus, the analysis of Dworczak et al. (2021) can be used in markets where the quality x^Q is the same.

The demand curve depends on quality x^Q and price p. We write demand with quality as

$$D_Q(x^Q, p) = \Pr(1\{v^K + v^Q x^Q - v^M p \geq 0\}),$$

where the probability is over the distribution of (v^K, v^Q, v^M) and we assume utility ties occur with probability zero. This specification uses the fact that the individual pays price p for the good and thus faces the dis-utility of expenditure $-v^M p$ regardless of monetary holdings. We give assumptions below that ensure identification of the distribution of values.

Assumption 1 The quality-price demand function D_Q is known for all values $x^Q \in \mathbb{R}$ and $p \in B \subseteq \mathbb{R}_+$, where B contains an open set.

Assumption 2 The value $v^Q = 1$ almost surely, the distribution of (v^K, v^M) is determined by its moments, and all absolute moments of (v^K, v^M) exist and are finite. For every x^Q and p, we have $Pr(v^K + x^Q - v^M p = 0) = 0$.

The assumption $v^Q = 1$ sets the scale of the latent utility model. It serves a key role in allowing us to define v^K and v^M as separate marginal values. In particular, Assumption (2) allows us to interpret v^M as the marginal utility of income *relative* to a benchmark value of quality. Thus, if we use distance to brewery as a quality shifter as in Miller and Weinberg (2017), we can coherently make the statement "A rich individual values money less than a poor individual, relative to a benchmark in which they both value travel distance the same." Note that this scale assumption sets utility in the same units for each person, and is a homogeneity condition concerning the value of quality.

When $v^Q > 0$ almost surely, the additional restriction $v^Q = 1$ does not have empirical content in general. That is, when $v^Q > 0$, we can divide by this term in the latent utility model and choices do not change. Setting $v^Q = 1$ is thus a normalization empirically, though it also sets the units that allow us to say that one individual values money more than another.

[6] In practice, quality shifters are often expressed through a utility index involving several observable covariates. We abstract from this and treat the quality measure as known. In practice, these results can be applied when the analyst has identified a quality measure by a previous argument. A large literature has established identification of utility indices, including (Matzkin 1993) for binary choice problems such as this.

Proposition 2 *Under Assumptions 1 and 2, the distribution of* (v^K, v^M) *is identified.*

Proof With the assumption $v^Q = 1$ almost surely, write

$$D_Q(x^Q, p) = Pr(1\{v^K - v^M p \geq -x^Q\}).$$

We recognize that varying x^Q over all of $\mathbb{R}$ recovers the distribution of $v^K - v^M p$. By varying p in an open ball contained in B, we recover the distribution of (v^K, v^M) from Masten (2018), Lemma 2. Fox (2021) presents a related identification result for discrete choice. □

There are two ways to view this result. The first is that in principle we can identify and estimate the distribution of (v^K, v^M) and then apply the analysis of Dworczak et al. (2021).[7] The second is more conceptual: (v^K, v^M) have a well-defined unique distribution in terms of observables, so there is clear meaning in the number v^K.

One natural concern is that adding quality changes the mechanism design problem. While this is a concern, we interpret quality as an exogenous shifter that cannot be controlled by the market designer and is constant within each (sub)market. Thus, the market design problem can be performed on each market using the variable $\tilde{v}^K = v^K + x^Q$ as the relevant marginal distribution of value for the good.

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[7] This is nontrivial in practice because the identification result requires variation in quality x^Q to all possible real numbers.