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Fault tracking sliding-mode controller design for fuzzy fractional-order system subject to actuator saturation

S. Senpagam[1] · P. Dhanalakshmi[1] · R. Mohanapriya[2]

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Abstract

This paper brings forth the fault tracking(estimation) problem for Fractional-Order Takagi-Sugeno Fuzzy(FOTSF) uncertain model subject to time-varying actuator fault and actuator saturation. In order to maintain the stability of considered system, fuzzy fault-tolerant sliding-mode controller is constructed based on fast adaptive fault estimation algorithm. Precisely, stability analysis is performed for the FOTSF model based on state and fault estimations by using the Lyapunov's stability theorem. More precisely, the sufficient constraints for stability of formulated model are built in proposed theorems. Eventually, two numerical exemplars including one chaotic model of Rossler are issued to support the proposed results and to demonstrate the efficacy of prescribed controller.

Keywords Fractional-Order dynamical model · Takagi-Sugeno model · Sliding-mode control · Fault tracking control

1 Introduction

Although the Fractional-Order(FO) calculus has a great history in the past, in recent years, scientists, researchers and engineers are extremely attracted by the field of FO dynamics due to its broad array of applications (see [1–7]). The significant difference between the FO dynamic model and the Integer Order(IO) dynamic model is that the IO derivative identifies the rate of change or specific properties of a machine process at a specific time, while the FO derivative identifies the same thing at all times. Moreover, only the regional properties of a certain position in a physical process can described by the IO derivatives, but FO derivatives can describe the properties of physical process in the whole space. Due to aforementioned aspects many practical and dynamical systems can be designed accurately by the FO derivatives, and these models are called by FO dynamical models. There are many fusion of FO with other dynamical models are presented in the research community, for example, FO neural networks with linear threshold neurons ([1,3]), FO fuzzy BAM model [5] and FO delayed fuzzy model [5]. Among them the fusion of FO model and Takagi-Sugeno Fuzzy(TSF) model is notable one. Because, the TSF modeling provides a well-build algorithm to design various classifications of complex nonlinear systems. In the algorithm of TSF construction, complex nonlinear systems can be effectively linearised via number of if...then-rules and membership functions (see [8]). Therefore in the past half decades, in research, there is a vast growth in the field of fuzzy dynamical models. However, these dynamical models may face some problems during the processing time, among them the instability and stabilization process are important issues to be noticeable [6,7]. In the process of stabilisation there are plenty of methods available, among them Lyapunov's stability method is one of an effective methods to deal the problem of stabilization [6]. More precisely, for TSF model, analysis of stability and control synthesis effects are discussed in [9–13]. Furthermore, the stability analysis and control synthesis for the combinations of fuzzy dynamical model with other models such as Markovian jump [14,15], neural network [16–18], neural network based stochastic systems [19] and FO system [20–24] are discussed. Among them, fuzzy FO models are to be noticeable due to its wide range of applications. Lan and Zhou(see [7]) were investigated the stability analysis under Lyapunov's method and designed a resilient robust controller for FO dynamical model. In [20–

✉ P. Dhanalakshmi
viga_dhanasekar@yahoo.co.in

1 Department of Applied Mathematics, Bharathiar University, Coimbatore 641046, India

2 Department of Mathematics, Sri Krishna Adithya college of Arts and Science, Affiliated to Bharathiar University, Coimbatore 641042, India

22], the authors discussed chaotic nature and stabilization problem for FO Rossler model, FO Lorenz model and FO permanent magnet synchronous motor model based on TSF model. Moreover, in [23] the authors discussed the synchronisation and Lyapunov's stability problem for uncertain FO model via TSF approach. Liu et. al. designed fuzzy back stepping controller for FO nonlinear system (see [24]). In very recent, the authors Pan et. al developed a new set of results for Lyapunov's stabilization of fuzzy systems under fixed-time fuzzy-controller [25] and the authors Liang et. al. designed the adaptive control based on event triggered mechanism for multi-agent systems [26].

In favour of improving productivity and performance of the system, engineering systems are undergo difficult operating conditions which drag the system to failure or fault. In order to handle this huddle situation, it is important to design a fault-tolerant control. Further, in past few years the fault diagnosis and estimation are became unavoidable problems due to the incremented claim for performance, reliability and protection in industrial actions. It is familiar that faults lay hold of many forms in a dynamic system, they are actuator faults, sensor faults, unforeseen instantaneous alters of some variable or even instantaneous formation changes. To ascertain where the fault and to surveil the system, fault detection and isolation are used. Then, to detect the vastness of the fault, fault tracking will be turn on (see [27,28]). As for the theme of fault tracking for continuous-time systems, advantageous upshot have been acquired throughout the past twenty. Fast adaptive fault estimation algorithm provided an effective fault tracking method through the fault estimator (see [29] and references therein) and the control which helps to achieve this tracking process is called fault tracking control. In [29] and [30] You et. al. discussed the problem of fault estimation for a time-varying delayed model and TSF model with sensor fault and the same problem was discussed for Markovian jump system in [31] by Liu et. al. Recently, a new adaptive fault tracking control is designed for discrete-time multi-agent system in [32]. N'Doye and Kirati made analysis on the problem of fault estimation for FO model with uncertainties in [33]. The robust fault tolerance control with estimation for FO model under quantization effect is analysed in [34].

Meanwhile, since that internal and external noises always occur in numerous experimental engineering models, it is necessary to design a vigorous controller for deal the aforementioned issues in practical applications. However, amid the various technique developed to control uncertain models, the Sliding-Mode-Control(SMC) has been greatly utilized. Recently, Wang et. al. [35] and Li et. al. [36] designed fuzzy SMC for continuous-time TSF model. In twenty o nine, an SMC for FO model is designed by Si-Ammour et. al.(see [37]). This is followed by in twenty eleven, Lin et. al. constructed the adaptive SMC for FO chaotic systems(see [38]). Moreover, very recently, Xu et. al. developed the FO fuzzy SMC for the real-time model thethered satellite system in [39].

Furthermore, in a large number of engineering models, actuator saturation causes spontaneously. This can triggered by dropping of performance and instability of control system which is disregarded in the design process (see [40] and references therein). In recent years, numerous researchers have made a research on the field of control system investigation and modelling with actuator saturation(see [41,42]). In favour of aforementioned introduction, the purpose of this study is inquired into robust stabilization problem of FOTSF model based on fault-estimation and actuator saturation via SMC. The central contributions of this study are in the following characteristics:

(1) The internal, external noise and time-varying fault against the FO dynamic model via the robust SMC is considered for first time in terms of TSF model under the actuator.
(2) A fresh Lyapunov function has been built and a class of Linear Matrix Inequality(LMI) is constructed to maintain the stability of the proposed FOTSF uncertain model with time-varying fault.
(3) The simulation studies let out the performance of proposed controller. It also reveals, the proposed controller does its best to achieve stability of recommended model, even in the presence of saturation and time-varying actuator fault.

Remark 1 As it remarks, SMC is recognized as a robust technique to handle the nonlinearities, uncertainties and faults in control systems. Particularly, the ideology of SMC is to stimulate the trajectories of selected system states onto some pre modeled sliding surfaces with the help of discontinuous controls. The sliding surfaces are designed as some stable ones, then the controlled system under the discontinuous controls can achieve desired performances instantly, such as tracking ability, nice transient behavior and disturbance/fault rejection capability. Hence, SMC has gained many theoretical and empirical researches, and more related applications have been accumulated.

Remark 2 Although a large number of fault-estimation based research articles have been published recently, the number of articles in the combination of FO system and fault-estimation based papers is still very low. The articles [29–32] discuss the problem of fault-estimation of IO dynamical system. Further, the articles [33,34] provide some results on fault-estimation for FO dynamical models. In the meantime, it is important to study this problem for the FOTSF model, because, as we have already discussed, many practical models are not linear and can be properly formatted using FO differential equations.

As well as, based on the Remark 1 the study of SMC with fault-estimation is meaningful one.

Notations:

$\mathfrak{D}^{\mathfrak{d}}$ denotes the Riemann derivative of the order $0 < \mathfrak{d} < 1$ with respect to the parameter of time t. $X(\mathfrak{h}) = \sum_{i=1}^{m} \mathfrak{h}_i(t) X_i$ for all matrices X_i. $sat(n)$ is saturation of the element n. $diag\{a, b, \ldots, c\}$ notates diagonal matrix with diagonal elements $\{a, b, \ldots, c\}$.

2 Problem formulation and preliminaries

This section explains the construction of addressed problem in presence of fault-tolerant SMC law with the aid of aforementioned fault-estimation algorithm. First, consider the FOTSF plant with IF-THEN rule in the following form:

RULE i: If $\mathfrak{g}_1(t)$ is μ_1^i; If $\mathfrak{g}_2(t)$ is μ_2^i; If $\mathfrak{g}_3(t)$ is μ_3^i; …If $\mathfrak{g}_l(t)$ is μ_l^i.

Then,

$$\mathfrak{D}^{\mathfrak{d}} x(t) = (\mathcal{A}_i + \Delta\mathcal{A}) x(t) + \mathfrak{B}_i u(t) + \mathcal{D}_i \varpi(t) + \mathcal{E}_i f(t),$$
$$y(t) = \mathfrak{C} x(t), \tag{1}$$

where $\mathfrak{g}(t) = [\mathfrak{g}_1^T(t)\ \mathfrak{g}_2^T(t)\ \ldots \mathfrak{g}_l^T(t)]^T$ is premises variable; μ_j^i are fuzzy sets, $\forall i \in \{1, 2, 3, \ldots, m\}$, $\forall j \in \{1, 2, 3, \ldots, l\}$; $x(t) \in \mathbb{R}^n$ represents state-space vector of the given system; $u(t) \in \mathbb{R}^k$ and $y(t) \in \mathbb{R}^h$ are given to indicate input and output vectors, respectively; $\varpi(t)$ and $f(t)$ are external disturbance and time-varying actuator fault, respectively; finally, $\mathcal{A}_i$, $\mathfrak{B}_i$, $\mathfrak{C}$, $\mathcal{D}_i$, $\mathcal{E}_i$ and $\Delta\mathcal{A} = M_i F(t) N_i$ are named for system parameter matrices of suitable dimension with unknown time-varying function satisfying $F^T(t) F(t) \leq I$.

Assumption 1 To reduce the complexity in calculation the parameters $\mathfrak{B}_i$'s and $\mathcal{E}_i$'s are assumed as follows: $\mathfrak{B} = \mathfrak{B}_1 = \mathfrak{B}_2 = \cdots = \mathfrak{B}_l = \mathcal{E}_1 = \mathcal{E}_2 = \cdots = \mathcal{E}_l$

Let the normalized membership function $\mathfrak{h}_i$ can be designed in the following form:

$$\mathfrak{h}_i(t) = \frac{\mathfrak{y}_i(\mathfrak{g}(t))}{\sum_{i=1}^{m} \mathfrak{y}_i(\mathfrak{g}(t))}, \tag{2}$$

where $\mathfrak{y}_i(t) = \Pi_{j=1}^{l} \mu_j^i(\mathfrak{g}_i(t))$ is inferred fuzzy set. Then the overall fuzzy set can be rewritten in the form of

$$\mathfrak{D}^{\mathfrak{d}} x(t) = (\mathcal{A}(\mathfrak{h}) + \Delta\mathcal{A}(\mathfrak{h})) x(t) + \mathfrak{B} u(t) + \mathcal{D}(\mathfrak{h}) \varpi(t) + \mathfrak{B} f(t),$$
$$y(t) = \mathfrak{C} x(t). \tag{3}$$

To estimate the given time-varying fault in (1), construct the overall fuzzy observer-model with estimated fault $\bar{f}(t)$ by following same steps in the construction of overall fuzzy state-model in the following form:

$$\mathfrak{D}^{\mathfrak{d}} \bar{x}(t) = (\mathcal{A}(\mathfrak{h}) + \Delta\mathcal{A}(\mathfrak{h})) \bar{x}(t) + \mathfrak{B} u(t) + \mathfrak{B} \bar{f}(t) + \mathfrak{L}(\mathfrak{h})[\mathfrak{C} x(t) - \bar{y}(t)],$$
$$\bar{y}(t) = \mathfrak{C} \bar{x}(t),$$
$$\mathfrak{D}^{\mathfrak{d}} \bar{f}(t) = \mathcal{R}^{-1} \mathcal{F} [\mathfrak{D}^{\mathfrak{d}} e_y(t) + e_y(t)]. \tag{4}$$

In the above consideration, $\mathfrak{L}_i$'s are observer gains; $\mathcal{R}^{-1}$ and $\mathcal{F}_i'$s are learning rate and fault unknowns, respectively; and $e_y(t) = y(t) - \bar{y}(t)$. In adjacent, by considering the actuator saturation effects in the construction of controller, the proportionate control-input can be formed as

$$u(t) = sat(u(t)) \tag{5}$$

and the saturation function is formed as $sat(u(t)) = [sat(u_1(t)), sat(u_1(t)), \ldots, sat(u_k(t))]^T$, here $sat(u_i(t)) = sgn(u_i(t)) min\{\hat{u}_i, |u_i(t)|\}$, $\forall i \in \{1, 2, \ldots, k\}$ and the known saturation level is named here as $\hat{u}_i$ of corresponding i^{th} actuator. Then, the control-input based on fuzzy dynamical model with the effect of actuator saturation under observer feedback is declared in the following form:

$$sat(u(t)) = sat\left(\mathcal{K}(\mathfrak{h}) \hat{x}(t)\right), \tag{6}$$

where $\mathcal{K}_i$'s are matrices of controller gains to be designed later. Consider the following lemma to handle the effect of actuator saturation.

Lemma 1 *[42] Let $\mathcal{W}$ be the set of diagonal matrices with the dimension $k \times k$ whose diagonal values are either 1 or zero. Let the elements of $\mathcal{W}$ be denoted by $\mathcal{W}_l^+$ and denote $\mathcal{W}_l^- = I - \mathcal{W}_l^+$, then $\mathcal{W}_l^- \in \mathcal{W}$. If $\mathcal{K}, \mathcal{H} \in \mathbb{R}^{k \times n}$, then for any $\bar{x}(t) \in L(\mathcal{H})$*

$$sat(\mathcal{K}\bar{x}) \in \text{convex hull} \left\{ \left(\mathcal{W}_l^+ \mathcal{K} + \mathcal{W}_l^- \mathcal{H}\right) \bar{x}(t),\ l = 1, 2, \ldots, 2^k \right\} \text{ or}$$

$$sat(\mathcal{K}\bar{x}(t)) = \sum_{l=1}^{2^k} \alpha_l \left(\mathcal{W}_l^+ \mathcal{K} + \mathcal{W}_l^- \mathcal{H}\right) \bar{x}(t),$$

$$\forall \alpha_l \in [0, 1] \text{ and } \sum_{l=1}^{2^k} \alpha_l = 1$$

Based on above Lemma and from (6), it can be obtained that

$$sat(u(t)) = \sum_{l=1}^{2^k} \alpha_l \left(\mathcal{W}_l^+ \mathcal{K}(\mathfrak{h}) + \mathcal{W}_l^- \mathcal{H}(\mathfrak{h})\right) \bar{x}(t) \tag{7}$$

where $\mathcal{K}_i$'s are matrices of controller feedback gains and $\mathcal{H}_i$ are controller auxiliary gains. For notational convenient, take $\sum_{l=1}^{2^k} \alpha_l \left(\mathcal{W}_l^+ \mathcal{K}(\mathfrak{h}) + \mathcal{W}_l^- \mathcal{H}(\mathfrak{h})\right)$ as $\bar{\mathcal{K}}(\mathfrak{h})$.

On the other hand the sliding mode controller can be derived by following steps (i) choosing the suitable sliding surface and (ii) constructing a controller law by taking the first order derivative for sliding mode surface and equating it to zero. First consider the sliding mode surface as in the following form:

$$s(t) = \mathcal{G}\mathcal{I}^{1-\mathfrak{d}}\bar{x}(t) + s_0, \tag{8}$$

where, $s_0 = -\mathcal{G}\bar{x}(0) - \mathcal{G}\int_0^t \left[\mathcal{A}(\mathfrak{h}) + \Delta\mathcal{A} + \mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})\right]\bar{x}(p)dp$. Then to design the sliding mode controller law as in [43] and solve $\dot{s}(t) = 0$ for controller. Consider,

$$\dot{s}(t) = \mathcal{G}\frac{d}{dt}\mathcal{I}^{1-\mathfrak{d}}\bar{x}(t) + \dot{s}_0(t) \tag{9}$$

$$= \mathcal{G}\mathfrak{D}^{\mathfrak{d}}\bar{x}(t) + \dot{s}_0(t). \tag{10}$$

Then based on $\dot{s}(t) = 0$ and (4) we have

$$u_e(t) = -(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G}\left[\mathcal{E}\bar{f}(t) + \mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t) - \mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})\bar{x}(t)\right], \tag{11}$$

where $e_x(t) = x(t) - \bar{x}(t)$. Then, by substituting (11) in (3), the proportionate sliding-mode dynamics of (4) can be expressed as follows:

$$\mathfrak{D}^{\mathfrak{d}}\bar{x}(t) = \left[(\mathcal{A}(\mathfrak{h}) + \Delta\mathcal{A}(\mathfrak{h})) + \mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})\right]\bar{x}(t) + (I - \mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})\mathfrak{L}(\mathfrak{h})\mathfrak{C}e_x(t) \tag{12}$$

Next it is necessary to construct the error system by using the definition of error state $e_x(t)$ as in the following form:

$$\mathfrak{D}^{\mathfrak{d}}e_x(t) = (\mathcal{A}(\mathfrak{h}) + \Delta\mathcal{A}(\mathfrak{h}))e_x(t) + \mathcal{D}(\mathfrak{h})\varpi(t) + \mathfrak{B}e_f(t) - \mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t), \tag{13}$$

here $e_f(t) = f(t) - \bar{f}(t)$.

To derive the required theoretical result, we consider the following assumption and lemma:

Definition 1 [43] For $\alpha > 0$, $\beta \in [0, 1]$, the error system of prescribed model FOTSF model (1) is said to have a mixed $\mathcal{H}_\infty$ and passivity asymptotically stable with zero initial condition if,

$$\int_0^t [-\alpha^{-1}\beta e_y^T(s)e_y(s) - 2(1-\beta)e_y^T(s)\varpi(s)]ds \geq -\int_0^t \alpha\varpi^T(s)\varpi(s)ds. \tag{14}$$

Assumption 2 $\mathfrak{D}^{\mathfrak{d}} f(t)$ is normed-bounded. (i.e) $||\mathfrak{D}^{\mathfrak{d}} f(t)|| \leq \mathfrak{F}$.

Assumption 3 $\mathfrak{B}$ is of full column rank.

Assumption 4 Invariant zeros of $(\mathcal{A}_i, \mathfrak{B}, \mathfrak{C})$ lie in open left plane $\forall i \in \{1, 2, \ldots, k\}$.

3 Main results

The central intention of this section is to guarantee the robust stability of contemplated FOTSF model (1) by resolving the gains $\mathcal{K}_i$, $\mathcal{H}_i$ and $\mathcal{L}_i$, such that the fuzzy SMC (11) have the skill to obtain the system states to the inception. For the purpose of do this, it is sufficient to prove error system (13) is robustly asymtotically stable with the considered mixed $\mathbf{H}_\infty$ and passivity performance index under resolved feedback controller parameters. More precisely three theorems are proven to show the asymptotic stability of the closed-loop of FOTSF model (1). In particular, initially, in the first theorem the controller and observer gain matrices are taken as known parameters; second theorem gives the result for unknown gains; finally, third theorem gives the SMC law and ensures the reachability of the state trajectories.

Theorem 1 *If the Assumptions 1–4 are hold and for the specified scalars α and β there exist a symmetry positive matrices $\mathcal{P}$, $\mathcal{Q}$ and $\mathcal{M}$ and positive scalar η such that the following LMIs hold*

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} \\ * & -\mathcal{Q} & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{Q} & 0 & 0 & 0 \\ * & * & * & -\eta & 0 & 0 \\ * & * & * & * & -\eta & 0 \\ * & * & * & * & * & -\alpha \end{bmatrix} < 0, \tag{15}$$

$$\mathcal{F}\mathfrak{C} = \mathfrak{B}^T\mathcal{Q}, \tag{16}$$

where

$$\phi_{11}=\begin{bmatrix}\phi_{11}^1 & 0 & 0 & 0\\ * & \mathcal{Q}\mathcal{A}(\mathfrak{h})+\mathcal{A}^T(\mathfrak{h})\mathcal{Q}-\mathcal{Q}\mathcal{L}(\mathfrak{h})\mathfrak{C}-\mathfrak{C}^T(\mathfrak{h})\mathcal{L}^T(\mathfrak{h})\mathcal{Q} & -\mathcal{A}^T(\mathfrak{h})\mathcal{Q}\mathfrak{B}+\mathfrak{C}^T\mathcal{L}^T(\mathfrak{h})\mathcal{Q}\mathfrak{B} & \mathcal{Q}\mathcal{D}(\mathfrak{h})-(1-\beta)\mathfrak{C}^T(\mathfrak{h})\\ * & * & -2\mathfrak{B}^T\mathcal{Q}\mathfrak{B}+\mathcal{M} & -\mathfrak{B}^T\mathcal{Q}\mathcal{D}(\mathfrak{h})\\ * & * & * & -\alpha I\end{bmatrix},$$

$\phi_{12}=\left[(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})^T\mathcal{P}\ 0\ 0\ 0\right]^T$, $\phi_{13}=[0\ \mathcal{Q}\mathfrak{L}(\mathfrak{h})\mathfrak{C}\ 0\ 0]^T$,

$\phi_{14}=\left[\eta M^T(\mathfrak{h})\mathcal{P}\ M^T(\mathfrak{h})\mathcal{Q}\ M^T(\mathfrak{h})\mathcal{Q}\mathfrak{B}\ 0\right]^T$ $\phi_{15}=\left[N(\mathfrak{h})\ \eta N(\mathfrak{h})\ 0\ 0\right]^T$,

$\phi_{16}=\left[0\ \sqrt{\beta}\mathfrak{C}\ 0\ 0\right]^T$, $\phi_{11}^1=\mathcal{P}\mathcal{A}(\mathfrak{h})+\mathcal{P}\mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})+\mathcal{A}^T(\mathfrak{h})\mathcal{P}+\bar{\mathcal{K}}^T(\mathfrak{h})\mathfrak{B}^T\mathcal{P}$.

Then the considered fuzzy model (1) and error systems are uniformly bounded if and only if the system is asymptotically stable.

Proof As in Lemma 1 from [34], the systems (12), (13) and error-fault can be re-assembled in the following form:

$$\begin{aligned}\frac{\partial Z_1(\Omega,t)}{\partial t}&=-\Omega Z_1(\Omega,t)\\&\quad+\left[(\mathcal{A}(\mathfrak{h})+\Delta\mathcal{A}(\mathfrak{h}))+\mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})\right]\bar{x}(t)\\&\quad+(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})\mathfrak{L}(\mathfrak{h})\mathfrak{C}e_x(t)\\ \bar{x}(t)&=\int_0^\infty\theta(\Omega)Z_1(\Omega,t)d\Omega\end{aligned}\tag{17}$$

$$\begin{aligned}\frac{\partial Z_2(\Omega,t)}{\partial t}&=-\Omega Z_2(\Omega,t)+(\mathcal{A}(\mathfrak{h})+\Delta\mathcal{A}(\mathfrak{h}))e_x(t)\\&\quad+\mathcal{D}(\mathfrak{h})\varpi(t)+\mathfrak{B}e_f(t)-\mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t)\\ e_x(t)&=\int_0^\infty\theta(\Omega)Z_2(\Omega,t)d\Omega\end{aligned}\tag{18}$$

$$\begin{aligned}\frac{\partial Z_3(\Omega,t)}{\partial t}&=-\Omega Z_3(\Omega,t)+\mathfrak{D}^{\mathfrak{d}}f(t)\\&\quad-\mathcal{R}^{-1}\mathcal{F}[\mathfrak{D}^{\mathfrak{d}}e_y(t)+e_y(t)]\\ e_f(t)&=\int_0^\infty\theta(\Omega)Z_3(\Omega,t)d\Omega\end{aligned}\tag{19}$$

To manifest the desire sequel of results in theorem statement, consider the following Lyapunov's functional candidate:

$$V(x,e_x,e_f,t)=\int_0^\infty\theta(\Omega)\mathcal{V}(Z_1,Z_2,Z_3,t)d\Omega\tag{20}$$

with the monochromatic Lyapunov's function $\mathcal{V}(Z_1,Z_2,Z_3,t)=Z_1^T(t)\mathcal{P}Z_1(t)+Z_2^T(t)\mathcal{Q}Z_2(t)+Z_3^T(t)\mathcal{R}Z_3(t)$ with corresponding elementary frequency Ω and the weighting function θ. To prove the Lyapunov's stability condition of given system, intially the time derivative of constructed Lyapunov's function (20) is taken as follows:

$$\begin{aligned}&\dot{V}(x,e_x,e_f,t)\\&\quad=\int_0^\infty\theta(\Omega)\dot{\mathcal{V}}(Z_1,Z_2,Z_3,t)d\Omega\\&\quad=2\int_0^\infty\theta(\Omega)\left[Z_1^T(t)\mathcal{P}\frac{\partial Z_1(\Omega,t)}{\partial t}\right.\\&\qquad\left.+Z_2^T(t)\mathcal{Q}\frac{\partial Z_2(\Omega,t)}{\partial t}+Z_3^T(t)\mathcal{R}\frac{\partial Z_3(\Omega,t)}{\partial t}\right]d\Omega\\&\quad=-2\int_0^\infty\theta(\Omega)Z_1^T(t)\mathcal{P}(\Omega Z_1(\Omega,t))d\Omega\\&\qquad-2\int_0^\infty\theta(\Omega)Z_2^T(t)\mathcal{Q}(\Omega Z_2(\Omega,t))d\Omega\\&\qquad-2\int_0^\infty\theta(\Omega)Z_3^T(t)\mathcal{R}(\Omega Z_3(\Omega,t))d\Omega\\&\qquad+2\int_0^\infty\theta(\Omega)Z_1(\Omega,t)d\Omega\mathcal{P}\{[(\mathcal{A}(\mathfrak{h})+\Delta\mathcal{A}(\mathfrak{h}))\\&\qquad+\mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})]\bar{x}(t)+(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})\mathfrak{L}(\mathfrak{h})\mathfrak{C}e_x(t)\}\\&\qquad+2\int_0^\infty\theta(\Omega)Z_2(\Omega,t)d\Omega\mathcal{Q}\{(\mathcal{A}(\mathfrak{h})\\&\qquad+\Delta\mathcal{A}(\mathfrak{h}))e_x(t)+\mathcal{D}(\mathfrak{h})\varpi(t)\\&\qquad+\mathfrak{B}e_f(t)-\mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t)\}\\&\qquad+2\int_0^\infty\theta(\Omega)Z_3(\Omega,t)d\Omega\mathcal{R}\{\mathfrak{D}^{\mathfrak{d}}f(t)\\&\qquad-\mathcal{R}^{-1}\mathcal{F}[\mathfrak{D}^{\mathfrak{d}}e_y(t)+e_y(t)]\}\\&\quad=-2\int_0^\infty\theta(\Omega)Z_1^T(t)\mathcal{P}(\Omega Z_1(\Omega,t))d\Omega\\&\qquad-2\int_0^\infty\theta(\Omega)Z_2^T(t)\mathcal{Q}(\Omega Z_2(\Omega,t))d\Omega\\&\qquad-2\int_0^\infty\theta(\Omega)Z_3^T(t)\mathcal{R}(\Omega Z_3(\Omega,t))d\Omega\\&\qquad+2\bar{x}^T(t)\mathcal{P}\{[(\mathcal{A}(\mathfrak{h})+\Delta\mathcal{A}(\mathfrak{h}))+\mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})]\bar{x}(t)\\&\qquad+(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})\mathfrak{L}(\mathfrak{h})\mathfrak{C}e_x(t)\}\\&\qquad+2e_x^T(t)\mathcal{Q}\{(\mathcal{A}(\mathfrak{h})+\Delta\mathcal{A}(\mathfrak{h}))e_x(t)+\mathcal{D}(\mathfrak{h})\varpi(t)\\&\qquad+\mathfrak{B}e_f(t)-\mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t)\}+2e_f^T(t)\mathcal{R}\{\mathfrak{D}^{\mathfrak{d}}f(t)\\&\qquad-\mathcal{R}^{-1}\mathcal{F}[\mathfrak{D}^{\mathfrak{d}}e_y(t)+e_y(t)]\}.\end{aligned}\tag{21}$$

As reported in Lyapunov's stability theory it is enough to prove that the time derivative of the energy equation (Lyapunov's candidates) is less than zero instead of proving the stability of a system. Since the first three terms of above Eq. (21) are negative terms, it remains to prove that the other terms of the Eq. (21) is less than zero (ie.)

$$\begin{aligned}&2\bar{x}^T(t)\mathcal{P}\{[(\mathcal{A}(\mathfrak{h})+\Delta\mathcal{A}(\mathfrak{h}))\\&\quad+\mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})]\bar{x}(t)+(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})\mathfrak{L}(\mathfrak{h})\mathfrak{C}e_x(t)\}\\&\quad+2e_x^T(t)\mathcal{Q}\{(\mathcal{A}(\mathfrak{h})\\&\quad+\Delta\mathcal{A}(\mathfrak{h}))e_x(t)+\mathcal{D}(\mathfrak{h})\varpi(t)\\&\quad+\mathfrak{B}e_f(t)-\mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t)\}\\&\quad+2e_f^T(t)\mathcal{R}\{\mathfrak{D}^{\mathfrak{d}}f(t)-\mathcal{R}^{-1}\mathcal{F}[\mathfrak{D}^{\mathfrak{d}}e_y(t)\\&\quad+e_y(t)]\}<0.\end{aligned}\tag{22}$$

Let $\tilde{\mathbf{T}}$ be left hand side of the above equation, then from the Eq. (13)

$$\begin{aligned}\tilde{\mathbf{T}}&=2\bar{x}^T(t)\mathcal{P}[(\mathcal{A}(\mathfrak{h})+\Delta\mathcal{A}(\mathfrak{h}))\\&\quad+\mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})]\bar{x}(t)\\&\quad+2\bar{x}^T(t)\mathcal{P}(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})\mathfrak{L}(\mathfrak{h})\mathfrak{C}e_x(t)\\&\quad+2e_x^T(t)\mathcal{Q}(\mathcal{A}(\mathfrak{h})+\Delta\mathcal{A}(\mathfrak{h}))e_x(t)\\&\quad+2e_x^T(t)\mathcal{Q}\mathcal{D}(\mathfrak{h})\varpi(t)\\&\quad+2e_x^T(t)\mathcal{Q}\mathfrak{B}e_f(t)-2e_x^T(t)\mathcal{Q}\\&\quad\times\mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t)\\&\quad+2e_f^T(t)\mathcal{R}\mathfrak{D}^{\mathfrak{d}}f(t)\\&\quad-2e_f^T(t)\mathcal{F}\mathfrak{C}[\mathfrak{D}^{\mathfrak{d}}e(t)+e(t)]\\&=2\bar{x}^T(t)\mathcal{P}[(\mathcal{A}(\mathfrak{h})+\Delta\mathcal{A}(\mathfrak{h}))\\&\quad+\mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})]\bar{x}(t)\\&\quad+2\bar{x}^T(t)\mathcal{P}(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})\mathfrak{L}(\mathfrak{h})\mathfrak{C}e_x(t)\\&\quad+2e_x^T(t)\mathcal{Q}(\mathcal{A}(\mathfrak{h})\\&\quad+\Delta\mathcal{A}(\mathfrak{h}))e_x(t)+2e_x^T(t)\mathcal{Q}\mathcal{D}(\mathfrak{h})\varpi(t)\\&\quad+2e_x^T(t)\mathcal{Q}\mathfrak{B}e_f(t)-2e_x^T(t)\mathcal{Q}\\&\quad\times\mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t)\\&\quad+2e_f^T(t)\mathcal{R}\mathfrak{D}^{\mathfrak{d}}f(t)\\&\quad-2e_f^T(t)\mathcal{F}\mathfrak{C}(\mathcal{A}(\mathfrak{h})\\&\quad+\Delta\mathcal{A}(\mathfrak{h}))e_x(t)\\&\quad-2e_f^T(t)\mathcal{F}\mathfrak{C}\mathcal{D}(\mathfrak{h})\varpi(t)\\&\quad-2e_f^T(t)\mathcal{F}\mathfrak{C}\mathfrak{B}e_f(t)\\&\quad+2e_f^T(t)\mathcal{F}\mathfrak{C}\mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t)\\&\quad-2e_f^T(t)\mathcal{F}\mathfrak{C}e(t).\end{aligned}\tag{23}$$

By substituting the Eq. (16) in (23), it is obtained that

$$\begin{aligned}\tilde{\mathbf{T}}&=2\bar{x}^T(t)\mathcal{P}[(\mathcal{A}(\mathfrak{h})+\Delta\mathcal{A}(\mathfrak{h}))\\&\quad+\mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})]\bar{x}(t)\\&\quad+2\bar{x}^T(t)\mathcal{P}(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})\mathfrak{L}(\mathfrak{h})\mathfrak{C}e_x(t)\\&\quad+2e_x^T(t)\mathcal{Q}(\mathcal{A}(\mathfrak{h})+\Delta\mathcal{A}(\mathfrak{h}))e_x(t)\\&\quad+2e_x^T(t)\mathcal{Q}\mathcal{D}(\mathfrak{h})\varpi(t)\\&\quad-2e_x^T(t)\mathcal{Q}\mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t)\\&\quad+2e_f^T(t)\mathcal{R}\mathfrak{D}^{\mathfrak{d}}f(t)-2e_f^T(t)\mathfrak{B}^T\mathcal{Q}(\mathcal{A}(\mathfrak{h})\\&\quad+\Delta\mathcal{A}(\mathfrak{h}))e_x(t)-2e_f^T(t)\mathfrak{B}^T\mathcal{Q}\mathcal{D}(\mathfrak{h})\varpi(t)\\&\quad-2e_f^T(t)\mathfrak{B}^T\mathcal{Q}\mathfrak{B}e_f(t)\\&\quad+2e_f^T(t)\mathfrak{B}^T\mathcal{Q}\mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t).\end{aligned}\tag{24}$$

The considered term in (24) as in the following has been taken as sum of two terms in the following form,

$$\begin{aligned}&2\bar{x}^T(t)\mathcal{P}(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})\mathfrak{L}(\mathfrak{h})\mathfrak{C}e_x(t)\\&\quad\le\bar{x}^T(t)\mathcal{P}(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})\mathcal{Q}^{-1}(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})T\mathcal{P}\bar{x}(t)\\&\quad\;+e_x^T(t)\mathfrak{C}^T(\mathfrak{h})\mathfrak{L}^T(\mathfrak{h})\mathcal{Q}\mathfrak{L}(\mathfrak{h})\mathfrak{C}e_x(t).\end{aligned}\tag{25}$$

Similarly from Assumption 2, there exist a positive definite matrix $\mathcal{M}$ for the term $2e_f^T(t)\mathcal{R}\mathfrak{D}^{\mathfrak{d}}f(t)$ in (24) such that,

$$\begin{aligned}&2e_f^T(t)\mathcal{R}\mathfrak{D}^{\mathfrak{d}}f(t)\\&\quad\le e_f^T(t)\mathcal{M}e_f(t)+\mathfrak{D}^{\mathfrak{d}}f^T(t)\mathcal{R}\mathcal{M}^{-1}\mathcal{R}\mathfrak{D}^{\mathfrak{d}}f(t),\\&\quad\le e_f^T(t)\mathcal{M}e_f(t)+||\mathfrak{D}^{\mathfrak{d}}f(t)||\lambda_{\mathcal{R}\mathcal{M}^{-1}\mathcal{R}},\\&\quad\le e_f^T(t)\mathcal{M}e_f(t)+\mathfrak{F}\lambda_{\mathcal{R}\mathcal{M}^{-1}\mathcal{R}},\end{aligned}\tag{26}$$

where $\lambda_{\mathcal{R}\mathcal{M}^{-1}\mathcal{R}}$ is maximum eigen value of $\mathcal{R}\mathcal{M}^{-1}\mathcal{R}$. Finally from the Lemma 3 in [34], the terms including uncertainty $\Delta\mathcal{A}(\mathfrak{h})$ can be rewritten as in the following forms:

$$\begin{aligned}2\bar{x}^T(t)\mathcal{P}\Delta\mathcal{A}(\mathfrak{h})\bar{x}(t)&=2\bar{x}^T(t)\mathcal{P}M(\mathfrak{h})F(t)N(\mathfrak{h})\bar{x}(t)\\&\le\bar{x}^T(t)\mathcal{P}M(\mathfrak{h})\eta M^T(\mathfrak{h})\mathcal{P}\bar{x}(t)\\&\quad+\bar{x}^T(t)N^T(\mathfrak{h})\eta^{-1}N(\mathfrak{h})\bar{x}(t),\end{aligned}\tag{27}$$

$$\begin{aligned}2e_x^T(t)\mathcal{Q}\Delta\mathcal{A}(\mathfrak{h})e_x(t)&=2e_x^T(t)\mathcal{Q}M(\mathfrak{h})F(t)N(\mathfrak{h})e_x(t)\\&\le e_x^T(t)\mathcal{Q}M(\mathfrak{h})\eta^{-1}M^T(\mathfrak{h})\mathcal{Q}e_x(t)\\&\quad+e_x^T(t)N^T(\mathfrak{h})\eta N(\mathfrak{h})e_x(t),\end{aligned}\tag{28}$$

$$\begin{aligned}-2e_f^T(t)\mathfrak{B}^T\mathcal{Q}\Delta\mathcal{A}(\mathfrak{h})e_x(t)&=-2e_f^T(t)\mathfrak{B}^T\mathcal{Q}M(\mathfrak{h})F(t)N(\mathfrak{h})e_x(t)\\&\le e_f^T(t)\mathfrak{B}^T\mathcal{Q}M(\mathfrak{h})\eta^{-1}M^T(\mathfrak{h})\mathcal{Q}\mathfrak{B}e_f^T(t)\\&\quad+e_x^T(t)N^T(\mathfrak{h})\eta N(\mathfrak{h})e_x(t).\end{aligned}\tag{29}$$

Then from (21)–(29) $\tilde{\mathbf{T}}$ becomes,

$$\begin{aligned}\dot{V}(x,e_x,e_f,t)\le\tilde{\mathbf{T}}&\le\zeta^T(t)\phi_1\zeta(t)\\&\quad+\Theta^T(t)\phi_2\Theta(t)+\mathfrak{F}\lambda_{\mathcal{R}\mathcal{M}^{-1}\mathcal{R}}.\end{aligned}\tag{30}$$

In the above equation

$$\zeta^T = \left[\Theta^T(t)\ \varpi^T(t)\right], \Theta^T(t) = \left[\bar{x}^T(t)\ e_x^T(t)\ e_f^T(t)\right] \text{ and}$$

$$\phi_1 = \begin{bmatrix} \mathcal{P}\mathcal{A}(\mathfrak{h})+\mathcal{P}\mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})+\mathcal{A}^T(\mathfrak{h})\mathcal{P}+\bar{\mathcal{K}}^T(\mathfrak{h})\mathfrak{B}^T\mathcal{P} & 0 & 0 & 0 \\ * & \mathcal{Q}\mathcal{A}(\mathfrak{h})+\mathcal{A}^T(\mathfrak{h})\mathcal{Q}-\mathcal{Q}\mathcal{L}(\mathfrak{h})\mathfrak{C}-\mathfrak{C}^T(\mathfrak{h})\mathcal{L}^T(\mathfrak{h})\mathcal{Q} & -\mathcal{A}^T(\mathfrak{h})\mathcal{Q}\mathfrak{B}+\mathfrak{C}^T\mathcal{L}^T(\mathfrak{h})\mathcal{Q}\mathfrak{B} & \mathcal{Q}\mathcal{D}(\mathfrak{h}) \\ * & * & -2\mathfrak{B}^T\mathcal{Q}\mathfrak{B}+\mathcal{M} & -\mathfrak{B}^T\mathcal{Q}\mathcal{D}(\mathfrak{h})\varpi(t) \\ * & * & * & 0 \end{bmatrix},$$

$$\phi_2 = \begin{bmatrix} \phi_2^{11} & 0 & 0 \\ * & \mathfrak{C}^T(\mathfrak{h})\mathfrak{L}^T(\mathfrak{h})\mathcal{Q}\mathfrak{L}(\mathfrak{h})\mathfrak{C}+\mathcal{Q}M(\mathfrak{h})\eta^{-1}M^T(\mathfrak{h})\mathcal{Q}+N^T(\mathfrak{h})\eta N(\mathfrak{h}) & 0 \\ * & * & \mathfrak{B}^T\mathcal{Q}M(\mathfrak{h})\eta^{-1}M^T(\mathfrak{h})\mathcal{Q}\mathfrak{B} \end{bmatrix},$$

$$\phi_2^{11} = \mathcal{P}(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})\mathcal{Q}^{-1}(I-\mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})^T\mathcal{P}+\mathcal{P}M(\mathfrak{h})\eta M^T(\mathfrak{h})\mathcal{P}+N^T(\mathfrak{h})\eta^{-1}N(\mathfrak{h}).$$

Then apply Schur complement Lemma for $\Theta^T(t)\phi_2\Theta(t)$, by taking $\varpi(t) = 0$ and the parameters of noise terms as zero in ϕ. From given LMI (15), it is obvious that $\dot{V}(x, e_x, e_f, t) \leq \tilde{\mathbf{T}} < 0$ when $\lambda_\phi||\theta(t)|| \geq \mathfrak{F}^2\lambda_{\mathcal{R}\mathcal{M}^{-1}\mathcal{R}}$ where $-\lambda_\phi$ is minimum eigen value of ϕ. Since Θ is uniformly bounded, $\lambda_\phi||\theta(t)|| \geq \mathfrak{F}^2\lambda_{\mathcal{R}\mathcal{M}^{-1}\mathcal{R}}$ is satisfied for some bound $\frac{\mathfrak{F}^2\lambda_{\mathcal{R}\mathcal{M}^{-1}\mathcal{R}}}{\lambda_\phi}$. So, by the Lyapunov's stability theorem the given system is asymptotically stable.

On the other case $\varpi(t) \neq 0$ it is necessary to reject or dominate them by using some performance. To prove the necessary condition of mixed $\mathbf{H}_\infty$ and passivity performance, the following consideration is taken

$$\begin{aligned} &\dot{V}(x, e_x, e_f, t) - \alpha\varpi^T(t)\varpi(t) \\ &\quad - 2(1-\beta)e_y^T(t)\varpi(t) + \alpha^{-1}\beta e_y^T(t)e_y(t) \\ &\leq \tilde{\mathbf{T}} - \alpha\varpi^T(t)\varpi(t) - 2(1-\beta)e_y^T(t)\varpi(t) \\ &\quad + \alpha^{-1}\beta e_y^T(t)e_y(t). \end{aligned} \tag{31}$$

Then again applying Schur complement Lemma for the term $\alpha^{-1}\beta y^T(t)y(t)$, the LMI (15) is obtained. Thus, the condition (15) hold and $\dot{V}(x, e_x, e_f, t) < 0$, then

$$\begin{aligned} &-\alpha\varpi^T(t)\varpi(t) - 2(1-\beta)e_y^T(t)\varpi(t) \\ &\quad + \alpha^{-1}\beta e_y^T(t)e_y(t) < 0. \end{aligned} \tag{32}$$

Then by the definition of mixed $\mathbf{H}_\infty$ and passivity performance, the given system is asymptotically stable with the mixed $\mathbf{H}_\infty$ and passivity performance. □

Remark 3 Optimisation problem is one of the easiest way to solve the Eq. (16). In this technique, an equation is moderated as an inequality in the following LMI formate:

$$\begin{bmatrix} -\rho I & \mathcal{F}\mathfrak{C} - \mathfrak{B}^T\mathcal{Q} \\ * & -\rho I \end{bmatrix} < 0. \tag{33}$$

Theorem 2 *Suppose the Assumptions 1–4 hold and for the specified scalars α and β, there exist matrices $\mathfrak{V}_{11i}$, $\mathfrak{V}_{12i}$ and $\mathfrak{V}_{2i}$, symmetry positive matrices $\bar{\mathcal{P}}$, $\mathcal{Q}$, and $\mathcal{M}$ and positive scalar η such that the following LMIs hold*

$$\begin{aligned} &\Phi_{ii} < 0, \quad \forall i \in \{1, 2, \ldots, m\} \\ &\Phi_{ij} + \Phi_{ji} < 0, \quad \forall i < j, \text{ and } i, j \in \{1, 2, \ldots, m\}, \end{aligned} \tag{34}$$

$$\begin{bmatrix} -\rho I & \mathcal{F}\mathfrak{C} - \mathfrak{B}^T\mathcal{Q} \\ * & -\rho I \end{bmatrix} < 0, \tag{35}$$

where

$$\Phi_{ij} = \begin{bmatrix} \Phi_{ij}^1 & \Phi_{ij}^2 & \Phi_{ij}^3 & \Phi_{ij}^4 & \Phi_{ij}^5 & \Phi_{ij}^6 \\ * & -\mathcal{Q} & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{Q} & 0 & 0 & 0 \\ * & * & * & -\eta I & 0 & 0 \\ * & * & * & * & -\eta I & 0 \\ * & * & * & * & * & -\alpha I \end{bmatrix},$$

$$\phi_{ij}{}^1 = \begin{bmatrix} \mathcal{A}_i\bar{\mathcal{P}} + \mathfrak{B}\mathfrak{V}_{1j} + \bar{\mathcal{P}}\mathcal{A}_i^T + \mathfrak{V}_{1j}\mathfrak{B}^T & 0 & 0 & 0 \\ * & \mathcal{Q}\mathcal{A}_i + \mathcal{A}_i^T\mathcal{Q} - \mathfrak{V}_{2i}\mathfrak{C} - \mathfrak{C}_j^T\mathfrak{V}_{2i}^T & -\mathcal{A}_i^T\mathcal{Q}\mathfrak{B} + \mathfrak{C}^T\mathfrak{V}_{2i}^T\mathfrak{B} & \mathcal{Q}\mathcal{D}_i - (1-\beta)\mathfrak{C}_i^T \\ * & * & -2\mathfrak{B}^T\mathcal{Q}\mathfrak{B} + \mathcal{M} & -\mathfrak{B}^T\mathcal{Q}\mathcal{D}_i \\ * & * & * & -\alpha I \end{bmatrix},$$

$$\Phi_{ij}^2 = \left[(I - \mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})^T \ 0\ 0\ 0\right]^T,$$
$$\Phi_{ij}^3 = \left[0\ \mathfrak{V}_i\mathfrak{C}\ 0\ 0\right]^T,$$
$$\Phi_{ij}^4 = \left[\eta M_i^T \ M_i^T\mathcal{Q}\ M_i^T\mathcal{Q}\mathfrak{B}\ 0\right]^T,$$
$$\Phi_{ij}^5 = \left[N_i\bar{\mathcal{P}}\ \eta N_i\ 0\ 0\right]^T,$$
$$\Phi_{ij}^6 = \left[0\ \sqrt{\beta}\mathfrak{C}\ 0\ 0\right]^T,$$
$$\mathfrak{V}_{1i} = \sum_{l=1}^{2^k}\alpha_l\left(\mathcal{W}_l^+\mathfrak{V}_{11i} + \mathcal{W}_l^-\mathfrak{V}_{12i}\right).$$

Then the FOTSF model (1) and error systems are uniformly bounded if and only if the system is asymptotically stable. Further, $\mathcal{K}_i = \mathfrak{V}_{11i}\bar{\mathcal{P}}^{-1}$, $\mathcal{H}_i = \mathfrak{V}_{12i}\bar{\mathcal{P}}^{-1}$ *and* $\mathcal{L}_i = \mathcal{Q}^{-1}\mathfrak{V}_{2i}$.

Proof The proof of this theorem is as comes behind from the proof of Theorem 1. For the same Lyapunov's functional candidates which leads to the same LMI as given in Theorem 1. Then take the pre- and post- multiplication by $\mathfrak{S} = diag\{\mathcal{S}, I, I, I, I\}$ (where $\mathcal{S} = diag\{\mathcal{P}^{-1}, I, I, I, I\}$) for the matrix given in (15), it is able to acquired the following:

$$\mathfrak{S}\phi\mathfrak{S} = \begin{bmatrix} \mathcal{S}\phi_{11}\mathcal{S} & \mathcal{S}\phi_{12} & \mathcal{S}\phi_{13} & \mathcal{S}\phi_{14} & \mathcal{S}\phi_{15} & \mathcal{S}\phi_{16} \\ * & -\mathcal{Q} & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{Q} & 0 & 0 & 0 \\ * & * & * & -\eta I & 0 & 0 \\ * & * & * & * & -\eta I & 0 \\ * & * & * & * & * & -\alpha I \end{bmatrix}, \quad (36)$$

where

$$\mathcal{S}\phi_{11}\mathcal{S} = \begin{bmatrix} \bar{\mathcal{P}}\phi_{11}^1\bar{\mathcal{P}} & 0 & 0 & 0 \\ * & \mathcal{Q}\mathcal{A}(\mathfrak{h}) + \mathcal{A}^T(\mathfrak{h})\mathcal{Q} - \mathcal{Q}\mathcal{L}(\mathfrak{h})\mathfrak{C} - \mathfrak{C}^T(\mathfrak{h})\mathcal{L}^T(\mathfrak{h})\mathcal{Q} & -\mathcal{A}^T(\mathfrak{h})\mathcal{Q}\mathfrak{B} + \mathfrak{C}^T\mathcal{L}^T(\mathfrak{h})\mathcal{Q}\mathfrak{B} & \mathcal{Q}\mathcal{D}(\mathfrak{h}) - (1-\beta)\mathfrak{C}^T(\mathfrak{h}) \\ * & * & -2\mathfrak{B}^T\mathcal{Q}\mathfrak{B} + \mathcal{M} & -\mathfrak{B}^T\mathcal{Q}\mathcal{D}(\mathfrak{h}) \\ * & * & * & -\alpha I \end{bmatrix},$$
$$\mathcal{S}\phi_{12} = \left[(I - \mathfrak{B}(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G})^T \ 0\ 0\ 0\right]^T,$$
$$\mathcal{S}\phi_{13} = \left[0\ \mathcal{Q}\mathfrak{L}(\mathfrak{h})\mathfrak{C}\ 0\ 0\right]^T,$$
$$\mathcal{S}\phi_{14} = \left[\eta M^T(\mathfrak{h})\ M^T(\mathfrak{h})\mathcal{Q}\ M^T(\mathfrak{h})\mathcal{Q}\mathfrak{B}\ 0\right]^T,$$
$$\mathcal{S}\phi_{15} = \left[N(\mathfrak{h})\bar{\mathcal{P}}\ \eta N(\mathfrak{h})\ 0\ 0\right]^T,$$
$$\phi_{16} = \left[0\ \sqrt{\beta}\mathfrak{C}\ 0\ 0\right]^T,$$
$$\bar{\mathcal{P}}\phi_{11}^1\bar{\mathcal{P}} = \mathcal{A}(\mathfrak{h})\bar{\mathcal{P}} + \mathfrak{B}\bar{\mathcal{K}}(\mathfrak{h})\bar{\mathcal{P}} + \bar{\mathcal{P}}\mathcal{A}^T(\mathfrak{h}) + \bar{\mathcal{P}}\bar{\mathcal{K}}^T(\mathfrak{h})\mathfrak{B}^T$$

and $\bar{\mathcal{P}} = \mathcal{P}^{-1}$. Rest of the proof can be reached by substituting $\bar{\mathcal{K}}(\mathfrak{h}) = \sum_{l=1}^{2^k}\alpha_l\left(\mathcal{W}_l^+\mathcal{K}(\mathfrak{h}) + \mathcal{W}_l^-\mathcal{H}(\mathfrak{h})\right)$, $\mathfrak{V}_{11}(\mathfrak{h}) = \mathcal{K}(\mathfrak{h})\bar{\mathcal{P}}$, $\mathfrak{V}_{12}(\mathfrak{h}) = \mathcal{H}(\mathfrak{h})\bar{\mathcal{P}}$, $\mathfrak{V}_2(\mathfrak{h}) = \mathcal{Q}\mathcal{L}(\mathfrak{h})$ and $X(\mathfrak{h}) = \sum_{i=1}^m \mathfrak{h}_i(t)X_i$ for all X. Then it leads to the matrix $\Phi = \sum_{i=1}^m\sum_{j=1}^m \mathfrak{h}_i(t)\mathfrak{h}_j(t)\Phi_{ij}$. Thus, the conditions (34),(35) and Remark 3 hold, the proof is obvious as similar as in Theorem 1. □

Theorem 3 *For the considered FOTSF model (1), make the assumption that the LMI conditions in Theorems 1 and 2 are hold. Then, sliding surface assumed in (8) where $\mathcal{G}$ is selected such that $|\mathcal{G}\mathfrak{B}| \neq 0$, the SMC condition can be satisfied control law in following form:*

$$u(t) = \sum_{i=1}^{k}\sum_{l=1}^{2^k} \mathfrak{h}_i(t)\left[\alpha_l\left(\mathcal{W}_l^{+}\mathcal{K}_i + \mathcal{W}_l^{-}\mathcal{H}_i\right)\bar{x}(t) - \bar{f}(t) - \mathfrak{p}_i(t)sgn(s(t))\right], \quad (37)$$

where $\mathfrak{p}_i(t) = \epsilon + ||(\mathcal{G}\mathcal{B})^{-1}\mathcal{G}||||\mathcal{L}(\mathfrak{h})\mathfrak{C}e(t)||$.

Proof Let the sliding surface parameter has chosen as $\mathcal{G} = \mathfrak{B}^T\mathcal{J}$ for some positive symmetric matrix $\mathcal{J}$. Then, it is trivial that $\mathcal{G}\mathfrak{B} = \mathfrak{B}^T\mathcal{J}\mathfrak{B}$ is non singular. To prove the reachability of SMC the following Lyapunov's candidate is considered:

$$\tilde{\mathcal{V}}(t) = \frac{1}{2}s^T(t)\left(\mathcal{G}\mathfrak{B}\right)^{-1}s(t). \quad (38)$$

By taking time derivative for $\tilde{\mathcal{V}}(t)$, it is easily obtained that

$$\dot{\tilde{\mathcal{V}}} = s(t)\left(\mathcal{G}\mathfrak{B}\right)^{-1}\dot{s}(t). \quad (39)$$

Then from the Eqs. (4), (10) and (37) it can be rewritten as follows,

$$\begin{aligned}\dot{\tilde{\mathcal{V}}} &= s^T(t)(\mathcal{G}\mathfrak{B})^{-1}\mathcal{G}\,[\mathfrak{B}u(t) + \mathcal{L}(\mathfrak{h})\mathfrak{C}e_x(t) - \mathfrak{B}\bar{\mathcal{K}}\bar{x}(t)] \\ &\leq \|s(t)\|\left\|(\mathcal{G}\mathcal{B})^{-1}\mathcal{G}\right\|\|\mathcal{L}(\mathfrak{h})\mathfrak{C}e(t)\| + s^T(t)\left[u(t) - \bar{\mathcal{K}}\bar{x}(t)\right] \\ &\leq -\epsilon\|s(t)\|, \quad \forall \|s(t)\| \neq 0 \end{aligned} \quad (40)$$

since the system state trajectories of (3) converge to the sliding surface which is already defined in (8) in a finite time duration for all subsequent time. This completes the proof. □

4 Numerical examples

This section contains an artificial numerical example and a simulated example of Rossler FO model to show the efficacy and reachability of suggested method in preceding sections.

Example 1 Consider the FOTSF model (3) of the order $\mathfrak{d} = 0.9$ with the controller law (37) and with the following parameters:

$$\mathcal{A}_1 = \begin{bmatrix} -3 & 2 & 2 \\ 1 & -2 & 0 \\ 1 & 2 & -5 \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} -5 & 1 & 3 \\ 0 & -1 & 0 \\ -1 & 3 & -4 \end{bmatrix}, \quad \mathfrak{B} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

$$\mathfrak{C} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \quad \mathcal{D}_1 = \mathcal{D}_2 = \begin{bmatrix} 0 \\ 0.1 \\ 0 \end{bmatrix},$$

$$M_1 = M_2 = 0.01\begin{bmatrix} -1.5 & 0 & 1 \\ 0 & -1 & 25 \\ 2 & 4 & -4 \end{bmatrix},$$

$$N_1 = N_2 = 0.1I_3 \text{ and } F(t) = \sin t.$$

Further, disturbance and fault are assumed as $0.8(\sin(2(t-1)))$ and $1.5\cos(15\pi t) - 0.3\sin(11\pi t)$, respectively. Furthermore, the other known constant scalar parameters are chosen as $\alpha_1 = 0.3$, $\alpha_2 = 0.7$, $\mathcal{R} = 0.0250$, $\alpha = 0.05$, $\beta = 0.3$ and $\rho = 0.3$. Finally the membership function is taken as $\mathfrak{h}_1(t) = 0.5\left(1 + \frac{x_1}{20}\right)$ and $\mathfrak{h}_2(t) = 1 - \mathfrak{h}_1(t)$. Then, solve the LMIs given in Theorem 2 by using LMI Toolbox in MATLAB, the following unknowns are obtained:

$$\mathcal{K}_1 = \begin{bmatrix} -2.8640 & -21.9594 & 3.9382 \end{bmatrix},$$
$$\mathcal{K}_2 = \begin{bmatrix} 3.0114 & -24.1852 & 1.4563 \end{bmatrix},$$
$$\mathcal{H}_1 = \begin{bmatrix} -1.2271 & -9.4107 & 1.6873 \end{bmatrix}$$
$$\mathcal{H}_2 = \begin{bmatrix} 1.2909 & -10.3646 & 0.6236 \end{bmatrix},$$
$$\mathcal{L}_1 = \begin{bmatrix} 0.8332 \\ -0.0311 \\ 0.4717 \end{bmatrix}, \quad \mathcal{L}_2 = \begin{bmatrix} 0.2570 \\ -0.1962 \\ 0.2078 \end{bmatrix},$$
$$\mathcal{F} = [2.5092] \text{ and } \eta = 4.6629.$$

In Fig. 1, the state trajectories of closed loop is displayed and Fig. 2 shows the state trajectories of open loop system. From the Figs. 1 and 2 it can be conclude that the proposed controller has the ability of stabilization before 1 second. The disturbance is taken as non periodic (i.e) $\varpi(t) = 1.5$ and the system-states are ploted with the considered disturbance in Fig. 3. Furthermore, Fig. 4 proves the effectiveness of proposed tracking control under fast adaptive fault estimation

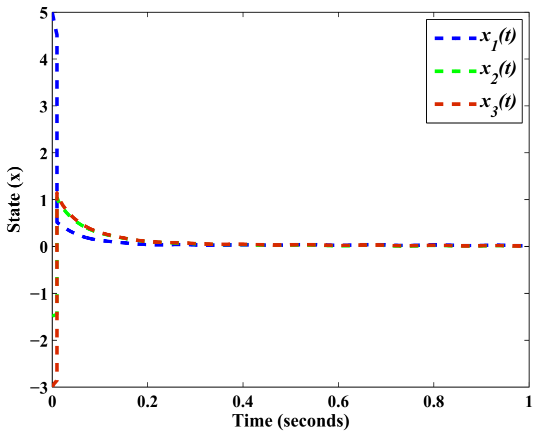

Fig. 1 State response for controller

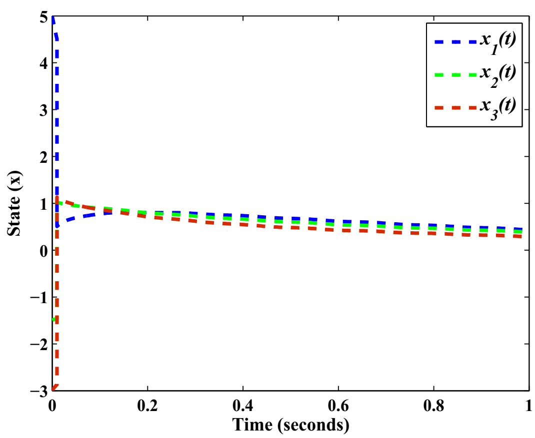

Fig. 2 Open-loop system-state trajectories

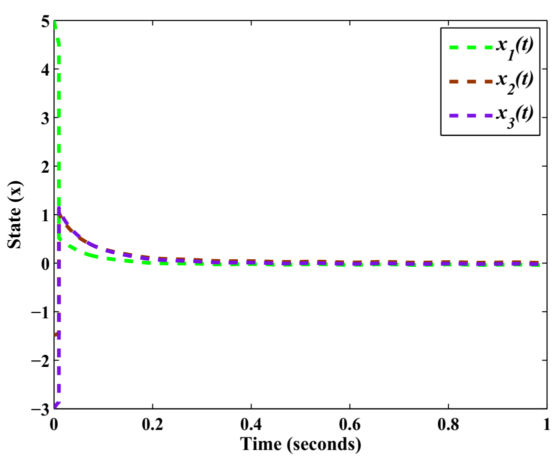

Fig. 3 State response for controller with non-periodic disturbance

algorithm. With the assumption parameter of sliding surface $\mathcal{G} = 0.01B^T$, the graph of sliding surface is plotted in the Fig. 5. Figure 6 shows the domain of attraction for saturation controller and the evaluation of state trajectories with different initial conditions which all are belonging in the integral domain.

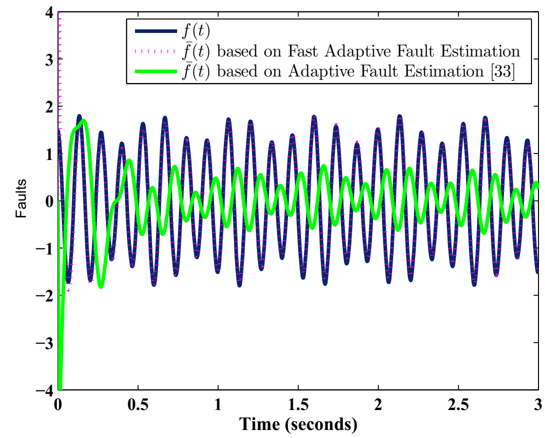

Fig. 4 Fault trajectory with its estimated fault trajectories

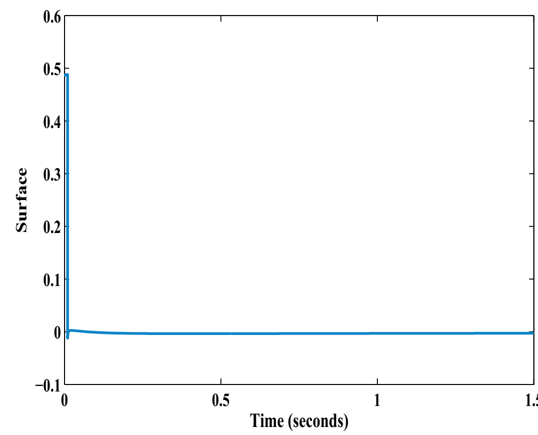

Fig. 5 Sliding surface state trajectories

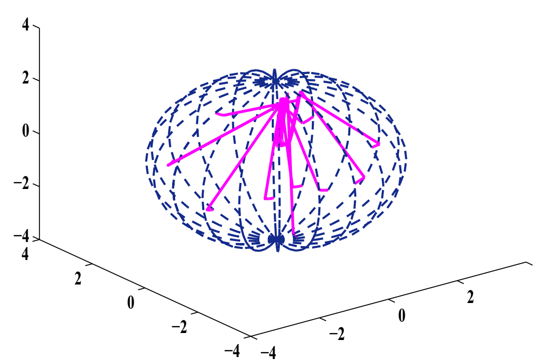

Fig. 6 Domain of attraction and system-state trajectories for different initial conditions

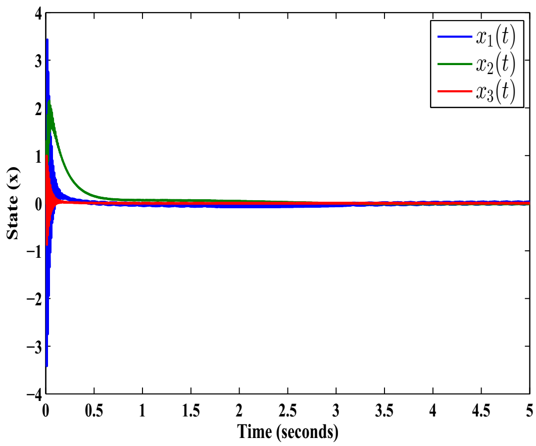

Fig. 7 State response for controller with actuator saturation

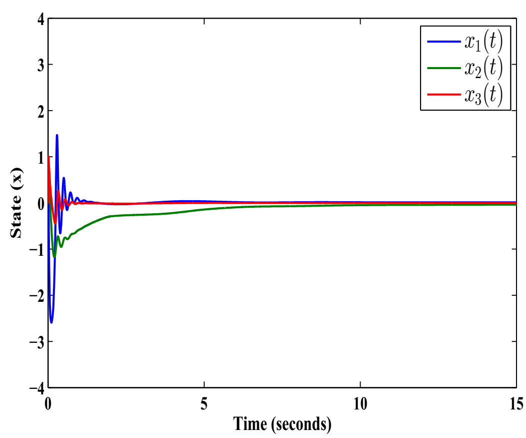

Fig. 8 State response for controller without actuator saturation

Example 2 The model of Rossler in FO is described as follows:

$$\mathfrak{D}^{0.99}x_1(t) = -x_2(t) - x_3(t),$$
$$\mathfrak{D}^{0.99}x_2(t) = x_1(t) + ax_2(t),$$
$$\mathfrak{D}^{0.99}x_3(t) = bx_1(t) - (c - x_1(t))x_3(t).$$

It can be rewritten as a fuzzy model as in [20] is given in the following form:
Rule 1: If $x_1(t)$ is $\mathfrak{h}_1(t)(x_1(t))$, then $\mathfrak{D}^{0.99}x(t) = A_1x(t)$,
Rule 2: If $x_1(t)$ is $\mathfrak{h}_2(t)(x_1(t))$, then $\mathfrak{D}^{0.99}x(t) = A_2x(t)$,
where

$$A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -d \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & d \end{bmatrix},$$
$\mathfrak{h}_1(t) = (1/2)(1 + (c - x_1(t))/d)$, $\mathfrak{h}_2(t) = 1 - \mathfrak{h}_1(t)$

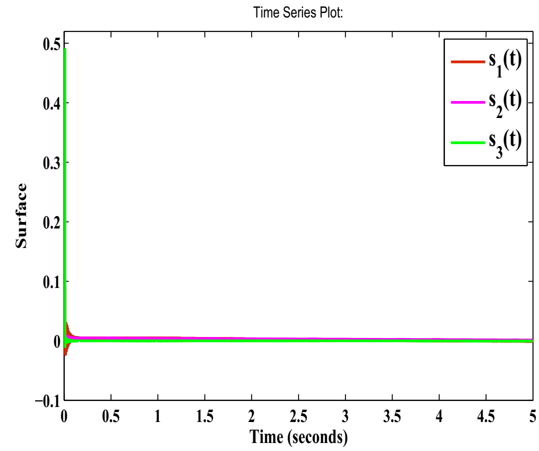

Fig. 9 Sliding surface state trajectories

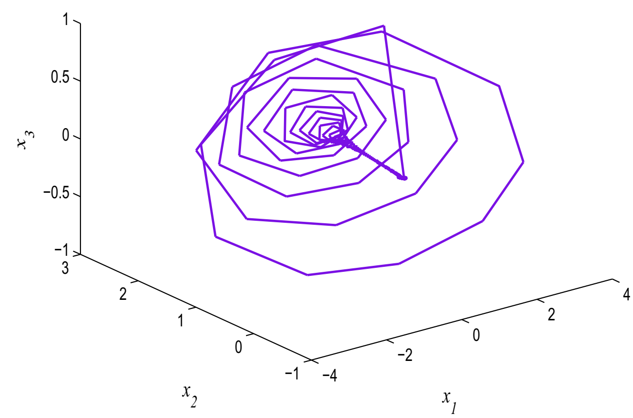

Fig. 10 Chaotic behaviour of Rossler models state trajectories

with $d = 10$, $a = 0.34$, $b = 0.4$ and $c = 4.5$. In order to check the efficiency of proposed controller, a controller element and the external disturbance is considered to the above said system. Then the system can be modified as follows:
Rule 1: If $x_1(t)$ is $\mathfrak{h}_1(t)(x_1(t))$, then $\mathfrak{D}^{0.99}x(t) = (\mathcal{A}_1 + \Delta\mathcal{A})x(t) + \mathfrak{B}(u(t) + f(t)) + \mathcal{D}_1\varpi(t)$,
Rule 2: If $x_1(t)$ is $\mathfrak{h}_2(t)(x_1(t))$, then $\mathfrak{D}^{0.99}x(t) = (\mathcal{A}_2 + \Delta\mathcal{A})x(t) + \mathfrak{B}(u(t) + f(t)) + \mathcal{D}_2\varpi(t)$, where $\mathcal{A}_1 = A_1$, $\mathcal{A}_2 = A_2$, $\Delta\mathcal{A} = 0$, $\mathfrak{B} = I_3$, $f(t) = 0$, $\mathcal{D}_1 = \mathcal{D}_2 = \begin{bmatrix} 0 \\ 0.1 \\ 0 \end{bmatrix}$ and $\varpi(t) = 0.8\cos(0.01t)e^{-0.28t^2}$. Then, solve the LMIs given in Theorem 2 by using LMI Toolbox in MATLAB, the following unknowns are obtained with the following considerations: saturation parameter $\alpha_l = 1/8$, $\forall l \in \{1, 2, 3, \ldots, 2^3\}$, performance parameters $\gamma = 0.2$ and $\theta = 0.4$:

$$\mathcal{K}_1 = 10^3 \begin{bmatrix} 0.1962 & 0.1866 & -5.2594 \\ 0.0836 & -0.0234 & 0.2805 \\ 0.3199 & -0.0166 & -0.1515 \end{bmatrix},$$

$$\mathcal{K}_2 = 10^3 \begin{bmatrix} -0.1153 & -0.2344 & 3.0873 \\ 0.0413 & -0.0522 & 0.8358 \\ -0.1817 & -0.0518 & 0.0288 \end{bmatrix},$$

$$\mathcal{H}_1 = 10^3 \begin{bmatrix} 0.1962 & 0.1866 & -5.2594 \\ 0.0836 & -0.0234 & 0.2805 \\ 0.3199 & -0.0166 & -0.1515 \end{bmatrix},$$

$$\mathcal{H}_2 = 10^3 \begin{bmatrix} -0.1153 & -0.2344 & 3.0873 \\ 0.0413 & -0.0522 & 0.8358 \\ -0.1817 & -0.0518 & 0.0288 \end{bmatrix},$$

$$\mathcal{L}_1 = \begin{bmatrix} 44.9807 \\ -31.9966 \\ 2.6293 \end{bmatrix} \text{ and } \mathcal{L}_2 = \begin{bmatrix} 44.9807 \\ -31.9966 \\ 22.6293 \end{bmatrix}.$$

Figures 7 and 8 show the effectiveness of saturated control. It is observed that even in Fig. 7 the state trajectories have large oscillations than the trajectories have in Fig. 8 but they are converged within short time period. The sliding surface trajectories are ploted in Fig. 10 for the sliding parameter $\mathcal{G} = \mathfrak{B}^T \begin{bmatrix} 0.015 & 0 & 0 \\ 0 & 0.015 & 0 \\ 0.0001 & 0 & 0.01 \end{bmatrix}$. The chaotic behaviour of Rossler model is shown in Fig. 9 by plotting the evaluation of state trajectories.

From the above-mentioned two examples it is well proven that the proposed controller mixed $\mathbf{H}_\infty$ and passivity FO sliding mode controller with actuator saturation have a high effective performance which can stabilize the system even it is a chaotic.

5 Conclusion

A novel mixed $\mathbf{H}_\infty$ and passivity fuzzy SMC of FOTSF uncertain model subject to time-varying actuator fault and actuator saturation is considered. The closed-loop of observer based FOTFS uncertain model with time varying actuator fault via SMC controller is formulated. Precisely, stability analysis is done for the FOTSF model based on state and fault estimations by using the Lyapunov's stability theorem and the sufficient constraints for stability of formulated model are built. Finally, two simulated exemplars including one Rossler chaotic model are issued to support the proposed results. Moreover, many real world systems are containing random process in it, so our next stand is to develop our current existing results for stochastic FO systems.

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