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Design of sliding mode control for structural vibration system with time-varying transmission delays

Dipesh Shah[1] · Devesh Soni[2]

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Abstract

This paper investigates the application of the robust control strategy for reducing structural vibrations using the hybrid protective system in the presence of network abnormalities. It focuses on the design of sliding mode control using a novel sliding variable and modified reaching law approach in the presence of time-varying transmission delay and matched uncertainties. The novel sliding variable is designed using compensated state information which nullifies the effect of time-varying or deterministic transmission delay and ensures finite-time convergence of state variables in the presence of system uncertainties. Further, the stability analysis of the proposed control algorithm with the closed-loop system in the presence of system uncertainties is also presented using the Lyapunov approach. The compound equation of motion of the hybrid protective structural system is formulated and solved in the time domain by the state-space approach. The simulation results are obtained for a typical massive storage structure equipped with a hybrid protective system under seismic excitation. To prove the efficacy of the proposed control algorithm, the results are compared with the power-rate reaching law and conventional delayed system. It is observed that the proposed control strategy is quite effective and robust in the presence of system uncertainties.

Keywords Sliding mode control · Transmission delays · MR damper · LNG tank · Lyapunov approach · Stability analysis

1 Introduction

The schemes to control structural vibration against an earthquake can be viewed as passive, active, semi-active and hybrid control. For over three decades seismic isolation technology and passive control scheme has been recognized as one of the promising alternatives for protecting the structures (see e.g. [1–4] and references therein). The active and semi-active control methods have also found increasing applications in civil engineering structures. In [5], a hybrid control method and a combination of passive and semi-active control systems have received considerable attention due to their advantages. In hybrid system, base-isolation helps in reducing superstructure responses where the semi-active control devices reduce the displacement at the isolation level. Several semi-active devices like magnetorheological (MR) dampers, electrorheological (ER) dampers, variable stiffness dampers, etc. have gained significant attention in recent years for the vibration control of structures. Amongst various semi-active control devices, MR dampers have been investigated extensively by researchers for its applications for structural vibration control.

The MR damper resembles an ordinary linear viscous damper except that the cylinder of the damper is filled with a special liquid consisting of magnetically polarizable micron-sized particles. The viscosity of the fluid can be altered very quickly from a liquid to a semi-solid and vice versa within 5 to 10 milliseconds by adjusting the magnitude of the magnetic field which is produced by a coil wrapped around the piston head of the damper. In absence of current to the coil, an MR damper behaves like an ordinary viscous damper. On the other hand, when current is supplied through the coil, liquid inside the damper changes to semi-solid state and its yield strength changes depending on the applied current. Since the damper does not apply the control force directly to the structure (only resistance of the damper is adjusted), control instability does not take place. Also the MR damper operates

✉ Dipesh Shah
dipeshshah.ic@gmail.com

Devesh Soni
devesh18@gmail.com

1 Department of Biological Engineering, Indian Institute of Technology, Gandhinagar, India

2 Department of Civil Engineering, Sardar Vallabhbhai Patel Institute of Technology, Vasad, India

at only few watts of power hence in case of the failure of control algorithm it continues to work as a passive device. Thus, the MR damper has proved to be a fail-safe and a reliable device.

To command the semi-active devices of hybrid control system, several control strategies have been developed like polynomial control [6,7], LQR [8], H_∞ control [9], instantaneous optimal control [10], LQG [11], neural [12,13], acceleration control [14], fuzzy control [15], control based on Lyapunov stability theory, decentralized bang-bang control [16], maximum energy dissipation, clipped-optimal control [17], modulated homogeneous friction, and sliding mode control [18–23] in the presence of system and model uncertainties.

Among above mentioned controllers, sliding mode control have received much attention in various applications such as robotics, networked control system, power electronics and industrial processes due to its robustness property in the presence of matched and unmatched disturbances [24–26]. In sliding mode control strategy, the methods for determining switching function and designing controller may be changeable for the same control object. Due to advancement in the network technology the usage of the communication medium is increased in various areas such as process industries, aerospace, military applications, control of structural vibration system and automobile sectors. The main reason is that it provides faster data rate transfer between the shared devices (plant and controller) and reduces the complexity of wiring as compared to point-to-point communication. Degradation in system performance due to the presence of communication medium is the major limitation of network based system. In the case of structural vibration system, whenever the controller is connected to the plant through some network medium, it suffers from time-varying or deterministic communication delay. If these delays are not handled properly they may lead to the major instability of the system. Moreover, till date none of the researchers have tried to design the control strategy for a network-based structural vibration system that addresses and overcomes the issues of communication. Thus, there is a need of designing a novel robust control staregy that overcomes the problem of communication delay. However, a robust control strategy having compensation effect of time-varying or deterministic transmission delay in the sliding variable have not yet been explored for structural vibration system.

1.1 Motivation

This research gap motivates authors to develop a novel type of robust control strategy for structural vibration system that compensates the effect of time-varying or deterministic transmission delay in the sliding variable in the presence of system uncertainties. The proposed technique results in an increase in the overall efficiency and performance of the network-based structural vibration system in the presence of transmission delays and matched uncertainty.

1.2 Contributions

The paper contributes mainly to the following:

- Mathematical modelling of time-varying transmission delays using exponential distribution function.
- Compensation of time-varying or deterministic transmission delay using Padé approximation.
- A design of novel sliding variable using compensated state information that nullifies the effect of time-varying or deterministic transmission delay which ensures the finite-time convergence of state variables in the presence of system uncertainties.
- A design of sliding mode control using modified reaching law and proposed sliding variable in the presence of transmission delay and matched uncertainty.
- Stability analysis of closed-loop system using proposed control algorithm in the presence of system uncertainties.
- Finally, the comprehensive simulation results on MR damper system with LNG tank test-bed platform under deterministic and time-varying transmission delay with system uncertainties.

1.3 Structure of the paper

The paper is organized as follows: Sect. 2 presents the detailed description of magnetorheological damper system in the presence of time-varying transmission delay. The mathematical modelling of MR damper system with time-varying transmission delay is presented in Sect. 3. The main contribution of the paper designing of compensated sliding variable using Padé approximation technique in the presence of deterministic or time-varying transmission delay is presented in Sect. 4. Section 5 describes the design of robust sliding mode controller with modified reaching law using proposed sliding variable in the presence of system uncertainties. Stability analysis of the closed-loop system using Lyapunov approach is presented in Sect. 6. Section 7 describes the detailed simulation results for MR damper system and LNG tank test-bed platform in the presence of system uncertainties. The concluding remarks of the paper and future possible direction of proposed control algorithm is discussed in Sect. 8.

2 Magnetorheological damper system with time-varying transmission delay

A magnetorheological (MR) damper is a system with shock absorber characteristics in which the damper is filled

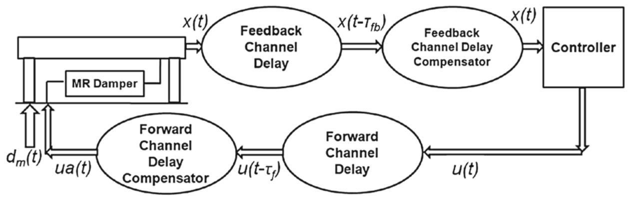

Fig. 1 MR damper system with transmission delay and matched uncertainty

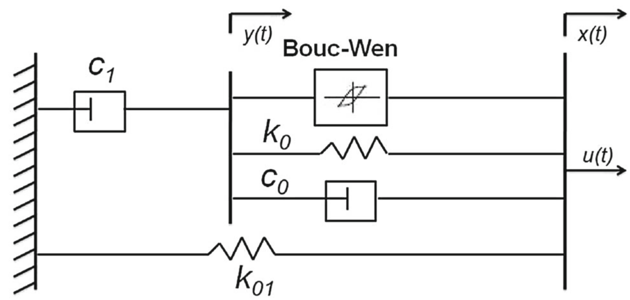

Fig. 2 Simplified mechanical model of MR damper [27]

with magnetorheological fluid. The output of the system is controlled by a magnetic field through electromagnets that are connected at the bottom of the MR damper system. Figure 1 shows the schematic diagram of the MR damper system with transmission delay and matched uncertainty. The output of the MR damper system is controlled through the communication network. The path through which the state information $x(t)$ is transmitted from plant to controller side is defined as a feedback channel. While the path through which the control signal $u(t)$ is transmitted from the controller to plant is defined as a forward channel. The state information $x(t)$ would experience the feedback channel transmission delay τ_{fb} while the control signal $u(t)$ would experience the forward channel transmission delay τ_f. These transmission delays are either deterministic or time-varying in nature depending on the characteristics of the network medium and location of the controller. The disturbance is applied at the input side of the channel in the form of an earthquake with different amplitudes which are defined as matched uncertainty. If these transmission delays and matched uncertainty are not handled properly it might deteriorate the performance of the system which even leads to instability. In order to avoid such a situation, the time delay compensator is connected in the forward and feedback channel that compensates the effect of time-varying or deterministic transmission delay in the presence of matched uncertainty.

3 Mathematical model of MR damper system with time-varying transmission delay

A simplified mechanical structure of the MR system with the Bouc-Wen model is shown in Fig. 2. It is considered as a test-bed platform for this work. The main reason for considering this model is that it accurately provides the behavior of the MR damper system over a broad range of inputs.

Remark 1 It is necessary to note that, this section describes the state-space model of the MR damper system with time-varying transmission delay. The detailed mathematical analysis of the system can be found in [27].
The structure presented in Fig. 2 is based on the response of a prototype MR damper which is obtained for evaluation from the Lord Corporation [27]. The equations governing the force $u(t)$ predicted by this model [27] are given by

$$u(t) = c_1\dot{y}(t) + k_{01}(x_d(t) - x_0(t)) \tag{1}$$

$$\dot{z}(t) = -\gamma|\dot{x_d}(t) - \dot{y}(t)|z(t)|z(t)|^{n-1} - \beta(\dot{x}(t) - \dot{y}(t))|z(t)|^n + A'(\dot{x}(t) - \dot{y}(t)) \tag{2}$$

$$\dot{y}(t) = \frac{1}{c_0 + c_1}[\alpha z(t) + c_0\dot{x_d}(t) + k_0(x_d(t) - y(t))] \tag{3}$$

where $z(t)$ is an evolutionary variable that accounts for the history dependence of the response, $x_0(t)$ is the initial displacement of the spring k_{01}, $x_d(t)$ is the final displacement of the spring k_{01}, c_1 is viscous damping coefficient of the fluid at low velocities, c_0 is viscous damping coefficient of the fluid at large velocities, k_{01} is the control stiffness of the accumulator, k_0 is the control stiffness at large velocities, $y(t)$ is the ouput of the system in terms of displacement and velocity of the MR damper system, α is variable tension of the system, and γ, β and A' are user defined parameters of the model which controls the linearity and smoothness of the transition from postyeild to preyeild region. In order to validate the model [27] in the presence of fluctuating magnetic field it is necessary to determine the functional dependence of parameters α, c_0 and c_1 on the applied voltage $v(t)$ to the current driver circuit which is given by,

$$\alpha = \alpha_a + \alpha_b w, c_1 = c_{1a} + c_{1b} w, c_0 = c_{0a} + c_{0b} w \tag{4}$$

where α_a, α_b, c_{0a}, c_{0b}, c_{1a} and c_{1b} are the positive real constants that are involved in the MR fluid for reaching at its rheological equilibrium. $w(t)$ is the output of the first-order filter which is represented as,

$$\dot{w}(t) = -\eta(w(t) - v(t)) \tag{5}$$

where η is viscousity of the fluid and $v(t)$ is the voltage applied to the current driver.

The mathematical model of filter mentioned in Eq. (5) plays a crucial role in reaching rheological to its equilibrium state and driving the electromagnet in the MR damper system.

Assumption 1 The forces generated through the MR damper system are assumed to be sufficient to drive the structure in the linear region.
Thus, considering a seismically excited structure controlled with a single MR damper system the equation of motion can be written as

$$M_s\ddot{x}(t) + C_s\dot{x}(t) + K_s x(t) = \delta u(t) - M_s \Gamma d_m(t) \tag{6}$$

where $x(t)$ is a vector of the relative displacements of the floors of the structure, $d_m(t)$ is a one-dimensional ground acceleration in the form of disturbance, $u(t)$ is the control input in the form of measured force which is defined by Eqns. (1)-(5), Γ is a column vector of ones, and δ is a vector determined by the position of the MR damper in the structure. Thus, using Eqs. (1)–(6) the state-space model of MR damper system is given by

$$\dot{x}(t) = Ax(t) + Bu(t) + D_s d_m(t) \tag{7}$$
$$y(t) = Cx(t) + Du(t) \tag{8}$$

where $x(t) \in R^n$ is system state vector, $u(t) \in R^m$ is control input, $y(t) \in R^p$ is system output vector, $d_m(t)$ is the matched uncertainty with an assumption that the $rank(B, D_s)$ is same and within it's input range, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, $D \in R^{n\times m}$, $D_s \in R^{n\times m}$ are the matrices of appropriate dimensions.

The closed-loop system defined in Eqs. (7) and (8) represents the conventional state-space model of the MR damper system with a single input and multiple outputs (SIMO) behavior. Depending on the computation of the control actions the displacement and velocity of the MR damper system are controlled. However, in order to make control problems more challenging and interesting the system is connected to the networked medium (see Fig. 1). The major advantage of the schematic diagram presented in Fig. 1 is that, the point to point cable connections are replaced by communication networks which provide faster data rate transfer with economical investments. Thus, the system defined in Eqs. (7) and (8) when connected to the communication network the system variables suffer from internal delays τ_i, forward channel transmission delay τ_f and feedback channel transmission delay τ_{fb}. The combination of these delays is defined as total transmission delay τ_t. It is represented as

$$\tau_t = \tau_i + \tau_f + \tau_{fb}. \tag{9}$$

The transmission delay (τ_f, τ_{fb}) in the forward and feedback channel can be deterministic or time-varying in nature depending on the characteristics of the network medium and the location of the controller. While the internal delays are always deterministic in nature. The computational delay at controller side τ_c, processing delay at the actuator side τ_a and processing delay at the sensor side τ_s combines to form the internal delay of the closed-loop system. It is given by

$$\tau_i = \tau_c + \tau_s + \tau_a. \tag{10}$$

Assumption 2 When the system is connected via real-time networks it is assumed that the effect of internal delays is negligible as compared to time-varying or deterministic transmission delay. So it can be neglected in comparison with transmission delays as it does not deteriorates the performance of the system.
Thus, satisfying **Assumption 2** the total transmission delay within the system is given by

$$\tau_t = \tau_f + \tau_{fb}. \tag{11}$$

Observing Eq. (11), it is noticed that the behavior of total transmission delay τ_t depends on the forward and feedback channel delay. In order to achieve satisfactory and stable output response, it is considered that the total transmission delay τ_t is bounded in nature which satisfies the following condition

$$\tau_{tl} \le \tau_t \le \tau_{tu} \tag{12}$$

where τ_{tl} and τ_{tu} indicates the lower and upper bounds of total transmission delay.

Remark 2 From Eq. (12), it should be noted that the values of upper and lower bounds of the total transmission delay would depend on the feedback and forward channel transmission delay which is computed using exponential distribution in the later section.

Assumption 3 Referring to Fig. 1 it can be noticed that a slow time-varying disturbance $d_m(t)$ applied to the system is assumed to be known and bounded in nature that satisfies the following condition

$$|d_m(t)| \le \varpi \tag{13}$$

where ϖ is a postive constant.

Thus considering the effect of total transmission delay, the closed-loop system in Eqs. (7) and (8) is given as

$$\dot{x}(t) = Ax(t) + Bu(t - \tau_t) + D_s d_m(t) \tag{14}$$
$$y(t) = Cx(t) + Du(t - \tau_t) \tag{15}$$

where τ_t is the total transmission delay in continuous-time domain.
Objective: The main objective of the paper is to design a robust sliding mode control using novel sliding variable and modified reaching law in the continuous-time domain for the

system (14), (15) in the presence of time-varying or deterministic transmission delay and matched uncertainty. The main advantage of the proposed control algorithm is that it provides faster convergence, negligible chattering and delay compensation even in the presence of system uncertainties such as transmission delay and matched uncertainty.

4 Sliding variable with time-varying transmission delay

In the control system, Taylor series [28], All pole approximation [29] and Padé approximation [30] techniques are widely used for processing of time delay in continuous-time domain. The major drawback of the Taylor approximation technique [28] is that it can approximate the lower range of delays that are deterministic. While in the case of all pole approximation technique [29] it generates smaller overshoots by adjusting time both in first and second-order of large delayed system. Thus to overcome these drawbacks, it is better to use the Padé approximation technique [30] as it approximates the time delay by rational models.
The effect of deterministic or time-varying transmission delay in the feedback channel is compensated at the sliding variable. The main advantage of this approach is that the reaching condition required for finite-time convergence of state variables is not affected even in the presence of transmission delay. The detailed analysis is presented in Lemma 1.

Lemma 1 *The compensated sliding variable $s(t)$ in the presence of deterministic or time-varying feedback channel transmission delay τ_{fb} and matched uncertainty is given by,*

$$s(t) = C_g(x'(t) - x(t)) \tag{16}$$

where C_g is the sliding gain computed using LQR approach, $x'(t)$ is the parameter to tackle feedback channel transmission delay τ_f, and $x(t)$ is the state information signal available from sensor side.

Proof In sliding mode control, the design of a sliding variable mainly depends on the computation of sliding gain and information signal available from the sensor. The state information signal $x(t)$ processed at the sensor side experiences the feedback channel transmission delay τ_{fb}. Thus, the sliding variable $s(t)$ at the controller side is given as

$$s(t) = C_g x(t - \tau_{fb}) \tag{17}$$

where $x(t - \tau_{fb})$ is the delayed state information signal. □

Remark 3 It is considered that deterministic transmission delay in forward and feedback channels is constant over an infinite interval of time. On the other hand, the time-varying transmission delay is random and unknown. So without loss of generality, it is necessary to model time-varying transmission delay using a stochastic approach in the continuous-time domain.

Remark 4 In the stochastic approach, the exponential distribution computes the random number based on the continuous generation of events in the process. However, in the proposed work it is considered that the state and control information signal is continuously received at the controller and plant side. So, considering this information signal in the form of event, the exponential distribution is the best suitable approach for modeling the time-varying transmission delay occurring in both sides of the channel.

Thus, the time-varying feedback channel transmission delay τ_{fb} modeled using exponential distribution [31] with probabilities is given by

$$Pr\{\tau_{fb} = d_v\} = E\{d_v\} = \beta_v;\ v = 1, 2, \ldots, q \tag{18}$$

where β_v is the positive scalar quantity, v is the event, $E\{d_v\}$ is the expectation of the stochastic variable d_v.
The mathematical representation of β_v with exponential distribution is given by,

$$\beta_v = \lambda - e^{-\lambda r} \tag{19}$$

where λ is the rate parameter with $\lambda > 0$, r is the random variable uniformly distributed over the interval [0,1] and e is the exponential term.
Using Padé approximation technique [30] and taking Laplace transform, the delayed state information signal in Eq. (17) is represented as

$$L\{x(t - \tau_{fb})\} = e^{-\tau_{fb}s} L\{x(t)\}. \tag{20}$$

Applying first order approximation, the above Eq. (20) is written as

$$L\{x(t - \tau_{fb})\} \approx \frac{1 - \frac{\tau_{fb}s}{2}}{1 + \frac{\tau_{fb}s}{2}} L\{x(t)\} \tag{21}$$

where $L\{x(t)\}$ is Laplace transform of $x(t)$ and s is the Laplace variable. Then the following variable $x'(t)$ is defined as

$$L\{x'(t)\} - L\{x(t)\} = \frac{1 - \frac{\tau_{fb}s}{2}}{1 + \frac{\tau_{fb}s}{2}} L\{x(t)\}. \tag{22}$$

where $x'(t)$ is the parameter to tackle time-varying or deterministic feedback channel transmission delay τ_{fb}.

Further solving Eq. (22), we get

$$L\{x(t)\} - L\{x(t)\}\frac{\tau_{fb}s}{2} = L\{x'(t)\} + \frac{\tau_{fb}s}{2}L\{x'(t)\}$$
$$-L\{x(t)\} - L\{x(t)\}\frac{\tau_{fb}s}{2}. \quad (23)$$

Applying Inverse Laplace transform to above Eq. (23), we get

$$x(t) - \frac{\dot{x}(t)\tau_{fb}}{2} = x'(t) + \frac{\dot{x}'(t)\tau_{fb}}{2} - x(t) - \frac{\dot{x}(t)\tau_{fb}}{2}. \quad (24)$$

On further simplification the above Eq. (24) is expressed in the form of $\dot{x}'(t)$ as

$$\dot{x}'(t) = -\zeta x'(t) + 2\zeta x(t) \quad (25)$$

where $\zeta = \frac{2}{\tau_{fb}}$.

Using Eqs. (22) and (25), the compensated sliding variable at the controller side in Eq. (17) is expressed as

$$s(t) = C_g(x'(t) - x(t)) \quad (26)$$

where $x'(t)=\int(-\zeta x'(t) + 2\zeta x(t))dt$.

Remark 5 The parameter $x'(t)$ is introduced in the sliding variable (26) which eliminates the effect of feedback channel transmission delay at the controller side.

5 Sliding mode control for MR damper system with time-varying transmission delay

Theorem 1 *The non-switching type sliding mode control law for the system (14), (15) in the presence of feedback channel transmission delay τ_{fb} and matched uncertainty $d_m(t)$ is given as*

$$u(t) = -(C_gB)^{-1}[C_gAx(t) - C_g\dot{x}'(t) - \chi s(t)$$
$$-k_2 sgn[s(t)] + c_n d'_m(t)] \quad (27)$$

where k_1, k_2, s_0 are positive contants, $k_2 \geq c_n\varpi$ and $c_n = C_gD_s$.

Proof The compensated sliding variable in (26) should satisfy following $'\Delta'$ reaching condition (28) that ensures the finite time convergence to $s(t) = 0$. It is given by,

$$s(t)\dot{s}(t) < -\Delta|s(t)|.\Delta > 0, \forall t. \quad (28)$$

The reaching law proposed in [32] that satisfy condition (28) is given as,

$$\dot{s}(t) = -\chi s(t) - k_2 sgn[s(t)] + c_n d_m(t) \quad (29)$$

where $\chi = \frac{k_1 s_0}{s_0 + |s(t)|}$.

Observing Eq. (29) it is noticed that, the disturbance term $d_m(t)$ suffers from feedback channel transmission delay τ_{fb} as it is applied from the plant side. Thus, the modified reaching law is represented as

$$\dot{s}(t) = -\chi s(t) - k_2 sgn[s(t)] + c_n d_m(t - \tau_{fb}). \quad (30)$$

According to Lemma 1, the term $d_m(t - \tau_{fb})$ is transformed as

$$d_m(t - \tau_{fb}) = (d'_m(t) - d_m(t)) \quad (31)$$

where $d'_m(t)=\int(-\zeta d'_m(t) + 2\zeta d_m(t))dt$. □

Remark 6 In order to tackle time-varying or deterministic feedback channel transmission delay τ_{fb} the delay-dependent parameter $d'_m(t)$ is introduced in (31) which eliminates the effect of feedback channel transmission delay at the controller side. Thus, it is noticed that the variable $d'_m(t)$ varies depending on the feedback channel transmission delay τ_{fb}.

Substituting Eq. (31) into Eq. (30) we have,

$$\dot{s}(t) = -\chi s(t) - k_2 sgn[s(t)] + c_n d'_m(t) - c_n d_m(t). \quad (32)$$

Remark 7 Equations (32) indicates the modified form of Eq. (29) in which the sliding variable and disturbance signal is replaced to the compensated signal. Thus, it is extended that in the presence of communication medium the reaching law in Eq. (29) would not hold with time delay compensation.

Using Eq. (26), the reaching law in Eq. (32) is expressed as

$$C_g(\dot{x}'(t) - \dot{x}(t)) = -\chi s(t) - k_2 sgn[s(t)] + c_n d'_m(t)$$
$$-c_n d_m(t). \quad (33)$$

Substituting the value of $\dot{x}(t)$ the above Eq. (33) is written as

$$C_g\dot{x}'(t) - C_g[Ax(t) + Bu(t - \tau_t) + D_s d_m(t)]$$
$$= c_n d'_m(t) - \chi s(t) - k_2 sgn[s(t)] - c_n d_m(t). \quad (34)$$

Remark 8 It is noticed from Eq. (26), that the feedback channel transmission delay τ_{fb} is compensated at the sliding variable. While forward channel transmission delay τ_f is compensated at the plant side. So, the effect of feedback channel transmission delay is not observed at the controller side for computing the control signal. Thus without loss of generality, the control signal in Eq. (34) is represented as

$$u(t - \tau_t) = u(t). \quad (35)$$

Thus using Eq. (35), Eq. (34) is written as

$$C_g\dot{x}'(t) - C_g Ax(t) - C_g Bu(t) - C_g D_s d_m(t) \\ = c_n d_m'(t) - \chi s(t) - k_2 sgn[s(t)] - c_n d_m(t). \quad (36)$$

Thus, the control law computed at the controller side in the presence of feedback channel transmission delay is given as

$$u(t) = -(C_g B)^{-1}[C_g Ax(t) - C_g\dot{x}'(t) - \chi s(t) \\ -k_2 sgn[s(t)] + c_n d_m'(t)]. \quad (37)$$

The control law computed in Eq. (37) generates the control actions for driving the current driver circuit at the plant side. These control actions suffer from forward channel transmission delay τ_f which is given by

$$u_a(t) = u(t - \tau_f). \quad (38)$$

Thus using Padé approximation technique [30], the compensated control signal computed at the plant side is given by

$$u_a(t) = u'(t) - u(t) \quad (39)$$

$$u'(t) = \int -\rho u'(t) + 2\rho u(t)dt \quad (40)$$

where $\rho = \frac{2}{\tau_f}$ and $u'(t)$ is a delay-dependent parameter that tackles time-varying or deterministic forward channel transmission delay τ_f.

6 Stability analysis

For a given time-varying or deterministic total transmission delay τ_t satisfying condition (12) and matched uncertainty $d_m(t)$ satisfying (13), the trajectories of the closed-loop system (14), (15) drives towards the designed sliding variable (26) for a designed controller (37) within a finite-time convergence such that the following condition in (41) must exists.

$$\Theta > 0 \quad (41)$$

where $\Theta = \chi s(t) + k_2 sgn[s(t)] + c_n(d_m(t) - d_m'(t))$.

Proof Consider the Lyapunov function as

$$V_s(t) = \frac{1}{2}s^2(t). \quad (42)$$

Taking first order derivative of Eq. (42) we have

$$\dot{V}_s(t) = s^T(t)\dot{s}(t). \quad (43)$$

Substituting the value of $\dot{s}(t)$, the above Eq. (43) can be written as

$$\dot{V}_s(t) = s^T(t)C_g(\dot{x}'(t) - \dot{x}(t)). \quad (44)$$

Futher substituting the value of $\dot{x}(t)$ we have

$$\dot{V}_s(t) = s^T(t)C_g(\dot{x}'(t) - [Ax(t) + Bu(t - \tau_t) + D_s d_m(t)]). \quad (45)$$

On simplification of Eq. (45), we get

$$\dot{V}_s(t) = s^T(t)[C_g\dot{x}'(t) - C_g[Ax(t) + Bu(t) + D_s d_m(t)]]. \quad (46)$$

Using Eq. (37), the above Eq. (46) is represented as

$$\dot{V}_s(t) = s^T[-\chi s(t) - k_2 sgn[s(t)] - c_n(d_m(t) - d_m'(t))] < 0. \quad (47)$$

The term $\dot{V}_s(t)$ is definitely negative as k_1, k_2 and s_0 are positive constants as well as $\chi > 0$ and $c_n(d(t) - d'(t)) < \chi s(t) + k_2 sgn[s(t)]$. So, the closed-loop system is asymtotically stable in the presence of time-varying or deterministic total transmission delay τ_t and matched uncertainty $d_m(t)$.

Thus observing Eq. (47), the 'Δ' reachability condition for finite-time convergence for proposed compensated sliding variable (26) and designed control law (37) that satisfies condition (28) is given by

$$0 < \Delta < \Theta. \quad (48)$$

This completes the $Proof$. □

7 Application on MR damper test-bed platform with LNG tank system

To prove the efficacy of the proposed robust control algorithm, a Liquefied Natural Gas (LNG) storage tank system of 165 million liters capacity, a massive storage structure as shown in Fig. 3 is considered as a test-bed. The LNG tank is structurally modeled into two layers. The outer portion of the tank is modeled based on Dunkerley's simplified model while the inner portion is modeled based on the mechanical analogy proposed in [2]. To reduce structural responses, the LNG tank is mounted on the High Damping Rubber Bearing (HDRB) type base-isolation system with a 2.0 s isolation period and 10% damping. The base-isolation system, however, reduces structural responses at the cost of inducing large displacements at the isolation level. Such a large isolator displacement will lead to very large isolators, costly flexible connections

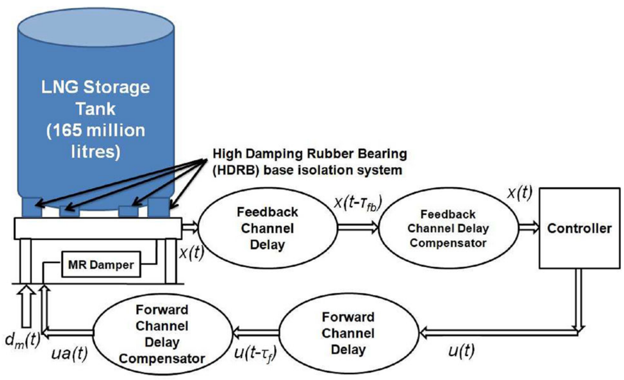

Fig. 3 Structure of network-based liquefied natural gas (LNG) storage tank system with MR damper

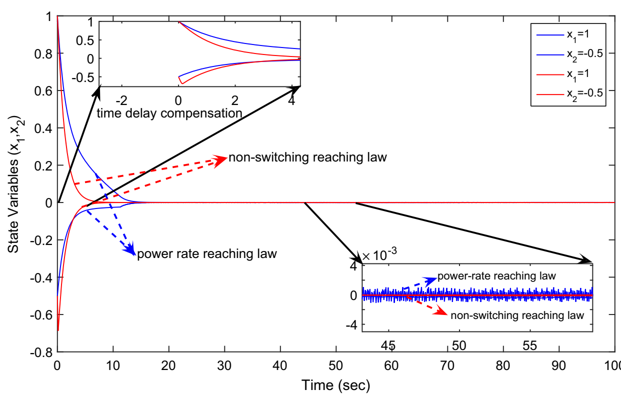

Fig. 4 State variables

for utilities, and might even cause instability of the structure. Therefore, control of isolator displacement is of great concern. Semi-active hybrid control using MR damper is one of the promising technologies to reduce isolator displacement (which is used in the study). To present the dynamics of MR damper, the mechanical phenomenological model proposed in [33,34] is used. The properties of MR damper considered are listed in "Annexure-I". To command the MR damper system in the presence of network abnormalities, sliding mode control (SMC) designed in this article using modified non-switching reaching law is validated. The proposed controller calculates a vector of desired control forces, based on measured structural response and applied force on the structure in the presence of system uncertainties. The 1940 Imperial Valley earthquake recorded at El Centro is considered as seismic input (peak ground acceleration = 0.349 g) to perform test simulations.

The simulation results discussed are presented in three sections (i) Figs. 4, 5, 6 shows the results of state variables, sliding variable, and control signal with time-delay compensation for the MR damper system. The results are compared with power-rate reaching law to show the efficiency of the proposed control algorithm, (ii) Figs. 7, 8, 9 show the results of system variables without time delay compensation and (iii)

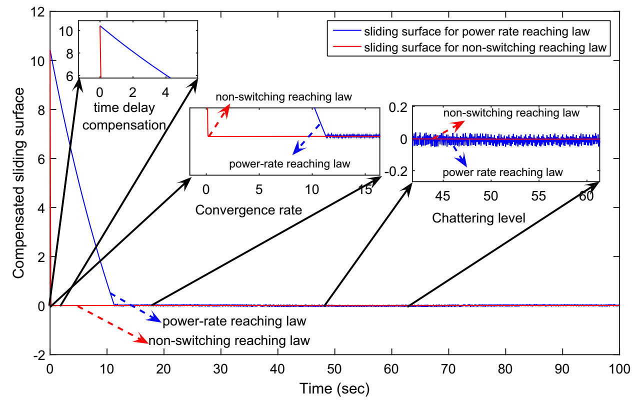

Fig. 5 Sliding variable

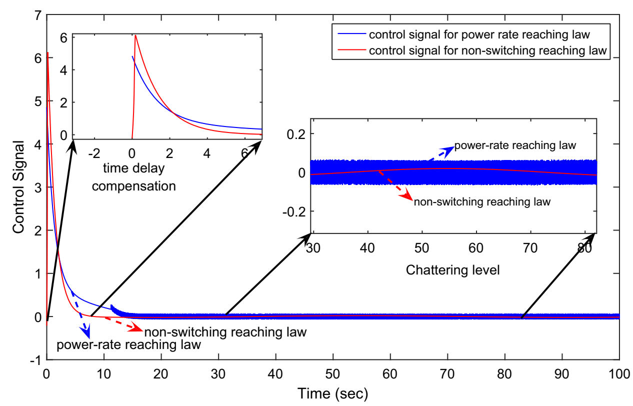

Fig. 6 Control signal computed at plant side

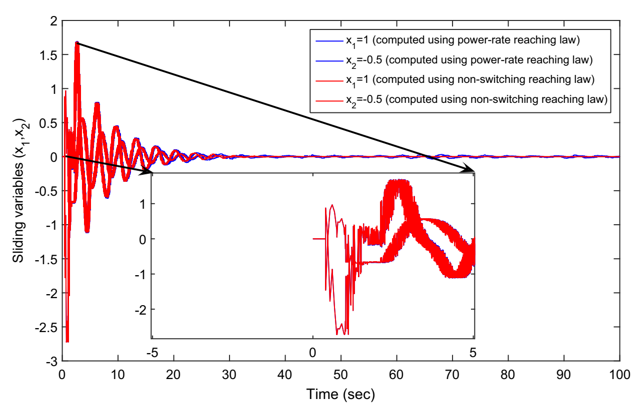

Fig. 7 State variables without time delay compensation

Figs. 10, 11, 12, 13, 14, 15 show the response LNG storage tank system in the terms of displacement of convective mass (upper portion of the liquid which is free to move during vibration), impulsive mass (lower portion of the liquid that remains adhered with the tank wall during vibration) and isolation displacement under deterministic and time-varying transmission delay. The comparative analysis is shown in the absence of delay, presence of delay, and time delay compensation technique.

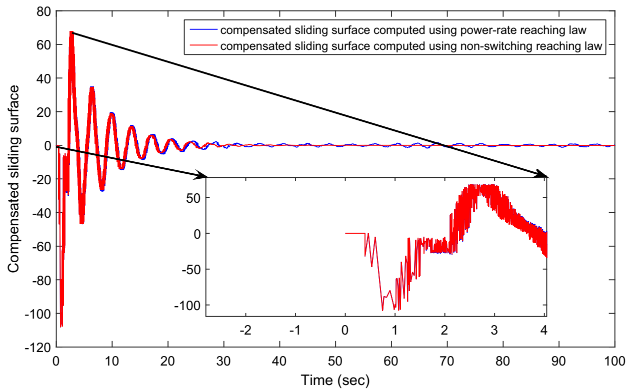

Fig. 8 Sliding variable without time delay compensation

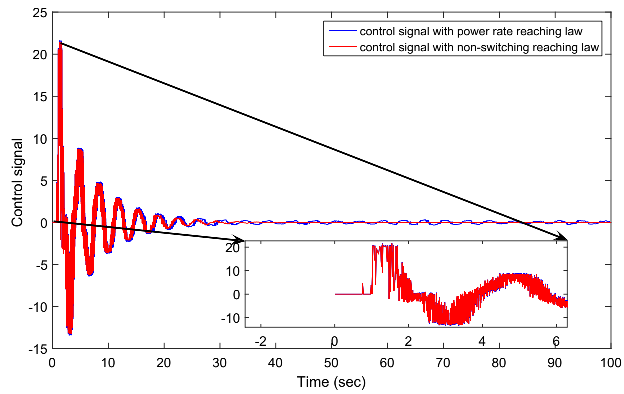

Fig. 9 Control signal computed at plant side without time delay compensation

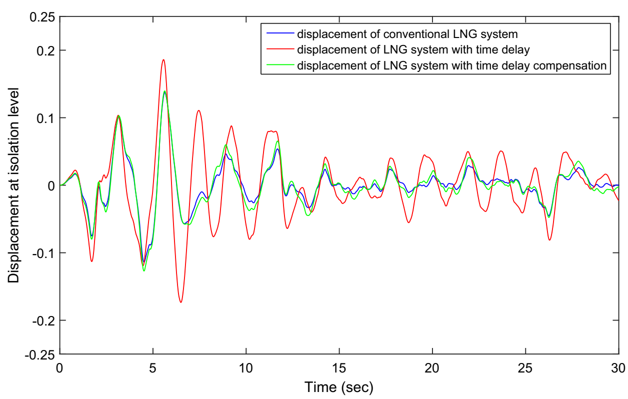

Fig. 10 Displacement history at isolation level for LNG system with deterministic transmission delay

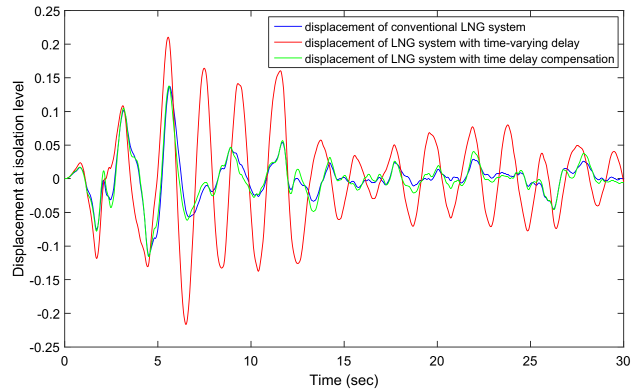

Fig. 11 Displacement history at isolation level for LNG system with time-varying transmission delay

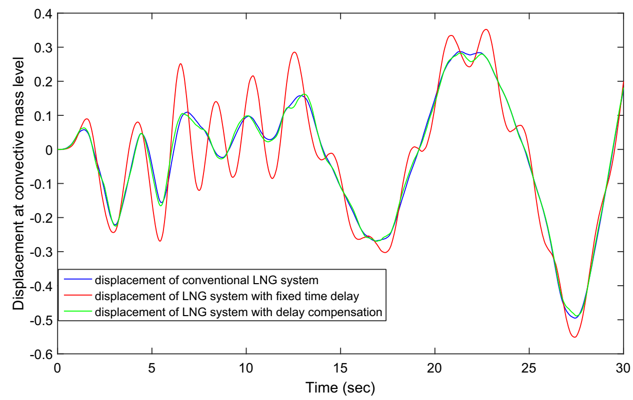

Fig. 12 Displacement history at convective mass level for LNG system with deterministic transmission delay

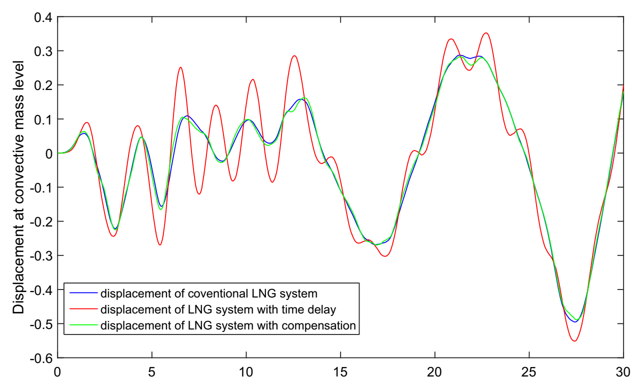

Fig. 13 Displacement history at convective mass level for LNG system with time-varying transmission delay

7.1 Results of state variables, sliding variable and control signal for MR damper system

In this section, the effectiveness of non-switching type sliding mode control law is discussed with time-delay compensation in the terms of state variables, sliding variable, and control signal for the MR damper system. The results are compared with SMC designed using power-rate reaching law. Thus, using **Theorem 1** the sliding mode control law derived using power-rate reaching law [35] is expressed as

$$u(t) = -(C_g B)^{-1}[-C_g \dot{x}'(t) + C_g A x(t) + k|s|^{\varsigma} sgn[s(t)] - C_g D_s d_m(t)] \quad (49)$$

where A, B, C, and D_s are the system matrix, control input matrix, output matrix and disturbance matrix (refer Annexure-II). In this section as the non-regulatory problem

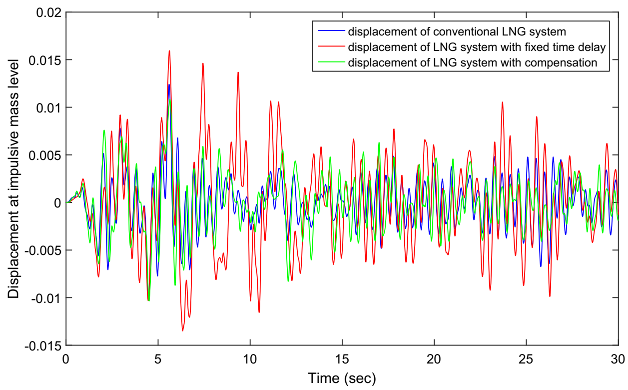

Fig. 14 Displacement history at impulsive mass level for LNG system with deterministic transmission delay

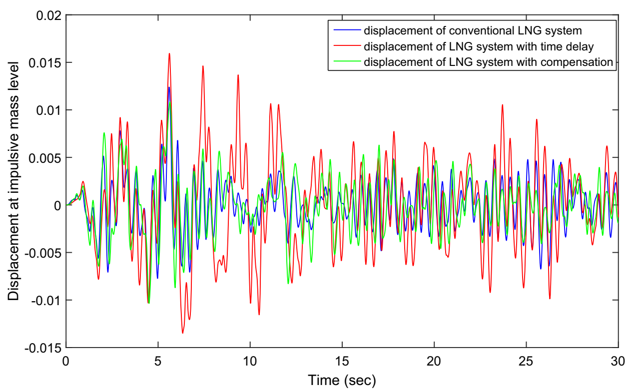

Fig. 15 Displacement history at impulsive mass level for LNG system with time-varying transmission delay

of MR damper system is considered, the system matrices A, B, and D_s are given as $A = \begin{bmatrix} 0 & 1 \\ -9.86 & -0.6283 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ -0.4327e-08 \end{bmatrix}$, $D_s = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$. The sliding gain C_g is calculated using continuous-time LQR method in order to have better optimal control with $Q = diag(1000, 1000)$ and $R = 1$ having $C_g = [-0.0110 - 0.1896]$. To justify the concept of the proposed control algorithm with time delay compensation, the non-regulatory behavior of the MR damper is studied in the presence of system uncertainties. So, in both the cases initial conditions are $x(t) = [1 - 0.5]^T$ respectively. The system variables shows proper response for the values of $k_1 = 2, c_n = 0.18, \varpi = 1, k_2 = 1, \varsigma = 0.5, k = 1$ and $s_0 = 10$ in the case of both the reaching laws. It is assumed that at regular time interval the state and control information signal is received at the controller and plant side with parameter $\lambda = 0.001$. So according to exponential distribution, the time-varying transmission delay in the forward channel is $0.1\,\text{s} \leq \tau_f \leq 0.35\,\text{s}$ and the feedback channel is $0.1\,\text{s} \leq \tau_{fb} \leq 0.35\,\text{s}$. Thus, the total time-varying transmission delay occurring within the communication medium is $0.2\,\text{s} \leq \tau_t \leq 0.7\,\text{s}$ satisfying condition (12). The slow time varying matched disturbance applied to the system is $d_m(t) = 0.02sin(0.086t)$.

Figure 4 shows the response of state variables for the specified initial conditions. It can be noticed that in both cases the state variables are computed from $t = 0$ (mentioned in the magnified window) and converges to the origin even in the presence of feedback and forward channel transmission delay. Thus it can be extended that in both cases, the control algorithm designed using a time delay approximation approach accurately compensates the effect of time-varying transmission delay in both the channels. Moreover, it can also be noticed that the convergence rate and oscillations in the state variables for power-rate reaching are comparatively high as compared to non-switching reaching law. This proves the robustness of the proposed control algorithm derived using modified non-switching reaching law in the presence of system uncertainties.

Figure 5 shows the response of the compensated sliding variable computed at the controller side for both the cases. It can be noticed that the sliding variable at the controller side is computed at $t = 0$ even in the presence of time-varying feedback channel transmission delay τ_{fb}. Thus it can be referred that, the delay-dependent parameter $x'(t)$ designed in (26) tackles the time-varying transmission delay τ_{fb} occurring in the feedback channel effectively. Moreover, it can also be noticed that the state variables in both the cases converge to sliding surface $s(t) = 0$ in the finite-time satisfying condition (26) and remains within its manifold even in the presence of system uncertainties. However, the convergence rate of non-switching reaching law is faster than the power-rate reaching law with negligible chattering. This indicates that the proposed control algorithm designed using novel sliding variable and modified reaching law is the most suitable approach for real-time applications where chattering and time-varying transmission delay are considered to be major concerns for the degradation of system performance.

Figure 6 shows the result of the control signal computed at the plant side for both the cases. The control information signal computed at the controller side based on the compensated sliding variable suffers from time-varying forward channel transmission delay. It can be observed that for both cases, the effect of this transmission delay is compensated at the plant side as the control signal is computed from $t = 0$. Thus it can be noticed that the delay-dependent parameter $u'(t)$ designed at the plant side compensates the effect of time-varying transmission delay τ_f precisely even in the presence of system uncertainties. Moreover, the convergence rate of the control signal computed using the proposed control algorithm is much better than power-rate reaching law with negligible chattering. This results in the improvement of transient response analysis in the time domain for the MR damper system and reduction of the heat losses in the current driver circuit which is used to drive the output of the MR damper system.

Figures 7, 8, and 9 show the response of state variables, sliding variable, and control signal at the plant side for both the cases without time delay compensation technique. Observing the results in the magnified window, it can be noticed that the system variables (plant states, sliding variable and control signal) are computed after a while due to which the response of the system becomes more oscillatory as compared to a compensated system which even leads to degradation in the output performance of the system.
So from the above analysis, it is noticed that the control law derived using modified reaching law with novel sliding variable proves to be more robust than a conventional system and power-rate reaching law as it provides faster convergence and negligible chattering in the system output performance. This increases the efficiency of the MR damper system even in the presence of time-varying transmission delay in both the channels and matched uncertainty.

7.2 Results of LNG storage tank system with deterministic and time-varying transmission delay

In the previous section, a hypothetical example of the MR damper system is used to have better clarity of time delay compensation technique in the presence of system uncertainties. In this section, a real-time example of the MR damper test-bed platform with an LNG tank system is considered to examine the robustness of the proposed control algorithm with the same delay parameters in the presence of system uncertainties. The system matrices of the MR damper system with a real-time LNG tank testbed platform are given in Annexure-III. In this case, the sliding gain is computed to be $C_g = 1.0e-0.6*$
$[-0.0205\ \ 0.0108\ \ 0.0110\ \ -0.0017\ \ -0.0172\ \ -0.0001\ \ -0.3722\ \ -0.4628]$
for $Q = diag(10, 10, 10, 10, 10, 10, 10, 10)$ and $R = 2$.

The time variation of displacement at the isolation level (which is directly connected to the MR damper commended by the control algorithm) of the LNG tank system is shown in Figs. 10 and 11 under deterministic and time-varying transmission delay respectively. The comparative analysis is shown in the absence of delay, presence of delay, and time delay compensation technique. From the results, it is observed that the response is amplified if the transmission delay is not taken into account explicitly and thereby the effectiveness of the controller is reduced. The amplification in response is more for the system with time-varying transmission delay as compared to deterministic time delay. Further, it is also seen that the isolator displacement history of the conventional LNG tank system (in absence of delay) and LNG tank system with time delay compensation matches closely. This shows the effectiveness of the time delay compensation technique and robustness of the proposed control scheme when it is connected through wired or wireless communication networks or when the controller is located at the geographical distance from the system.

The superstructure responses i.e., convective and impulsive displacements are plotted in Figs. 12, 13, 14, 15 for both type of delays. It is interesting to observe that the amplification of response due to negligence of transmission delay is relatively smaller. Comparing Figs. 12 and 13, the amplification of response is found to be the same for the system with time-varying and deterministic transmission delay. A similar trend is observed for an impulsive response as shown in Figs. 14 and 15. This shows that the superstructure response is relatively less sensitive to the effect of transmission delay of any type.

8 Conclusion

In this paper, the designing of the robust sliding mode control strategy using modified reaching law and a novel sliding variable in the presence of time-varying or deterministic transmission delay and matched uncertainty is presented. The novel sliding variable is designed using the concept Padé approximation technique while time-varying transmission delay is modeled using an exponential distribution. The designed sliding variable uses compensated state information that nullifies the effect of transmission delay and ensures finite-time convergence of state variables in the presence of system uncertainties. The proposed control algorithm is also analyzed for its stability in the presence of system uncertainties by the Lyapunov approach. The compound equation of motion of the hybrid protective structural system is formulated and solved by the Runga-Kutta method for a typical massive storage structure under seismic excitation. From the simulation results, the following conclusions are drawn

1. Neglecting time-varying transmission delay for a real-time system in the presence of the communication medium, the effectiveness of the controller reduces significantly.
2. The degradation of system performance is more for the system with time-varying transmission delay as compared to deterministic time delay,
3. The superstructure response is found less sensitive to the effect of transmission delay.
4. The proposed control algorithm designed using modified reaching law is found a most suitable approach for real-time application where chattering and time-varying transmission delay is considered as major concerns for the degradation of system performance.

5. The control law derived using modified reaching law with the novel sliding variable is observed more robust than a conventional system and power-rate reaching law. The robustness is attributed to the faster convergence, transmission delay compensation, and negligible chattering in the system performance.

In the future, the proposed control strategy can be extended for single and multiple packet loss in the presence of system uncertainties.

8.1 Annexure-1

The specifications of MR damper system are: $\alpha_{0a} = 8.7$kN/m/V; $\gamma = 496m^{-2}$; $\alpha_{0b} = 6.40\,kN/m/V$; $\beta = 496\,m^{-2}$; $c_{0a} = 50.30\,kN\,s/m$; $\eta = 195\,sec^{-1}$; $c_{0b} = 8.3\,kN\,s/m/V$; $k_0 = 0.0054\,kN/m$; $c_{1a} = 8106.2\,kN\,s/m$; $k_1 = 0.0087\,kN/m$; $c_{1b} = 7807.9\,kN\,s/m/V$; $x_0 = 0.18\,m$; $A_d = 810.50$; $n = 2$.

8.2 Annexure-II

$$A = \begin{bmatrix} [0] & [I] \\ -[M_s]^{-1}[K_s] & -[M_s]^{-1}[C_s] \end{bmatrix}, B = \begin{bmatrix} 0 \\ [M_s]^{-1}\{\delta\} \end{bmatrix},$$

$$C = \begin{bmatrix} [I] & [0] \\ -[M_s]^{-1}[K_s] & -[M_s]^{-1}[C_s] \end{bmatrix}, D = \begin{bmatrix} \{0\} \\ [M_s]^{-1}\{\delta\} \end{bmatrix},$$

$$D_s = \begin{bmatrix} 0 \\ -\{\Gamma\} \end{bmatrix}$$

where $[M_s]$, $[K_s]$, $[C_s]$ are the mass, stiffness and damping matrices of the LNG tank system. $\{\delta\}$ is vector of location of control devices and $\{\Gamma\}$ is the influence coefficient vector.

8.3 Annexure-III

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \\ -491.1092 & -44.5056 & -0.1119 & 9.8696 & -2.5715 & -0.1326 & -0.0018 & 0.6283 \\ -126.3822 & -224.7526 & -0.1119 & 9.8696 & -0.6618 & -0.6696 & -0.0018 & 0.6283 \\ -126.3822 & -44.5056 & -0.5139 & 9.8696 & -0.6618 & -0.1326 & -0.0081 & 0.6283 \\ 126.3822 & 44.5056 & 0.1119 & -9.8696 & 0.6618 & 0.1326 & 0.0018 & -0.6283 \end{bmatrix},$$

$$B = 1.0e-08 * \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.6600 \\ 0.6600 \\ 0.6600 \\ -0.6600 \end{bmatrix}, D = 1.0e-08 * \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.6600 \\ 0.6600 \\ 0.6600 \\ -0.6600 \end{bmatrix},$$

$$D_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \\ -491.1092 & -44.5056 & -0.1119 & 9.8696 & -2.5715 & -0.1326 & -0.0018 & 0.6283 \\ -126.3822 & -224.7526 & -0.1119 & 9.8696 & -0.6618 & -0.6696 & -0.0018 & 0.6283 \\ -126.3822 & -44.5056 & -0.5139 & 9.8696 & -0.6618 & -0.1326 & -0.0081 & 0.6283 \\ 126.3822 & 44.5056 & 0.1119 & -9.8696 & 0.6618 & 0.1326 & 0.0018 & -0.6283 \end{bmatrix}.$$

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