Int. J. Dynam. Control (2018) 6:873–880
https://doi.org/10.1007/s40435-017-0323-9

A general characterization of the stochastic optimal combined control of mean field stochastic systems with application

Shahlar Meherrem[1] · Mokhtar Hafayed[2] · Deniz H. Gucoglu[1] · Saban Eren[1]

Received: 7 January 2017 / Revised: 24 March 2017 / Accepted: 30 March 2017 / Published online: 12 April 2017

Abstract In this paper, a general characterization of the optimal stochastic combined control for mean-field jump-systems is derived by applying mixed convex-spike perturbation method. The diffusion coefficient depends on the continuous control variable and the control domain is not necessary convex. In our combined mean-field control problem, we discuss two classes of jumps for the state processes, the inaccessible jumps which caused by Poisson martingale measure and the predictable ones which caused by the singularity of the control variable. Markowitz's mean–variance portfolio selection problem with intervention control is discussed.

Keywords Singular stochastic control · Maximum principle · Second-order variational equation · Convex-spike perturbations · Markowitz's mean–variance portfolio

Mathematics Subject Classification 60H10 · 93E20

✉ Shahlar Meherrem
maharramsf@yahoo.com

Mokhtar Hafayed
hafa.mokh@yahoo.com

Deniz H. Gucoglu
denizhasan09@gmail.com

Saban Eren
ur.saba16@gmail.com

1 Department of Mathematics, Yasar University, University aven, Agacli Yol No. 35-57, Izmir, Turkey

2 Laboratory of Applied Mathematics, Biskra University, PO. Box 145, 07000 Biskra, Algeria

1 Introduction

We consider a class of combined control problem for a stochastic jump-systems governed by controlled nonlinear mean-field stochastic differential equations (MFSDEJs) driven by Poisson martingale measures and Brownian motion of the form

$$\begin{aligned} dX^{u,\eta}(t) &= f(t, X^{u,\eta}(t), E(X^{u,\eta}(t)), u(t))dt \\ &\quad + \sigma(t, X^{u,\eta}(t), E(X^{u,\eta}(t)), u(t))dB(t) \\ &\quad + \int_{\Theta} g(t, X^{u,\eta}(t_-), u(t), z)N(dz, dt) \\ &\quad + G(t)d\eta(t), \end{aligned} \tag{1}$$

$$X^{u,\eta}(0) = X_0,$$

where f, σ, g and $G(\cdot)$ are given deterministic functions, $B(\cdot)$ is a standard Brownian motion, $N(\cdot,\cdot)$ is a Poisson martingale measure, $\eta(\cdot)$ is the singular part of the control. The control variable consists of a combination of continuous stochastic control $u(\cdot)$ and a singular control $\eta(\cdot)$.

The expected cost on the time interval $[0, T]$ is defined by

$$\begin{aligned} &J_0(X_0, u(\cdot), \eta(\cdot)) \\ &\quad = E\Big\{\int_0^T \ell(t, X^{u,\eta}(t), E(X^{u,\eta}(t)), u(t))dt \\ &\qquad + h(X^{u,\eta}(T), E(X^{u,\eta}(T))) \\ &\qquad + \int_{[0,T]} \mathcal{M}(t)d\eta(t)\Big\}, \end{aligned} \tag{2}$$

where ℓ, h and $\mathcal{M}(\cdot)$ are given maps and $\int_{[0,T]} \mathcal{M}(t)d\eta(t)$ called the intervention cost.

An admissible control $(u^*(\cdot), \eta^*(\cdot))$ is called optimal if it satisfies

$$J_0\left(X_0,u^*(\cdot),\eta^*(\cdot)\right)=\inf_{(u(\cdot),\eta(\cdot))\in\mathcal{A}_1\times\mathcal{A}_2([0,T])}J_0\left(X_0,u(\cdot),\eta(\cdot)\right). \tag{3}$$

Mean-field stochastic control problems have been investigated by many authors, see for instance, [1–14]. Mean-field type stochastic maximum principle for optimal control under partial information has been investigated in Wang et al. [1]. Maximum principle for mean-field type stochastic differential equations with correlated state and observation noises have been established in Zhang [2]. Stochastic optimal control of mean-field jump-diffusion systems with delay has been studied by Meng and Shen [3]. A general necessary and sufficient conditions of optimality and near-optimality for continuous-singular control for mean-field SDE have been established in Hafayed and Abbas [4]. Under partial information, necessary and sufficient conditions for optimal control for stochastic systems driven by Lévy processes have been derived by Hafayed et al. [5]. Maximum principle for optimal singular control of mean-field stochastic systems governed by Lévy processes, associated with Teugels martingales measures have been investigated by Hafayed et al. [6]. Optimality necessary and sufficient conditions for singular control of mean-field forward-backward stochastic have been derived by Hafayed [7]. A McKean-Vlasov optimal mixed regular-singular control problems, for nonlinear stochastic systems with Poisson jump processes have been studied in Hafayed et al. [8]. A general mean-field type maximum principle was introduced in Buckdahn et al. [9]. A sufficient conditions for optimal control of mean-field SDEs have been obtained by Shi [10]. A general stochastic maximum principle for optimal control for mean-field jump diffusions was proved in Hafayed and Abbas [11]. Under the conditions that the control domains are convex, a various local maximum principles have been studied in [12–14]. Feedback control problems for stochastic systems have been investigated in [15,16].

The stochastic control problems have attracted much attention because of their practical applications in many areas such as economics and finances. The optimal combined control problems with applications have been studied by many authors including [17–20]. Optimal intervention control problem with application in the exchange rate has been studied by Mundaca and Øksendal [17]. A good account on stochastic optimal control for jump diffusions and mixed singular stochastic control problems in jump-systems with applications in finance can be found in [18,19]. Stochastic maximum principle for near-optimal singular control of jump-systems has been derived by Hafayed and Abbas [20]. The optimal singular control problems have been considered by many authors, see for instance [21–24] and the references therein. The general necessary conditions for optimal control of stochastic systems with jumps have been investigated in Tang and Li [25].

Our purpose in this paper is to establish a general characterization of the optimal combined control of mean-field jump-systems by maximum principle approach, where the coefficients of the system and the performance functional depend not only on the state process but also its marginal law of the state process through its expected value. The diffusion coefficient depends on the control variable and the control domain is not assumed to be convex. In our mean-field stochastic control problem (1–2), the predictable and inaccessible representations of the jumps are studied. Markowitz's mean–variance portfolio selection problem control with intervention control is studied to illustrate our theoretical results.

The rest of this paper is organized as follows. The assumptions, notations and some basic definitions are given in Sect. 2. Section 3 is devoted to prove our main result. As an illustration, Markowitz's mean–variance portfolio selection problem with interventions is discussed in Sect. 4.

2 Statement of the control problem

Let $T>0$ be a fixed time horizon and $(\Omega,\mathcal{F},\mathcal{F}_t,\mathbb{P})$ be a filtered probability space equipped with a $\mathbb{P}-$ completed right continuous filtration on which a *one*$-$dimensional Brownian motion $B=(B(t))_{t\in[0,T]}$ is defined. Let μ be a homogeneous $\mathcal{F}_t$-Poisson point process independent of $B(\cdot)$. We denote by $\widetilde{N}(dz,dt)$ the random counting measure induced by μ, defined on $\Theta\times\mathbb{R}_+$, where Θ is a fixed nonempty subset of $\mathbb{R}$ with its Borel $\sigma-$ field $\mathcal{B}(\Theta)$. Further, let $m(dz)$ be the local characteristic measure of μ, i.e. $m(dz)$ is a σ-finite measure on $(\Theta,\mathcal{B}(\Theta))$ with $m(\Theta)<+\infty$. We denote by $\mathbb{L}^2_{\mathcal{F}}([0,T];\mathbb{R})=\{f(\cdot)$ is an $\mathcal{F}_t-$adapted $\mathbb{R}-$valued measurable process on $[0,T]$ such that $E\int_0^T|f(t)|^2dt<\infty\}$ and $\mathbb{M}^2_{\mathcal{F}}([0,T];\mathbb{R})=\{f(\cdot,\cdot)$ is an $\mathcal{F}_t-$adapted $\mathbb{R}-$valued measurable process on $[0,T]\times\Theta$ such that $E\int_0^T\int_\Theta|f(t,z)|^2m(dz)dt<\infty\}$. We then define $N(dz,dt)=\widetilde{N}(dz,dt)-m(dz)dt$, where $N(\cdot,\cdot)$ is Poisson martingale measure on $\mathcal{B}(\Theta)\times\mathcal{B}(\mathbb{R}_+)$ with local characteristics $m(dz)dt$. We denote by I_A the indicator function of A and $X^{u,\eta}(t_-)=\lim_{s\to t,s<t}X^{u,\eta}(s)$, $t\in[0,T]$. We assume that $\mathcal{F}_t$ is $\mathbb{P}-$augmentation of the natural filtration $(\mathcal{F}_t^{(B,N)})_{t\in[0,T]}$ defined as follows

$$\mathcal{F}_t^{(B,N)}=\sigma\{B(s):0\le s\le t\}\vee\sigma\left\{\int_0^s\int_A N(dz,dr):0\le s\le t,A\in\mathcal{B}(\Theta)\right\}\vee\mathcal{F}_0,$$

where $\mathcal{F}_0$ denotes the totality of $\mathbb{P}-$null sets, and $\mathcal{F}_1\vee\mathcal{F}_2$ denotes the σ-field generated by $\mathcal{F}_1\cup\mathcal{F}_2$. Consider the following sets: $\mathbb{A}_1$ is a nonempty subset of $\mathbb{R}$ and $\mathbb{A}_2=\mathbb{R}^+$.

An admissible control is a pair $(u(\cdot), \eta(\cdot))$ of measurable $\mathbb{A}_1 \times \mathbb{A}_2-$ valued, $\mathcal{F}_t^B-$adapted processes, such that

(1) $\eta(\cdot)$ is stochastic process of bounded variation, non-decreasing continuous on the left with right limits and $\eta(0_-) = 0$. The process $\eta(\cdot)$ is called intervention control, where $\eta(t_-) = \lim_{s\to t, s<t} \eta(s)$, $t > 0$.

(2) $E\left[\sup_{t\in[0,T]} |u(t)|^2 + |\eta(T)|^2\right] < \infty$.
Notice that the jumps of a singular control $\eta(\cdot)$ at any jumping time t_j denote by $\Delta\eta(t_j) \triangleq \eta(t_j)-\eta(t_{j-})$ and we define the continuous part of the singular control by $\eta^{(c)}(t) = \eta(t) - \sum_{0\leq t_j\leq t} \Delta\eta(t_j)$, i.e., the process obtained by removing the jumps of $\eta(t)$. We denote $\mathcal{A}_1 \times \mathcal{A}_2$ ([0, T]) the set of all admissible controls. The corresponding state processes, solution of MFSDEJs-(1) is denoted by $X^*(t) = X^{u^*,\xi^*}(t)$.

Throughout this paper, we distinguish between the jumps obtained by the singular control $\eta(\cdot)$ at any jumping time t defined by $\Delta_\eta X^{u,\eta}(t) = G(t)\Delta\eta(t) = G(t)(\eta(t) - \eta(t_-))$ and the jumps of $X^{u,\eta}(t)$ obtained by the Poisson martingales measure $\widetilde{N}(z, t)$ given by

$$\Delta_N X^{u,\eta}(t) = \int_\Theta g\left(t, X^{u,\eta}(t_-), u(t_-), z\right) \widetilde{N}(dz, \{t\})$$
$$= \begin{cases} g\left(t, X^{u,\eta}(t_-), u(t), z\right) : \text{if } \eta \text{ has a jump of size } z \text{ at } t. \\ 0 : \text{otherwise,}\end{cases}$$

where $\widetilde{N}(dz, \{t\})$ means the jump in the Poisson random measure occurring at time t. The general jump of the state processes at any jumping time t is given by

$$\Delta X^{u,\eta}(t) = X^{u,\eta}(t) - x^{u,\eta}(t_-)$$
$$= \Delta_\eta X^{u,\eta}(t) + \Delta_N X^{u,\eta}(t).$$

For convenience, we will use the following notations throughout the paper. For $\varphi = f, \sigma, \ell$:

$$\varphi_x(t) = \frac{\partial\varphi}{\partial x}(t, X^*(t), E(X^*(t)), u^*(t)),$$
$$\delta\varphi(t) = \varphi(t, X^*(t), E(X^*(t)), u(t)) - \varphi(t, X^*(t), E(X^*(t)), u^*(t)),$$
$$g_x(t, z) = g_x(t, x(t_-), u(t), z),$$
$$g_{xx}(t, z) = g_{xx}(t, x(t_-), u(t), z),$$
$$\varphi_{xx}(t) = \frac{\partial^2\varphi}{\partial x^2}(t, X^*(t), E(X^*(t)), u^*(t)),$$
$$\mathbb{W}_t(\varphi, y) = \frac{1}{2}\varphi_{xx}(t, X^*(t), E(X^*(t)), u^*(t))y^2,$$
$$\mathbb{W}_{t,z}(g, y) = \frac{1}{2}g_{xx}(t, X^*(t), u^*(t), z)y^2.$$

We denote by

$$\delta H(t) = \Psi^*(t)\delta f(t) + K^*(t)\delta\sigma(t) + \int_\Theta \delta g(t, z)\gamma^*(t, z)m(dz) - \delta\ell(t),$$
$$H_x(t) = f_x(t)\Psi^*(t) + \sigma_x(t) K^*(t) + \int_\Theta g_x(t, z)\gamma^*(t, z)m(dz) - \ell_x(t),$$
$$H_{xx}(t) = f_{xx}(t)\Psi^*(t) + \sigma_{xx}(t) K^*(t) + \int_\Theta g_{xx}(t, z)\gamma^*(t, z)m(dz) - \ell_{xx}(t).$$

Throughout this paper, we assume the following

Hypothesis (H1) The functions $f : [0, T]\times\mathbb{R}\times\mathbb{R}\times\mathbb{A}_1\to\mathbb{R}$, $\sigma : [0, T]\times\mathbb{R}\times\mathbb{R}\times\mathbb{A}_1\to\mathbb{R}$, $\ell : [0, T]\times\mathbb{R}\times\mathbb{R}\times\mathbb{A}_1\to\mathbb{R}$ and $h : \mathbb{R}\times\mathbb{R}\to\mathbb{R}$ are twice continuously differentiable with respect to (x, y). Moreover, f, σ, h and ℓ and all their derivatives up to second-order with respect to (x, y) are continuous in (x, y, u) and bounded.

Hypothesis (H2) The function $g : [0, T]\times\mathbb{R}\times\mathbb{A}_1\times\Theta\to\mathbb{R}$ is twice continuously differentiable in x. Moreover, g_x is continuous, $\sup_{z\in\Theta} |g_x(t, z)| < +\infty$ and there exists a constant $C > 0$ such that

$$\sup_{z\in\Theta}\left|g(t, x, u, z) - g\left(t, x', u, z\right)\right| + \sup_{z\in\Theta}\left|g_x(t, x, u, z) - g_x\left(t, x', u, z\right)\right| \leq C\left|x - x'\right|. \tag{4}$$
$$\sup_{z\in\Theta}|g(t, x, u, z)| \leq C(1 + |x|). \tag{5}$$

Hypothesis (H3) The functions $G(\cdot) : [0, T]\to\mathbb{R}$, and $\mathcal{M}(\cdot) : [0, T]\to\mathbb{R}^+$ are continuous and bounded.
Under the hypotheses (H1–H3), Eq. (1) has a unique solution $x^{u,\eta}(\cdot)$ given by

$$X^{u,\eta}(t) = X_0 + \int_0^t f(s, X^{u,\eta}(s), E(X^{u,\eta}(s)), u(s))ds + \int_0^t \sigma(s, X^{u,\eta}(s), E(X^{u,\eta}(s)), u(s))dB(s) + \int_0^t\int_\Theta g(s, X^{u,\eta}(s_-), u(s), z)N(dz, ds) + \int_{[0,t]} G(s)d\eta(s),$$

such that $E\left[\sup_{t\in[0,T]} |X^{u,\eta}(t)|^n\right] < C_n$, where C_n is a constant depending only on n and the functional $J(X_0, \cdot, \cdot)$ is well defined.

We introduce the adjoint equations involved in the stochastic maximum principle for our mean-field control problem, which are independent to singular control.

(1) First-order adjoint equation:

$$\begin{cases} d\Psi(t) = -\left\{ f_x(t)\,\Psi(t) + E\left(f_y(t)\Psi(t)\right) \right.\\ \quad +\sigma_x(t)\,K(t) + E\left(\sigma_y(t)K(t)\right) + \ell_x(t) + E\left(\ell_y(t)\right) \\ \quad \left. + \int_\Theta g_x(t,z)\,\gamma(t,z)m(dz)\right\} dt + K(t)dB(t) \\ \quad + \int_\Theta \gamma(t,z)N(dt,dz) \\ \Psi(T) = -\left(h_x(X(T), E(X(T)) + E\left(h_y(X(T), E(X(T))\right)\right). \end{cases} \tag{6}$$

(2) Second-order adjoint equation: classical linear backward SDEJs (see [11,25])

$$\begin{cases} dQ(t) = -\left\{2 f_x(t)\,Q(t) + \sigma_x^2(t)\,Q(t) + 2\sigma_x(t)\,R(t) \right.\\ \quad + \int_\Theta (\psi(t,z) + Q(t))\,(g_x(t,z))^2\, m(dz) \\ \quad +2\int_\Theta \psi(t,z) g_x(t,z)\, m(dz) \\ \quad \left. + H_{xx}(t))\right\} dt + R(t)dB(t) + \int_\Theta \psi(t,z)N(dz,dt) \\ Q(T) = -h_{xx}(x(T), E(x(T))). \end{cases} \tag{7}$$

As it is well known that under conditions (H1) and (H2), the first-order adjoint Eq. (6) admits one and only one $\mathcal{F}_t-$adapted solution pair $(\Psi(\cdot), K(\cdot), \gamma(\cdot,\cdot)) \in \mathbb{L}^2_{\mathcal{F}}([0,T];\mathbb{R}) \times \mathbb{L}^2_{\mathcal{F}}([0,T];\mathbb{R}) \times \mathbb{M}^2_{\mathcal{F}}([0,T];\mathbb{R})$. Also the second-order adjoint Eq. (7) admits one and only one $\mathcal{F}_t-$adapted solution pair $(Q(\cdot), R(\cdot), \psi(\cdot,\cdot)) \in \mathbb{L}^2_{\mathcal{F}}([0,T];\mathbb{R}) \times \mathbb{L}^2_{\mathcal{F}}([0,T];\mathbb{R}) \times \mathbb{M}^2_{\mathcal{F}}([0,T];\mathbb{R})$ (See [9,11]).
We define the usual Hamiltonian associated with the mean-field stochastic control problem (1–2) as follows

$$\begin{aligned} &H(t, X, Y, u, \Psi(t), K(t), \gamma(t,z)) \\ &\triangleq \Psi(t) f(t, X, Y, u) + K(t)\sigma(t, X, Y, u) \\ &\quad + \int_\Theta \gamma(t,z) g(t, X(t), u(t), z)\, m(dz) \\ &\quad -\ell(t, X, Y, u), \end{aligned} \tag{8}$$

where $(t, X, u) \in [0,T] \times \mathbb{R} \times \mathbb{A}_1$ and $(\Psi(t), K(t), \gamma(t,z)) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ the adjoint processes given by Eq. (6).

3 Main result

In this section, we establish a set of mean-field type necessary conditions for optimal continuous-singular control in jump-system, where the system evolves according to controlled MFSDEJs. Our result is proved by applying *spike variation method* for continuous parts of the control and *convex perturbation technique* for singular parts. The following theorem constitutes the main contribution of this paper.

Let $(u^*(\cdot), \eta^*(\cdot), X^*(\cdot))$ is an optimal solution of the mean-field continuous-singular control problem (1–2).

Theorem 3.1 *Let hypotheses (H1), (H2) and (H3) hold. Then there are two triple of $\mathcal{F}_t-$ adapted processes $(\Psi^*(\cdot), K^*(\cdot), \gamma^*(\cdot,\cdot))$ and $(Q^*(\cdot), R^*(\cdot), \psi^*(\cdot,\cdot))$ that satisfy (6) and (7) respectively, such that for all $(u,\eta) \in \mathbb{A}_1 \times \mathbb{A}_2$, we have*

$$\begin{aligned} &H(t, X^*(t), E(X^*(t)), u, \Psi^*(t), K^*(t), \gamma^*(t,z)) \\ &-H(t, X^*(t), E(X^*(t)), u^*(t), \Psi^*(t), K^*(t), \gamma^*(t,z)) \\ &+\frac{1}{2}[\sigma\left(t, X^*(t), E(X^*(t)), u\right) \\ &-\sigma\left(t, X^*(t), E(X^*(t)), u^*(t)\right)]^2 Q^*(t) \\ &+\frac{1}{2}\int_\Theta (g\left(t, X^*(t), u, z\right) - g\left(t, X^*(t), u^*(t), z\right))^2 \\ &\times \left(Q^*(t) + \psi^*(t,z)\right) m(dz) \\ &\le 0. \mathbb{P} - a.s.,\ a.e.t \in [0,T], \end{aligned} \tag{9}$$

and

$$\begin{aligned} &E\int_{[0,T]} (\mathcal{M}(t) + G(t)\Psi^*(t)) d\eta^*(t) \\ &\le E\int_{[0,T]} (\mathcal{M}(t) + G(t)\Psi^*(t)) d\eta(t). \end{aligned} \tag{10}$$

Proof of Theorem 3.1 In our mean-field control problem (1–2), since the control domain is not necessarily convex, we must obtain the maximum principle in its general form. A classical way of treating such a problem is to use the *spike variation method* for the continuous part of the control, and *convex variation method* for the singular part. More precisely, if $(u^*(\cdot), \eta^*(\cdot))$ is an optimal control and $(u(\cdot), \eta(\cdot))$ is an arbitrary element of $\mathcal{F}_t-$measurable random variable with values in $\mathbb{A}_1 \times \mathbb{A}_2$ which we consider as fixed from now on. We define an admissible control as follows:

$$\left(u^\varepsilon(t), \eta^\varepsilon(t)\right) = \begin{cases} \left(u, \eta^*(t) + \varepsilon\left(\eta(t) - \eta^*(t)\right)\right) : s \le t \le s + \varepsilon, \\ \left(u^*(t), \eta^*(t) + \varepsilon\left(\eta(t) - \eta^*(t)\right)\right) : \text{otherwise}, \end{cases} \tag{11}$$

where ε a sufficiently small $\varepsilon > 0$.and $s \in [0,T]$. Then, we derive the variational inequalities (9) and (10) in several steps, from the fact that

$$J_0\left(X_0, u^*(\cdot), \eta^*(\cdot)\right) \le J_0\left(X_0, u^\varepsilon(\cdot), \eta^\varepsilon(\cdot)\right). \tag{12}$$

Let $J_1 = J_0(X_0, u^\varepsilon(\cdot), \eta^\varepsilon(\cdot)) - J_0(X_0, u^\varepsilon(\cdot), \eta^*(\cdot))$, and $J_2 = J_0(X_0, u^\varepsilon(\cdot), \eta^*(\cdot)) - J_0(X_0, u^*(\cdot), \eta^*(\cdot))$. We introduce the following new variational equations for our control problem, which have a mean-field type.

First-order variational equation: Let $x_1^\varepsilon(t) \equiv x_1^{u^\varepsilon,\eta^\varepsilon}(t)$, and $\mathcal{D}_\varepsilon = [s, s+\varepsilon]$:

$$\begin{cases} dx_1^\varepsilon(t) = \left\{f_x(t)x_1^\varepsilon(t) + f_y(t)E\left(x_1^\varepsilon(t)\right) + \delta f(t) I_{\mathcal{D}_\varepsilon}(t)\right\} dt \\ \qquad + \left\{\sigma_x(t)x_1^\varepsilon(t) + \sigma_y(t)E\left(x_1^\varepsilon(t)\right) + \delta\sigma(t) I_{\mathcal{D}_\varepsilon}(t)\right\} dB(t) \\ \qquad + \int_\Theta \left\{g_x(t_-, z)\, x_1^\varepsilon(t) + \delta g(t_-, z) I_{\mathcal{D}_\varepsilon}(t)\right\} N(dz, dt) \\ \qquad + G(t)d(\eta^\varepsilon - \eta^*)(t), \\ x_1^\varepsilon(0) = 0. \end{cases} \tag{13}$$

Second-order variational equation:

$$\begin{cases} dx_2^{\varepsilon}(t) = \{f_x(t)x_2^{\varepsilon}(t) + f_y(t)E\left(x_2^{\varepsilon}(t)\right) \\ \qquad +\mathbb{W}_t(f, x_1^{\varepsilon}) + \delta f_x(t)I_{\mathcal{D}_\varepsilon}(t)\}dt \\ \qquad +\{\sigma_x(t)x_2^{\varepsilon}(t) + \sigma_y(t)E\left(x_2^{\varepsilon}(t)\right) \\ \qquad +\mathbb{W}_t(\sigma, x_1^{\varepsilon}) + \delta\sigma_x(t)I_{\mathcal{D}_\varepsilon}(t)\}dB(t) \\ \qquad +\int_\Theta\{g_x(t_-, z)x_2^{\varepsilon}(t) + \mathbb{W}_{t,e}(g, x_1^{\varepsilon}) \\ \qquad +\delta g_x(t_-, z)I_{\mathcal{D}_\varepsilon}(t)\}N(dz, dt), \\ x_2^{\varepsilon}(0) = 0. \end{cases} \tag{14}$$

Since $J_2 = J_0(X_0, u^{\varepsilon}(\cdot), \eta^*(\cdot)) - J_0(X_0, u^*(\cdot), \eta^*(\cdot))$ is independent to singular part, then the proof of (9) is similar as in ([11], Theorem 3.1). This completes the proof of (9). □

To prove the second variational inequality (10). We need the following technical Lemmas.

Lemma 3.1 *Let $x_1^{u^{\varepsilon},\eta^*}(\cdot)$ be the solution of mean-field Eq. (13), corresponding to $(u^{\varepsilon}(\cdot), \eta^*(\cdot))$ then the following estimation holds*

$$\lim_{\varepsilon\to 0} E\left[\sup_{0\le t\le T}\left|\frac{X^{u^{\varepsilon},\eta^*}(t) - X^*(t)}{\varepsilon} - x_1^{u^{\varepsilon},\eta^*}(t)\right|^2\right] = 0.$$

Proof We set $t \in [0, T]$,

$$\beta^{\varepsilon}(t) = \frac{1}{\varepsilon}\left[X^{u^{\varepsilon},\eta^*}(t) - X^*(t)\right] - x_1^{u^{\varepsilon},\eta^*}(t), \tag{15}$$

by Tylor's formula, we get

$$\begin{aligned} &\frac{X^{u^{\varepsilon},\eta^*}(t) - X^*(t)}{\varepsilon} \\ &= \int_0^t\int_0^1 f_x(r, X^*(r) + \lambda\varepsilon(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r)), E(X^*(r) \\ &\quad + \lambda\varepsilon(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r))), u^{\varepsilon}(r))(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r))d\lambda dr \\ &\quad + \int_0^t\int_0^1 f_y(r, X^*(r) + \lambda\varepsilon(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r)), E(X^*(r) \\ &\quad + \lambda\varepsilon(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r))), u^{\varepsilon}(r))E(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r))d\lambda dr \\ &\quad + \int_0^t\int_0^1 \sigma_x(r, X^*(r) + \lambda\varepsilon(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r)), E(X^*(r) \\ &\quad + \lambda\varepsilon(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r))), u^{\varepsilon}(r))(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r))d\lambda dr \\ &\quad + \int_0^t\int_0^1 \sigma_y(r, X^*(r) + \lambda\varepsilon(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r)), E(X^*(r) \\ &\quad + \lambda\varepsilon(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r))), u^{\varepsilon}(r))E(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r))d\lambda dr \\ &\quad + \int_0^t\int_\Theta\int_0^1 g_x(r, X^*(r) + \lambda\varepsilon(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r)), E(X^*(r) \\ &\quad + \lambda\varepsilon(\beta^{\varepsilon}(r) + x_1^{u^{\varepsilon},\eta^*}(r))), u^{\varepsilon}(r), z)(\beta^{\varepsilon}(r) \\ &\quad + x_1^{u^{\varepsilon},\eta^*}(r))d\lambda m(dz)dr, \end{aligned}$$

then from the above equation and (15) we conclude that $\beta^{\varepsilon}(t)$ is independent to singular part, then we can using similar method developed in Li [12] for the rest of proof. □

Lemma 3.2 *Let $x_1^*(t)$ solution of Eq. (13) correspondent to $(u^*(\cdot), \eta^*(\cdot))$. Then we have*

$$\begin{aligned} 0 &\le E[h_x(X^*(T), E(X^*(T)))x_1^*(t) \\ &\quad + h_y(X^*(T), E(X^*(T)))E(x_1^*(t))] \\ &\quad + E\int_0^T [\ell_x(t, X^*(T), E(X^*(T)), u^*(t))x_1^*(t) \\ &\quad + \ell_y(t, X^*(t), E(X^*(t)), u^*(t))E(x_1^*(t))]dt \\ &\quad + E\int_{[0,T]} \mathcal{M}(t)d\left(\eta - \eta^*\right)(t). \end{aligned}$$

Proof By a simple computations, we get

$$\begin{aligned} \frac{J_1}{\varepsilon} &= \frac{1}{\varepsilon}\left[J\left(X_0, u^{\varepsilon}(\cdot), \eta^{\varepsilon}(\cdot)\right) - J\left(X_0, u^{\varepsilon}(\cdot), \eta^*(\cdot)\right)\right] \\ &= \frac{1}{\varepsilon}E[[h(X^{u^{\varepsilon},\eta^{\varepsilon}}(T), E(X^{u^{\varepsilon},\eta^{\varepsilon}}(T))) \\ &\quad -h(X^{u^{\varepsilon},\eta^*}(T), E(X^{u^{\varepsilon},\eta^*}(T)))] \\ &\quad + \frac{1}{\varepsilon}E\int_0^T [\ell(t, X^{u^{\varepsilon},\eta^{\varepsilon}}(t), E(X^{u^{\varepsilon},\eta^{\varepsilon}}(t)), u^{\varepsilon}(t)) \\ &\quad -\ell(t, X^{u^{\varepsilon},\eta^*}(t), E(X^{u^{\varepsilon},\eta^*}(t)), u^{\varepsilon}(t))]dt \\ &\quad + \frac{1}{\varepsilon}E\int_{[0,T]} \mathcal{M}(t)d\left(\eta^{\varepsilon} - \eta^*\right)(t). \end{aligned}$$

By Tylor's formula, and the fact that $\dfrac{\eta^{\varepsilon}(t) - \eta^*(t)}{\varepsilon} = (\eta(t) - \eta^*(t))$, we get: for any $\eta(\cdot) \in \mathcal{A}_2([0, T])$

$$\begin{aligned} \frac{J_1}{\varepsilon} &= E\int_0^1 h_x(X^{u^{\varepsilon},\eta^{\varepsilon}}(T) + \lambda\varepsilon(\gamma^{\varepsilon}(T) \\ &\quad + x_1^{\varepsilon}(T)), E[X^{u^{\varepsilon},\eta^{\varepsilon}}(T) \\ &\quad + \lambda\varepsilon(\gamma^{\varepsilon}(T) + x_1^{\varepsilon}(T))])\left(\gamma^{\varepsilon}(T) + x_1^{\varepsilon}(T)\right)d\lambda \\ &\quad + E\int_0^1 h_y(X^{u^{\varepsilon},\eta^{\varepsilon}}(T) + \lambda\varepsilon(\gamma^{\varepsilon}(T) + x_1^{\varepsilon}(T)), E[X^{u^{\varepsilon},\eta^{\varepsilon}}(T) \\ &\quad + \lambda\varepsilon(\gamma^{\varepsilon}(T) + x_1^{\varepsilon}(T))])E\left(\gamma^{\varepsilon}(T) + x_1^{\varepsilon}(T)\right)d\lambda \\ &\quad + E\int_0^T\int_0^1 \ell_x(t, X^{u^{\varepsilon},\eta^{\varepsilon}}(t) \\ &\quad + \lambda\varepsilon(\gamma^{\varepsilon}(t) + x_1^{\varepsilon}(t)), E[X^{u^{\varepsilon},\eta^{\varepsilon}}(t) \\ &\quad + \lambda\varepsilon(\gamma^{\varepsilon}(t) + x_1^{\varepsilon}(t))])\left(\gamma^{\varepsilon}(t) + x_1^{\varepsilon}(t)\right)d\lambda dt \\ &\quad + E\int_0^T\int_0^1 \ell_y(t, X^{u^{\varepsilon},\eta^{\varepsilon}}(t) + \lambda\varepsilon(\gamma^{\varepsilon}(t) \\ &\quad + x_1^{\varepsilon}(t)), E[X^{u^{\varepsilon},\eta^{\varepsilon}}(t) \\ &\quad + \lambda\varepsilon(\gamma^{\varepsilon}(t) + x_1^{\varepsilon}(t))])E\left(\gamma^{\varepsilon}(t) + x_1^{\varepsilon}(t)\right)d\lambda dt \\ &\quad + E\int_{[0,T]} \mathcal{M}(t)d\left(\eta - \eta^*\right)(t). \end{aligned}$$

Finally, since the derivatives h_x, h_y, ℓ_x and ℓ_y are bounded, the result follows from Lemma 3.1 and by letting ε going to zero. This completes the proof of Lemma 3.2. □

Now, from Lemma 3.2 and by applying similar arguments developed in [4], we get

$$\lim_{\varepsilon\to 0}\frac{J_1}{\varepsilon}=E\int_{[0,T]}\left(\mathcal{M}(t)+G(t)\Psi^*(t)\right)d\left(\eta-\eta^*\right)(t)\geq 0.$$

This completes the proof of (10) and Theorem 3.1. □

4 Application: Markowitz's mean–variance problem

In this section, we apply our general maximum principle of optimality to study a Markowitz's mean–variance portfolio selection with interventions and we derive the explicit expression of the optimal portfolio selection strategy in feedback form. We consider a market consisting of stock and a bond whose prices are stochastic processes $\mathbb{S}_i : i=0,1$, governed by the following equations: $t\in[0,T]$

$$\begin{cases} d\mathbb{S}_0(t)=\mathbb{S}_0(t)\,\mu_0(t)dt,\ \mathbb{S}_0(0)>0.\\ d\mathbb{S}_1(t)=\mathbb{S}_1(t)\,\mu_1(t)dt+\sigma_t\mathbb{S}_1(t)\,dB(t)\\ \quad+\mathbb{S}_1(t)\int_\Theta A_t(z)\,N(dz,dt)\,,\ \mathbb{S}_1(0)>0,\end{cases}\tag{16}$$

where $\mu_1(t)$: is the appreciation rate process of the stock, $\mu_0(t)$ is the interest rate process, σ_t and $A_t(z)$ are bounded deterministic functions such that $\mu_1(t)\neq 0$, $\sigma_t\neq 0$ and $\mu_1(t)>\mu_0(t)$. In order to ensure that $\mathbb{S}_1(t)>0$ for all $t\in[0,T]$ we assume that $A_t(z)>-1$ for any $z\in\Theta$, and the function $t\to\int_\Theta A_t^2(z)\,m(dz)$ is a locally bounded. We denote by $u(t)$ the amount invested in the stock. Now, we introduce the wealth dynamics as follows

$$\begin{cases} dX^{u,\eta}(t)=\left[\mu_0(t)X^{u,\eta}(t)+(\mu_1(t)-\mu_0(t))u(t)\right]dt\\ \quad+\sigma_t u(t)dB(t)+\int_\Theta A_{t_-}(z)\,u(t)N(dz,dt)+G(t)d\eta(t),\\ X^{u,\eta}(0)=X_0,\end{cases}\tag{17}$$

If the corresponding wealth process $X^{u,\eta}(\cdot)$ given by Eq. (17) is square integrable, the control variable $(u(\cdot),\eta(\cdot))$ is called tame. We denote $\mathcal{A}_1\times\mathcal{A}_2\,([0,T])$ the set of admissible portfolio valued in $\mathbb{A}_1\times\mathbb{A}_2$. Consider the following controlled system (17) along with the cost functional.

$$J_0(X_0,u(\cdot),\eta(\cdot))=E\left[(X^{u,\eta}(T)-E(X^{u,\eta}(T)))^2\right]+\int_{[0,T]}\mathcal{M}(t)d\eta(t),\tag{18}$$

We may interpret the function $\mathcal{M}(\cdot)$ as a *cost rate* for the use of the singular control $\eta(\cdot)$. Our objective is to find an admissible portfolio $(u^*(\cdot),\eta^*(\cdot)$ which minimizes the cost function (18). The Hamiltonian H gets the form

$$H\left(t,X,E\left(X\right),u(t),\Psi(t),K(t),\gamma_t\left(e\right)\right)$$

$$=-\Psi(t)\mu_0(t)X(t)-u(t)\Big[\Psi(t)(\mu_1(t)-\mu_0(t))+K(t)\sigma_t+\int_\Theta\gamma_t\left(z\right)A_t\left(z\right)m(dz)\Big].$$

Consequently, since this is a linear expression of $u(\cdot)$ then it is clear that the supremum is attained at $u^*(t)$ satisfying

$$\Psi^*(t)(\mu_1(t)+\mu_0(t))+K^*(t)\sigma_t+\int_\Theta\gamma_t^*\left(z\right)A_t\left(z\right)m(dz)=0.\tag{19}$$

By simple computation, the first-order adjoint Eq. (6) associated with $u^*(t)$ gets the form

$$\begin{cases} d\Psi^*(t)=-\mu_0(t)\Psi^*(t)dt+K^*(t)dB(t)\\ \qquad+\int_\Theta\gamma_t^*(z)N(dt,dz),\\ \Psi^*(T)=2\left(X^*(T)-E(X^*(T))\right),\end{cases}\tag{20}$$

and the second-order adjoint equation being

$$\begin{cases} dQ^*(t)=-2\mu_0(t)Q^*(t)dt+R^*(t)dB(t)\\ \qquad+\int_\Theta\psi_t^*(z)N(dz,dt),\\ Q^*(T)\ =2.\end{cases}$$

By uniqueness of the solution of the above classical backward SDE it is easy to show that

$$Q^*(t)=2\exp[2\int_t^T\mu_0(r)dr],$$
$$R^*(t)=0,\ \text{for any } t\in[0,T]\,,$$
$$\psi_t^*(z)=0,\ \text{for any } z\in\Theta.$$

In order to solve the above Eq. (20) and to find the expression of $u^*(t)$ we conjecture a process $\Psi^*(\cdot)$ of the form

$$\Psi^*(t)=\varphi_1(t)X^*(t)+\varphi_2(t)E\left(X^*(t)\right)+\varphi_3(t),\tag{21}$$

where $\varphi_1(\cdot)$, $\varphi_2(\cdot)$ and $\varphi_3(\cdot)$ are deterministic differentiable functions. Applying Itô's formula to (21), in virtue of SDE-(17), we get

$$\begin{aligned}&-\mu_0(t)\Psi^*(t)=X^*(t)\varphi_1(t)\\&\quad+\varphi_1(t)\left[\mu_0(t)X^*(t)+(\mu_1(t)-\mu_0(t))u^*(t)\right]\\&\quad+\varphi_2(t)\left[\mu_0(t)E(X^*(t))+(\mu_1(t)-\mu_0(t))u^*(t)\right]\\&\quad+\varphi_2(t)E\left(X^*(t)\right)+\varphi_3(t),\end{aligned}\tag{22}$$

$$\begin{aligned}&K^*(t)=\varphi_1(t)\sigma_t u^*(t),\\&\gamma_t^*(z)=\varphi_1(t)u^*(t)A_t\left(z\right),\end{aligned}\tag{23}$$

and

$$\varphi_1(T)=2,\ \varphi_2(T)=-2,\ \varphi_3(T)=0.\tag{24}$$

Combining (23) and (24) together with (19), we get

$$u^*(t) = \frac{-(\mu_1(t)-\mu_0(t))\Psi^*(t)}{\varphi_1(t)\left[\sigma_t^2 + \int_\Theta A_t^2(z)\,m(dz)\right]}. \tag{25}$$

We denote

$$M(t) = \int_\Theta A_t^2(z)\,m(dz) + \sigma_t^2, \tag{26}$$

by using (19) together with (25) and (26) then we get

$$\begin{aligned}
&\varphi_3(t) = 0 \text{ for } t \in [0,T],\\
&u^*(t) = (\mu_0(t)-\mu_1(t))\,(M(t)\varphi_1(t))^{-1}\\
&\qquad\times\left(\varphi_1(t)X^*(t) + \varphi_2(t)E\left(X^*(t)\right)\right)\\
&\qquad= \left\{(\mu_0(t)-\mu_1(t))\,(M(t))^{-1}\right\}X^*(t)\\
&\qquad+ \left\{(\mu_0(t)-\mu_1(t))\,(M(t))^{-1}\varphi_2(t)\,(\varphi_1(t))^{-1}\right\}E\left(X^*(t)\right).
\end{aligned} \tag{27}$$

Now combining (22) with (21), we deduce

$$\begin{aligned}
&u^*(t)\,(\varphi_1(t)+\varphi_2(t))\,(\mu_0(t)-\mu_1(t))\\
&\quad= [2\mu_0(t)\varphi_1(t)+\varphi_1(t)]\,X^*(t)\\
&\quad+[2\mu_0(t)\varphi_2(t)+\varphi_2(t)]\,E(X^*(t)).
\end{aligned} \tag{28}$$

By comparing the terms containing $X^*(t)$ and $E\left(X^*(t)\right)$, we obtain from (27) with (28) the two ordinary differential equations (ODEs in short):

$$\begin{aligned}
&\left[(\mu_0(t)-\mu_1(t))^2\,(M(t))^{-1} - 2\mu_0(t)\right]\varphi_1(t)\\
&\quad+(\mu_0(t)-\mu_1(t))^2\,(M(t))^{-1}\varphi_2(t) = \varphi_1(t).\\
&\quad\left[(\mu_0(t)-\mu_1(t))^2\,(M(t))^{-1} - 2\mu_0(t)\right]\varphi_2(t)\\
&\quad+(\mu_0(t)-\mu_1(t))^2\,(M(t))^{-1}\frac{\varphi_2^2(t)}{\varphi_1(t)} = \varphi_2(t).
\end{aligned} \tag{29}$$

Let us turn to calculate explicitly $\varphi_1(t)$ and $\varphi_2(t)$. Since $\varphi_1(T)=2, \varphi_2(T)=-2$, [see (24)], then by dividing the first ODE in (29) by $\varphi_1(t)$ and the second ODE by $\varphi_2(t)$ we get

$$\begin{aligned}
&\varphi_1(t) = 2\exp\left[\int_t^T \mu_0(s)ds\right], \varphi_1(T)=2\\
&\varphi_2(t) = -2\exp\left[\int_t^T \mu_0(s)ds\right], \varphi_2(T)=-2.
\end{aligned} \tag{30}$$

From (30) we conclude that $u^*(t)$ is given by

$$\begin{aligned}
u^*(t) &= \left[(\mu_0(t)-\mu_1(t))\,(M(t))^{-1}\right]X^*(t)\\
&\quad-\left[(\mu_0(t)-\mu_1(t))\,(M(t))^{-1}\right]E\left(X^*(t)\right).
\end{aligned} \tag{31}$$

The first-order adjoint processes are given by

$$\begin{aligned}
&\Psi^*(t) = \varphi_1(t)X^*(t) + \varphi_2(t)E\left(X^*(t)\right),\\
&K^*(t) = \sigma_t\varphi_1(t)u^*(t),\\
&\gamma_t^*(z) = \varphi_1(t)u^*(t)A_t(z),
\end{aligned}$$

and the second-order adjoint processes are given by

$$\begin{aligned}
&Q^*(t) = 2\exp[2\int_t^T \mu_0(r)dr],\\
&R^*(t) = 0,\ \psi_t^*(z) = 0,
\end{aligned}$$

satisfying the adjoint Eq. (6). Now, let $\eta(\cdot) \in \mathcal{A}_2([0,T])$ such that

$$d\eta(t) = \begin{cases} 0 \text{ if } t \in \{(w,t)\in\Omega\times[0,T]:\\ \quad \mathcal{M}(t)+G(t)\Psi^*(t)\geq 0\},\\ d\eta^*(t) \text{ if } t\in\{(w,t)\in\Omega\times[0,T]:\\ \quad \mathcal{M}(t)+G(t)\Psi^*(t)<0\},\end{cases} \tag{32}$$

then by a simple computations, it is easy to see that

$$\begin{aligned}
0 &\leq E\int_{[0,T]}(\mathcal{M}(t)+G(t)\Psi^*(t))d\left(\eta-\eta^*\right)(t)\\
&= E\int_{[0,T]}(\mathcal{M}(t)+G(t)\Psi^*(t))d\eta(t)\\
&\quad -E\int_{[0,T]}(\mathcal{M}(t)+G(t)\Psi^*(t))d\eta^*(t)\\
&= E\int_{[0,T]}(\mathcal{M}(t)+G(t)\Psi^*(t))\\
&\quad\times I_{\{(w,t)\in\Omega\times[0,T]:\mathcal{M}(t)+G(t)\Psi^*(t)\geq 0\}}d\left(-\eta^*\right)(t)\\
&= -E\int_{[0,T]}(\mathcal{M}(t)+G(t)\Psi^*(t))\\
&\quad\times I_{\{(w,t)\in\Omega\times[0,T]:\mathcal{M}(t)+G(t)\Psi^*(t)\geq 0\}}d\eta^*(t),
\end{aligned}$$

which implies that $\eta^*(t)$ satisfies for any $t\in[0,T]$

$$\begin{aligned}
&E\int_{[0,T]}(\mathcal{M}(t)+G(t)\Psi^*(t))\\
&\quad\times I_{\{(w,t)\in\Omega\times[0,T]:\mathcal{M}(t)+G(t)\Psi^*(t)\geq 0\}}d\eta^*(t)\\
&\quad= 0.
\end{aligned} \tag{33}$$

By applying (32) and (33), we have

$$\begin{aligned}
\eta^*(t) &= \int_0^t I_{\{(w,s)\in\Omega\times[0,T]:\mathcal{M}(s)+G(s)\Psi^*(s)\geq 0\}^c}(w,s)ds + \eta(t),\\
&= \int_0^t I_{\{(w,s)\in\Omega\times[0,T]:\mathcal{M}(s)+G(s)\Psi^*(s)<0\}}(w,s)ds + \eta(t).
\end{aligned} \tag{34}$$

5 Conclusion and future works

In this paper, general necessary conditions of optimal continuous-singular control for mean-field jump diffusion have been discussed. The control domain is not necessary convex. These results extend some existing models. If we assume $G(\cdot) \equiv \mathcal{M}(\cdot) \equiv 0$, (without singular parts), Theorem 3.1 reduces to maximum principle proved in [11]. If we assume $g \equiv 0$ (without Poisson jump), our result (Theorem 3.1) reduces to Theorem 6.1 proved in [4].

An open question is to study continuous-singular optimal control for mean-field jump-systems, where the coefficients $G(\cdot)$ and $\mathcal{M}(\cdot)$ of the singular parts depend to state and continuous control processes (i.e, the singular parts have the form: $\int_{[0,t]} G(t, X^{u,\eta}(t), u(t))d\eta(t)$ and $\int_{[0,t]} \mathcal{M}(t, X^{u,\eta}(t), u(t)))d\eta(t)$. This topic will be included in our future article.

Acknowledgements The authors would like to thank, the associate editor, and anonymous referees for their constructive corrections and valuable suggestions that improved the manuscript considerably. This work was supported by the Tubitak project (Grant 2221), Turkey.

References

1. Wang G, Zhang C, Zhang W (2014) Stochastic maximum principle for mean-field type optimal control under partial information. IEEE Trans Autom Control 59(2):522–528
2. Zhang H (2016) A necessary conditions for mean-field type stochastic differential equations with correlated state and observation noises. J Ind Manag Optim 12(4):1287–1301
3. Meng Q, Shen Y (2015) Optimal control of mean-field jump-diffusion systems with delay: a stochastic maximum principle approach. J Comput Appl Math 279:13–30
4. Hafayed M, Abbas S (2014) On near-optimal mean-field stochastic singular controls: necessary and sufficient conditions for near-optimality. J Optim Theory Appl 160(3):778–808
5. Hafayed M, Abbas S, Abba A (2015) On mean-field partial information maximum principle of optimal control for stochastic systems with Lévy processes. J Optim Theory Appl 167:1051–1069
6. Hafayed M, Abba A, Abbas S (2016) On partial-information optimal singular control problem for mean-field stochastic differential equations driven byTeugels martingales measures. Int J Control 89(2):397–410
7. Hafayed M (2014) Singular mean-field optimal control for forward-backward stochastic systems and applications to finance. Int J Dyn Control 2(4):542–554
8. Hafayed M, Boukaf S, Shi Y, Meherrem S (2016) A McKean-Vlasov optimal mixed regular-singular control problem, for nonlinear stochastic systems with Poisson jump processes. Neurocomputing 182(19):133–144
9. Buckdahn R, Djehiche B, Li J (2011) A general stochastic maximum principle for SDEs of mean-field type. Appl Math Optim 64:197–216
10. Shi J (2012) Sufficient conditions of optimality for mean-field stochastic control problems. In: 12th international conference on control, automation, robotics & vision, Guangzhou, P.R. China, December 5–7, pp 747–752
11. Hafayed M, Abbas S (2013) A general maximum principle for stochastic differential equations of mean-field type with jump processes. arXiv: 1301.7327v4
12. Li J (2012) Stochastic maximum principle in the mean-field controls. Automatica 48:366–373
13. Shen Y, Meng Q, Shi P (2014) Maximum principle for mean-field jump-diffusions to stochastic delay differential equations and its applications to finance. Automatica 50(6):1565–1579
14. Hafayed M (2014) A mean-field necessary and suffucient conditions for optimal singular stochastic control. Commun Math Stat 1(4):417–435
15. Su X, Wu L, Shi P, Song YD (2014) A novel approach to output feedback control of fuzzy stochastic systems. Automatica 50(12):3268–3275
16. Wu L, Su X, Shi P (2014) Output feedback control of Markovian jump repeated scalar nonlinear systems. IEEE Trans Autom Control 59(1):199–204
17. Mundaca G, Øksendal B (1998) Optimal stochastic intervention control with application to the exchange rate. J Math Econ 29:225–243
18. Menaldi J, Robin M (1984) On singular stochastic control problems for diffusions with jumps. IEEE Trans Autom Control 29(11):991–1004
19. Øksendal B, Sulem A (2007) Applied stochastic control of jump diffusions, 2nd edn. Springer, Berlin
20. Hafayed M, Abbas S (2013) On stochastic near-optimal singular controls for jumps diffusions: necessary and sufficient conditions. J Dyn Control Syst 19(4):503–517
21. Cadenillas A, Haussmann U (1994) The stochastic maximum principle for singular control problem. Stoch Stoch Rep 49(3–4):211–237
22. Dufour F, Miller B (2006) Maximum principle for singular stochastic control problem. SIAM J Control Optim 45(2):668–698
23. Haussmann UG, Suo W (1995) Singular optimal control I, II. SIAM J Control Optim 33(3):916–959
24. Aghayeva C, Morali N (2008) Necessary condition for some singular stochastic control systems with variable delay. Theory Stoch Process 14(30):108–115
25. Tang SL, Li XJ (1994) Necessary conditions for optimal control of stochastic systems with random jumps. SIAM J Control Optim 32:1447–1475