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RESEARCH ARTICLE

Optimal licensing contracts with a downstream oligopoly: insider versus outsider innovation

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Abstract
In the literature that deals with cost-reduction technology licensing in an oligopolistic downstream market, the paper by Sen and Tauman (Games Econ Behav 59:163–186, 2007) has been a milestone in that it thoroughly characterizes the optimal licensing contracts for both cases of insider and outsider innovation under complete information. However, when determining the licensee's fee payment to obtain the license through an auction, their treatments for different numbers of licensees are inconsistent. We instead use a consistent approach that can be applied to all numbers of licensees, in which a firm's reservation payoff is determined by its Cournot profit if it rejects the contract. We find that the optimal contract is for both the insider and outsider innovator to sell the license to all downstream firms. We also show that an insider innovator sets a (weakly) higher royalty rate and generates a (weakly) lower social welfare than an outsider innovator.

Keywords Technology licensing · Oligopoly · Insider innovator · Outsider innovator

JEL Classification D43 · D45 · L13 · L24

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1 Introduction

In the literature on technology licensing, the forms of the optimal contract and the optimal number of licensees to whom the innovator should sell the license have drawn a lot of attention. An important paper by Sen and Tauman (2007, hereafter, ST) analyzes the situation where an innovator who owns a cost-reducing technology can transfer its technology through licensing contracts to the other n downstream firms in an oligopolistic industry. They construct a general model that incorporates both cases of an insider and an outsider innovator, and show that, at the optimum, the innovator will sell the license to either $n-1$ or n firms. It is by far the most complete analysis in the literature that generally solves the optimal licensing contracts for both cases of insider and outsider innovation under complete information.

One special setup in ST is that the innovator sells the licenses through an *auction with a uniform linear royalty*, which is referred to as the *AR* policy. By contrast, in the literature, an alternative way is to sell licenses through an *upfront fixed fee with a uniform linear royalty* (which is referred to as the FR policy hereafter). Such a setup is adopted by, for example, Kamien and Tauman (1986), and Erutku and Richelle (2007), and the latter has provided a general solution for the case of an outsider innovator. However, to the best of our knowledge, the general solution for the case of insider innovation under the FR policy has yet to be provided.

One problem in ST, however, is that the firm's payments to obtain the licenses are determined in an inconsistent way for the cases of $k \leq n-1$ and $k = n$, where k is the number of the licensees. When $k \leq n-1$, the licenses are distributed to those who submit the highest bid and the payment for the license will be determined by the bid; however, when $k = n$, since all firms will obtain the license for sure, every firm will bid 0 in the auction and use the new technology without paying anything. To avoid this trivial situation, ST then adopt a different way to determine the reservation payoff of a licensee in the case where $k = n$, which is the profit when a firm rejects the contract, given that the other firms accept it. In the optimal contract, the innovator will sell either $k^* = n-1$ or $k^* = n$ contracts. However, we think that this dichotomous outcome is implausible because it is based on an inconsistent way to determine a licensee's willingness to pay.

In this paper, we instead consider the situation where a license is sold through an upfront fixed fee, which is the difference in a firm's profit between accepting and rejecting the contract, given the other firms' decision. By doing so, the determination of the optimal contract can be consistently applied to any $k \leq n$. We find that it is optimal to sell the license to all downstream firms, i.e., $k^* = n$, for both cases of insider and outsider innovation. That is, the innovator chooses "no exclusion" to generate the largest size of the "pie" (i.e., the industry profit obtained by using the new technology). The optimal royalty rate is set to resolve the trade-off between the industry profit enjoyed by the innovator and the licensees and the reservation payoff paid to the downstream firms. The forms of the optimal contract are similar for both insider and outsider innovators when $n \geq 3$: it is a fee-only contract when either the size of the innovation or the number of the firms is small; otherwise, it is a two-part tariff (i.e., an upfront fixed fee plus a royalty).[1] We also show that an insider innovator sets a (weakly) higher royalty rate and generates a (weakly) lower social welfare than an outsider innovator.

The early literature on licensing mainly considers the case of an outsider innovator. The formal analysis is initiated by Arrow (1962). He suggests that a competitive industry can provide more incentives for firms to innovate than a monopoly. Kamien and Tauman (1984, 1986) and Katz and Shapiro (1985, 1986) are the pioneers that analyze licensing under an oligopoly. By comparing licensing with a royalty, an upfront fixed fee, and an auction, they conclude that royalty licensing is dominated by both the auction and upfront fee in the case of an outsider innovator. Amir et al. (2014) consider an outsider patent holder offering a license to many potential buyers who can challenge the patent's validity if they do not accept the contract. The licensing contract is either a per-unit royalty or a fixed fee. They show that a royalty is superior to a fixed fee for the licensing of weak patents. More recently, Colombo et al. (2021) consider an outsider innovator with many potential buyers in an oligopolistic market. There are two schemes to sell the license: a fixed fee and an ad valorem profit royalty, determined by either an auction or posted prices. Under very general conditions for the equilibrium profit functions, they show that these two schemes are in fact equivalent to the innovator in that they generate the same licensing revenue. Most of the above literature focuses on a comparison of various licensing mechanisms.

Erutku and Richelle (2007) construct by far the most complete model that analyzes the licensing contract in an oligopoly for the case of an outsider innovator with complete information. They allow all types of contracts to be offered, including two-part tariff, fee-only, and pure-royalty contracts. They show that the optimal contract should be a two-part tariff, and that it is optimal to sell the license to *all* downstream firms. We obtain a similar result to Erutku and Richelle (2007) in the case of an outsider innovator. However, there are differences between our analysis and theirs: first, they only consider the case of an outsider innovator, while we solve the optimal contracts for both insider and outsider innovation. Second, they consider the possibility of negative royalty rates while we restrict our attention to the non-negative ones.

The early literature that analyzes the case of insider innovation includes Shapiro (1985), Wang (1998), and Wang and Yang (1999). They find the opposite result to the case of an outsider innovator: pure-royalty contracts are better than fee-only contracts. Since the insider innovator cares not only about the revenue but also its competitive position in the product market, charging a royalty can provide the innovator with a cost advantage over its rivals and thus becomes profitable. Kamien and Tauman (2002) extend Wang (1998) to a general Cournot oligopoly market of any number of firms, and again show that royalty contracts are better than fixed-fee contracts as long as there are sufficient firms. However, none of the above papers considers contracts with a two-part tariff.

By combining both upfront fees determined by auction and royalties, Sen and Tauman (2007) construct a general model to consider both cases of an insider and an outsider innovator under complete information. They show that the license is sold to

[1] For $n = 2$, the optimal contract is either fee-only or a two-part tariff for the case of outsider innovation, while it is royalty-only for the case of insider innovation.

either $n-1$ or n firms for both cases. Moreover, the forms of the optimal licensing contracts depend on the parameters of the model in a complicated way (see their Tables A5 and A6). As has been mentioned before, we consider that their approach is not consistent in determining the payment for a firm to obtain the license. We therefore modify ST's analysis by adopting the upfront fixed fee system, and argue that their solution does not completely hold under the FR system, and that the innovator should sell the license to *all* firms.

The remainder of this paper is organized as follows. Section 2 presents the baseline model. In Sects. 3 and 4, we characterize the optimal contract for both the cases of insider and outsider innovation under the *FR* system, respectively. In Sect. 5, we compare the optimal royalty rate and social welfare under these two types of innovation. Section 6 concludes the paper.

2 Model

An innovator owns a cost-reduction technology that can reduce the marginal cost of producing some good by ε. Through licensing contracts, the innovator can transfer its new technology to other firms which originally used the old technology to produce the good with the same marginal cost c. The level of innovation $\varepsilon < c$ is assumed to be common knowledge to all the players. We assume that the market demand function is $p(Q) = a - Q$, where Q is the total quantity and p is the market price. Firms compete with each other in Cournot fashion. In the case of an outsider innovator, there are $n \geq 2$ firms that can produce the product while the innovator can neither produce the good nor compete with the other firms. In the case of an insider innovator, there are $n + 1$ firms in the market, including the innovator and n other firms, all of which can produce and compete with each other. Throughout the paper, we also make the following assumption in that the cost-reduction innovation is non-drastic, i.e., $\varepsilon < a - c$.

The timing of the game is as follows. In Stage 1, the innovator chooses a licensing contract (r, F), where $r \in [0, \varepsilon]$ is the per-unit royalty rate and $F \geq 0$ is the upfront fixed fee. In Stage 2, given the contract (r, F), firms simultaneously decide whether to accept or to reject the contract. The licensees then pay the fixed fee. The contract will endogenously determine k, the number of downstream firms that accept the contract (i.e., the licensees), where $1 \leq k \leq n$. In Stage 3, in the case of an insider innovator, the innovator and the rival firms simultaneously decide the outputs, while in the case of an outsider innovator, the firms other than the innovator simultaneously decide the outputs. After their outputs have been sold in the market, the licensee firm pays the royalties, and the profits of all firms are realized.

The equilibrium concept applied here is the subgame perfect Nash equilibrium. We then solve the game by backward induction. Under complete information, the innovator will choose a fixed fee such that each licensee receives its reservation payoff. Thus, choosing a contract (r, F) is equivalent to choosing (k, δ), where $\delta \equiv \varepsilon - r \in [0, \varepsilon]$ is the *effective magnitude of the innovation*. Let $\Phi_L(k, \delta)$ and $\Phi_N(k, \delta)$ denote the Cournot profit obtained by a licensee and a non-licensee in Stage 3, respectively. We have

$$\Phi_L(k,\delta) = [q_L(k,\delta)]^2 \quad \text{and} \quad \Phi_N(k,\delta) = [q_N(k,\delta)]^2,$$

where $q_L(k,\delta)$ and $q_N(k,\delta)$ are the licensee's and the non-licensee's output.

In Stage 2, the innovator offers a contract (r, F) that is accepted by k downstream firms. It is required that the firms' strategies in which k licensees accept the contract and $n-k$ non-licensees reject that contract can constitute a Nash equilibrium in this stage. There are two conditions that need to be met: first, given that the other $k-1$ firms accept the contract, a firm will accept it if

$$\Phi_L(k,\delta) - F \geq \Phi_N(k-1,\delta), \tag{1}$$

where $\Phi_N(k-1,\delta)$ is the Cournot profit if the firm deviates and rejects the contract (and becomes a non-licensee), which is the reservation payoff for a licensee firm to accept the contract. Define

$$w^{FR}(k,\delta) \equiv \Phi_L(k,\delta) - \Phi_N(k-1,\delta), \tag{2}$$

then (1) is equivalent to $F \leq w^{FR}(k,\delta)$.

Second, a non-licensee firm will reject the contract (r, F) if

$$\Phi_N(k,\delta) \geq \Phi_L(k+1,\delta) - F, \tag{3}$$

where $\Phi_L(k+1,\delta)$ is the Cournot profit for the firm if it deviates and accepts the contract (and then turns into a licensee), given that the other k firms accept the contract. Following (2), $w^{FR}(k+1,\delta) = \Phi_L(k+1,\delta) - \Phi_N(k,\delta)$, and so (3) is equivalent to $F \geq w^{FR}(k+1,\delta)$.

Based on (1) and (3), for the downstream firms' strategies to be supported in a Nash equilibrium in this stage, it is required that

$$w^{FR}(k+1,\delta) \leq F \leq w^{FR}(k,\delta). \tag{4}$$

We have the following lemma:

Lemma 1 $w^{FR}(k+1,\delta) < w^{FR}(k,\delta)$ *for* $1 \leq k \leq n-1$.[2]

Proof See the Appendix. □

By Lemma 1, there exists a fixed fee F satisfying (4). It is then straightforward to see that the innovator will optimally choose an F such that

$$F = w^{FR}(k,\delta). \tag{5}$$

This is indeed the unique equilibrium fixed fee.

[2] Erutku and Richelle (2007) have a similar claim; see their Claim 1. However, they only consider the outsider case. By contrast, our Lemma 1 applies to both the insider and outsider case.

When the fixed fee is determined by (5), it is referred to as the FR system. Alternatively, ST specify the licensee's willingness to pay as

$$w^{AR}(k,\delta) = \Phi_L(k,\delta) - \Phi_N(k,\delta) \tag{6}$$

for $1 \le k \le n-1$, where the fee is determined by an auction. Given that k licensees have been offered, $w^{AR}(k,\delta)$ is the highest bid that each firm will place in equilibrium, and among the highest bidders, k firms will be chosen randomly to be the licensees. Nevertheless, when $k = n$, since all firms will be licensees, an auction is of no use, and so ST adopt the same approach as the FR system, that is,

$$w^{AR}(n,\delta) = \Phi_L(n,\delta) - \Phi_N(n-1,\delta). \tag{7}$$

As we can see, the willingness to pay in (6) and (7) is determined in an inconsistent way. By contrast, our approach can be applied to any number of k and is consistent in determining the licensee's willingness to pay to obtain the license.

In Stage 1, the innovator chooses (k,δ) to maximize its payoff, denoted by Π_λ^{FR}, with the upfront fixed fee w^{FR} determined in (2):

$$\begin{aligned}\Pi_\lambda^{FR}(k,\delta) &= \lambda\Phi_I(k,\delta) + k[(\varepsilon-\delta)q_L(k,\delta) + w^{FR}(k,\delta)]\\ &= \lambda\Phi_I(k,\delta) + k[(\varepsilon-\delta)q_L(k,\delta) + \Phi_L(k,\delta) - \Phi_N(k-1,\delta)]\\ &= [p(k,\delta) - c + \varepsilon][\lambda q_I(k,\delta) + kq_L(k,\delta)] - k\Phi_N(k-1,\delta), \end{aligned} \tag{8}$$

where $\Phi_I(k,\delta)$ is the Cournot profit obtained by the innovator, and $q_I(k,\delta)$ is its output. $\lambda = 1$ ($\lambda = 0$) represents the case of insider (outsider) innovation, which depends on whether or not the Cournot profit yielded from the innovator's own production is included.

Equation (8) has another intuitive interpretation. The payoff of the innovator can indeed be decomposed into two parts: the first term represents the share of the industry profit enjoyed by the innovator and the licensees by using the new technology, and the second term is the reservation payoff that the innovator leaves to the licensees in order for them to accept the contract. In other words, the innovator tries to maximize the industry profit generated by using the new technology, net of the reservation payoff left to the licensees.

It is also convenient to rewrite (8) as the following expression:

$$\Pi_\lambda^{FR}(k,\delta) = \Pi_\lambda^{AR}(k,\delta) + k[\Phi_N(k,\delta) - \Phi_N(k-1,\delta)], \tag{9}$$

where

$$\Pi_\lambda^{AR}(k,\delta) = [p(k,\delta) - c + \varepsilon]Q(k,\delta) - n\Phi_N(k,\delta) - (n-k)\varepsilon q_N(k,\delta). \tag{10}$$

Equation (10) is the innovator's payoff under the *AR* policy for $k \le n-1$ shown in ST, in which the reservation payoff of each licensee is $\Phi_N(k,\delta)$. By contrast, in our setup, the reservation payoff is $\Phi_N(k-1,\delta)$. Thus, (9) reflects the difference between these two systems. Nevertheless, when $k = n$, these two systems have the same outcome, i.e., $\Pi_\lambda^{AR}(n,\delta) = \Pi_\lambda^{FR}(n,\delta)$. Again, this is because ST adopt a different treatment for the case of $k = n$.

Based on the payoff function of each player, we can solve the typical Cournot equilibrium outputs in Stage 3. Define

$$\tilde{\delta}_\lambda(k) \equiv \frac{a-c-\lambda\varepsilon}{k}$$

as a threshold that separates two cases: when $\delta \geq \tilde{\delta}_\lambda(k)$, this is a $(k+\lambda)$-*drastic* case, where $k+\lambda$ firms that obtain the new technology survive and the other non-licensed firms drop out of the market. Thus, a natural oligopoly with $k+\lambda$ firms is formed. Otherwise, when $\delta < \tilde{\delta}_\lambda(k)$, it is a non-drastic case, where all the licensees and non-licensees produce positive outputs.[3]

For the non-drastic case, when $1 \leq k \leq n-1$, the equilibrium outputs are

$$\begin{aligned} q_N(k,\delta) = p(k,\delta) - c = \frac{k[\tilde{\delta}_\lambda(k)-\delta]}{n+1+\lambda}, \\ q_L(k,\delta) = q_N(k,\delta)+\delta, \quad \text{and} \quad q_I(k,\delta) = \lambda[q_N(k,\delta)+\varepsilon], \end{aligned} \tag{11}$$

and when $k = n$,

$$\begin{aligned} q_L(n,\delta) = p(n,\delta) - c + \delta = \frac{n\tilde{\delta}_\lambda(n)+(1+\lambda)\delta}{n+1+\lambda}, \quad \text{and} \\ q_I(n,\delta) = \lambda[q_L(n,\delta)+\varepsilon-\delta]. \end{aligned} \tag{12}$$

On the other hand, for the $(k+\lambda)$-drastic case,

$$\begin{aligned} q_N(k,\delta) = 0, q_L(k,\delta) = p(k,\delta) - c + \delta = \frac{k\tilde{\delta}_\lambda(k)+(1+\lambda)\delta}{k+1+\lambda}, \\ \text{and} \quad q_I(k,\delta) = \lambda[q_L(k,\delta)+\varepsilon-\delta]. \end{aligned} \tag{13}$$

It follows that

$$k[\Phi_N(k,\delta) - \Phi_N(k-1,\delta)] = \begin{cases} -\frac{k\delta[2B-(2k-1)\delta]}{(n+1+\lambda)^2} \leq 0, & \text{when } \delta < \tilde{\delta}_\lambda(k), \\ -k\Phi_N(k-1,\delta) \leq 0, & \text{when } \delta \geq \tilde{\delta}_\lambda(k), \end{cases} \tag{14}$$

where $B = a - c - \lambda\varepsilon = k\tilde{\delta}_\lambda(k)$.[4]

[3] According to Arrow (1962), a cost-reducing innovation is *drastic* if it is significant enough for only one firm that uses the new technology to become a monopoly. Otherwise, it is non-drastic. Sen and Tauman (2007) define an innovation to be k-*drastic* "if k is the minimum number such that if k firms have the innovation, all other firms drop out of the market and a k-firm natural oligopoly is created." Indeed, "a drastic innovation is 1-drastic and any non-drastic innovation is k-drastic for some integer $k \geq 2$" (page 169). Here, we follow the definition of Sen and Tauman (2007).

[4] To see the negativity, note that $2B - (2k-1)\delta \geq 0$ if and only if $\delta \leq \frac{2k\tilde{\delta}_\lambda(k)}{2k-1}$. Hence, $k[\Phi_N(k,\delta) - \Phi_N(k-1,\delta)] \leq 0$ for the non-drastic case. On the other hand, for the $(k+\lambda)$-drastic case, $\Phi_N(k,\delta) = 0$ and $\Phi_N(k-1,\delta) \geq 0$ when $\delta \geq \tilde{\delta}_\lambda(k)$. Hence, $k[\Phi_N(k,\delta) - \Phi_N(k-1,\delta)] = -k\Phi_N(k-1,\delta) \leq 0$.

According to (9) and (14), it is easy to see that

$$\Pi_{\lambda}^{FR}(k,\delta) \leq \Pi_{\lambda}^{AR}(k,\delta). \tag{15}$$

That is, the FR system yields a lower payoff to the innovator than the AR system. Intuitively, if a firm fails in an auction, it obtains $\Phi_N(k,\delta)$, given that there are k firms having the license; while under the FR system, if a firm rejects the contract, there are $k-1$ firms having the license. Thus, a firm can enjoy a higher reservation payoff $\Phi_N(k-1,\delta)$ under the FR system, and so the innovator obtains less.

In the following important lemma, we show that when a contract with a non-drastic δ is offered, the innovator (both the insider and outsider) will sell the license to all downstream firms. Before stating the lemma, we observe that $q_N(k,\delta)$, $p(k,\delta)$ and $\Phi_N(k,\delta)$ depend only on the product $k\cdot\delta$ for non-drastic innovations where $\delta < \tilde{\delta}_{\lambda}(k) = \frac{a-c-\lambda\varepsilon}{k}$. To see this, when $1 \leq k \leq n-1$ (the case $k=n$ can be similarly derived), according to (11), we know that $q_N(k,\delta) = p(k,\delta) - c = \frac{k[\tilde{\delta}_{\lambda}(k)-\delta]}{n+1+\lambda} = \frac{a-c-\lambda\varepsilon-k\delta}{n+1+\lambda}$, which means that it depends only on $k\cdot\delta$ given the parameters $(a,c,\varepsilon,n,\lambda)$, and so does $p(k,\delta)$. Since $p=a-Q$ and $\Phi_N(k,\delta)=[q_N(k,\delta)]^2$, it means that both $Q(k,\delta)$ and $\Phi_N(k,\delta)$ depend only on $k\cdot\delta$, too.

Lemma 2 *When the innovator offers a contract* (k,δ) *with a non-drastic* $\delta < \tilde{\delta}_{\lambda}(k)$, *the optimal contract is such that it is accepted by all the downstream firms, i.e.,* $k^* = n$.

Proof When $n\cdot\delta' = k\cdot\delta$, we have $\delta' < \delta < \tilde{\delta}_{\lambda}(k)$ for any $1 \leq k < n$, which means that the contract (n,δ') still belongs to a non-drastic case. Then based on (9), (10) and (14), we have

$$\begin{aligned}
\Pi_{\lambda}^{FR}(k,\delta) &= [p(k,\delta)-c+\varepsilon]Q(k,\delta) - n\Phi_N(k,\delta) - (n-k)\varepsilon q_N(k,\delta) \\
&\quad - \frac{k\delta[2B-(2k-1)\delta]}{(n+1+\lambda)^2} \\
&< [p(n,\delta')-c+\varepsilon]Q(n,\delta') - n\Phi_N(n,\delta') - \frac{n\delta'[2B-2n\delta'+\delta]}{(n+1+\lambda)^2} \\
&< [p(n,\delta')-c+\varepsilon]Q(n,\delta') - n\Phi_N(n,\delta') - \frac{n\delta'[2B-2n\delta'+\delta']}{(n+1+\lambda)^2} \\
&= \Pi_{\lambda}^{FR}(n,\delta').
\end{aligned} \tag{16}$$

The first inequality in (16) is obtained by observing that $p(k,\delta)=p(n,\delta')$, $Q(k,\delta)=Q(n,\delta')$ and $\Phi_N(k,\delta)=\Phi_N(n,\delta')$ when $n\cdot\delta'=k\cdot\delta$, and that $(n-k)\varepsilon q_N(k,\delta)>0$ for $k<n$. The second inequality is due to the fact that $\delta'<\delta$. Since $\Pi_{\lambda}^{FR}(k,\delta) < \Pi_{\lambda}^{FR}(n,\delta')$, selling n licenses dominates selling $k<n$ licenses. Thus, $k^*=n$ for all non-drastic cases. □

Intuitively, the innovator optimally chooses "no exclusion" so that all firms can use the better technology $(c-\varepsilon)$ to produce, i.e., the diffusion of the innovation is at its maximum. This generates the largest industry profit. Furthermore, choosing a smaller δ means a higher royalty rate r and a lower output, which makes it closer to the monopolistic output. This also increases the industry profit. However, a smaller δ makes the reservation payoff higher. Hence, the optimal δ is set to resolve the trade-off between the industry profit $(p - c + \varepsilon)Q$ enjoyed by the innovator and the licensees and the reservation payoff $n\Phi_N(n-1, \delta)$ paid to the downstream firms.

Lemma 2 differs from the result obtained in ST in the following way. ST also show that the payoff function $\Pi_\lambda^{AR}(k, \delta)$ under the AR policy has the same property in that it depends only on the product $k \cdot \delta$, and the innovator will choose the largest number of licensees. However, their argument only applies to $k \leq n - 1$. When $k = n$, since all firms will obtain the license for sure, every firm will bid 0 in an auction and obtain the new technology by paying nothing. To avoid this trivial situation, ST then adopt a different (and thus inconsistent) way to determine the reservation payoff for a licensee, which is in fact the same as the *FR* policy for $k = n$. Lemma 2 shows that under the *FR* policy, the argument can apply all the way to $k \leq n$, and thus the innovator will choose $k^* = n$. This will rule out the solution $k^* = n - 1$ in ST, as will be argued in the following section.

3 The optimal contract for the insider innovator

In this section, we analyze the case of insider innovation, i.e., $\lambda = 1$. When the innovator offers a contract with a non-drastic δ, it will choose $k = n$ according to Lemma 2. Given this fact, the innovator then chooses a δ to maximize the payoff. Let $\delta_1^*(n)$ be the optimal effective cost reduction and $\Pi_1^{FR}(n, \delta_1^*(n))$ be the value function under the optimal contract. Alternatively, the innovator can offer a $(k+1)$-drastic contract (k, δ) with $\delta \geq \tilde{\delta}_1(k)$. In the following lemma, we show that this kind of contract is sub-optimal:

Lemma 3 *The insider innovator can at most obtain $\varepsilon(a - c)$ when it offers a $(k+1)$-drastic contract (k, δ) with $\delta \geq \tilde{\delta}_1(k)$ for any $1 \leq k \leq n - 1$, which is dominated by the royalty-only contract $(n, 0)$, i.e., $\Pi_1^{FR}(n, 0) > \varepsilon(a - c) \geq \Pi_1^{FR}(k, \delta)$.*

Proof When $\delta = \tilde{\delta}_1(k)$, all the non-licensees drop out, i.e., $q_N(k, \delta) = 0$, and the equilibrium market price is equal to the original marginal cost, i.e., $p(k, \delta) = c$. Then, we have

$$\begin{aligned}\Pi_1^{AR}(k, \delta) &= [p(k, \delta) - c + \varepsilon]Q(k, \delta) - n\Phi_N(k, \delta) - (n - k)\varepsilon q_N(k, \delta) \\ &= \varepsilon(a - c).\end{aligned}$$

If $\delta > \tilde{\delta}_1(k)$, then $p(k, \delta) < c$, which leads to $\Pi_1^{AR}(k, \delta) < \varepsilon(a - c)$. Since $\Pi_1^{FR}(k, \delta) \leq \Pi_1^{AR}(k, \delta)$, the innovator can obtain at most $\varepsilon(a - c)$ from a $(k+1)$-drastic contract.

On the other hand, when offering the royalty-only contract, the innovator can obtain

$$\Pi_1^{FR}(n, 0) = q_I^2(n, 0) + n\varepsilon q_L(n, 0).$$

$q_I(n, 0)$ and $q_L(n, 0)$ are the Cournot equilibrium quantities for the innovator and the licensees, respectively, where all firms but the innovator use the marginal cost c while the innovator uses $c-\varepsilon$ under the royalty-only contract. Then we have:

$$\begin{aligned}\Pi_1^{FR}(n,0)-\varepsilon(a-c) &= \left[\frac{a-c+(n+1)\varepsilon}{n+2}\right]^2 + n\varepsilon\left(\frac{a-c-\varepsilon}{n+2}\right) - \varepsilon(a-c) \\ &= \left(\frac{a-c-\varepsilon}{n+2}\right)^2 > 0.\end{aligned}$$

Hence, a $(k+1)$-drastic contract is dominated by the royalty-only contract. □

With Lemmas 2 and 3, we can obtain the following result:

Proposition 1 *The optimal contract for an insider innovator is* $(k^*,\delta^*) = \left(n, \delta_1^*(n)\right)$, *where* $\delta_1^*(n) = \min\{\frac{(n-2)(a-c-\varepsilon)}{2(n^2-n+1)}, \varepsilon\}$, *which dominates any* $(k+1)$*-drastic contract with* $\delta \geq \tilde{\delta}_1(k)$. *The optimal royalty rate is*

$$r_1^* = \varepsilon - \delta_1^*(n) = \begin{cases} \frac{n(2n-1)\varepsilon-(n-2)(a-c)}{2(n^2-n+1)} > 0, & \textit{if } \varepsilon > \frac{(n-2)(a-c)}{n(2n-1)}, \\ 0, & \textit{if } \varepsilon \leq \frac{(n-2)(a-c)}{n(2n-1)}. \end{cases}$$

Proof We first show that $\Pi_1^{FR}(n,\delta^*(n)) \geq \Pi_1^{FR}(n,0)$ where $\delta_1^*(n) < \tilde{\delta}_1(n)$. Then by Lemma 3, we can conclude that $(n,\delta^*(n))$ dominates contract (k,δ) with a $(k+1)$-drastic $\delta \geq \tilde{\delta}_1(k)$ for any $k \leq n-1$.

Since $p(n,\delta)-c = \frac{n[\tilde{\delta}_\lambda(n)-\delta]}{n+1+\lambda} = \frac{a-c-\lambda\varepsilon-n\delta}{n+1+\lambda}$, the innovator's payoff function can be written as

$$\begin{aligned}\Pi_1^{FR}(n,\delta) &= [p(n,\delta)-c+\varepsilon]Q(n,\delta) - n\Phi_N(n-1,\delta) \\ &= \left[\frac{n(\tilde{\delta}_1(n)-\delta)}{n+2}+\varepsilon\right]\left[a-\frac{n(\tilde{\delta}_1(n)-\delta)}{n+2}-c\right] \\ &\quad -n\left[\frac{(n-1)(\tilde{\delta}_1(n-1)-\delta)}{n+2}\right]^2 \\ &= \left[\frac{a-c-\varepsilon-n\delta}{n+2}+\varepsilon\right]\left[a-\frac{a-c-\varepsilon-n\delta}{n+2}-c\right] \\ &\quad -n\left[\frac{a-c-\varepsilon-(n-1)\delta}{n+2}\right]^2. \end{aligned} \tag{17}$$

By taking the derivative with respect to (17), we obtain

$$\delta_1^*(n) = \frac{(n-2)(a-c-\varepsilon)}{2(n^2-n+1)}. \tag{18}$$

By the second-order condition, $\frac{d^2}{d\delta^2}\Pi_1^{FR}(n,\delta) < 0$, and $\frac{d}{d\delta}\Pi_1^{FR}(n,\delta)|_{\delta=0} > 0$ for $n > 2$, we know that the optimal payoff $\Pi_1^{FR}(n,\delta^*(n)) > \Pi_1^{FR}(n,0)$ for $n > 2$. On the other hand, when $n=2$, the optimal contract is $\delta_1^*(2) = 0$ or $r_1^* = \varepsilon$, which is a royalty-only contract. In this case, $\Pi_1^{FR}(n, \delta^*(n)) = \Pi_1^{FR}(n, 0)$, which again dominates any contract with a $(k+1)$-drastic δ.

Since we consider the case where the royalty rate $r \geq 0$, it is required that the solution satisfy $\delta_1^* \in [0, \varepsilon]$. According to (18), $\delta_1^*(n) \geq \varepsilon$ if and only if $\varepsilon \leq \frac{(n-2)(a-c)}{n(2n-1)}$. When $\varepsilon \leq \frac{(n-2)(a-c)}{n(2n-1)}$, since the payoff function is concave in δ, this means that the payoff function is always increasing for $\delta \in [0, \varepsilon]$, and so the solution is at the corner, i.e., $\delta_1^*(n) = \varepsilon$, or equivalently, $r_1^* = 0$. On the other hand, if $\varepsilon > \frac{(n-2)(a-c)}{n(2n-1)}$, the solution (18) is interior, i.e., $\delta_1^*(n) < \varepsilon$. It follows that $r_1^* = \varepsilon - \delta_1^*(n) = \frac{n(2n-1)\varepsilon-(n-2)(a-c)}{2(n^2-n+1)}$. $\square$

Proposition 1 indicates that when $n = 2$, the optimal contract is royalty-only, while when $n \geq 3$, it is either fee-only (for a small ε or n) or a two-part tariff (for a large ε or n). By decreasing δ, or equivalently, by increasing the royalty rate r, both the industry profit $(p - c + \varepsilon)Q$ and the reservation payoff $n\Phi_N(n, \delta)$ will increase. When $n = 2$, the increase in the industry profit dominates the increase in the reservation payoff, so that the solution is chosen at the maximum level, $r_1^* = \varepsilon$. However, when $n \geq 3$, a royalty-only contract gives up too much of the reservation payoff to the firms, and so the optimal contract will be $r_1^* < \varepsilon$, which is either fee-only or a two-part tariff.

In Proposition 1, we show that $k^* = n$ is always the case under the *FR* system. By contrast, ST adopt two different ways to determine the fee payments for the cases $k \leq n-1$ and $k = n$. They then compare the payoffs of $(n-1, \delta^*(n-1))$ under the AR system with those of $(n, \delta^*(n))$ under the FR system, and they argue that, given different ranges of the parameters, either $k^* = n-1$ or $k^* = n$ can be the optimal contract. Instead, our approach consistently applies to any $1 \leq k \leq n$, and it is shown that their result does not completely hold under the FR system, in that $k = n-1$ is sub-optimal.

4 The optimal contract for the outsider innovator

In this section, we analyze the optimal contract for the case of the outsider innovation. Since Lemma 2 still applies to this case, the outsider innovator will choose $k^* = n$ when it offers a contract that induces a non-drastic δ. We are left with analyzing contracts with a k-drastic δ. Here, however, Lemma 3 cannot apply because the royalty-only contract does not necessarily dominate contracts with a k-drastic δ. Specifically, the royalty-only contract can only yield a payoff equal to $\Pi_0^{FR}(n, 0) = \frac{n\varepsilon(a-c)}{n+1}$ for the outsider innovator, which is lower than that for an insider innovator, which is at least $\varepsilon(a-c)$. Therefore, the argument for using the AR policy in ST cannot directly apply to the case of an outsider innovator. We then need to compare the payoff of the contract that induces a k-drastic δ with that which induces a non-drastic δ. In the following lemma, we show that the outsider innovator will still sell the license to all downstream firms when offering a contract with a k-drastic δ:

Lemma 4 *When a k-drastic contract* (k, δ) *with* $\delta \geq \tilde{\delta}_0(k) = \frac{a-c}{k}$ *is offered, there is a contract with* $k = n$ *that can dominate all the contracts with* $2 \leq k \leq n-1$.[5]

Proof We consider three possible scenarios:
Case (1): $\delta = \tilde{\delta}_0(k) = \frac{a-c}{k}$.

In this case, all non-licensees drop out and the equilibrium market price is $p(k, \delta) = c$. The reservation payoff for the licensees has $\Phi_N(k-1, \delta) > 0$ (which requires $q_N(k-1, \delta) > 0$ or $\delta < \tilde{\delta}_0(k-1)$). Then we have

$$\begin{aligned}\Pi_0^{FR}\left(k, \frac{a-c}{k}\right) &= (p-c+\varepsilon)Q - k\Phi_N\left(k-1, \frac{a-c}{k}\right) \\ &= \varepsilon(a-c) - \frac{1}{k}\left(\frac{a-c}{n+1}\right)^2, \end{aligned} \tag{19}$$

which increases in k. This means that the contract with $k = n$ is better than any other contract with $k < n$ in this case.

Case (2): $\tilde{\delta}_0(k) < \delta < \tilde{\delta}_0(k-1) = \frac{a-c}{k-1}$.

In this case, $p(k, \delta) < c$ and the reservation payoff for the licensees has $\Phi_N(k-1, \delta) > 0$. The innovator's payoff is

$$\Pi_0^{FR}(k, \delta) = k\left[a - k\left(\frac{a-c+\delta}{k+1}\right) - c + \varepsilon\right]\left(\frac{a-c+\delta}{k+1}\right) - k\left[\frac{a-c-(k-1)\delta}{n+1}\right]^2. \tag{20}$$

Let $\delta^{\#}(k)$ be the maximizer of (20). Then by the first-order condition $\frac{\partial}{\partial \delta}\Pi_0^{FR}(k, \delta^{\#}) = 0$, it can be checked that $\delta^{\#}(k)$ satisfies:

$$\begin{aligned}&\left[\frac{k-1}{(n+1)^2} + \frac{k}{(k-1)(k+1)^2}\right]\delta^{\#} \\ &\quad = \frac{\varepsilon}{2(k-1)(k+1)} + \left[\frac{1}{(n+1)^2} - \frac{1}{2(k+1)^2}\right](a-c). \end{aligned} \tag{21}$$

By the envelope theorem, it can be checked that

$$\begin{aligned}&\frac{d}{dk}\Pi_0^{FR}(k, \delta^{\#}(k)) \\ &= \frac{\{(k+1)[(a-c)-\varepsilon] + 2[k\delta - (a-c)]\}\{(a-c) + \delta + 3k[k\delta - (a-c)]\}}{2(k+1)^3(k-1)} > 0.\end{aligned}$$

The terms in the brackets of the numerator are positive because $k\delta > a-c$ (or equivalently, $\delta > \tilde{\delta}_0(k)$) in this range and $a - c > \varepsilon$. Thus, $\Pi_0^{FR}(n, \delta^{\#}(n)) > \Pi_0^{FR}(k, \delta^{\#}(k))$ for $k = 2, \ldots, n-1$. This implies that the innovator is better off by choosing $k = n$ when there is a $\delta^{\#}$ such that $\tilde{\delta}_0(k) < \delta^{\#} < \tilde{\delta}_0(k-1)$.

[5] Since we assume that $a - c > \varepsilon$, there is no δ that satisfies $\delta > \tilde{\delta}_0(1) = a - c$ because $\delta \in [0, \varepsilon]$. That is, the 1-drastic case is infeasible in the outsider case.

Case (3): $\delta \geq \tilde{\delta}_0(k-1)$. We will show that the solution is at the corner, i.e., $\delta = \frac{a-c}{k-1}$, and that $\Pi_0^{FR}(k, \frac{a-c}{k-1})$ increases in k. Therefore, $\Pi_0^{FR}(n, \frac{a-c}{n-1}) > \Pi_0^{FR}(k, \frac{a-c}{k-1})$ for $k = 2, \ldots, n-1$.

First, when $\delta \geq \tilde{\delta}_0(k-1)$, both $q_N(k, \delta) = q_N(k-1, \delta) = 0$ and all non-licensees drop out. Furthermore, the reservation payoff for the licensees $\Phi_N(k-1, \delta) = [q_N(k-1, \delta)]^2 = 0$, so the innovator enjoys all the industry profit $(p(k, \delta) - c + \varepsilon)Q$. Since the total quantity $Q(k, \delta) = k(\frac{a-c+\delta}{k+1})$ is increasing in δ, reducing δ will make Q be closer to the monopolistic quantity and further increase the industry profit. Hence, the optimal effective cost reduction within this range is chosen to be as small as possible, i.e., $\delta^{\#} = \tilde{\delta}_0(k-1) = \frac{a-c}{k-1}$.[6]

By substituting this $\delta^{\#}$, $Q(k, \frac{a-c}{k-1}) = \frac{(a-c)k^2}{k^2-1}$, which is decreasing in k. Since $Q(n, \frac{a-c}{n-1})$ is the closest to the monopolistic quantity, it is best for the innovator to choose $k = n$. That is, $\Pi_0^{FR}(n, \frac{a-c}{n-1}) > \Pi_0^{FR}(k, \frac{a-c}{k-1})$ for $k = 2, \ldots, n-1$.

Accordingly, when a contract with a k-drastic δ is offered, there is a contract with $k = n$ that can dominate all contracts with $k < n$ for the outsider innovator. □

With Lemmas 2 and 4, in the following proposition, we show that the outsider innovator will sell its license to all downstream firms, i.e., $k^* = n$, and the optimal contract is either fee-only or a two-part tariff:

Proposition 2 *The optimal contract for an outsider innovator is* $(k^*, \delta^*) = \left(n, \delta_0^*(n)\right)$, *where* $\delta_0^*(n) = \min\{\frac{(n-1)(a-c)+(n+1)\varepsilon}{2(n^2-n+1)}, \varepsilon\}$. *The optimal royalty rate is*

$$r_0^* = \varepsilon - \delta_0^*(n) = \begin{cases} \frac{(n-1)[(2n-1)\varepsilon-(a-c)]}{2(n^2-n+1)} > 0, & \text{if } \varepsilon > \frac{a-c}{2n-1}, \\ 0, & \text{if } \varepsilon \leq \frac{a-c}{2n-1}. \end{cases}$$

Proof We separate three cases:

Case 1: $\varepsilon \leq \frac{a-c}{2n-1}$.

First, we note that since the royalty rate $r \geq 0$, it is required that the solution satisfy $\delta_0^* \in [0, \varepsilon]$. Thus, when $\varepsilon \leq \frac{a-c}{2n-1}$, we can rule out the k-drastic case for any k, because it only occurs when $\delta \geq \tilde{\delta}_0(n) = \frac{a-c}{n-1}$, which is not feasible since $\frac{a-c}{n-1} > \frac{a-c}{2n-1}$ for any n. Then according to Lemma 2, when a contract with a non-drastic δ is offered, the innovator will always choose the one with $k = n$. Moreover, since $\varepsilon < \frac{a-c}{n-1}$ in this case, the reservation payoff for the licensees is always positive, i.e., $\Phi_N(n-1, \delta) > 0$. Thus, the innovator's payoff function is:

$$\Pi_0^{FR}(n, \delta) = n\left[a - n\left(\frac{a-c+\delta}{n+1}\right) - c + \varepsilon\right]\left(\frac{a-c+\delta}{n+1}\right) - n\left[\frac{a-c-(n-1)\delta}{n+1}\right]^2, \tag{22}$$

[6] Note that the monopolistic quantity is $Q^M = \frac{a-c+\varepsilon}{2}$. Then when $\delta \geq \tilde{\delta}_0(k-1) = \frac{a-c}{k-1}$, $Q(k, \delta) = \frac{k(a-c+\delta)}{k+1} \geq \frac{k(a-c+\frac{a-c}{k-1})}{k+1} = \frac{k^2(a-c)}{k^2-1} > (a-c) > \frac{(a-c+\varepsilon)}{2} = Q^M$ for $k \geq 2$. Thus, the closer $Q(k, \delta)$ is to Q^M, the higher the industry profit is.

which is in fact (20) when $k = n$. The payoff maximizer to (22) is

$$\delta_0^*(n) = \frac{(n-1)(a-c) + (n+1)\varepsilon}{2(n^2 - n + 1)}. \tag{23}$$

However, $\delta_0^*(n) \geq \varepsilon$ if and only if $\varepsilon \leq \frac{a-c}{2n-1}$. Since the payoff function is concave in δ, it means that the payoff function is always increasing for $\delta \in [0, \varepsilon]$, and so the solution is at the corner, i.e., $\delta_0^*(n) = \varepsilon$, or equivalently, $r_0^* = 0$.

Case 2: $\frac{a-c}{2n-1} < \varepsilon < \frac{a-c}{n-1}$.

Based on (23), $\delta_0^*(n) < \varepsilon$ in this case. The k-drastic case does not occur, either, and the reservation payoff for the licensees is still positive. Similar to the previous case, the optimal solution is again (23), except that now the solution is interior. Thus, $r_0^* = \varepsilon - \delta_0^*(n) = \frac{(n-1)[(2n-1)\varepsilon-(a-c)]}{2(n^2-n+1)} > 0$.

Case 3: $\varepsilon \geq \frac{a-c}{n-1}$.

In this case, the k-drastic case can occur. Then by Lemma 4, if the innovator considers a k-drastic contract with $\delta(k) \geq \frac{a-c}{k}$ for $k = 2, \ldots, n-1$, it is dominated by a contract with $k = n$.

By focusing on the contracts with $k = n$, there are two subcases to consider: If $\delta < \tilde{\delta}_0(n-1) = \frac{a-c}{n-1}$, the reservation payoff for the licensees is again positive. Then the solution is (23) as in the previous Case 2 of this proposition. On the other hand, if $\delta \geq \frac{a-c}{n-1}$, the reservation payoff for the licensees is 0. Then the analysis is the same as Case 3 in the proof of Lemma 4, in which case the innovator will choose $\delta = \frac{a-c}{n-1}$.

The remaining job is to compare the payoffs of these two subcases and to determine the optimal $\delta_0^*(n)$. It should be noted that the innovator's payoff is maximized at $\delta_0^*(n)$ defined by (23) for the range $\delta < \frac{a-c}{n-1}$. Since the payoff function is concave and continuous in δ, $\Pi_0^{FR}(n, \delta_0^*(n)) > \lim_{\delta \to (\frac{a-c}{n-1})^-} \Pi_0^{FR}(n, \delta) = \Pi_0^{FR}(n, \frac{a-c}{n-1})$, which means that the global maximizer is still $\delta_0^*(n) = \frac{(n-1)(a-c)+(n+1)\varepsilon}{2(n^2-n+1)}$. Equivalently, $r_0^* = \frac{(n-1)[(2n-1)\varepsilon-(a-c)]}{2(n^2-n+1)}$. □

Proposition 2 indicates that the optimal contract is fee-only when either ε or n is small, and is a two-part tariff otherwise. Differing from the case of insider innovation, the royalty-only contract will never be optimal. This is because the outsider innovator cannot produce by itself and thus has to pay relatively more to the downstream firms. If the royalty rate is set too high, the reservation payoff for the licensees will be too high to cover the gain in the industry profit, especially when $n = 2$. Therefore, a royalty-only contract is not optimal.

This result is similar to Erutku and Richelle (2007), where they also show that an outsider innovator induces all firms to accept the contract at the optimum. Although they also consider an upfront fee system like ours, there are still some differences between these two approaches. First, they consider that $r \in [\varepsilon - c, \varepsilon]$ and allow for a negative royalty rate, while we focus on non-negative royalty rates, $r \in [0, \varepsilon]$. Therefore, the fixed fee is also non-negative, i.e., $F \geq 0$. Second, they only consider the case of an outsider innovator, while we can compare the cases of the insider and outsider innovator, as will be shown in the following section.

5 Comparison

Based on the results obtained in the previous sections, we can compare the optimal contract and the social welfare under these two kinds of innovation. In order to make a legitimate comparison, we fix the same industry structure with $n \geq 3$ producing firms.

First, we note that comparing r_1^* with r_0^* is equivalent to comparing $\delta_1^*(n-1)$ with $\delta_0^*(n)$. Second, the social welfare is defined as the sum of the industry profit and the consumer's surplus:

$$SW = [p(Q) - c + \varepsilon]\, Q + \int_0^Q [p(q) - p(Q)]\, dq = \int_0^Q [p(q) - c + \varepsilon] dq.$$

It can be seen that the social welfare increases in the total quantity Q. Note that the total quantity produced in the case of the insider innovation with $n-1$ firms is

$$\begin{aligned} Q_1(n-1) &= q_I(n-1, \delta_1^*(n-1)) + (n-1)q_L(n-1, \delta_1^*(n-1)) \\ &= n\left[\frac{a-c}{n+1} + \frac{\varepsilon + (n-1)\delta_1^*(n-1)}{n(n+1)}\right], \end{aligned}$$

and the total quantity produced in the case of the outsider innovation with n firms is

$$Q_0(n) = nq_L(n, \delta_0^*(n)) = n\left[\frac{a-c}{n+1} + \frac{\delta_0^*(n)}{n+1}\right].$$

Thus, it is equivalent to comparing $Q_1^*(n-1)$ with $Q_0^*(n)$ to determine which case can generate a higher social welfare.

We have the following proposition:

Proposition 3 *Under the same market structure with $n \geq 3$, an insider innovator sets a (weakly) higher royalty rate and generates a (weakly) lower social welfare than an outsider innovator.*

Proof Regarding the royalty rate, we know that, given the same n, the optimal $\delta_1^*(n-1)$ and $\delta_0^*(n)$ are:

$$\delta_1^*(n-1) = \begin{cases} \frac{(n-3)(a-c-\varepsilon)}{2(n^2-3n+3)}, & \text{if } \varepsilon > \frac{(n-3)(a-c)}{(n-1)(2n-3)}, \\ \varepsilon, & \text{if } \varepsilon \leq \frac{(n-3)(a-c)}{(n-1)(2n-3)}, \end{cases}$$

$$\delta_0^*(n) = \begin{cases} \frac{(n-1)(a-c)+(n+1)\varepsilon}{2(n^2-n+1)}, & \text{if } \varepsilon > \frac{a-c}{2n-1}, \\ \varepsilon, & \text{if } \varepsilon \leq \frac{a-c}{2n-1}. \end{cases}$$

When $n = 3$, $\delta_1^*(2) = 0$ while $\delta_0^*(3) > 0$. Hence, $\delta_1^*(2) - \delta_0^*(3) < 0$.

When $n \geq 4$, there are three cases to consider:

(1) When $\varepsilon \leq \frac{(n-3)(a-c)}{(n-1)(2n-3)}$, $\delta_1^*(n-1) = \delta_0^*(n) = \varepsilon$.

(2) When $\frac{(n-3)(a-c)}{(n-1)(2n-3)} < \varepsilon \leq \frac{a-c}{2n-1}$, $\delta_1^*(n-1) < \varepsilon = \delta_0^*(n)$.

(3) When $\varepsilon > \frac{a-c}{2n-1}$, $\delta_1^*(n-1) = \frac{(n-3)(a-c-\varepsilon)}{2(n^2-3n+3)}$ and $\delta_0^*(n) = \frac{(n-1)(a-c)+(n+1)\varepsilon}{2(n^2-n+1)}$. Then it can be immediately shown that

$$\delta_1^*(n-1) - \delta_0^*(n) = -\frac{n\left[a-c+(n^2-3n+2)\varepsilon\right]}{\left(n^2-n+1\right)\left(n^2-3n+3\right)} < 0.$$

Since $r_1^* \geq r_0^*$ if and only if $\delta_1^* \leq \delta_0^*$, this means that the insider innovator sets a (weakly) higher royalty rate than the outsider innovator.

Regarding the social welfare, we observe that $Q_1(n-1) < Q_0(n)$ if and only if

$$\varepsilon + (n-1)\delta_1^*(n-1) < n\delta_0^*(n). \tag{24}$$

When $n = 3$, $\delta_1^*(2) = 0$. On the other hand, when $\varepsilon \leq \frac{a-c}{2n-1} = \frac{a-c}{5}$, $\delta_0^*(3) = \varepsilon$. Clearly, the condition in (24) holds. When $\varepsilon > \frac{a-c}{5}$, $\delta_0^*(3) = \frac{(n-1)(a-c)+(n+1)\varepsilon}{2(n^2-n+1)}$. One can check that $\varepsilon + 2\delta_1^*(2) - 3\delta_0^*(3) = \frac{1}{7}(3c - 3a + \varepsilon) < 0$, and so (24) holds again. Thus, we have $Q_1(2) < Q_0(3)$.

When $n \geq 4$, there are three cases to consider:

(1) When $\varepsilon \leq \frac{(n-3)(a-c)}{(n-1)(2n-3)}$, $\delta_1^*(n-1) = \delta_0^*(n) = \varepsilon$. Then (24) holds as an equality. That is, $Q_1(n-1) = Q_0(n)$.

(2) When $\frac{(n-3)(a-c)}{(n-1)(2n-3)} < \varepsilon \leq \frac{a-c}{2n-1}$, $\delta_1^*(n-1) = \frac{(n-3)(a-c-\varepsilon)}{2(n^2-3n+3)}$ and $\delta_0^*(n) = \varepsilon$. Then $\varepsilon + (n-1)\delta_1^*(n-1) - n\delta_0^*(n) = -\frac{(n-1)[-(n-3)(a-c)+(2n^2-5n+3)\varepsilon]}{2(n^2-3n+3)} < 0$. Again, (24) holds.

(3) When $\varepsilon > \frac{a-c}{2n-1}$, $\delta_1^*(n-1) = \frac{(n-3)(a-c-\varepsilon)}{2(n^2-3n+3)}$ and $\delta_0^*(n) = \frac{(n-1)(a-c)+(n+1)\varepsilon}{2(n^2-n+1)}$. Then $\varepsilon + (n-1)\delta_1^*(n-1) - n\delta_0^*(n) = -\frac{(n-1)\left[(n^2-n+3)(a-c)+(n^2-5n+3)\varepsilon\right]}{2(n^2-n+1)(n^2-3n+3)}$. The numerator is always positive for any $n \geq 4$ given that $a - c > \varepsilon$. That is, the condition (24) holds in this case.

Therefore, we can conclude that $Q_1(n-1) \leq Q_0(n)$, which means that the social welfare induced by an outsider innovator is (weakly) higher than that induced by an insider innovator. □

The intuition for this result is the following. Since the insider innovator can produce by itself, it has an extra incentive to control the output and its Cournot profit. Thus, it prefers to set a higher royalty rate and thus a lower δ^* than an outsider innovator, in order to push the output level closer to the monopolistic quantity. This makes the total quantity lower, which further reduces the social welfare so that it is lower than that under the outsider innovator.

6 Conclusion

In the literature that deals with cost-reduction technology licensing with complete information, Sen and Tauman (2007) provide by far the most complete analysis in characterizing the optimal licensing contracts for both cases of the insider and outsider innovation in an oligopolistic industry. However, in our opinion, their auction approach is not consistent in determining the licensee's willingness to pay for the cases of $k < n$ and $k = n$ licensees. We therefore modify their analysis by adopting the upfront fixed fee system which can be consistently applied to all possible numbers of licensees, in which the firm's reservation payoff is determined by its Cournot profit if it rejects the contract. Differing from their argument that both contracts in which either $n-1$ or n downstream firms accept can be optimal, we find that it is optimal for both the insider and the outsider innovator to sell the license to *all* the downstream firms. Moreover, given the same market structure where there are at least three producing firms, an insider innovator sets a (weakly) higher royalty rate and generates a (weakly) lower social welfare than an outsider innovator.

Appendix

Proof of Lemma 1 We first note that:

$$q_N(k,\delta) = \frac{k[\tilde{\delta}_\lambda(k)-\delta]}{n+1+\lambda} = \begin{cases} \frac{a-c-\lambda\varepsilon-k\delta}{n+1+\lambda}, & \text{when } \delta < \tilde{\delta}_\lambda(k) \\ 0, & \text{when } \delta \geq \tilde{\delta}_\lambda(k). \end{cases}$$

$$q_L(k,\delta) = q_N(k,\delta)+\delta = \begin{cases} \frac{a-c-\lambda\varepsilon+(n+1+\lambda-k)\delta}{n+1+\lambda}, & \text{when } \delta < \tilde{\delta}_\lambda(k) \\ \frac{a-c-\lambda\varepsilon+(1+\lambda)\delta}{k+1+\lambda}, & \text{when } \delta \geq \tilde{\delta}_\lambda(k). \end{cases}$$

We also observe that, when $\delta < \tilde{\delta}_\lambda(k)$, $q_N(k,\delta)$ and $q_L(k,\delta)$ are decreasing in k.

By definition,

$$\begin{aligned} &w^{FR}(k+1,\delta) - w^{FR}(k,\delta) \\ &= [\Phi_L(k+1,\delta)-\Phi_N(k,\delta)] - [\Phi_L(k,\delta)-\Phi_N(k-1,\delta)] \\ &= [\Phi_L(k+1,\delta)-\Phi_L(k,\delta)] - [\Phi_N(k,\delta)-\Phi_N(k-1,\delta)]. \end{aligned} \tag{A.1}$$

To check if it is negative, there are two cases to consider:

<u>Case 1</u>. $\delta < \tilde{\delta}_\lambda(k)$:

In this case, the Cournot profits $\Phi_L(k+1,\delta)$, $\Phi_L(k,\delta)$, $\Phi_N(k,\delta)$ and $\Phi_N(k-1,\delta)$ are positive. Then:

$$\begin{aligned} \Phi_L(k+1,\delta)-\Phi_L(k,\delta) &= [q_L(k+1,\delta)]^2 - [q_L(k,\delta)]^2 \\ &= -\delta\left[\frac{2(a-c-\lambda\varepsilon)-(2k+1)\delta+2\delta(n+1+\lambda)}{(n+1+\lambda)^2}\right] < 0. \end{aligned} \tag{A.2}$$

$$\begin{aligned} \Phi_N(k,\delta)-\Phi_N(k-1,\delta) &= [q_N(k,\delta)]^2 - [q_N(k-1,\delta)]^2 \\ &= -\delta\left[\frac{2(a-c-\lambda\varepsilon)-(2k-1)\delta}{(n+1+\lambda)^2}\right] < 0. \end{aligned} \tag{A.3}$$

Hence, by (A.2) and (A.3), we have

$$w^{FR}(k+1,\delta) - w^{FR}(k,\delta) = -\frac{2(n+\lambda)\delta^2}{(n+1+\lambda)^2} < 0.$$

Case 2: $\delta \geq \tilde{\delta}_\lambda(k)$:

In this case, $\Phi_N(k,\delta) = 0$, while $\Phi_L(k+1,\delta)$, $\Phi_L(k,\delta)$ and $\Phi_N(k-1,\delta)$ are positive. Then $w^{FR}(k+1,\delta) - w^{FR}(k,\delta) = \Phi_L(k+1,\delta) - \Phi_L(k,\delta) + \Phi_N(k-1,\delta)$. Note that

$$\begin{aligned}
&\Phi_L(k+1,\delta) - \Phi_L(k,\delta) \\
&\quad = \left[\frac{a-c-\lambda\varepsilon+(1+\lambda)\delta}{k+2+\lambda}\right]^2 - \left[\frac{a-c-\lambda\varepsilon+(1+\lambda)\delta}{k+1+\lambda}\right]^2 \\
&\quad = -\frac{(2k+2\lambda+3)\,[a-c-\lambda\varepsilon+(1+\lambda)\delta]^2}{(k+1+\lambda)^2(k+2+\lambda)^2} \\
&\quad \leq -\frac{(2k+2\lambda+3)\,[a-c-\lambda\varepsilon+(1+\lambda)\tilde{\delta}_\lambda(k)]^2}{(k+1+\lambda)^2(k+2+\lambda)^2} \\
&\quad = -\frac{(2k+2\lambda+3)\,[a-c-\lambda\varepsilon+(1+\lambda)(\frac{a-c-\lambda\varepsilon}{k})]^2}{(k+1+\lambda)^2(k+2+\lambda)^2} \\
&\quad = -\frac{(2k+2\lambda+3)\,(a-c-\lambda\varepsilon)^2}{k^2(k+2+\lambda)^2},
\end{aligned} \tag{A.4}$$

and

$$\begin{aligned}
\Phi_N(k-1,\delta) &= \left[\frac{a-c-\lambda\varepsilon-(k-1)\delta}{n+1+\lambda}\right]^2 \leq \left[\frac{a-c-\lambda\varepsilon-(k-1)\tilde{\delta}_\lambda(k)}{n+1+\lambda}\right]^2 \\
&= \frac{(a-c-\lambda\varepsilon)^2}{k^2(n+1+\lambda)^2}.
\end{aligned} \tag{A.5}$$

The inequalities in (A.4) and (A.5) are obtained by using $\delta \geq \tilde{\delta}_\lambda(k)$ in this case. Hence, for any $1 \leq k \leq n-1$, we have

$$\begin{aligned}
&w^{FR}(k+1,\delta) - w^{FR}(k,\delta) \\
&\quad \leq \left[-\frac{(2k+2\lambda+3)}{k^2(k+2+\lambda)^2} + \frac{1}{k^2(n+1+\lambda)^2}\right](a-c-\lambda\varepsilon)^2. \\
&\quad \leq \left[-\frac{(2k+2\lambda+3)}{k^2(n+1+\lambda)^2} + \frac{1}{k^2(n+1+\lambda)^2}\right](a-c-\lambda\varepsilon)^2 \\
&\quad = -\frac{2\,(k+1+\lambda)}{k^2(n+1+\lambda)^2}(a-c-\lambda\varepsilon)^2 < 0,
\end{aligned} \tag{A.6}$$

The second inequality in (A.6) is obtained from knowing that $k+2+\lambda \leq n+1+\lambda$, or $k \leq n-1$. Therefore, we have shown that $w^{FR}(k+1,\delta) < w^{FR}(k,\delta)$. □

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