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Multistability and hidden chaotic attractors in a new simple 4-D chaotic system with chaotic 2-torus behaviour

Jay Prakash Singh[1] · B. K. Roy[1]

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Abstract The finding of hidden attractors in a chaotic/hyperchaotic system is more important, interesting and difficult than a self-excited attractor. This paper reports a new simple 4-D chaotic system with no equilibrium point and having hidden attractors with the coexistence of attractors (i.e. multistability). The proposed system has a total of eight terms including only one nonlinear term and hence, it is simple. It has only one bifurcation parameter. The system has complex dynamical behaviour. It exhibits 3-torus, 2-torus, chaotic and chaotic 2-torus behaviours. The coexistence of hidden attractors in the proposed system is analysed with phase portrait, Finite time Lyapunov spectrum, bifurcation diagram, Poincare map, instantaneous phase plot and 0–1 test. The system has chaotic behaviour with $(+, 0, -, -)$ sign of distinct Lyapunov exponents although the Jacobian matrix has rank less than four. Electronic circuit realisation is shown to validate the chaotic behaviour of the proposed system.

Keywords Hidden attractors · 4-D chaotic system · Chaotic 2- torus · Simple chaotic system · Multistability

1 Introduction

Advancement in the numerical methods and computer simulation techniques has helped us to develop chaotic systems with the desired characteristics. The available chaotic systems can be classified into two main parts: self-excited attractors or hidden attractors [1–4]. The examples of self-excited attractors are: Lorenz [5], Rossler [6], Chen [7], Lu systems [8] and system in [9–13]. The hidden attractors may be grouped into three parts; these are the system with (1) no equilibrium point [14–17], (2) only stable equilibrium points [18,19] and (3) system with many equilibria [20–22]. However, the basin of attraction may touch the equilibrium point in case of a system with many equilibria [20–22]. The dynamical systems with no equilibrium point are first introduced in Nose-Hoover [23,24] and Sommerfield effect [25]. The finding of the hidden attractors is difficult compared with the self-excited attractors because the behaviours of hidden attractors do not depend on the location of the equilibrium points [1,2,15,26–28]. The study of hidden attractors is important because it can lead to unexpected and potentially disastrous behaviours in many physical systems [29,30].

Few papers are available on 4-D or 5-D chaotic/hyperchaotic systems with no equilibrium point. The available 4-D or 5-D chaotic/hyperchaotic systems with no equilibrium point are shown in Table 1.

It is clear from Table 1 that no simple 4-D system is reported with no equilibrium point which has coexistence of asymmetric attractors. The system in [42] has a total of seven terms with two nonlinear terms. The new system has eight terms with only one nonlinear term which is simpler than the available similar 4-D chaotic/hyperchaotic systems.

The paper reports a new simple 4-D dissipative autonomous chaotic system with no equilibrium point and coexistence of attractors.

Following points explain the unique and interesting properties of the system.

1. The system consists of a total eight terms including only one nonlinear term and one constant term. Therefore, the

✉ Jay Prakash Singh
Jayprakash1261@gmail.com

B. K. Roy
bkr_nits@yahoo.co.in

1 Department of Electrical Engineering, National Institute of Technology Silchar, Silchar, Assam 788010, India

Table 1 Categorisation of reported 4-D or 5-D chaotic/hyperchaotic systems with no equilibrium points

Sl. No.	4-D/5-D System	Nature of system	References of papers
1.	4-D Chaotic system	No equilibrium point attractors with coexistence of attractors	[31]
		No equilibrium point attractors with asymmetric multistability	this work
		Multi-scroll or multi-attractor with no equilibrium point	[32,33]
2.	4-D Hyperchaotic system	No equilibrium point attractors	[34–39]
		Multi-scroll or multi-attractor with no equilibrium points	[40,41]
		Coexistence of attractors with no equilibrium points	[42]
3.	5-D Hyperchaotic system	No equilibrium point attractors	[43]
		Coexistence of attractors with no equilibrium points	[44]

proposed system is a simple compared with the similar type of systems available in the literature.
2. The system has coexistence (multistability) of attractors.
3. The findings of the bifurcation diagram and Lyapunov spectrum reveal that the system has chaotic 2-torus behaviour for the bifurcation parameter. This type of behaviour in a chaotic system is rare in the literature.
4. The system exhibits only quasi-periodic (2-torus, 3-torus) behaviour other than the chaotic behaviour.
5. Although the rank of the Jacobian matrix of the system is less than four, but the system has four distinct $(+, 0, -, -)$ values of Lyapunov exponents for some values of the parameter.

The above points also indicate the novelty and contribution of the paper.

In 2011, Sprott [45] proposed some standard for the publication of a new chaotic system. The new system should satisfy at least one of the following criteria [21,45]:

1. The system should model an important unsolved problem in nature and analyse the problem.
2. The system should exhibit some novel behaviour.
3. The system should be simpler than the similar available system in the literature.

The new system satisfies the second and third criteria.

The outline of the paper is as follows. Section 2 describes the dynamics of the proposed 4-D chaotic system. Some basic properties of the new system are analysed in Sect. 3. Section 4 represents the findings of the numerical tools used for analyses of the system. Lyapunov spectrum analysis and bifurcation diagram of the proposed system are presented in Sect. 6. Circuit design and its results of the proposed system are given in Sect. 7. The conclusions of the paper are given in Sect. 8.

2 Dynamics of the new 4-D chaotic system with no equilibrium point

This section describes the dynamics of the new 4-D chaotic system. We consider the following simple 4-D chaotic system for our study.

$$\begin{aligned}
\dot{x}_1 &= -x_2 - x_3 \\
\dot{x}_2 &= x_1 + x_4 \\
\dot{x}_3 &= a\left(1 - x_2^2\right) - bx_3 \\
\dot{x}_4 &= -cx_2
\end{aligned} \tag{1}$$

where a, b and c are the constant positive parameters and x_1, x_2, x_3 and x_4 are the state variables of the system. Here, the parameters $a = 0.5$ and $b = 0.6$ are kept fixed whereas c is considered as the only bifurcation parameter. The system has chaotic behaviour for $c = 0.0303$ where the Lyapunov exponents are $L_i = (0.0568, 0, -0.0003, -0.6565)$ and Lyapunov dimension (Kaplan–Yorke dimension) is $D_{KY} = 3.0860$. The dynamic behaviours of the system for other values of the bifurcation parameter are discussed in Sect. 5. Here, all the simulations are carried out with the initial conditions $x(0) = (0.01, 0.001, 0.001, 0, 0.1)^T$ using ode45 solver in MATLAB simulation environment.

Detailed theoretical and numerical analyses of system (1) are presented in the subsequent sections.

3 Some basic properties of system (1)

This section analyses some common basic properties of system (1).

3.1 Dissipative, existence of the attractor and symmetrical property

It is not hard to prove that the system is a dissipative chaotic system. The divergence of the vector field of system (1) is given as

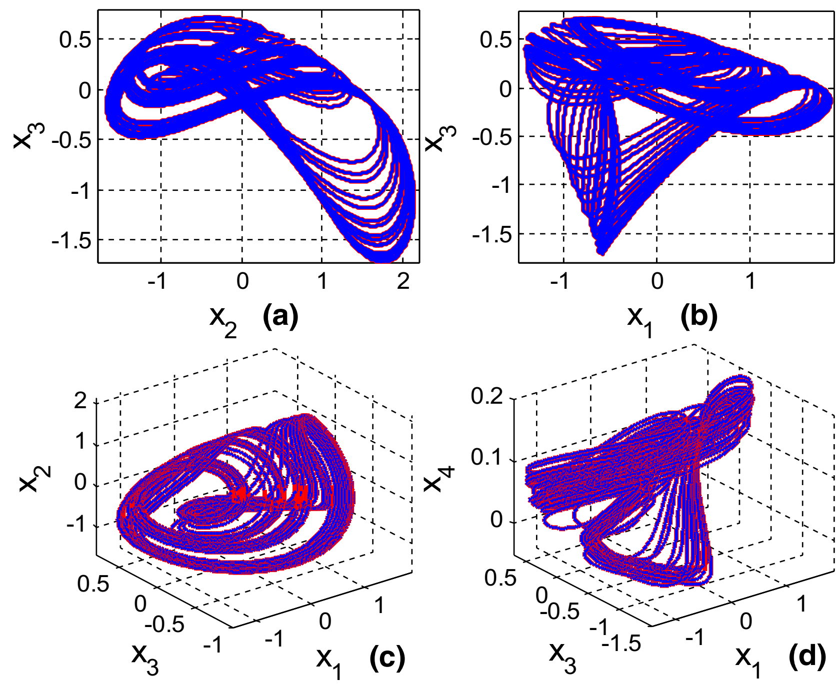

Fig. 1 Chaotic attractors of system (1) with $c = 0.0303$ and $x(0) = (0.01, 0.001, 0.001, 0, 0.1)^T$

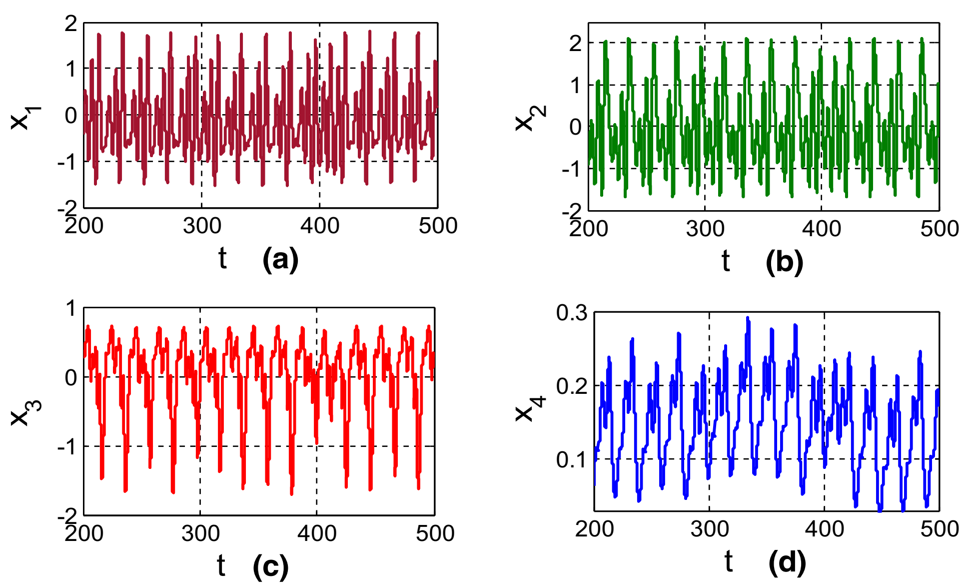

Fig. 2 Time responses of states of system (1) with $c = 0.0303$ and $x(0) = (0.01, 0.001, 0.001, 0, 0.1)^T$

$$\nabla v = \frac{\partial \dot{x}_1}{x_1} + \frac{\partial \dot{x}_2}{x_2} + \frac{\partial \dot{x}_3}{x_3} + \frac{\partial \dot{x}_4}{x_4} = -b \tag{2}$$

Thus, system (1) is dissipative chaotic flow for $b > 0$. System (1) has rate of state space contraction equal to -0.6 for $b = 0.6$. Therefore, attractors may exist for the new system. System (1) has asymmetry to its coordinates, plane and spaces. The system is not invariant under any coordinate, plane and space transformations. The boundedness of system (1) is proved using the approach given in [46].

Theorem 1 *Suppose the parameters of the system are positive, then all orbits of system* (1) *will be confined in a bounded region.*

Proof Consider a Lyapunov function candidate as

$$v(x) = \frac{1}{2}\left(cx_1^2 + cx_2^2 + x_3^2 + x_4^2\right) \tag{3}$$

Using system dynamics (1), the time derivative of (3) can be written as

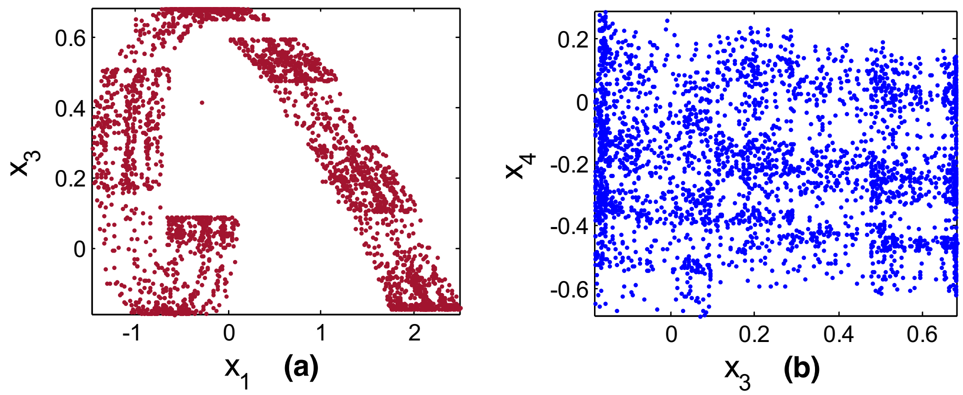

Fig. 3 Poincaré maps of system (1) with $x_2 = 0$ in: **a** $x_1 - x_3$ plane and **b** $x_3 - x_4$ plane

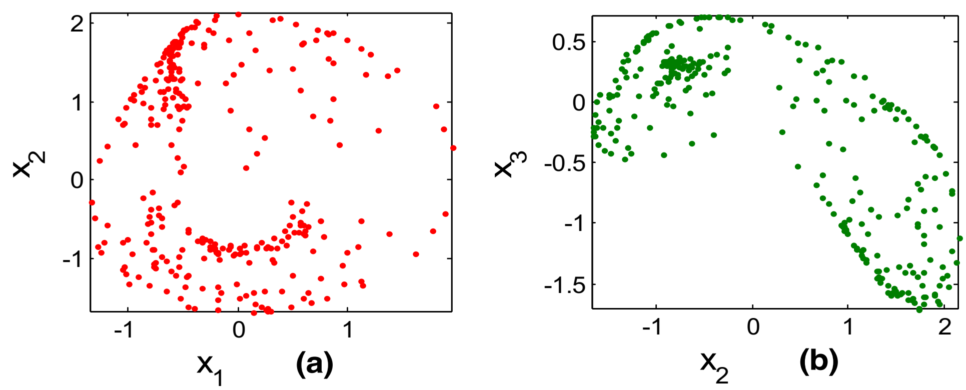

Fig. 4 Poincare maps of system (1) with $x_4 = 0$ in: **a** $x_1 - x_2$ plane and **b** $x_2 - x_3$ plane

$$\dot{v}(x) = -\left(\frac{c}{2\sqrt{b}}x_1 + \sqrt{b}x_3\right)^2 + \left(\frac{c}{2\sqrt{b}}x_1\right)^2 - \left(\sqrt{ax_3}x_2 - \frac{a}{2y}\sqrt{x_3}\right)^2 + \left(\frac{a}{2y}\sqrt{x_3}\right)^2 \tag{4}$$

Let, $S_0 > 0$ be the sufficiently large region where all the state trajectories satisfy $v(x) = S$ for $S > S_0$ along with the following condition

$$\left[\left(\frac{c}{2\sqrt{b}}x_1 + \sqrt{b}x_3\right)^2 + \left(\sqrt{ax_3}x_2 - \frac{a}{2y}\sqrt{x_3}\right)^2\right] > \left[\left(\frac{c}{2\sqrt{b}}x_1\right)^2 + \left(\frac{a}{2y}\sqrt{x_3}\right)^2\right]$$

On the hypersurface $\{x|v(x)\} = S$ with $S > S_0$ we can write as $v(x) < 0$. Thus, we can say that the set $\{x|v(x) \le S\}$ $\{x|v(x) \le S\}$ is a confined region for all the trajectories of system (1).

3.2 Equilibrium point

The equilibrium point of system (1) can be obtained by equating derivative of the each state variable to zero i.e. $\dot{x}_1 = 0, \dot{x}_2 = 0, \dot{x}_3 = 0, \dot{x}_4 = 0$ and solving them to find the solution. It is observed that there is no solution of system (1) and hence, system (1) has no equilibrium point. Thus, we confirm that system (1) has no equilibrium point.

4 Dynamical behaviour of the new system

In this section, the dynamic behaviours of the proposed system are analysed using various numerical methods.

4.1 Chaotic attractors and Poincaré map

The chaotic attractors and time response of system (1) with $c = 0.0303$ are shown in Figs. 1 and 2, respectively. The Poincaré maps of system (1) with $c = 0.0303$ across different sections of planes are shown in Figs. 3 and 4. The random location of dots on the Poincare maps (Figs. 3, 4) indicates the chaotic behaviour of the new system.

4.2 Instantaneous phase (IP) and frequency spectrum

To validate the existence of chaotic attractors in system (1), the instantaneous phase (Ø) is plotted. The Hilbert transformation method is used for the generation of instantaneous phase (Ø). The IP of a chaotic signal increases monotonically with respect to time, whereas for a periodic signal it remains constant. Suppose, $s(t)$ is an analytical or a com-

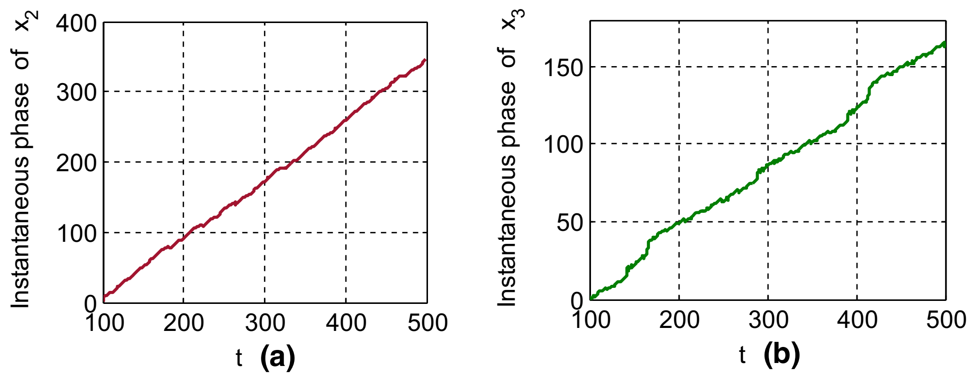

Fig. 5 Instantaneous phase of system (1) with $c = 0.0303$ for: **a** $x_2(t)$ and **b** $x_3(t)$ signal

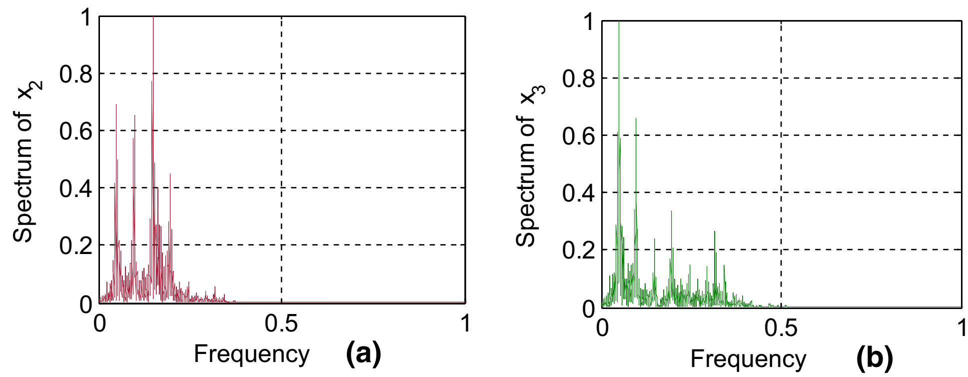

Fig. 6 Frequency spectrum of the signal of system (1) with $c = 0.0303$ for: **a** $x_2(t)$ and **b** $x_3(t)$ signal

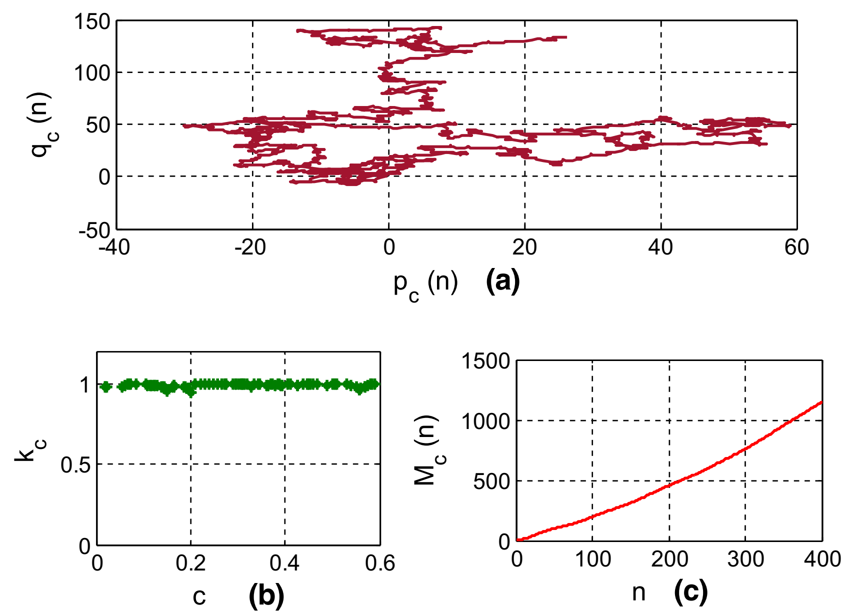

Fig. 7 0-1 test of $x_2(t)$ state of system (1) with $c = 0.111$, $x(0) = (0.01, 0.001, 0.001, 0.1)^T$, $\Delta t = 1$: **a** translation components $(p_c(n), q_c(n))$, **b** asymptotic growth rate (k_c) and **c** mean square displacement $M_c(n)$

plex signal and is generated using a chaotic signal y(t). The amplitude (A) and phase $(\emptyset)$ of the signal $s(t)$ can be written as [10]:

$$s(t) = y(t) + i\tilde{y}(t) = A(t)\, e^{j\emptyset(t)} \tag{5}$$

$$\text{where} \quad \tilde{y}(t) = \frac{1}{\pi} P.V \left(\int_{-\infty}^{\infty} \frac{y(t')}{t - t'} dt \right) \tag{6}$$

where $P.V$ is the Cauchy principle value in the Hilbert transform (HT) [47]. Here, HT is calculated using the technique given in [47] with the help of MATLAB. The instantaneous phase of $x_2(t)$ and $x_3(t)$ signals of system (1) after ignoring the transient part is shown in Fig. 5. It is clear from Fig. 5 that the increase in the instantaneous phase $(\emptyset)$ of the signal is monotonic with respect to time. This indicates the chaotic behaviour of system (1).

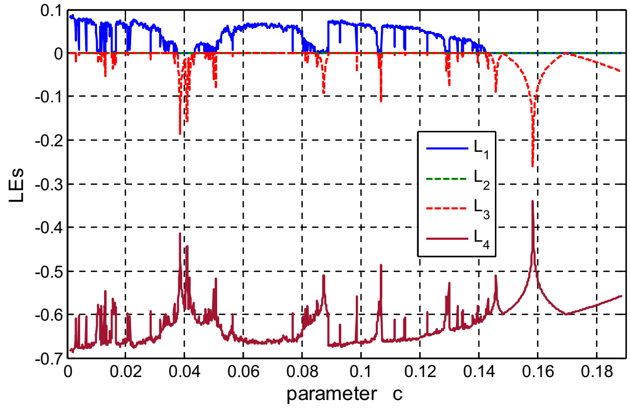

Fig. 8 Finite time Lyapunov spectrum of system (1) with $x(0) = (0.01, 0.001, 0.001, 0.1)^T$

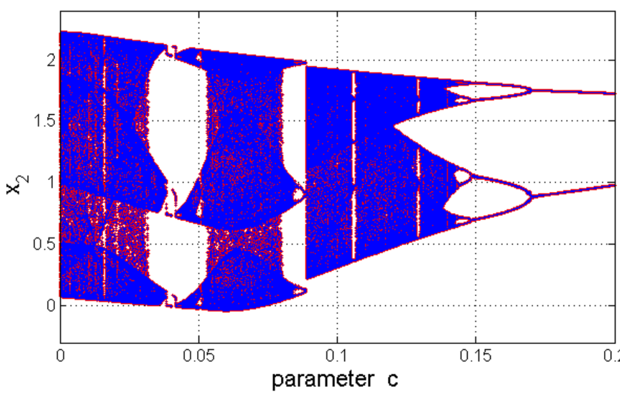

Fig. 9 Bifurcation diagram of system (1) with $x(0) = (0.01, 0.001, 0.001, 0.1)^T$

The frequency spectrum of $x_2(t)$ and $x_3(t)$ of system (1) with $c = 0.0303$ is shown in Fig. 6. The random location of peaks in the frequency spectrum indicates the chaotic nature of the system.

4.3 0–1 Test analysis

It is a binary value (0 or 1) test used to classify the chaotic or periodic nature of a system [48]. In this test, the dynamics of the system is transformed into a space of translation variable and asymptotic growth rate (k_c) of the mean square displacement $M_c(n)$ of the trajectories. The value of k_c define the

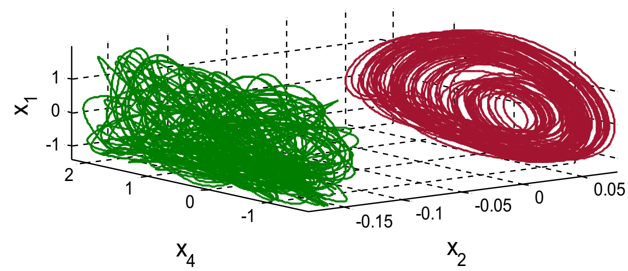

Fig. 10 Coexistence of chaotic attractors of system (1) with $c = 0.0011$ for $x(0) = (0.01, 0.001, 0.001, \pm 0.1)^T$

chaotic or periodic behaviour of the system. The translation variables (p_c, q_c) can be written as [48–50]:

$$\begin{cases} \mathrm{p_c}(\mathrm{n}) = \sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{x}(\mathrm{k}) \cos \mathrm{kc} \\ \mathrm{q_c}(\mathrm{n}) = \sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{x}(\mathrm{k}) \sin \mathrm{kc} \end{cases} \tag{7}$$

where c is an arbitrarily chosen variable in the range $(0 - 2\pi)$ and $x(k)$ is the time series of any state variable of the system [48–50]. For the chaotic nature, $p_c(n)$ and $q_c(n)$ represent a random Brownian like motion, whereas for the periodic solution, the plane of the translation variables is a bounded motion. The mean square displacement $M_c(n)$ obtained using $p_c(n)$ and $q_c(n)$ can be defined as in [48–50].

$$M_c(n) = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \left\{ [p_c(j+n) - p_c(j)]^2 + [q_c(j+n) - q_c(j)]^2 \right\} \tag{8}$$

$M_c(n)$ grows exponentially for the chaotic behaviour, whereas it varies periodically for periodic nature. The asymptotic growth rate (k_c) is defined as given in [48–50]

$$k_c = \lim_{n\to\infty} \frac{\log M_c(n)}{\log n} \tag{9}$$

The output of $k_c \approx 1$ represent the chaotic nature, and $k_c \approx 0$ represents the periodic behaviour. The translation variable (p_c, q_c), asymptotic growth rate (k_c) and mean square displacement $(M_c(n))$ of system (1) with $c = 0.0303$ are shown in Fig. 7. We got $k_c = 0.9971 \approx 1$ for $c = 0.0303$ which indicates chaotic behaviour.

Table 2 Four distinct nature of Lyapunov exponents $(+, 0, -, -)$ for some values of parameter c

Sl. No.	Parameter c	LE1	LE2	LE3	LE4	Summation of LEs
1.	0.0038	0.0771	0	−0.0006	−0.6765	−0.6
2.	0.0041	0.0714	0	−0.0005	−0.6709	−0.6
3.	0.0074	0.0724	0	−0.0005	−0.6719	−0.6
4.	0.0122	0.0678	0	−0.0004	−0.6674	−0.6
5.	0.025	0.0671	0	−0.0004	−0.6667	−0.6

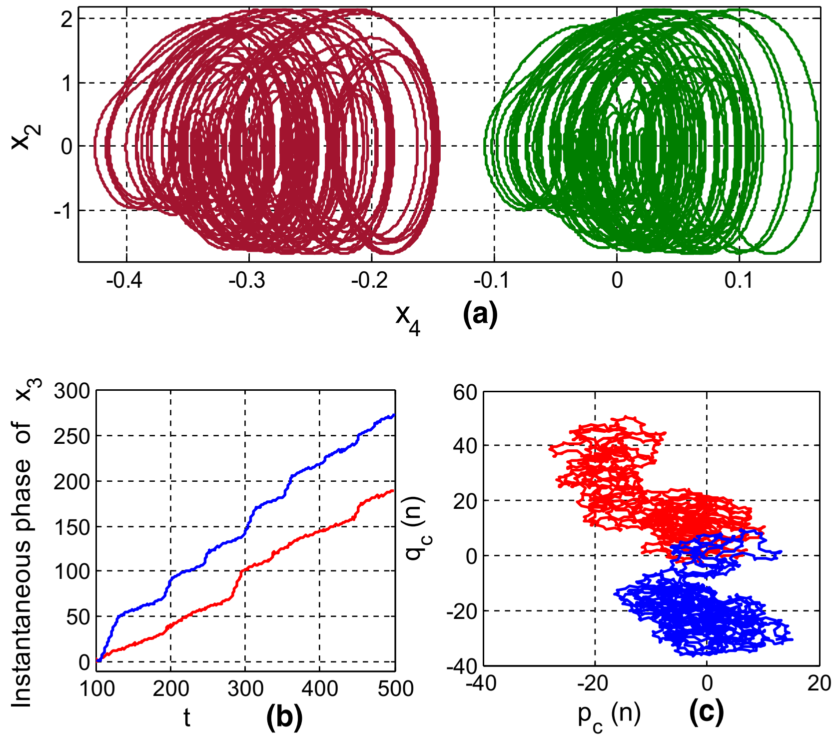

Fig. 11 Coexistence of chaotic attractors of system (1) with $c = 0.0259$ and $x(0) = (0.01, 0.001, 0, \pm 0.1)^T$: **a** phase portrait **b** instantaneous phase plot of x_3 state and **c** translation components of 0-1 test for $x_2(t)$ variable

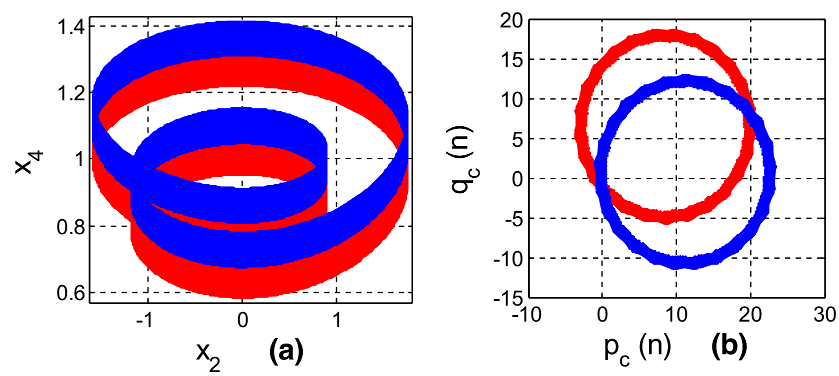

Fig. 12 Coexistence of 3-torus (behaviour) of system (1) with $c = 0.1699$ and $x(0) = (0.01, 0.001, 0, \pm 0.1)^T$ where LEs are $(0, 0, 0, \ -0.5998)$: **a** phase portrait and **b** translation components of 0-1 test for $x_2(t)$ variable

5 Bifurcation and finite time Lyapunov spectrum analyses

The existence of the quasi-periodicity (2-torus, 3-torus), chaotic and chaotic 2-torus nature of the system is discovered using the variation of the bifurcation parameter and keeping other fixed. This is achieved by using Lyapunov spectrum and bifurcation diagrams plot. Here, finite-time Lyapunov exponents (LEs) are calculated by using Wolf et al. algorithm [51] with observation time $T = 20000$, step size $\Delta t = 0.02$ and initial conditions $x(0) = (0.01, 0.001, 0.001, 0.1)^T$ in MATLAB simulation environment. Lyapunov spectrums for parameter c is generated with fixed step size $\Delta c = 0.0001$. The observation time, step size and initial conditions for plotting the bifurcation diagram of the system for bifurcation parameter c is the same as that of Lyapunov spectrum. Lyapunov spectrum and bifurcation diagram of system (1) with $c \in [0.001, 0.19]$ are shown in Figs. 8 and 9, respectively. It is seen from Figs. 8 and 9 that system (1) has chaotic, chaotic 2-torus, 3-torus and 2-torus behaviours for the different values of parameter c. The chaotic 2- torus nature of the system is considered when the sign of Lyapunov exponents is $(+, 0, -, -)$ [52]. The four distinct nature $(+, 0, -, -)$ of Lyapunov exponents of system (1) for some values of some parameter c are given in Table 2. Thus, the system can be considered as a four dimensional for these set of parameters [39].

6 Coexistence of asymmetric hidden chaotic attractors

The proposed system has coexistence (multistability) of asymmetric hidden chaotic attractors with the change in the initial conditions from $x(0) = (0, 0, 0, 0.1)^T$ to $x(0) =$

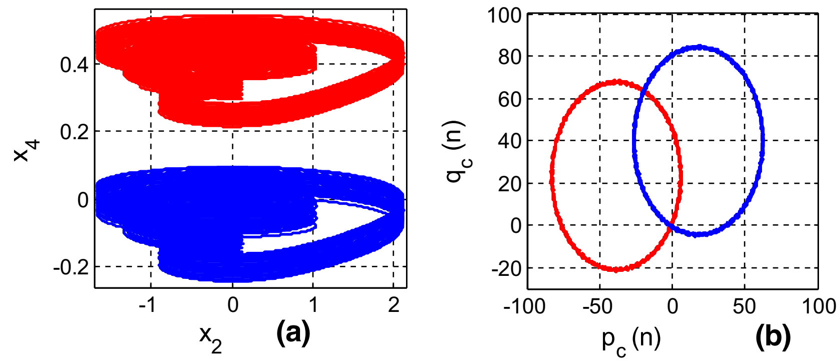

Fig. 13 Coexistence of 2-torus (behaviour) of system (1) with $c = 0.0376$ and $x(0) = (0.01, 0.001, 0.001, \pm 0.1)^T$ where LEs are $(0, 0, -0.0047, -0.5959)$: **a** phase portrait and **b** translation components of 0-1 test for $x_2(t)$ variable

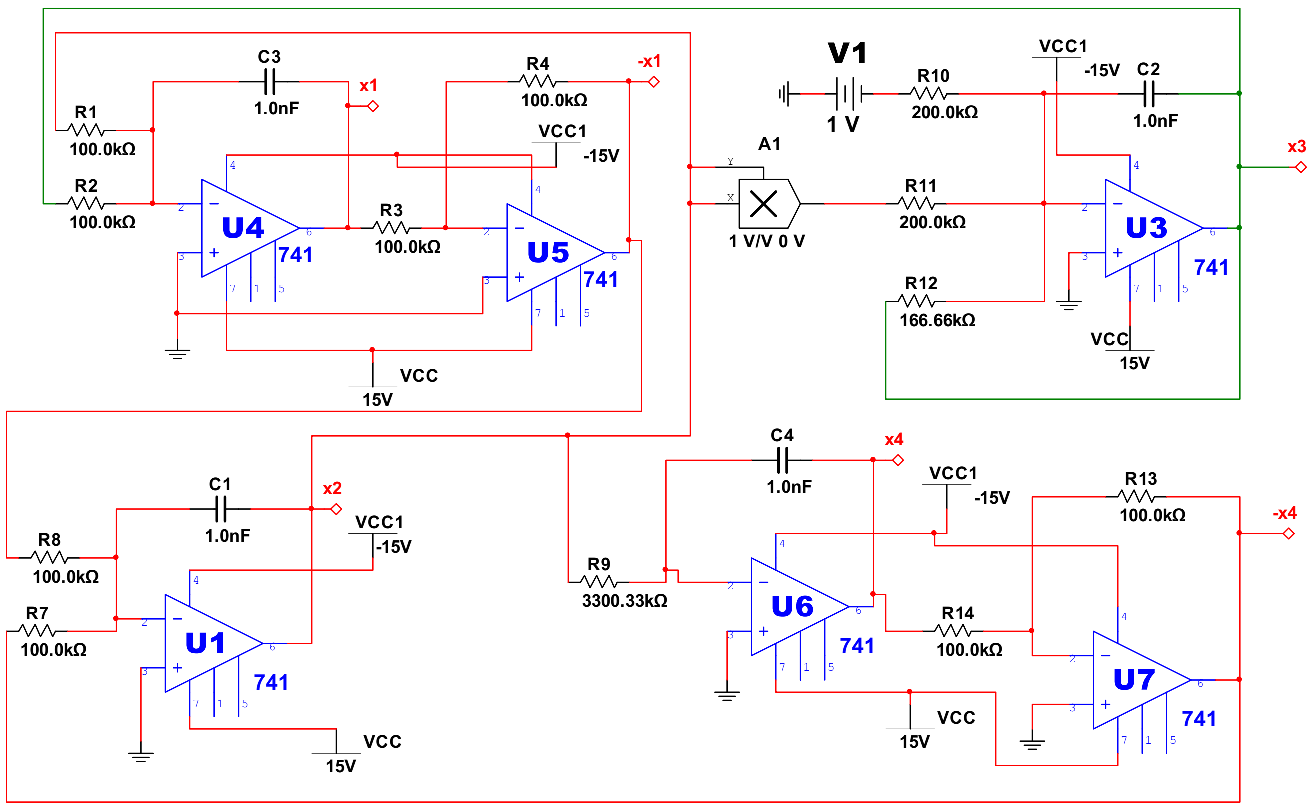

Fig. 14 Circuit implementation of system (1) with $a = 0.5$, $b = 0.5$, $c = 0.0303$

$(0, 0, 0, -0.1)^T$. The system exhibits this phenomenon for all ranges of the bifurcation parameters. The coexistence of the different dynamic behaviours of the new system is presented using phase portrait and translation components of the 0-1 test. The coexistence of chaotic 2-torus attractors of system (1) with $c = 0.0011$ and $c = 0.0259$, and $x(0) = (0.01, 0.001, 0.001, \pm 0.1)^T$ are shown in Figs. 10 and 11, respectively. The 3-torus behaviour of system (1) with $c = 0.1699$ and $x(0) = (0.01, 0.001, 0.001, \pm 0.1)^T$ is shown in Fig. 12. Figure 13 shows 2-torus behavior of system (1) with $c = 0.0376$ and $x(0) = (0.01, 0.001, 0.001, \pm 0.1)^T$

7 Circuit validation

It this section, circuit design and implementation of system (1) are presented to validate the chaotic behaviour of system (1). The circuit design of system (1) with $a = 0.5$, $b = 0.5$, $c = 0.0303$ is shown in Fig. 14. It is seen from Fig. 14 that the circuit is designed with six number of Op-Amp, one multiplier and fewer components (resistors, capacitors). Attractors plot obtained using circuit design of system (1) is shown in Fig. 15. It is seen from Fig. 15 that attractors plot obtained using circuit implementation matches with the results obtained using MATLAB simulation.

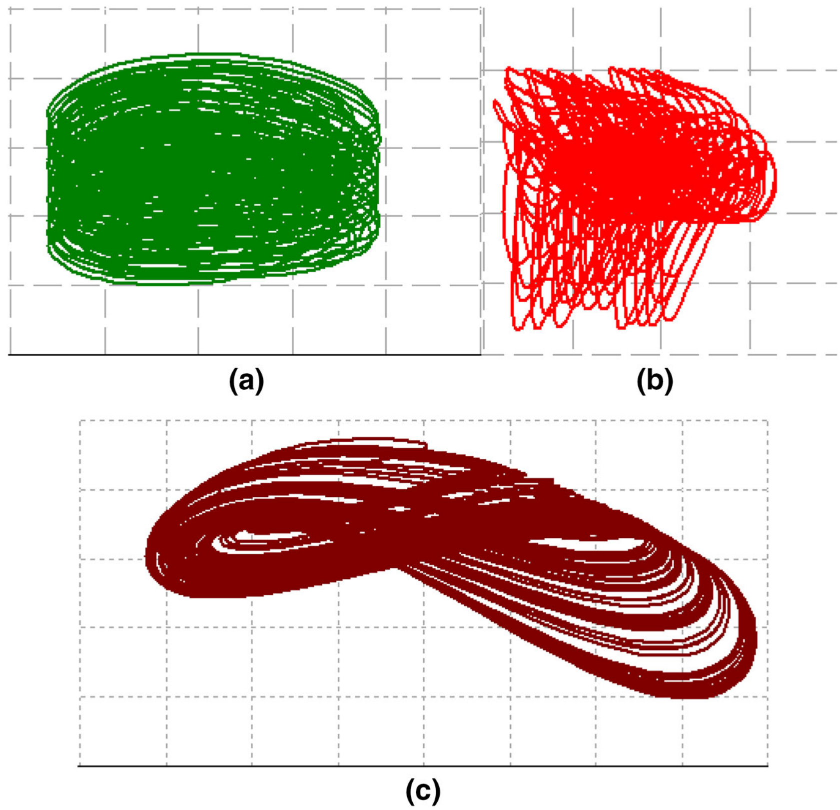

Fig. 15 Chaotic attractors of system (1) using circuit simulation with $c = 0.0303$ across: **a** $x_1 - x_2$, **b** $x_1 - x_3$ and **c** $x_2 - x_3$ plane

8 Conclusions

This paper reports a new simple 4-D chaotic system with no equilibrium point and coexistence of hidden chaotic attractors. The system consists of one bifurcation parameter. The system has a total of eight terms including only one nonlinear term. Therefore, the system is simple compared with the other similar available 4-D chaotic/hyperchaotic systems. The rank of the Jacobian matrix of the new system is less than four. However, the system has four distinct Lyapunov exponents with $(+, 0, -, -)$ sign for some values of the parameters. Thus, the system can be considered as four-dimensional for these set of parameter. Further, the system has chaotic 2-torus, chaotic, 3-torus and 2-torus behaviours. The system exhibits multistability of the chaotic attractor, 3-torus and 2-torus behaviours for the bifurcation parameter. The complex and rich dynamical behaviours of the system are analysed using theoretical and numerical techniques like phase portrait, instantaneous phase plot, Poincaré map, bifurcation diagram, Lyapunov spectrum and 0–1 test. The results obtained by circuit design and implementation of the proposed system validate the MATLAB simulation results.

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